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Billette de Villemeur, Etienne and Cea-Echenique, Sebastián and Cuevas, Conrado

Université de Lille, France, Universidad de los Andes, Chile, INCAE, Costa Rica

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Etienne Billette de Villemeur[a,b], Sebastián Cea-Echenique[c,*], Conrado Cuevas[d]

[a] *Université de Lille, Lille, France*
[b] *Lille Econommics Management, Lille, France*
[c] *Universidad de los Andes, Chile, Facultad de Ingeniería y Ciencias Aplicadas, Santiago, Chile*
[d] *INCAE, Alajuela, Costa Rica*

Abstract

We revisit the consequences of uncertainty in the private provision of a public good. We show that, despite the risk aversion of agents and the decreasing returns to scale in the production function of the public good, uncertainty may *improve* welfare. This may hold true even if uncertainty leads to a reduction in the aggregate amount of donations for the production of the public good. This may also hold true when uncertainty makes the production of the public good more costly on average. Our findings suggest that regulation and control over the production process for public goods might not always be a desirable policy.

Keywords: Public goods, Uncertainty, Control

1. Introduction

We study the consequences for welfare that stem from uncertainty in the production of a public good.

In recent years there has been a boom in investments that take into account environmental, social, and governance (ESG) considerations. ESG investments, however, are typically associated with both important externalities and significant risks. For example, investments in clean technologies or

*Corresponding author
Email address: scea@miuandes.cl (Sebastián Cea-Echenique)

"cleantech" are mostly motivated by the private provision of a public good, namely the reduction of polluting emissions. Yet, investments that improve energy efficiency do not necessarily result in decreased emissions because they also contribute to an increase in energy use. Moreover, the actual impact of any emissions reductions over current and future welfare is highly uncertain.

The existence of societal benefits not accounted for by private investors justifies the statement that public goods are generally under-provided and that their production should be enhanced. Economic agents are generally risk averse and have imperfect information about the actual benefits of the provision of public goods and the costs of providing them; this may result in inefficiently low investment levels. Thus, it is natural to assume that reducing uncertainty in the production process should increase welfare.

However, this intuition may not be valid. For example, when Gradstein et al. [7] introduced uncertainty into the standard model of voluntary provision of public goods, they identified conditions under which uncertainty alleviates the free-rider problem by inducing economic agents to increase their donations.

Since then, the consequences of uncertainty on the provision of public goods have been explored by several authors and in different contexts. For instance, McBride [9] considers circumstances where the public good production is discrete, i.e., the provision only occurs if contributions exceed a threshold, imperfectly known (rather than being a continuous function of individual contributions). Tamai [12] considers a dynamic general equilibrium model and shows that uncertainty may increase individual contributions in the short run while capital accumulation may decrease it in the long run.

Keenan et al. [8] bridge with previous literature on public good provision by comparing (symmetric) Nash-equilibrium contributions with those arising in a setup where donors choose their contribution by considering its possible impact on others' strategies, as modeled with conjectural equilibria. More recently, Banerjee and Gravel [1]) looked at the role of subjective uncertainty by considering the impact of beliefs on equilibrium strategies.

Eichberger and Kelsey [6] show that ambiguity may induce increased contributions. More complex changes in risk are considered in Nocetti and Smith [11] to pinpoint circumstances where an increase in risk can induce an increase in equilibrium strategies (contributions).

Bramoullé and Treich [4] go beyond the impact of uncertainty on equilibrium strategies in that, ultimately, they aim at signing its overall impact on social welfare. In their model, framed in the context of climate change, climate uncertainty always damages welfare. At the same time, reducing emissions always reduces the exposition of countries to that risk. There are thus two components to the impact of uncertainty on welfare: a (positive) strategic effect and a (negative) risk-averse effect. The first effect dominates when the risk is small enough, and uncertainty results in higher welfare.

Rather than looking at (Nash) competition, Boucher and Bramoullé [3] look at (endogenous) cooperation in a model similar to Bramoullé and Treich [4]. For a public good, they show that uncertainty reduces the effort of cooperating agents but increases the number of cooperating agents. Again, they identify circumstances under which the second effect dominates, i.e., welfare increases with uncertainty. In both contributions, strategies and uncertainty are co-monotone.

All these contributions point to circumstances in which uncertainty results in more donations. In this paper, we show that the impact of uncertainty on donations is unrelated to its impact on social welfare. More precisely, uncertainty may be beneficial for welfare, regardless of whether it induces increased or decreased (amount of) donations for the provision of the public good.

We work with a simple, standard model of the provision of a public good. In the model, there are two goods, a private and a public one. Provision of the public good increases with total donations. We introduce uncertainty by allowing for stochastic costs, and we compare the impact on consumers' welfare in two types of economies: one with and one without uncertainty. We show that, in some circumstances, welfare may increase with uncertainty, even if the amount of donations decreases and even if costs are, on average, higher.

To understand the impact of uncertainty on welfare, we identify the various channels that affect the provision of the public good in equilibrium. Along with Bramoullé and Treich [4], we use the term "strategic effect" to refer to the impact of uncertainty on donors' contributions (strategies) for the production of public goods. While the threat of a "bad outcome" may indeed induce donors to contribute more, we show that this is not the sole explanation for a possible positive impact of uncertainty on welfare.

We coin the term "spread effect" to refer to the impact of uncertainty on welfare, for a given level of aggregate amount of donations. Our contribution consists in pointing out that this effect may also lead to higher consumer welfare.

Consumers enjoy the public good, not donations directly. A fixed level of contributions is transformed across states of nature into inputs for the production process. The amount of inputs is stochastic because the cost of such inputs is stochastic. A technology transforms this stochastic input into units of the public good. Ultimately, the stochastic amount of the public good gives rise to welfare.

When the marginal productivity of the input is non-increasing (there are decreasing returns to scale), mean-preserving uncertainty in its amount results in a lower level of public good than what would prevail with the average amount of input. If the marginal utility of the public good decreases with its provision, then there is risk-aversion. In that case, mean-preserving uncertainty in the level of the public good yields a lower welfare than that which would result from having the average level of the public good. Both types of uncertainty are thus detrimental to consumers' welfare.

Yet, the amount of input that can be acquired to feed the production process is a decreasing and, typically, *convex* function of the (possibly implicit) price of the input. Thus, it is fair to assume that the production of the public good is convex in its costs. If so, uncertainty in (the cost of) the production process can be associated, *ceteris paribus*, to a higher level of expected input, and thus higher: i) expected production of the public good and ii) expected social welfare.

Overall, it is possible to identify circumstances in which the positive effect dominates all the others. More precisely, a mean-preserving spread in costs may increase the expected production of the public good, even if it is associated with a lower amount of donations; and, depending on the risk aversion of consumers, this larger expected production may compensate for the costs of risks to the economic agents, and thus result in higher welfare.

Our results suggest that it is not possible to infer *a priori* the impact of uncertainty on welfare by studying the consequence in terms of donations alone. We point to the fact that, on average, the benefits that stream from "good luck" (in terms of the productivity of the process of providing the public good) override the costs of "bad luck" (as associated with inefficiencies in the production process). This suggests that regulations and controls that aim to reduce uncertainty over the process for providing a public good should be considered very cautiously – because they could easily reduce welfare.

The remainder of this short paper presents a formal exposition of these arguments.

2. The economy

The economy consists of I consumers, one private good, and one public good. Each consumer has wealth $w_i > 0$, and may donate an amount d_i, satisfying $0 \leq d_i \leq w_i$, to produce the public good; she consumes the rest, $x_i = w_i - d_i \geq 0$, in the form of the private good. The sum of the donations of all consumers is D, and D_{-i} denotes the sum of all donations by consumers other than i.

Assume that an amount D of donations results in D/c inputs for the production of the public good, so that c denotes the (unit) cost of the public good. From these inputs, $G = D/c$ units of the public good are produced.[1] Finally, consumers get utility from their consumption of the private good and from the units of the public good.

Consumer i's utility function is:

$$U_i\left(x_i, D/c\right) = x_i + \beta_i u\left(D/c\right).$$

We assume $\beta_i \geq 0$ for each $i \in I$, with strict inequality for at least two consumers.[2] We also assume that the function u is strictly increasing, twice continuously differentiable, and strictly concave. Adopting a quasi-linear framework removes income effects. Nevertheless, the results extend to a more general setup.

Consumers are uncertain about the cost, at least when taking their decision about donations; c takes values from $C := \{c_1, ..., c_S\}$, and π_s denotes the probability of occurrence of c_s.[3] The vector $\pi := (\pi_1, ..., \pi_S)$ summarizes the probability distribution.

Definition 1. *A Nash equilibrium in the public-good game with cost-uncertainty, $NEPGU(C, \pi)$, is a vector of ex-ante donations $d^* \in \mathbb{R}^I_+$, such that for each $i \in I$, d^*_i solves*

$$\max_{d_i \in [0, w_i]} \quad E_\pi\left[U_i\left(w_i - d_i, \left(d_i + D^*_{-i}\right)/c_s\right)\right].$$

[1] All results hold if we generalize the production of the public good to $G = F(D/c)$, with F increasing and concave.

[2] If there is only one i such that $\beta_i > 0$, the private provision and the efficient provision of the public good coincide; there is no free-rider problem.

[3] Gradstein et al. [7] interpret c_s as the price of the public good, while Nocetti and Smith [11] call it a productivity shifter.

Equilibrium existence follows from Bergstrom et al. [2]. We refer to d^* as a cost-uncertain equilibrium. We let $\bar{c} := E[c_s]$ be the expected cost under probability π. We assume that there exists $c_{s'} \in C$ such that $c_{s'} = \bar{c}$. We let $\bar{\pi}$ be a probability vector that assigns probability one to cost $c_{s'}$. We define $\bar{d} \in \mathbb{R}^I_+$ as a $NEPGU(C, \bar{\pi})$. We refer to $\bar{d}$ as a cost-certain equilibrium, and we define $\bar{D} := \sum_i \bar{d}_i$.

3. Results

3.1. Set of contributors

If wealth is large enough, uncertainty does not affect the set of contributing consumers (those i with $d_i^* > 0$).

To see this, observe that the problem faced by consumer i can be rewritten as:

$$\max_{D \in [D_{-i}, w_i + D_{-i}]} \; w_i + D_{-i} - D + \beta_i E\left[u\left(D/c_s\right)\right].$$

The optimal choice of D by a contributor is implicitly defined by:

$$-1 + \beta_i E\left[u'\left(\frac{D^*}{c_s}\right)\frac{1}{c_s}\right] = 0. \tag{1}$$

For i to be a contributor, her optimal choice D^* must verify $D^* > D^*_{-i}$. Let $\beta^* := \max_{j \in \{1,\ldots,I\}} \beta_j$ and I^* be the set of all i agents such that $\beta_i = \beta^*$. Only members of I^* contribute to the public good. Suppose indeed that there exist two contributors ι and j such that $\beta_\iota < \beta_j$. If equation (1) is satisfied for β_j, then the left-hand side of (1) is negative for β_ι, meaning that consumer ι is willing to decrease her contribution to the public good, a contradiction.

The non-negativity constraint on the consumption of the private good imposes that $d_i^* \leq w_i$. When binding, the contribution is set to its maximum, and the left-hand side of (1) is strictly positive. Unless this constraint binds for all $i \in I^*$,[4] the only contributors are the members of I^*. If the constraint binds for some $j \in I^*$, then this agent contributes w_j. For simplicity we assume in what follows that $\sum_{i \in I^*} w_i > \max\left\{D^*, \bar{D}\right\}$.

[4] If the non-negativity constraint on the consumption of the private good binds for all $i \in I^*$, then consumers with a lower β may contribute.

3.2. Impact of uncertainty: donations and welfare

Let V_i^* denote the indirect utility function of consumer i in a cost-uncertain equilibrium, and let $\bar{V}_i$ denote the indirect utility function in a cost-certain equilibrium. We divide the consequences of uncertainty on welfare into two components. The first is a strategic effect that refers to the impact of uncertainty on aggregate (*ex ante*) donations. The second is a spread effect for the impact of uncertainty on the level of the public good produced and, ultimately, on welfare for a given aggregate donation.

In what follows, we study the sign of the expression $V_i^* - \bar{V}_i$. We can write this expression as:

$$V_i^* - \bar{V}_i = -d_i^* + \beta_i Eu\left(D^*/c_s\right) - \left(-\bar{d}_i + \beta_i Eu\left(\bar{D}/\bar{c}\right)\right). \tag{2}$$

Adding and subtracting $\beta_i Eu(\bar{D}/c_s)$ to (2) and rearranging terms:

$$V_i^* - \bar{V}_i = \underbrace{\bar{d}_i - d_i^* + \beta_i \left(Eu\left(\frac{D^*}{c_s}\right) - Eu\left(\frac{\bar{D}}{c_s}\right)\right)}_{\text{Strategic effect}} + \underbrace{\beta_i \left(Eu\left(\frac{\bar{D}}{c_s}\right) - u\left(\frac{\bar{D}}{\bar{c}}\right)\right)}_{\text{Spread effect}}. \tag{3}$$

We can visualize the strategic and spread effects in equation (3). In the strategic effect, welfare changes come only from variations in the amount of donations, while in the spread effect, changes in welfare are only due to variability in costs.

As evidenced by Gradstein et al. [7], uncertainty may either increase or decrease aggregate donations, depending on the curvature of the utility function. More precisely, in our setup:

LEMMA 1. *If $u'(D/c)(1/c)$ is strictly convex in c, the strategic effect is positive: total donations increase with uncertainty. If $u'(D/c)(1/c)$ is concave in c, the strategic effect is negative: total donations do not increase with uncertainty.*

Proof. The first-order condition defining equilibrium contributions – that is, equation (1) – can be rewritten as:

$$\beta^i u'(\bar{D}/\bar{c})(1/\bar{c}) = 1;$$

$$\beta^i E\left[u'(D^*/c_s)(1/c_s)\right] = 1$$

under certainty and uncertainty, respectively.

As $\bar{c} = E[c_s]$ and u' is decreasing in D, then:

(i) if $E[u'(D/c_s)(1/c_s)] > u'(D/\bar{c})(1/\bar{c})$ for a fixed D, then $D^* > \bar{D}$ to satisfy both first order conditions;

(ii) if $E[u'(D/c_s)(1/c_s)] \leq u'(D/\bar{c})(1/\bar{c})$ for a fixed D, then $D^* \leq \bar{D}$ to satisfy both first order conditions.

□

Conditional on a value of donations D, uncertainty also affects welfare. There are two countervailing effects. On one hand, risk aversion as reflected in this model by the concavity of the function $u(\cdot)$ implies that (mean-preserving) uncertainty in the level of a public good result in a lower welfare than otherwise. On the other hand, uncertainty into the price of the inputs (and/or the marginal productivity of donations, as modeled here by c) results in a higher level of expected production and can enhance welfare.

Technically, if $u(D/c)$ is strictly concave in c, then

$$E\left[u(D/c_s)\right] < u(D/E\left[c_s\right]),$$

and the spread effect is negative; conditional on a value of D, welfare decreases with uncertainty. Similarly, if $u(D/c)$ is strictly convex in c, the spread effect is positive; conditional on a value of D, welfare increases with uncertainty. Given the linear-production function, it is not difficult to establish:

LEMMA 2. *The spread effect is negative (i.e., given donations, uncertainty along the production process decreases welfare) if and only if the coefficient of relative risk aversion (σ) is larger than two.*

Proof. The second derivative of $u(D/c)$ with respect to c is:

$$u''(D/c)(D/c^2)^2 + 2u'(D/c)(D/c^3).$$

Or, equivalently,

$$u'(D/c)(D/c^3) \cdot \left(\underbrace{(D/c)u''(D/c)/u'(D/c)}_{-\sigma} + 2 \right).$$

□

To the best of our knowledge, the fact that the spread effect can be positive (*i.e.*, that uncertainty in the production process might result in a higher welfare, despite consumer risk aversion and non-increasing returns to scale) has, up to now, escaped attention. Of course, the effect of uncertainty upon welfare follows from both the spread and the strategic effect. We shall see however that, in most circumstances, at least in our setting, the spread effect dominates.

3.3. Changes in donations and changes in welfare are actually unrelated

In what follows, to display clear-cut results, we make two further assumptions.

First, we assume u takes a constant-relative-risk-aversion (CRRA) functional form, *i.e.*

$$u\left(D/c\right) = \left(\left(D/c\right)^{1-\sigma} - 1\right) / \left(1-\sigma\right),$$

with $\sigma > 0$. The CRRA assumption is unnecessary for the following results to hold, but it simplifies the analysis. In particular, Lemma 1, which establishes the impact of uncertainty upon aggregate donations (the strategic effect), simplifies to:

Corollary 1. *If utilities are CRRA, then when $\sigma \in (1,2)$, total donations decrease with uncertainty. If $\sigma \in \{1,2\}$, uncertainty has no impact upon total donations. When $\sigma \notin [1,2]$, total donations increase with uncertainty.*

Second, observe that in Equation (1), which defines the donations of each contributor, all aim at the same level of aggregate donations D^*. This says that individual contributions adjust to match what is missing from the others' donations D_{-i} to reach that goal. In other words, there is a multiplicity of equilibria; in fact, there is actually a continuum of equilibria.[5]

A consequence of this multiplicity is that, when comparing two $NEPGU$s, a larger (resp. smaller) aggregate contribution in one case, does not imply that all individual contributions attached to that case are also larger (resp. smaller) than those attached to the other case. Formally:

$$\left[D > D'\right] \nRightarrow \left[d_i > d_i' \quad \forall i \in I^*\right].$$

[5]Morgan [10] pointed out this fact earlier.

To avoid making strong assumptions on equilibrium selection (such as, for example, imposing symmetry) while avoiding paradoxical behavior, we assume that aggregate and individual donations move in the same direction.

Assumption Same Direction (SD). *When total donations increase with uncertainty, all contributors donate weakly more than they would with certainty. When total donations do not increase with uncertainty, all contributors donate weakly less.*

This technical assumption, together with the CRRA functional form, allows us to prove that a positive spread effect ($\sigma < 2$) is a necessary and sufficient condition for uncertainty to improve welfare in our economy. In other words, changes in donations resulting from uncertainty (through the strategic effect) are actually unrelated to the impact of uncertainty on welfare.

Proposition 1. *Under assumption SD and when utilities are CRRA, every consumer is better off under uncertainty if and only if the coefficient of relative risk aversion is less than two.*

Proof. From Equation (1), contributions in presence or absence of uncertainty are, respectively:

$$\begin{aligned} D^* &= \beta^* E\left[\left(\frac{D^*}{c_s}\right)^{1-\sigma}\right], \\ \bar{D} &= \beta^* \left(\frac{\bar{D}}{\bar{c}}\right)^{1-\sigma}. \end{aligned} \tag{4}$$

The latter allows us to rewrite $E\left[u(D/c_s)\right]$ at each equilibrium as:

$$\begin{aligned} E\left[u\left(\frac{D^*}{c_s}\right)\right] &= \frac{D^*/\beta^* - 1}{1-\sigma}, \\ u\left(\frac{\bar{D}}{\bar{c}}\right) &= \frac{\bar{D}/\beta^* - 1}{1-\sigma}. \end{aligned}$$

Hence, for both those who do and do not contribute, the indirect (expected) utility function is linear in D:

$$\begin{aligned} V_i^* &= w_i - d_i^* + \left(\frac{\beta_i}{\beta^*}\right)\left(\frac{D^* - \beta^*}{1-\sigma}\right), \\ \bar{V}_i &= w_i - \bar{d}_i + \left(\frac{\beta_i}{\beta^*}\right)\left(\frac{\bar{D} - \beta^*}{1-\sigma}\right). \end{aligned}$$

- If $\sigma < 1$: $D^* > \bar{D}$. Computing the difference $V_i^* - \bar{V}_i$ for contributors:

$$V_i^* - \bar{V}_i = \frac{D^* - \bar{D}}{1-\sigma} - \left(d_i^* - \bar{d}_i\right).$$

  Assumption SD implies $d_i^* - \bar{d}_i$ is bounded above by $D^* - \bar{D}$ for all i. As $0 < 1-\sigma < 1$, then $V_i^* - \bar{V}_i \geq \left(D^* - \bar{D}\right) / (1-\sigma) - \left(D^* - \bar{D}\right) > 0$ for all contributors. Those who do not contribute are also better off because the change in their indirect utility function is simply obtained by setting their contribution to zero. More precisely,

$$V_i^* - \bar{V}_i = \left(D^* - \bar{D}\right) / (1-\sigma) > 0.$$

- If $\sigma = 1$: $D^* = \bar{D}$ and from Assumption SD, no individual contribution is changed. Nevertheless, the spread effect is positive: $\log D^* - E[\log c_s] > \log \bar{D} - \log \bar{c}$, by strict concavity of the logarithmic function.

- If $\sigma \in (1,2)$: $D^* < \bar{D}$, $1-\sigma < 0$, and $V_i^* - \bar{V}_i > 0$ for $i \notin I^*$ ($d_i^* = \bar{d}_i = 0$ for non-contributors). Because non-contributors are better off under uncertainty and the strategic effect is negative, then from Assumption SD, all contributors reduce their contributions, and consume more private good, so that they are also better off under uncertainty.

- If $\sigma = 2$: both the strategic and the spread effect are null, and from Assumption SD, no individual contribution changes. Therefore, welfare remains the same in both scenarios.

- If $\sigma > 2$: total donations increase with uncertainty, and from Assumption SD, none of the individual contributions decreases. However, $1-\sigma < 0$. Hence, both non-contributors and contributors are worse off.

$\square$

Proposition 1 shows that uncertainty may lead to higher welfare in an economy with public goods.[6] The proposition also shows that welfare may

[6] Observe that Pareto improvements with uncertainty can be achieved without Assumption SD if wealth transfers are allowed. Assume total donations increase with uncertainty, but some contributors reduce their donations. The sum of indirect utility functions across

improve even if total donations decline with uncertainty. Given the free-rider problem around donations for the production of the public good, the result may seem surprising. Separating the total effect of uncertainty into the strategic and spread effects helps clarify understanding. A positive spread effect (as following from the possibility of facing costs c_s below their expected value) may dominate a negative strategic effect, resulting in higher welfare.

Observe that these theoretical results *do not* refer to unlikely circumstances. Estimates by Chetty [5] indeed suggest that two is an upper-bound on the coefficient of relative risk aversion.

3.4. The free-rider problem under uncertainty: improvements in allocative efficiency and in welfare are actually unrelated

We next show that uncertainty may alleviate the free-rider problem – the difference between the private and efficient provision of the public good – even when it results in a lower amount of donations. Moreover, we show that the free-rider problem may also worsen under uncertainty despite higher welfare. In other words, we demonstrate that focusing on allocative efficiency can be misleading as welfare and efficiency are *not* co-monotone

PROPOSITION 2. *The distance between the efficient provision and the private provision of the public good is smaller under uncertainty if and only if the coefficient of relative aversion is greater than one and less than two.*

Proof. To obtain the efficient level of the provision of a public good a planner solves:

$$\max_{D\in[0,\sum_i w_i]} \sum_i w_i - D + \sum_i \frac{\beta_i}{1-\sigma}\left(D^{1-\sigma}E[c^{\sigma-1}] - 1\right).$$

The first-order condition is (assuming an interior solution):

$$-1 + D^{-\sigma}E[c^{\sigma-1}]\sum_i \beta_i = 0. \tag{5}$$

consumers is higher under uncertainty when $\sigma < 2$; thus, if we transfer enough wealth from contributors who donate less to contributors who donate more, we can achieve a Pareto improvement. As there is no income effect, donations do not change if the Planner leaves consumers enough wealth after transfers.

Equation (5) defines the efficient level of provision, D^e. Solving for D^e in (5), solving for D^* in (4), and taking the difference yields:

$$D^e - D^* = \left(E[c^{\sigma-1}]\right)^{1/\sigma} \left(\left(\sum_{i \in \mathcal{I}} \beta_i \right)^{1/\sigma} - (\beta^*)^{1/\sigma} \right). \qquad (6)$$

The second term in parentheses is strictly positive and does not vary with uncertainty. The first term in parentheses is higher under certainty if and only if the function $g(x) := x^{\sigma-1}$ is strictly concave, i.e., if and only if $\sigma \in (1, 2)$. □

Allocative efficiency (*i.e.*, the difference between the efficient provision and the equilibrium level of private provision) is a frequent policy concern. Proposition 2 shows that the impact of uncertainty on allocative efficiency differs from the impact on welfare. In Proposition 1, uncertainty is beneficial if the spread effect is positive. In Proposition 2, both a positive spread effect and a negative strategic effect are necessary to obtain an efficiency improvement.

To understand, observe from equation (6) that uncertainty has a bigger impact (in absolute value) on the efficient level of a public good than on the equilibrium level. When the strategic effect is positive, both levels increase, ending up further apart. Hence, for $\sigma < 1$, while every consumer prefers uncertainty, the free-rider problem is exacerbated. By contrast, when the strategic effect is negative, i.e., $\sigma \in (1, 2)$, an increase in risk induces the efficient solution to decrease more than the private provision. Thus, both levels move closer. It follows that for $\sigma \in (1, 2)$, every consumer prefers uncertainty, and the free-rider problem across contributors is alleviated.

3.5. Benefits from flexibility may overcome its costs

In propositions 1 and 2 we compare a cost-uncertain economy with a cost certain economy with cost $\bar{c} = E[c_s]$. Assume now that cost-certainty is attached to a cost $\widehat{c} < E[c_s]$. Nevertheless, we prove that all agents may still prefer uncertainty.

Take some s' such that $c_{s'} < E[c_s]$. Define $\hat{c} = c_{s'}$ and $\hat{\pi}$ as the probability vector that assigns probability one to $c_{s'}$. In the next proposition, we work with $\hat{\pi}$ to compute the cost-certain equilibrium.

Proposition 3. *There are economies in which every consumer is better off under uncertainty than in a cheaper (on average) and cost-certain economy.*

Proof. From Proposition 1 the result holds for $\sigma = 1$ because the difference in the indirect utility function is: $V_i^* - \bar{V}_i = \log \hat{c} - E\left[\log c_s\right] > 0$ for $\hat{c} < E[c_s]$, but sufficiently close to the expected cost. By continuity, the result also holds for CRRA utility functions with σ sufficiently close to one. □

Proposition 3 highlights the possible benefits of flexibility, which can be interpreted as a lack of control resulting in cost uncertainty. Uncertainty's positive effect on welfare may dominate small efficiency gains in a cost-certain economy. In an economy with public goods, a reduction in costs and a reduction in uncertainty do not imply a Pareto improvement, and may even be Pareto-dominated.

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