A Two-Stage Duopoly Game with Ethical Labeling and Price Competition when Consumers differ in Preferences

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Abstract

We study a *two-stage duopoly game*, where, at the first stage, firms choose if adopting or not a *social responsibility label.* The firm who adopts the social responsibility label (the ethical firm) has high marginal costs, while the firm who doesn't adopt it (the standard firm), supports low marginal costs. After the first stage, each firm knows the choice made by its rival and, at the second stage of the game, chooses *prices*. Consumers are divided into two groups: the group of consumers who prefers buying the good by the ethical firm and the group of consumers who prefers buying the good by the lowest price firm. Depending on the difference between the high and the low marginal cost and on the proportions of the two groups of consumers, the game has two *asymmetric* or two *symmetric Sub-game Perfect Nash Equilibria*. Symmetric Nash Equilibria imply that both firms makes the same choice at the first stage of the game (both decide to be ethical or standard), while asymmetric Nash Equilibria imply different choices at the first stage of the game: one of the two firms chooses to be ethical and the other standard. We analyzed the same model of Davies (2005) changing one of its assumption: the proportions of the two groups of consumers are not fixed a priori. With this new assumption, results of Davies (2005) are no more satisfied. In Davies (2005), ethical labeling cannot eliminate standard production when there are two firms and the marginal cost of ethical firm is higher than the marginal cost of standard firm: in equilibrium, one of the two firms always chooses to be standard at the first stage of the game. In our model (a duopoly where marginal cost of the ethical firm is higher than marginal cost of standard firm) instead it exists a condition on the model's parameters such that ethical labeling, in equilibrium, can eliminate standard production: if that particular condition is satisfied, it exists a symmetric subgame perfect Nash Equilibrium where both firms chooses, at the first stage of the game, to be ethical.

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1 INTRODUCTION

In the last decades, an increasing rate of consumers is modifying its purchase decisions: consumers are becoming more attentive to the environment, to the labour conditions of workers and, more generally, to any social aspect concerning the production of goods.

There are several local and international organizations which inform consumers about conditions of production used by firms, which organize boycotts and promote positive buying (the so-called "buycott") with respect of that goods which are certified with a particular *social responsibility label* (hereafter *SRL*). One of the most famous example of international SRL is the *International Fairtrade Certification Mark*, which is an independent certification mark used in over 50 countries. To obtain the Fairtrade Certification, producers have to respect several productive standards (International Fairtrade standards) set by FLO International (Fairtrade Labelling Organizations International). The Fairtrade standards "guarantee a minimum price considered as fair to producers. They provide a Fairtrade Premium that the producer must invest in projects enhancing its social, economic and environmental development. They strive for mutually beneficial long term trading relationships. They set clear minimum and developmental criteria and objectives for social, economic and environmental sustainability" (see http://www.fairtrade.net/generic_standards.html).

Considering the national labels, there are several local SRLs born in several countries: an italian example of SRL is *Valore Sociale* (see http://www.valoresociale.it), in England one of the most famous SRL is instead the *Ethical Consumer Best Buy Label* (launched by Ethical Consumer Magazine, see http://www.ethicalconsumer.org/magazine/best-buys-label.htm). In Belgium, the Belgian Parliament has approved a law that promotes social responsible production. To reach this objective, a label is established for products brought on the market that respect several basic rights (see http://www.epsu.org/a/128). A great part of SRLs are enviromental labels (the so-called *eco-labels*) which certifies if goods are produced with methods friendly to the environment; examples of eco-labels include the *European Union's Ecoflower*, the *German Blue Angel* and the *Nordic Swan*.

The recent growth of firms who are adopting SRLs is naturally due to an increasing demand of ethical products by a specific share of the consumers' population (the share of "ethical" consumers) which is concerned about the methods adopted by firms in the production of goods. Ethical consumers differ from "standard" ones since they prefer paying a price sightly higher than the "standard" one to acquire a product with a SRL (hereafter an *ethical good*): in particular, it has been shown that ethical consumers prefer paying a *price premium* for a product that is known to be produced by methods friendly to the environment and to any other social aspect. As an example, in *Biørner et al. (2004)*, authors found that "the Nordic Swan Label has had a significant effect on consumers' brand choices for toilet paper, corresponding to a marginal willingness to pay for the certified environmental label of 13-18% of the price…" (see Biørner et al., 2004).

Ethical consumption may be seen *as a source of product differentiation* generated by the preferences of ethical consumers: in some oligopolistic markets, firms adoptes a SRL as a strategic variable to differentiate their products from rivals' ones and gaining the ethical consumers' share of the market. In the Industrial Organization literature, some recent contributions introduce ethical consumption and social responsibility labeling into the traditional product differentiation models. These works may be divided into two different branches: the first one who considers *ethical consumption as a source of vertical product differentiation* (see, as an example, *Amacher et al., 2004* and *Uchida, 2007*), and the second one who considers *ethical consumption as a source of horizontal product differentiation* (see, as an example, *Becchetti and Solferino, 2003*; and *Conrad, 2005*). Both branches of literature analyze a duopoly game where firms has to choose if producing an "ethical" or a "standard" good and choose prices. The first branch of literature assumes that all the consumers are "ethical" in the sense that prefer buying the ethical good if ethical firm adopts a price lower or equal than the price of the standard one (*vertical product differentiation*); if instead the ethical firm adopts a higher price than the standard's one, some consumers buy the ethical good while others the standard one depending on the willingness to pay (hereafter *w.t.p.*) of each consumer, where w.t.p. is represented by a parameter uniformly distributed on a closed interval. *All the consumers have the same preferences represented by a unique utility function and it doesn't exists a group of consumers which is not concerned with the methods adopted in the production of goods*. The second branch of literature assumes instead that each consumer has a different level of ethical conduct which is uniformly distributed on a line $[0,1]$. To a higher level of ethical conduct corresponds a higher level of social responsibility desired by consumers: the less ethical consumer is positioned on point 0 of $[0,1]$, while the most ethical consumer is positioned at point 1. Consumers positioned on point 0 are the standard consumers while consumers positioned on point 1 represent the most ethical consumers. This is the typical case of *horizontal product differentiation* as in a traditional Hotelling model.

A work which is a combination of the two branches of literature is *Davies (2005)*. In Davies (2005), consumers' w.t.p. is uniformly distributed on a closed interval; consumers positioned at the lowest value of the w.t.p.'s interval prefer buying the lowest price good without caring about its social content, while consumers positioned on the remaining values of the w.t.p.'s interval constitute the group of the "potentially" ethical consumers: if the ethical firm practices a price which is lower or equal than the price of the standard firm, potentially ethical consumers buy the ethical good, if instead the ethical firm adopts a higher price than the standards one, it exists a share of potentially ethical consumers which buys the standard good and a share which buys the ethical good, depending on the w.t.p. of each potentially ethical consumer. Since consumers are uniformly distributed on the w.t.p.'s interval, only a very small quota of consumers belongs to the group of consumers who prefer buying the lowest price good (i.e. only that consumers located on the lowest value of the w.t.p.'s interval), while the potentially ethical consumers represent the great majority of the consumers' population: the two groups of consumers are then fixed in a given proportion.

We analyze the same model of Davies (2005) changing one of its assumption. As in Davies (2005), we assume consumers' population is split into two different exogenously given groups of consumers with different preferences: the group of *convinced standard consumers* and the group of *potentially ethical consumers*. Convinced standard consumers represent the group of consumers which is uncorcerned or simply uninformed about the methods adopted in the production of goods; they do not care if the purchased good is labeled with a SRL or not and, between products of the same physical characteristics, they prefer the lowest price good; potentially ethical consumers instead prefer paying a *price premium* for an ethical product, i.e. have a higher w.t.p. the ethical good than the convinced standards' ones. However, differently from Davies (2005), *we assume that the size of those two groups is not fixed a priori*; as an example, the group of the convinced standard consumers may represent the majority (and not the minority as in Davies, 2005) of the consumers' population.

In our paper we assume a *two-stage duopoly game* where, at the *first stage*, firms choose, simultaneously and independently, if adopting or not the SRL. The adoption of the SRL is connected with a high marginal cost of production, while, a low marginal cost of production is connected with the absence of the SRL. This is due to the fact that the SRL represents the respect of an ethical code of conduct which the firm decides to adopt in the production of goods: as an example, we may think to the adoption of an enviromental friendly production system or to the respect of a minimum wage for workers. The goods produced by the two firms have the same physical characteristics but may differ in its social content represented by the presence or not of the SRL (the goods produced by the two firms are different in its social content if, at the first stage of the game, firms make different choices). Consumers recognize the ethical firm (the firm who chooses to adopt the SRL) by a logo posted on the package of its products which is not present on standard goods. After having observed the choice made by its rival (SRL or not SRL, i.e. high or low marginal costs), each firm chooses, simultaneously and independently, prices. If, at the first stage, firms make the same choices, then all the consumers choose the lowest price good, if instead one of the two firms chooses to be ethical and the other standard, then each consumer behaves differently depending on the share of the population she (he) belongs and on the prices adopted by the two firms; in this last case, the pricing decisions of firms determine the shares of *standard* and *ethical consumers*, where the standard consumers are that consumers who decide to buy the standard good and the ethical consumers are that consumers who buy the ethical good. Since convinced standards and potentially ethical consumers' have different utility functions, we obtain a different market demand function for each group of consumers: a *demand function for standard consumers* and a *demand function for ethical consumers*. We analyze the existence of Subgame-Perfect Nash Equilibria and, assuming that the size of the convinced standard and potentially ethical consumers' groups is not fixed a priori, we show that results of Davies (2005) are no more satisfied. In Davies (2005), the author shows that, in equilibrium, ethical labeling cannot eliminate standard production when there are two firms and the marginal cost of the ethical firm is higher than the marginal cost of the standard firm: in equilibrium, one of the two firms always chooses to be standard at the first stage of the game. In our model (a duopoly where marginal cost of the ethical firm is higher than marginal cost of standard firm) instead it exists a condition on the model's parameters such that ethical labeling, in equilibrium, can eliminate standard production: if that particular condition is satisfied, it exists a subgame perfect Nash Equilibrium where both firms choose, at the first stage, to be ethical. The paper is structured as follows: in section two we present the Model, in section three the Equilibrium Results, in section four the Comparative Statics and in section five the Conclusions. References and the Appendix end the paper.

2 THE MODEL

2.1 The market

The market is a *duopoly*. The set of firms is $I=\{e,s\}$, where e is the *ethical firm* and s is the *standard firm*, with $i=e,s$ and $i\in I$. The *good* produced by the two firms is identical in its physical characteristics (goods are pefect substitutes) but may differ in its social content: the ethical firm is the firm who produces an ethical good (a good with the SRL), while the standard firm produces a standard good (a good without the SRL).

Firms compete in a *two stage game*, where, at the *first stage*, each firm i chooses, simultaneously and independently, if adopting or not the *SRL* (the firm who decide to adopt the SRL is firm e while the firm who doesn't adopt it is firm s).

Different marginal costs of production are related to different choices of firms at the first stage: the adoption of the SRL implies a high marginal cost of production $c_e=c>0$ while the absence of the SRL implies a low marginal cost of production, $c_s=0$. Then: $c_e>c_s$ and $c_e-c_s=c$. The cost function of a generic firm i is defined as

$$C_i\left(q_i\right)=c_iq_i=\begin{cases} cq_e & \text{if firm } i \text{ adopt the SRL } (i=e) \\ 0 & \text{if firm } i \text{ doesn't adopt the SRL } (i=s) \end{cases} \tag{2.1.1}$$

where q_i is the output sold by firm i.

After the first period, each firm knows the choice made by its rival, and, at the *second stage* of the game, each firm i chooses, simultaneously and independently, prices within the interval $p_i\in[c_i,\infty)$ $\forall i\in I$.

2.2 Consumers preferences and individual firms' demand functions

We assume that the consumers' population is split into two different groups: the group of *"convinced" standard consumers* $\overline{\theta}_{cs}\in(0,1)$ and the group of *"potentially" ethical consumers* $\overline{\theta}_{pe}=1-\overline{\theta}_{cs}\in(0,1)$. The group of convinced standard consumers prefer buying the product by the lowest price firm without caring about the social content of the purchased good; while the potentially ethical consumers prefer buying the *ethical good*.

The utility function of a generic *potentially ethical consumer* $j\in\overline{\theta}_{pe}$ is represented by the following expression:

$$u_j^{pe}=\begin{cases} \gamma^j\sigma-p_e & \text{if consumer } j \text{ buy an ethical good} \\ \sigma-p_s & \text{if consumer } j \text{ buy a standard good} \end{cases}$$

while the utility function of a generic *convinced standard consumer* $j \in \bar{\theta}_{cs}$ is given by:

$$u_j^{cs} = \begin{cases} \sigma - p_e & \text{if consumer } j \text{ buy an ethical good} \\ \sigma - p_s & \text{if consumer } j \text{ buy a standard good} \end{cases}.$$

γ^j is the willingness to pay of a generic consumer *j*: if $j \in \bar{\theta}_{pe}$, then $\gamma^j \in (1, \bar{\gamma}]$ which is the interval on which potentially ethical consumers' willingness to pay γ^j is uniformly distributed; if instead $j \in \bar{\theta}_{cs}$ then $\gamma^j = 1$; the w.t.p. an ethical good is higher for potentially ethical consumers than for convinced standards' ones since potentially ethical consumers give a higher value to the ethical goods than to the standard ones; p_s and p_e are respectively the prices adopted by the standard and the ethical firm. Each consumer buys one unit of the good and σ is large enough to assure that each consumer obtains a positive utility $u_j \geq 0 \;\; \forall j$ whatever is the price adopted by firms.

Moreover, we define the *ethical consumers*' group as the share $\theta_e \in \left[0, \bar{\theta}_{pe}\right]$ of the potentially ethical consumers who decides to buy the ethical good, while the *standard consumers* as the share $\theta_s \in [0,1]$ of the population who decides to acquire the good from the standard firm. Since each consumer buys one unit of the good, it will be $\theta_e = 1 - \theta_s$.

At the second stage of the game, the choice of the SRL is given and firms choose prices. Depending on the choices made by the two firms at the first stage of the game, there are the following two relevant cases to study:

a) One firm decides to adopt the SRL, while the other doesn't adopt it (i.e. both ethical and standard firm co-exists into the market): $c_e = c$ and $c_s = 0$.

b) Both firms decides to adopt the SRL (i.e. both firms are ethical): $c_i = c_e = c, \; \forall i \in I$; or both firms decides not to adopt the SRL (i.e. both firms are standard): $c_i = c_s = 0, \; \forall i \in I$.

Case (a)

If, at the first stage of the game, one of the two firms chooses to be ethical and the other standard, each consumer behaves differently depending on the group of the population she (he) belongs and on the prices adopted by the two firms.

If firm *e* practices a price lower than firm *s*, $p_e < p_s$, all the potentially ethical and convinced standard consumers become ethical (i.e. buy an ethical good): all the potentially ethical consumers buy the ethical good since $\gamma^j \sigma - p_e > \sigma - p_s$ for each *j* ($\gamma^j \sigma > \sigma$ and $p_e < p_s$), while all the convinced standard consumers become ethical since $\sigma - p_e > \sigma - p_s$; then: $\theta_e = 1$ and $\theta_s = 0$.

If firms *e* and *s* practice the same price $p_e = p_s = p$, all the potentially ethical consumers become ethical since $\gamma^j\sigma - p > \sigma - p \Leftrightarrow \gamma^j > 1$ which is true by assumption ($\gamma^j \in (1,\bar{\gamma}]$) while half of the group of convinced standard consumers becomes ethical and the remaining half standard, where this is due to the fact that all the convinced standard consumers are indifferent between the two firms: $\sigma - p_e = \sigma - p_s$. Then: $\theta_e = \bar{\theta}_{pe} + \bar{\theta}_{cs}/2 = 1 - \bar{\theta}_{cs} + \bar{\theta}_{cs}/2 = 1 - \bar{\theta}_{cs}/2$ and $\theta_s = \bar{\theta}_{cs}/2$.

If instead $p_e > p_s$, all the convinced standard consumers becomes standard since $\sigma - p_s > \sigma - p_e$, while the potentially ethical consumers are indifferent between the two firms if $\gamma^j\sigma - p_e = \sigma - p_s \Leftrightarrow \gamma^* = \dfrac{\sigma - p_s + p_e}{\sigma}$, with $\gamma^* > 1$ since $p_e > p_s$.

In particular, $\gamma^* \geq \bar{\gamma}$ and hence $\gamma^j \leq \gamma^*\ \forall j \in \bar{\theta}_{pe}$ if and only if $p_e \geq p_s + \sigma(\bar{\gamma} - 1)$: all the potentially ethical consumers becomes standard and $\theta_s = 1$ and $\theta_e = 0$. If instead $p_s < p_e < p_s + \sigma(\bar{\gamma} - 1)$, we have that $\gamma^* < \bar{\gamma}$ and the share of the potentially ethical consumers s.t. $1 < \gamma^j < \gamma^*$ buys the standard good while the share of potentially ethical consumers s.t. $\gamma^* \leq \gamma^j < \bar{\gamma}$ buys the ethical good. This means that, if $p_s < p_e < p_s + \sigma(\bar{\gamma} - 1)$, a share $1 - \dfrac{p_e - p_s}{\sigma(\bar{\gamma} - 1)}$ of the potentially ethical consumers becomes ethical and the remaining share $\dfrac{p_e - p_s}{\sigma(\bar{\gamma} - 1)}$ of the potentially ethical consumers becomes standard; then:

$$\theta_e = \bar{\theta}_{pe}\left(1 - \frac{p_e - p_s}{\sigma(\bar{\gamma} - 1)}\right) = \left(1 - \bar{\theta}_{cs}\right)\left(1 - \frac{p_e - p_s}{\sigma(\bar{\gamma} - 1)}\right) \text{ and}$$

$$\theta_s = \bar{\theta}_{cs} + \bar{\theta}_{pe}\frac{p_e - p_s}{\sigma(\bar{\gamma} - 1)} = \bar{\theta}_{cs} + \left(1 - \bar{\theta}_{cs}\right)\frac{p_e - p_s}{\sigma(\bar{\gamma} - 1)}.$$

We can then define the *ethical* and the *standard demand functions*:

$$q_e = \begin{cases} 1 & \text{if } p_e < p_s \\ 1 - \dfrac{\bar{\theta}_{cs}}{2} & \text{if } p_e = p_s \\ \left(1 - \bar{\theta}_{cs}\right)\left(1 - \dfrac{p_e - p_s}{\sigma(\bar{\gamma} - 1)}\right) & \text{if } p_s < p_e < p_s + \sigma(\bar{\gamma} - 1) \\ 0 & \text{if } p_e \geq p_s + \sigma(\bar{\gamma} - 1) \end{cases} \quad \text{and}$$

$$q_s = \begin{cases} 1 & \text{if } p_s \le p_e - \sigma(\bar{\gamma}-1) \\ \bar{\theta}_{cs} + (1-\bar{\theta}_{cs})\dfrac{p_e - p_s}{\sigma(\bar{\gamma}-1)} & \text{if } p_e - \sigma(\bar{\gamma}-1) < p_s < p_e \\ \dfrac{\bar{\theta}_{cs}}{2} & \text{if } p_s = p_e \\ 0 & \text{if } p_s > p_e \end{cases}.$$

The ethical and the standard profit functions are

$$\pi_e = \begin{cases} (p_e - c) & \text{if } c \le p_e < \max\{p_s, c\} \\ (p_e - c)\left(1 - \dfrac{\bar{\theta}_{cs}}{2}\right) & \text{if } p_e = \max\{p_s, c\} \\ (p_e - c)(1-\bar{\theta}_{cs})\left(1 - \dfrac{p_e - p_s}{\sigma(\bar{\gamma}-1)}\right) & \text{if } \max\{p_s, c\} < p_e < \max\{p_s + \sigma(\bar{\gamma}-1), c\} \\ 0 & \text{if } p_e \ge \max\{p_s + \sigma(\bar{\gamma}-1), c\} \end{cases} \text{ and}$$

$$\pi_s = \begin{cases} p_s & \text{if } 0 \le p_s \le \max\{0, p_e - \sigma(\bar{\gamma}-1)\} \\ p_s\left[\bar{\theta}_{cs} + (1-\bar{\theta}_{cs})\dfrac{p_e - p_s}{\sigma(\bar{\gamma}-1)}\right] & \text{if } \max\{0, p_e - \sigma(\bar{\gamma}-1)\} < p_s < p_e \\ p_s\dfrac{\bar{\theta}_{cs}}{2} & \text{if } p_s = p_e \\ 0 & \text{if } p_s > p_e \end{cases}.$$

Case (b)

If, at the first stage, firms make the same choices (i.e. both firms choose to be ethical or standard), then all the consumers (both the convinced standard and the potentially ethical consumers) choose the lowest price good, while, if firms adopts the same price, consumers are indifferent between the two firms and firms share equally the market: half of the consumers' population buys the good from one firm and the remaining half from its rival. Each firm obtains the same individual demand function and the same profit function.

3 EQUILIBRIUM RESULTS

3.1 Second Stage

At the second stage of the game, the choice of the SRL is given and firms choose prices. To find the equilibria of the second stage we study separately cases (a) and (b).

Case (a) – Stage 2

Let's analyze the profit functions of firm e and s to build the reaction functions of the two firms.

Firm e: analysis and reaction funtion

The profit function π_e is composed by four separate functions:

- $\pi_e = p_e - c$ if $c \le p_e < \max\{p_s, c\}$, which is represented by a segment increasing with respect of p_e if $\max\{p_s, c\} = p_s$, or by the point $(p_e, \pi_e) = (c, 0)$ if $\max\{p_s, c\} = c$.
- $\pi_e = (p_e - c)\left(1 - \frac{\bar{\theta}_{cs}}{2}\right)$ if $p_e = \max\{p_s, c\}$, which is represented by the point

$$(p_e, \pi_e) = \begin{cases} (c, 0) & \text{if } \max\{p_s, c\} = c \\ \left(p_s, (p_s - c)\left(1 - \frac{\bar{\theta}_{cs}}{2}\right)\right) & \text{if } \max\{p_s, c\} = p_s \end{cases}.$$

- $\pi_e = (p_e - c)(1 - \bar{\theta}_{cs})\left(1 - \frac{p_e - p_s}{\sigma(\bar{\gamma} - 1)}\right)$ if $\max\{p_s, c\} < p_e < \max\{p_s + \sigma(\bar{\gamma} - 1), c\}$.

If $p_s + \sigma(\bar{\gamma} - 1) > c$, π_e is represented by a section of a concave parabola intersecting the horizontal axis p_e at $p_e = c, p_s + \sigma(\bar{\gamma} - 1)$ and whose argmax is given by

$$p_e^{**} = \frac{1}{2}\left[p_s + c + \sigma(\bar{\gamma} - 1)\right];$$

If instead $p_s + \sigma(\bar{\gamma} - 1) \le c$, π_e is represented by the point $(p_e, \pi_e) = (c, 0)$.

- $\pi_e = 0$ if $p_e \ge \max\{p_s + \sigma(\bar{\gamma} - 1), c\}$, which is represented by an half line.

Case (a.1): firm e

If $c - \sigma(\bar{\gamma} - 1) \le 0$, then $\max\{p_s + \sigma(\bar{\gamma} - 1), c\} = p_s + \sigma(\bar{\gamma} - 1)$ (i.e. $p_s + \sigma(\bar{\gamma} - 1) > c \Leftrightarrow$ $p_s > c - \sigma(\bar{\gamma} - 1)$ which is always satisfied). The segment $p_e - c$ intersects the parabola at points

$$p_e = c \text{ and } p_e = p_s - \frac{\bar{\theta}_{cs}\sigma(\bar{\gamma}-1)}{(1-\bar{\theta}_{cs})} \text{ with } p_s - \frac{\bar{\theta}_{cs}\sigma(\bar{\gamma}-1)}{(1-\bar{\theta}_{cs})} < p_s .$$

The intersection $p_e = p_s - \frac{\bar{\theta}_{cs}\sigma(\bar{\gamma}-1)}{(1-\bar{\theta}_{cs})}$ lies on the increasing side of the parabola if

$$p_s < \tilde{p}_s = \frac{(1+\bar{\theta}_{cs})\sigma(\bar{\gamma}-1)+c(1-\bar{\theta}_{cs})}{1-\bar{\theta}_{cs}} > 0$$

while it lies on the maximum point of the parabola if $p_s = \tilde{p}_s$ and on its decreasing side if $p_s > \tilde{p}_s$. The price p_e, in correspondence of which the value asssumed by $p_e - c$ is equal to the maximum point of the parabola, is equal to

$$\tilde{p}_e = \frac{p_s^2(1-\bar{\theta}_{cs})+2p_s(1-\bar{\theta}_{cs})\left[\sigma(\bar{\gamma}-1)+c\right]-R}{4\sigma(\bar{\gamma}-1)}$$

with

$R = 2\sigma c(\bar{\gamma}-1)(1+\bar{\theta}_{cs})-(1-\bar{\theta}_{cs})\left[\sigma^2(\bar{\gamma}-1)^2+c^2\right]$, where $\tilde{p}_e$ is a quadratic function of p_s, represented by a convex parabola.

In figure 3.1.1, we represent the four components of π_e when $\max\{p_s, c\} = p_s$ and $p_s < \tilde{p}_s$.

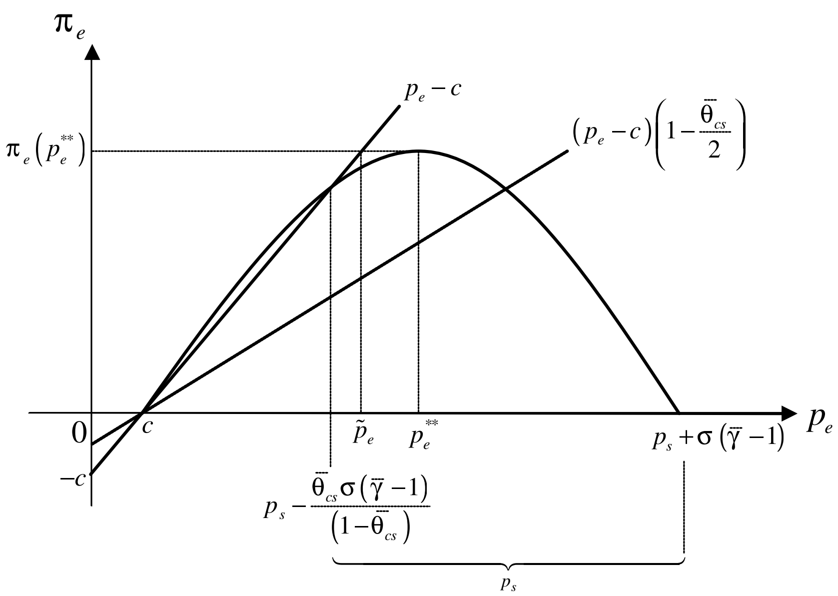

Figure 3.1.1

If $p_s \geq \tilde{p}_s$, profit function π_e has only a superior given by $p_s - c$ and not a maximum point; if instead $p_s < \tilde{p}_s$, π_e may have a maximum point or not depending on the difference $\tilde{p}_e - p_s$: if $\tilde{p}_e - p_s < 0$ (i.e. $p_s > \tilde{p}_e$) then π_e has only a superior given by $p_s - c$ and not a maximum point, while if $\tilde{p}_e - p_s \geq 0$ (i.e. $p_s \leq \tilde{p}_e$) π_e has a maximum point $\pi_e\left(p_e^{**}\right)$ and the argmax is given by p_e^{**}. The difference $\tilde{p}_e - p_s$ is represented by a convex parabola which intersects the horizontal axis p_s at points $p_s = \tilde{p}_s \pm \frac{2\sigma\left(\bar{\gamma}-1\right)\sqrt{\bar{\theta}_{cs}}}{1-\bar{\theta}_{cs}}$, with

$$\tilde{p}_s + \frac{2\sigma\left(\bar{\gamma}-1\right)\sqrt{\bar{\theta}_{cs}}}{1-\bar{\theta}_{cs}} > \tilde{p}_s - \frac{2\sigma\left(\bar{\gamma}-1\right)\sqrt{\bar{\theta}_{cs}}}{1-\bar{\theta}_{cs}},$$

$$\tilde{p}_s - \frac{2\sigma\left(\bar{\gamma}-1\right)\sqrt{\bar{\theta}_{cs}}}{1-\bar{\theta}_{cs}} > 0 \text{ (by assumptions)}$$

and

$$0 < c < \tilde{p}_s - \frac{2\sigma\left(\bar{\gamma}-1\right)\sqrt{\bar{\theta}_{cs}}}{1-\bar{\theta}_{cs}} \text{ (by assumptions).}$$

Then, in order to build the reaction function of firm *e*, there are two relevant cases to study:

i. $0 \leq p_s \leq \tilde{p}_s - \frac{2\sigma\left(\bar{\gamma}-1\right)\sqrt{\bar{\theta}_{cs}}}{1-\bar{\theta}_{cs}}$; and

ii. $p_s > \tilde{p}_s - \frac{2\sigma\left(\bar{\gamma}-1\right)\sqrt{\bar{\theta}_{cs}}}{1-\bar{\theta}_{cs}}$.

i. If $0 \leq p_s \leq \tilde{p}_s - \frac{2\sigma\left(\bar{\gamma}-1\right)\sqrt{\bar{\theta}_{cs}}}{1-\bar{\theta}_{cs}}$, π_e has a maximum point $\pi_e\left(p_e^{**}\right)$ and the argmax is given by $p_e = p_e^{**}$ since $\pi_e\left(p_e^{**}\right) \geq p_s - c$.

ii. If $p_s > \tilde{p}_s - \frac{2\sigma\left(\bar{\gamma}-1\right)\sqrt{\bar{\theta}_{cs}}}{1-\bar{\theta}_{cs}}$, π_e has a superior given by $p_s - c$ since $p_s - c > \pi_e\left(p_e^{**}\right)$.

Case (a.2): firm e

If $c - \sigma\left(\bar{\gamma}-1\right) > 0$, then it can be $p_s + \sigma\left(\bar{\gamma}-1\right) > c$ or $p_s + \sigma\left(\bar{\gamma}-1\right) \leq c$. If $p_s > c - \sigma\left(\bar{\gamma}-1\right) > 0$, then the analysis of π_e is identical to case a.1:

- if $c-\sigma(\bar{\gamma}-1)<p_s\le\tilde{p}_s-\dfrac{2\sigma(\bar{\gamma}-1)\sqrt{\bar{\theta}_{cs}}}{1-\bar{\theta}_{cs}}$, π_e has a maximum point $\pi_e\left(p_e^{**}\right)$ and the argmax is given by $p_e=p_e^{**}$;
- if $p_s>\tilde{p}_s-\dfrac{2\sigma(\bar{\gamma}-1)\sqrt{\bar{\theta}_{cs}}}{1-\bar{\theta}_{cs}}$, π_e has a superior and not a mximum point;

if instead $0\le p_s\le c-\sigma(\bar{\gamma}-1)$, then π_e is represented only by the half line $\pi_e=0$ for $p_e\in[c,\infty)$. In this case, the profit function π_e is constant and in correspondence of each value $p_e\in[c,\infty)$, π_e reaches its maximum value (zero). For each value of p_s such that $0\le p_s\le c-\sigma(\bar{\gamma}-1)$, the argmax of π_e is represented by the interval $[c,\infty)$.

The *reaction function* of firm *e* is then represented by the following expressions.

If $c-\sigma(\bar{\gamma}-1)\le 0$:

$$p_e(p_s)=p_e^{**}\qquad \text{if } 0\le p_s\le\tilde{p}_s-\frac{2\sigma(\bar{\gamma}-1)\sqrt{\bar{\theta}_{cs}}}{1-\bar{\theta}_{cs}}.$$

If $c-\sigma(\bar{\gamma}-1)>0$:

$$p_e(p_s)=\begin{cases}[c,\infty) & \text{if } 0\le p_s\le c-\sigma(\bar{\gamma}-1)\\ p_e^{**} & \text{if } c-\sigma(\bar{\gamma}-1)<p_s\le\tilde{p}_s-\dfrac{2\sigma(\bar{\gamma}-1)\sqrt{\bar{\theta}_{cs}}}{1-\bar{\theta}_{cs}}\end{cases}\cdot$$

Firm s: analysis and reaction funtion

The profit function π_s is composed by four separate functions:

- p_s if $0\le p_s\le\max\{0,p_e-\sigma(\bar{\gamma}-1)\}$, which is represented by a segment increasing with respect of p_s if $\max\{0,p_e-\sigma(\bar{\gamma}-1)\}=p_e-\sigma(\bar{\gamma}-1)$, or by the point $(p_s,\pi_s)=(0,0)$ if $\max\{0,p_e-\sigma(\bar{\gamma}-1)\}=0$.

- $p_s\left[\bar{\theta}_{cs}+\left(1-\bar{\theta}_{cs}\right)\frac{p_e-p_s}{\sigma\left(\bar{\gamma}-1\right)}\right]$ if $\max\left\{0,p_e-\sigma\left(\bar{\gamma}-1\right)\right\}<p_s<p_e$, which is represented by a section of a concave parabola intersecting the horizontal axis p_s at $p_s=0, p_e+\frac{\bar{\theta}_{cs}\sigma\left(\bar{\gamma}-1\right)}{1-\bar{\theta}_{cs}}$ and whose argmax is given by

$$p_s^{**}=\frac{1}{2}\left[p_e+\frac{\bar{\theta}_{cs}\sigma\left(\bar{\gamma}-1\right)}{1-\bar{\theta}_{cs}}\right].$$

- $p_s\frac{\bar{\theta}_{cs}}{2}$ if $p_s=p_e$, which is represented by the point $\left(p_s,\pi_s\right)=\left(p_e,p_e\frac{\bar{\theta}_{cs}}{2}\right)$.

- $\pi_s=0$ if $p_s>p_e$, which is represented by an half line.

Case (a.1): firm s

If $p_e>\sigma\left(\bar{\gamma}-1\right)$ (i.e. $\max\left\{0,p_e-\sigma\left(\bar{\gamma}-1\right)\right\}=p_e-\sigma\left(\bar{\gamma}-1\right)$), the segment p_s intersects the parabola at points $p_s=0$ and $p_s=p_e-\sigma\left(\bar{\gamma}-1\right)$. The intersection $p_s=p_e-\sigma\left(\bar{\gamma}-1\right)$ lies on the increasing side of the parabola if

$$p_e-\sigma\left(\bar{\gamma}-1\right)<p_s^{**}\Leftrightarrow p_e<p_s^{**}+\sigma\left(\bar{\gamma}-1\right)=\frac{1}{2}\left[p_e+\frac{\bar{\theta}_{cs}\sigma\left(\bar{\gamma}-1\right)}{1-\bar{\theta}_{cs}}\right]+\sigma\left(\bar{\gamma}-1\right)\Leftrightarrow$$

$$\Leftrightarrow p_e<\frac{\left(2-\bar{\theta}_{cs}\right)\sigma\left(\bar{\gamma}-1\right)}{1-\bar{\theta}_{cs}}.$$

while it lies on the maximum point of the parabola if

$$p_e-\sigma\left(\bar{\gamma}-1\right)=p_s^{**}\Leftrightarrow p_e=\frac{\left(2-\bar{\theta}_{cs}\right)\sigma\left(\bar{\gamma}-1\right)}{1-\bar{\theta}_{cs}}$$

and on its decreasing side if

$$p_e-\sigma\left(\bar{\gamma}-1\right)>p_s^{**}\Leftrightarrow p_e>\frac{\left(2-\bar{\theta}_{cs}\right)\sigma\left(\bar{\gamma}-1\right)}{1-\bar{\theta}_{cs}}.$$

In correspondence of $p_s=p_e$, the value assumed by π_s is always lower than the value assumed by the parabola:

$$\pi_s\left(p_s=p_e\right)=p_s\frac{\bar{\theta}_{cs}}{2}<p_s\left[\bar{\theta}_{cs}+\left(1-\bar{\theta}_{cs}\right)\frac{p_e-p_s}{\sigma\left(\bar{\gamma}-1\right)}\right]\Leftrightarrow p_e\frac{\bar{\theta}_{cs}}{2}<p_e\bar{\theta}_{cs}$$

In figure 3.1.2, we represent the four components of π_s when $p_e > \sigma(\bar{\gamma}-1)$ and $p_e < \dfrac{(2-\bar{\theta}_{cs})\sigma(\bar{\gamma}-1)}{1-\bar{\theta}_{cs}}$.

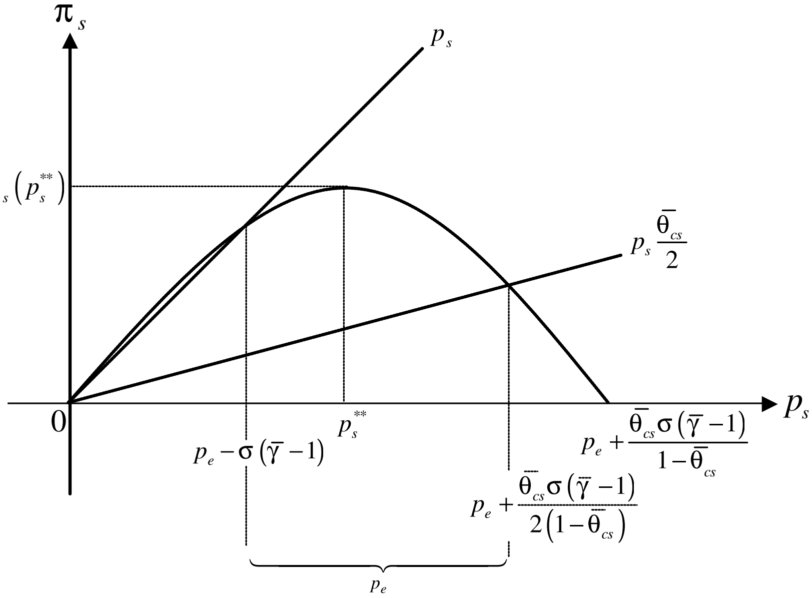

Figure 3.1.2

If $p_e < \dfrac{(2-\bar{\theta}_{cs})\sigma(\bar{\gamma}-1)}{1-\bar{\theta}_{cs}}$ (as in figure 3.1.2), then:

- if $p_e \le p_s^{**}$, profit function π_s has only a superior given by the value assumed by the parabola in $p_s = p_e$, $p_e\bar{\theta}_{cs}$;
- if instead $p_e > p_s^{**}$, then π_s has a maximum point given by the maximum of the parabola $\pi_s(p_s^{**})$ and the argmax is given by p_s^{**}.

If instead $p_e \ge \dfrac{(2-\bar{\theta}_{cs})\sigma(\bar{\gamma}-1)}{1-\bar{\theta}_{cs}}$, π_s has a maximum point given by $p_s = p_e - \sigma(\bar{\gamma}-1)$ because the segment p_s intersects the parabola on its decreasing side (or on its maximum point).

Now, since

$$p_e > p_s^{**} \Leftrightarrow p_e > \frac{\bar{\theta}_{cs}\sigma(\bar{\gamma}-1)}{1-\bar{\theta}_{cs}}$$

and

$$\frac{\bar{\theta}_{cs}\sigma(\bar{\gamma}-1)}{1-\bar{\theta}_{cs}} < \frac{(2-\bar{\theta}_{cs})\sigma(\bar{\gamma}-1)}{1-\bar{\theta}_{cs}},$$

to build the reaction function of firm s, there are three relevant cases to study:

i. $p_e \geq \max\left\{\frac{(2-\bar{\theta}_{cs})\sigma(\bar{\gamma}-1)}{1-\bar{\theta}_{cs}}, c\right\}$;

ii. $\max\left\{\sigma(\bar{\gamma}-1), \frac{\bar{\theta}_{cs}\sigma(\bar{\gamma}-1)}{1-\bar{\theta}_{cs}}, c\right\} < p_e < \max\left\{\frac{(2-\bar{\theta}_{cs})\sigma(\bar{\gamma}-1)}{1-\bar{\theta}_{cs}}, c\right\}$; and

iii. $\max\{\sigma(\bar{\gamma}-1), c\} < p_e \leq \max\left\{\sigma(\bar{\gamma}-1), \frac{\bar{\theta}_{cs}\sigma(\bar{\gamma}-1)}{1-\bar{\theta}_{cs}}, c\right\}$.

i. If $p_e \geq \max\left\{\frac{(2-\bar{\theta}_{cs})\sigma(\bar{\gamma}-1)}{1-\bar{\theta}_{cs}}, c\right\}$, π_s has a maximum point given by $p_s = p_e - \sigma(\bar{\gamma}-1)$.

ii. If $\max\left\{\sigma(\bar{\gamma}-1), \frac{\bar{\theta}_{cs}\sigma(\bar{\gamma}-1)}{1-\bar{\theta}_{cs}}, c\right\} < p_e < \max\left\{\frac{(2-\bar{\theta}_{cs})\sigma(\bar{\gamma}-1)}{1-\bar{\theta}_{cs}}, c\right\}$, we are in the case in which $p_e < \frac{(2-\bar{\theta}_{cs})\sigma(\bar{\gamma}-1)}{1-\bar{\theta}_{cs}}$ and $p_e > p_s^{**}$, i.e. the profit function π_s has a maximum point given by the maximum of the parabola $\pi_s(p_s^{**})$ and the argmax is given by p_s^{**}.

iii. If $\max\{\sigma(\bar{\gamma}-1), c\} < p_e \leq \max\left\{\sigma(\bar{\gamma}-1), \frac{\bar{\theta}_{cs}\sigma(\bar{\gamma}-1)}{1-\bar{\theta}_{cs}}, c\right\}$, we are in the case in which $p_e < \frac{(2-\bar{\theta}_{cs})\sigma(\bar{\gamma}-1)}{1-\bar{\theta}_{cs}}$ and $p_e \leq p_s^{**}$, i.e. the profit function π_s has only a superior given by the value assumed by the parabola in $p_s = p_e$, $p_e\bar{\theta}_{cs}$.

Case (a.2): firm s

If $p_e \leq \sigma(\bar{\gamma}-1)$ (i.e. $\max\{0, p_e - \sigma(\bar{\gamma}-1)\} = 0$), π_s is represented only by a section of the parabola, the point $(p_s, \pi_s) = \left(p_e, p_e\frac{\bar{\theta}_{cs}}{2}\right)$ and the half line $\pi_s = 0$. Then in order to build the reaction function of firm s, there are two relevant cases to study:

i. $\max\left\{\min\left\{\sigma(\bar{\gamma}-1), \frac{\bar{\theta}_{cs}\sigma(\bar{\gamma}-1)}{1-\bar{\theta}_{cs}}\right\}, c\right\} < p_e \leq \max\{\sigma(\bar{\gamma}-1), c\}$;

ii. $c \le p_e \le \max\left\{\min\left\{\sigma(\bar{\gamma}-1), \frac{\bar{\theta}_{cs}\sigma(\bar{\gamma}-1)}{1-\bar{\theta}_{cs}}\right\}, c\right\}$.

i. If $\max\left\{\min\left\{\sigma(\bar{\gamma}-1), \frac{\bar{\theta}_{cs}\sigma(\bar{\gamma}-1)}{1-\bar{\theta}_{cs}}\right\}, c\right\} < p_e \le \max\{\sigma(\bar{\gamma}-1), c\}$, we are in the case in which $p_e > p_s^{**}$, i.e. the profit function π_s has a maximum point given by the maximum of the parabola $\pi_s(p_s^{**})$ and the argmax is given by p_s^{**}.

ii. If $c \le p_e \le \max\left\{\min\left\{\sigma(\bar{\gamma}-1), \frac{\bar{\theta}_{cs}\sigma(\bar{\gamma}-1)}{1-\bar{\theta}_{cs}}\right\}, c\right\}$, we are in the case in which $p_e \le p_s^{**}$, i.e. the profit function π_s has only a superior given by the value assumed by the parabola in $p_s = p_e$, $p_e\bar{\theta}_{cs}$.

The *reaction function* of firm *s* is then represented by the following expression

$$p_s(p_e) = \begin{cases} p_s^{**} & \text{if } \max\left\{\sigma(\bar{\gamma}-1), \frac{\bar{\theta}_{cs}\sigma(\bar{\gamma}-1)}{1-\bar{\theta}_{cs}}, c\right\} < p_e < \max\left\{\frac{(2-\bar{\theta}_{cs})\sigma(\bar{\gamma}-1)}{1-\bar{\theta}_{cs}}, c\right\} \\ p_e - \sigma(\bar{\gamma}-1) & \text{if } p_e \ge \max\left\{\frac{(2-\bar{\theta}_{cs})\sigma(\bar{\gamma}-1)}{1-\bar{\theta}_{cs}}, c\right\} \end{cases}.$$

Then, we can conclude that:

<u>Lemma 1</u>*: Equilibrium Results – Second Stage – case (a)*. In case (a), at the second stage of the game:

$\rightarrow$ if cost c is higher than $(2-\bar{\theta}_{cs})\sigma(\bar{\gamma}-1)/1-\bar{\theta}_{cs}$ (and for each $\bar{\theta}_{cs} \in (0,1)$), it exists a unique pure strategy Nash equilibrium where firm *s* practices $p_s^* = c - \sigma(\bar{\gamma}-1)$ and firm *e* $p_e^* = c$; equilibrium profits are $\pi_e^* = 0$ and $\pi_s^* = c - \sigma(\bar{\gamma}-1) > 0$ (firm *s* gains the entire market).

$\rightarrow$ if cost *c* is lower than $(2-\bar{\theta}_{cs})\sigma(\bar{\gamma}-1)/1-\bar{\theta}_{cs}$, the equilibrium results depend on both the size of the convinced standard consumers' group and *c*. In particular

- If $0 < \bar{\theta}_{cs} < 7-3\sqrt{5}/2$ (with a small number of convinced standard consumers) or if $7-3\sqrt{5}/2 \le \bar{\theta}_{cs} < 1$ and

$$\frac{\sigma\left(\bar{\gamma}-1\right)\left(3\sqrt{\bar{\theta}_{cs}}-1-\bar{\theta}_{cs}\right)}{1-\bar{\theta}_{cs}} \le c < \frac{\sigma\left(\bar{\gamma}-1\right)\left(2-\bar{\theta}_{cs}\right)}{1-\bar{\theta}_{cs}}$$

it exists a unique pure strategy Nash Equilibrium where firm *e* and *s* shares the market, practice

$$\left(p_e^*, p_s^*\right) = \left(\frac{2c\left(1-\bar{\theta}_{cs}\right)+\sigma\left(\bar{\gamma}-1\right)\left(2-\bar{\theta}_{cs}\right)}{3\left(1-\bar{\theta}_{cs}\right)}, \frac{c\left(1-\bar{\theta}_{cs}\right)+\sigma\left(\bar{\gamma}-1\right)\left(1+\bar{\theta}_{cs}\right)}{3\left(1-\bar{\theta}_{cs}\right)}\right)$$

and gain

$$\left(\pi_e^*, \pi_s^*\right) = \left(\frac{\left[c\left(1-\bar{\theta}_{cs}\right)+\sigma\left(\bar{\gamma}-1\right)\left(\bar{\theta}_{cs}-2\right)\right]^2}{9\left(1-\bar{\theta}_{cs}\right)\sigma\left(\bar{\gamma}-1\right)}, \frac{\left[c\left(1-\bar{\theta}_{cs}\right)+\sigma\left(\bar{\gamma}-1\right)\left(1+\bar{\theta}_{cs}\right)\right]^2}{9\left(1-\bar{\theta}_{cs}\right)\sigma\left(\bar{\gamma}-1\right)}\right).$$

- If instead $7-3\sqrt{5}/2 \le \bar{\theta}_{cs} < 1$ and $0 < c < \frac{\sigma\left(\bar{\gamma}-1\right)\left(3\sqrt{\bar{\theta}_{cs}}-1-\bar{\theta}_{cs}\right)}{1-\bar{\theta}_{cs}}$, it doesn't exists any equilibrium in pure strategy.

Proof.
See the Appendix.

The interpretation of results is the following: when adopting the SRL increase "too much" the cost of production of ethical firm, all the potentially ethical consumers prefer buying the standard good, profits of ethical firm are equal to zero for each price adopted by the standard firm and the standard firm behaves as a monopolist on the entire market, if adopting the SRL does not increase "too much" the cost of production, the equilibrium results depend also on the number of convinced standard consumers:

- if the share of the convinced standard consumers is lower than a given small positive value, then it exists a unique equilibrium where both firm adopts a positive price, a group of the potentially ethical consumers becomes standard and the remaining ethical, firms share the market and both earns a positive profit;
- if instead the share of the convinced standard consumers is higher than the small positive value, the previous equilibrium exists if and only if the cost of SRL lies into an interval where the upper bound is given by that value in correspondence of which adopting the SRL increase "too much" the cost of production of ethical firm, while the lower bound is a positive value; if cost of SRL is lower than the lower bound, the price at which standard firm is able to maximize its profits is such that firm *e* is unable to maximize its profits.

Case (b) – Stage 2

In case (b), firms' competition is exactly the same as in a traditional Bertrand Duopoly with symmetric marginal costs (the second stage of the game is, in fact, identical to the traditional Bertrand Duopoly Game). Then, we can conclude that:

<u>Lemma 2</u>*: Equilibrium Results – Second Stage – case (b).* In case (b), at the second stage of the game, it exists a unique pure strategy Nash Equilibrium:

$\rightarrow$ If both firms choose to adopt the SRL, each firm i adopts an equilibrium price $p_i^* = c$, $\forall i \in I$.

$\rightarrow$ If both firms choose not to adopt the SRL, each firm i adopts an equilibrium price $p_i^* = 0$, $\forall i \in I$.

$\rightarrow$ In both cases, each firm i obtains an equilibrium profit $\pi_i^* = 0$, $\forall i \in I$.

<u>Proof</u>.

See Tirole (1988).

3.2 First Stage and Sub-Game Perfect Equilibria

In the first Stage of the game, each firm i chooses, simultaneously and independently, if adopting or not the SRL (i.e. the marginal costs of production): $c_i = \{0, c\}$.

Since we are looking for a subgame perfect Nash Equilibrium, we have to analyze the following three cases:

$\rightarrow$ $c \geq \dfrac{(2-\bar{\theta}_{cs})\sigma(\bar{\gamma}-1)}{1-\bar{\theta}_{cs}}$ and $\forall \bar{\theta}_{cs} \in (0,1)$;

$\rightarrow$ $0 < \bar{\theta}_{cs} < \dfrac{7-3\sqrt{5}}{2}$ and $c < \dfrac{(2-\bar{\theta}_{cs})\sigma(\bar{\gamma}-1)}{1-\bar{\theta}_{cs}}$, or $\dfrac{7-3\sqrt{5}}{2} \leq \bar{\theta}_{cs} < 1$ and $\dfrac{\sigma(\bar{\gamma}-1)\left(3\sqrt{\bar{\theta}_{cs}}-1-\bar{\theta}_{cs}\right)}{1-\bar{\theta}_{cs}} \leq c < \dfrac{\sigma(\bar{\gamma}-1)(2-\bar{\theta}_{cs})}{1-\bar{\theta}_{cs}}$; and

$\rightarrow$ $\dfrac{7-3\sqrt{5}}{2} \leq \bar{\theta}_{cs} < 1$ and $0 < c < \dfrac{\sigma(\bar{\gamma}-1)\left(3\sqrt{\bar{\theta}_{cs}}-1-\bar{\theta}_{cs}\right)}{1-\bar{\theta}_{cs}}$.

If $c \geq \frac{\left(2-\bar{\theta}_{cs}\right)\sigma\left(\bar{\gamma}-1\right)}{1-\bar{\theta}_{cs}}$ and $\forall \bar{\theta}_{cs} \in (0,1)$, we have that the choice at the first stage is represented by the following 2x2 matrix (figure 3.2.1) where each firm (firm 1 and 2) can choose if adopting or not the SRL. The payoffs are represented by the equilibrium profits obtainable by the two firms at the second stage of the game.

		Firm 2	
		srl (e)	*not srl (s)*
Firm 1	*srl (e)*	0 , 0	0 , >0
	not srl (s)	>0 , 0	0 , 0

Figure 3.2.1

Observing the matrix, we can conclude that there are two asymmetric Subgame Perfect Nash Equilibria:

- $(1,2)=(s,e)$, $\left(c_1^*,c_2^*\right)=(0,c)$, $\left(p_1^*,p_2^*\right)=\left(c-\sigma\left(\bar{\gamma}-1\right),c\right)$ with $\left(\pi_1^*,\pi_2^*\right)=\left(c-\sigma\left(\bar{\gamma}-1\right),0\right)$; and
- $(1,2)=(e,s)$, $\left(c_1^*,c_2^*\right)=(c,0)$, $\left(p_1^*,p_2^*\right)=\left(c,c-\sigma\left(\bar{\gamma}-1\right)\right)$ with $\left(\pi_1^*,\pi_2^*\right)=\left(0,c-\sigma\left(\bar{\gamma}-1\right)\right)$.

If $0<\bar{\theta}_{cs}<\frac{7-3\sqrt{5}}{2}$ and $c<\frac{\left(2-\bar{\theta}_{cs}\right)\sigma\left(\bar{\gamma}-1\right)}{1-\bar{\theta}_{cs}}$, or $\frac{7-3\sqrt{5}}{2}\leq\bar{\theta}_{cs}<1$ and $\frac{\sigma\left(\bar{\gamma}-1\right)\left(3\sqrt{\bar{\theta}_{cs}}-1-\bar{\theta}_{cs}\right)}{1-\bar{\theta}_{cs}}\leq c<\frac{\sigma\left(\bar{\gamma}-1\right)\left(2-\bar{\theta}_{cs}\right)}{1-\bar{\theta}_{cs}}$, the choice at the first stage is represented by the following 2x2 matrix (figure 3.2.2)

		Firm 2	
		srl (e)	*not srl (s)*
Firm 1	*srl (e)*	0 , 0	>0 , >0
	not srl (s)	>0 , >0	0 , 0

Figure 3.2.2

As in the previous case, there are two asymmetric Subgame Perfect Nash Equilibria where:

- $(1,2)=(s,e)$, $(c_1^*,c_2^*)=(0,c)$, with

$$\left(p_1^*,p_2^*\right)=\left(\frac{c\left(1-\bar{\theta}_{cs}\right)+\sigma\left(\bar{\gamma}-1\right)\left(1+\bar{\theta}_{cs}\right)}{3\left(1-\bar{\theta}_{cs}\right)},\frac{2c\left(1-\bar{\theta}_{cs}\right)+\sigma\left(\bar{\gamma}-1\right)\left(2-\bar{\theta}_{cs}\right)}{3\left(1-\bar{\theta}_{cs}\right)}\right) \text{ and}$$

$$\left(\pi_1^*,\pi_2^*\right)=\left(\frac{\left[c\left(1-\bar{\theta}_{cs}\right)+\sigma\left(\bar{\gamma}-1\right)\left(1+\bar{\theta}_{cs}\right)\right]^2}{9\left(1-\bar{\theta}_{cs}\right)\sigma\left(\bar{\gamma}-1\right)},\frac{\left[c\left(1-\bar{\theta}_{cs}\right)+\sigma\left(\bar{\gamma}-1\right)\left(\bar{\theta}_{cs}-2\right)\right]^2}{9\left(1-\bar{\theta}_{cs}\right)\sigma\left(\bar{\gamma}-1\right)}\right);$$

- $(1,2)=(e,s)$, $(c_1^*,c_2^*)=(c,0)$ with

$$\left(p_1^*,p_2^*\right)=\left(\frac{2c\left(1-\bar{\theta}_{cs}\right)+\sigma\left(\bar{\gamma}-1\right)\left(2-\bar{\theta}_{cs}\right)}{3\left(1-\bar{\theta}_{cs}\right)},\frac{c\left(1-\bar{\theta}_{cs}\right)+\sigma\left(\bar{\gamma}-1\right)\left(1+\bar{\theta}_{cs}\right)}{3\left(1-\bar{\theta}_{cs}\right)}\right) \text{ and}$$

$$\left(\pi_1^*,\pi_2^*\right)=\left(\frac{\left[c\left(1-\bar{\theta}_{cs}\right)+\sigma\left(\bar{\gamma}-1\right)\left(\bar{\theta}_{cs}-2\right)\right]^2}{9\left(1-\bar{\theta}_{cs}\right)\sigma\left(\bar{\gamma}-1\right)},\frac{\left[c\left(1-\bar{\theta}_{cs}\right)+\sigma\left(\bar{\gamma}-1\right)\left(1+\bar{\theta}_{cs}\right)\right]^2}{9\left(1-\bar{\theta}_{cs}\right)\sigma\left(\bar{\gamma}-1\right)}\right).$$

If, finally, $\frac{7-3\sqrt{5}}{2}\leq\bar{\theta}_{cs}<1$ and $0<c<\frac{\sigma\left(\bar{\gamma}-1\right)\left(3\sqrt{\bar{\theta}_{cs}}-1-\bar{\theta}_{cs}\right)}{1-\bar{\theta}_{cs}}$, firms are indifferent between choosing both to be or not to be ethical: in both cases, firms obtain zero equilibrium profits; in fact, no equilibrium in the second stage exists if firms make different choices on the SRL. Then, there are two Sub-Game Perfect Nash Equilibria given by:

- $(1,2)=(s,s)$, $(c_1^*,c_2^*)=(0,0)$, $(p_1^*,p_2^*)=(0,0)$ with $(\pi_1^*,\pi_2^*)=(0,0)$; and
- $(1,2)=(e,e)$, $(c_1^*,c_2^*)=(c,c)$, $(p_1^*,p_2^*)=(c,c)$ with $(\pi_1^*,\pi_2^*)=(0,0)$.

By the previous analysis we can conclude that

Proposition 1: Equilibrium Results. If cost c is higher than $\left(2-\bar{\theta}_{cs}\right)\sigma\left(\bar{\gamma}-1\right)/1-\bar{\theta}_{cs}$ (and for each $\bar{\theta}_{cs}\in(0,1)$), the two-stage game has *two asymmetric subgame perfect Nash Equilibria* where one of the two firms chooses to be ethical and the other standard; if cost c is lower than $\left(2-\bar{\theta}_{cs}\right)\sigma\left(\bar{\gamma}-1\right)/1-\bar{\theta}_{cs}$, equilibrium results depend on both the size of the convinced standard consumers' group and c:

→ with a very small number of convinced standard consumers ($0 < \bar{\theta}_{cs} < 7 - 3\sqrt{5}/2$), the two-stage game has *two asymmetric subgame perfect Nash Equilibria* where one of the two firms chooses to be ethical and the other standard;

→ if the number of convinced standard consumers is $7 - 3\sqrt{5}/2 \le \bar{\theta}_{cs} < 1$, the two-stage game has two *asymmetric subgame perfect Nash Equilibria* (where one of the two firms chooses to be ethical and the other standard) if and only if cost c belongs to the interval

$$\frac{\sigma(\bar{\gamma}-1)\left(3\sqrt{\bar{\theta}_{cs}}-1-\bar{\theta}_{cs}\right)}{1-\bar{\theta}_{cs}} \le c < \frac{\sigma(\bar{\gamma}-1)\left(2-\bar{\theta}_{cs}\right)}{1-\bar{\theta}_{cs}};$$

if instead cost c is

$$0 < c < \frac{\sigma(\bar{\gamma}-1)\left(3\sqrt{\bar{\theta}_{cs}}-1-\bar{\theta}_{cs}\right)}{1-\bar{\theta}_{cs}}$$

the two-stage game has two *symmetric subgame perfect Nash Equilibria* where both firms chooses to be ethical or standard.

<u>*Proof*</u>.
See above.

In figure 3.2.3, we represent the areas in which the game has two asymmetric Sub-Game Perfect Nash Equilibria and two symmetric Sub-Game Perfect Nash Equilibria.

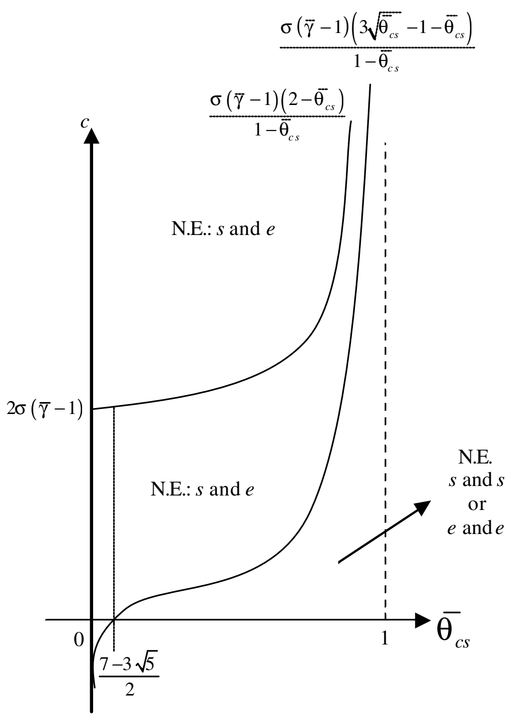

Figure 3.2.3

Assuming that the proportions of convinced standard and potentially ethical consumers' groups are not fixed a priori (as in Davies, 2005), equilibrium results differ from Davies (2005)'s ones: if

$$\frac{7-3\sqrt{5}}{2} \leq \bar{\theta}_{cs} < 1 \text{ and } 0 < c < \frac{\sigma\left(\bar{\gamma}-1\right)\left(3\sqrt{\bar{\theta}_{cs}}-1-\bar{\theta}_{cs}\right)}{1-\bar{\theta}_{cs}}$$

both firms may choose, in equilibrium, to be ethical. If this condition on parameters is satisfied, ethical labeling may be seen as a method to obtain a market in which both firms choose to be ethical. In particular we can show that

Proposition 2: Sufficient Conditions to eliminate standard production. If $7-3\sqrt{5}/2 \leq \bar{\theta}_{cs} < 1$ and $0 < c < \sigma\left(\bar{\gamma}-1\right)\left(3\sqrt{\bar{\theta}_{cs}} - 1-\bar{\theta}_{cs}\right)/1-\bar{\theta}_{cs}$, an institution can eliminate standard production transfering any strictly positive sum of money to the firms who choose to adopt the SRL at the first stage of the game.

Proof.

When $7-3\sqrt{5}/2 \leq \bar{\theta}_{cs} < 1$ and $0 < c < \sigma\left(\bar{\gamma}-1\right)\left(3\sqrt{\bar{\theta}_{cs}} - 1-\bar{\theta}_{cs}\right)/1-\bar{\theta}_{cs}$, the game has two symmetric subgame perfect Nash equilibria. This means that, at the first stage of the game, each firm makes the same choice: both adopt the SRL or both doesn't do it. To obtain a coordination on the adoption of the SRL (i.e. to eliminate standard production), it is sufficient that an institution transfers even one euro to the firms who chooses to adopt the SRL at the first stage of the game. Choosing to adopt or not the SRL, firms obtain zero equilibrium profits. Giving any strictly positive sum of money to the firms who chooses to adopt the SRL, both firms finds it convenient to adopt the SRL at the first stage since

$$\begin{array}{c} \pi_1^*(s) = 0 < 1 = \pi_1^*(e) \\ \pi_2^*(s) = 0 < 1 = \pi_2^*(e) \end{array}.$$

Q.E.D.

4 COMPARATIVE STATICS

In this section we analyze the differences between the equilibrium results in the three relevant cases:

1) $c \geq \dfrac{\left(2-\bar{\theta}_{cs}\right)\sigma\left(\bar{\gamma}-1\right)}{1-\bar{\theta}_{cs}}$ and each $\bar{\theta}_{cs} \in (0,1)$;

2) $0 < \bar{\theta}_{cs} < \dfrac{7-3\sqrt{5}}{2}$ and $c < \dfrac{\left(2-\bar{\theta}_{cs}\right)\sigma\left(\bar{\gamma}-1\right)}{1-\bar{\theta}_{cs}}$ or $\dfrac{7-3\sqrt{5}}{2} \leq \bar{\theta}_{cs} < 1$ and $\dfrac{\sigma\left(\bar{\gamma}-1\right)\left(3\sqrt{\bar{\theta}_{cs}}-1-\bar{\theta}_{cs}\right)}{1-\bar{\theta}_{cs}} \leq c < \dfrac{\sigma\left(\bar{\gamma}-1\right)\left(2-\bar{\theta}_{cs}\right)}{1-\bar{\theta}_{cs}}$;

3) $\dfrac{7-3\sqrt{5}}{2} \leq \bar{\theta}_{cs} < 1$ and $0 < c < \dfrac{\sigma\left(\bar{\gamma}-1\right)\left(3\sqrt{\bar{\theta}_{cs}}-1-\bar{\theta}_{cs}\right)}{1-\bar{\theta}_{cs}}$.

CASE 1

If cost c is higher than $\left(2-\bar{\theta}_{cs}\right)\sigma\left(\bar{\gamma}-1\right)\big/1-\bar{\theta}_{cs}$ (and for each $\bar{\theta}_{cs} \in (0,1)$), the two-stage game has two asymmetric subgame perfect Nash Equilibria where one of the two firms chooses to be ethical and the other standard, equilibrium prices are

$$p_e^* = c > 0$$
$$p_s^* = c - \sigma\left(\bar{\gamma}-1\right) > 0$$

with

$$p_e^* > p_s^*,$$

equilibrium quantities sold by the two firms are

$$\begin{matrix} q_e^* = 0 \\ q_s^* = 1 \end{matrix} \text{ and}$$

equilibrium profits are

$$\pi_e^* = 0$$
$$\pi_s^* = c - \sigma\left(\bar{\gamma}-1\right) > 0$$

with

$$\pi_e^* < \pi_s^*.$$

At equilibrium, each consumer becomes standard (i.e. buy a standard good) and obtains an utility

$$u_j = \sigma - p_s = \sigma - c + \sigma(\bar{\gamma} - 1) = \sigma\bar{\gamma} - c$$

which is decreasing in c and increasing in σ and $\bar{\gamma}$.

To sum up we have that

Proposition 3. If cost c is higher than $(2-\bar{\theta}_{cs})\sigma(\bar{\gamma}-1)/1-\bar{\theta}_{cs}$ (and for each $\bar{\theta}_{cs} \in (0,1)$), at equilibrium, standard firm obtains the entire market, behaves as a monopolist and obtains a positive profit, while ethical firm obtains zero demand, zero equilibrium profit and practices a price equal to its marginal costs. All the consumers become standard and obtains an utility $u_j(s) = \sigma\bar{\gamma} - c$, which is decreasing in c and increasing in σ and $\bar{\gamma}$.

Proof.
See above.

CASE 2

If $0 < \bar{\theta}_{cs} < 7 - 3\sqrt{5}/2$ and $c < \dfrac{\sigma(\bar{\gamma}-1)(2-\bar{\theta}_{cs})}{1-\bar{\theta}_{cs}}$ or $7 - 3\sqrt{5}/2 \le \bar{\theta}_{cs} < 1$ and $\dfrac{\sigma(\bar{\gamma}-1)\left(3\sqrt{\bar{\theta}_{cs}} - 1 - \bar{\theta}_{cs}\right)}{1-\bar{\theta}_{cs}} \le c < \dfrac{\sigma(\bar{\gamma}-1)(2-\bar{\theta}_{cs})}{1-\bar{\theta}_{cs}}$ the two-stage game has two asymmetric subgame perfect Nash Equilibria where one of the two firms chooses to be ethical and the other standard, equilibrium prices are

$$p_e^* = \frac{2c(1-\bar{\theta}_{cs}) + \sigma(\bar{\gamma}-1)(2-\bar{\theta}_{cs})}{3(1-\bar{\theta}_{cs})}$$

$$p_s^* = \frac{c(1-\bar{\theta}_{cs}) + \sigma(\bar{\gamma}-1)(1+\bar{\theta}_{cs})}{3(1-\bar{\theta}_{cs})}$$

with

$$p_e^* > p_s^* \Leftrightarrow c > \frac{\sigma(\bar{\gamma}-1)(2\bar{\theta}_{cs}-1)}{1-\bar{\theta}_{cs}}.$$

Then, if $\bar{\theta}_{cs} \le \dfrac{1}{2}$

$$c > \frac{\sigma(\bar{\gamma}-1)(2\bar{\theta}_{cs}-1)}{1-\bar{\theta}_{cs}} \text{ is always satisfied since } \frac{\sigma(\bar{\gamma}-1)(2\bar{\theta}_{cs}-1)}{1-\bar{\theta}_{cs}} \leq 0,$$

if instead $\bar{\theta}_{cs} > \frac{1}{2}$

$$\frac{\sigma(\bar{\gamma}-1)(2\bar{\theta}_{cs}-1)}{1-\bar{\theta}_{cs}} > 0$$

and

$$\frac{\sigma(\bar{\gamma}-1)(2\bar{\theta}_{cs}-1)}{1-\bar{\theta}_{cs}} < \frac{\sigma(\bar{\gamma}-1)\left(3\sqrt{\bar{\theta}_{cs}}-1-\bar{\theta}_{cs}\right)}{1-\bar{\theta}_{cs}}$$

then

$$c > \frac{\sigma(\bar{\gamma}-1)(2\bar{\theta}_{cs}-1)}{1-\bar{\theta}_{cs}}$$

is always satisfied in the interval

$$\frac{\sigma(\bar{\gamma}-1)\left(3\sqrt{\bar{\theta}_{cs}}-1-\bar{\theta}_{cs}\right)}{1-\bar{\theta}_{cs}} \leq c < \frac{\sigma(\bar{\gamma}-1)(2-\bar{\theta}_{cs})}{1-\bar{\theta}_{cs}}$$

Equilibrium quantities sold by the two firms are

$$q_e^* = \frac{\sigma(\bar{\gamma}-1)(2-\bar{\theta}_{cs}) - c(1-\bar{\theta}_{cs})}{3\sigma(\bar{\gamma}-1)}$$

$$q_s^* = \frac{\sigma(\bar{\gamma}-1)(1+\bar{\theta}_{cs}) - c(1-\bar{\theta}_{cs})}{3\sigma(\bar{\gamma}-1)}$$

with

$$q_e^* > (\leq)\, q_s^* \Leftrightarrow \bar{\theta}_{cs} < (\geq) \frac{1}{2}.$$

Equilibrium profits are equal to

$$\pi_e^* = \frac{\left[c(1-\bar{\theta}_{cs}) + \sigma(\bar{\gamma}-1)(\bar{\theta}_{cs}-2)\right]^2}{9(1-\bar{\theta}_{cs})\sigma(\bar{\gamma}-1)} > 0$$

$$\pi_s^* = \frac{\left[c(1-\bar{\theta}_{cs}) + \sigma(\bar{\gamma}-1)(1+\bar{\theta}_{cs})\right]^2}{9(1-\bar{\theta}_{cs})\sigma(\bar{\gamma}-1)} > 0$$

with

$$\pi_e^* > \pi_s^* \Leftrightarrow c < \frac{\sigma(\bar{\gamma}-1)(1-2\bar{\theta}_{cs})}{2(1-\bar{\theta}_{cs})}.$$

In particular

$$\frac{\sigma(\bar{\gamma}-1)(1-2\bar{\theta}_{cs})}{2(1-\bar{\theta}_{cs})} < \frac{\sigma(\bar{\gamma}-1)(2-\bar{\theta}_{cs})}{1-\bar{\theta}_{cs}}$$

and

$$\frac{\sigma(\bar{\gamma}-1)(1-2\bar{\theta}_{cs})}{2(1-\bar{\theta}_{cs})} > \frac{\sigma(\bar{\gamma}-1)\left(3\sqrt{\bar{\theta}_{cs}}-1-\bar{\theta}_{cs}\right)}{1-\bar{\theta}_{cs}} \Leftrightarrow$$

$$\Leftrightarrow \bar{\theta}_{cs} < \frac{1}{4}.$$

Then, if $0 < \bar{\theta}_{cs} < 7 - 3\sqrt{5}/2$,

$$\pi_e^* > (\leq)\pi_s^* \Leftrightarrow c < (\geq)\frac{\sigma(\bar{\gamma}-1)(1-2\bar{\theta}_{cs})}{2(1-\bar{\theta}_{cs})}.$$

If instead $7 - 3\sqrt{5}/2 \leq \bar{\theta}_{cs} < 1/4$,

$$\pi_e^* > \pi_s^* \Leftrightarrow \frac{\sigma(\bar{\gamma}-1)\left(3\sqrt{\bar{\theta}_{cs}}-1-\bar{\theta}_{cs}\right)}{1-\bar{\theta}_{cs}} \leq c < \frac{\sigma(\bar{\gamma}-1)(1-2\bar{\theta}_{cs})}{2(1-\bar{\theta}_{cs})}$$

and

$$\pi_e^* \leq \pi_s^* \Leftrightarrow \frac{\sigma(\bar{\gamma}-1)(1-2\bar{\theta}_{cs})}{2(1-\bar{\theta}_{cs})} \leq c < \frac{\sigma(\bar{\gamma}-1)(2-\bar{\theta}_{cs})}{(1-\bar{\theta}_{cs})}.$$

If, finally, $1/4 \leq \bar{\theta}_{cs} < 1$,

$$\pi_e^* \leq \pi_s^*$$

because

$$c \geq \frac{\sigma(\bar{\gamma}-1)(1-2\bar{\theta}_{cs})}{2(1-\bar{\theta}_{cs})}$$

is always satisfied.

Finally, at equilibrium, all the convinced standard consumers become standard and gain an utility

$$u_j(s) = \sigma - \frac{c(1-\bar{\theta}_{cs})+\sigma(\bar{\gamma}-1)(1+\bar{\theta}_{cs})}{3(1-\bar{\theta}_{cs})},$$

all the potentially ethical consumers s.t.

$$1 < \gamma^j < \frac{(1-\bar{\theta}_{cs})(3\sigma+c)+\sigma(\bar{\gamma}-1)(1-2\bar{\theta}_{cs})}{3\sigma(1-\bar{\theta}_{cs})}$$

become standard and obtain an utility of

$$u_j(s) = \sigma - \frac{c(1-\bar{\theta}_{cs}) + \sigma(\bar{\gamma}-1)(1+\bar{\theta}_{cs})}{3(1-\bar{\theta}_{cs})} \text{ and}$$

all the potentially ethical consumers s.t.

$$\frac{(1-\bar{\theta}_{cs})(3\sigma + c) + \sigma(\bar{\gamma}-1)(1-2\bar{\theta}_{cs})}{3\sigma(1-\bar{\theta}_{cs})} < \gamma^j < \bar{\gamma}$$

become ethical (i.e. buy an ethical good) and gain an utility of

$$u_j(e) = \gamma^j \sigma - \frac{2c(1-\bar{\theta}_{cs}) + \sigma(\bar{\gamma}-1)(2-\bar{\theta}_{cs})}{3(1-\bar{\theta}_{cs})}.$$

$u_j(s)$ is decreasing in c and increasing in $\bar{\gamma}$ and σ, while $u_j(e)$ is decreasing in c and increasing in γ^j, $\bar{\gamma}$ and σ. Depending on the values of $\bar{\gamma}$ and c, u_j is increasing or decreasing in $\bar{\theta}_{cs}$.

To sum up we have that

<u>*Proposition 4*</u>. If $0 < \bar{\theta}_{cs} < 7 - 3\sqrt{5}/2$ and $c < \frac{\sigma(\bar{\gamma}-1)(2-\bar{\theta}_{cs})}{1-\bar{\theta}_{cs}}$ or $7 - 3\sqrt{5}/2 \le \bar{\theta}_{cs} < 1$ and $\frac{\sigma(\bar{\gamma}-1)\left(3\sqrt{\bar{\theta}_{cs}} - 1 - \bar{\theta}_{cs}\right)}{1-\bar{\theta}_{cs}} \le c < \frac{\sigma(\bar{\gamma}-1)(2-\bar{\theta}_{cs})}{1-\bar{\theta}_{cs}}$, at equilibrium, standard and ethical firms sell respectively a quantity q_s^* and q_e^*, where $q_e^* > (\le) q_s^* \Leftrightarrow \bar{\theta}_{cs} < (\ge) \frac{1}{2}$, practice a price p_s^* and p_e^*, with $p_s^* < p_e^*$, and obtain a positive profit π_e^* and π_s^*, where

$$\pi_e^* > (\le) \pi_s^* \Leftrightarrow c < (\ge) \frac{\sigma(\bar{\gamma}-1)(1-2\bar{\theta}_{cs})}{2(1-\bar{\theta}_{cs})} \text{ if } 0 < \bar{\theta}_{cs} \le 1/4$$

and

$$\pi_e^* \le \pi_s^* \text{ if } 1/4 \le \bar{\theta}_{cs} < 1.$$

A share of the consumers become standard and the remaining ethical, standard consumers obtain an utility $u_j(s) = \sigma - \frac{c(1-\bar{\theta}_{cs}) + \sigma(\bar{\gamma}-1)(1+\bar{\theta}_{cs})}{3(1-\bar{\theta}_{cs})}$, while ethical consumers obtain $u_j(e) = \gamma^j \sigma - \frac{2c(1-\bar{\theta}_{cs}) + \sigma(\bar{\gamma}-1)(2-\bar{\theta}_{cs})}{3(1-\bar{\theta}_{cs})}$. $u_j(s)$ is decreasing in c and increasing in $\bar{\gamma}$ and σ, while $u_j(e)$ is decreasing in c and increasing in γ^j, $\bar{\gamma}$ and σ. Depending on the values of $\bar{\gamma}$ and c, u_j is increasing or decreasing in $\bar{\theta}_{cs}$.

<u>*Proof*</u>.

See above.

CASE 3

If $7-3\sqrt{5}/2 \le \bar{\theta}_{cs} < 1$ and $0 < c < \dfrac{\sigma(\bar{\gamma}-1)\left(3\sqrt{\bar{\theta}_{cs}}-1-\bar{\theta}_{cs}\right)}{1-\bar{\theta}_{cs}}$ the two-stage game has two symmetric subgame perfect Nash Equilibria where both firms choose to be ethical or standard, equilibrium prices are

$$p_1^* = c$$
$$p_2^* = c$$

if both firms choose to be ethical and

$$p_1^* = 0$$
$$p_2^* = 0$$

if both firms choose to be standard; in both cases

$$p_1^* = p_2^*.$$

Equilibrium quantities sold by the two firms are

$$q_1^* = \frac{1}{2}$$
$$q_2^* = \frac{1}{2}$$

with

$$q_1^* = q_2^*.$$

Equilibrium profits are equal to

$$\pi_1^* = 0$$
$$\pi_2^* = 0$$

with

$$\pi_1^* = \pi_2^*.$$

Finally, at equilibrium, all the consumers becomes standard and gains an utility

$$u_j(s) = \sigma$$

if both firms chooses to be standard, while all the consumers becomes ethical if both firms chooses to be ethical and obtains an utility

$$u_j(e) = \sigma - c \quad \text{if } j \in \bar{\theta}_{cs}$$
$$u_j(e) = \gamma^j \sigma - c \quad \text{if } j \in \bar{\theta}_{pe}$$

where potentially ethical consumers obtain a higher utility than convinced standard consumers

$$\gamma^j \sigma - c > \sigma - c.$$

$u_j(s)$ is increasing in σ, while $u_j(e)$ is decreasing in c and increasing in σ if $j \in \bar{\theta}_{cs}$ and $u_j(e)$ is decreasing in c and increasing in σ and γ^j if $j \in \bar{\theta}_{pe}$.

Moreover

- convinced standard consumers earn a higher utility if both firms chooses to be standard:

$$u_j(s) = \sigma > \sigma - c = u_j(e); \text{ and}$$

- potentially ethical consumers earn a higher utility if both firms chooses to be ethical

$$u_j(s) = \sigma < \gamma^j \sigma - c = u_j(e) \Leftrightarrow \gamma^j > \frac{c+\sigma}{c} \text{ while}$$

$$u_j(s) = \sigma \geq \gamma^j \sigma - c = u_j(e) \Leftrightarrow \gamma^j \leq \frac{c+\sigma}{c}.$$

To sum up we have that

<u>*Proposition 5*</u>. If $7 - 3\sqrt{5}/2 \leq \bar{\theta}_{cs} < 1$ and $0 < c < \dfrac{\sigma(\bar{\gamma}-1)\left(3\sqrt{\bar{\theta}_{cs}} - 1 - \bar{\theta}_{cs}\right)}{1-\bar{\theta}_{cs}}$, at equilibrium, firms shares equally the market, $q_1^* = q_2^* = \dfrac{1}{2}$, practice the same price $p_1^* = p_2^* = p^*$, with $p^* = 0$ if both firms chooses to be standard and $p^* = c$ if both chooses to be ethical. Both firms obtains zero equilibrium profits $\pi_1^* = \pi_2^* = 0$. All the consumers becomes standard and gains an utility $u_j(s) = \sigma$ if both firms chooses to be standard, while all the consumers become ethical if both firms chooses to be ethical and obtains an utility $u_j(e) = \sigma - c$ if $j \in \bar{\theta}_{cs}$ and $u_j(e) = \gamma^j \sigma - c$ if $j \in \bar{\theta}_{pe}$. $u_j(s)$ is increasing in σ, while $u_j(e)$ is decreasing in c and increasing

in σ if $j \in \bar{\theta}_{cs}$ and $u_j(e)$ is decreasing in c and increasing in σ and γ^j if $j \in \bar{\theta}_{pe}$. Convinced standard consumers obtain a higher utility when both firms choose to be standard while potentially ethical consumers may obtain a higher utility when both firms chooses to be ethical: if $\gamma^j > c + \sigma/c$ then potentially ethical consumers gains more with two ethical firms, while if $\gamma^j \leq c + \sigma/c$ then potentially ethical consumers earn more with two standard firms.

Proof.

See above.

We conclude this section analyzing which of the three cases is the best for firms in terms of equilibrium profits and the best for consumers in terms of utilities.

The best case for firms

Firm e

Case *2* is the best case for firm *e* and, in particular,

$$\pi_e^*(case\ 2) > \pi_e^*(case\ 1) = \pi_e^*(case\ 3) = 0.$$

Firm s

The worst case for firm s is case *3*

$$\pi_s^*(case\ 3) < \pi_s^*(case\ 1)$$
$$\pi_s^*(case\ 3) < \pi_s^*(case\ 2)$$

while firm *s* obtains the highest equilibrium profits in case *1* if and only if

$$\pi_s^*(case\ 1) - \pi_s^*(case\ 2) \geq 0 \Leftrightarrow$$

$$\Leftrightarrow \frac{\sigma(\bar{\gamma}-1)\left(13-3\sqrt{13}-2\bar{\theta}_{cs}\right)}{2\left(1-\bar{\theta}_{cs}\right)} \leq c \leq \frac{\sigma(\bar{\gamma}-1)\left(13+3\sqrt{13}-2\bar{\theta}_{cs}\right)}{2\left(1-\bar{\theta}_{cs}\right)}$$

while

$$\pi_s^*(case\ 1) - \pi_s^*(case\ 2) < 0 \Leftrightarrow$$

$$\Leftrightarrow c > \frac{\sigma(\bar{\gamma}-1)\left(13+3\sqrt{13}-2\bar{\theta}_{cs}\right)}{2\left(1-\bar{\theta}_{cs}\right)} \text{ and } c < \frac{\sigma(\bar{\gamma}-1)\left(13-3\sqrt{13}-2\bar{\theta}_{cs}\right)}{2\left(1-\bar{\theta}_{cs}\right)}$$

Now since

$$\frac{\sigma\left(\bar{\gamma}-1\right)\left(13-3\sqrt{13}-2\bar{\theta}_{cs}\right)}{2\left(1-\bar{\theta}_{cs}\right)}<\frac{\sigma\left(\bar{\gamma}-1\right)\left(2-\bar{\theta}_{cs}\right)}{1-\bar{\theta}_{cs}}<\frac{\sigma\left(\bar{\gamma}-1\right)\left(13+3\sqrt{13}-2\bar{\theta}_{cs}\right)}{2\left(1-\bar{\theta}_{cs}\right)}$$

and

$$\frac{\sigma\left(\bar{\gamma}-1\right)\left(13-3\sqrt{13}-2\bar{\theta}_{cs}\right)}{2\left(1-\bar{\theta}_{cs}\right)}\geq\frac{\sigma\left(\bar{\gamma}-1\right)\left(3\sqrt{\bar{\theta}_{cs}}-1-\bar{\theta}_{cs}\right)}{1-\bar{\theta}_{cs}}\Leftrightarrow$$

$$\Leftrightarrow\bar{\theta}_{cs}\leq\left(\frac{15-3\sqrt{13}}{6}\right)^{2}\cong 0.48,$$

we have that:

- if we are in case *1*, equilibrium profit of firm s in case *1* is the highest profit of the three cases - $\pi_s^*\left(case\ 1\right)>\pi_s^*\left(case\ 2\right)$ - if and only if

$$\frac{\sigma\left(\bar{\gamma}-1\right)\left(2-\bar{\theta}_{cs}\right)}{1-\bar{\theta}_{cs}}\leq c<\frac{\sigma\left(\bar{\gamma}-1\right)\left(13+3\sqrt{13}-2\bar{\theta}_{cs}\right)}{2\left(1-\bar{\theta}_{cs}\right)}$$

while if

$$c\geq\frac{\sigma\left(\bar{\gamma}-1\right)\left(13+3\sqrt{13}-2\bar{\theta}_{cs}\right)}{2\left(1-\bar{\theta}_{cs}\right)}$$

we have that

$$\pi_s^*\left(case\ 1\right)\leq\pi_s^*\left(case\ 2\right);$$

- if we are in case *2*, we have that

→ if $0<\bar{\theta}_{cs}<7-3\sqrt{5}/2$, firm s obtains the highest equilibrium profit in case *2* - $\pi_s^*\left(case\ 2\right)>\pi_s^*\left(case\ 1\right)$ - if and only if

$$0<c<\frac{\sigma\left(\bar{\gamma}-1\right)\left(13-3\sqrt{13}-2\bar{\theta}_{cs}\right)}{2\left(1-\bar{\theta}_{cs}\right)}$$

while $\pi_s^*\left(case\ 2\right)\leq\pi_s^*\left(case\ 1\right)$ if

$$\frac{\sigma\left(\bar{\gamma}-1\right)\left(13-3\sqrt{13}-2\bar{\theta}_{cs}\right)}{2\left(1-\bar{\theta}_{cs}\right)}\leq c<\frac{\sigma\left(\bar{\gamma}-1\right)\left(2-\bar{\theta}_{cs}\right)}{1-\bar{\theta}_{cs}};$$

→ if $7-3\sqrt{5}/2\leq\bar{\theta}_{cs}\leq\left(\frac{15-3\sqrt{13}}{6}\right)^{2}$, firm s obtains the highest equilibrium profit in case *2* - $\pi_s^*\left(case\ 2\right)>\pi_s^*\left(case\ 1\right)$ - if and only if

$$\frac{\sigma\left(\overline{\gamma}-1\right)\left(3\sqrt{\overline{\theta}_{cs}}-1-\overline{\theta}_{cs}\right)}{1-\overline{\theta}_{cs}} \leq c < \frac{\sigma\left(\overline{\gamma}-1\right)\left(13-3\sqrt{13}-2\overline{\theta}_{cs}\right)}{2\left(1-\overline{\theta}_{cs}\right)}$$

while $\pi_s^*\left(case\ 2\right) \leq \pi_s^*\left(case\ 1\right)$ if

$$\frac{\sigma\left(\overline{\gamma}-1\right)\left(13-3\sqrt{13}-2\overline{\theta}_{cs}\right)}{2\left(1-\overline{\theta}_{cs}\right)} \leq c < \frac{\sigma\left(\overline{\gamma}-1\right)\left(2-\overline{\theta}_{cs}\right)}{1-\overline{\theta}_{cs}};$$

$\rightarrow$ if $\left(\frac{15-3\sqrt{13}}{6}\right)^2 < \overline{\theta}_{cs} < 1$, firm *s*, in case *2*, obtains always an equilibrium profit lower than the equilibrium profit of case *1* - $\pi_s^*\left(case\ 2\right) < \pi_s^*\left(case\ 1\right)$, since

$$\frac{\sigma\left(\overline{\gamma}-1\right)\left(13-3\sqrt{13}-2\overline{\theta}_{cs}\right)}{2\left(1-\overline{\theta}_{cs}\right)} < \frac{\sigma\left(\overline{\gamma}-1\right)\left(3\sqrt{\overline{\theta}_{cs}}-1-\overline{\theta}_{cs}\right)}{1-\overline{\theta}_{cs}}.$$

The best case for consumers

CASE 1

Case 1 with respect of case 3

In case *1*, each consumer obtains an utility which is always greater than the utility obtained by consumers in case *3* if both firms choose to be ethical:

$$u_j\left(case\ 1\right) = \overline{\gamma}\sigma - c > \gamma^j\sigma - c > \sigma - c;$$

if instead, in case *3*, firms choose to be standard, in case *1*, consumers obtain a lower utility than the utility obtained by consumers in case *3*, since in case *1* we have $c \geq \sigma\left(\overline{\gamma}-1\right)$

$$u_j\left(case\ 1\right) = \overline{\gamma}\sigma - c > \sigma \Leftrightarrow c < \sigma\left(\overline{\gamma}-1\right).$$

Case 1 with respect of case 2

In case *1*, each consumer obtains a lower utility than the utility obtained by *ethical* consumers in case *2* because

$$u_j\left(case\ 1\right) = \overline{\gamma}\sigma - c < \gamma^j\sigma - \frac{2c\left(1-\overline{\theta}_{cs}\right)+\sigma\left(\overline{\gamma}-1\right)\left(2-\overline{\theta}_{cs}\right)}{3\left(1-\overline{\theta}_{cs}\right)} = u_j\left(e, case\ 2\right) \Leftrightarrow$$

$$\bar{\gamma}\sigma-\gamma^{j}\sigma<c-\frac{2c\left(1-\bar{\theta}_{cs}\right)+\sigma\left(\bar{\gamma}-1\right)\left(2-\bar{\theta}_{cs}\right)}{3\left(1-\bar{\theta}_{cs}\right)}\Leftrightarrow$$

$$\Leftrightarrow\bar{\gamma}\sigma-\gamma^{j}\sigma<c-\frac{2c\left(1-\bar{\theta}_{cs}\right)+\sigma\left(\bar{\gamma}-1\right)\left(2-\bar{\theta}_{cs}\right)}{3\left(1-\bar{\theta}_{cs}\right)}\Leftrightarrow$$

$$\Leftrightarrow\gamma^{j}>\frac{\left(1-\bar{\theta}_{cs}\right)\left(3\sigma\bar{\gamma}-c\right)+\sigma\left(\bar{\gamma}-1\right)\left(2-\bar{\theta}_{cs}\right)}{3\sigma\left(1-\bar{\theta}_{cs}\right)}$$

is always verified: in fact γ^{j} is such that

$$\frac{\left(1-\bar{\theta}_{cs}\right)\left(3\sigma+c\right)+\sigma\left(\bar{\gamma}-1\right)\left(1-2\bar{\theta}_{cs}\right)}{3\sigma\left(1-\bar{\theta}_{cs}\right)}<\gamma^{j}<\bar{\gamma}$$

and

$$\frac{\left(1-\bar{\theta}_{cs}\right)\left(3\sigma\bar{\gamma}-c\right)+\sigma\left(\bar{\gamma}-1\right)\left(2-\bar{\theta}_{cs}\right)}{3\sigma\left(1-\bar{\theta}_{cs}\right)}\leq\frac{\left(1-\bar{\theta}_{cs}\right)\left(3\sigma+c\right)+\sigma\left(\bar{\gamma}-1\right)\left(1-2\bar{\theta}_{cs}\right)}{3\sigma\left(1-\bar{\theta}_{cs}\right)}\Leftrightarrow$$

$$\Leftrightarrow c\geq\frac{\sigma\left(\bar{\gamma}-1\right)\left(2-\bar{\theta}_{cs}\right)}{1-\bar{\theta}_{cs}}$$

then

$$\gamma^{j}>\frac{\left(1-\bar{\theta}_{cs}\right)\left(3\sigma\bar{\gamma}-c\right)+\sigma\left(\bar{\gamma}-1\right)\left(2-\bar{\theta}_{cs}\right)}{3\sigma\left(1-\bar{\theta}_{cs}\right)}$$

is always satisfied.

Finally in case *1*, each consumer obtains a lower or equal utility than the utility obtained by *standard* consumers in case *2* because

$$u_{j}\left(case\ 1\right)=\bar{\gamma}\sigma-c\leq\sigma-\frac{c\left(1-\bar{\theta}_{cs}\right)+\sigma\left(\bar{\gamma}-1\right)\left(1+\bar{\theta}_{cs}\right)}{3\left(1-\bar{\theta}_{cs}\right)}=u_{j}\left(s,case\ 2\right)\Leftrightarrow$$

$$\bar{\gamma}\sigma-\sigma\leq c-\frac{c\left(1-\bar{\theta}_{cs}\right)+\sigma\left(\bar{\gamma}-1\right)\left(1+\bar{\theta}_{cs}\right)}{3\left(1-\bar{\theta}_{cs}\right)}\Leftrightarrow$$

$$\Leftrightarrow c\geq\frac{\sigma\left(\bar{\gamma}-1\right)\left(2-\bar{\theta}_{cs}\right)}{1-\bar{\theta}_{cs}}.$$

CASE 2

Case 2 with respect of case 1

In case *2*, each *ethical* consumer obtains a lower utility than the utility obtained by consumers in case *1*

$$u_j(case\ 1) = \overline{\gamma}\sigma - c > \gamma^j\sigma - \frac{2c\left(1-\overline{\theta}_{cs}\right)+\sigma\left(\overline{\gamma}-1\right)\left(2-\overline{\theta}_{cs}\right)}{3\left(1-\overline{\theta}_{cs}\right)} = u_j(e, case\ 2) \Leftrightarrow$$

$$\Leftrightarrow \overline{\gamma}\sigma - \gamma^j\sigma > c - \frac{2c\left(1-\overline{\theta}_{cs}\right)+\sigma\left(\overline{\gamma}-1\right)\left(2-\overline{\theta}_{cs}\right)}{3\left(1-\overline{\theta}_{cs}\right)}$$

which is always satisfied since

$$\overline{\gamma}\sigma - \gamma^j\sigma \geq 0$$

and

$$c - \frac{2c\left(1-\overline{\theta}_{cs}\right)+\sigma\left(\overline{\gamma}-1\right)\left(2-\overline{\theta}_{cs}\right)}{3\left(1-\overline{\theta}_{cs}\right)} < 0$$

because

$$c < \frac{\sigma\left(\overline{\gamma}-1\right)\left(2-\overline{\theta}_{cs}\right)}{1-\overline{\theta}_{cs}}.$$

In case *2*, each *standard* consumer obtains a lower utility than the utility obtained by consumers in case *1* since:

$$u_j(case\ 1) = \overline{\gamma}\sigma - c > \sigma - \frac{c\left(1-\overline{\theta}_{cs}\right)+\sigma\left(\overline{\gamma}-1\right)\left(1+\overline{\theta}_{cs}\right)}{3\left(1-\overline{\theta}_{cs}\right)} = u_j(s, case\ 2) \Leftrightarrow$$

$$\Leftrightarrow \overline{\gamma}\sigma - \sigma > c - \frac{c\left(1-\overline{\theta}_{cs}\right)+\sigma\left(\overline{\gamma}-1\right)\left(1+\overline{\theta}_{cs}\right)}{3\left(1-\overline{\theta}_{cs}\right)} \Leftrightarrow$$

$$\Leftrightarrow c < \frac{\sigma\left(\overline{\gamma}-1\right)\left(1+\overline{\theta}_{cs}\right)}{2\left(1-\overline{\theta}_{cs}\right)}$$

which is always satisfied.

Case 2 with respect of case 3

In case *2*, each *standard* consumer obtains a lower utility than the utility obtained by consumers in case *3* when firms choose to be standard

$$u_j\left(case\ 3\right)=\sigma>\sigma-\frac{c\left(1-\overline{\theta}_{cs}\right)+\sigma\left(\overline{\gamma}-1\right)\left(1+\overline{\theta}_{cs}\right)}{3\left(1-\overline{\theta}_{cs}\right)}=u_j\left(s,case\ 2\right)\text{; and}$$

each *ethical* consumers obtains a lower utility than the utility obtained by consumers in case *3* when firms choose to be standard if and only if

$$u_j\left(case\ 3\right)=\sigma>\gamma^j\sigma-\frac{2c\left(1-\overline{\theta}_{cs}\right)+\sigma\left(\overline{\gamma}-1\right)\left(2-\overline{\theta}_{cs}\right)}{3\left(1-\overline{\theta}_{cs}\right)}=u_j\left(e,case\ 2\right)\Leftrightarrow$$

$$\Leftrightarrow\gamma^j\sigma-\sigma<\frac{2c\left(1-\overline{\theta}_{cs}\right)+\sigma\left(\overline{\gamma}-1\right)\left(2-\overline{\theta}_{cs}\right)}{3\left(1-\overline{\theta}_{cs}\right)}\Leftrightarrow$$

$$\Leftrightarrow\gamma^j<\frac{\left(1-\overline{\theta}_{cs}\right)\left(2c+3\sigma\right)+\sigma\left(\overline{\gamma}-1\right)\left(2-\overline{\theta}_{cs}\right)}{3\sigma\left(1-\overline{\theta}_{cs}\right)}$$

Now since

$$\frac{\left(1-\overline{\theta}_{cs}\right)\left(2c+3\sigma\right)+\sigma\left(\overline{\gamma}-1\right)\left(2-\overline{\theta}_{cs}\right)}{3\sigma\left(1-\overline{\theta}_{cs}\right)}>\frac{\left(1-\overline{\theta}_{cs}\right)\left(c+3\sigma\right)+\sigma\left(\overline{\gamma}-1\right)\left(1-2\overline{\theta}_{cs}\right)}{3\sigma\left(1-\overline{\theta}_{cs}\right)}$$

is always verified, we have that:

- if $\frac{\left(1-\overline{\theta}_{cs}\right)\left(c+3\sigma\right)+\sigma\left(\overline{\gamma}-1\right)\left(1-2\overline{\theta}_{cs}\right)}{3\sigma\left(1-\overline{\theta}_{cs}\right)}<\gamma^j<\frac{\left(1-\overline{\theta}_{cs}\right)\left(2c+3\sigma\right)+\sigma\left(\overline{\gamma}-1\right)\left(2-\overline{\theta}_{cs}\right)}{3\sigma\left(1-\overline{\theta}_{cs}\right)}$,

  $u_j\left(case\ 3\right)>u_j\left(e,case\ 2\right)$;
- if instead $\frac{\left(1-\overline{\theta}_{cs}\right)\left(2c+3\sigma\right)+\sigma\left(\overline{\gamma}-1\right)\left(2-\overline{\theta}_{cs}\right)}{3\sigma\left(1-\overline{\theta}_{cs}\right)}\le\gamma^j<\overline{\gamma}$, $u_j\left(case\ 3\right)\le u_j\left(e,case\ 2\right)$.

If, in case *3*, firms choose to be ethical, we have that:

- each *standard* consumer obtains (in case *2*) a lower utility than the utility obtained by *convinced standard* consumers in case *3* if and only if $c<\frac{\sigma\left(\overline{\gamma}-1\right)\left(1+\overline{\theta}_{cs}\right)}{2\left(1-\overline{\theta}_{cs}\right)}$ with

$\frac{\sigma\left(\overline{\gamma}-1\right)\left(3\sqrt{\overline{\theta}_{cs}}-1-\overline{\theta}_{cs}\right)}{1-\overline{\theta}_{cs}}<\frac{\sigma\left(\overline{\gamma}-1\right)\left(1+\overline{\theta}_{cs}\right)}{2\left(1-\overline{\theta}_{cs}\right)}<\frac{\sigma\left(\overline{\gamma}-1\right)\left(2-\overline{\theta}_{cs}\right)}{1-\overline{\theta}_{cs}}$:

$$u_j\left(case\ 3\right)=\sigma-c>\sigma-\frac{c\left(1-\overline{\theta}_{cs}\right)+\sigma\left(\overline{\gamma}-1\right)\left(1+\overline{\theta}_{cs}\right)}{3\left(1-\overline{\theta}_{cs}\right)}=u_j\left(s,case\ 2\right)\Leftrightarrow$$

$$\Leftrightarrow c-\frac{c\left(1-\bar{\theta}_{cs}\right)+\sigma\left(\bar{\gamma}-1\right)\left(1+\bar{\theta}_{cs}\right)}{3\left(1-\bar{\theta}_{cs}\right)}=\frac{2c\left(1-\bar{\theta}_{cs}\right)-\sigma\left(\bar{\gamma}-1\right)\left(1+\bar{\theta}_{cs}\right)}{3\left(1-\bar{\theta}_{cs}\right)}<0\Leftrightarrow$$

$$\Leftrightarrow c<\frac{\sigma\left(\bar{\gamma}-1\right)\left(1+\bar{\theta}_{cs}\right)}{2\left(1-\bar{\theta}_{cs}\right)}$$

if instead

$$c\geq\frac{\sigma\left(\bar{\gamma}-1\right)\left(1+\bar{\theta}_{cs}\right)}{2\left(1-\bar{\theta}_{cs}\right)}$$

we have that $u_j\left(case\ 3\right)\leq u_j\left(s,case\ 2\right)$.

- each *standard* consumer obtains a lower utility than the utility obtained by *potentially ethical* consumers in case *3* if and only if

$$u_j\left(case\ 3\right)=\gamma^j\sigma-c>\sigma-\frac{c\left(1-\bar{\theta}_{cs}\right)+\sigma\left(\bar{\gamma}-1\right)\left(1+\bar{\theta}_{cs}\right)}{3\left(1-\bar{\theta}_{cs}\right)}=u_j\left(s,case\ 2\right)\Leftrightarrow$$

$$\Leftrightarrow\gamma^j>\frac{\left(1-\bar{\theta}_{cs}\right)\left(2c+3\sigma\right)+\sigma\left(\bar{\gamma}-1\right)\left(1+\bar{\theta}_{cs}\right)}{3\sigma\left(1-\bar{\theta}_{cs}\right)}$$

Now since

$$\frac{\left(1-\bar{\theta}_{cs}\right)\left(2c+3\sigma\right)+\sigma\left(\bar{\gamma}-1\right)\left(1+\bar{\theta}_{cs}\right)}{3\sigma\left(1-\bar{\theta}_{cs}\right)}>\frac{\left(1-\bar{\theta}_{cs}\right)\left(c+3\sigma\right)+\sigma\left(\bar{\gamma}-1\right)\left(1-2\bar{\theta}_{cs}\right)}{3\sigma\left(1-\bar{\theta}_{cs}\right)}$$

we have that:

if $\frac{\left(1-\bar{\theta}_{cs}\right)\left(2c+3\sigma\right)+\sigma\left(\bar{\gamma}-1\right)\left(1+\bar{\theta}_{cs}\right)}{3\sigma\left(1-\bar{\theta}_{cs}\right)}<\gamma^j<\bar{\gamma}$, $u_j\left(case\ 3\right)>u_j\left(s,case\ 2\right)$,

if instead $\frac{\left(1-\bar{\theta}_{cs}\right)\left(c+3\sigma\right)+\sigma\left(\bar{\gamma}-1\right)\left(1-2\bar{\theta}_{cs}\right)}{3\sigma\left(1-\bar{\theta}_{cs}\right)}<\gamma^j\leq\frac{\left(1-\bar{\theta}_{cs}\right)\left(2c+3\sigma\right)+\sigma\left(\bar{\gamma}-1\right)\left(1+\bar{\theta}_{cs}\right)}{3\sigma\left(1-\bar{\theta}_{cs}\right)}$,

$u_j\left(case\ 3\right)\leq u_j\left(s,case\ 2\right)$.

- each *ethical* consumer obtains a lower utility than the utility obtained by *convinced standard* consumers in case *3* if and only if

$$u_j\left(case\ 3\right)=\sigma-c>\gamma^j\sigma-\frac{2c\left(1-\bar{\theta}_{cs}\right)+\sigma\left(\bar{\gamma}-1\right)\left(2-\bar{\theta}_{cs}\right)}{3\left(1-\bar{\theta}_{cs}\right)}=u_j\left(e,case\ 2\right)\Leftrightarrow$$

$$\Leftrightarrow \gamma^j\sigma - \sigma < \frac{2c\left(1-\bar{\theta}_{cs}\right)+\sigma\left(\bar{\gamma}-1\right)\left(2-\bar{\theta}_{cs}\right)}{3\left(1-\bar{\theta}_{cs}\right)} - c \Leftrightarrow$$

$$\Leftrightarrow \gamma^j < \frac{\left(1-\bar{\theta}_{cs}\right)\left(3\sigma - c\right)+\sigma\left(\bar{\gamma}-1\right)\left(2-\bar{\theta}_{cs}\right)}{3\sigma\left(1-\bar{\theta}_{cs}\right)}.$$

Now since

$$\frac{\left(1-\bar{\theta}_{cs}\right)\left(3\sigma - c\right)+\sigma\left(\bar{\gamma}-1\right)\left(2-\bar{\theta}_{cs}\right)}{3\sigma\left(1-\bar{\theta}_{cs}\right)} > \frac{\left(1-\bar{\theta}_{cs}\right)\left(c+3\sigma\right)+\sigma\left(\bar{\gamma}-1\right)\left(1-2\bar{\theta}_{cs}\right)}{3\sigma\left(1-\bar{\theta}_{cs}\right)} \Leftrightarrow$$

$$c < \frac{\sigma\left(\bar{\gamma}-1\right)\left(1+\bar{\theta}_{cs}\right)}{2\left(1-\bar{\theta}_{cs}\right)}$$

we have that

if $c < \frac{\sigma\left(\bar{\gamma}-1\right)\left(1+\bar{\theta}_{cs}\right)}{2\left(1-\bar{\theta}_{cs}\right)}$ and

$$\frac{\left(1-\bar{\theta}_{cs}\right)\left(c+3\sigma\right)+\sigma\left(\bar{\gamma}-1\right)\left(1-2\bar{\theta}_{cs}\right)}{3\sigma\left(1-\bar{\theta}_{cs}\right)} < \gamma^j < \frac{\left(1-\bar{\theta}_{cs}\right)\left(3\sigma - c\right)+\sigma\left(\bar{\gamma}-1\right)\left(2-\bar{\theta}_{cs}\right)}{3\sigma\left(1-\bar{\theta}_{cs}\right)},$$

$u_j\left(case\ 3\right) > u_j\left(e, case\ 2\right)$

if $c < \frac{\sigma\left(\bar{\gamma}-1\right)\left(1+\bar{\theta}_{cs}\right)}{2\left(1-\bar{\theta}_{cs}\right)}$ and $\frac{\left(1-\bar{\theta}_{cs}\right)\left(3\sigma - c\right)+\sigma\left(\bar{\gamma}-1\right)\left(2-\bar{\theta}_{cs}\right)}{3\sigma\left(1-\bar{\theta}_{cs}\right)} \leq \gamma^j < \bar{\gamma}$,

$u_j\left(case\ 3\right) \leq u_j\left(e, case\ 2\right)$,

if finally $c \geq \frac{\sigma\left(\bar{\gamma}-1\right)\left(1+\bar{\theta}_{cs}\right)}{2\left(1-\bar{\theta}_{cs}\right)}$ we have that $u_j\left(case\ 3\right) \leq u_j\left(e, case\ 2\right)$.

- each *ethical* consumer obtains a lower utility than the utility obtained by *potentially ethical* consumers in case *3* since

$$u_j\left(case\ 3\right) = \gamma^j\sigma - c > \gamma^j\sigma - \frac{2c\left(1-\bar{\theta}_{cs}\right)+\sigma\left(\bar{\gamma}-1\right)\left(2-\bar{\theta}_{cs}\right)}{3\left(1-\bar{\theta}_{cs}\right)} = u_j\left(e, case\ 2\right) \Leftrightarrow$$

$$\Leftrightarrow c < \frac{2c\left(1-\bar{\theta}_{cs}\right)+\sigma\left(\bar{\gamma}-1\right)\left(2-\bar{\theta}_{cs}\right)}{3\left(1-\bar{\theta}_{cs}\right)} \Leftrightarrow$$

$$\Leftrightarrow c < \frac{\sigma\left(\bar{\gamma}-1\right)\left(2-\bar{\theta}_{cs}\right)}{1-\bar{\theta}_{cs}}$$

is always satisfied.

CASE 3

Case 3 – when both firms chooses to be standard - with respect of case 1

In case 3, consumers obtain an utility greater than the utility obtained by consumers in case *1* if and only if

$$u_j\left(case\ 3\right)=\sigma>\overline{\gamma}\sigma-c=u_j\left(case\ 1\right)\Leftrightarrow$$

$$\Leftrightarrow\sigma\left(\overline{\gamma}-1\right)<c,$$

if instead

$$c\le\sigma\left(\overline{\gamma}-1\right)$$

we have that $u_j\left(case\ 3\right)\le u_j\left(case\ 1\right)$.

Case 3 – when both firms chooses to be ethical - with respect of case 1

In case *3*, *convinced standard* consumers obtain an utility lower than the utility obtained by consumers in case *1* since

$$\begin{aligned}&u_j\left(case\ 3\right)=\sigma-c<\overline{\gamma}\sigma-c=u_j\left(case\ 1\right)\Leftrightarrow\\&\Leftrightarrow\overline{\gamma}\sigma-\sigma>0\end{aligned}$$

is always satisfied.

In case *3*, *potentially ethical* consumers obtain an utility lower or equal than the utility obtained by consumers in case *1* since

$$\begin{aligned}&u_j\left(case\ 3\right)=\gamma^j\sigma-c\le\overline{\gamma}\sigma-c=u_j\left(case\ 1\right)\Leftrightarrow\\&\Leftrightarrow\gamma^j\sigma\le\overline{\gamma}\sigma\end{aligned}$$

is always satisfied.

Case 3 – when both firms chooses to be standard - with respect of case 2

In case 3, consumers obtain an utility greater than the utility obtained by *standard* consumers in case *2*:

$$u_j\left(case\ 3\right)=\sigma>\sigma-\frac{c\left(1-\overline{\theta}_{cs}\right)+\sigma\left(\overline{\gamma}-1\right)\left(1+\overline{\theta}_{cs}\right)}{3\left(1-\overline{\theta}_{cs}\right)}=u_j\left(s,case\ 2\right)$$

while consumers obtain an utility greater than the utility obtained by *ethical consumers* in case *2* if and only if

$$u_j(case\ 3)=\sigma>\gamma^j\sigma-\frac{2c\left(1-\bar{\theta}_{cs}\right)+\sigma\left(\bar{\gamma}-1\right)\left(2-\bar{\theta}_{cs}\right)}{3\left(1-\bar{\theta}_{cs}\right)}=u_j(e,case\ 2)\Leftrightarrow$$

$$\Leftrightarrow\gamma^j\sigma-\sigma<\frac{2c\left(1-\bar{\theta}_{cs}\right)+\sigma\left(\bar{\gamma}-1\right)\left(2-\bar{\theta}_{cs}\right)}{3\left(1-\bar{\theta}_{cs}\right)}\Leftrightarrow$$

$$\Leftrightarrow\gamma^j<\frac{\left(1-\bar{\theta}_{cs}\right)\left(2c+3\sigma\right)+\sigma\left(\bar{\gamma}-1\right)\left(2-\bar{\theta}_{cs}\right)}{3\sigma\left(1-\bar{\theta}_{cs}\right)}.$$

Now since

$$\frac{\left(1-\bar{\theta}_{cs}\right)\left(2c+3\sigma\right)+\sigma\left(\bar{\gamma}-1\right)\left(2-\bar{\theta}_{cs}\right)}{3\sigma\left(1-\bar{\theta}_{cs}\right)}>\frac{\left(1-\bar{\theta}_{cs}\right)\left(c+3\sigma\right)+\sigma\left(\bar{\gamma}-1\right)\left(1-2\bar{\theta}_{cs}\right)}{3\sigma\left(1-\bar{\theta}_{cs}\right)}$$

we have that

if $$\frac{\left(1-\bar{\theta}_{cs}\right)\left(c+3\sigma\right)+\sigma\left(\bar{\gamma}-1\right)\left(1-2\bar{\theta}_{cs}\right)}{3\sigma\left(1-\bar{\theta}_{cs}\right)}<\gamma^j<\frac{\left(1-\bar{\theta}_{cs}\right)\left(2c+3\sigma\right)+\sigma\left(\bar{\gamma}-1\right)\left(2-\bar{\theta}_{cs}\right)}{3\sigma\left(1-\bar{\theta}_{cs}\right)},$$

$u_j(case\ 3)>u_j(e,case\ 2)$,

if instead $\frac{\left(1-\bar{\theta}_{cs}\right)\left(2c+3\sigma\right)+\sigma\left(\bar{\gamma}-1\right)\left(2-\bar{\theta}_{cs}\right)}{3\sigma\left(1-\bar{\theta}_{cs}\right)}\le\gamma^j<\bar{\gamma}$, $u_j(case\ 3)\le u_j(e,case\ 2)$.

Case 3 – when both firms chooses to be ethical - with respect of case 2

In case *3*, *convinced standard* consumers obtain an utility greater than the utility obtained by *standard* consumers in case *2*:

$$u_j(case\ 3)=\sigma-c>\sigma-\frac{c\left(1-\bar{\theta}_{cs}\right)+\sigma\left(\bar{\gamma}-1\right)\left(1+\bar{\theta}_{cs}\right)}{3\left(1-\bar{\theta}_{cs}\right)}=u_j(s,case\ 2)\Leftrightarrow$$

$$\Leftrightarrow c<\frac{c\left(1-\bar{\theta}_{cs}\right)+\sigma\left(\bar{\gamma}-1\right)\left(1+\bar{\theta}_{cs}\right)}{3\left(1-\bar{\theta}_{cs}\right)}$$

which is always satisfied since

$$c\le\frac{\sigma\left(\bar{\gamma}-1\right)\left(3\sqrt{\bar{\theta}_{cs}}-1-\bar{\theta}_{cs}\right)}{1-\bar{\theta}_{cs}}<\frac{\sigma\left(\bar{\gamma}-1\right)\left(1+\bar{\theta}_{cs}\right)}{2\left(1-\bar{\theta}_{cs}\right)}.$$

In case *3*, *potentially ethical* consumers obtain an utility greater than the utility obtained by *standard* consumers in case *2*:

$$u_j\left(case\ 3\right)=\gamma^j\sigma-c>\sigma-\frac{c\left(1-\bar{\theta}_{cs}\right)+\sigma\left(\bar{\gamma}-1\right)\left(1+\bar{\theta}_{cs}\right)}{3\left(1-\bar{\theta}_{cs}\right)}=u_j\left(s,case\ 2\right)\Leftrightarrow$$

$$\Leftrightarrow\gamma^j\sigma-\sigma>c-\frac{c\left(1-\bar{\theta}_{cs}\right)+\sigma\left(\bar{\gamma}-1\right)\left(1+\bar{\theta}_{cs}\right)}{3\left(1-\bar{\theta}_{cs}\right)}$$

which is always satisfied since

$$\gamma^j\sigma-\sigma>0$$

and

$$c\le\frac{\sigma\left(\bar{\gamma}-1\right)\left(3\sqrt{\bar{\theta}_{cs}}-1-\bar{\theta}_{cs}\right)}{1-\bar{\theta}_{cs}}<\frac{\sigma\left(\bar{\gamma}-1\right)\left(1+\bar{\theta}_{cs}\right)}{2\left(1-\bar{\theta}_{cs}\right)}.$$

In case *3*, *convinced standard* consumers obtain an utility greater than the utility obtained by *ethical* consumers in case *2* if and only if

$$u_j\left(case\ 3\right)=\sigma-c>\gamma^j\sigma-\frac{2c\left(1-\bar{\theta}_{cs}\right)+\sigma\left(\bar{\gamma}-1\right)\left(2-\bar{\theta}_{cs}\right)}{3\left(1-\bar{\theta}_{cs}\right)}=u_j\left(e,case\ 2\right)\Leftrightarrow$$

$$\Leftrightarrow\gamma^j\sigma-\sigma<\frac{2c\left(1-\bar{\theta}_{cs}\right)+\sigma\left(\bar{\gamma}-1\right)\left(2-\bar{\theta}_{cs}\right)}{3\left(1-\bar{\theta}_{cs}\right)}-c\Leftrightarrow$$

$$\Leftrightarrow\gamma^j<\frac{\left(1-\bar{\theta}_{cs}\right)\left(3\sigma-c\right)+\sigma\left(\bar{\gamma}-1\right)\left(2-\bar{\theta}_{cs}\right)}{3\sigma\left(1-\bar{\theta}_{cs}\right)}.$$

Now since

$$\frac{\left(1-\bar{\theta}_{cs}\right)\left(3\sigma-c\right)+\sigma\left(\bar{\gamma}-1\right)\left(2-\bar{\theta}_{cs}\right)}{3\sigma\left(1-\bar{\theta}_{cs}\right)}>\frac{\left(1-\bar{\theta}_{cs}\right)\left(c+3\sigma\right)+\sigma\left(\bar{\gamma}-1\right)\left(1-2\bar{\theta}_{cs}\right)}{3\sigma\left(1-\bar{\theta}_{cs}\right)}\Leftrightarrow$$

$$c<\frac{\sigma\left(\bar{\gamma}-1\right)\left(1+\bar{\theta}_{cs}\right)}{2\left(1-\bar{\theta}_{cs}\right)}$$

which is always satisfied, we have that

if $\dfrac{\left(1-\bar{\theta}_{cs}\right)\left(c+3\sigma\right)+\sigma\left(\bar{\gamma}-1\right)\left(1-2\bar{\theta}_{cs}\right)}{3\sigma\left(1-\bar{\theta}_{cs}\right)}<\gamma^j<\dfrac{\left(1-\bar{\theta}_{cs}\right)\left(3\sigma-c\right)+\sigma\left(\bar{\gamma}-1\right)\left(2-\bar{\theta}_{cs}\right)}{3\sigma\left(1-\bar{\theta}_{cs}\right)}$,

$u_j\left(case\ 3\right)>u_j\left(e,case\ 2\right)$,

if instead $\dfrac{\left(1-\bar{\theta}_{cs}\right)\left(3\sigma-c\right)+\sigma\left(\bar{\gamma}-1\right)\left(2-\bar{\theta}_{cs}\right)}{3\sigma\left(1-\bar{\theta}_{cs}\right)}\le\gamma^j<\bar{\gamma}$, $u_j\left(case\ 3\right)\le u_j\left(e,case\ 2\right)$.

In case *3*, *potentially ethical* consumers obtain an utility greater than the utility obtained by *ethical* consumers in case *2*:

$$u_j\left(case\ 3\right)=\gamma^j\sigma-c>\gamma^j\sigma-\frac{2c\left(1-\bar{\theta}_{cs}\right)+\sigma\left(\bar{\gamma}-1\right)\left(2-\bar{\theta}_{cs}\right)}{3\left(1-\bar{\theta}_{cs}\right)}=u_j\left(e,case\ 2\right)\Leftrightarrow$$

$$\Leftrightarrow c<\frac{2c\left(1-\bar{\theta}_{cs}\right)+\sigma\left(\bar{\gamma}-1\right)\left(2-\bar{\theta}_{cs}\right)}{3\left(1-\bar{\theta}_{cs}\right)}$$

which is always satisfied since

$$c\leq\frac{\sigma\left(\bar{\gamma}-1\right)\left(3\sqrt{\bar{\theta}_{cs}}-1-\bar{\theta}_{cs}\right)}{1-\bar{\theta}_{cs}}<\frac{\sigma\left(\bar{\gamma}-1\right)\left(2-\bar{\theta}_{cs}\right)}{1-\bar{\theta}_{cs}}.$$

5 CONCLUSIONS

In this paper, we analyzed the same model of Davies (2005) changing one of its assumption. In particular, we have assumed that the proportions of convinced standard and potentially ethical consumers' groups are not fixed a priori as in Davies (2005) and we have shown that with this new assumption, equilibrium results change: if a particular condition on the model's parameters is satisfied, results of Davies (2005) are not satisfied, i.e. in equilibrium, even with a positive cost of the SRL, both firms may choose to be ethical. If that condition on parameters is satisfied, ethical labeling may be seen as a method to obtain a market in which both firms choose to be ethical.

In particular we have found that if the number of convinced standard consumers is elevate and the cost of the SRL is positive but lower than a given value, the two-stage game has two symmetric subgame perfect Nash Equilibria where both firms may choose to be ethical or standard. The option where both firms choose to be ethical is then realizable; however to be sure to eliminate standard production the intervention of an institution seems to be necessary: with each strictly positive transfer of money to the firms who choose to adopt the SRL, at the first stage of the game, the institution guarantees the existence of two ethical firms in a given market, where this is, of course, valid if and only if the condition on parameters we found is satisfied.

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APPENDIX

Proof of Lemma 1

We find the equilibria of the second stage when, at the first stage of the game, one firm chooses to be ethical and the rival standard. There are seven case to analize, depending on the value of c and $\bar{\theta}_{cs}$.

1st Case

If $c \geq \frac{\left(2-\bar{\theta}_{cs}\right)\sigma\left(\bar{\gamma}-1\right)}{1-\bar{\theta}_{cs}} > \sigma\left(\bar{\gamma}-1\right)$, the reaction functions of the two firms are:

$$p_s\left(p_e\right) = p_e - \sigma\left(\bar{\gamma}-1\right) \qquad \forall p_e \text{ and}$$

$$p_e\left(p_s\right) = \begin{cases} \left[c,\infty\right) & \text{if } 0 \leq p_s \leq c-\sigma\left(\bar{\gamma}-1\right) \\ p_e^{**} & \text{if } c-\sigma\left(\bar{\gamma}-1\right) < p_s \leq \tilde{p}_s - \dfrac{2\sigma\left(\bar{\gamma}-1\right)\sqrt{\bar{\theta}_{cs}}}{1-\bar{\theta}_{cs}} \end{cases}.$$

It exists a unique equilibrium in pure strategies given by

$$\begin{aligned} p_e^* &= c \\ p_s^* &= c-\sigma\left(\bar{\gamma}-1\right) \end{aligned}$$

where this equilibrium derives from the intersection of $p_s\left(p_e\right) = p_e - \sigma\left(\bar{\gamma}-1\right)$ and $p_e\left(p_s\right) = \left[c,\infty\right)$ (i.e. $\pi_e = 0$ for each p_e) and the equilibrium profits of the two firms are

$$\begin{aligned} \pi_e^* &= 0 \\ \pi_s^* &= c-\sigma\left(\bar{\gamma}-1\right) > 0 \end{aligned}.$$

2nd Case

If $\bar{\theta}_{cs} \geq \frac{1}{2}$ (i.e. $\frac{\bar{\theta}_{cs}\sigma\left(\bar{\gamma}-1\right)}{1-\bar{\theta}_{cs}} \geq \sigma\left(\bar{\gamma}-1\right)$) and $\frac{\bar{\theta}_{cs}\sigma\left(\bar{\gamma}-1\right)}{1-\bar{\theta}_{cs}} < c < \frac{\left(2-\bar{\theta}_{cs}\right)\sigma\left(\bar{\gamma}-1\right)}{1-\bar{\theta}_{cs}}$, the reaction functions of the two firms are:

$$p_s\left(p_e\right) = \begin{cases} p_s^{**} & \text{if } c \leq p_e < \dfrac{\left(2-\bar{\theta}_{cs}\right)\sigma\left(\bar{\gamma}-1\right)}{1-\bar{\theta}_{cs}} \\ p_e - \sigma\left(\bar{\gamma}-1\right) & \text{if } p_e \geq \dfrac{\left(2-\bar{\theta}_{cs}\right)\sigma\left(\bar{\gamma}-1\right)}{1-\bar{\theta}_{cs}} \end{cases} \quad \text{and}$$

$$p_e(p_s)=\begin{cases}[c,\infty) & \text{if } 0\le p_s\le c-\sigma(\bar{\gamma}-1)\\ p_e^{**} & \text{if } c-\sigma(\bar{\gamma}-1)<p_s\le \tilde{p}_s-\dfrac{2\sigma(\bar{\gamma}-1)\sqrt{\bar{\theta}_{cs}}}{1-\bar{\theta}_{cs}}\end{cases}.$$

It exists a unique equilibrium in pure strategies given by

$$p_e^*=\frac{2c\left(1-\bar{\theta}_{cs}\right)+\sigma(\bar{\gamma}-1)\left(2-\bar{\theta}_{cs}\right)}{3\left(1-\bar{\theta}_{cs}\right)}$$

$$p_s^*=\frac{c\left(1-\bar{\theta}_{cs}\right)+\sigma(\bar{\gamma}-1)\left(1+\bar{\theta}_{cs}\right)}{3\left(1-\bar{\theta}_{cs}\right)}$$

if and only if

$$\frac{\left(-1-\bar{\theta}_{cs}+3\sqrt{\bar{\theta}_{cs}}\right)\sigma(\bar{\gamma}-1)}{1-\bar{\theta}_{cs}}\le c<\frac{\left(2-\bar{\theta}_{cs}\right)\sigma(\bar{\gamma}-1)}{1-\bar{\theta}_{cs}}.$$

This equilibrium derives from the intersection of $p_s(p_e)=p_s^{**}$ and $p_e(p_s)=p_e^{**}$.

3rd Case

If $\bar{\theta}_{cs}\ge\frac{1}{2}$ (i.e. $\frac{\bar{\theta}_{cs}\sigma(\bar{\gamma}-1)}{1-\bar{\theta}_{cs}}\ge\sigma(\bar{\gamma}-1)$) and $\sigma(\bar{\gamma}-1)< c\le\frac{\bar{\theta}_{cs}\sigma(\bar{\gamma}-1)}{1-\bar{\theta}_{cs}}$, the reaction functions of the two firms are:

$$p_s(p_e)=\begin{cases}p_s^{**} & \text{if } \dfrac{\bar{\theta}_{cs}\sigma(\bar{\gamma}-1)}{1-\bar{\theta}_{cs}}\le p_e<\dfrac{\left(2-\bar{\theta}_{cs}\right)\sigma(\bar{\gamma}-1)}{1-\bar{\theta}_{cs}}\\ p_e-\sigma(\bar{\gamma}-1) & \text{if } p_e\ge\dfrac{\left(2-\bar{\theta}_{cs}\right)\sigma(\bar{\gamma}-1)}{1-\bar{\theta}_{cs}}\end{cases}\quad\text{and}$$

$$p_e(p_s)=\begin{cases}[c,\infty) & \text{if } 0\le p_s\le c-\sigma(\bar{\gamma}-1)\\ p_e^{**} & \text{if } c-\sigma(\bar{\gamma}-1)<p_s\le \tilde{p}_s-\dfrac{2\sigma(\bar{\gamma}-1)\sqrt{\bar{\theta}_{cs}}}{1-\bar{\theta}_{cs}}\end{cases}.$$

It doesn't exists any equilibrium in pure strategies.

4th Case

If $\bar{\theta}_{cs}\ge\frac{1}{2}$ (i.e. $\frac{\bar{\theta}_{cs}\sigma(\bar{\gamma}-1)}{1-\bar{\theta}_{cs}}\ge\sigma(\bar{\gamma}-1)$) and $0<c\le\sigma(\bar{\gamma}-1)$, the reaction functions of the two firms are:

$$p_s(p_e)=\begin{cases} p_s^{**} & \text{if } \frac{\bar\theta_{cs}\sigma(\bar\gamma-1)}{1-\bar\theta_{cs}} \le p_e < \frac{(2-\bar\theta_{cs})\sigma(\bar\gamma-1)}{1-\bar\theta_{cs}} \\ p_e-\sigma(\bar\gamma-1) & \text{if } p_e \ge \frac{(2-\bar\theta_{cs})\sigma(\bar\gamma-1)}{1-\bar\theta_{cs}} \end{cases} \quad \text{and}$$

$$p_e(p_s)=p_e^{**} \quad \text{if } 0\le p_s \le \tilde p_s - \frac{2\sigma(\bar\gamma-1)\sqrt{\bar\theta_{cs}}}{1-\bar\theta_{cs}}.$$

It doesn't exists any equilibrium in pure strategies.

5th Case

If $\bar\theta_{cs} < \frac{1}{2}$ and $\sigma(\bar\gamma-1) < c < \frac{(2-\bar\theta_{cs})\sigma(\bar\gamma-1)}{1-\bar\theta_{cs}}$, the reaction functions are

$$p_s(p_e)=\begin{cases} p_s^{**} & \text{if } c \le p_e < \frac{(2-\bar\theta_{cs})\sigma(\bar\gamma-1)}{1-\bar\theta_{cs}} \\ p_e-\sigma(\bar\gamma-1) & \text{if } p_e \ge \frac{(2-\bar\theta_{cs})\sigma(\bar\gamma-1)}{1-\bar\theta_{cs}} \end{cases} \quad \text{and}$$

$$p_e(p_s)=\begin{cases} [c,\infty) & \text{if } 0\le p_s \le c-\sigma(\bar\gamma-1) \\ p_e^{**} & \text{if } c-\sigma(\bar\gamma-1) < p_s \le \tilde p_s - \frac{2\sigma(\bar\gamma-1)\sqrt{\bar\theta_{cs}}}{1-\bar\theta_{cs}} \end{cases}$$

It exists a unique equilibrium in pure strategies given by

$$p_e^* = \frac{2c(1-\bar\theta_{cs})+\sigma(\bar\gamma-1)(2-\bar\theta_{cs})}{3(1-\bar\theta_{cs})}$$

$$p_s^* = \frac{c(1-\bar\theta_{cs})+\sigma(\bar\gamma-1)(1+\bar\theta_{cs})}{3(1-\bar\theta_{cs})}$$

if and only if

$$\frac{\left(-1-\bar\theta_{cs}+3\sqrt{\bar\theta_{cs}}\right)\sigma(\bar\gamma-1)}{1-\bar\theta_{cs}} \le c < \frac{(2-\bar\theta_{cs})\sigma(\bar\gamma-1)}{1-\bar\theta_{cs}} \quad \text{when } \frac{4}{9} < \bar\theta_{cs} < \frac{1}{2}.$$

For $0<\bar\theta_{cs}\le\frac{4}{9}$, p_e^*, p_s^* is an equilibrium in the relevant interval of c.

This equilibrium derives from the intersection of $p_s(p_e)=p_s^{**}$ and $p_e(p_s)=p_e^{**}$.

6th Case

If $\bar{\theta}_{cs} < \frac{1}{2}$ and $\frac{\bar{\theta}_{cs}\sigma(\bar{\gamma}-1)}{1-\bar{\theta}_{cs}} < c \leq \sigma(\bar{\gamma}-1)$, the reaction functions are

$$p_s(p_e) = \begin{cases} p_s^{**} & \text{if } c \leq p_e < \frac{(2-\bar{\theta}_{cs})\sigma(\bar{\gamma}-1)}{1-\bar{\theta}_{cs}} \\ p_e - \sigma(\bar{\gamma}-1) & \text{if } p_e \geq \frac{(2-\bar{\theta}_{cs})\sigma(\bar{\gamma}-1)}{1-\bar{\theta}_{cs}} \end{cases} \quad \text{and}$$

$$p_e(p_s) = p_e^{**} \quad \text{if } 0 \leq p_s \leq \tilde{p}_s - \frac{2\sigma(\bar{\gamma}-1)\sqrt{\bar{\theta}_{cs}}}{1-\bar{\theta}_{cs}}.$$

It exists a unique equilibrium in pure strategies given by

$$p_e^* = \frac{2c(1-\bar{\theta}_{cs}) + \sigma(\bar{\gamma}-1)(2-\bar{\theta}_{cs})}{3(1-\bar{\theta}_{cs})}$$

$$p_s^* = \frac{c(1-\bar{\theta}_{cs}) + \sigma(\bar{\gamma}-1)(1+\bar{\theta}_{cs})}{3(1-\bar{\theta}_{cs})}$$

if and only if

$$\frac{\left(-1-\bar{\theta}_{cs}+3\sqrt{\bar{\theta}_{cs}}\right)\sigma(\bar{\gamma}-1)}{1-\bar{\theta}_{cs}} \leq c \leq \sigma(\bar{\gamma}-1) \text{ when } \frac{1}{4} \leq \bar{\theta}_{cs} \leq \frac{4}{9}.$$

For $0 < \bar{\theta}_{cs} \leq \frac{1}{4}$, p_e^*, p_s^* is an equilibrium in the relevant interval of *c*, while for $\frac{4}{9} < \bar{\theta}_{cs} < \frac{1}{2}$ it doesn't exists any equilibrium.

This equilibrium derives from the intersection of $p_s(p_e) = p_s^{**}$ and $p_e(p_s) = p_e^{**}$.

7th Case

If $\bar{\theta}_{cs} < \frac{1}{2}$ and $0 < c \leq \frac{\bar{\theta}_{cs}\sigma(\bar{\gamma}-1)}{1-\bar{\theta}_{cs}}$, the reaction functions are

$$p_s(p_e) = \begin{cases} p_s^{**} & \text{if } \frac{\bar{\theta}_{cs}\sigma(\bar{\gamma}-1)}{1-\bar{\theta}_{cs}} \leq p_e < \frac{(2-\bar{\theta}_{cs})\sigma(\bar{\gamma}-1)}{1-\bar{\theta}_{cs}} \\ p_e - \sigma(\bar{\gamma}-1) & \text{if } p_e \geq \frac{(2-\bar{\theta}_{cs})\sigma(\bar{\gamma}-1)}{1-\bar{\theta}_{cs}} \end{cases} \quad \text{and}$$

$$p_e(p_s) = p_e^{**} \quad \text{if } 0 \leq p_s \leq \tilde{p}_s - \frac{2\sigma(\bar{\gamma}-1)\sqrt{\bar{\theta}_{cs}}}{1-\bar{\theta}_{cs}}.$$

It exists a unique equilibrium in pure strategies given by

$$p_e^* = \frac{2c\left(1-\bar{\theta}_{cs}\right)+\sigma\left(\bar{\gamma}-1\right)\left(2-\bar{\theta}_{cs}\right)}{3\left(1-\bar{\theta}_{cs}\right)}$$
$$p_s^* = \frac{c\left(1-\bar{\theta}_{cs}\right)+\sigma\left(\bar{\gamma}-1\right)\left(1+\bar{\theta}_{cs}\right)}{3\left(1-\bar{\theta}_{cs}\right)}$$

if and only if

$$\frac{\left(-1-\bar{\theta}_{cs}+3\sqrt{\bar{\theta}_{cs}}\right)\sigma\left(\bar{\gamma}-1\right)}{1-\bar{\theta}_{cs}} \le c \le \sigma\left(\bar{\gamma}-1\right) \text{ when } \frac{7-3\sqrt{5}}{2} \le \bar{\theta}_{cs} \le \frac{1}{4}.$$

For $0<\bar{\theta}_{cs}<\frac{7-3\sqrt{5}}{2}$, p_e^*, p_s^* is an equilibrium in the relevant interval of *c*, while for $\frac{1}{4}<\bar{\theta}_{cs}<\frac{1}{2}$ it doesn't exists any equilibrium.

This equilibrium derives from the intersection of $p_s\left(p_e\right)=p_s^{**}$ and $p_e\left(p_s\right)=p_e^{**}$.

To sum up we obtain the following results:

If $c \ge \frac{\sigma\left(\bar{\gamma}-1\right)\left(2-\bar{\theta}_{cs}\right)}{\left(1-\bar{\theta}_{cs}\right)}$, at the second stage of the game, it exists a unique equilibrium given by

$$p_e^* = c$$
$$p_s^* = c-\sigma\left(\bar{\gamma}-1\right)$$

If instead $c < \frac{\sigma\left(\bar{\gamma}-1\right)\left(2-\bar{\theta}_{cs}\right)}{\left(1-\bar{\theta}_{cs}\right)}$, the existence of an equilibrium, at the second stage of the game depends on $\bar{\theta}_{cs}$ and *c*, and the unique equilibrium is given by

$$p_e^* = p_e^{**} = \frac{2c\left(1-\bar{\theta}_{cs}\right)+\sigma\left(\bar{\gamma}-1\right)\left(2-\bar{\theta}_{cs}\right)}{3\left(1-\bar{\theta}_{cs}\right)}$$
$$p_s^* = p_s^{**} = \frac{c\left(1-\bar{\theta}_{cs}\right)+\sigma\left(\bar{\gamma}-1\right)\left(1+\bar{\theta}_{cs}\right)}{3\left(1-\bar{\theta}_{cs}\right)}$$

- If $0<\bar{\theta}_{cs}<\frac{7-3\sqrt{5}}{2}$, it exists the equilibrium.

- If $\frac{7-3\sqrt{5}}{2} \le \bar{\theta}_{cs} < 1$, it exists the equilibrium if and only

$$\frac{\sigma\left(\bar{\gamma}-1\right)\left[3\sqrt{\bar{\theta}_{cs}}-1-\bar{\theta}_{cs}\right]}{\left(1-\bar{\theta}_{cs}\right)} \le c < \frac{\sigma\left(\bar{\gamma}-1\right)\left(2-\bar{\theta}_{cs}\right)}{\left(1-\bar{\theta}_{cs}\right)}. \qquad \textit{Q.E.D.}$$