Can Discounting Alone Resolve the Forward Guidance Puzzle?

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Abstract

Numerically, we examine the expansionary effects of forward guidance at different horizons in Gabaix's (2020) model and find that the role of discounting is limited and that unrealistic assumptions about parameter calibration are also required to resolve the forward guidance puzzle. We extend Gabaix's (2020) model to include wage stickiness and find that discounting alone can resolve the puzzle even assuming standard calibration.

JEL Classification: E12, E21, E23, E52, E62.

Keywords: Forward Guidance Puzzle, Discounting, New Keynesian Model, Wage Stickiness.

1 Introduction

When the economy falls into a liquidity trap, key macroeconomic variables decline sharply, and the monetary authority cannot continue to lower the nominal interest rate because of the zero lower bound (ZLB). There is a large body of literature examining various policy responses to get the economy out of the liquidity trap and back to normal as soon as possible. Policy interventions can be broadly classified into three categories. The government can increase fiscal spending to make up for the shortfall in effective demand, reform the structure of the economy to make it more efficient, and implement quantitative easing or forward guidance. In this paper, we focus on the effectiveness of forward guidance.

Since the Great Recession, Del Negro et al. (2015), Carlstrom et al. (2015) and many others examine the effectiveness of forward guidance in a standard new Keynesian model and find that the more distant the policy is implemented in the future, the greater its effect on current output and inflation, which is the so-called "forward guidance puzzle." The reason is that a cut in the nominal interest rate T periods in the future raises inflation expectations, which lowers the real interest rate as the nominal interest rate is either pegged or at the ZLB prior to period T[1]. This reduction in the real interest rate further raises inflation expectations, which further lowers the real interest rate. The effect of this self-reinforcing process is greater the longer the time interval between the announcement and the implementation of the cut.

A strand of recent work is engaged in constructing various microeconomic foundations to reduce the forward-looking nature of agents.[2] In these models, the effect of forward guidance is limited because the policy-induced rise in inflation expectations is dampened. It is worth noting that some of these models can be reduced to a three-equation new Keynesian model, from which it is clear that the reduction in agents' forward-looking nature is achieved by increasing their discounting of the future. In the standard new Keynesian model, if focusing only on the IS curve, one can see that a change in the real interest rate in the future has the same effect on the current output gap as an equally large change to the current real interest rate.[3] Del Negro et al. (2015) and McKay et al. (2016), among many others, point out that this is the source of the forward guidance puzzle.

In this paper, we explore what exactly resolves the forward guidance puzzle in Gabaix's (2020) model.[4] Our

[1]The modeling of forward guidance in the literature can be broadly grouped into two categories. The first category explores the effectiveness of forward guidance in a liquidity trap with the ZLB binding (e.g., Eggertsson and Woodford (2003), Werning (2012), Del Negro et al. (2015), Carlstrom et al. (2015), and Hagedorn et al. (2019)). The second category models forward guidance by assuming that the nominal interest rate falls T periods in the future and follows an interest-rate peg in all other periods prior to period T (e.g., Del Negro et al. (2015), Kiley (2016), Campbell et al. (2017), Farhi and Werning (2017), and Angeletos and Lian (2018)).

[2]More specifically, the theoretical literature modifies the underlying assumptions of the standard new Keynesian framework, such as infinitely-lived agents, rational expectations, complete markets, sticky prices, and homogeneous agents. Del Negro et al. (2015) assume finitely-lived agents; Angeletos and Lian (2018), García-Schmidt and Woodford (2019), and Gabaix (2020) assume bounded rationality; McKay et al. (2016; 2017) and Hagedorn et al. (2019) assume incomplete markets; Farhi and Werning (2019) emphasize the interaction between bounded rationality and incomplete markets; Carlstrom et al. (2015) and Kiley (2016) assume sticky information; Kaplan et al. (2018) assume heterogeneous agents.

[3]The standard new Keynesian IS curve is given by $x_t = E_t x_{t+1} - \sigma\left(i_t - E_t\pi_{t+1} - r_t^n\right) = E_t x_{t+1} - \sigma\left(r_t - r_t^n\right)$, where x_t denotes the output gap, i_t the nominal interest rate, π_t inflation, r_t^n the natural rate of interest, $r_t = i_t - E_t\pi_{t+1}$ the real interest rate, and σ the intertemporal elasticity of substitution. Solving this equation forward yields $x_t = -\sigma \sum_{j=0}^{\infty} E_t\left(r_{t+j} - r_{t+j}^n\right)$. It can be seen that the effects on the current output gap of future real interest rates are not discounted.

[4]Please note that Angeletos and Lian's (2018) proposition 5 and the proposition 10 of their online appendix prove that their model

choice of this model is motivated by the following consideration. Although the model has a novel microeconomic foundation, it can be reduced to IS and Phillips curves by log-linearization, thus allowing comparison with the standard new Keynesian model. It can be seen that discounting appears in both IS and Phillips curves. McKay et al. (2017) also simplify McKay et al.'s (2016) model by log-linearization, but discounting appears only in the IS curve. Nakata et al. (2019) study the optimal design of the forward guidance policy with ad hoc discounting in both IS and Phillips curves.[5]

Compared with the standard new Keynesian model, we find that in addition to introducing discounting, Gabaix (2020) adjusts the calibration of two structural parameters. One is used to measure consumers' elasticity of intertemporal substitution (i.e., the forward-looking nature of consumers), and the other is used to measure the degree of price stickiness. Compared with the textbook calibration (Galí, 2015), the former is lowered while the latter is raised, and both are adjusted by a large margin, which can significantly reduce the degree of responsiveness of inflation and expected inflation to forward guidance, partially weakening the policy's effect.[6] Thus, it is necessary to isolate the introduction of discounting from the changes in parameter calibration. We find that discounting alone cannot resolve the forward guidance puzzle in Gabaix's (2020) model without unrealistic calibration of these two parameters.

To address the calibration problem, we incorporate wage stickiness into Gabaix's (2020) model, thereby reducing the responsiveness of inflation expectations through the model's endogenous mechanism. Since Gabaix's (2020) model assumes only price stickiness, expansionary monetary policy leads to a larger rise in nominal wages than in prices, which leads to a rise in real wages. The rise in real wages raises consumers' purchasing power and firms' marginal costs, both of which further raise inflation expectations. The presence of wage stickiness, on the other hand, reduces the rise in nominal wages, which in turn reduces the rise in real wages, and may even lead to a fall in real wages, which dampens the rise in inflation expectations. We find that discounting alone can resolve the forward guidance puzzle in the extended model even under textbook calibration.

is equivalent to Gabaix's (2020) model.

[5]Structurally, all these models are three-equation new Keynesian models and McKay et al.'s (2017) model is a special case of Gabaix's (2020) and Nakata et al.'s (2019) models. The main difference between Gabaix's (2020) and Nakata et al.'s (2019) models is that Gabaix's (2020) model provides a microeconomic foundation for discounting so that the value of discounting is supported by relevant empirical studies, whereas Nakata et al.'s (2019) model sets the value of discounting arbitrarily. Therefore, for the sake of not losing generality, it is more appropriate to study Gabaix's (2020) model.

[6]Both a low intertemporal elasticity of substitution and a high degree of price stickiness hinder the aforementioned self-reinforcing process between inflation expectations and the real interest rate, thus reducing the stimulative effect of forward guidance. A low intertemporal elasticity of substitution makes consumers insensitive to changes in the real interest rate. When the real interest rate falls, consumers are less willing to consume more, which dampens the rise in inflation expectations. A high degree of price stickiness reduces the slope of the Phillips curve. When inflation expectations rise, firms become reluctant to raise prices, which in turn dampens the rise in inflation expectations.

2 Gabaix's (2020) model: Can discounting alone resolve the forward guidance puzzle?

The microeconomic foundation of Gabaix's (2020) model is identical to that of the standard new Keynesian framework (Galí 2015, Chapter 3) except for the difference in the modeling of expectations. When agents make forecasts of the future, Gabaix (2020) assumes that they are cognitively myopic. So, what is the difference between rational expectations and cognitive myopia? Rational expectations assume that agents know exactly what events are likely to occur in the future and the correct probability of each event. Therefore, rational expectations are modeled by taking mathematical expectations ($E_t x_{t+k}$ for instance, where E_t is the rational expectations operator and x_{t+k} can be any variable in period $t+k$). In Gabaix's (2020) model, agents still form rational expectations but discount the future, so that the importance of events likely to occur in the distant future can be arbitrarily reduced ($E_t^{BR} x_{t+k} = \bar{m}^k E_t x_{t+k}$, where E_t^{BR} denotes the subjective expectations operator and $\bar{m} \in [0,1]$). This modeling of myopia is manifested in a new Keynesian model in the form of increasing agents' degree of discounting to the future. Using Gabaix's (2020) notations, the IS and Phillips curves are given by

$$x_t = ME_t x_{t+1} - \sigma \left(i_t - E_t \pi_{t+1} - r_t^n \right), \tag{1}$$

$$\pi_t = \beta M^f E_t \pi_{t+1} + \kappa x_t, \tag{2}$$

where x_t denotes the output gap, i_t the nominal interest rate, π_t inflation, r_t^n the natural rate of interest, β the rate of time preference, $\sigma = \frac{1}{\gamma R}$ the sensitivity of the output gap to interest rates, γ the coefficient of risk aversion, $R = \frac{1}{\beta}$ the steady state of the gross real interest rate, $\kappa = \frac{(1-\theta)(1-\beta\theta)}{\theta} (\gamma + \phi)$ the slope of the Phillips curve, θ the survival rates of prices, ϕ the inverse of Frisch elasticity, $M = \bar{m} \in [0,1]$ discounting by consumers, and $M^f = \bar{m} \left[\theta + \frac{1-\beta\theta}{1-\beta\theta\bar{m}} (1-\theta) \right] \in [0,1]$ discounting by firms. The appendix provides detailed derivations. It is worth noting that M and M^f are increasing functions of $\bar{m}$. The closer $\bar{m}$ is to 1, the more rational the subjective expectations are. When $\bar{m}$ equals 1, $M = M^f = 1$ and equations (1) and (2) turn back to the textbook IS and Phillips curves. We assume that monetary policy follows a Taylor-type rule

$$i_t = \phi_\pi \pi_t + \phi_x x_t, \tag{3}$$

where ϕ_π and ϕ_x are monetary policy's response to inflation and the output gap, respectively.

Tables 1 and 2 reproduce the corresponding tables in Gabaix (2020), which contain the calibration of the parameters needed for solving the model. In addition to the parameters used to characterize discounting ($M = \bar{m}$ and $M^f = \bar{m} \left[\theta + \frac{1-\beta\theta}{1-\beta\theta\bar{m}} (1-\theta) \right]$), we find that the values assigned by Gabaix (2020) to γ (the coefficient of risk aversion) and θ (the survival rates of prices) are much different from the standard calibration of the new Keynesian model (Galí, 2015). More specifically, the inverse of γ determines the magnitude of $\sigma = \frac{1}{\gamma R} = \frac{\beta}{\gamma}$, which measures the degree of intertemporal substitution in the IS curve (1). As can be seen, Gabaix (2020) sets it too small ($\sigma = 0.198$) by assuming $\gamma = 5$, which diminishes the forward-looking nature of agents to a certain extent. In addition, Gabaix (2020) assigns too large a value to the degree of price stickiness ($\theta = 0.875$), which is far beyond the reasonable range of values measured by empirical studies. To test that discounting is the only reason that the forward guidance puzzle is resolved, we re-calibrate γ and θ in this paper. According to Galí (2015), we assume a log utility function (i.e., set γ equal to 1) as well as reduce the degree of price stickiness (θ) from 0.875 to 0.75 (which is essentially at the upper limit of the range of values measured by empirical studies). The results are presented in the third column of Tables 1 and 2. As can be seen, consumers' discounting remains unchanged, while firms' discounting becomes even smaller since M^f increases with θ, resulting in smaller overall discounting. In addition, the degree of intertemporal substitution (σ) increases significantly and the slope of the Phillips curve (κ) becomes slightly larger. We examine the effect of forward guidance on the output gap and inflation based on both the calibration of Gabaix (2020) (second column) and the standard calibration (third column), respectively.

Table 1: Key Parameter Inputs

	Gabaix's (2020) Calibration	Standard Calibration
Cognitive discounting by consumers and firms	$M = 0.85$, $M^f = 0.7976$	$M = 0.85$, $M^f = 0.7858$
Sensitivity to interest rates	$\sigma = 0.198$	$\sigma = 0.99$
Slope of the Phillips curve	$\kappa = 0.1146$	$\kappa = 0.1717$
Rate of time preference	$\beta = 0.99$	$\beta = 0.99$
Interest rate rule coefficients	$\phi_\pi = 1.5$, $\phi_x = 0.5/4$	$\phi_\pi = 1.5$, $\phi_x = 0.5/4$

Notes: This table reports the coefficients used in the model. Units are quarterly.

Table 2: Ancillary Parameters

	Gabaix's (2020) Calibration	Standard Calibration
Coefficient of risk aversion	$\gamma = 5$	$\gamma = 1$
Inverse of Frisch elasticity	$\phi = 1$	$\phi = 1$
Survival rates of prices	$\theta = 0.875$	$\theta = 0.75$
Cognitive discounting	$\bar{m} = 0.85$	$\bar{m} = 0.85$

Notes: This table reports the coefficients used in the model to generate the parameters of Table 1. Units are quarterly.

Another parameter of interest is $\bar{m}$. It is used to measure the degree of consumers' cognitive myopia ($M = \bar{m}$) and, together with other parameters, to measure the degree of firms' cognitive myopia ($M^f = \bar{m}\left[\theta + \frac{1-\beta\theta}{1-\beta\theta\bar{m}}(1-\theta)\right]$). We can indirectly infer that M^f equals 0.8 from Galí and Gertler's (1999) estimation of the Phillips curve. According to Gabaix's (2020) calibration, $\bar{m}$ equals 0.85. But we can also indirectly infer that M equals 0.65 from Fuhrer and Rudebusch's (2004) estimation of the IS curve. Gabaix (2020) believes that $\bar{m}$ equals 0.65 is quite extreme and thus sets it to 0.85. In addition, Gabaix (2020) examines the empirical literature on expectations (Coibion and Gorodnichenko, 2015) in the online appendix and finds that $\bar{m}$ equals 0.73. Angeletos et al. (2021) rerun Coibion and Gorodnichenko's (2015) regression using various samples, from which we can infer that $\bar{m}$ takes on a range of values between 0.73 and 0.92.

Following Del Negro et al. (2015), we consider four monetary-policy experiments: (1) in period 1 (i.e., in the present), the nominal interest rate falls by 1% - no forward guidance; (2) in period 4, the nominal interest rate is expected to fall by 1%, and from the present to period 3, the nominal interest rate is pegged to its steady-state value - 4-quarter forward guidance; (3) in period 8, the nominal interest rate is expected to fall by 1%, and from the present to period 7, the nominal interest rate is pegged to its steady-state value - 8-quarter forward guidance; (4) in period 12, the nominal interest rate is expected to fall by 1%, and from the present to period 11, the nominal interest rate is pegged to its steady-state value - 12-quarter forward guidance. As can be seen, there is no interest-rate peg in scenario (1), while scenarios (2), (3) and (4) assume 3-period, 7-period, and 11-period pegs, respectively. We study the effects of forward guidance on the output gap, inflation, and the real interest rate in Gabaix's (2020) model. We not only assume discounting but also follow the calibration of Gabaix (2020) (second column of Tables 1 and 2). The upper panels of Figure 1 show the results. Under different forward-guidance experiments, the responses of the output gap and inflation remain essentially unchanged - the forward guidance puzzle seems to be resolved. Is this caused by discounting alone? To answer this question, we only change the calibration of Gabaix (2020) to the standard calibration (third column of Tables 1 and 2), i.e., the value of γ (the coefficient of risk aversion) is reduced from 5 to 1 and the value of θ (the survival rates of prices) is reduced from 0.875 to 0.75. It should be noted that we still assume discounting. The lower panels report the results, and one can clearly see the forward guidance puzzle, the economic intuition of which is that people's expectations of expansionary monetary policy in the future raise the current output gap and inflation, and with an interest-rate peg at work, the real interest rate falls, which in turn further increases the current output gap and inflation. This reinforcing process is strengthened with longer duration of the peg and can lead to implausibly large responses of the output gap and inflation. We can thus draw the following conclusion. In Gabaix's (2020) model, that the forward guidance puzzle is resolved is not only due to discounting, but also requires both lower intertemporal substitution and a higher degree of price stickiness.

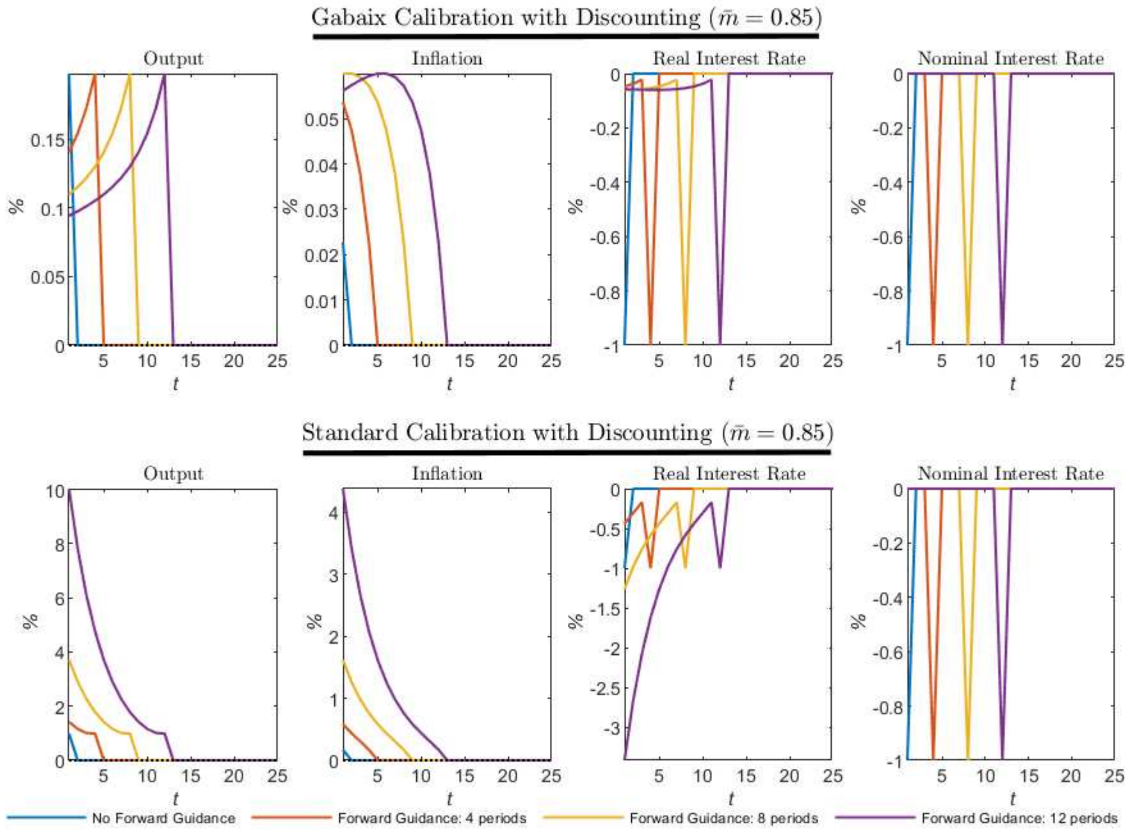

Figure 1: Effects of forward guidance in Gabaix's (2020) model. Note: We assume discounting in both cases. According to Tables 1 and 2, we set γ to 5 and θ to 0.875 under Gabaix's (2020) calibration, whereas under the standard calibration we set γ to 1 and θ equal to 0.75. The other parameters take the same values under both calibrations.

Is there a criterion in the literature that can be used to judge whether the forward guidance puzzle is resolved or not? The answer is yes. Carlstrom et al. (2015), Kiley (2016), McKay et al. (2016), Farhi and Werning (2017), Hagedorn et al. (2019), and Gabaix (2020) among many others study the relationship between the initial responses of the output gap and inflation and the horizon of forward guidance. They argue that the more distant the announced cut in the nominal interest rate occurs in the future, the smaller the output gap and inflation's initial responses should be. Conversely, there exists the forward guidance puzzle. As shown by the upper panel of Figure 2, if Gabaix's (2020) calibration is followed (second column of Tables 1 and 2), then the output gap and inflation's initial responses become smaller as the implementation of the announced cut becomes more distant in the future, i.e., there is no forward guidance puzzle. However, as seen from the middle panel, simply changing Gabaix's (2020) calibration to the standard one (third column of Tables 1 and 2) turns the relationship between the initial responses of the output gap and inflation and the horizon of forward guidance from decreasing to increasing. By comparing upper and middle panels, one can find that calibration plays a dominant role in resolving the forward guidance puzzle, while discounting plays only a limited role. To further illustrate this point, we increase the degree of discounting to what is deemed extreme by Gabaix (2020), i.e., reducing $\bar{m}$ from 0.85 to 0.65. The results are shown by the lower panel. Under the standard calibration, even with extreme discounting, the output gap and inflation's initial responses still become larger the more distant the announced cut occurs. With the analysis in Figures 1 and 2 combined, we can conclude that it is Gabaix's (2020) calibration, rather than discounting, that plays a dominant role in the resolution of the forward guidance puzzle.

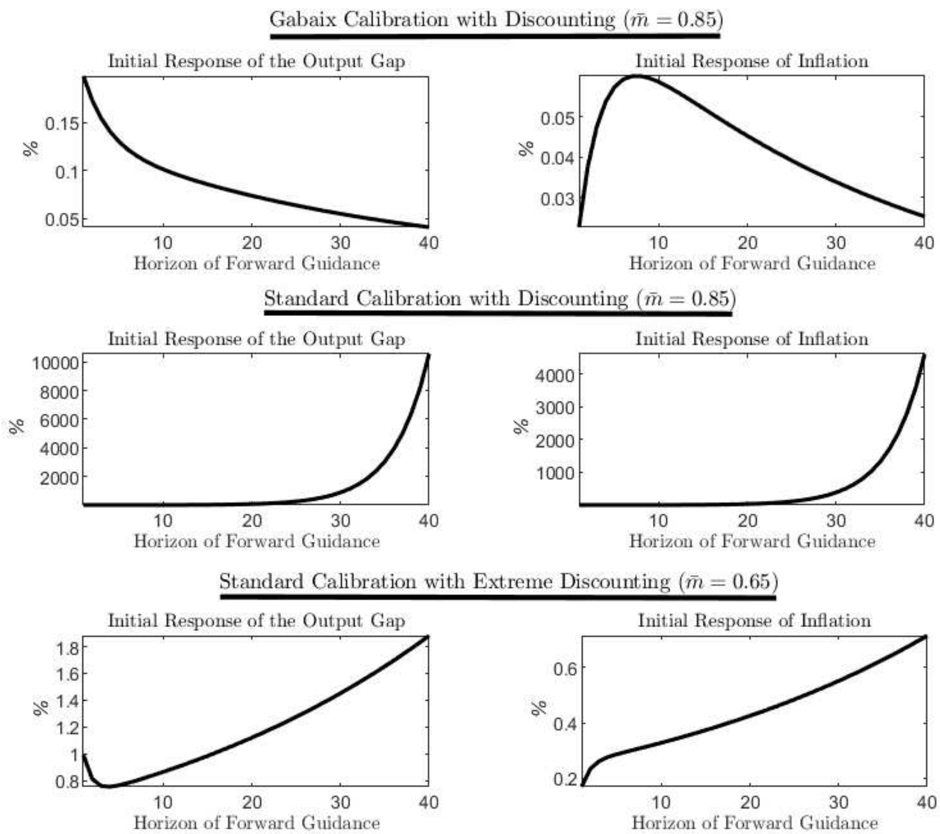

Figure 2: Effects on the output gap and inflation's initial responses of forward guidance at different horizons in Gabaix's (2020) model. Note: We assume discounting in all three cases. According to Tables 1 and 2, we set γ to 5 and θ to 0.875 under Gabaix's (2020) calibration, whereas under the standard calibration we set γ to 1 and θ equal to 0.75. The other parameters take the same values under both calibrations.

Gabaix (2020) argues that it is discounting that resolves the forward guidance puzzle. We, on the other hand, show that the resolution of the forward guidance puzzle relies mainly on parameter calibration. So, how to understand these two seemingly contradictory assertions? To answer this question, we look at how Gabaix (2020) models forward guidance. Following papers such as McKay et al. (2016), Gabaix (2020) assumes that the monetary authority reduces the real interest rate rather than the nominal interest rate in future period T and keeps it fixed in all other periods. As can be seen in the upper panels of Figures 3 and 4, the forward guidance puzzle is resolved with discounting. Interestingly, Gabaix's (2020) results are insensitive to parameter calibration. As the lower panels of Figures 3 and 4 show, even if the value of γ (the coefficient of risk aversion) is reduced from 5 to 1 and the value of θ (the survival rates of prices) is reduced from 0.875 to 0.75, the responses of the output gap and inflation are almost identical to that under Gabaix's (2020) calibration, in stark contrast to what is shown in Figures 1 and 2.

We argue that McKay et al. (2016), Gabaix (2020) and many others, in resolving the forward guidance puzzle, artificially close off the important channel of intertemporal substitution by fixing the real interest rate rather than the nominal interest rate, while the calibration of the two parameters, intertemporal elasticity of substitution and price stickiness, plays an important role in this channel. Thus, they only partially address the puzzle.

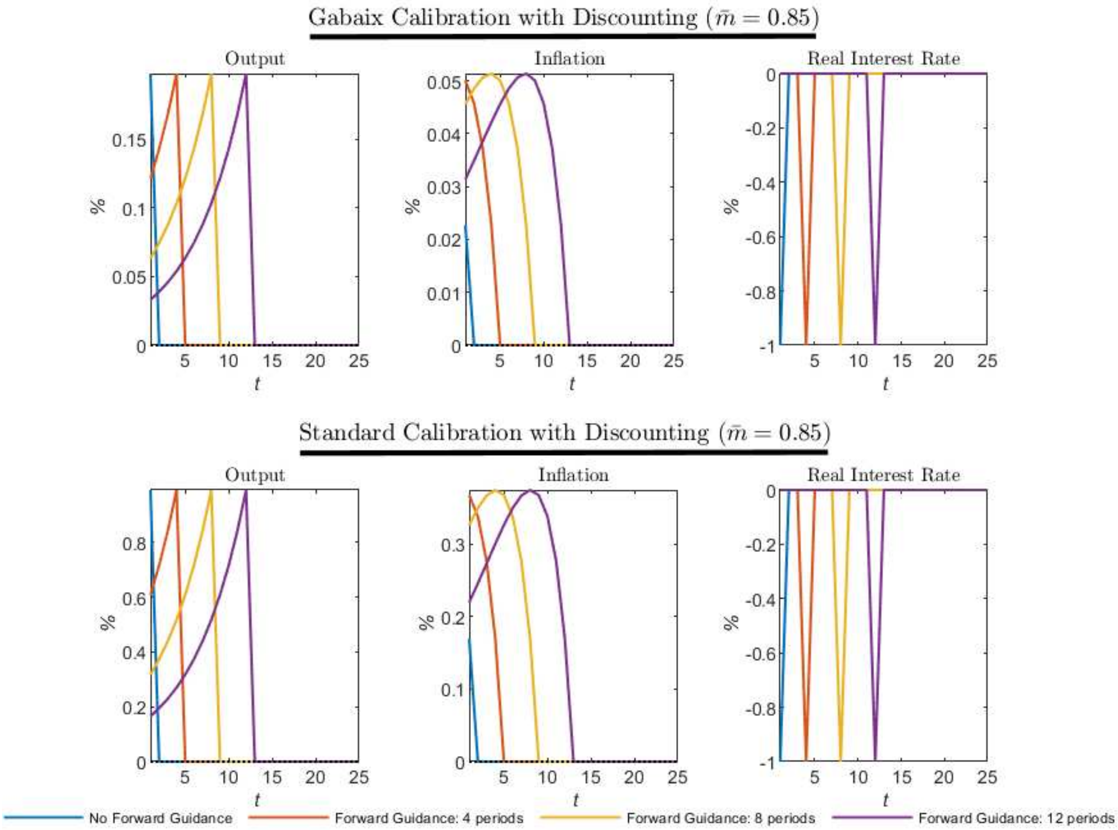

Figure 3: Effects of forward guidance in Gabaix's (2020) model. Note: We assume discounting in both cases. According to Tables 1 and 2, we set γ to 5 and θ to 0.875 under Gabaix's (2020) calibration, whereas under the standard calibration we set γ to 1 and θ equal to 0.75. The other parameters take the same values under both calibrations.

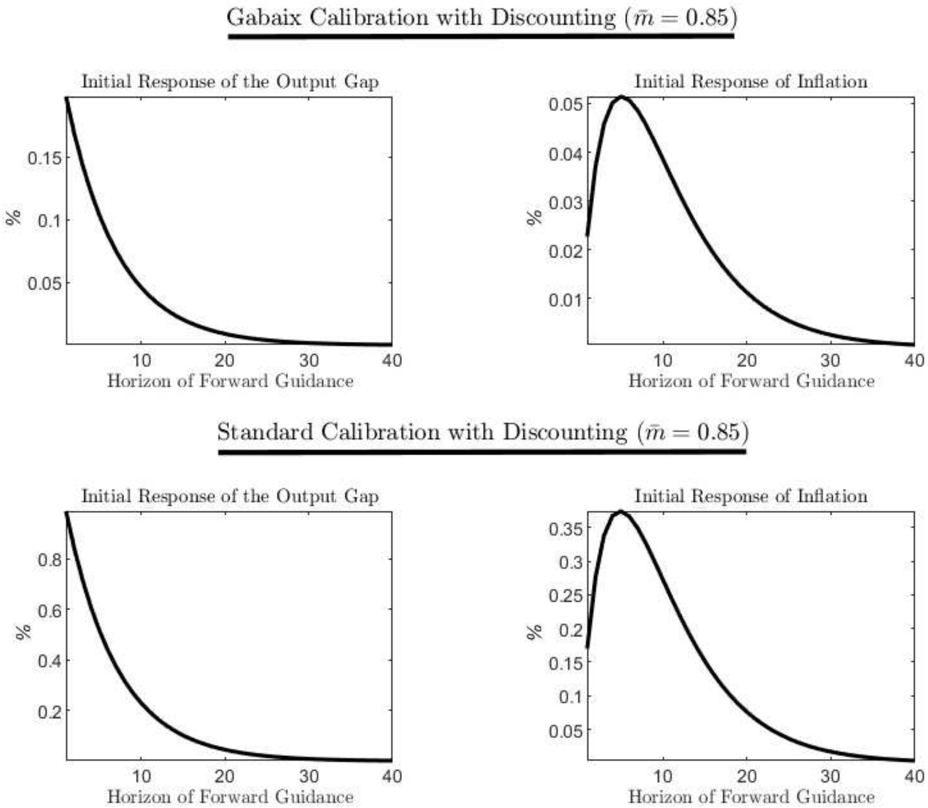

Figure 4: Effects on the output gap and inflation's initial responses of forward guidance at different horizons in Gabaix's (2020) model. Note: We assume discounting in both cases. According to Tables 1 and 2, we set γ to 5 and θ to 0.875 under Gabaix's (2020) calibration, whereas under the standard calibration we set γ to 1 and θ equal to 0.75. The other parameters take the same values under both calibrations.

3 A proposed resolution

To address the calibration problem described above, we follow Chapter 6 of Galí (2015) in extending Gabaix's (2020) model to include wage stickiness. Please refer to the appendix for detailed derivations. The extended model includes an IS curve, a price Phillips curve, a wage Phillips curve, a law of motion for real wages, and a Taylor-type rule, which is given by the following five equations

$$x_t = ME_t x_{t+1} - \sigma \left(i_t - E_t \pi^p_{t+1} - r^n_t \right), \tag{4}$$

$$\pi^p_t = \beta M^f E_t \pi^p_{t+1} + \kappa_p x_t + \lambda_p w_t, \tag{5}$$

$$\pi^w_t = \beta M^w E_t \pi^w_{t+1} + \kappa_w x_t - \lambda_w w_t, \tag{6}$$

$$w_t = w_{t-1} + \pi_t^w - \pi_t^p, \tag{7}$$

$$i_t = \phi_\pi \pi_t + \phi_x x_t, \tag{8}$$

where x_t denotes the output gap, i_t the nominal interest rate, π_t^p the price inflation rate, r_t^n the natural rate of interest, w_t the real wage, π_t^w the wage inflation rate, $M = \bar{m} \in [0,1]$ discounting by consumers, $\sigma = \frac{1}{\gamma R}$ the sensitivity of the output gap to interest rates, γ the coefficient of risk aversion, $R = \frac{1}{\beta}$ the steady state of the gross real interest rate, β the rate of time preference, $M^f = \bar{m}\left[\theta_p + \frac{1-\beta\theta_p}{1-\beta\theta_p\bar{m}}(1-\theta_p)\right] \in [0,1]$ discounting by firms, θ_p the survival rates of prices, $\kappa_p = \frac{\alpha\lambda_p}{1-\alpha}$, $\lambda_p = \frac{(1-\theta_p)(1-\beta\theta_p)(1-\alpha)}{\theta_p(1-\alpha+\alpha\epsilon_p)}$, ϵ_p the elasticity of substitution between differentiated goods, $1-\alpha$ the share of income paid to labor, $M^w = \bar{m}\left[\theta_w + \frac{1-\beta\theta_w}{1-\beta\theta_w\bar{m}}(1-\theta_w)\right] \in [0,1]$ discounting by workers, $\kappa_w = \lambda_w[\gamma + \frac{\phi}{1-\alpha}]$, $\lambda_w = \frac{(1-\theta_w)(1-\beta\theta_w)}{\theta_w(1+\epsilon_w\phi)}$, θ_w the survival rates of nominal wages, ϵ_w the elasticity of substitution between labor varieties, ϕ the inverse of Frisch elasticity, ϕ_p the monetary policy response to price inflation, and ϕ_y the monetary policy response to the output gap.

According to Chapter 6 of Galí (2015), we calibrate the parameter values as follows: $\beta = 0.99$, $\alpha = \frac{1}{4}$, $\epsilon_p = 9$, $\epsilon_w = 4.5$, $\gamma = 1$, $\phi = 5$, $\phi_p = 1.5$, $\phi_y = 0.125$, $\theta_p = 0.75$, and $\theta_w = 0.75$. As for discounting, we set $\bar{m}$ to 0.85.

We examine the expansionary effects of forward guidance on the output gap, inflation, real wages, and the real interest rate in the extended model. It is important to note that the relative stickiness of prices and wages plays a key role in the results. In Figures 5 and 6, we fix price stickiness (θ_p) and consider the following three cases by varying wage stickiness (θ_w): (1) wage stickiness is lower than price stickiness ($\theta_w = 0.55 < \theta_p = 0.75$), (2) wage and price have the same stickiness ($\theta_w = \theta_p = 0.75$), and (3) wage stickiness is higher than price stickiness ($\theta_w = 0.95 > \theta_p = 0.75$).[7] As shown by these two graphs, the forward guidance puzzle cannot be resolved in case (1), but is resolved in cases (2) and (3). The economic intuition is that in a standard new Keynesian model, prices are sticky while nominal wages are not. Prices rise due to forward guidance, while nominal wages can rise by an even larger amount, which makes real wages rise. The rise in real wages raises consumers' purchasing power and firms' marginal costs, both of which raise inflationary expectations, which, with an interest rate peg at work, lowers the real interest rate, which in turn further raises prices, thus further increasing the expansionary effect of forward guidance. As seen in the third column of Figure 5, any increase in wage stickiness can reduce the rise in real wages; when wage stickiness is high enough, real wages can even fall. We can thus conclude that discounting alone can resolve the forward guidance puzzle in the extended model even assuming the standard calibration, as long as a reasonable degree of wage stickiness is allowed.

[7]The empirical literature finds that nominal wages are typically stickier than prices. Bils and Klenow (2004) find that prices are adjusted on average between 1.3 and 2 quarters, corresponding to the value of θ_p between 0.23 and 0.5; if sales are excluded, Nakamura and Steinsson (2008) find that prices are adjusted on average between 2.6 and 3.7 quarters (or $0.62 \leq \theta_p \leq 0.73$). Druant et al. (2012) and Barattieri et al. (2014), on the other hand, find that the average frequency of wage adjustment is usually more than one year (or $\theta_\omega \geq 0.75$). There is also downward nominal wage rigidity, where wages are essentially fixed (Bewley 1999; Dickens et al. 2007).

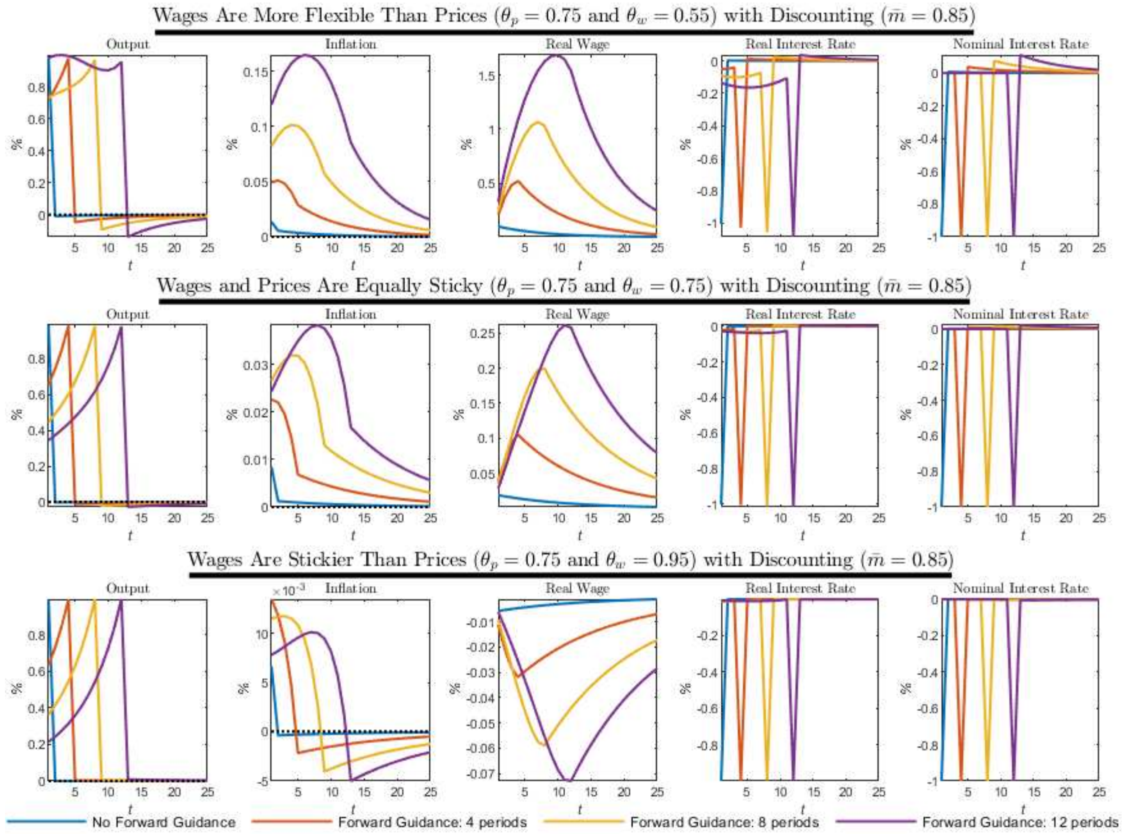

Figure 5: Effects of forward guidance in the extended model. Note: We assume discounting in all three cases.

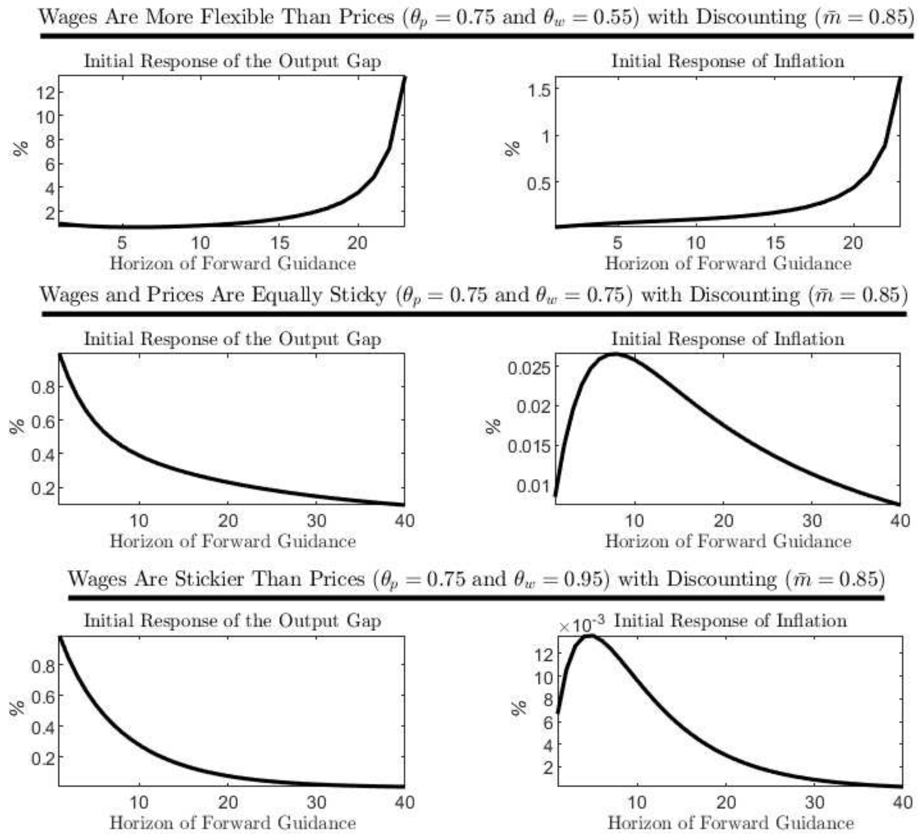

Figure 6: Effects on the output gap and inflation's initial responses of forward guidance at different horizons in the extended model. Note: We assume discounting in all three cases.

4 Conclusion

We investigate whether discounting alone can resolve the forward guidance puzzle in a standard new Keynesian model. Numerically, we examine the expansionary effects of forward guidance at different horizons in Gabaix's (2020) model. In addition to discounting, we find that Gabaix (2020) assumes a smaller intertemporal elasticity of substitution and higher price stickiness, both of which reduce the response of inflation and thus the expansionary effects of forward guidance to some extent. If the standard calibration is used, discounting alone cannot resolve the forward guidance puzzle. We extend Gabaix's (2020) model to include wage stickiness. Wage stickiness can dampen the rise in real wages and thus the expansionary effect of forward guidance. We find that discounting alone can resolve the forward guidance puzzle in the extended model even assuming the standard calibration, as long as a reasonable degree of wage stickiness is allowed.

Appendix

We base the derivations of the two models shown in the main text on Chapters 3 and 6 of Galí (2015). Since Gabaix's (2020) assumption of cognitive myopia affects only agents' expectations but not their behavior in decision making, the presence of discounting does not affect the functional form of optimal conditions. Therefore, we skip their derivations in this appendix. However, in deriving the Phillips curves for prices and wages, it is necessary to convert infinite-horizon optimal conditions into recursive ones, and as we explain in detail in this appendix, different assumptions about expectations lead to different results.

A. Consumer's utility maximization problem

Suppose that there exist an infinite number of consumers, uniformly distributed between 0 and 1. Each consumer j chooses a basket of differentiated goods ($C_t \equiv \left[\int_0^1 C_t(i)^{\frac{\epsilon_p-1}{\epsilon_p}} di\right]^{\frac{\epsilon_p}{\epsilon_p-1}}$) and provides labor of type j ($N_t(j)$). The utility maximization problem for a typical consumer is

$$E_0^{BR} \sum_{t=0}^{\infty} \beta^t \left[\frac{C_t^{1-\gamma}-1}{1-\gamma} - \int_0^1 \frac{N_t(j)^{1+\phi}}{1+\phi} dj \right], \tag{9}$$

$$\text{s.t.} \int_0^1 P_t(i) C_t(i) \, di + Q_t B_t \leq B_{t-1} + \int_0^1 W_t(j) N_t(j) \, dj + D_t, \tag{10}$$

where C_t denotes consumption, $C_t(i)$ the quantity of good i consumed, $N_t(j)$ hours worked of type-j labor, $P_t(i)$ the price of good i, B_t one-period bond holdings, Q_t one-period bond prices, $W_t(j)$ the nominal wage of type-j labor, D_t dividends from the ownership of firms, ϵ_p the elasticity of substitution between differentiated goods, β the rate of time preference, γ the coefficient of risk aversion, and ϕ the inverse of Frisch elasticity. E_0^{BR} is the subjective expectations operator.

By solving the first-order conditions, the demand curve for good i and the consumption Euler equation are obtained as follows

$$C_t(i) = \left[\frac{P_t(i)}{P_t} \right]^{-\epsilon_p} C_t, \tag{11}$$

$$Q_t = \beta E_t^{BR} \left[\left(\frac{C_{t+1}}{C_t} \right)^{-\gamma} \left(\frac{P_t}{P_{t+1}} \right) \right], \tag{12}$$

where $P_t \equiv \left[\int_0^1 P_t(i)^{1-\epsilon_p} di\right]^{\frac{1}{1-\epsilon_p}}$ denotes prices (or the aggregate price level).

The stochastic discount factor is defined as

$$\Lambda_{t,t+k} \equiv \beta^k \left(\frac{C_{t+k}}{C_t} \right)^{-\gamma}, \tag{13}$$

for $k = 1, 2, ...$

It is important to note that consumers are suppliers of labor, but their choice of labor supply depends on the competitiveness of the labor market. The standard new Keynesian model assumes a perfectly competitive labor market, where all types of labor are identical, i.e., labor is homogeneous. Under this assumption, the labor supply curve from the utility maximization problem is given by

$$\frac{N_t^{\phi}}{C_t^{-\gamma}} = \frac{W_t}{P_t}, \tag{14}$$

where W_t denotes nominal wages.

A perfectly competitive labor market implies flexible nominal wages. Wage stickiness, on the other hand, requires the assumption of a monopolistically competitive labor market such that each consumer has a choice over the price (that is, the nominal wage) of the type of labor he or she supplies. In this case, consumers can choose whether to adjust nominal wages or not. We model wage stickiness in subsection D.

B. Firm's profit maximization problem

Suppose that there exist an infinite number of firms producing intermediate goods, uniformly distributed between 0 and 1. Each firm j has monopoly power over good j that it produces. Assume the Calvo price-setting model in which each firm keeps its previous price with probability θ_p and resets its price with probability $1-\theta_p$ each period. The profit maximization problem for a typical firm that can reset its price is

$$\max_{P_t^{*}} E_t^{BR} \sum_{k=0}^{\infty} \theta_p^k \left[\Lambda_{t,t+k} Y_{t+k|t} \left(\frac{1}{P_{t+k}} \right) \left(P_t^{*} - \Psi_{t+k|t} \right) \right], \tag{15}$$

$$\text{s.t. } Y_{t+k|t} = \left(\frac{P_t^{*}}{P_{t+k}} \right)^{-\epsilon_p} C_{t+k}, \tag{16}$$

where P_t^{*} denotes the optimal price set by the firm, $Y_{t+k|t}$ the quantity of the intermediate good produced by the firm, $\Psi_{t+k|t} \equiv \frac{W_{t+k}}{MPN_{t+k|t}}$ the nominal marginal cost, and $MPN_{t+k|t}$ the marginal product of labor.

Solving the first-order conditions yields the following infinite-horizon optimal price-setting condition

$$E_t^{BR} \sum_{k=0}^{\infty} \theta_p^k \left[\Lambda_{t,t+k} Y_{t+k|t} \left(\frac{1}{P_{t+k}} \right) \left(P_t^{*} - M_p \Psi_{t+k|t} \right) \right] = 0, \tag{17}$$

where $M_p \equiv \frac{\epsilon_p}{\epsilon_p - 1}$ denotes the steady state of the price markup.

C. Derivation of Gabaix's (2020) model

Log-linearizing the consumption Euler equation (12) and the optimal price-setting condition (17) yields

$$c_t = E_t^{BR} c_{t+1} - \sigma r_t, \tag{18}$$

$$p_t^{*} = \mu^p + (1 - \beta\theta_p) E_t^{BR} \sum_{k=0}^{\infty} (\beta\theta_p)^k \psi_{t+k|t}, \tag{19}$$

where the lowercase letter denotes the log-deviation of a variable from its steady state, $r_t \equiv i_t - E_t \pi_{t+1}^p$ the real interest rate, i_t the nominal interest rate, π_t^p the price inflation rate, $\sigma = \frac{1}{\gamma R}$ the sensitivity of the output gap to interest rates, $R = \frac{1}{\beta}$ the steady state of the gross real interest rate, and $\mu^p \equiv \log M_p$.

Assuming a concave production function for intermediate-good producers ($Y_t(i) = N_t(i)^{1-\alpha}$) and using the market clearing conditions for both intermediate and final goods ($Y_t(i) = C_t(i)$ and $Y_t = C_t$) yields

$$x_t = E_t^{BR} x_{t+1} - \sigma \left(i_t - E_t \pi_{t+1}^p - r_t^n\right), \tag{20}$$

$$p_t^{*} = (1 - \beta\theta_p) E_t^{BR} \sum_{k=0}^{\infty} (\beta\theta_p)^k \left(p_{t+k} - \Theta \mu_{t+k}^p\right), \tag{21}$$

where x_t denotes the output gap, r_t^n the natural rate of interest, μ_t^p the price markup, $1 - \alpha$ the share of income paid to labor, and $\Theta \equiv \frac{1-\alpha}{1-\alpha+\alpha\epsilon_p}$.

So far, we have closely followed Chapter 3 of Galí (2015) in the derivation of a standard new Keynesian model. The only difference is that agents' expectations are subjective rather than rational. If cognitive myopia is assumed, according to Gabaix (2020), subjective expectations diverge from rational expectations. For any variable x, the relationship between the two is defined as

$$E_t^{BR} x_{t+k} = \bar{m}^k E_t x_{t+k}, \tag{22}$$

where $\bar{m} \in [0, 1]$. It can be seen that the closer $\bar{m}$ is to 1, the more rational the subjective expectations are. When $\bar{m}$ equals 1, subjective expectations are equivalent to rational expectations.

For the IS curve, since it is only necessary to forecast one period ahead, equation (20) can be rewritten as

$$x_t = M E_t x_{t+1} - \sigma \left(i_t - E_t \pi_{t+1}^p - r_t^n\right), \tag{23}$$

where $M = \bar{m} \in [0, 1]$ denotes discounting by consumers. This is Gabaix's (2020) IS curve.

We next convert the infinite-horizon optimal price-setting condition (21) into a recursive form, where we highlight how to handle cognitive myopia

$$
\begin{aligned}
p_t^{*} &= (1-\beta\theta_p)\, E_t^{BR} \sum_{k=0}^{\infty} (\beta\theta_p)^k \left(p_{t+k} - \Theta\mu_{t+k}^p\right) \\
&= p_t - p_t + (1-\beta\theta_p)\, E_t^{BR} \sum_{k=0}^{\infty} (\beta\theta_p)^k \left(p_{t+k} - \Theta\mu_{t+k}^p\right) \\
&= p_t - (1-\beta\theta_p)\, E_t^{BR} \sum_{k=0}^{\infty} (\beta\theta_p)^k\, p_t + (1-\beta\theta_p)\, E_t^{BR} \sum_{k=0}^{\infty} (\beta\theta_p)^k \left(p_{t+k} - \Theta\mu_{t+k}^p\right) \\
&= p_t + (1-\beta\theta_p)\, E_t^{BR} \sum_{k=0}^{\infty} (\beta\theta_p)^k \left(p_{t+k} - p_t - \Theta\mu_{t+k}^p\right) \\
&= p_t + (1-\beta\theta_p)\, E_t^{BR} \sum_{k=0}^{\infty} (\beta\theta_p)^k (p_{t+k} - p_t) - (1-\beta\theta_p)\, \Theta E_t^{BR} \sum_{k=0}^{\infty} (\beta\theta_p)^k \mu_{t+k}^p \\
&= p_t + (1-\beta\theta_p)\, E_t \sum_{k=1}^{\infty} (\beta\theta_p\bar{m})^k (p_{t+k} - p_t) - (1-\beta\theta_p)\, \Theta E_t \sum_{k=0}^{\infty} (\beta\theta_p\bar{m})^k \mu_{t+k}^p \\
&= p_t + (1-\beta\theta_p)\, E_t \sum_{k=1}^{\infty} (\beta\theta_p\bar{m})^k \left(\pi_{t+1}^p + \pi_{t+2}^p + \cdots + \pi_{t+k}^p\right) - (1-\beta\theta_p)\, \Theta E_t \sum_{k=0}^{\infty} (\beta\theta_p\bar{m})^k \mu_{t+k}^p, \qquad (24)
\end{aligned}
$$

where we use the fact that $p_t = E_t^{BR} p_t = (1-\beta\theta_p)\, E_t^{BR} \sum_{k=0}^{\infty} (\beta\theta_p)^k p_t$, $E_t^{BR} \sum_{k=0}^{\infty} (\beta\theta_p)^k (p_{t+k} - p_t) = E_t \sum_{k=0}^{\infty} (\beta\theta_p\bar{m})^k (p_{t+k} - p_t) = E_t \sum_{k=1}^{\infty} (\beta\theta_p\bar{m})^k (p_{t+k} - p_t)$, $E_t^{BR} \sum_{k=0}^{\infty} (\beta\theta_p)^k \mu_{t+k}^p = E_t \sum_{k=0}^{\infty} (\beta\theta_p\bar{m})^k \mu_{t+k}^p$, and $p_{t+k} - p_t = \pi_{t+1}^p + \pi_{t+2}^p + \cdots + \pi_{t+k}^p$.

Note that

$$
\begin{aligned}
E_t \sum_{k=1}^{\infty} (\beta\theta_p\bar{m})^k \left(\pi_{t+1}^p + \pi_{t+2}^p + \cdots + \pi_{t+k}^p\right) &= E_t \left\{ \left[\beta\theta_p\bar{m} + (\beta\theta_p\bar{m})^2 + \cdots\right] \pi_{t+1}^p + \left[(\beta\theta_p\bar{m})^2 + (\beta\theta_p\bar{m})^3 + \cdots\right] \pi_{t+2}^p + \cdots \right\} \\
&= E_t \left[\frac{\beta\theta_p\bar{m}}{1-\beta\theta_p\bar{m}} \pi_{t+1}^p + \frac{(\beta\theta_p\bar{m})^2}{1-\beta\theta_p\bar{m}} \pi_{t+2}^p + \cdots \right] + \\
&= \frac{1}{1-\beta\theta_p\bar{m}} E_t \sum_{k=1}^{\infty} (\beta\theta_p\bar{m})^k \pi_{t+k}^p. \qquad (25)
\end{aligned}
$$

Plugging equation (25) into equation (24) gives

$$
\begin{aligned}
p_t^{*} - p_t &= \frac{1-\beta\theta_p}{1-\beta\theta_p\bar{m}} E_t \sum_{k=1}^{\infty} (\beta\theta_p\bar{m})^k \pi_{t+k}^p - (1-\beta\theta_p)\, \Theta E_t \sum_{k=0}^{\infty} (\beta\theta_p\bar{m})^k \mu_{t+k}^p \\
&= \frac{1-\beta\theta_p}{1-\beta\theta_p\bar{m}} E_t \left[\beta\theta_p\bar{m}F + (\beta\theta_p\bar{m}F)^2 + \cdots\right] \pi_t^p - (1-\beta\theta_p)\, \Theta E_t \left[1 + \beta\theta_p\bar{m}F + (\beta\theta_p\bar{m}F)^2 + \cdots\right] \mu_t^p \\
&= \frac{1-\beta\theta_p}{1-\beta\theta_p\bar{m}} E_t \frac{\beta\theta_p\bar{m}}{1-\beta\theta_p\bar{m}F} \pi_{t+1}^p - (1-\beta\theta_p)\, \Theta E_t \frac{1}{1-\beta\theta_p\bar{m}F} \mu_t^p, \qquad (26)
\end{aligned}
$$

where we follow Gabaix (2020) in using the forward operator (F) to handle summation terms.

The price dynamics under Calvo pricing ($\Pi_t^{1-\epsilon_p} = \theta_p + (1-\theta_p)\left(\frac{P_t^*}{P_t}\right)$) implies

$$\begin{aligned}
\pi_t^p &= (1-\theta_p)\left(p_t^* - p_{t-1}\right) \\
&= (1-\theta_p)\left[\left(p_t^* - p_t\right) + (p_t - p_{t-1})\right] \\
&= (1-\theta_p)\left(p_t^* - p_t\right) + (1-\theta_p)\,\pi_t^p \\
&= \frac{1-\theta_p}{\theta_p}\left(p_t^* - p_t\right), \qquad (27)
\end{aligned}$$

where $\Pi_t \equiv \frac{P_t}{P_{t-1}}$.

Combining equations (26) and (27) gives

$$\begin{aligned}
\pi_t^p &= \frac{1-\theta_p}{\theta_p}\frac{1-\beta\theta_p}{1-\beta\theta_p\bar{m}}E_t\frac{\beta\theta_p\bar{m}}{1-\beta\theta_p\bar{m}F}\pi_{t+1}^p - \frac{(1-\beta\theta_p)(1-\theta_p)}{\theta_p}\Theta E_t\frac{1}{1-\beta\theta_p\bar{m}F}\mu_t^p \\
&= \beta\bar{m}\left[\theta_p + \frac{1-\beta\theta_p}{1-\beta\theta_p\bar{m}}(1-\theta_p)\right]E_t\pi_{t+1}^p - \frac{(1-\beta\theta_p)(1-\theta_p)}{\theta_p}\Theta\mu_t^p. \qquad (28)
\end{aligned}$$

It can be seen in the derivation of equation (28) that we do not involve anything related to the competitiveness of the labor market, which is why Chapter 6 of Galí (2015) starts directly from this equation when deriving a new Keynesian model with both sticky prices and wages.

Using the relationship between the price markup and the marginal cost, and labor demand and supply curves, it follows that

$$\mu_t^p = -\left(\gamma + \frac{\phi+\alpha}{1-\alpha}\right)x_t. \qquad (29)$$

Combining equations (28) and (29) yields Gabaix's (2020) Phillips curve

$$\pi_t^p = \beta M^f E_t\pi_{t+1}^p + \kappa x_t, \qquad (30)$$

where $M^f = \bar{m}\left[\theta_p + \frac{1-\beta\theta_p}{1-\beta\theta_p\bar{m}}(1-\theta_p)\right]$ denotes discounting by firms and $\kappa = \frac{(1-\beta\theta_p)(1-\theta_p)}{\theta_p}\Theta\left(\gamma + \frac{\phi+\alpha}{1-\alpha}\right) = \frac{(1-\beta\theta_p)(1-\theta_p)}{\theta_p}(\gamma+\phi)$ because Gabaix (2020) assumes a linear production function ($\alpha = 0$).

D. Worker's wage-setting problem

Suppose a monopolistically competitive labor market in which there are an infinite number of labor types, uniformly distributed between 0 and 1. Labor of type j is supplied by consumer j. Each intermediate goods producer employs the following basket of labor types

$$N_t(i) \equiv \left[\int_0^1 N_t(i,j)^{\frac{\epsilon_w-1}{\epsilon_w}} dj\right]^{\frac{\epsilon_w}{\epsilon_w-1}}, \tag{31}$$

where $N_t(i,j)$ denotes the quantity of type-j labor employed by firm i and ϵ_ω the elasticity of substitution between labor varieties.

Solving the firm's cost minimization problem yields the demand curve for each type of labor

$$N_t(i,j) = \left[\frac{W_t(j)}{W_t}\right]^{-\epsilon_w} N_t(i), \tag{32}$$

where $W_t \equiv \left[\int_0^1 W_t(j)^{1-\epsilon_w} dj\right]^{\frac{1}{1-\epsilon_w}}$ denotes nominal wages.

Suppose the Calvo wage-setting model in which each worker has probability $1-\theta_w$ of adjusting the nominal wage and probability θ_w of keeping the nominal wage fixed each period. The maximization problem for a typical worker who can adjust the nominal wage is

$$\max_{W_t^{\ast}} E_t^{BR} \sum_{k=0}^{\infty} (\beta\theta_w)^k \left(C_{t+k}^{-\gamma}\frac{W_t^{\ast}}{P_{t+k}} N_{t+k|t} - \frac{N_{t+k|t}^{1+\phi}}{1+\phi}\right), \tag{33}$$

$$\text{s.t. } N_{t+k|t} = \left(\frac{W_t^{\ast}}{W_{t+k}}\right)^{-\epsilon_w} \left(\int_0^1 N_t(i)\, di\right), \tag{34}$$

where $W_t^{\ast}$ denotes the optimal nominal wage set by the worker.

Solving the first-order conditions yields the following infinite-horizon optimal wage-setting condition

$$E_t^{BR} \sum_{k=0}^{\infty} (\beta\theta_w)^k \left[N_{t+k|t} C_{t+k}^{-\gamma}\left(\frac{W_t^{\ast}}{P_{t+k}} - M_w MRS_{t+k|t}\right)\right] = 0, \tag{35}$$

where $M_w \equiv \frac{\epsilon_w}{\epsilon_w-1}$ denotes the steady state of the wage markup and $MRS_{t+k|t} \equiv C_{t+k}^{\gamma} N_{t+k|t}^{\phi}$ the marginal rate of substitution between consumption and employment.

Log-linearizing equation (35) and doing some manipulations yields

$$w_t^{\ast} = (1-\beta\theta_w) E_t^{BR} \sum_{k=0}^{\infty} (\beta\theta_w)^k \left(w_{t+k} - \frac{1}{1+\epsilon_w\phi}\mu_{t+k}^w\right). \tag{36}$$

E. Derivation of the extended model

It follows from Chapter 6 of Galí (2015) that the competitiveness of the labor market does not affect the IS curve and thus

$$x_t = M E_t x_{t+1} - \sigma\left(i_t - E_t \pi_{t+1}^p - r_t^n\right). \tag{37}$$

The firms' optimal price-setting condition is exactly the same as that in a standard new Keynesian model (Galí 2015, Chapter 3), the only difference being the equation for price markups, and so the derivation of the price Phillips curve can start from equation (28)

$$\begin{aligned}
\pi_t^p &= \beta\bar{m}\left[\theta_p + \frac{1-\beta\theta_p}{1-\beta\theta_p\bar{m}}\left(1-\theta_p\right)\right]E_t\pi_{t+1}^p - \frac{\left(1-\beta\theta_p\right)\left(1-\theta_p\right)}{\theta_p}\Theta\mu_t^p \\
&= \beta M^f E_t\pi_{t+1}^p + \frac{\left(1-\beta\theta_p\right)\left(1-\theta_p\right)}{\theta_p}\Theta\left(\frac{\alpha}{1-\alpha}x_t + \omega_t\right) \\
&= \beta M^f E_t\pi_{t+1}^p + \frac{\left(1-\beta\theta_p\right)\left(1-\theta_p\right)}{\theta_p}\frac{1-\alpha}{1-\alpha+\alpha\epsilon_p}\frac{\alpha}{1-\alpha}x_t + \frac{\left(1-\beta\theta_p\right)\left(1-\theta_p\right)}{\theta_p}\frac{1-\alpha}{1-\alpha+\alpha\epsilon_p}\omega_t \\
&= \beta M^f E_t\pi_{t+1}^p + \kappa_p x_t + \lambda_p\omega_t, \qquad (38)
\end{aligned}$$

where $M^f = \bar{m}\left[\theta_p + \frac{1-\beta\theta_p}{1-\beta\theta_p\bar{m}}\left(1-\theta_p\right)\right]$, $\kappa_p = \frac{\alpha\lambda_p}{1-\alpha}$, $\lambda_p = \frac{(1-\theta_p)(1-\beta\theta_p)(1-\alpha)}{\theta_p(1-\alpha+\alpha\epsilon_p)}$ and we use $\mu_t^p = -\frac{\alpha}{1-\alpha}x_t - \omega_t$. Equation (38) is the price Phillips curve.

In addition, due to wage stickiness, there is an identity linking real wages, price inflation, and wage inflation

$$w_t = w_{t-1} + \pi_t^\omega - \pi_t^p. \qquad (39)$$

In this subsection, we focus on deriving the wage Phillips curve for the extended model. We start from equation (36)

$$\begin{aligned}
w_t^* &= \left(1-\beta\theta_w\right)E_t^{BR}\sum_{k=0}^{\infty}\left(\beta\theta_w\right)^k\left(w_{t+k} - \frac{1}{1+\epsilon_w\phi}\mu_{t+k}^w\right) \\
&= w_t - w_t + \left(1-\beta\theta_w\right)E_t^{BR}\sum_{k=0}^{\infty}\left(\beta\theta_w\right)^k\left(w_{t+k} - \frac{1}{1+\epsilon_w\phi}\mu_{t+k}^w\right) \\
&= w_t - \left(1-\beta\theta_w\right)E_t^{BR}\sum_{k=0}^{\infty}\left(\beta\theta_w\right)^k w_t + \left(1-\beta\theta_w\right)E_t^{BR}\sum_{k=0}^{\infty}\left(\beta\theta_w\right)^k\left(w_{t+k} - \frac{1}{1+\epsilon_w\phi}\mu_{t+k}^w\right) \\
&= w_t + \left(1-\beta\theta_w\right)E_t^{BR}\sum_{k=0}^{\infty}\left(\beta\theta_w\right)^k\left(w_{t+k} - w_t - \frac{1}{1+\epsilon_w\phi}\mu_{t+k}^w\right) \\
&= w_t + \left(1-\beta\theta_w\right)E_t^{BR}\sum_{k=0}^{\infty}\left(\beta\theta_w\right)^k\left(w_{t+k} - w_t\right) - \frac{1-\beta\theta_w}{1+\epsilon_w\phi}E_t^{BR}\sum_{k=0}^{\infty}\left(\beta\theta_w\right)^k\mu_{t+k}^w \\
&= w_t + \left(1-\beta\theta_w\right)E_t\sum_{k=1}^{\infty}\left(\beta\theta_w\bar{m}\right)^k\left(w_{t+k} - w_t\right) - \frac{1-\beta\theta_w}{1+\epsilon_w\phi}E_t\sum_{k=0}^{\infty}\left(\beta\theta_w\bar{m}\right)^k\mu_{t+k}^w \\
&= w_t + \left(1-\beta\theta_w\right)E_t\sum_{k=1}^{\infty}\left(\beta\theta_w\bar{m}\right)^k\left(\pi_{t+1}^w + \pi_{t+2}^w + \cdots + \pi_{t+k}^w\right) - \frac{1-\beta\theta_w}{1+\epsilon_w\phi}E_t\sum_{k=0}^{\infty}\left(\beta\theta_w\bar{m}\right)^k\mu_{t+k}^w, \qquad (40)
\end{aligned}$$

where we use the fact that $w_t = E_t^{BR}w_t = \left(1-\beta\theta_w\right)E_t^{BR}\sum_{k=0}^{\infty}\left(\beta\theta_w\right)^k w_t$, $E_t^{BR}\sum_{k=0}^{\infty}\left(\beta\theta_w\right)^k\left(w_{t+k}-w_t\right) = E_t\sum_{k=0}^{\infty}$

$(\beta\theta_w\bar{m})^k (w_{t+k} - w_t) = E_t \sum_{k=1}^{\infty} (\beta\theta_w\bar{m})^k (w_{t+k} - w_t)$, $E_t^{BR} \sum_{k=0}^{\infty} (\beta\theta_w)^k \mu_{t+k}^w = E_t \sum_{k=0}^{\infty} (\beta\theta_w\bar{m})^k \mu_{t+k}^w$, and $w_{t+k} - w_t = \pi_{t+1}^w + \pi_{t+2}^w + \cdots + \pi_{t+k}^w$.

Note that

$$
\begin{aligned}
E_t \sum_{k=1}^{\infty} (\beta\theta_w\bar{m})^k \left(\pi_{t+1}^w + \pi_{t+2}^w + \cdots + \pi_{t+k}^w\right) &= E_t \left\{ \left[\beta\theta_w\bar{m} + (\beta\theta_w\bar{m})^2 + \cdots\right] \pi_{t+1}^w + \left[(\beta\theta_w\bar{m})^2 + (\beta\theta_w\bar{m})^3 + \cdots\right] \pi_{t+2}^w + \cdots \right\} \\
&= E_t \left[\frac{\beta\theta_w\bar{m}}{1-\beta\theta_w\bar{m}} \pi_{t+1}^w + \frac{(\beta\theta_w\bar{m})^2}{1-\beta\theta_w\bar{m}} \pi_{t+2}^w + \cdots \right] \\
&= \frac{1}{1-\beta\theta_w\bar{m}} E_t \sum_{k=1}^{\infty} (\beta\theta_w\bar{m})^k \pi_{t+k}^w. \qquad (41)
\end{aligned}
$$

Plugging equation (41) into equation (40) gives

$$
\begin{aligned}
w_t^{\ast} - w_t &= \frac{1-\beta\theta_w}{1-\beta\theta_w\bar{m}} E_t \sum_{k=1}^{\infty} (\beta\theta_w\bar{m})^k \pi_{t+k}^w - \frac{1-\beta\theta_w}{1+\epsilon_w\phi} E_t \sum_{k=0}^{\infty} (\beta\theta_w\bar{m})^k \mu_{t+k}^w \\
&= \frac{1-\beta\theta_w}{1-\beta\theta_w\bar{m}} E_t \left[\beta\theta_w\bar{m}F + (\beta\theta_w\bar{m}F)^2 + \cdots\right] \pi_t^w - \frac{1-\beta\theta_w}{1+\epsilon_w\phi} E_t \left[1 + \beta\theta_w\bar{m}F + (\beta\theta_w\bar{m}F)^2 + \cdots\right] \mu_t^w \\
&= \frac{1-\beta\theta_w}{1-\beta\theta_w\bar{m}} E_t \frac{\beta\theta_w\bar{m}}{1-\beta\theta_w\bar{m}F} \pi_{t+1}^w - \frac{1-\beta\theta_w}{1+\epsilon_w\phi} E_t \frac{1}{1-\beta\theta_w\bar{m}F} \mu_t^w, \qquad (42)
\end{aligned}
$$

where we again follow Gabaix (2020) in using the forward operator (F) to handle summation terms.

The wage dynamics under Calvo wage-setting ($W_t = \left[\theta_w W_{t-1}^{1-\epsilon_w} + (1-\theta_w)\left(W_t^{\ast}\right)^{1-\epsilon_w}\right]^{\frac{1}{1-\epsilon_w}}$) implies

$$
\begin{aligned}
\pi_t^w &= (1-\theta_w)\left(w_t^{\ast} - w_{t-1}\right) \\
&= (1-\theta_w)\left[\left(w_t^{\ast} - w_t\right) + (w_t - w_{t-1})\right] \\
&= (1-\theta_w)\left(w_t^{\ast} - w_t\right) + (1-\theta_w)\pi_t^w \\
&= \frac{1-\theta_w}{\theta_w}\left(w_t^{\ast} - w_t\right), \qquad (43)
\end{aligned}
$$

where we use the fact that $\pi_t^w = w_t - w_{t-1}$.

Combining equations (42) and (43) gives

$$
\begin{aligned}
\pi_t^w &= \frac{1-\theta_w}{\theta_w} \frac{1-\beta\theta_w}{1-\beta\theta_w\bar{m}} E_t \frac{\beta\theta_w\bar{m}}{1-\beta\theta_w\bar{m}F} \pi_{t+1}^w - \frac{1-\theta_w}{\theta_w} \frac{1-\beta\theta_w}{1+\epsilon_w\phi} E_t \frac{1}{1-\beta\theta_w\bar{m}F} \mu_t^w \\
&= \beta\bar{m}\left[\theta_w + \frac{1-\beta\theta_w}{1-\beta\theta_w\bar{m}}(1-\theta_w)\right] E_t \pi_{t+1}^w - \frac{(1-\theta_w)(1-\beta\theta_w)}{\theta_w(1+\epsilon_w\phi)} \mu_t^w. \qquad (44)
\end{aligned}
$$

Finally, plugging the equation for wage markups ($\mu_t^w = w_t - \left(\gamma + \frac{\phi}{1-\alpha}\right) x_t$) into equation (44) gives the wage Phillips curve

$$\begin{aligned}
\pi_t^w &= \beta \bar{m} \left[\theta_w + \frac{1-\beta\theta_w}{1-\beta\theta_w \bar{m}} (1-\theta_w)\right] E_t \pi_{t+1}^w - \frac{(1-\theta_w)(1-\beta\theta_w)}{\theta_w(1+\epsilon_w \phi)} \mu_t^w \\
&= \beta M^w E_t \pi_{t+1}^w + \frac{(1-\theta_w)(1-\beta\theta_w)}{\theta_w(1+\epsilon_w \phi)} \left(\gamma + \frac{\phi}{1-\alpha}\right) x_t - \frac{(1-\theta_w)(1-\beta\theta_w)}{\theta_w(1+\epsilon_w \phi)} w_t \\
&= \beta M^w E_t \pi_{t+1}^w + \kappa_w x_t - \lambda_w w_t, \qquad (45)
\end{aligned}$$

where $M^w = \bar{m}\left[\theta_w + \frac{1-\beta\theta_w}{1-\beta\theta_w \bar{m}}(1-\theta_w)\right]$ denotes discounting by workers, $\kappa_w = \lambda_w[\gamma + \frac{\phi}{1-\alpha}]$, and $\lambda_w = \frac{(1-\theta_w)(1-\beta\theta_w)}{\theta_w(1+\epsilon_w\phi)}$.

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