Two's not company: mis-aggregation and "supply-induced" unemployment increases

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Abstract

The seminal work by Blanchard and Quah (1989) identifying long-run shocks finds that the unemployment rate increases following a "supply" shock. This puzzling result engenders a lively debate between leading schools of macroeconomic thought. But should we employ this model to study macroeconomic fluctuations? Faust and Leeper (1997) warn that the model is not reliable for structural inference, nor is the related Bayoumi and Eichengreen (1993) model. In this paper, I revisit the disputed result from the viewpoint of IRF's bias. Using a novel methodology to parse the sources of IRF's bias, I find that "supply-induced" unemployment increases reflect mis-aggregation of technology and labor-supply shocks.

JEL Classification Numbers: C32, C52, E3, E5

Keywords: Vector autoregression, Permanent and temporary shocks to output, Missing-variables, Fundamentalness, Mis-aggregated shocks, Co-mingled shocks, Impulse response function bias, Moving-average representation, Aggregate demand and supply shocks, Long-run neutrality, Labor-supply shocks, Contractionary technology shocks

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I. INTRODUCTION

Blanchard and Quah (1989) (henceforth, BQ) seminal work in identifying long-run shocks engenders a vast literature.[2] But almost from the outset, controversy surrounded one of its findings: the unemployment rate, ur_t , increases following a positive "supply" shock.[3] BQ explain that "*…in response to a positive supply shock, say an increase in productivity, aggregate demand does not initially increase enough to match the increase in output needed to maintain constant unemployment; real wage rigidities can explain why increases in productivity can lead to a decline"* over time (BQ, page 663).[4]

Two separate branches of literature stem from "supply-induced" ur_t increases. One prompted by Bayoumi and Eichengreen (1993) (henceforth, BE), "corrects" the BQ model by pairing output growth, Δy_t , with inflation, Δp_t , instead of ur_t .[5] Keating and Nye (1998) claim the purported

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[2] This includes tri-variate models exploring differences in macroeconomic fluctuations under the Gold Standard (see Bordo et al, 2010) and larger models for the U.S. economy (Gali, 1992 and 1999). King et al. (1991) and Pagan and Pesaran (2008) discuss the implications of cointegration in identifying temporary and permanent structural shocks. In addition, the long-run recursive identification strategy was extended to small-open economies by Lastrapes (1992) and Hoffmaister and Roldós (2001). For a comprehensive review the literature and a discussion of the limitations of long-run restrictions in identifying structural shocks see Kilian and Lutkepohl (2017, pp. 278–96).

[3] In other G7 countries, Keating and Nye (1999) also find that the BQ model results in "supply-induced" ur_t increases in France, Germany, Italy, and the U.K. Moreover, Blanchard (1989) finds supply-induced ur_t increases even if long-run restrictions are eschewed. That study notes that positive supply shocks "*…may well increase unemployment in the short-run if aggregate demand does not increase enough to maintain employment in the face of productivity innovations, or to increase employment in the case of increase in the labor force*." (ibid, page 1158).

[4] Burnichon (2010) also finds similar results using a model where Δy_t (*per capita*) is paired with ur_t using more recent U.S. data. While that study is closely related to BQ, it differs in its use an HP-filter to extract the "cyclical" component of both series and, following Shapiro and Watson (1988), uses IV estimation to recover technology and aggregate demand shocks in a model estimated with four (not eight) lags. The shorter lag length could explain that, while Burnichon (2010) replicates the supply-induced ur_t increases in the short-run, it does not show subsequent declines in ur_t (*ibid*, page 1017, Figure 2). Indeed, those IRF's align with those of the BQ variable pairing when the model is estimated with (shorter) optimal lags (see IRF's depicted in green, Figure A1, Appendix II.C).

[5] BE considers whether European countries (and U.S. regions) constitute an optimal currency area (OCA). For cross-country comparisons, that study stresses the need to avoid the implicit assumption of a common size (normalization)

superiority of this pairing arguing that: "…*the use of unemployment rate makes it difficult to find impulse responses to permanent output shocks that are ~~in~~consistent with the effects of aggregate supply shocks.*" (strike-out added, ibid, page 234).[6] That study stresses that, in principle, (positive) technology and labor-supply shocks can have opposing effects on ur_t —the former decreasing ur_t over time and the latter increasing ur_t on impact—thus biasing impulse response functions (IRF's). Moreover, Keating and Nye (1998) further argue that the BE variable pairing sidesteps this problem as both supply and labor-supply shocks reduce p_t .[7, 8]

The other branch of the literature, in contrast, embraces the BQ variable pairing and, in doing so, begets its own controversy. Specifically, a growing literature dating back to Gali (1999) notes that ur_t can increase following technological advances because, from a new-Keynesian view, technology shocks may decrease employment and hours worked. That study pairs Δy_t (per capita) with the *change* in hours worked and finds that hours decline following a positive supply

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in the identified structural shocks and use the reduced-from residuals' correlation matrix to recover structural shocks. For a recent discussion of the implications of such approach see Binet and Pentecote (2015). In addition, the BE model is used to explore whether: (1) macroeconomic fluctuations in G7 countries changed after the Bretton Woods System collapsed (Bayoumi and Eichegreen, 1994); (2) pre-World War I and post-World War II fluctuations differ in G7 countries (plus Denmark, Norway, and Sweden) (Keating and Nye, 1998); and, more recently, whether (3) earlier OCA evidence has changed (Campos and Machiarello, 2016, and Bayoumi and Eichengreen, 2020).

[6] This sentence comes at the end of a paragraph that makes clear that the authors meant to say "…*that are* ***consistent*** *with the effects of aggregate supply shocks*."

[7] This reasoning focuses on the impact effects and does not recognize the potential that, in the presence of wage rigidities, supply shocks can have opposing effects on prices over time. It also fails to account for the role of labor market information in the DGP for the U.S. post-World War II period. These issues are evident in the missing-variable test results and the augmented Keynesian framework discussed below.

[8] Keating (2013) notes two additional strengths of the BE variable pairing. First, it has a natural advantage because demand and supply models are formulated in terms of output and prices. But FL (page 350) note that the BE variable pairing needs to contend with potentially large price movements (such as those associated with the Korea War in 1950's), while the BQ variable pairing does not. And second, the BE variable pairing is ideal for "historical" studies because of the lack of robust unemployment data for earlier time periods. "*The unemployment data, even when collected, did not take on special significance until the Great Depression and the ascendance of the Keynesian theory of economic policy*." (Keating and Nye, 1999, page 266).

shock.[9] But Christiano, Eichenbaun, and Vigfusson (2003) argue that hours worked do not contain a unit root. That study finds an "expansionary-employment" effect when Δy_t is paired with (log-levels of) hours, highlighting the consistency with an alternative view of the economy, the real-business cycle model of Kydland and Prescott (1982).

A lively scholarly debate ensues from these divergent views fueled by contradictory empirical evidence, often reflecting the time-series properties of labor market variables (see Kilian and Lutkepohl, 2017, pp. 590–608). The debate has considered alternative empirical strategies and massive data efforts, notably Basu, Fernald, and Kimball (2006) (henceforth BFK). That study, using a modified growth-accounting framework to refine the measurement of the Solow residual, finds that directly measured "pure" technology shocks, Δbfk_t , are contractionary (on inputs).[10] Christiano, et al. (2004) provide however evidence against two key assumptions made in that study, namely the difference stationarity of hours and the exogeneity of technology shocks. Relaxing these assumptions reverses the effect of technology on hours.

But should the BQ and BE bivariate models be used to study macroeconomic fluctuations? Faust and Leaper (1997) (henceforth, FL) warn that stringent conditions are needed for multiple permanent and temporary shocks to be correctly sorted in these models.[11] Indeed, FL uncover a disturbing fact about the bivariate models' structural shocks: permanent shocks from one and temporary shocks from the other are more correlated (0.36 and 0.34) than permanent shocks across models (0.30) (Table 1)! FL interpret the "cross-type" correlation as evidence of mis-

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[9] Gali (1999) finds these results are robust to the inclusion of four (unidentified) temporary shocks.

[10] In stripping out cyclical effects from the Solow residual, BFK account for time-varying utilization of capital and labor, imperfect competition and returns to scale, and sectoral aggregation effects.

[11] In fairness, BQ acknowledge the potential for "co-mingling" of temporary and permanent shocks and discuss the conditions for multiple shocks to be sorted correctly.

aggregated shocks: "…*we see no strong a priori or empirical grounds for selecting between the two models and conclude that neither provides a reliable basis for structural inference*" (FL, page 351).

Table 1. Correlation of Structural Shocks

	Blanchard-Quah	
	Permanent	Temporary
Bayoumi-Eichengreen:		
Permanent	0.30	0.34
Temporary	0.36	0.63

Note. These calculations replicate Table 2 in Faust and Leeper (1997). Structural shocks are identified using the long-run restrictions in Blanchard and Quah (1989) and Bayoumi and Eichengreen (1993). VAR models are estimated with eight lags using quarterly data from 1950:Q2 to 1987:Q4.

Can mis-aggregation explain "supply-induced" ur_t increases? In this paper, I re-visit the reliability of the BQ and BE models and consider "supply-induced" ur_t increases from this point of view. Specifically, by taking advantage of advances in the understanding of, and testing for, missing-variables and fundamentalness, I trace the puzzling ur_t response back to FL's criticism and find evidence of a missing third variable and non-fundamentalness in both bivariate models.[12] I form a tri-variate model encompassing the BQ and BE models and identifying labor-supply shocks as well as technology and aggregate demand shocks (the identification combines long-run and short-run restrictions).[13] I find that the correlation between the structural shocks of the tri-variate and the bivariate models are suggestive of mis-aggregation (Table 2). Specifically, permanent shocks from the BQ model are correlated with the tri-variate model's technology

[12] These test results are robust to alternative deterministic specifications, optimally selected lag length, and extending the sample period to include the 1990's low-inflation period and the 2007–08 financial crisis.

[13] Blanchard (1989) argues that "supply" shocks variation is mostly due to technology not labor-supply and to "*deal separately with labor-supply and productivity disturbances…is beyond the scope of this paper*." (ibid., page 1149). Brinca et al. (2021) suggests labor-supply shocks explain a large share of the movements in hours worked in the U.S., particularly during the COVID-19 outbreak; for an alternative view see Fernald and Li (2021).

shock (0.48) and labor-supply shocks (0.83). Cross-type correlation with the tri-variate model's shocks is moderate for the BQ's permanent shock (0.20) but substantial for BQ's temporary shock (0.63). For the BE model, I find that its permanent and temporary shocks are highly correlated with their tri-variate counterparts but exhibit cross-type correlation.[14]

Table 2. Correlation of Structural Shocks including Labor Supply Shocks

	Tri-variate model		
	Permanent	Labor supply	Temporary
Blanchard-Quah:			
Permanent	0.48	0.83	0.20
Temporary	0.63	-0.50	0.46
Bayoumi-Eichengreen:			
Permanent	0.81	0.00	-0.34
Temporary	0.33	0.00	0.92

Note. For the tri-variate model, structural shocks are identified using the long- and short-run restrictions discussed in the main text; structural shocks for the Blanchard-Quah and Bayoumi-Eichengreen models are identified as in Table 1. All VAR models are estimated using eight lags using quarterly data from 1950:Q2 to 1987:Q4.

I also find that the tri-variate model's IRF's conform to their structural interpretation. Technology and labor-supply shocks persistently increase y_t and decrease p_t, and have opposite effects on ur_t: the former decreases ur_t and the latter increases ur_t. In addition, I find that predictions for wage responses from the illustrative Keynesian framework are borne out by the data. Using the tri-variate model as my benchmark, I document that BQ model's IRF's bias is small and the propagation of its shocks mimics the tri-variate model, except for the disputed "supply-induced" ur_t increases. In this case, the magnitude of the bias reverses the sign of the

[14] Note that the BE structural shocks are "empirically orthogonal" to labor-supply shocks (zero up to six decimals). This reflects the fact that BE shocks are highly correlated with the tri-variate model's permanent and temporary shocks that are, by construction, orthogonal to labor-supply shocks.

impact of technology in the short run. For the BE model, I find that the IRF's bias to be large for supply shocks but small for demand shocks.

And employing a novel decomposition of IRF's bias, I uncover distinct afflictions in the BQ and the BE variable pairings: the latter model endures missing-variable bias and the former model exhibits mis-aggregated supply shocks. Moreover, mis-aggregation does a good job of explaining "supply-induced" ur_t increases: the ur_t response to a combined technology and labor-supply shock from the tri-variate model replicates the puzzling BQ result (Figure 1).[15] In addition, the tri-variate model's key results are robust to a stochastic trend in ur_t .

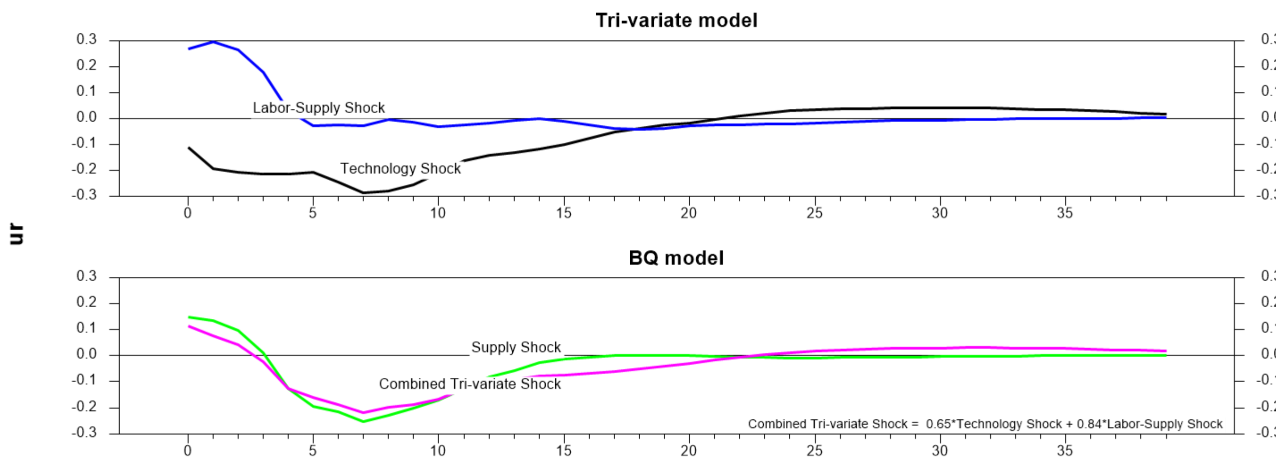

Figure 1. IRF's for the Tri-variate and BQ models

Given the new-Keynesian versus real business cycle debate noted, I also examine the relation between SVAR-based structural shocks and directly measured Δbfk_t . Two stylized facts emerge. First, SVAR-model demand shocks are not correlated to Δbfk_t suggesting an agreement between

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[15] The combined response is the fitted value of the OLS regression of the BQ model's IRF's for ur_t following a "supply" shock on the tri-variate model's IRF's for technology and labor-supply shocks. The regression's coefficient of determination is 0.88 and the coefficients for the tri-variate model's IRF's are 0.65 and 0.84 respectively for technology and labor-supply shocks; the null hypothesis of equal "weights" is rejected at the 1 percent level.

the methodologies in separating out shocks with permanent effects on output. And second, Δbfk_t is most correlated to BQ model's permanent and the tri-variate model's labor-supply shocks.

The main contributions of this paper are twofold. First, I propose a novel method to parse the IRF's bias into mis-aggregated shocks and MA-representation distortion. These distortions are related to non-fundamentalness and missing variables discussed in Lutkepohl (1991), Giannone and Reichlin (2006), Forni and Gambetti (2014), and Canova and Hamidi Sahneh (2018). The methodology can gauge the effects of these pathologies on the propagation of shocks at different time horizons and thereby complement the measurement of the "severity" of non-fundamentalness in Beaudry, Feve, Guay, and Portier (2019).[16]

Second by decomposing the IRF's bias in the BQ and BE models, I provide new insights into "supply-induced" ur_t increases that are independent of a stochastic trend in ur_t. For the BQ model, the overall bias reflects the effect of comingled *supply* shocks (with opposing effects on the pairing variable); the bias follows directly from that predicted by FL's *Proposition 2 part 2* (FL, page 349). As noted, a combined technology and labor-supply shock can reproduce BQ's "supply-induced" ur_t increases. Supportive evidence in this regard follows from the variance decomposition of ur_t pointing to the importance of labor-supply shocks in the short-run and supply shocks at longer horizons. For the BE model, the bias reflects the exclusion of relevant labor market information, and the consequent missing-variable bias. An ancillary result emerges for Δbfk. Not only is Δbfk correlated to labor-supply shocks as noted, but its propagation aligns

[16] That study shows that the R^2 of the regression of the structural shocks on the missing variable(s) provides a straightforward measure of the severity of the non-fundamentalness problem in models "testing positive" for non-fundamentalness. (The higher the R^2 the more information missing from the model, that is, the further from being fundamental.) In an illustrative example, Beaudry et al. (2019) show that an R^2 as small as 2 percent can trigger a rejection of the null hypothesis of fundamentalness.

well with labor-supply shocks. While surprising, this is consistent with the finding in Christiano et al. (2004) that hours Granger-cause Δbfk .

Granted, the empirical evidence in this paper is based solely on the U.S. post-World War II data and will need to be confirmed for earlier time periods and across countries. By the same token, further research will be needed to verify the usefulness of tri-variate model in studying macroeconomic fluctuations, including examining its: fundamentalness; explanation for the disputed "supply-induced" ur_t increases; and relation to Δbfk .

Methodologically, this study builds on the missing-variables IRF's bias literature, namely Braun and Mittnik (1993) (henceforth, BM), and follows Canova (2008) in recasting "co-mingling" in terms of that literature. The latter study stresses that missing-variables and mis-aggregation can provide an informative view of VAR representation problems.[17] Specifically, I derive a formula to parse the sources of IRF's bias—MA-representation distortion and mis-aggregation of shocks—using standard time-series notation. I show how this formula relates to the IRF's bias expression in BM, which is derived using (compact) lower-triangular Toeplitz matrix notation to simultaneously represent the IRF's for all shocks and up to a specific horizon of interest.[18] It seems fair to say that BM's non-recursive notation may have discouraged researchers from exploring the sources of IRF's bias to date. To my knowledge, this is the first study to do so.

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[17] Canova (2008) also highlights a deeper question for "small-scale" VAR models, associated with a growing literature going back to Hansen and Sargent (1980, 1991). Can structural shocks be recovered from the information contained in linear combinations of current and past values of the variables included in the model, that is, are these models "fundamental?" Fernandez-Villaverde et al. (2007) derive formal conditions for VAR modeling techniques to recover structural shocks, and Canova and Hamidi (2018) discuss empirically testing "fundamentalness."

[18] Appendix I details the correspondence between BM's *Proposition 2* (page 328) and the time-series notation formula in this paper; the latter can be implemented directly with available econometric software packages.

This paper also relates to the empirical literature studying the effects of labor-supply shocks. Specifically, it relates to earlier work associated with Shapiro and Watson (1988) that explores a related (larger) SVAR model that relies exclusively on long-run identification restrictions.[19] That study assumes hours worked to be exogenous in the long-run and characterized by a stochastic trend.[20] Following a technology shock, Shapiro and Watson (1988) find that hours decrease initially and increase thereafter (ibid. page 126, Figure 2) consistent with BQ's "supply-induced" ur_t increases. At first, this seems to run counter to my claim that mis-aggregation underlies the puzzling ur_t response (that study, after all, distinguishes between technology and labor-supply shocks). But the long-run exogeneity of hours assumption requires restricting to zero its long-run response to technology. I find that replacing the long-run exogeneity of labor-supply with a less a restrictive identification assumption—namely ur_t does not respond on impact to technology (or demand) shocks—resolves the apparent contradiction; this holds whether or not ur_t contains a stochastic trend. In other words, the "supply-induced" ur_t increases in Stock and Watson (1988) do not reflect mis-aggregation but the restriction that technology not affect ur_t in the long run.

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[19] Note that that study's complete model contains two (not separately identified) "demand" shocks that "…*allow output and labor to move independently of the labor and productivity shocks in the short-run*…" (ibid. page 115) as well as an ad-hoc exogenous oil shock that is found to play, on average, a minor role in explaining macroeconomic fluctuations (ibid. page 144).

[20] In addition, Δp_t is also modeled as an integrated process, and thus that study's core model comprises: $\Delta hours_t$, Δy_t, and $\Delta^2 p_t$. And its identification is based on the following long-run-restrictions: (1) aggregate demand shocks not affect output (as in this paper) nor (2) do they affect hours, and (3) technology shocks not affect hours. The last two restrictions impose the assumption of an exogenous long-run (level of the) labor-supply and thus real wage growth does not affect labor-supply in the long-run. That study notes that relaxing assumption (3) would not affect "*…the decomposition between supply and demand…*" and would "*…**only** affect the decomposition…into labor-supply and technology…*" (emphasis added, ibid page 114). Indeed, as noted below, the variance decomposition in this paper (Table 4) confirms that the supply-demand divide for y_t and p_t is unaffected (and almost identical to Table 2, ibid, page 128). But the imposition of (3) downplays the role technology (versus labor-supply) in explaining hours. That study's conclusion that "*Increases in technology have little effect on hours*." (ibid. page 127) follows directly from the resulting IRF's depicted in Figure 2 and the (miniscule) fraction the variance of hours associated with technology in Table 2 (ibid. pages 126–8).

In addition, this paper relates to more recent literature associated with Brinca, Duarte, and Faria-e-Castro (2021). That study finds that labor-supply underlies recent movements in hours worked (using a bivariate VAR model for changes in both real wages and hours worked). Besides using sectoral (panel) data, that study employs a Bayesian methodology to identify structural shocks using sign restrictions.[21] This agnostic approach is broadly consistent with Keating (2013) that stresses that many well-regarded macro models exhibit non-neutrality of aggregated demand shocks.[22] But by using (static) sign restrictions, Brinca et al. (2021) implicitly assume that wages react contemporaneously to shocks, otherwise wage rigidities can result in time-smearing.[23] In addition, the identification of labor-supply versus labor-demand shocks can run into trouble with technology shocks unless technology and hours are positively correlated (as predicted by RBC-type models).[24] Whether agnostic sign restrictions or the long-run and short-run restrictions (favored in this study) are better able to identify labor-supply shocks will depend on the validity of the restrictions. As noted, I provided evidence of the tri-variate model's ability to recover structural shock independently of its estimation and identification.

[21] Brinca, et al. (2021) follow the general Baumeister and Hamilton (2015) methodology applicable to any market with price and quantity data. Brinca et al (2021) assume that labor-demand and labor-supply shocks have standard (sign) effects on real wages and hours. Their informative prior borrows available slope estimates to calibrate a truncated Student's *t* distributions describing slope parameters. Using Bayesian techniques, the posterior distribution combines the informative prior and the data.

[22] That study stresses the non-equivalency of temporary shocks and demand shocks and notes that avoiding long-run restriction on output (neutrality of demand shocks) can deepen our understanding of the effects and importance of permanent and temporary shocks.

[23] Consider a stylized wage-setting mechanism where wages are set one period in advance, as in BQ and consistent with Calvo-type wage setting and "equilibrium" wage setting in Hall (2005). In this context, (static) sign restrictions hours will reflect the current shock but wages will reflect last period's shock. This timing error can distort the relative effect of shocks on hours and wages unless successive shocks have the same sign and size.

[24] When technology and hours are negatively correlated (the new-Keynesian view), static sign restrictions can be hard pressed to distinguish labor-supply from labor-demand shocks associated with technology shocks. The sign identification breaks down as labor-supply and technology shocks both reduce hours and increase wages.

The paper is organized as follows. Section II provides evidence of missing-variables and fundamentalness for the BQ and BE models. Section III discusses the tri-variate model, its identification and IRF's and contrasts these to the bivariate models' IRF's; the effect of a stochastic trend in ur_t is also discussed. Section IV derives the novel expression for the sources of IRF's bias and applies the methodology to the bivariate models. Section V contrasts SVAR-based structural shocks with the directly measured Δbfk_t. Section IV briefly concludes.

II. Missing Third Variable and Fundamentalness

This section focuses on the BQ and BE models' ability to characterize the data generating process (DGP) by testing for a missing (third) variable and for fundamentalness. Of note, the evidence in this section requires taking a stand only on the specification and identification of these bivariate models and is robust to alternative specifications of the deterministic components, optimally selected lags, and in an extended sample period.[25]

A. Data and VAR Preliminaries

Test are performed (and underlying the bivariate models are estimated) using BQ's original quarterly data and sample period. These data comprise the U.S. post-World War II data from 1950:Q2 to 1987:Q4 for seasonally adjusted GNP and the unemployment rate for males, ages 20 and over from "*BLS, February issue, 1982, Table A-39*" (BQ, page 661, footnote 5); the seasonally adjusted GDP deflator is used to deflate GNP.[26] Note that the lag length for all VAR models is set to eight following BQ and FL and, throughout this study, the base-case results are

[25] Appendix II contains a robustness exercise for the main results in this paper.

[26] The data are from the RATS replication code: https://www.estima.com/procs_perl/700/blanchardquahaer1989.zip. The extended sample for the robustness exercise runs through 2009:Q4; these data are downloaded from the FRED database (October 2021 vintage): GNP, UNRATE (number of people 16 and over actively searching for a job as a percentage of total labor force), and GDPDF. The quarterly unemployment series averages the monthly series.

computed when the means of all variables are extracted before and after the 1973–74 oil shock.[27] While this treatment of the deterministic components for ur_t deviates from BQ and FL, it avoids the pathologies documented for the detrended ur_t case.[28]

B. Missing-variable and non-fundamentalness tests[29]

To fix ideas, consider a general *k*-variable VAR model (of order *p*) in Y_t partitioned as:

$$\begin{bmatrix} I - C^{(1,1)}(L) & -C^{(1,2)}(L) \\ -C^{(2,1)}(L) & I - C^{(2,2)}(L) \end{bmatrix} \cdot \begin{bmatrix} Y_{1,t} \\ Y_{2,t} \end{bmatrix} = \begin{bmatrix} U_{1,t} \\ U_{2,t} \end{bmatrix}$$

where $Y_{1,t}$ and $Y_{2,t}$ denote respectively k_1 variables included in the sub-model under review and k_2 potentially missing variables; $k = k_1 + k_2$ and $E\left[U \cdot U'\right] = \Omega$.[30]

Missing-variable tests. $Y_{2,t}$ is missing from the k_1-variable sub-model for $Y_{1,t}$ when $Y_{2,t}$ Granger-causes (GC) $Y_{1,t}$. This can be tested directly as $H_0 : C^{(1,2)}(L) = 0$ or as a Sims test. Sims (1972) proves that in the regression:

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[27] The original BE study uses annual data and sets the lag length equal to two lags.

[28] These problems are discussed in the robustness exercise (see Appendix II). It is important to note nonetheless that base-case results in BQ are robust to alternative deterministic specifications, including, as in this paper, the removal of means before and the after the 1973–74 oil shock (BQ, page 661).

[29] These tests are performed with the information set spun by the BQ and BE models. Thus, the implementation of fundamentalness tests here differs from their typical application based on information comprising dozens, if not hundreds, of time series. Those series are used to extract principal components and test for fundamentalness by examining their statistical significance. Still, a rejection of fundamentalness with a single time series (as is the case in this paper) is telling as it means that the series is useful to forecast and/or predict the structural (or reduced-form) shocks from the k_1 -variable VAR model in question. But a non-rejection is less trustworthy because it is conditional on the information in a single series and subject to being overturned in a richer information set.

[30] For simplicity, I focus the discussion on "system-wide" tests as the "equation-by-equation" or "shock-by-shock" tests stem directly from these expressions. See Appendix III for details.

$$Y_{2,t} = \left\{ I - D_1(L) \right\} \cdot Y_{1,t} + B_1(F) \cdot Y_{1,t} + \xi_t$$

where $B_1(F)$ denotes (not a lag polynomial but) a "forward" polynomial, $Y_{2,t}$ GC $Y_{1,t}$ if and only if $B_1(F) \neq 0$. The null hypothesis for the Sims test is thus $H_0 : B_1(F) = 0$, that is, future values of $Y_{1,t}$ (conditioned on past and present values of $Y_{1,t}$) do not help predict $Y_{2,t}$.

Non-fundamentalness tests. Forni and Gambetti (2014) and Canova and Hamidi Sahneh (2018) propose to test for fundamentalness—defined heuristically as the structural shocks from the k_1-variable VAR model can be recovered from the information spun by $Y_{1,t}$—respectively with GC-type and Sims-type tests where $Y_{1,t}$ is suitably redefined.[31] Specifically, the former study proposes an orthogonality (OR) test based on a GC-type test where $Y_{1,t}$ denotes *structural* shocks from the k_1-variable model. In this case, $H_0 : C^{(1,2)}(L) = 0$ implies that the information in $Y_{2,t}$ is not useful in forecasting the model's *structural* shocks and thus the k_1-variable VAR model is fundamental. The latter study proposes the Canova-Hamidi (CH) test based on a Sims-type test where $Y_{1,t}$ denotes *reduced-form* shocks from the k_1-variable VAR model. In this case $H_0 : B_1(F) = 0$ implies that the future values of the reduced-form shocks from the k_1-variable VAR model are not useful in predicting $Y_{2,t}$ so that the k_1-variable VAR model is fundamental.

[31] In the typical implementation of these tests, $Y_{2,t}$ would also be redefined as one or more principal components obtained from large set of relevant time-series.

Empirical results. I use all four tests, thus taking an agnostic view on Canova and Hamidi Sahneh (2018).[32] That study notes the use of GC-type test in the (empirical) fundamentalness literature (Lutkepohl, 1991, and Giannone and Reichlin, 2006). But, while conceding the usefulness of these tests to examine missing variables, CH argue that GC-type test are not reliable tests of fundamentalness. This is particularly troublesome when a model includes cross-sectionally aggregated or proxy variables; this issue also afflicts the (GC-type) OR test for the unpredictability of structural shocks.[33] Forni, Gambetti, and Sala (2018) argue however that a truly structural "*aggregate*" model does not exist.

Model-wide and equation-by-equation test results (for $k_1 = 2$ and $k_2 = 1$) are reported in Table 3. Consider the BQ variable pairing tests (Panel A), $Y_{1,t} = \left[\Delta y_t, \quad ur_t\right]'$ and $Y_{2,t} = \left[\Delta p_t\right]$. At the 5 percent significance level, model-wide missing-variable results are mixed: the GC test of the lags of Δp_t are significant but the Sims test of the leads of the BQ variables are not (in the "Sims-regression" for Δp_t). Regarding fundamentalness tests, $Y_{1,t} = \left[\varepsilon_t^{BQ,S}, \quad \varepsilon_t^{BQ,D}\right]'$, the OR test rejects the null: the lags of Δp_t in the corresponding bivariate VAR model are not zero. In contrast, the CH test, $Y_{1,t} = \left[\mu_t^{BQ,\Delta y}, \quad \mu_t^{BQ,ur}\right]'$, does not reject the null as the leads of the BQ model's reduced-form residuals in the corresponding Sims-regression are not statistically

[32] To account for potential serial correlation in the original Sims test formulation, I use the Geweke, Meese, and Dent (1983) version of the Sims test (adding lagged dependent variables); I also use this specification for the CH test as in Forni, Gambetti, and Sala (2018). Either way, the tests results are (qualitatively) unaltered.

[33] Consider a VAR model with a cross-sectionally aggregated variable. Each variable forming the aggregate will Granger-cause the VAR model, regardless of whether the model is fundamental or not (see CH, page 1070).

different from zero. The equation-by-equation test results consistently point to missing-variables in the Δy_t equation and "non-fundamentalness" in the ur_t equation.[34]

Table 3. Missing Third Variable and Fundamentalness Tests for BQ and BE models.

A. BQ model (Third variable: Δp_t)								***B. BE model (Third variable: ur_t)***						
Model-wide			Equation					Model-wide			Equation			
				Δy_t		ur_t						Δy_t		Δp_t
1-Granger-Causality test for:														
F(16,101)	21.39		F(8,127)	2.45		1.79		F(16,101)	16.27		F(8,127)	3.20		1.85
M-Signif	0.00	*	M-Signif	0.02	*	0.08		M-Signif	0.00	*	M-Signif	0.00	*	0.07
2-Sims-exogeneity test of:														
F(16,100)	1.46		F(8,117)	2.22		1.42		F(16,100)	2.09		F(8,117)	3.88		1.13
M-Signif	0.13		M-Signif	0.03	*	0.19		M-Signif	0.01	*	M-Signif	0.00	*	0.35
3-OR (Forni-Gambetti) test for:														
	$\varepsilon_t^{BQ,S}+\varepsilon_t^{BQ,D}$			$\varepsilon_t^{BQ,S}$		$\varepsilon_t^{BQ,D}$			$\varepsilon_t^{BE,S}+\varepsilon_t^{BE,D}$			$\varepsilon_t^{BE,S}$		$\varepsilon_t^{BE,D}$
F(16,93)	2.61		F(8,126)	1.57		2.27		F(16,93)	2.88		F(8,126)	3.86		0.88
M-Signif	0.00	*	M-Signif	0.14		0.03	*	M-Signif	0.01	*	M-Signif	0.00	*	0.53
4-CH (Canova-Hamidi) test for:														
	$\mu_t^{BQ,\Delta y}+\mu_t^{BQ,ur}$			$\mu_t^{BQ,\Delta y}$		$\mu_t^{BQ,ur}$			$\mu_t^{BE,\Delta y}+\mu_t^{BE,\Delta p}$			$\mu_t^{BE,\Delta y}$		$\mu_t^{BE,\Delta p}$
F(16,92)	1.65		F(8,109)	0.63		2.39		F(16,92)	1.76		F(8,109)	2.88		0.90
M-Signif	0.07		M-Signif	0.75		0.02	*	M-Signif	0.05	*	M-Signif	0.01	*	0.52

Note. The Granger-causality tests the lags of the corresponding missing third variable in an augmented bivariate VAR model, which is estimated with eight lags. The Sims-exogeneity tests the leads of the BQ or BE variables in the regression where the third variable is regressed on its lags plus the leads and lags of the corresponding bivariate model. The OR test is a GC-type test for the third variable in a VAR model composed of the structural shocks in the corresponding bivariate model. The CH test is a Sims-exogeneity-type test that tests the leads of the BQ or BE variables in a the regression where the the third variable is regressed on its lags plus the leads and lags of the reduced-form residuals from the corresponding bivariate model. For all tests, the reduced-form residuals and structural shocks are obtained from the estimated BQ and BE models using eight lags. An "asterik" indicates rejection of the null hypothesis at the 5 percent significance level.

[34] Note that here I use the fundamentalness concept in the spirit of Forni and Gambetti (2014). That study argues that it is possible that a model-wide test reject fundamentalness but the information in the VAR could be "sufficient" to recover a single or sub-set structural shocks.

Consider next the BE variable pairing tests (Panel B), $Y_{1,t} = \left[\Delta y_t ,\quad \Delta p_t \right]'$ and $Y_{2,t} = \left[ur_t \right]$. Model-wide tests do not favor the BE variable pairing: both tests for a missing third variable reject the null hypothesis that the third variable (ur_t) is not in the model, and both fundamentalness tests $\left(Y_{1,t} = \left[\varepsilon_t^{BE,S},\quad \varepsilon_t^{BE,D} \right]' \text{ and } Y_{1,t} = \left[\mu_t^{BE,\Delta y},\quad \mu_t^{BE,\Delta p} \right]' \right)$ reject fundamentalness. These problems also appear in the equation-by-equation test results for Δy_t but not for Δp_t .

I conclude that ur_t cannot be excluded from the DGP without biasing the bivariate model's VAR coefficient estimates and, within the information set spun by these bivariate models, Δp_t should not be excluded as this can hinder the recovery of structural shocks. I thus explore a tri-variate model combining these models.[35]

III. IRF'S: TRI-VARIATE MODEL VERSUS BQ AND BE MODELS

I propose a straightforward extension of BQ's illustrative Keynesian framework to identify the structural shocks of the tri-variate model. The modified framework implies less restrictive assumptions on labor-supply compared to a long-run recursive model.[36] The corresponding IRF's

[35] I also find evidence in favor of combining the BQ and BE models from unreported non-nested hypothesis tests based the *C*-test and the *J*-test from the artificial-regression methodology in Mackinnon (1983). The tests reject the DGP for the common dependent variable, Δy_t , with the *J*-test rejections are "in the direction" of the competing model, thus favoring a combined model. See Pesaran and Weeks (2001) for a discussion of non-nested tests.

[36] The long-run recursiveness implies that aggregate demand shocks not have (1) long-run effects on y_t , but also imply that: (2) demand shocks not have an effect on the "*accumulated sum*" of ur_t , and (3) labor-supply shocks not have a long-run effect on y_t . To avoid (2) and (3), one could assume that the labor market variable is exogenous in long-run, as in Shapiro and Watson (1988). Specifically, by including a unit root in ur_t and swapping its position with Δy_t , $Y_t \equiv \left[\Delta ur_t,\ \Delta y_t ,\ \Delta p_t \right]'$, the long-run recursiveness now implies that: (1) technology and

are computed and compared to those from (replicated) BQ and BE models; the impact of a unit root in ur_t is discussed. Note that the empirical results in this section (and in Section IV) are conditional on the identification strategy of the tri-variate and bivariate models but are robust to optimally selected lags and an extended sample (see Appendix II).

A. An Illustrative Three-Shock Keynesian Model[37]

To identify the tri-variate model's shocks, while maintaining close comparability to the BQ and BE models, I make two straightforward modifications to BQ's illustrative Keynesian framework. I introduce labor-supply shocks by replacing the constant full employment level with the stylized assumption that the labor force follows a random walk driven by labor-supply shocks. And I relax the (implicit) assumption that aggregate demand shocks not have an impact effect on prices by including money directly in the price-setting equation.[38] The modified framework borrows

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(2) aggregate demand shocks not have long-run effects on ur_t ; and (3) aggregate demand does not have long-effects on y_t . Consistent with BQ, the identification in this paper eschews a unit root in, and restrictions on, ur_t . Regardless, the main conclusions from the tri-variate model are robust to a stochastic trend in ur_t .

[37] Compared to the prototypical three-equation new-Keynesian model in Gali (2018), this framework does not model monetary policy shocks, nor does it include an interest-rate rule. Instead, the framework implicitly assumes a simplified money market where shocks to nominal money are, in essence, shocks to (the inverse of) the velocity of money. These shocks may have short-run macroeconomic effects but are taken as neutral in the long-run. The "three-shock" model in this paper is thus not equipped to consider monetary policy and its macroeconomic effects. Specifically, the question of whether expansionary monetary policy can offset the contractionary effects of technology cannot be meaningfully addressed nor inferred from the results below.

[38] Note that, while grounded in the velocity theory of money, adding money to the price-setting equation is essential for technical reasons. Otherwise, the solution for Δp_t imposes the restriction that demand shocks not have an impact effect on p_t . This would imply a non-sensical (and over-identifying) restriction that the variance of demand shocks is zero! This can be verified by setting the elasticity of p_t with respect to nominal balances, λ, equal to zero in the solution for p_t below (see also Appendix IV). This issue does not emerge in the BQ variable pairing because Δp_t is not part of the model. Note further that I implicitly assume that λ is between zero and one.

BQ's long-run neutrality assumption and its wage-setting mechanism; the latter assumes wages are set one period in advance and consistent with expected full employment.[39]

Tri-variate model solution. The augmented illustrative model can be solved as the tri-variate system:[40]

$$\Delta y_t = e_{s,t} + \theta \cdot \Delta e_{s,t} + e_{\tilde{n},t-1} + (1-\lambda) \cdot \Delta e_{d,t}$$

$$ur_t = -\theta \cdot e_{s,t} + e_{\tilde{n},t} - (1-\lambda) \cdot e_{d,t}$$

$$\Delta p_t = -e_{s,t} + \theta \cdot e_{s,t-1} - e_{\tilde{n},t-1} + \lambda \cdot e_{d,t} + (1-\lambda) \cdot e_{d,t-1}$$

where $e_{s,t}$ and $e_{d,t}$ denote the "supply" (or technology) and "demand" (temporary) shocks in BQ and $e_{\tilde{n},t}$ denotes labor-supply shocks. The latter can be interpreted as a shock emerging from labor-participation decisions in an involuntary unemployment model as in Christiano, Trabandt, and Walentin (2021).[41] Viewed from this prism, a (positive) $e_{\tilde{n},t}$ shock reflects a low "aversion-to-work" realization prompting increases in the labor force that, given the prevailing wage, increases (involuntary) ur_t. Note that the modified framework here is not rich enough to discuss business cycle effects on labor participation (and ur_t) discussed in that study. Here, labor

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[39] This stylized wage-setting mechanism is broadly consistent with a Calvo-type wage setting and recent "equilibrium" wage stickiness in Hall (2005). The latter study shows that in a with equilibrium wage stickiness, technology advances reduces ur_t ; Hall (2005) documents this effect empirically using a univariate model for ur_t . Of note, the propagation of technology advances in that study is consistent with the results in this paper.

[40] Appendix IV details the modified model and its solution (including for its "*fourth*" variable). Note that the tri-variate model's solution reproduces the BQ model solution when $\lambda = 0$ and $e_{\tilde{n},t} = 0$.

[41] In that study, unemployment reflects the lower utility of the unemployed (versus the employed) inducing workers to embark on costly job-search activities whenever their aversion-to-work does not exceed a threshold. Each period, a worker privately observes a random draw of their "aversion-to-work" and workers whose realization does not exceed their threshold pursue costly job search activities (becoming part of the labor force) and with a specific probability find a job. In expected terms the unemployment rate equals 1 minus this probability. Those whose "adversion-to-work" realization is high exit the labor force (so-called discouraged workers).

participation decisions are associated exclusively to $e_{\tilde{n},t}$, which is orthogonal to $e_{d,t}$. The latter would however capture the intensification of job search of those already in the labor force.[42, 43]

It is important to note that, just as the original BQ framework, the modified framework provides solutions for four variables. In this paper, the fourth variable is wage growth:

$$\Delta w_t = \theta \cdot e_{s,t-1} - e_{\tilde{n},t-1} + (1-\lambda) \cdot e_{d,t-1} .$$

It is useful to keep this solution in mind for the intuition of the identification restrictions. I also use this solution to verify the consistency of the tri-variate model's structural shocks, independently of the model's estimation and identification.[44] This is possible because Δw_t is not part of the tri-variate model's estimation nor identification and thus constitutes an independent "duck-test" for the structural shocks.[45]

[42] The orthogonality does not capture wealth effects on labor participation from $e_{s,t}$ and $e_{d,t}$ in Gali (2011) and Gali et al. (2011). Christiano et al. (2021) note however that the resulting labor-supply declines are counterfactual.

[43] Alternatively, viewed from the prism of Hall (2008), $e_{\tilde{n},t}$ combines shocks to the employment supply and labor-force supply curves. The illustrative framework solution suggests that $e_{\tilde{n},t}$ increases ur_t, implying that reductions in ur_t from an employment shock are more than offset by increases from a labor-force supply shock. Also note that Hall's labor-demand shock combines $e_{d,t}$ and $e_{s,t}$ that both reduce ur_t (and increase wages as discussed below).

[44] Note that since $\Delta w_t = -ur_{t-1}$, an alternative tri-variate model could be defined using Δw_{t+1} or Δw_t as the labor market variable. Neither is appealing: the former introduces endogeneity in the corresponding VAR equation; the latter introduces three overidentifying restrictions. More importantly, neither provides direct comparison to BQ.

[45] The duck test relies on "abductive reasoning" yielding a plausible conclusion but not definitive proof: "If it looks like a duck, walks like a duck, and quacks like a duck, it's *probably* a duck."

Identifying restrictions. Three identifying restrictions emerge from the tri-variate model's solution:

$$A_0 = \begin{bmatrix} * & 0 & * \\ * & * & * \\ * & 0 & * \end{bmatrix} \qquad A(1) = C(1) \cdot A_0 = \begin{bmatrix} * & * & 0 \\ * & * & * \\ * & * & * \end{bmatrix}$$

where, using Hamilton (1984) notation, A_0 and $A(1)$ denote respectively arrays grouping the contemporaneous impact and the long-run effects of structural shocks (with their typical elements denoted by $a(0)(i,j)$ and $a(1)(i,j)$).[46] Here asterisks represent unrestricted elements and zeros indicate the identifying restrictions, which together with the orthogonality and normalization of the structural shocks exactly identify the model.[47,48]

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[46] Specifically, I use Hamilton's equation 11.4.22 (page 323) to express the underlying structural tri-variate and reduced-form models respectively as: $Y_t^{3V} = A(L) \cdot E_t^{3V}$ and $Y_t^{3V} = C(L) \cdot U_t^{3V}$ with $Y_t^{3V} = \left[\Delta y_t , \quad ur_t , \quad \Delta p_t \right]'$, $C(L) = \left[I - C_1 \cdot L - C_2 \cdot L^2 - \cdots - C_p \cdot L^p \right]^{-1}$, and $E_t^{3V} = \left[e_{s,t}, \quad e_{\tilde{n},t}, \quad e_{d,t} \right]'$. C_s and E_t^{3V} contain respectively the VAR model's coefficients for lag *s* and the orthogonal structural shocks obtained by post-multiplying their reduced-form counterparts by A_0; the latter satisfies $A_0 \cdot A_0' = \Omega^{3V}$ where Ω^{3V} denotes the VAR model's reduced-form covariance matrix.

[47] The tri-variate model conforms to the necessary and sufficient rank conditions to just-identify VAR models in Rubio-Ramirez, et al. (2010). Specifically, for VAR models with both short- and long-run restrictions, these amount to a "restriction-counting" exercise: the combined sum of the number of restrictions on the structural shocks in the columns of A_0 and $A(1)$ must be (in any order) exactly 0, 1, and 2.

[48] I obtain A_0 as the solution to the following set of (nine) non-linear equations:

$$\left[\mathrm{vech}\left(A_0 \cdot A_0 ' - \Omega^{3V} \right), \quad a(0)(1,2), \quad a(0)(3,2), \quad a(1)(1,3) \right]' = \left[0, \quad 0, \quad 0, \quad \cdots, 0 \right]',$$

where $\mathrm{vech}\left(A_0 \cdot A_0 ' - \Omega^{3V} \right)$, $a(0)(.\ ,\ .)$, and $a(1)(.\ ,\ .)$ denote respectively the "vectorization" of the six conditions associated with orthogonality of structural shocks (and their normalized variances) and the tri-variate model's restrictions. To solve these equations, I use the Newton-Raphson method (see Kilian and Lutkephol, 2017, pp. 310–16) starting the algorithm with an initial $\hat{A}_0$ equal to the Cholesky decomposition of Ω^{3V} and an initial $\hat{A}(1)$ equal to the Cholesky decomposition of $C(1) \cdot \Omega^{3V} \cdot C(1)'$ (the factor for a long-run recursive tri-variate model). The solution algorithm iterates until the squared norm of the nine non-linear equations is less than 0.000001.

Consider the identifying restrictions from the augmented framework. The solution for Δy_t implies that $e_{d,t}$ not have long-run effects on y_t (long-run neutrality). This solution also imposes that $e_{\tilde{n},t}$ shocks do not have an *impact* effect on y_t reflecting the fact that $e_{\tilde{n},t}$ only increases the labor force (and ur_t) on impact but not employment; increases in y_t and employment follow once w_t are revised down. The solution for Δp_t provides a third restriction: $e_{\tilde{n},t}$ shocks not have an impact effect on p_t . This is because $e_{\tilde{n},t}$ affect prices indirectly through their effect on w_t (that take one period to adjust). Note that the wage-setting assumption also implies that declines in p_t following $e_{s,t}$ shocks are (partially) offset by the delayed w_t effects; $e_{s,t}$ shocks thus result in "under-shooting" p_t . Note that the delayed effects of $e_{s,t}$ and $e_{\tilde{n},t}$ are of opposite signs.[49]

Before turning to the tri-variate model's IRF's, it is important to note that although the ur_t solution does not provide identifying restrictions, as in the BQ model, it implies that both $e_{s,t}$ and $e_{d,t}$ reduce ur_t . These shocks reflect shifts in the labor-demand curve that, for an unchanged w_t , increases employment. But $e_{\tilde{n},t}$ increases ur_t because it increases the labor force (shifts out labor-supply)—that, as noted, can arise from a low "aversion-to-work" realization—while employment (quantity of labor demanded) remains unchanged for a given w_t .

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[49] What does the tri-variate model's solution imply for the BQ's and BE's models pairings? In both models $e_{s,t}$ shocks (scaled by θ) are combined (mis-aggregated) with $e_{\tilde{n},t}$ shocks, which have opposing effects on the pairing variable. As noted, FL would argue that because in the reference model $e_{s,t}$ and $e_{\tilde{n},t}$ have opposing effects on ur_t , the bivariate BQ solution does not meet the conditions in Proposition 2, part 2 (FL, page 349). An analogous problem emerges for the BE solution, though the opposing effects are for the dynamics.

B. Impulse Response Functions

Impulse response functions are defined and computed in the standard way,

$$\frac{\partial Y_{t+s}^{3V}}{\partial e_{j,t}} = \Psi_s^{3V} \cdot p_j^{3V}, \text{ for } s = 0, 1, 2, 3, ...$$

where Y_{t+s}^{3V} and $e_{j,t}$ denote respectively the variables in the tri-variate model (Δy_t, ur_t, and Δp_t), and the j^{th} structural shock; Ψ_s^{3V} and p_j^{3V} denote respectively the (3×3) matrix grouping the lag-*s* coefficients of the VAR model's MA-representation and the j^{th} column from the tri-variate model's identification factor, A_0. Foreshadowing the discussion of the sources of IRF's bias, I express the IRF's as the following function:

$$IRF_{j,s}\left(\Psi_s^{3V}, p_j^{3V}\right) = \Psi_s^{3V} \cdot p_j^{3V}, \text{ for } s = 0, 1, 2, 3, ...$$

Figure 2 depicts the IRF's for the tri-variate's structural shocks (labeled *3V*); the model is estimated with eight lags over the BQ sample period noted above. Using the tri-variate model as the DGP, bootstrapping methods are used to generate 2,000 quasi-data samples and repeatedly estimate the tri-variate model, compute IRF's, and generate confidence bands at the 68 and 95 percent levels. To facilitate comparisons, Figure 2 also depicts the IRF's (point estimates) for the replicated BQ and BE models (labeled *BQ* and *BE*).

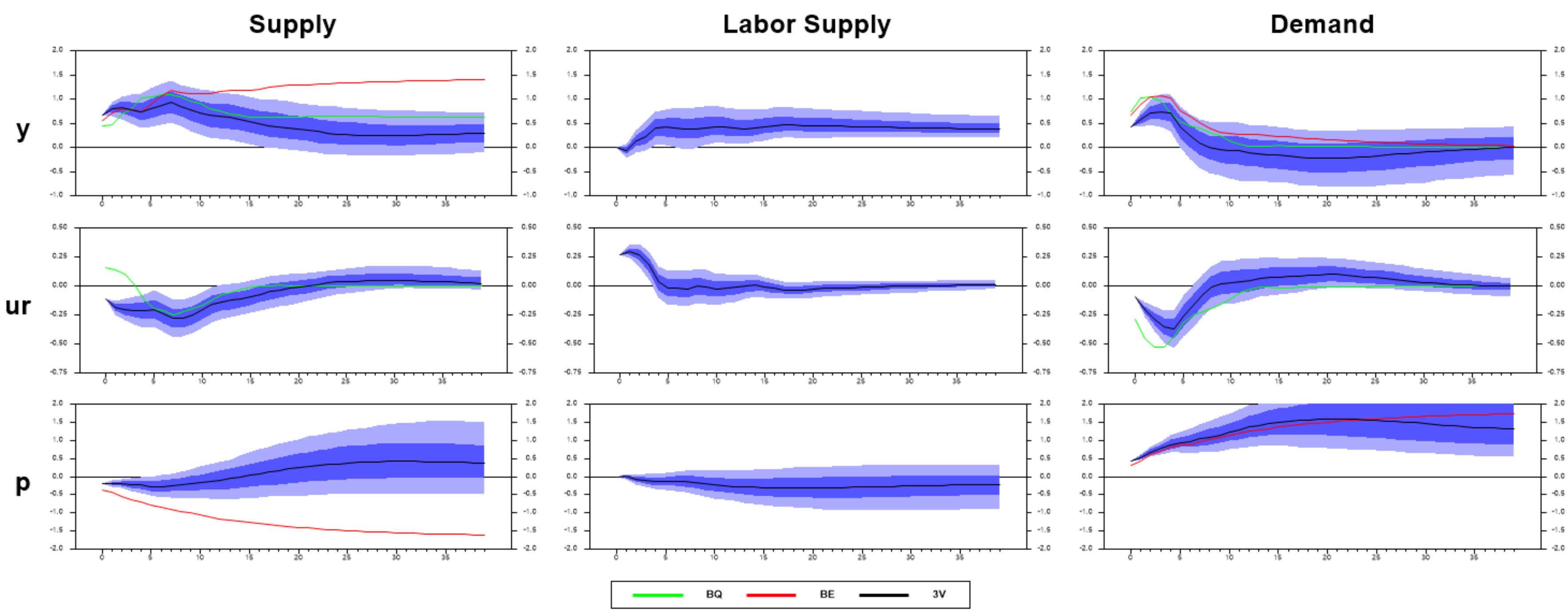

Figure 2. IRF's for the Tri-variate model (shaded 68% and 95% bands), BQ, and BE models

The IRF's are consistent with the structural shocks' interpretation from the modified Keynesian framework.[50] A supply shock increases y_t persistently and lowers ur_t in the short run. p_t decline on impact and further decline slightly thereafter but tend to increase over the long term. A labor-supply shock temporarily increases ur_t and does not affect y_t nor p_t on impact (by construction). Subsequently, a labor-supply shock results in lasting increases in y_t and reductions in p_t. A demand shock results in the familiar hump-shaped response in y_t (vanishing by construction) and a virtual mirror-image response for ur_t. p_t increase through the medium term and falls back slightly in the long run.

Contrasting these to the bivariate models IRF's, I note that for:

- *Supply shocks*. The BQ model's y_t responses are qualitatively the same to the tri-variate model, though understating short-run and overstating long-run effects. As noted, "supply-induced" ur_t increases appear to combine the opposing forces in its "comingled" permanent shock. The BE's model's y_t and p_t responses differ markedly with increases resulting in large deviations over time.
- *Demand shocks*. The BQ and BE model's y_t responses exhibit standard hump-shaped responses that vanish over time. Regarding the pairing variables, the BQ model's ur_t response is qualitatively the same as that of the tri-variate response,

[50] Shapiro and Watson (1988) find technological advances decrease hours in the short-run. FG report a consistent (though more persistent) result for ur_t when the information content of a BQ-related bivariate model is augmented (with at least one principal component) (FG, Figure 2, page 134). In both studies, the propagation is reminiscent of that of the BQ model and consistent with Gali (1999) where, as noted, hours include a stochastic trend.

though it overstates declines; the BE model's p_t response is qualitatively the same as the tri-variate model but overstates p_t increases in the long-run.[52]

S***tructural shock propagation for wages and the labor force***. Further evidence of the validity of the interpretation of the tri-variate model's structural shocks stems from the responses of variables not included in the model. I use BFL's near-VAR framework to trace the dynamic responses of a single variable of interest, x_t, to exogenously determined shocks, ξ_t. The near-VAR model can be expressed as:

$$\begin{bmatrix} \Delta S_t \\ x_t \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ -c^s_{2,1} & 1-c_{22}(L) \end{bmatrix}^{-1} \cdot \begin{bmatrix} \xi_t \\ \mu_t \end{bmatrix}$$

where ΔS_t denotes a vector of random walks associated with tri-variate model's structural shocks, $\xi_t = \left[e_{s,t}, e_{\tilde{n},t}, e_{d,t}\right]'$, and μ_t is the reduced-form shock of x_t. Note that the near-VAR is not used to identify structural shocks: ξ_t is identified (exogenously) by the tri-variate model.[53]

Figure 3 depicts the propagation of the tri-variate model's structural shocks for two variables of particular interest: $x_t = w_t$, and labor force; I include ur_t as a reference to the IRF's in Figure 2. The near-VAR model captures qualitatively the broad dynamics responses of ur_t from the tri-variate model's IRF's: ur_t decreases following $e_{s,t}$ or $e_{d,t}$ and increases following $e_{\tilde{n},t}$. For w_t, the propagation aligns with the augmented illustrative Keynesian framework's predictions: w_t increase following $e_{s,t}$ and $e_{d,t}$, and decrease following $e_{\tilde{n},t}$ (as predicted by the framework's

[52] BE's IRF's fare better with two lags, but fails GC, Sims, FG and CH tests (Appendix II, Figure A1 and Table A3).

[53] Also note that for notational simplicity, I omit constants included near-VAR models; the latter contain eight lags of x_t and are with quarterly data over the sample period noted in the main text.

solution, these are of the opposite sign as ur_t responses). For the labor force, it too responds as expected, exhibiting larger increases to $e_{\tilde{n},t}$ than to either $e_{s,t}$ and $e_{d,t}$, but these are imprecisely measured.[54]

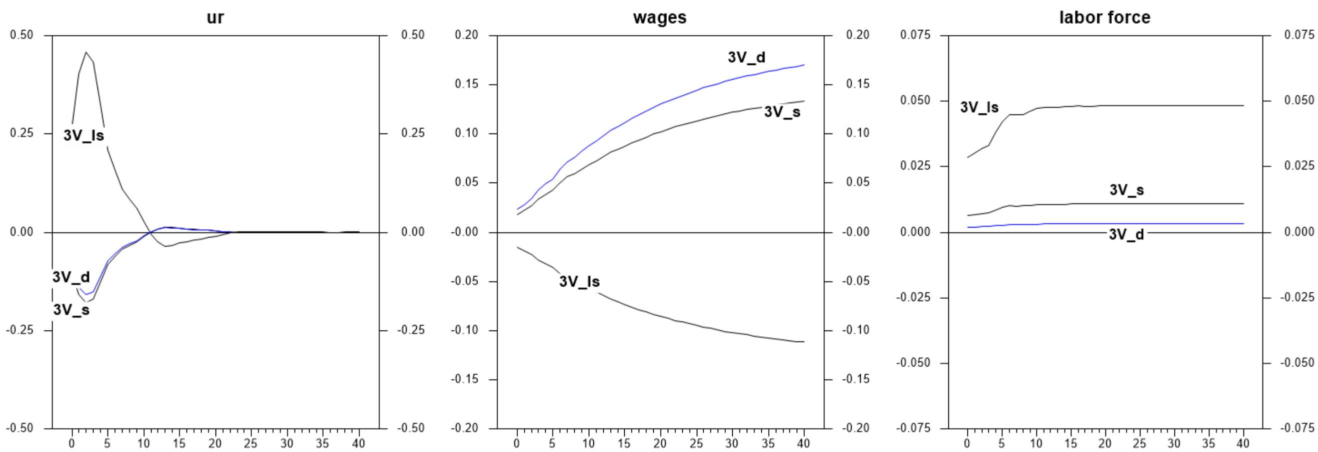

Figure 3. Labor market response to tri-variate shocks (near-VAR)

Stochastic trend in ur_t. The key IRF's results are robust to a stochastic trend in ur_t (Figure 4). I introduce the stochastic trend by replacing ur_t with Δur_t in the tri-variate model (and in the BQ model for illustrative purposes).[55] The corresponding IRF's for all three shocks remain consistent with their structural interpretation and reproduce qualitatively those without the unit root. But

[54] Note that the estimated $-c^{s}_{2,1}$ for ur_t are precise, the ratio of the estimate to its standard error exceed 6 for all three shocks; for wages the precision is lower 1.6, 1.5, and 2.3 respectively for $e_{s,t}$, $e_{\tilde{n},t}$, and $e_{d,t}$. The labor force coefficients are not estimated with precision (the ratios are less than 1).

[55] The IRF's for ur_t are the accumulated sums of the Δur_t responses for the tri-variate model, where the model is estimated and identified as in the main text but the model includes Δur_t instead of ur_t. As noted, the tri-variate model's ur_t solution does not imply short-run or long-run restrictions so the unit-root specification does not alter the identification but can alter the estimation. Moreover, if ur_t is stationary (as assumed in BQ), the resulting IRF's will reflect "over-differencing" and introduce spurious persistence in the responses.

$e_{s,t}$ results in persistent decreases in ur_t : the 95% confidence bands do not include zero (at all horizons). While confidence bands are not a formal statistical test, it stands to reason that restricting this long-run effect to zero will clash with the data. Moreover, imposing this restriction will mechanically require that decreases in ur_t be offset by "supply-induced" ur_t increases as found in Shapiro and Watson (1988). Of note, the BQ model's "supply-induced" ur_t increases vanish when ur_t is contains a stochastic trend, as widely reported in the literature.

I conclude that the time-series properties of the labor market variable are not responsible for a negative or positive relation between technology and employment. Below I argue that the mis-aggregation of technology and labor-supply effects is.[56] When these effects are combined, as in BQ model, introducing an unwarranted stochastic trend in the labor market variable undermines the role of the information with more persistence (technology) in ur_t fluctuations, leaving behind shorter-run information (labor-supply and aggregate demand). The tri-variate model disentangles technology and labor-supply shocks and paves the way for a different view on "supply-induced" ur_t increases, one that does not hinge on the time series properties of the labor market variable.

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[56] As noted, the opposing effects of technology and labor-supply shocks on ur_t constitute a textbook example of how the bivariate BQ variable pairing fails to meet the conditions for proper aggregation of shocks: FL's Proposition 2, part 2 (FL, page 349) "…require that each underlying shock of a given type affects the economy in the same way up to a scale factor."

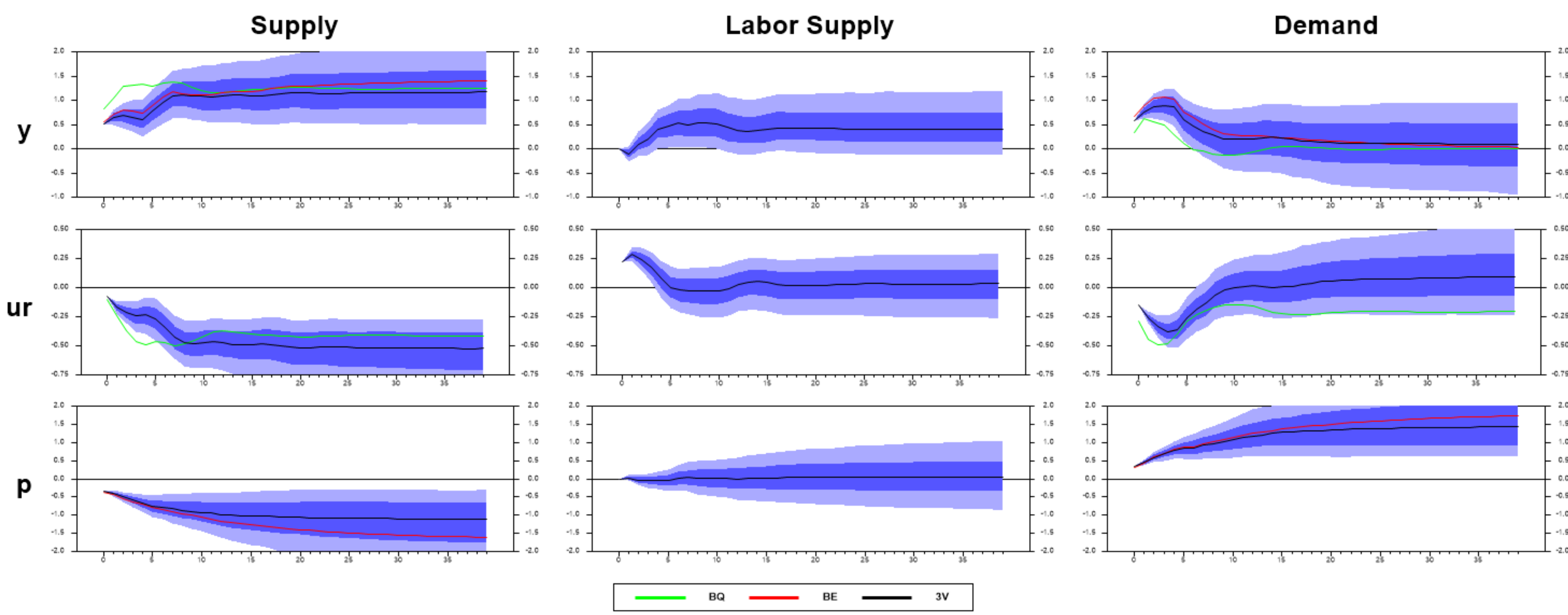

Figure 4. IRF's for the Tri-variate model (shaded 68% and 95% bands), BQ, and BE models (ur stochstic trend)

III. Sources of Impulse Response Function Missing-Variable Bias.

To shed light on the bivariate models' IRF's bias and gauge the underlying sources of bias, I derive an expression to parse the sources of bias. As noted, this expression is consistent with *Proposition 2* in BM but obtained using time-series notation (see Appendix I).

To begin, I partition the IRF's for the tri-variate model into a bivariate pairing and a third variable, $Y_t^{3V} = \begin{bmatrix} Y_{1,t} & Y_{2,t} \end{bmatrix}'$. Without loss of generality, I define $Y_{1,t}$ to contain the BQ variables with $Y_{2,t}$ denoting the missing third variable, $Y_{1,t} = \begin{bmatrix} \Delta y_t & ur_t \end{bmatrix}'$ and $Y_{2,t} = \begin{bmatrix} \Delta p_t \end{bmatrix}$. Next, I partition the tri-variate model's IRF's as follows:

$$IRF_{j,s}^{3V}\left(\Psi_s^{3V}, p_j^{3V}\right) = \begin{bmatrix} IRF_{j,s}^{3V,BQ}\left(\Psi_s^{3V,BQ}, p_j^{3V}\right) \\ IRF_{j,s}^{3V,\Delta p}\left(\Psi_s^{3V,\Delta p}, p_j^{3V}\right) \end{bmatrix} = \begin{bmatrix} \Psi_s^{3V,BQ} \\ \Psi_s^{3V,\Delta p} \end{bmatrix} \cdot p_j^{3V}$$

where the tri-variate model's MA representation Ψ_s^{3V} (3×3) is divided into $\Psi_s^{3V,BQ}$ (2×3) and $\Psi_s^{3V,\Delta p}$ (1×3). At the risk of stating the obvious, since the object $IRF_{j,s}^{3V,BQ}$ is identical to IRF's for $Y_{1,t}$ from $IRF_{j,s}^{3V}$ (the IRF's from the unpartitioned model), I can focus on this object when comparing the IRF's for $Y_{1,t}$ to the IRF's obtained from the BQ model; the latter expressed as:

$$IRF_{j,s}^{BQ,BQ}\left(\hat{\Psi}_s^{BQ}, \hat{p}_j^{BQ}\right) \equiv \frac{\partial Y_{t+s}^{BQ}}{\partial e_{j,t}^{BQ}} = \hat{\Psi}_s^{BQ} \cdot \hat{p}_j^{BQ}, \quad \text{for } s = 0, 1, 2, 3, \ldots$$

where $\hat{\Psi}_s^{BQ}$ and $\hat{p}_j^{BQ}$ are respectively of dimensions (2×2) and (2×1).

Following BM (Section 4, pp. 330–8), I define the BQ model's IRF's missing-variable bias as the difference between the IRF's of the bivariate model and (benchmark) tri-variate model:

$$\Delta_{IFR_{j,s}} = IRF_{j,s}^{BQ,BQ}\left(\hat{\Psi}_s^{BQ}, \hat{p}_j^{BQ}\right) - IRF_{j,s}^{3V,BQ}\left(\Psi_s^{3V,BQ}, p_j^{3V}\right).$$

Note that the bias corresponds to the vertical distance between the IRF's from the BQ and the tri-variate models (at horizon *s*).

Next, take $\hat{\Psi}_s^{BQ} = \Psi_s^{3V,BQ} + \Delta_\Psi$ and $\hat{p}_j^{BQ} = p_j^{3V} + \Delta_{p_j}$ where Δ_Ψ and Δ_{p_j} are respectively (implicitly) defined as the change in the MA-representation and in the j^{th} column of A_0 that follows when $Y_{2,t}$ is excluded from the tri-variate model to obtain the bivariate model.[57] That is, Δ_Ψ and Δ_{p_j} capture respectively the bivariate model's MA-representation distortion and mis-aggregation (compared to the tri-variate model).

The IRF bias can now be re-expressed as:

$$\Delta_{IFR_{j,s}} = IRF_{j,s}\left(\Psi_s^{3V,BQ} + \Delta_\Psi,\ p_j^{3V} + \Delta_{p_j}\right) - IRF_{j,s}\left(\Psi_s^{3V,BQ},\ p_j^{3V}\right)$$

and carrying out the implied multiplication results in the sources of bias expression:[58]

$$\Delta_{IFR_{j,s}} = IRF_{j,s}\left(\Psi_s^{3V,BQ}, \Delta_{p_j}\right) + IRF_{j,s}\left(\Delta_{\Psi_s}, p_j^{3V}\right) + IRF_{j,s}\left(\Delta_{\Psi_s}, \Delta_{p_j}\right).$$

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[57] To avoid addition notation, I implicitly "re-dimension" $\hat{\Psi}_s^{BQ}$ and $\hat{p}_j^{BQ}$ to conform to the dimensions of the tri-variate model. Specifically, these arrays are expanded respectively from (2×2) to (2×3) and from (2×1) to (3×1) by adding a third column of zeros to $\hat{\Psi}_s^{BQ}$ and a third element equal to zero to $\hat{p}_j^{BQ}$. The IRF's from these expanded arrays replicate those from the bivariate BQ model, but also contain zeros for the responses for a notional "third variable" and to a notional "third shock."

[58] In essence, this is the product-rule for discrete changes. Intuitively, it emerges because omitting variables in a VAR model can simultaneously biases the two arrays underlying the IRFs calculation: $C(L)$ and Ω. In deriving the bias expression, I use the distributive property of matrix multiplication that $IRF_{j,s}(\cdot,\cdot)$ inherits from the IRF's calculation. Namely, for conformable matrices A, B, C, and D:
$IRF_{j,s}(A+B, C+D) = IRF_{j,s}(A,C) + IRF_{j,s}(A,D) + IRF_{j,s}(B,C) + IRF_{j,s}(B,D)$.

This expression divides the overall bias in three. The first term captures the bias from mis-aggregated shocks as it computes the IRF's when the correct MA-representation, $\Psi_s^{3V,BQ}$, is post-multiplied by the bias in the identification factor, Δ_{p_j}. The second term reflects the bias from the MA-representation distortion as it computes the IRF's with the correct identification factor, p_j^{3V}, but pre-multiplied by the bias in the MA-representation, Δ_{Ψ_s}. And the third term captures the interaction of these biases as each is computed holding the other constant.[59]

With this expression, I am now equipped to examine the IRF's biases in the BQ and BE models and assess the role of mis-aggregation bias in "supply-induced" ur_t increases. Figures 5 and 6 depict respectively the overall bias of these models in the first column, and the sources of bias in subsequent columns (supply and demand shocks are in the top-two and bottom-two rows). Confidence bands for the 68 and 95 percent levels are computed using bootstrapping methods taking the tri-variate model as the DGP to obtain 2,000 quasi-data samples. These samples are used to repeatedly estimate the BQ, BE, and tri-variate models, and compute their respective IRF's, IRF's biases, and sources of bias.

[59] Note that while Δ_{IRF} is not a closed-form expression for the IRF's bias—Δ_{Ψ_s} and Δ_{p_j} are not derived analytically—it shows how to compute the sources of IRF bias from the empirical counterparts of those objects. Specifically, Δ_{Ψ_s} and Δ_{p_j} can be obtained respectively as the difference between the bivariate and tri-variate models' (estimated) MA-representations and (estimated) j^{th} columns of the identification factors. For the closed-form expression of Δ_{Ψ_s} see BM's equations 25 and 26 (BM, page 328).

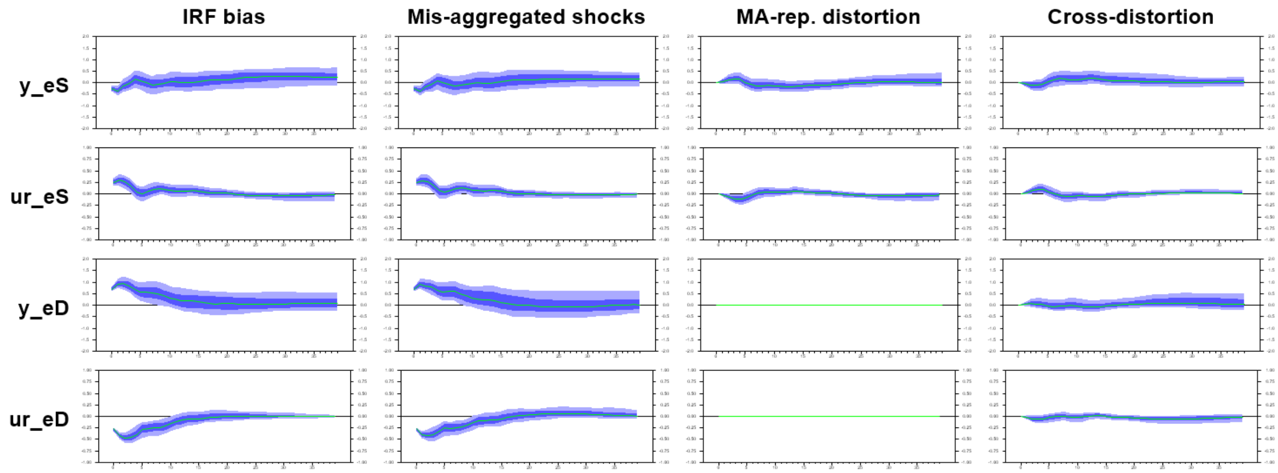

Figure 5. BQ model's IRFs bias decomposition (shaded 68% and 95% bands)

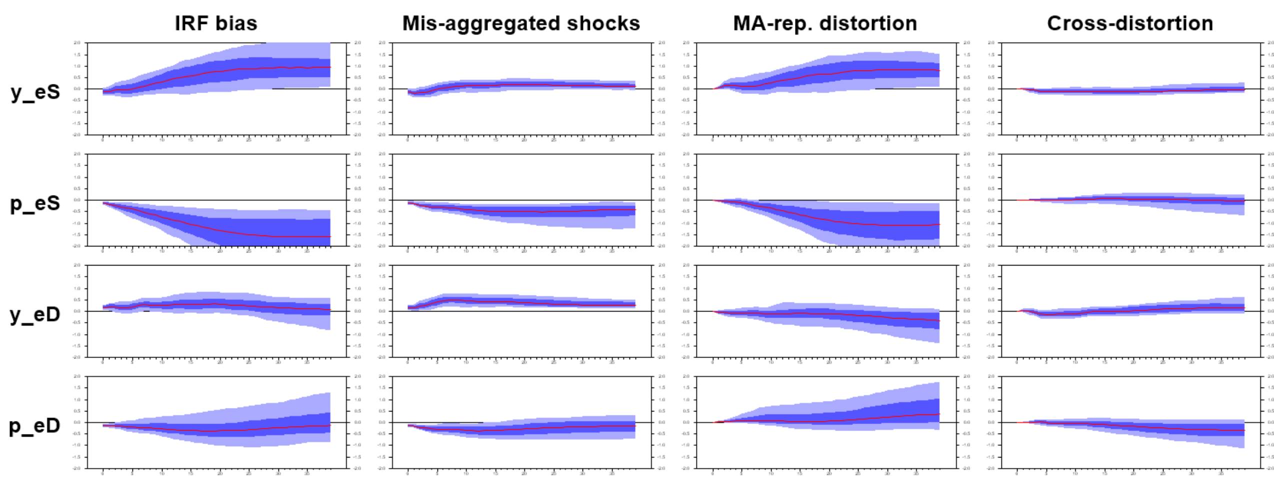

Figure 6. BE model's IRFs bias decomposition (shaded 68% and 95% bands)

The bias decompositions provide a clear-cut picture of the overall IRF's bias and diagnosis of the principal pathology afflicting these bivariate models. Specifically,

- the BQ model's IRF's bias reflects primarily mis-aggregation of shocks with a lesser bias from MA-representation distortions (Figure 5, columns 2 and 3). Of particularly interest, mis-aggregation clearly underlies the "positive" IRF's bias of ur_t following a supply shock in the BQ model (row 2, columns 1 and 2). The magnitude of the mis-aggregation bias is such that it reverses the sign of the ur_t responses from the tri-variate model, particularly in the first four quarters. Note also that the prevalence of mis-aggregation bias aligns with the evidence from cross-type correlations in Table 2 and, to a lesser extent, with the test results in Table 3. (Note that for demand shocks, the MA-representation distortion essentially drops out of the bias decomposition suggesting that excluding p_t does "substantially" affect the dynamics associated with demand shocks.)

- the BE model's IRF's bias—overstating y_t and understating p_t—reflect mis-aggregation in the short run, but MA-representation distortions dominate at longer horizons (Figure 6, columns 2 versus 3). For demand shocks, the overall bias is smaller (column 1, rows 3 and 4) reflecting partially offsetting effects of mis-aggregation and MA-representation distortions (columns 2 and 3, rows 3 and 4). The MA-representation distortion mirrors the GC and Sims test results in Table 3.

Of note, the size of the IRF's biases align with measure of the "severity" of non-fundamentalness in Beaudry et al. (2019). Consider the IRF's bias in each model: larger biases in the BQ and the BE model follow demand shocks in the former and supply shocks in the latter. This suggests more "severe" non-fundamentalness for demand shocks in the BQ model and supply shocks in the BE model. Indeed, for the BQ model the R^2 of a regression of demand shocks on the "missing" third variable (0.11) is almost three times larger than for supply shocks (0.04). And for the BE model the R^2 of a regression of supply shocks on the "missing" third variable (0.16) is eight times that of demand shocks (0.02).

IV. ADDITIONAL QUESTIONS

To what extent do mis-aggregated shocks and MA-representation distortions alter assessments of the relative importance of supply versus demand factors in macroeconomic fluctuations in these bivariate models? Can we rely on VAR-based structural shocks to infer the effect of technology shocks on ur_t (or employment) or should we favor instead directly measured "pure" Δbfk_t shocks? To address these questions, I turn to variance decompositions, FL cross-type correlations, and BFK's near-VAR propagation methodology noted above.

Variance decompositions (Table 4). Consider the tri-variate model. Up to 4 quarters, the dominant source of variation of y_t and ur_t fluctuations are supply ($e_{s,t}$ and $e_{\tilde{n},t}$) shocks while p_t fluctuations are dominated by $e_{d,t}$. In contrast, both the BQ and BE overstates (understates) the importance of $e_{d,t}$ ($e_{s,t}$) shocks in explaining y_t fluctuations through forecasting horizons up to 24 quarters. Regarding their respectively pairing variables, the BQ model consistently *overstates* the importance of $e_{d,t}$ shocks in explaining ur_t fluctuations, while the BE model consistently *understates* the importance of $e_{d,t}$ shocks in explaining p_t fluctuations. At face value and compared to the tri-variate model, ignoring the missing-variable and non-fundamentalness a researcher would infer greater roles than justified for aggregate demand and demand management policies in explaining macroeconomic fluctuations, except for p_t where lesser roles would be inferred.

Table 4. Variance Decompositions (percent due to structural shocks)

	Permanent					***Demand***		
	Trivariate			BQ	BE	Trivariate	BQ	BE
Horizon	Supply	Labor	Sum					
				y_t				
0	71.6	0.0	71.6	27.0	42.0	28.4	73.0	58.0
1	68.2	0.4	68.6	21.2	40.6	31.4	78.8	59.4
2	63.5	0.8	64.2	25.5	39.3	35.8	74.5	60.7
3	59.9	1.8	61.7	30.2	38.1	38.3	69.8	61.9
4	55.6	5.6	61.2	38.7	37.2	38.8	61.3	62.8
8	62.6	13.9	76.5	59.6	55.4	23.5	40.4	44.6
12	63.2	19.2	82.4	67.3	66.7	17.6	32.7	33.3
24	57.7	28.8	86.5	75.3	82.5	13.5	24.7	17.5
40	51.7	37.2	88.8	81.3	90.2	11.2	18.7	9.8
				ur_t				
Horizon								
0	13.0	76.2	89.2	21.2	...	10.8	78.8	...
1	16.2	63.3	79.5	12.2	...	20.5	87.8	...
2	16.8	53.5	70.3	8.0	...	29.7	92.1	...
3	17.3	43.9	61.2	5.5	...	38.8	94.5	...
4	18.5	35.4	53.9	5.9		46.1	94.1	
8	33.8	24.4	58.2	17.0	...	41.9	83.0	...
12	41.6	21.9	63.5	21.0	...	36.5	79.0	...
24	41.7	20.6	62.4	21.2	...	37.6	78.8	...
40	42.0	20.0	62.0	21.2	...	38.0	78.8	...
				p_t				
Horizon								
0	15.8	0.0	15.8	...	54.8	84.2	...	45.2
1	12.7	0.1	12.8	...	51.3	87.2	...	48.7
2	10.3	0.5	10.8	...	48.1	89.2	...	51.9
3	9.4	1.0	10.3	...	46.3	89.7	...	53.7
4	8.6	1.6	10.2		45.0	89.8		55.0
8	6.7	1.7	8.5	...	46.4	91.5	...	53.6
12	4.0	2.4	6.3	...	46.5	93.7	...	53.5
24	2.9	2.9	5.7	...	46.3	94.3	...	53.7
40	6.3	2.5	8.9	...	46.3	91.1	...	53.7

Note. All VAR models are estimated using eight lags over the sample period 1950:02 to 1987:04. Before estimation I remove the sample means for the three variables before and after the first OPEC oil shock (1973:04). This treatment of the data series is consistent with the BE sample split but differs from BQ's base case. Nonetheless, differences are quantitative in nature. Structural shocks are identified using short- and long-run restrictions as discussed in the main text.

Beyond the broad demand-supply divide, the relative importance of technology versus labor-supply shocks is informative. For y_t fluctuations, $e_{\tilde{n},t}$ shocks come into play in the medium-term, explaining roughly a third of the variance. For ur_t fluctuations, the reverse is true: $e_{\tilde{n},t}$ shocks play a dominant role in the shorter term and $e_{s,t}$ shocks become more important at longer horizons.[60] This pattern aligns with the literature's conjecture about "supply-induced" ur_t increases in BQ model: the initial increase is associated with the importance of labor-supply shocks in short-run variations and subsequently decreases as supply shocks take hold. For price fluctuations, supply plays a minor role in the first four quarters and even a smaller role subsequently; $e_{\tilde{n},t}$ does not play a role.

Δbfk_t versus VAR-model based structural shocks

FL cross-type correlations (Table 5). To compute the correlations between Δbfk_t and VAR-based structural shocks, I use the quarterly version of Δbfk_t, from Fernald (2014). That study does not correct for deviations from constant returns to scale nor from perfect competition due to data availability constraints, but the quarterly series "*…goes a long way towards cleansing the Solow residual of non-technological cyclicality*." (ibid, page 15).

[60] The demand-supply divide for y_t and p_t closely replicates that in Shapiro and Watson (1988) (Table 2, page 128). Not surprisingly, it does not for the alternative labor market variable (hours). A more notable difference arises in the apportionment of fluctuations between technology and labor-supply, a divide that, as noted, Shapiro and Watson (1988) acknowledge reflects that study's identifying restriction that: technology shocks do not affect hours in the long-run (ibid 114). In this connection, that study also finds that labor-supply shocks explain about half of output fluctuations in the first year and 40 percent at eight quarters; in Hall's comments to that study, he calls out this surprising result that: "*…shifts in labor-supply are an important determinant of output in business cycle frequencies*" (page 148). The variance decompositions in this paper, reflecting the less restrictive identification of the tri-variate model, point to a negligible role for labor-supply shocks in explaining output fluctuations at four quarters and only about 15 percent at eight quarters (Table 4).

Table 5. Correlation of BFK shocks and SVAR-based shocks

	Permanent	Labor supply	Temporary
Blanchard-Quah	0.18	…	-0.06
Bayoumi-Eichengreen	0.09	…	-0.05
Tri-variate model	0.11	0.15	-0.07

Note. Quarterly BFK shocks are taken from Fernald (2014). VAR-based structural shocks are identified using the long- and short-run restrictions discussed in the main text for the tri-variate model and the long-run restrictions in Blanchard-Quah and Bayoumi-Eichengreen. All VAR models are estimated using eight lags using quarterly data from 1950:Q2 to 1987:Q4

The correlations are low, not surprisingly, as Δbfk_t is obtained from a different empirical tradition.[61] Nonetheless as noted, two stylized facts are noteworthy. First, VAR-model based demand shocks are not correlated to Δbfk_t (these are slightly negatively correlated). And second, Δbfk_t is most highly correlated with BQ's supply and the tri-variate model's labor-supply shocks; and to a lesser extent with the supply shocks from the BE and the tri-variate model.

These stylized facts point to a general agreement between the BFK's direct measurement and SVAR approaches in separating demand (cyclical) from supply (long-run) factors; this is also noted in Gali and Rabanal (2004). Moreover, that study's conclusion: "…*VAR-based permanent shocks may indeed be capturing exogenous variations in technology, in a way consistent with the interpretation in Gali (1999)*…" (ibid, page 245) is not fully at odds with the correlations in Table 5. But the fact remains that Δbfk_t is more correlated to labor-supply shocks than to technological shocks.

[61] Gali and Rabanal (2004) report higher correlations, which may reflect that study's use of annual data (that study averages quarterly VAR-model shocks.

BFK's near-VAR propagation. I use BFK's near-VAR methodology as a common framework in the comparison of VAR-based supply shocks to BFK's technology shock. But I now define ξ_t to be scalar denoting one of five *supply* shocks: those recovered from BQ and BE models, the technology and labor-supply shocks from the tri-variate model, and the BFK's technology shock, Δbfk_t . The near-VAR models are estimated using quarterly data with eight lags over the sample period used above; following BFK, I estimate the near-VAR model with SUR techniques.

Figure 7 depicts the dynamic responses for the labor force and wages for the five supply shocks noted.[63] The labor force increases persistently following all shocks. At face value, this is inconsistent with the modified Keynesian framework that does not accommodate labor force movements beyond labor-supply shocks.[64] But in the real world, it is not unreasonable for labor participation decisions to reflect other factors, notably information regarding future wages implicit in technology shocks; this information may underlie the labor force propagation.[65] More telling, however, are wage responses to supply shocks. Save one of the five supply shocks, wages persistently decrease in response to "supply" shocks in a manner consistent with labor-supply shocks. This suggests that the supply shocks recovered from the BQ and BE model and

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[63] I use changes in the log of the "civilian labor force, 16 years-old and over" (CLF16OV) (thousands of persons, seasonally adjusted) and in the log of the "average hourly earnings of production and nonsupervisory employees, total private" (AHETPI) (dollars per hour seasonally adjusted) from FRED (June 2022 vintage).

[64] Note also, that, to the extent that hours worked and the labor force are correlated, this propagation clashes with the restriction of long-run exogeneity of hours in Shapiro and Watson (1988).

[65] I find supportive evidence of this conjecture using a simple extension of the near-VAR used to obtain the labor force responses. Specifically, I include both technology and labor-supply shocks in the near-VAR model and focus on the estimates of the impact effects of these shocks, respectively $c_{2,1}^{s}$ and $c_{2,1}^{\tilde{n}}$, in the near-VAR model. In this case, the estimate for $c_{2,1}^{s}$ is cut in half compared to its estimate when only the technology shock is included. This suggests that, in the near-VAR model, the technology shock indeed contains information about the labor force. The reverse however is not true: estimates for $c_{2,1}^{\tilde{n}}$ remains virtually unchanged whether the technology shock is included or not in the near-VAR model.

Δbfk_t suffer from a degree of mis-aggregation.[66] And, to the extent that hours reflect labor-supply, wage responses to the Δbfk_t shock are consistent with the finding in Christiano, et al. (2004) that hours Granger-cause Δbfk_t . Wages increase persistently only to the supply shock from the tri-variate model: $e_{s,t}$ "quacks" like a technology shock!

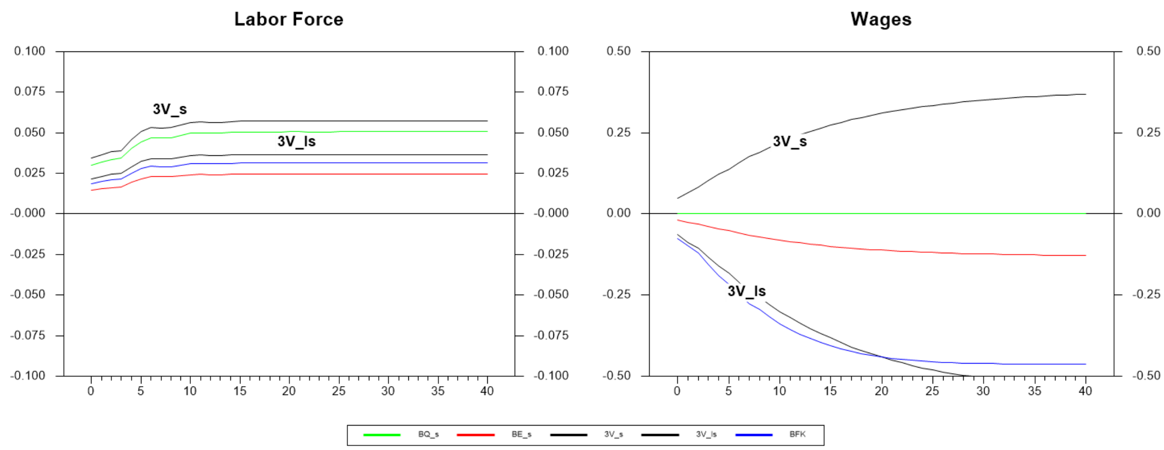

Figure 7. Labor Force and Wage responses to supply shocks (univariate near-VAR)

[66] I also explore MA-representation distortions as an alternative explanation for the propagation of Δbfk_t shocks depicted in Figure 7. In unreported results, I take Δbfk_t shocks to be correctly "identified" and extend BFK's near-VAR approach to encompass the tri-variate model. I do this by adding to tri-variate VAR model an equation for an exogenously determined Δbfk_t shock. (I also use an analogous approach to form near-VAR models for the BQ and BE variable pairings; for completeness I also used univariate near-VAR to trace out the propagation.) The dynamic responses show that, regardless of the MA-representation, ur_t increases following Δbfk_t shocks. This suggests that increases in ur_t are an intrinsic feature of the BFK shock not associated with MA-representation distortions.

V. Concluding Remarks

Using a tri-variate model to identify labor-supply along with technology and aggregate demand shocks, I find that the IRF's bias in the bivariate BQ and BE models are primarily due to mis-aggregation of structural shocks in the former and MA-representation distortions in the latter. These assessments are supported by FL cross-type correlations, missing-variables and non-fundamentalness test results, variance decompositions, and a novel calculation of the sources of IRF's bias. The parsing methodology provides a full account of the IRF's biases with an added benefit of measuring the sources of bias at different time horizons. This can be useful in understanding differences in IRF's from alternative VAR models.

For the U.S. post-World War II data, I come to two broad conclusions. First, labor market information should be included to properly assess macroeconomic fluctuations and price information is needed to account for the demand-side factors. At the very least a tri-variate model is required, echoing FL's stern warning about the reliability of the bivariate BQ and BE models and their usefulness to study macroeconomic fluctuations. And second, "supply-induced" ur_t increases reflect mis-aggregated technology and labor-supply shocks, regardless of whether the labor-market variable contains a stochastic trend. I also find that direct measurements of technology are more correlated to, and their propagation more aligned with, labor-supply shocks.

Granted, further evidence is needed for earlier time periods and across countries to judge more broadly the weaknesses of bivariate models, evaluate the tri-variate model's ability to assess macroeconomic fluctuations, and confirm the role of labor-supply shocks in ur_t responses. For now, I conclude that for the BQ and BE variable pairings "two's *not* company" and future research may determine that "*three's company*."

Appendix I. Δ_{IRF} and BM's *Proposition 2*

To fix notation, I briefly review IRF's non-recursive notation and its relation to textbook time-series notation.

Non-recursive notation. BM express IRF's in non-recursive form (Mittnik, 1987) using lower-triangular block Toeplitz matrix notation. Specifically, for a *k*-dimensional VARMA model and up to horizon *h*, IRF's are expressed compactly in BM equation (24) (page 328) as:

$$\Psi(\Theta) = M^{-1}(\Theta) \cdot B(\Theta) \cdot \Sigma^{1/2}(\Theta)$$

where $\Psi(\cdot)$ $(k \cdot (h+1) \times k)$ contains the IRF's for all *k* variables to all *k* structural shocks up to horizon *h*.[67] These IRF's are computed as the product of three arrays: (1) the $(k \cdot (h+1) \times k \cdot (h+1))$ Toeplitz matrix containing the MA-representation of the VAR part of the VARMA model,

$$M^{-1}(\cdot) = \begin{bmatrix} I & 0 & 0 & 0 & \cdots & 0 & 0 \\ C_1 & I & 0 & 0 & \cdots & 0 & 0 \\ C_2 & C_1 & I & 0 & \cdots & 0 & 0 \\ C_3 & C_2 & C_1 & I & \cdots & 0 & 0 \\ \vdots & \vdots & \vdots & \vdots & \ddots & \vdots & \vdots \\ C_{h-1} & C_{h-2} & C_{h-3} & C_{h-4} & \cdots & I & 0 \\ C_h & C_{h-1} & C_{h-2} & C_{h-3} & \cdots & C_1 & I \end{bmatrix}$$

(2) the $(k \cdot (h+1) \times k)$ MA part of the VARMA model, $B(\cdot)$,

$$B(\cdot)' = \begin{bmatrix} B_0, & B_1, & B_2, & B_3, & \cdots, & B_{h-1}, & B_h \end{bmatrix}$$

[67] A reader familiar with panel data might find useful to think about $\Psi(\cdot)$ in terms of *time-specific* formulation where each variable is a "panel unit" and its columns stack the responses for the *k* "units" for each time period (horizon).

and (3) the $(k \times k)$ "square-root" of the model's covariance matrix, $\Sigma^{1/2}(\cdot)$.[68] Each array is evaluated at Θ, the vector collecting the VARMA model's coefficients. Note that the block-elements in $\Psi(\cdot)$, $M^{-1}(\cdot)$, and $B(\cdot)$ are of dimension $(k \times k)$.

To focus on VAR models, I assume $B^{I}(\Theta)' = \begin{bmatrix} I & I & I & I & \cdots & I & I \end{bmatrix}$ and express $\Psi(\cdot)$ explicitly as:

$$\begin{aligned}\Psi(\cdot) &= M^{-1}(\cdot) \cdot B^{I}(\cdot) \cdot \Sigma^{1/2}(\cdot) \\ &= \left[\Sigma^{1/2}, \ \left[I + C_1 \right] \cdot \Sigma^{1/2}, \ \left[I + C_1 + C_2 \right] \cdot \Sigma^{1/2}, \ \left[I + C_1 + C_2 + C_3 \right] \cdot \Sigma^{1/2}, \ \cdots \ , \ \left[I + \sum_{s=1}^{h} C_s \right] \cdot \Sigma^{1/2} \right]'\end{aligned}$$

where each block-element in $\Psi(\cdot)$ contains the IRF's for all k shocks for a specific horizon.

Relation to time-series notation. The block-elements in $\Psi(\cdot)$ correspond to $\Sigma^{1/2}$ pre-multiplied by the $\tilde{C}_i$ arrays defined implicitly in:

$$\begin{aligned}C(L) &= \left[I - C_1 \cdot L - C_2 \cdot L^2 - \cdots - C_p \cdot L^p \right]^{-1} \\ &= I + \tilde{C}_1 \cdot L + \tilde{C}_2 \cdot L^2 + \tilde{C}_3 \cdot L^3 + \cdots\end{aligned}$$

Thus, $\Psi(\cdot)$ can be expressed as:

$$\Psi(\cdot) = \left[I \cdot \Sigma^{1/2}, \ \tilde{C}_1 \cdot \Sigma^{1/2}, \ \tilde{C}_2 \cdot \Sigma^{1/2}, \ ..., \ \tilde{C}_h \cdot \Sigma^{1/2} \right]',$$

which contains the IRF's for a VAR model for all shocks up to horizon h. Note that

$$\tilde{C}_0 = I \ \text{ and } \ \tilde{C}_i = I + \sum_{ii=1}^{h} C_{ii}, \ h \geq 1.$$

[68] BM's IRF's are the product of three elements because these are for VARMA models.

Standard time-series IRF's notation however focuses on a single shock, a period at a time. To pick out to the j^{th} shock (the j^{th} column in $\Psi(\cdot)$) define the $(k\times 1)$ unit vector q_j (with "1" in the j^{th} position). Using q_j, the IRF's to the j^{th} shock can be expressed as:

$$\begin{aligned}\Psi(\cdot)_j &= \Psi(\cdot)\ \cdot q_j \\ &= \left[p_j,\quad \tilde{C}_1\cdot p_j,\quad \tilde{C}_2\cdot p_j,\quad \tilde{C}_3\cdot p_j,\quad \cdots\quad \tilde{C}_{h-1}\cdot p_j,\quad \tilde{C}_h\cdot p_j\right]'\end{aligned}$$

where $\Psi(\cdot)_j$ is of dimension $(k\cdot(h+1)\times 1))$ as it stacks the responses to the j^{th} shock from s= 0, 1, 2, 3, …, h. To pick out the responses for period s, define the $(k\times k\cdot(h+1))$ matrix $R_s = \left[0,\quad 0,\quad 0,\quad \cdots,\quad I,\quad \cdots,\quad 0\right]$ that contains "h+1" block elements of dimension $(k\times k)$, all of zero except for a single $(k\times k)$ identity matrix located in rows $(k\times s+1)$ to $(k\times(s+1))$. Using R_s, the IRF's for shock j and period s can be expressed as:

$$\Psi(\cdot)_{j,s} = R_s\cdot\Psi(\cdot)_j = \tilde{C}_s\cdot p_j \text{ for } s = 0, 1, 2, 3, ...$$

where $\Psi(\cdot)_{j,s}$ is of dimension $(k\times 1)$, that is, the standard IRF's for the k variables to the j^{th} shock for horizon s.

BM's *Proposition 2*.[69] To start, I first reproduce (in BM notation) equation (26) from *Proposition 2* (BM, page 328) detailing the closed-form analytical solution for IRF's biases:

$$\delta_{IRF}^{BM} = \Delta_c\cdot\Sigma^{1/2}\left(\Theta\right) + M^{-1}\left(\text{plim}\ \hat{\Theta}_T\right)\cdot B\left(\text{plim}\ \hat{\Theta}_T\right)\cdot\tilde{\Sigma}^{1/2}\ ,$$

where Δ_c, $\tilde{\Sigma}^{1/2}$, and $\Sigma^{1/2}\left(\Theta\right)$ denote respectively, for all shocks and up to horizon h, the inconsistencies of the MA-representation $(k\cdot(h+1)\times k)$, the bias of the "square-root" of

[69] *Proposition 2* covers a broader range of VAR model (coefficient) misspecification than those considered in the main text as it also includes "omitted lags" and "omitted MA-components."

covariance matrix $(k \times k)$, and the unbiased "square-root" of the covariance matrix $(k \times k)$.[70]

$M^{-1}(\cdot)$ and $B(\cdot)$ are defined above. Note that this expression for the IRF's bias corresponds to a VARMA model. I focus on VAR models below and thus assume no missing MA components, $B\left(\text{plim}\,\hat{\Theta}_T\right) = B^I$.

To obtain the one-to-one mapping of δ_{IRF}^{BM} and $\Delta_{IFR_{j,s}}$, I take BM's equation (26) above and express it as the (non-recursive) product rule, and then extract the j^{th} shock and the corresponding response for the s-period.[71]

I do this by using BM's general plim expression and write $M^{-1}\left(\text{plim}\,\hat{\Theta}_T\right) \cdot B^I$ as the equivalent expression $M^{-1}(\Theta) \cdot B^I + \Delta_c$ and substituting into BM's equation (26) above: [72]

$$
\begin{aligned}
\delta_{IRF}^{BM} &= \Delta_c \cdot \Sigma^{1/2}(\Theta) + \left\{ M^{-1}(\Theta) \cdot B^I + \Delta_c \right\} \cdot \tilde{\Sigma}^{1/2} \\
&= \Delta_c \cdot \Sigma^{1/2}(\Theta) + M^{-1}(\Theta) \cdot B^I \cdot \tilde{\Sigma}^{1/2} + \Delta_c \cdot \tilde{\Sigma}^{1/2}
\end{aligned}
$$

where the three terms are matrices $(k \cdot (h+1) \times k)$ comprising the product-rule (for all shocks and up to period h). Note that these terms capture respectively the MA-representation distortion, mis-aggregation, and the cross-product distortion.

[70] The equivalency between *Proposition 2* and the expressing in main text can be obtained without reference to the explicit analytical expressions for Δ_c and $\tilde{\Sigma}^{1/2}$ detailed in BM equation (25) and Corollary 2 (BM, page 329). As noted in the main text, I use the empirical counterparts to compute the sources of IRF's biases in this paper.

[71] Of note, BM do not express equation (26) in terms of the product rule. I do so here to show the relation to the sources of IRF's bias in the main text.

[72] Specifically, BM (page 328) define inconsistencies as "…*the quantities derived from the plim* $\hat{\Theta}_T$. *For example,* $\text{plim}\Psi(\hat{\Theta}_T) = \Psi(\text{plim}\hat{\Theta}_T) = \Psi(\Theta + \tilde{\Theta}) = \Psi(\Theta) + \Delta_\Psi$, …". This definition allows me to express the term $M^{-1}\left(\text{plim}\,\hat{\Theta}_T\right) \cdot B^I$ as follows: $M^{-1}(\Theta) \cdot B^I + \Delta_c$, where $\Delta_c \equiv \Delta_\Psi \cdot B^I$.

Next, I post-multiply δ_{IRF}^{BM} by the unit vector q_j to extract the expression for the bias corresponding to the j^{th} shock (up to period h) and pre-multiply by the $(k \times k \cdot (h+1))$ R_s matrix to pick out the responses for the s-period. Specifically,

$$\begin{aligned}\delta_{IRFj,s}^{BM} &= R_s \cdot \left[\Delta_c \cdot \Sigma^{1/2}\left(\Theta\right) + \left\{ M^{-1}\left(\Theta\ \right) \cdot B^I + \Delta_c \right\} \cdot \tilde{\Sigma}^{1/2} \right] \cdot q_j \\ &= \Delta_{c,s} \cdot p_j(\Theta) + M^{-1}\left(\Theta\ \right)_s \cdot \Delta_{\tilde{p}_j} + \Delta_{c,s} \cdot \Delta_{\tilde{p}_j}\end{aligned},$$

where $\Delta_{c,s}$, $p_j(\Theta)$, $\Delta_{\tilde{p}_j}$, and $M^{-1}\left(\Theta\right)_s$ denote respectively the s-period responses contained in Δ_c, the j^{th} columns of $\Sigma^{1/2}\left(\Theta\right)$ and $\tilde{\Sigma}^{1/2}$, and the s^{th} block from $M^{-1}\left(\Theta\right)$.

And using the vector IRF's function notation from the main text:

$$\delta_{IRFj,s}^{BM} = IRF_{j,s}\left(\Delta_{c,s}, p_j(\Theta)\right) + IRF_{j,s}\left(M^{-1}\left(\Theta\right)_s, \Delta_{\tilde{p}_j}\right) + IRF_{j,s}\left(\Delta_{c,s}, \Delta_{\tilde{p}_j}\right),$$

where the terms on the right-hand side reflect respectively the MA-representation distortion, mis-aggregation bias, and the cross-product distortion. Save the order of the terms and obvious differences in the nomenclature, this expression provides the one-to-one correspondence of BM's *Proposition 2* and IRF's bias decomposition in the main text

APPENDIX II. ROBUSTNESS

Here I discuss the robustness of the results of cross-type FL correlations, the missing-variable and fundamentalness tests, the IRF's of the bivariate and tri-variate models, variance decompositions, and comparisons to Δbfk_t. I recompute these for three cases. First, all VAR models contain two lags in line with the Hannan-Quinn (HQ) Criterion (Table A1). Ivanov and Kilian (2005) find that the HQ criterion is best able to select the lag length that minimizes the distortions in the resulting IRF's when the estimation is based on at least 120 quarterly observations; for shorter quarterly samples the Schwarz Information Criterion (SIC) is preferred. Second, the relevant VAR models include deterministically detrended ur_t consistent with the base-case results in BQ and FL. Note that in this case, even though the estimates for the BE model do not change (as it omits ur_t), the BE model's correlations and statistical tests can because these reference ur_t. And third, the models are estimated using an extended sample period through 2009:04 using data from the FRED (October 2021 vintage) database: GNP, UNRATE (number of people 16 and over actively searching for a job as a percentage of total labor force), and GDPDF. The quarterly unemployment series averages the monthly series.

Table A1. Lag Order Selection Analysis for BQ, BE, and tri-variate VAR models.

	Akaike Information Criterion (AIC)			Hannan-Quinn Criterion (HQ)			Schwarz Information Criterion (SIC)		
	BQ	BE	Tri-variate	BQ	BE	Tri-variate	BQ	BE	Tri-variate
Lag length									
1	-2.461	-1.290	-3.697	-2.381	-1.210	**-3.517** *	-2.429	-1.257	-3.624
2	-2.544	-1.375	-3.841	**-2.384** *	**-1.215** *	-3.482	**-2.479** *	**-1.310** *	**-3.695** *
3	**-2.557** *	-1.374	-3.842	-2.317	-1.134	-3.303	-2.459	-1.277	-3.623
4	-2.523	-1.336	-3.772	-2.203	-1.016	-3.053	-2.393	-1.206	-3.480
5	-2.521	**-1.402** *	**-3.869** *	-2.122	-1.003	-2.970	-2.359	-1.240	-3.504
6	-2.512	-1.356	-3.852	-2.033	-0.876	-2.773	-2.317	-1.161	-3.414
7	-2.482	-1.335	-3.815	-1.923	-0.775	-2.556	-2.255	-1.107	-3.304
8	-2.467	-1.312	-3.732	-1.828	-0.673	-2.293	-2.207	-1.052	-3.147

Note. The criterium are calculated for a fixed sample of 150 quarterly observations (1950:Q2 to 1987:Q4) using formulas detailed in Ivanov and Kilian (2005). I rely on HQ because Monte Carlo experiments suggest it results in the most accurate IRF's for a quarterly data sample exceeding 120 observations (see Ivanov and Kilian, 2005) . An asterisk symbol denotes the selected lag length for individual criteria and VAR models.

A. FL cross correlation of structural shock.

(*Case 1*) Setting the number of lags to two, I find structural shock correlations that are very similar to the base-case results (Table A2, panel A). As above, the correlations for the BQ variable pairing point to high correlation of its supply shock with the tri-variate model's "technology" and labor-supply, with limited correlation to temporary shocks. Also, its demand shock is correlated with all three of the tri-variate models structural shocks. Likewise, the correlations for the BE variable pairing point to high correlation of the model's supply and demand shocks with their tri-variate model's counterparts and limited cross-type correlation; the BE model's shocks remain "empirical" orthogonal to labor-supply shocks. (*Case 2*) Using detrended (instead of means-removed) ur_t in the relevant VAR models results in puzzling revisions to the structural shock correlations (Table A2, panel B). The BQ model's supply shocks are highly correlated with labor-supply shocks, with limited cross-type correlation, while its demand shocks exhibit substantial cross-type correlation with the tri-variates supply (technology) shocks. The BE model's supply and demand shocks exhibit high cross-type correlation, reversing the pattern of correlations in the base-case! The missing-variable and fundamentalness tests results discussed below go a long way to explain these changes. (*Case 3*) For the extended sample period, the bivariate models virtually replicate the base-case results in the main text.

Table A2. Correlation of Structural Shocks, Robustness

	Tri-variate model		
	Permanent	Labor Supply	Temporary
A. VAR models with two lags			
Blanchard-Quah:			
Permanent	0.40	0.90	0.14
Temporary	0.74	-0.40	0.54
Bayoumi-Eichengreen:			
Permanent	0.89	0.00	0.27
Temporary	-0.27	0.00	0.96
B. Detrended unemployment rate			
Blanchard-Quah:			
Permanent	0.10	0.94	0.26
Temporary	0.78	-0.17	0.42
Bayoumi-Eichengreen:			
Permanent	0.09	0.00	0.88
Temporary	0.94	0.00	-0.15
C. Extended sample			
Blanchard-Quah:			
Permanent	0.53	0.79	0.27
Temporary	0.62	-0.59	0.45
Bayoumi-Eichengreen:			
Permanent	0.93	0.00	0.08
Temporary	-0.11	0.00	0.97

Note. Structural shocks are identified using the long- and short-run restrictions discussed in the main text for the tri-variate model and long-run restrictions in Blanchard-Quah and Bayoumi-Eichengreen. The VAR models used for Panels A and B are estimated using the sample period in Blanchard-Quah (1950:02 to 1987:04). The VAR models used in Panel C are estimated using the sample period 1950:02 to 2009:04.

B. Missing-variable and fundamentalness tests (Table A3)

(*Case 1*) Decreasing the number of lags to two, results in very similar missing-variable and fundamentalness test results as those in the main text. But now the BQ model only fails the GC test and the BE model continues to fail all four tests. (*Case 2*) Linear detrending of ur_t wreaks havoc on the test results: both the BQ and the BE models reject three out of four tests! (The common exception is the non-rejection of the CH test.) These results can explain the dramatic changes in the correlation of structural shocks above and serve as a flag not to place too much credence on the results based on a linearly detrended ur_t discussed below. It is precisely because of this that I take my base-case to be those results obtained using "means-removed" ur_t .

(*Case 3*) Extending the sample through 2009:04 the BQ model now fails three of the four test (it passes the Sims test) and the BE model continues to fail all four tests.

Table A3. Missing Third Variable and Fundamentalness Tests for BQ and BE models, Robustness

A. BQ model (Third variable: Δp)						***B. BE model (Third variable: ur_t)***					
Two lags		Detrended ur_t		Extended Sample		Two lags		Detrended ur_t		Extended Sample	
						1-Granger causality tests for:					
F(4,143)	4.792	F(16,101)	36.257	F(16,181)	27.129	F(4,101)	24.617	F(16,101)	15.514	F(16,181)	63.345
M-Signif	0.00 *	M-Signif	0.00 *	M-Signif	0.00 *	M-Signif	0.00 *	M-Signif	0.00 *	M-Signif	0.00 *
						2-Sims-exogeneity test of:					
F(4,143)	1.114	F(16,101)	2.2794	F(16,181)	1.045	F(4,101)	5.506	F(16,101)	1.974	F(16,181)	3.064
M-Signif	0.35	M-Signif	0.01 *	M-Signif	0.41	M-Signif	0.00 *	M-Signif	0.02 *	M-Signif	0.00 *
						3-Orthogonality (Forni-Gambetti) test for:					
$\varepsilon_t^{BQ,S} + \varepsilon_t^{BQ,D}$						$\varepsilon_t^{BE,S} + \varepsilon_t^{BE,D}$					
F(4,141)	0.454	F(16,93)	3.65	F(16,181)	2.020	F(4,141)	7.345	F(16,93)	3.154	F(16,181)	5.445
M-Signif	0.77	M-Signif	0.00 *	M-Signif	0.01 *	M-Signif	0.00 *	M-Signif	0.00 *	M-Signif	0.00 *
						4-Canova-Hamidi test for:					
$\mu_t^{BQ,\Delta y} + \mu_t^{BQ,ur}$						$\mu_t^{BE,\Delta y} + \mu_t^{BE,\Delta p}$					
F(4,141)	1.104	F(16,93)	1.535	F(16,181)	1.896	F(4,141)	4.989	F(16,93)	1.1712	F(16,181)	3.084
M-Signif	0.36	M-Signif	0.10	M-Signif	0.02 *	M-Signif	0.00 *	M-Signif	0.31	M-Signif	0.00 *

Note. The Granger-causality tests the lags of the corresponding missing third variable in an augmented bivariate VAR model. The Sims-exogeneity tests the leads of the BQ or BE variables in the regression where the third variable is regressed on its lags and the leads and lags of the corresponding bivariate model. The Forni-Gambetti test is a type of Granger-causality test of the third variable in VAR model whose variables are the corresponding structural shocks. The Canova-Hamidi test is a type of Sims-exogeneity test that tests the leads of the BQ or BE variables in a the regression where the the third variable is regressed on its lags and the leads and lags of the reduced-form residuals from the corresponding bivariate model. The reduced-form residuals and the structural shocks are obtained by estimating the BQ and BE models with eight lags or two lags and detrended ur (both over the sample 1950:02 to 1987:04); the extended sample covers data from 1950:02 to 2009:04. An "asterik" indicates

C. IRF's relative to the tri-variate model (Figures A1 to A3)

(Case 1) Decreasing lags to two in all VAR models, three out of four of the BQ model's IRF's compared to the tri-variate model remain broadly unchanged (Figure A1). The exception is the y_t response to supply shocks, which now lies outside of the tri-variate model's confidence bands. For the BE model, the IRF's are comparable to the base-case, though the p_t response to supply shocks no longer shows the persistent deviation from the tri-variate model's IRF's. (*Case 2*) For completeness, I provide the IRF's using (in the relevant VAR models) detrended ur_t (Figure A2). But, as noted, these results are best taken with a grain of salt as both bivariate models exhibit missing variables and fail fundamentalness tests. Note that since the BE does not include ur_t, its IRF's are unchanged. Note further, that the tri-variate model's IRF's continue to show that ur_t declines (increases) following supply (labor-supply) shocks but p_t fail to decline as expected. (*Case 3*) Extending the sample through 2009:04 does not qualitatively change the conclusions for the BQ model as its IRF's closely resemble those in the base-case results. Though a bit more deviation is observed in the IRF's following demand shocks (Figure A3). For the BE model, its IRF's are more aligned to the tri-variate model's IRF's, particularly its p_t response to supply shocks.

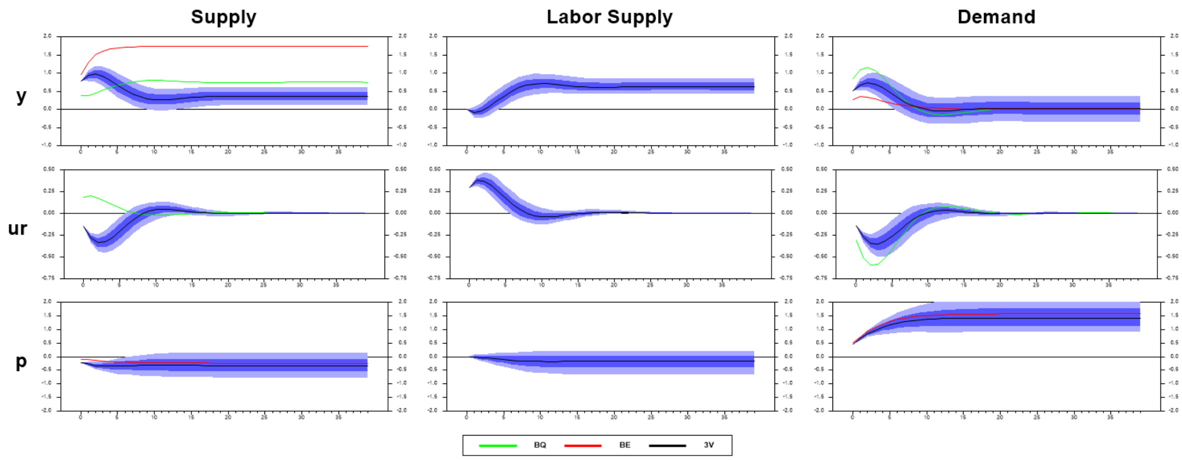

Figure A1. IRF's for the Tri-variate model (with shaded 68% and 95% bands), BQ, and BE models (two lags)

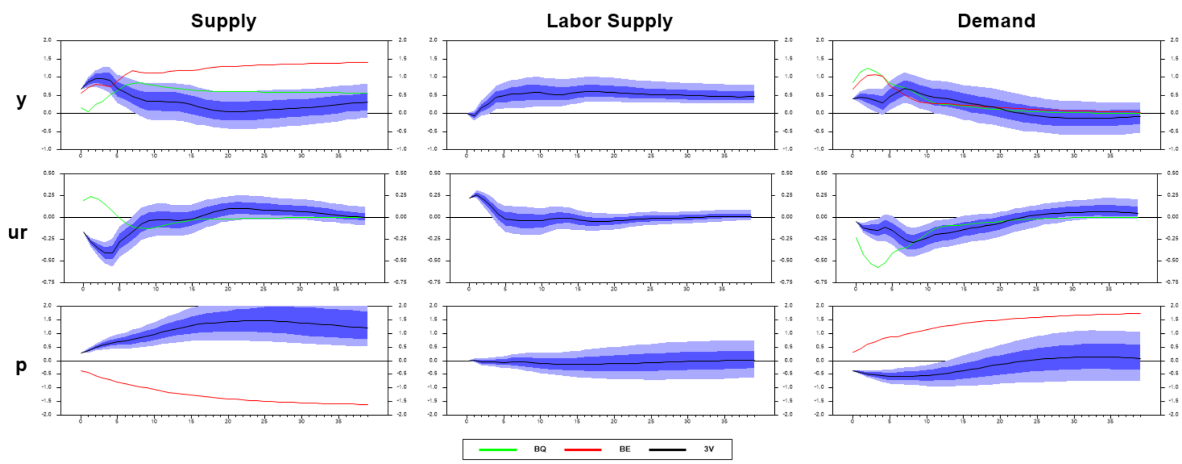

Figure A2. IRF's for the Tri-variate model (with shaded 68% and 95% bands), BQ, and BE models (ur detrend)

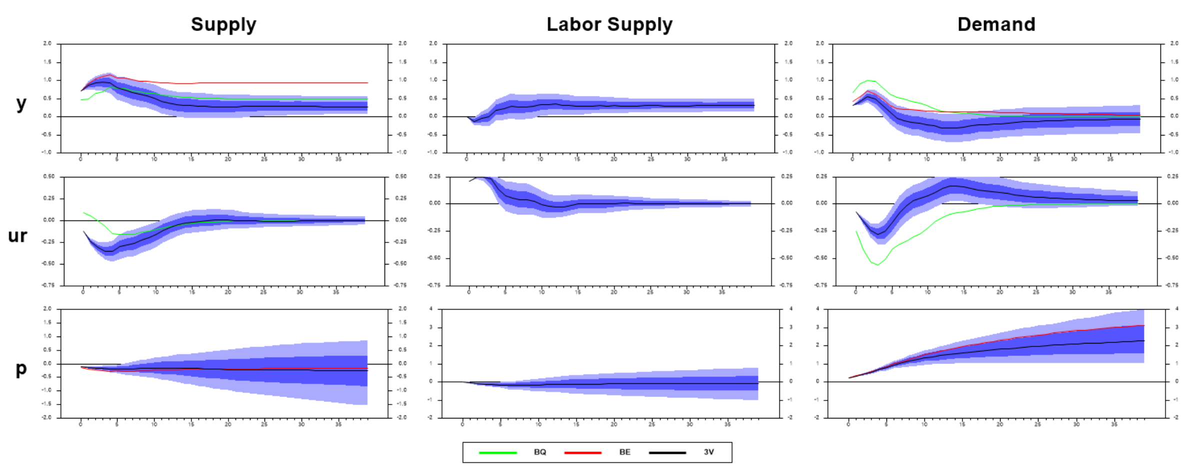

Figure A3. IRF's for the Tri-variate model (with shaded 68% and 95% bands), BQ, and BE models (extended sample)

D. Variance decompositions (Table A4)

(*Case 1*) Decreasing the number of lags to two, does not qualitatively change the stylized fact that BQ and BE models overstate the importance of demand shocks (Table A4, panel A). In contrast to the base-case, the BE model now also overstates their importance for p_t fluctuations. As in the base-case, the variance decomposition for the tri-variate model continues to point to the importance of labor-supply shocks in the short-run and supply shocks as the forecast horizon is extended in explaining movements in ur_t. (*Case 2*) For completeness, I provide the variance decomposition when detrended ur_t is used in the relevant VAR models results (Table A4, panel B). Qualitatively, these show similar results as those form the base-case, that is, BQ and BE overstate the importance of demand shocks. But now the BE model also overstates demand shocks for p_t fluctuations. However, as noted, these results should be taken with a grain of salt as both bivariate models fail missing variables and fundamentalness tests. Regarding the tri-variate model, movements in ur_t are associated with labor-supply shocks in the short-run with the importance of supply shocks increase (but not as pronouncedly) at longer horizons. (*Case 3*). Extending the sample through 2009:04 does not qualitatively change the conclusions regarding the importance of demand shocks being overstated in the BQ and BE models for all variables in the models. And for the tri-variate model, fluctuations in ur_t continue to be dominated by labor-supply and supply shocks respectively in the short- and longer-term.

Table A4 Variance Decompositions (percent due to structural shocks)

A. Two lags

	Permanent					***Demand***		
	Trivariate			BQ	BE	Trivariate	BQ	BE
	Supply	Labor	Sum					
				y_t				
Horizon								
0	70.17	0.00	70.2	16.5	93.1	29.8	83.45	6.91
1	66.53	0.34	66.9	13.2	93.0	33.1	86.81	7.03
2	65.17	0.25	65.4	13.2	94.2	34.6	86.79	5.84
3	64.30	0.26	64.6	14.9	95.1	35.4	85.08	4.90
4	63.58	0.87	64.5	17.9	95.9	35.5	82.14	4.10
8	56.98	12.75	69.7	35.3	97.7	30.3	64.66	2.28
12	47.96	28.07	76.0	48.9	98.5	24.0	51.13	1.51
24	38.03	46.78	84.8	67.3	99.3	15.2	32.72	0.74
40	32.94	56.85	89.8	77.9	99.6	10.2	22.12	0.44
				ur_t				
Horizon								
0	18.43	66.22	84.6	26.2	...	15.4	73.76	...
1	23.64	53.29	76.9	16.7	...	23.1	83.30	...
2	26.41	45.99	72.4	12.3	...	27.6	87.68	...
3	27.51	41.65	69.2	10.0	...	30.8	89.96	...
4	27.87	38.89	66.8	8.8		33.2	91.23	
8	27.41	35.34	62.8	7.5	...	37.3	92.49	...
12	27.45	35.36	62.8	7.6	...	37.2	92.42	...
24	27.51	35.33	62.8	7.5	...	37.2	92.46	...
40	27.51	35.33	62.8	7.5	...	37.2	92.46	...
				p_t				
Horizon								
0	19.96	0.00	20.0	...	4.0	80.0	...	95.95
1	15.70	0.00	15.7	...	2.7	84.3	...	97.35
2	14.61	0.02	14.6	...	2.4	85.4	...	97.56
3	13.44	0.06	13.5	...	2.3	86.5	...	97.71
4	12.42	0.15	12.6		2.2	87.4		97.75
8	8.95	0.73	9.7	...	2.2	90.3	...	97.80
12	6.97	1.19	8.2	...	2.2	91.8	...	97.80
24	5.32	1.54	6.9	...	2.2	93.1	...	97.80
40	4.84	1.63	6.5	...	2.2	93.5	...	97.79

Note. All VAR models are estimated using eight lags over the sample period 1950:02 to 1987:04. Before estimation I remove the sample means for the three variables before and after the first OPEC oil shock (1973:04). This treatment of the data series is consistent with the BE sample split but differs from BQ's base case. Nonetheless, differences are quantitative in nature. Structural shocks are identified using short- and long-run restrictions as discussed in the main text.

Table A4 Variance Decompositions (percent due to structural shocks) (continued)

B. Detrended ur

	Permanent					Demand		
	Trivariate			BQ	BE	Trivariate	BQ	BE
	Supply	Labor	Sum					
				y_t				
Horizon								
0	74.3	0.0	74.3	3.2		25.7	96.84	58.02
1	76.3	0.8	77.1	1.3		22.9	98.71	59.39
2	79.2	0.9	80.1	2.3		19.9	97.72	60.66
3	80.5	2.1	82.6	3.7		17.4	96.30	61.91
4	79.1	5.9	85.1	6.9		14.9	93.07	62.77
8	57.8	18.9	76.7	25.7		23.3	74.33	44.62
12	48.0	26.6	74.6	37.1		25.4	62.87	33.34
24	37.0	41.7	78.7	52.2		21.3	47.84	17.47
40	32.2	49.8	81.9	62.6		18.1	37.36	9.82
				ur_t				
Horizon								
0	35.5	61.4	96.8	38.7	...	3.2	61.29	...
1	42.9	50.8	93.7	28.1	...	6.3	71.85	...
2	52.4	40.1	92.5	20.7	...	7.5	79.31	...
3	60.4	31.2	91.6	15.6	...	8.4	84.45	...
4	66.1	25.2	91.3	12.6		8.7	87.44	
8	58.4	16.8	75.3	10.8	...	24.7	89.22	...
12	49.4	14.8	64.1	12.1	...	35.9	87.89	...
24	45.8	13.3	59.1	12.2	...	40.9	87.75	...
40	47.0	12.3	59.3	12.3	...	40.7	87.74	...
				p_t				
Horizon								
0	36.0	0.0	36.0	...	54.8	64.0	...	45.23
1	40.8	0.1	40.9	...	51.3	59.1	...	48.74
2	44.8	0.2	45.0	...	48.1	55.0	...	51.93
3	47.9	0.3	48.2	...	46.3	51.8	...	53.68
4	50.6	0.3	50.9		45.0	49.1		55.03
8	58.3	0.2	58.5	...	46.4	41.5	...	53.61
12	68.1	0.2	68.2	...	46.5	31.8	...	53.53
24	88.1	0.1	88.2	...	46.3	11.8	...	53.67
40	93.0	0.1	93.1	...	46.3	6.9	...	53.74

Note. All VAR models are estimated using eight lags over the sample period 1950:02 to 1987:04. Before estimation I remove the sample means for the three variables before and after the first OPEC oil shock (1973:04). This treatment of the data series is consistent with the BE sample split but differs from BQ's base case. Nonetheless, differences are quantitative in nature. Structural shocks are identified using short- and long-run restrictions as discussed in the main text.

Table A4 Variance Decompositions (percent due to structural shocks) (concluded)

C. Extended sample

y_t

	Permanent					***Demand***		
	Trivariate			BQ	BE	Trivariate	BQ	BE
	Supply	Labor	Sum					
Horizon								
0	70.32	0.00	70.3	35.4	79.9	29.68	64.59	20.13
1	66.45	0.88	67.3	27.9	77.3	32.66	72.09	22.72
2	62.11	0.55	62.7	28.6	74.2	37.34	71.36	25.79
3	61.73	0.39	62.1	30.1	74.2	37.89	69.91	25.78
4	62.26	1.08	63.3	34.7	75.2	36.66	65.28	24.84
8	68.59	5.19	73.8	46.3	82.6	26.22	53.72	17.37
12	68.50	9.72	78.2	51.8	86.2	21.78	48.17	13.82
24	62.54	19.00	81.5	60.9	91.3	18.46	39.11	8.74
40	57.46	27.27	84.7	68.3	94.4	15.28	31.72	5.57

ur_t

Horizon								
0	19.29	67.91	87.2	9.3	...	12.81	90.69	...
1	25.80	52.51	78.3	3.4	...	21.69	96.64	...
2	29.02	41.88	70.9	1.7	...	29.10	98.35	...
3	33.19	34.17	67.4	2.1	...	32.64	97.91	...
4	36.99	28.15	65.1	4.2		34.86	95.77	
8	48.82	20.56	69.4	9.5	...	30.62	90.48	...
12	53.60	18.11	71.7	11.0	...	28.29	89.02	...
24	50.45	16.03	66.5	11.1	...	33.51	88.86	...
40	48.95	15.47	64.4	11.1	...	35.59	88.86	...

p_t

Horizon								
0	36.34	0.00	36.3	...	28.5	63.66	...	71.53
1	34.71	0.04	34.8	...	26.0	65.25	...	74.02
2	30.30	0.64	30.9	...	22.0	69.07	...	77.98
3	27.90	1.05	28.9	...	19.5	71.06	...	80.52
4	25.34	1.41	26.8		16.9	73.25		83.07
8	22.65	1.40	24.0	...	13.8	75.96	...	86.23
12	22.73	0.93	23.7	...	13.4	76.34	...	86.61
24	22.54	0.68	23.2	...	13.1	76.79	...	86.95
40	21.11	0.66	21.8	...	12.9	78.23	...	87.08

Note. All VAR models are estimated using eight lags over the sample period 1950:02 to 1987:04. Before estimation I remove the sample means for the three variables before and after the first OPEC oil shock (1973:04). This treatment of the data series is consistent with the BE sample split but differs from BQ's base case. Nonetheless, differences are quantitative in nature. Structural shocks are identified using short- and long-run restrictions as discussed in the main text.

E. Δbfk_t and VAR-based Structural Shocks

FL "cross-type" correlations (Table A5). (*Case 1*) The pattern of the FL structural shock correlations using two lags remains unchanged: the highest correlation of Δbfk_t is with the BQ supply shock, and Δbfk_t remains more correlated to the tri-variate model's labor-supply shocks than to technological shocks. Also, it continues to exhibit limited correlation with the temporary/demand shocks. (*Case 2*) Though the pattern remains when using detrended ur_t, higher correlation is found with the tri-variate model's demand shock. As noted above, these results should be taken with a grain of salt. (*Case 3*) The highest correlation remains to the BQ supply shock but now the correlation with the supply shocks from the BE and tri-variate models have increased. And because of these increases, these correlations now exceed those with the labor-supply shock, which remains as strong as before.

Table A5. Correlation of BFK shocks with VAR-based shocks, Robustness

	Permanent	Labor supply	Temporary
A. Two lags			
Blanchard-Quah	0.24	...	0.00
Bayoumi-Eichengreen	0.09	...	-0.07
Tri-variate model	0.14	0.19	-0.04
B. Detrended ur			
Blanchard-Quah	0.19	...	0.03
Bayoumi-Eichengreen	0.09	...	-0.05
Tri-variate model	-0.01	0.18	0.15
C. Extended sample			
Blanchard-Quah	0.28	...	0.00
Bayoumi-Eichengreen	0.23	...	-0.09
Tri-variate model	0.23	0.19	-0.10

Note. Quarterly BFK shocks are taken from Fernald (2014). VAR-based structural shocks are identified using the long- and short-run restrictions discussed in the main text for the tri-variate model and the long-run restrictions in Blanchard-Quah and Bayoumi-Eichengreen. VAR models are estimated using eight lags using quarterly data from 1950:Q2 to 1987:Q4

Propagation of Δbfk_t shocks

The dynamic responses of ur_t (Figures A4 to A6): (*Case 1*) Decreasing lags to two in the univariate near-VAR models, does not qualitatively change the results: the propagation is similar to that in Figure 2 and ur_t increases following Δbfk_t, BQ supply, and tri-variate labor-supply shocks. (*Cases 2 and 3*) The propagation patterns remain qualitatively unchanged.

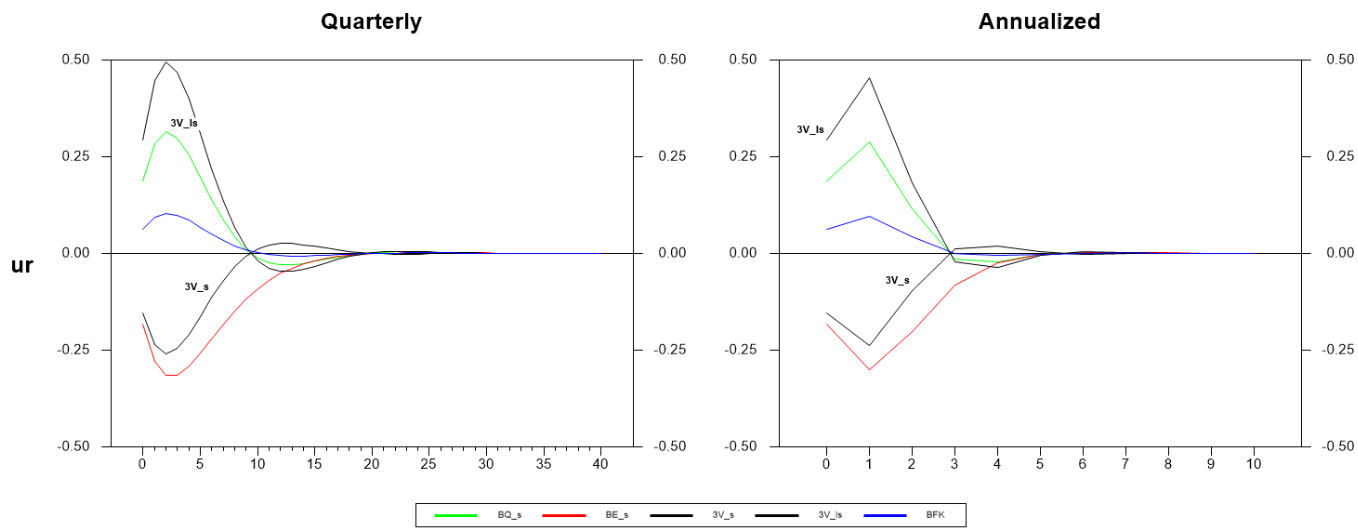

Figure A4. Unemployment responses to supply shocks (univariate near-VAR models) (two lags)

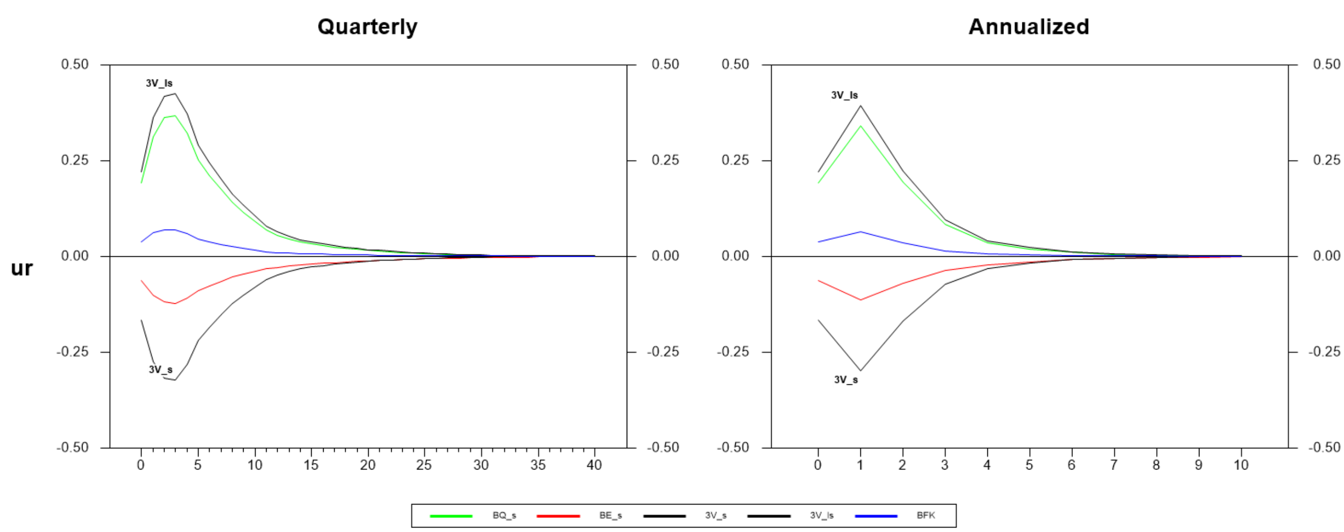

Figure A5. Unemployment responses to supply shock (univariate near-VAR models) (ur detrended)

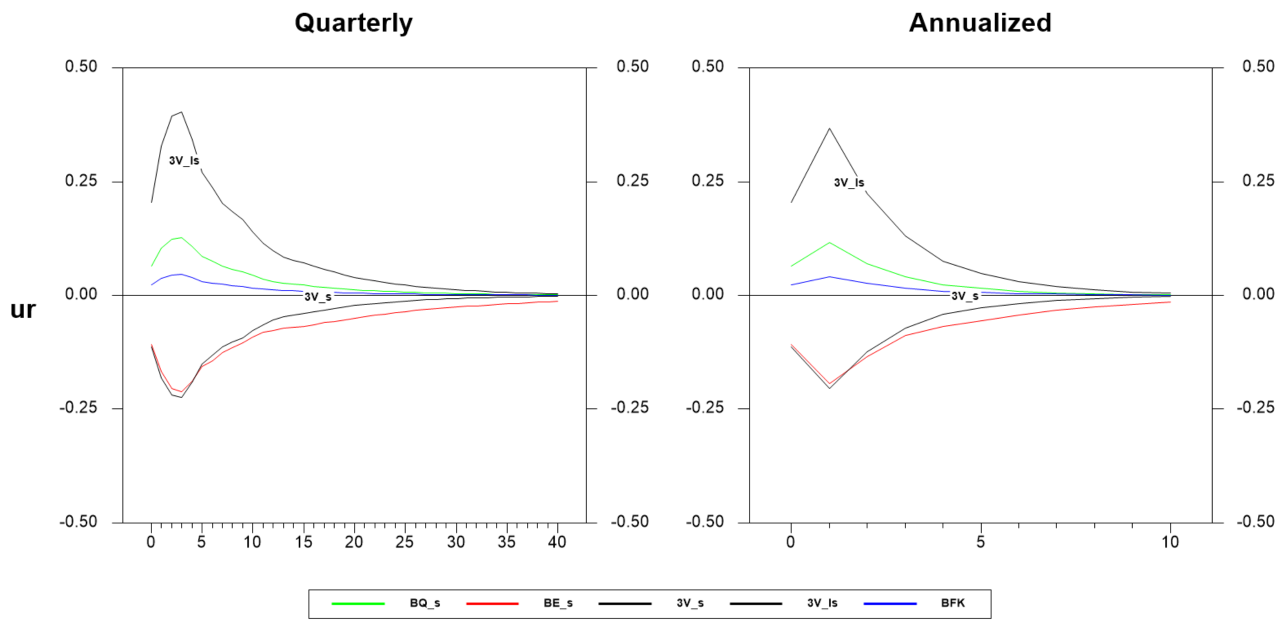

Figure A6. Unemployment responses to supply shocks (univariate near-VAR) (extended sample)

The dynamic responses of the labor force and wages. (Figures A7 to A9): (*Case 1*) Reducing the lags to two, does not change the bottom line for the tri-variate model: the propagation remains consistent with the wage "duck test." Note that the tri-variate model's technology and the BE supply shock now reduce the labor force. And except for the Δbfk_t shock and the labor-supply shock, wages increase as expected following technology shocks from the BQ and BE models. (*Case 2*) Using de-trended ur_t results in propagation qualitatively the same as the base-case: wages suggest that structural shocks pass the duck test and the labor forces increase following a labor-supply shock. The implications for BFK are unchanged. (*Case 3*) Extending the sample period results in propagation results that are qualitative the same as in the base-case: the wage "duck test" show that wages decline following a labor-supply shock as well as following a pure Δbfk_t shock and increase for other supply shocks. Regarding the labor force propagation, the main difference is that the labor force declines following the BE model's supply shock.

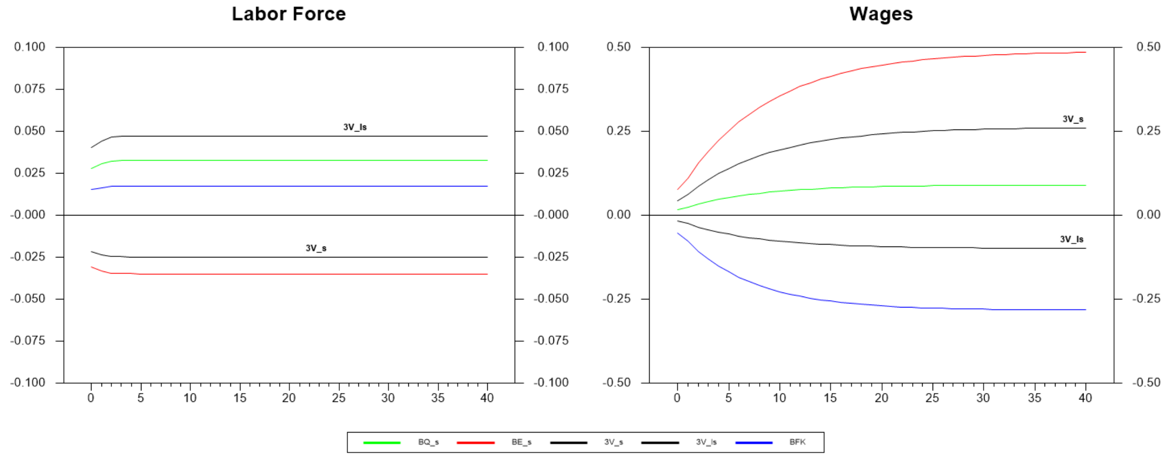

Figure A7. Labor Force and Wage responses to supply shocks (univariate near-VAR) (two lags)

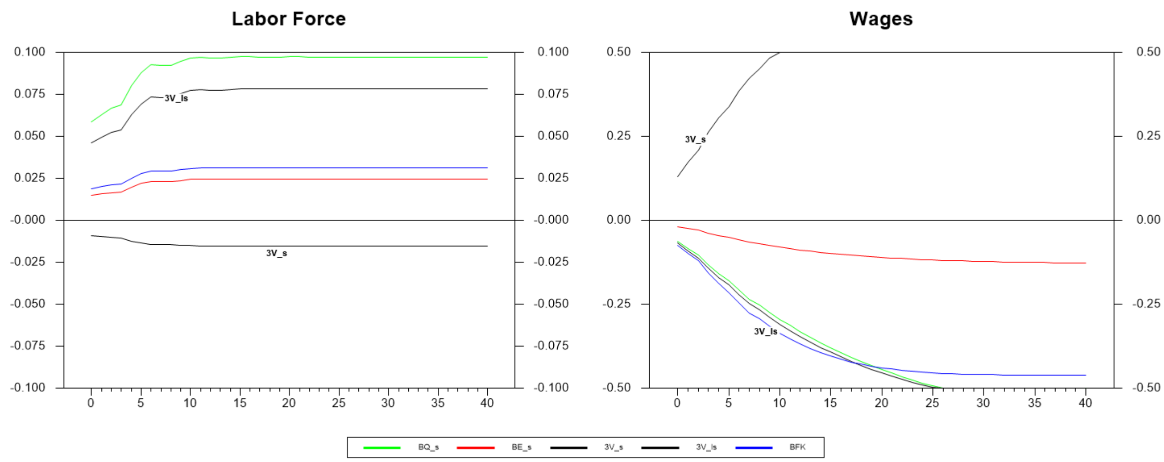

Figure A8. Labor Force and Wage responses to supply shocks (univariate near-VAR) (extended sample)

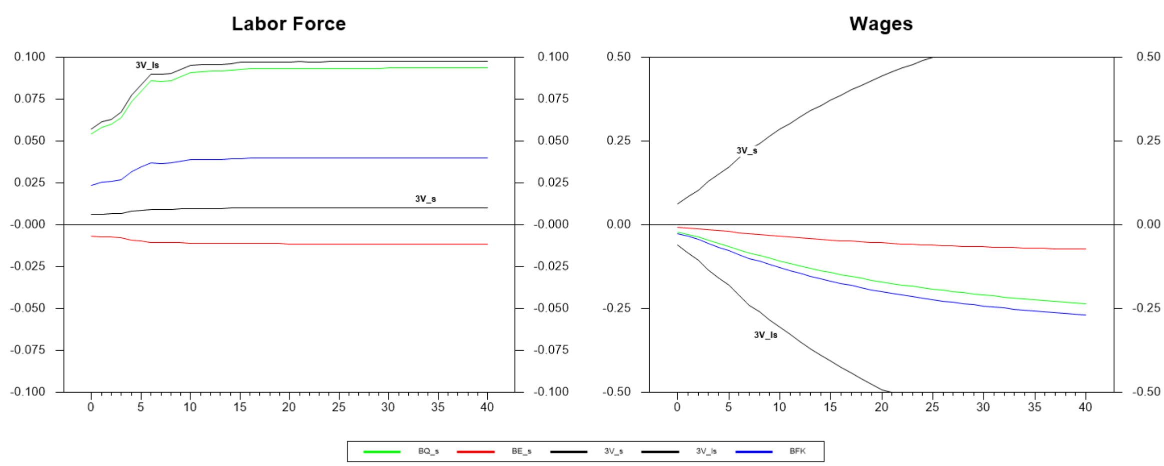

Figure A9. Labor Force and Wage responses to supply shocks (univariate near-VAR) (extended sample)

Appendix III. Missing-Variable and Fundamentalness Tests

Here I detail the formulas of the four tests reported in Tables 3 and A3. Namely, the three tests discussed in CH to check for missing variables and assess fundamentalness, plus the Sims test as modified by Geweke, Meese, and Dent (1982) (henceforth GMD).

Consider the partitioned tri-variate model in the main text:

$$\left[I - C(L)\right] \cdot \begin{bmatrix} Y_{1,t} \\ Y_{2,t} \end{bmatrix} = U_t^{3V},$$

where $Y_{1,t}$ and $Y_{2,t}$ contain respectively the BQ (or BE) variable pairing and the corresponding "third" variable. As noted, the information set is limited to that spun by $Y_{1,t}$ and $Y_{2,t}$.

***Granger-causality test**:*

System-wide:

$$Y_{1,t} = C^{(1,1)}(L) \cdot Y_{1,t-1} + C^{(1,2)}(L) \cdot Y_{2,t-1}$$

F-test for $C^{(1,2)}(L) = 0$

Equation-by-equation:

$$Y_{1,t}(i) = C_i^{(1,1)}(L) \cdot Y_{1,t-1}(i) + C_i^{(1,2)}(L) \cdot Y_{2,t-1} \text{ for } i\text{=1, 2}$$

F-test for $C_i^{(1,2)}(L) = 0$

Sims-exogeneity test (GMD version):[74]

System-wide:

$$Y_{2,t} = C^{(2,2)}(L) \cdot Y_{2,t-1} + C^{(2,1)}(L) \cdot Y_{1,t} + D^{(2,1)}(F) \cdot Y_{1,t+1}$$

F-test for $D^{(2,1)}(F) = 0$

Equation-by-equation:

$$Y_{2,t} = C^{(2,2)}(L) \cdot Y_{2,t-1} + C_i^{(2,1)}(L) \cdot Y_{1,t}(i) + D_i^{(2,1)}(F) \cdot Y_{1,t+1}(i) \text{ for } i\text{=1, 2}$$

F-test for $D_i^{(2,1)}(F) = 0$

Orthogonality (Forni-Gambetti) test:

System-wide:

$$E_{1,t} = H(L) \cdot E_{1,t-1} + G(L) \cdot Y_{2,t-1}$$

F-test for $G(L) = 0$

Equation-by-equation:

$$e_{i,t} = H_i\,(L) \cdot e_{i,t-1} + G_i\,(L) \cdot Y_{2,t-1} \text{ for } i\text{= s, d}$$

F-test for $G_i\,(L) = 0$

[74] Note that the lead elements in $C_i^{(1,2)}$ and $C^{(1,2)}$ are respectively 1 and *I*.

Canova-Hamidi test (GMD version):

System-wide:

$$Y_{2,t} = C^{(2,2)}(L) \cdot Y_{2,t-1} + G(L) \cdot U_{1,t} + \tilde{G}(F) \cdot U_{1,t+1}$$

F-test for $\tilde{G}(F) = 0$

Equation-by-equation:

$$Y_{2,t} = C^{(2,2)}(L) \cdot Y_{2,t-1} + G_i\,(L) \cdot \mu_{i,t} + \tilde{G}_i\,(F) \cdot \mu_{i,t+1} \text{ for } i=1,\,2$$

F-test for $\tilde{G}_i\,(F) = 0$

APPENDIX IV. TRI-VARIATE MODEL SOLUTION

The augmented illustrative Keynesian model can be expressed as:

$$y_t = m_t - p_t + \theta \cdot a_t \qquad \text{(aggregate demand equation)}$$

$$y_t = n_t + a_t \qquad \text{(production function equation)}$$

$$p_t = w_t - a_t + \lambda \cdot m_t \qquad \text{(price-setting equation)}$$

$$w_t = w| \ \left\{E_{t-1}\left[\left(\tilde{n}_t - n_t\right)\right] = 0\right\} \qquad \text{(wage-setting equation)}$$

$$ur_t = \tilde{n}_t - n_t \qquad \text{(unemployment definition)}$$

where y_t, m_t, p_t, a_t, and n_t denote the logs of output, money, prices, productivity, and employment, with (the log of) full employment denoted by $\tilde{n}_t$ and thus ur_t defines the unemployment rate. As in the original BQ model, aggregate demand is a function of real balances and productivity, while production reflects employment and productivity. But prices, in addition to reflecting wages and productivity, now also reflects m_t. Wages are set one-period in advance, as in the BQ model, such that full employment is achieved (in expected terms). I define $\Delta a_t = e_{s,t}$ and $\Delta m_t = e_{d,t}$ as the technology and demand shocks consistent with BQ, and add a labor-supply shock by replacing the original model's (implicit assumption), $\Delta \tilde{n}_t = 0$, with the assumption that it follows a random walk, $\Delta \tilde{n}_t = e_{\tilde{n},t}$, where $e_{\tilde{n},t}$ denotes labor-supply shocks.

Note that, in terms of the involuntary unemployment model in Christiano et al. (2021) $e_{\tilde{n},t}$ can be interpretated as a low "adversion-to-work" realization prompting increases in labor participation and the labor force that, given the prevailing wage, increases (involuntary) ur_t. Also, in terms of the model's discussed in Hall (2008), $e_{\tilde{n},t}$ could be interpretated as a "labor-*force* supply" shock. This requires re-interpreting unemployment, for a given wage rate, as the difference between the labor-force supply (n^{LF}) and the equilibrium employment (n_t^* ,) from the labor market (the equality of labor demand and employment supply). In this case, "equilibrium" wages would be set to be consistent with a labor market equilibrium that (in expected terms) results in a constant "natural" rate of unemployment:

$$w_t = w| \ \left\{ E_{t-1}\left[\left(n_t^{NF} - n_t^*\right)\right] = ur^{(natural)} \right\}$$

Regardless, the augmented illustrative Keynesian model can be expressed in tri-variate form as:

$$\begin{bmatrix} 1 & 0 & 1 \\ 1 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} \cdot \begin{bmatrix} y_t \\ ur_t \\ p_t \end{bmatrix} = \begin{bmatrix} m_t + \theta \cdot a_t \\ \tilde{n}_t + a_t \\ w_t - a_t + \lambda \cdot m_t \end{bmatrix}$$

or:

$$\begin{bmatrix} y_t \\ ur_t \\ p_t \end{bmatrix} = \begin{bmatrix} 1 & 0 & -1 \\ -1 & 1 & 1 \\ 0 & 0 & 1 \end{bmatrix} \cdot \begin{bmatrix} m_t + \theta \cdot a_t \\ \tilde{n}_t + a_t \\ w_t - a_t + \lambda \cdot m_t \end{bmatrix}$$

so that the tri-variate model's equations can be expressed as:

$$y_t = \left(1-\lambda\right) \cdot m_t + \left(1+\theta\right) \cdot a_t - w_t$$

$$ur_t = -m_t - \theta \cdot a_t + \tilde{n}_t + w_t + \lambda \cdot m_t$$

$$p_t = w_t - a_t + \lambda \cdot m_t$$

The full solution follows from replacing w_t with model-consistent wage expectations, w_t^*. The latter can be obtained by taking the expectation of w_t (at time t-1) from the ur_t equation above (solved for wages),

$$w_t = ur_t + \left(m_t + \theta \cdot a_t - \tilde{n}_t - \lambda \cdot m_t\right),$$

that assuming full employment (in expected terms) can be written as:

$$w_t^* = E_{t-1}\left[ur_t + m_t - \tilde{n}_t + \theta \cdot a_t - \lambda \cdot m_t \mid E_{t-1}\left[\tilde{n}_t - n_t\right] = 0\right]$$

Taking the expectation and using the definitions $a_t = a_{t-1} + e_{s,t}$, $\tilde{n}_t = \tilde{n}_{t-1} + e_{\tilde{n},t}$, and $m_t = m_{t-1} + e_{d,t}$ where $e_{\cdot,t}$ denotes a zero-mean structural shock:

$$w_t^* = \left(1-\lambda\right) \cdot m_{t-1} - \tilde{n}_{t-1} + \theta \cdot a_{t-1}$$ [75]

Setting $w_t = w_t^*$, the solution of the illustrative tri-variate structural model can be expressed as follows:

(1) Δy_t:

$$\begin{aligned} y_t &= \left(1-\lambda\right) \cdot m_t + \left(1+\theta\right) \cdot a_t - \left\{\left(1-\lambda\right) \cdot m_{t-1} - \tilde{n}_{t-1} + \theta \cdot a_{t-1}\right\} \\ &= \left(1-\lambda\right) \cdot m_t - \left(1-\lambda\right) \cdot m_{t-1} + \left(1+\theta\right) \cdot a_t + \tilde{n}_{t-1} - \theta \cdot a_{t-1} \end{aligned}$$

and taking the first difference:

[75] Note that the expression for w_t^* (and the corresponding solution for ur_t) are augmented by an additive constant, $ur^{(natural)}$ when $e_{\tilde{n},t}$ is interpreted as a labor *force* supply shock. This constant cancels out in the solutions of other variables, including for its fourth variable, Δw_t .

$$\Delta y_t = (1-\lambda)\cdot\Delta e_{d,t} + (1+\theta)\cdot e_{s,t} - \theta\cdot e_{s,t-1} + \varepsilon_{\tilde{n},t-1}$$
$$= e_{s,t} + \theta\cdot\Delta e_{s,t} + \varepsilon_{\tilde{n},t-1} + (1-\lambda)\cdot\Delta e_{d,t}$$

(2) ur_t:

$$ur_t = -m_t - \theta\cdot a_t + \tilde{n}_t + \left\{(1-\lambda)\cdot m_{t-1} - \tilde{n}_{t-1} + \theta\cdot a_{t-1}\right\} + \lambda\cdot m_{t-1}$$
$$= -(1-\lambda)\cdot\Delta m_t - \theta\cdot\Delta a_t + \Delta\tilde{n}_t$$
$$= -\theta\cdot e_{s,t} + e_{\tilde{n},t} - (1-\lambda)\cdot e_{d,t}$$

and (3) Δp_t:

$$p_t = \left\{(1-\lambda)\cdot m_{t-1} - \tilde{n}_{t-1} + \theta\cdot a_{t-1}\right\} - a_t + \lambda\cdot m_t$$
$$= \lambda\cdot m_t + (1-\lambda)\cdot m_{t-1} - \tilde{n}_{t-1} - a_t + \theta\cdot a_{t-1}$$

and taking the first difference:

$$\Delta p_t = \lambda\cdot\Delta m_{t-1} + (1-\lambda)\cdot\Delta m_{t-1} - \Delta\tilde{n}_{t-1} - \Delta a_t + \theta\cdot\Delta a_{t-1}$$
$$= -e_{s,t} + \theta\cdot e_{s,t-1} - e_{\tilde{n},t-1} + \lambda\cdot e_{d,t} + (1-\lambda)\cdot e_{d,t-1}$$

Note that the implicit solution for the tri-variate model's "fourth" can be obtained from the solution for $w_t\,(=w_t^*)$ and taking the first difference:

$$\Delta w_t = \Delta w_t^* = \theta\cdot e_{s,t-1} - e_{\tilde{n},t-1} + (1-\lambda)\cdot e_{d,t-1}.$$

Note further that $\Delta w_t = -ur_{t-1}$.

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