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Notes on contributor

I graduated from the Moscow Institute of Physics and Technology and received the degree of Candidate of Physical and Mathematical Sciences there. The second scientific degree of Doctor of Phys. and Math. Sciences I obtained at the Computer Center of the Siberian Branch of the USSR Academy of Sciences. My main works until the early 2000s were devoted to optimal control and ill-posed problems of data processing for physical and geophysical experiments. Since the late1990th, my research interests turn to the problem of the production economy and consumer markets. In view of crisis state of economic theory, I also delved in the problem of this theory and economic methodology.

ABSTRACT

The problem of consumer demand in modern (micro)economic theory is that this theory contains a normative theory of individual demand, but does not contain a positive theory of market demand – an object of real interest for economists-practitioners and governments. This failure has led to failures in equilibrium theory and applied demand analysis. The article presents a methodological analysis of this demand problem, created within the framework of general scientific methodology and with rejection of the individual demand theory. The studying object of this theory, is a fuzzy set of market buyers regarded as a holistic object and called the 'statistical ensemble of consumers.' This theory formally retains the individual demand theory, but it is a positive theory verifiable by trade statistics. The verification method of the theory is a development of the Afriat-Varian non-parametric one with the simultaneous calculation of economic indexes reflecting the population preferences.

Keywords*:* Market demand problem, methodological analysis, statistical ensemble of consumers, verification, Konüs indexes

JEL Classification*:* D11 – Consumer Economics: Theory; B59 – Other Heterodox Approaches; C51 Model Construction and Estimation; C43 – Index Numbers and Aggregation;

Such complicated laws as those of economy cannot be accurately traced in individual cases. Their operation can only be detected in aggregates and by the method of averages.

W.S. Jevons (1866) [1]

1. Introduction

The modern neoclassical economic theory (orthodox Economics) is constructed within the framework of a radical form of methodological individualism and forms the basis of economic education and most economic research of the world. It contains a normative individual demand theory (IDTh), but does not contain a positive theory of final multi-commodity market demand – an object of real interest for producers of goods and services, trade and governments. This Economics failure has led to failures in Walrasian equilibrium theory, applied demand analysis, the construction of economic price and quantity indexes of market demand that reflect the population preferences. These indexes are a generalization of the cost-of-living (COL) index, introduced by Alexander Konüs in a Russian article of 1924 (1939 – translation), as a consumer price index (CPI). These indexes, defined within IDTh, currently refer only to individuals / households (Diewert, 1993; CPIM, 2004, Chs. 17-18), but Konüs' COL serves as the conceptual framework for the notion of CPI in some countries (Triplett, 2001).

In the last three decades, this and other unresolved basic questions of economic theory have caused many critical works on the Economics' state and its methodology, which include, among others, the book by Geoffrey Hodgson (1988), articles by Maurice Allais (1990), Viktor Polterovich (1998), Alan Kirman (2006, 2010) and Claud Hillinger (2008). In recent years, journals and educational literature have emerged that recognize or present heterodox approaches to economic research (e.g. Cambridge Journal of Economics; Hayashi, 2021; Petri, 2021). Microeconomics by Takashi Hayashi is written strictly within methodological individualism, but the author already recognizes the legitimacy of heterodox economic research and intends to convince its authors of the scientific productivity of the neoclassical mainstream as well. Microeconomics by Fabio Petri goes beyond individualist methodology. It discusses the failures of neoclassical microeconomics, attempts to correct them, and presents alternative approaches to economic problems – classical (Smith, Ricardo, Marx) and Keynesian – to overcome the inertia of specialists and students who believe that methodological individualism is '*the only possible way to explain the functioning of the market economy*.' (p. vi).

Despite the critical works of authoritative economists on the negative role of the radical form of methodological individualism in the formation of positive economic theory, this methodology continues to form mainly scholastic researches allowed in high-ranking journals, blocking heterodox approaches, particularly scientific one. This justifies the need to deepen a critical exploring of mainstream methodology through formal-logical analysis and to propose a constructive theoretical alternative that resolves the generally recognized practical problems on a scientific basis. Accordingly, the purpose of this article is methodological analysing the microeconomic problem of consumer demand and to present a constructive alternative – a positive theory of market demand,

[1] 18th thesis of Jevons' speech at the British Science Association in 1862 (printed in 1866).

built within a general scientific methodology in my Russian works (Gorbunov, 2004, 2015) and the verification method of this theory with simultaneous constructing Konüs (economic / analytical) indexes (Gorbunov, Lvov, 2019; Gorbunov, Kozlova, Lvov, 2020; Gorbunov, Lvov, 2022)[2].

The rest of the article is organized as follows. Section 2 discusses the methodology of positive economic theory. Section 3 analyses the microeconomic problem of demand theory and the 'applied demand analysis' of Stone-Deaton. Section 4 presents the meaningful and formal basics of the holistic theory of market demand. Section 5 briefly discusses the parametric verification method of the market demand theory in the Deaton setting, and presents our nonparametric verification method. Section 6 concludes.

2. On the methodology of positive economic theory

2.1. The program 'Economics as a science'

The well-known textbook by Boumans and Davies (2016) describes the history, basic principles and facts of the methodology of economic theory, understood as a science, and here I add some methodological issues specific to the social sciences and related with the market demand problem.

The founders of the mathematized neoclassical approach to Economics that are William Stanley Jevons (1835-l882) and Leon Walras (1834-1910) intended to reconsider Economics in general and create a demand theory on general scientific principles used in the natural sciences, firstly, mechanics and physics. The coined essence of these principles is *objectivity, provability and verifiability by facts*. The article's epigraph says that Jevons understood that the demand theory of interest to any economic activity is the theory of aggregate market demand. However, he and Walras started by constructing a theory of individual demand. Perhaps because of the ideas of the classics Adam Smith and John Stuart Mill about Man as an independent egoist (Homo Economicus) that were established in Europe in the 19th century. The second reason for the creation of an IDTh could be the uncommon productivity of the *reductionist approach* in the natural sciences in the study of complex objects. I discuss this approach below.

Some way or other, they both, but independently, suggested the rationality principle in consumer behaviour that had been earlier conjectured by Hermann Gossen (1810-1858) in a general problem of obtaining satisfaction with limited resources. Walras formulated this principle as maximizing the buyer's 'utility' of a commodity bundle under budgetary constraint. Significantly, that this principle was based on watching the collective market behaviour, but in their seminal 'Political Economies' (Jevons, 1957/1871; Walras, 1954/1877), it was imposed on an individual consumer.

The programs to revise economic theories along the line of natural sciences met with stiff resistance from most research economists already at the late 19th century, and this resistance continues to this day. Opponents of the 'scientization' of economic theory explain their position by the significant differences in natural and social phenomena. In doing so, they deny the legitimacy of a (natural)scientific approach to economic problems.

The opponents' arguments about the significant differences between natural and social phenomena are not in doubt. The feature of the natural sciences is the identity of elementary objects in their classes, complete in physics, chemistry, and molecular biology and limited, but sufficient for

[2] The first English presentations of our Russian works are in (Gorbunov: 2018, 2021a, 2021b).

productive analysis, in the biology of living organisms. Here main processes are usually reproduced experimentally, and this simplifies the formalization of their research and the application of mathematical and statistical methods to create and verify many descriptive laws and predictive theories.

The feature of the social sciences is that the 'atoms' of the objects - persons - have *a psyche* and *an active mind*, an ability to work and transform the environment and society. People behave, especially when making decisions under uncertainty, in a poorly predictable manner and often spontaneously. Accordingly, economic processes at the personal (when buying), meso and macro levels are unique, which limits the possibilities of experiments, error assessment, and this feature complicates the verification of the proposed theories.

However, the Natural Sciences and Social Sciences (Humanities) are Sciences, so there should be commonality in their definitions and methodology, consistent with the root concept of Science. We understand 'Science' as *a system of non-trivial knowledge, justified logically and empirically, about some system of real objects*. Next, I consider specific characteristics of the scientific methodology that are required for exploring the demand problem.

2.2. Normativity and positivity in the contemporary social sciences

For social sciences it is essential that people differ not only in abilities, characters, consumer preferences, but also in preferences regarding the principles of social organization. The latter reveals itself in the description of a socio-economic problem not *as it is*, but *as it should be*. In 1890, while discussing this phenomenon, John Neville Keynes (1998) proposed to distinguish in socio-economic researches a *positive approach* studying phenomena '*as they are'* - and a *normative approach* – formulating '*what they should be*.'

So, theories of real social phenomena whose conclusions are justified logically and empirically, that is, scientific theories, are positive theories. The book by Boumans and Davis (2016) is devoted mainly to the methodology of positive economic theory, understood by the authors 'as a science'.

Political economy in its understanding today is the study of the relationship between economic and political systems and institutions. Here the influence of subjective ideological and political preferences on research and proposed theories of 'social constructivism' is inevitable. Objectivity is rare here, and political economies are usually based on the dogmatic principles of some ideology representing the interests of a part of society. So, political economy is the set of normative theories.

The founder of monetarism, Milton Friedman, understood the need to develop positive economic theories as a tool for analysing normative theories and noted the resistance to this process from experts guided by their normative (ideological) biases (1953, p. 4):

> The conclusions of positive economics seem to be, and are, immediately relevant to important normative problems, to questions of what ought to be done and how any given goal can be attained. Laymen and experts alike are inevitably tempted to shape positive conclusions to fit strongly held normative preconceptions and to reject positive conclusions if their normative implications - or what are said to be their normative implications - are unpalatable.

Add also the inertia of thinking and career advantages of following the mainstream as reasons for the resistance of highly ranked journals to publications of a positive nature, which are inconsistent with mainstream methodology – *methodological individualism.*

The concept and basic principles of methodological individualism were formulated in the early 20th century by Joseph Schumpeter (1908, p. 8), beginning with the denial of the social character of '*total demand'* and '*total supply'* – concepts fundamental to any economic activity. Kenneth Arrow formulated the essence of this methodology and its influence on the nature of economic research (1994, p. 1):

> It is a touchstone of accepted economics that *all explanations must run in terms of the actions and reactions of individuals* [*our emphasis*]. Our behavior in judging economic research, in peer review of papers and research, and in promotions, includes the criterion that in principle the behavior we explain and the policies we propose are explicable in terms of individuals, not of other social categories.

Hayashi succinctly expressed the research technology within methodological individualism in the Preface to his textbook (2021): '*Professional work in economic theory is presented as a sequence of definitions, assumptions and their implications*.' (p. v). The verification problem is simply not considered here, and this is a technology of mathematics, which is not a science, but the most effective tool of proof in science. It is clear that this methodology sets very severe limits to creating positive theories of social / collective phenomena, in particular market demand, since the behaviour of individuals is not regular and the corresponding theory adequate to reality is still absent.

But the positive and normative approaches are not fundamentally mutually exclusive alternatives to theoretical constructions, see (Davis, 2013; Colander & Su, 2015). Really, in the theories of complex economic phenomena it may be impossible to avoid the entanglement of positive issues of presenting the object under study and the normative direction of the desirable character of its functioning. Equilibrium theory gives such an example.

The goal of equilibrium theory is to determine the prices at which market demand equals the supply of commodities produced by the economy. The mainstream variant of Walrasian equilibrium theory is developed by K. Arrow and G. Debreu (1954) within the methodological individualism with the axiomatic representation of individual consumers independently maximizing their 'utility' under budget constraints, and independent firms maximizing their profits on the sets of 'possible production plans.'[3] These representations of the demand and supply sides are normative, and the corresponding equilibrium theory is also normative.

The article (Gorbunov, 2018) presents an equilibrium model, which is a development of the Cassel-Wald model (Wald, 1951). Here, market demand is a whole entity verifiable by standard trade statistics of prices and quantities of commodities sold over several periods. The production side of the economy is represented by the Leontief model with linear resource constraints. The production model parameters that compile the technological and resource matrixes, are also provided by statistical services. Thus, the used demand and production theories approximately represent the real economy '*as it is*', and they are positive. But posing an equilibrium problem of practical interest requires defining the rationality of the production system more realistically than in the Arrow-Debreu model, given the government's ability to regulate and manage the system. This is a normative problem, and it is resolved in this equilibrium model as maximizing the value of gross output.

[3] As it is known, this unrealistic theory turned out to be 'a journey down the wrong road' (Kirman, 2006; Polterovich, 1998).

2.3. Reductionism and holism in science and economic theory

Reductionism and *holism*[4] are two methods of studying complex systems consisting of many interrelated elements and possessing *emergent* properties that are absent in their elements. Reductionism reduces the study of a complex system to the study of its elements, if possible. A completer study of complex systems, as a rule, requires systemic approach, taking into account the interactions of the elements of the system and emergent characteristics. From the mid-20th century, the term 'holism' is used to analyse the methodologies of various disciplines.

In physics, reductionism is a natural method, since the main problem here is the study of the basic laws of organization and functioning of the simplest forms of the material world. However, the physicist Philip Anderson (Nobel laureate 1977), in his famous article (1972) '*More is different...*', analysing the excessive enthusiasm for reductionist methodology in physics and other sciences, argued (p. 393) that '*The reductionist hypothesis ... is accepted without question.*' He convincingly showed by the example of condensed matter physics that '*more'* often leads to emergent phenomena. Anderson considered two lists of sciences, from 'particle physics' to 'social sciences', arranged in a hierarchy X ← Y, such that *the elementary entities of science X obey the lows of science Y.* He noted that *this hierarchy does not imply that science X is 'just applied Y.*' According to this list, *the social sciences are not applied psychology*. In conclusion, he recalled the dialectical law of Engels (calling Marx the author) about *the transition of quantitative changes into qualitative ones* and gave its illustration as a dialog:

> Fitzgerald: '*The rich are different from us.*'
> Hemingway: '*Yes, they have more money.*'

Another physicist Erwin Schrödinger (Nobel laureate 1933) in his lecture of 1943 (1944), analysing the connections between the basic laws of physics, chemistry, molecular biology and genetics, came to the conclusion about the irreducibility of the analysis of living systems to the laws of physics and chemistry.

It is worth noting that in the natural sciences the reductionist approach usually supports the holistic one when creating complex theories. An example is the positive molecular-kinetic theory of the explaining macroprocesses in continuous media. The latter are also investigated within a holistic methodology that provides possibilities to find emergent phenomena such as magnetohydrodynamics. Such productivity is explained by the validity of the molecular-kinetic theory.

In the social sciences, 'holism' is usually understood as 'collectivism' and its antonym is 'individualism.' In line with the essence of methodological individualism, this antonym is analogue to 'reductionism.'

[4] The term 'holism' was coined by the South African philosopher and politician Jan Smuts (1927) in 1925 as a development of Aristotle's principle "the whole is greater than the sum of its parts" and creation on this basis holistic system of knowledge about universum.

3. Microeconomic problem 'aggregation over consumers'

3.1. Model of individual demand

Consider the microeconomic model of individual consumer demand in a market of *n* goods, whose quantities are represented by the vector *x* of the non-negative orthant of Euclidean space E_+^n and the price by the vector $p \in E_+^{n*}$[5] (Mas-Colell et al., Ch. 3). The model will also represent virtual market demand, understood as the sum of the individual demands of a fixed set of *H* individuals/households (reductionism)[6], and as the initial object (holism).

Each of the *H* consumers indexed by *h* has the budget w_h and preferences represented by an ordinal utility function $u_h : E_+^n \to R_+$, $h = \overline{1,H}$. The analytical demand theory is derived under the *regularity assumptions*, when the utility function $u_h(\cdot)$ is twice-differentiable, increasing, and strictly quasi-concave. *Rationality of the consumer* is understood as the maximization of the utility function on the set of goods available to her/him at given prices *p* and budget w_h. The regularity assumptions provide the single-valuedness and differentiability of the demand function

$$x^h(p, w_h) = arg\ \max\left\{u_h(x) : \langle p, x\rangle \le w_h,\ \ x \ge 0\right\}. \tag{1}$$

Models like (1) say that the *utility function* $u_h(\cdot)$ *rationalizes demand* $x^h(\cdot,\cdot)$.

Model (1) means that each individual is able to solve the problems of determining the utility function $u_h(\cdot)$ and maximizing this function on the set of goods available at the prices of the total market known to him, absorbing his entire budget. Such a model is obviously far from reality and can only be seen as a normative axiom of an artificial economy. This seems sufficient to abandon this individualistic model as the basis of a positive theory of real market demand. However, model (1) is the cornerstone of modern Economics, and this induces to present a formal argument of its unsuitability for scientific knowledge.

3.2. Aggregation over consumers

The problem of aggregation over consumers arose in the first half of 20th century in order to analyze formally the belief by many economics' theorists that '*Market demand has almost exactly the same properties as individual demand.'* (Hicks, 1975, p. 34). Accordingly, market demand can be defined through the model like (1).

Methodological individualism dictates that market demand is the sum of a fixed set of H individual demands (1):

$$\widehat{x}(p, w_1, ..., w_H) = \sum_{h=1}^{H} x^h(p, w_h). \tag{2}$$

The budget distribution $\{w_1, ..., w_H\}$ among consumers is not known, and the problem of consumer aggregation is to find conditions for individual preferences, in which there is a *collective utility*

[5] The asterisk * denotes a conjugate space.

[6] The reductionist variant is virtual, since the corresponding model cannot be the basis of a positive theory of real demand for the reasons given below.

function $u_h(\cdot)$ that rationalizes additive market demand (2), depending on prices p and the total budget $w = w_1 + ... + w_H$. This means that additive demand (2) can be represented by a model of class (1):

$$\hat{x}(p, w_1, ..., w_H) \equiv x(p, w) \triangleq arg \max \left\{ u(x) : \langle p, x \rangle \le w, \quad x \ge 0 \right\}. \tag{3}$$

Thus, the problem under consideration is to clarify the question of which properties of individual preferences or corresponding demand functions $x^h(\cdot,\cdot)$ ensure the equality of demand - *additive* (2) and *collective market* (3), i.e.

$$\hat{x}(p, w_1, ..., w_H) = x(p, w) \, ? \tag{4}$$

In 1953, William Gorman (1953) has published the result: *to fulfill equality* (4) *it is necessary and sufficient that all individual Engel trajectories* $E^h(p) = \left\{ x = x^h(p, w_h) : w_h \ge 0 \right\}$ *are parallel lines*. This is expressed analytically by individual demand functions affine with respect to budget

$$x_i^h(p, w_h) = a_i^h(p) + b_i(p) w_h, \quad i = \overline{1,n}, \ h = \overline{1,H}, \tag{5}$$

called the 'Gorman form'.

The same coefficients $b_i(p)$ for all consumers mean the same response from all of them regarding incremental purchases as their budgets increase! Summing equalities (5) by h gives a similar affine structure of market demand (3):

$$x_i(p, w) = a_i(p) + b_i(p) w, \quad with \quad a_i(p) = \sum_{h=1}^{H} a_i^h(p). \tag{6}$$

Gorman's result does not take into account the case when all budgets $w_h = 0$ and, accordingly, purchases $x^h(p, 0) = 0$. In this case, the individual Engel trajectories leave the centre of coordinates and, due to their parallelism, merge into one ray! Analyzing the Gorman's result, Paul Samuelson (1956) noted this. The corollary of this refinement is that in formulas (5) and (6) should be put $a_i^h(p) = a_i(p) = 0, \quad i = \overline{1,n}, \ h = \overline{1,H}$, that is, *all demand functions – individual* (1) *and market* (3) *are homogeneous relative to budgets and have the same price structures*:

$$x^h(p, w_h) = x(p) w_h, \quad h = \overline{1,H}, \quad x(p, w) = x(p) w. \tag{7}$$

In terms of preferences, equalities (7) mean that *all individual preferences are the same and homothetic!* In view of the unreality of this conclusion, the logical consequence of the Gorman-Samuelson result is that models of class (1) cannot be used to formalize both individual and collective market behaviour of consumers.

Thus, Gorman-Samuelson's analysis of the microeconomic problem ‘aggregation over consumers’ does not apply to real markets. Nevertheless, Samuelson solved the logical contradiction in favour of the individualistic model (1), denying the economic legitimacy of the model (3) for market demand. And modern Economics has accepted this unreasonable conclusion as a postulate.

It is worth noting that the real market demand (3) must be verified by trade statistics collected over a certain period of time, and these statistics are formed not as the sum of individual purchases (2) of a uncertain set of buyers, but *as the sum of sales of a known set of sellers of the market under study*.

3.3. Applied Stone-Deaton Demand Analysis

Richard Stone laid down 'Applied demand analysis' (1945, 1954) in the 'naive' period of Economics, before Gorman's article (1953), when it was supposed that the IDTh could be applied to market demand considered as an initial object. In the 1945 article, Stone used multiple regression analysis to construct separate functions of market demand for 10 groups of goods depending on the average price of a good, the aggregate price of other goods, and the income of the population.

In the 1954 article, Stone laid down a heuristic (based on the non-justified supposition) method for creating, on trade statistics of prices and quantities sold over several periods, market demand functions defined by the regular model of class (1)

$$x(p,e) = arg\max\left\{u(x) : \langle p,x\rangle \le e,\ \ x \ge 0\right\}, \tag{8}$$

where the utility function $u(\cdot)$ refers to the set of the market buyers, and e is the total expenditures of all the buyers. Implicitly, because Stone did not consider model (8) and the utility function, but imposed on the desired differentiable demand function, belonging to some parametric class, so-called *integrability conditions:* homogeneity of the zero-degree, expenditure identity (Walras' Law), negative semi-definiteness and symmetry of the Slutsky matrix (Mas-Colell et al., sec. 3.H). Fulfilment of these conditions for vector-function $x(\cdot,\cdot)$ is equivalent to fulfilment of (8).

As the parametric class, he used the Linear Expenditure System (LES), satisfying the integrability conditions, that was introduced by L. Klein and H. Rubin (1948) as *individual demand functions* with the goal '*to express the index of the cost of living in terms of measurable phenomena which are independent of the subjective concepts of utility*'[7]:

$$x_i(p,e) = \frac{\alpha_i}{p_i}\left[e - \sum_{i=1}^{n}\gamma_k p_k\right] + \gamma_i, \quad i = \overline{1,n}\,. \tag{9}$$

The development of Stone's approach to demand analysis was carried out after the Gorman-Samuelson' 'impossibility' result. The main contributor here was Angus Deaton (Deaton, 1974, 1986; Deaton and Muellbauer, 1980, Ch. 6). Deaton developed Stone's approach already taking into account the Gorman result, but without the Samuelson correction (7), imposing in his 'market demand theory' on *virtual* individual demand functions, in addition to integrability, the conditions for *'aggregation over consumers'* that ensure the fulfillment of conditions (5) with replacing budget w_h by individual expenses e_h:

$$x_i^h(p,e_h) = a_i^h(p) + b_i(p)e_h, \quad i = \overline{1,n},\ \ h = \overline{1,H}\,. \tag{10}$$

[7] R. Geary (1951) found a utility function that rationalized the LES. It is the function $u(x) = \prod_{i=1}^{n}(x_i - \gamma_i)^{\alpha_i}$, $0 < \alpha_i < 1,\ \ 0 \le \gamma_i$.

Like Gorman's 'pure theory' of consumer aggregation discussed in subsection 3.2, aggregate demand takes a form affine on expenses:

$$x_i(p,e) = a_i(p) + b_i(p)e, \quad i = \overline{1,n}, \tag{11}$$

where $e = e_1 + ... + e_H$, $a_i(p) = a_i^1(p) + ... + a_i^H(p)$. But the arguments about the properties of unobservable individual demand functions (10) are redundant, and the operational consequence of Deaton's condition for 'aggregation over consumers' is the limitation of the desired market demand to the class of integrable quasi-homogeneous relative to costs *e* systems of demand functions (11) generated (virtually) by individual functions (10).

The LES (9) belongs to the functional class (11), but constructing more flexible nonlinear demand systems satisfying the Deaton's condition has proved rather difficult. The works of H. Theil, W. Barnet, D. Jorgenson, A. Deaton and others[8] represent such systems (Rotterdam model, Translog functions, Almost Ideal Demand System, etc.) and their restoring methods.

Thus, the problem of aggregation over consumers has very complicated and restricted the market demand analysis, but ignoring this problem makes this analysis heuristic within the orthodox Economics.

4. Holistic theory of market demand

This section presents the reconsideration of the neoclassical IDTh as the MDTh, and the words of Frisch (1992/1929, p. 391) are very relevant:

> The raison d'etre of the following observations lies not in the originality of the formulae but in their economic interpretation.

4.1. Market complexity denies reductionism

The frequent productivity of the reductionistic approach in the natural sciences is due to the identity of the elements of complex objects not endowed with the psyche, and the repeatability of processes. The market system is much more complicated than the systems studied in sciences. The market is inherently collective; people are different relative to their tastes, behaviour and are subject to mutual and external influences: traditions, fashion, mood, advertising. The assumption of individual rationality has been criticized for decades in the research literature (but never in the education one), beginning with an article Herbert Simon's (1972), where the notion of *bounded rationality* was introduced.

Thus, the failures of the modern neoclassical demand theory make it natural to conclude that here the reductionist approach is unsuitable for constructing a scientific theory adequate to reality.

4.2. Statistical ensemble of consumers and collective rationality

Unlike microeconomics' IDTh, in the presenting MDTh no assumptions that people are rational and independent agents of the economy, knowing all about market prices and commodities. To replace this unrealistic object of research with a real one, it is relevant to define the changeable population of market buyers in terms of the fuzzy sets' theory by Lotfi Zadeh (1965). The notion of 'fuzzy set'

[8] See (Deaton and Muellbauer: 1980a, 1980b; Deaton, 1986; Barnett and Serletis, 2008; Boysen, 2019) and references within.

reflects a situation when some elements of a given 'universal set' U can be thought of elements, belonging to some subset $C \subseteq U$, possibly, not completely, but in some grade (like a random event can take place with some probability).

Definition 1. A subset *C* of the *universal set U* is called a *fuzzy subset* of *U*, if an element $u \in U$ belongs to *C with degree* $\mu_C(u)$, and the function $\mu_C : U \to [0, 1]$ is called the '*degree of membership*' of *u* to *C*.

If $\mu_C(u) = m = 0$, then *u* does not belong to *C*, if $0<m<1$, then *u* belong to *C* partially, in degree *m*, and if $m=1$, then *u* belong to *C* completely.

Based on this Zadeh concept, the next definition of a fuzzy set of the market's consumers (Gorbunov, 2015) has introduced, where the universal set *U* is the population of a region / country.

Definition 2. The *"Statistical Ensemble of Consumers"* (SEC) of the studied market is a fuzzy subset *C* of the set of all potential consumers *U*, and the membership function $\mu_C(\cdot)$ represents for each person $u \in U$ the share of her/his expenses in this market from all his/her expenses.

The notion SEC is only conceptual and not observable for real markets, as are individual utility functions. There is no need to reveal fuzzy characteristics of SEC, because market demand is the origin object of MDTh observable through trade statistics. This holistic alternative to neoclassical IDTh based on the following assumptions.

Assumption 1. There is statistical stability in the studied market regarding the dependence of the commodity quantities' sales on their prices and the total expenditure of all consumers in the market.

The dependence in Assumption 1 is the *market demand*, and this corresponds to Jevons's original conjecture about the nature of the laws of economics in the epigraph of the article.

Rejecting the assumption of individual rationality, it is necessary to explain assumptions about collective rationality. A more realistic assumption respectively individual market awareness is that *the whole consumer community knows all about the market!* Accordingly, the next assumption about the rationality of averaged collective preferences will be more grounding than for individual ones.

Assumption 2. Most of individuals only want to be rational, these wants determine the dominant behaviour of market's consumers, and SEC is the bearer of a collective preference relation that can be recovered using trade statistics.

These assumptions have the status of *hypotheses*, and they are sufficient to construct the classical variant of MDTh, considering the given market as a black box, represented by trade statistics.

4.3. The general collective utility maximization problem

The *hypothetical model of market demand* is a holistic model of type (8), but with a more general class of quasi-concave increasing collective utility functions $u(\cdot)$ allowing a multi-valued solving the extreme problem. Accordingly, the demand model is

$$x(p,e) \in D(p,e) = Arg\max\left\{u(x) : \langle p, x\rangle \le e,\ \ x \ge 0\right\}, \tag{12}$$

where $D(\cdot,\cdot): E_+^{n+1} \to 2^{E_+^n}$ denotes the *market demand correspondence*.

The MDTh derived from the model (12) coincides with the formal classical IDTh (Mas-Colell et al., Ch. 3). According to Jevons' belief (1866) problem (12) in the holistic setting turns individual chaos into collective order.

Under sufficient verification experience, MDTh solves, in particular, the problem of calculating analytical indexes of market demand, determined through the *expenditure function* (Ibid., 3.E) with values

$$e(p,c) = \min\left\{\left\langle p,x\right\rangle : u(x) \geq c,\ x \geq 0\right\}. \tag{13}$$

The value of $e(p,c)$ provides at prices p the minimal expenditures that ensure a given level of consumption c. Analytical indexes play an important role in the nonparametric verification method below.

5. Verification of MDTH and applied demand analysis

The verification of MDTh is to clarify the question: is there a utility function $u(\cdot)$ that rationalizes demand correspondence $D(\cdot,\cdot)$ when a finite set of statistical data of 'prices – quantities' pairs

$$\left\{p^t, x^t : t = \overline{0,T}\right\} \tag{14}$$

is known? These data also determine total expenditures of all buyers over the t-period $e_t = \left\langle p^t, x^t \right\rangle$. This verification is factually an applied demand analysis based on MDTh and is considered successful if the model (12) proves to be adequate to statistics (14).

The notion of '*statistics* (14) *rationalization by a utility function'* is specified in two variants – the parametric applied demand analysis, has used by Stone and Deaton, and nonparametric, developed in our works (Gorbunov, 2015, sec. 8.5; Gorbunov, & Lvov, 2019; Gorbunov, Kozlova, & Lvov, 2020; Gorbunov, & Lvov, 2022). The answer to the question can be only in principle, and can be constructive.

The real trade statistics (14) only approximate a set of exact values formed in complex processes of collecting, averaging, and aggregating prices and quantities of elementary commodities. There are usually no probabilistic characteristics of trade statistics' errors. Therefore, standard econometric methods for verifying MDTh are inapplicable, but it is possible to develop the methods of the *ill-posed problems' theory*[9] (Tikhonov, & Arsenin, 1977; Gorbunov, 2003).

Here I will confine some remarks on the parametric method of constructing Stone-Deaton's 'demand systems', as well as a brief presentation of our verification method, developed on the basis of a nonparametric demand analysis by Sidney Afriat (1967) and Hal Varian. (1982).

[9] A calculation problem is called a 'well-posed' or 'correct problem' if it has a unique solution that depends on the initial data continuously. If this condition doesn't fulfil, the problem is called the 'ill-posed' or 'incorrect'. An ill-posed problem must be *regularized*, which means its approximation by a well-posed problem, involving some additional information about the desired solution.

5.1. Parametric verification

The Stone-Deaton's applied demand analysis, summarized in subsection 3.3 use;s directly the demand functions of some integrable parametric class $x(p,e;w)$, where $w=(w_1,...,w_k)$ is a parameter vector from the admissible set *W*. The verification problem is reduced to determining the parameters at which the estimated demand approximates the statistical demand (14) in the sense that the approximate equality holds 'sufficiently well':

$$x\left(p^t,e_t;w\right)\simeq x^t,\quad t=\overline{0,T}\,. \tag{15}$$

This problem is well studied in applied mathematical modeling, and to solve it, the method of least squares is more often used. However, in view of the lack of reliable probabilistic information and statistical errors of (14), the justified application of the usual statistical criteria for the selection of 'good' systems of demand functions is excluded. Here the conditions of integrability play a stabilizing role as an additional information about the desired solution, like the 'quasi-solution method' of solving ill-posed problems of natural science (Tikhonov, & Arsenin, 1977), and the 'wellness' of equality (15) should be estimated by experts, possible, with the help of computer modelling.

We noted above that Angus Deaton complicated Stone's integrable demand functions by conditions of 'aggregation over consumer' for virtual functions of individual demand as Gorman functions (10) not involved in the computations, but, by the Economics axiom, defining desired market demand in the similar class (11). But even with this undue complication, *the examples of heuristic analysis of real market demand by Stone, Deaton, and their followers are examples of successful verification of the MDTh, created within the framework of scientific methodology*.

5.2. Nonparametric verification

The main shortcoming of parametric verification, even without Deaton's limitations, is that the unsuccessful outcomes of the given trade statistics analysis with a finite set of parameterized demand systems do not give grounds to reject the rationalizability hypothesis of this statistics.

Revealing the question about rationalizing a given trade statistics in the general class of non-decreasing nonsatiated utility functions in principle, as well as constructive, is possible within the nonparametric demand analysis of Afriat-Varian. This analysis was created within IDTh, but it can be used as a logical tool for the market statistics.

5.2.1. Afriat's theorem, inequalities and the function. Konüs' indexes

Hall Varian in his articles (1982, 1983), based on the fundamental article of Sydney Afriat (1967), presented several criteria for rationalizing trade statistics in a very wide class of nondecreasing unsatiated utility functions without assuming their quasiconcavity and differentiability.

Definition 3. (Varian, 1982, p. 946) A utility function $u(\cdot)$ *rationalizes data* (14), if the equalities hold:

$$u\left(x^t\right)=\max\left\{u(x):\left\langle p^t,x\right\rangle\le e_t,\,x\ge 0\right\},\quad t=\overline{0,T}\,. \tag{16}$$

Let's introduce the 'cross-expenditures' $e_{ts} = \left\langle p^t, x^s \right\rangle$, coefficients $a_{ts} = e_{ts} - e_t$ and a system of inequalities

$$u_s - u_t - \lambda_t a_{ts} \le 0, \quad s,t = \overline{0,T} \wedge s \ne t \,. \tag{17}$$

Inequalities (17) are called *Afriat's inequalities,* and the numbers $\{u_\tau, \lambda_\tau\}$ are the *Afriat's numbers* defined as $u_t = u(x^t)$ and the Lagrange multipliers of extremum problems (17) $\lambda_t \equiv \lambda(p^t, e_t)$.

We present Afriat's theorem partially and modified relatively (Varian, 1982) – only the facts that are used in the verification method.

Afriat's theorem: *Trade statistics (14) are rationalizable by a nondecreasing unsatiated utility function if and only if there exist a positive solution to the inequality system (17). If* $\{u_t, \lambda_t > 0 : t = \overline{0,T}\}$ *is a solution to this system, then the function*

$$\overline{u}(x) = \min_\tau \left\{ u_\tau + \lambda_\tau \left\langle p^\tau, x - x^\tau \right\rangle \right\} \tag{18}$$

rationalizes the data (14).

In cited works of Afriat and Varian, the piecewise linear concave *Afriat's function* (18) plays only technical role in proving the theorem. But this function very simplifies calculating expenditure function (13) that defines the economic indexes of prices and quantities. The general definition of these indexes (Samuelson and Swamy, 1974) is as follows:

$$P(p^t, p^s; x) = \frac{e(p^t, u(x))}{e(p^s, u(x))}, \quad Q(x^t, x^s; p) = \frac{e(p, u(x^t))}{e(p, u(x^s))}. \tag{19}$$

Here, the vectors of quantities *x* and prices *p* are the *reference situations*, and the article (Gorbunov, & Lvov, 2019) shows that calculating the values of the expenditure function $e(\cdot,\cdot)$ generated by the function (18) reduces to linear programming (LP).

Reference situations in (19) must be specified for the applications. The pairs of indexes $\left(P_{st}^{KL} \triangleq P(p^t, p^s; x^s),\ Q_{st}^{KL} \triangleq Q(x^t, x^s; p^s)\right)$ and $\left(P_{st}^{KP} \triangleq P(p^t, p^s; x^t),\ Q_{st}^{KP} \triangleq Q(x^t, x^s; p^t)\right)$ are called *Konüs-Laspeyres* and *Konüs-Paasche* price and quantity indexes, respectively. We use for the MDTh verification the *Konüs-Fisher* (KF) indexes $P_{st}^{KF} \triangleq (P_{st}^{KL} P_{st}^{KP})^{1/2}$, $Q_{st}^{KF} \triangleq (Q_{st}^{KL} Q_{st}^{KP})^{1/2}$. Due to the definitions (19) and the equality $e(p^s, u(x^s)) = e_s$, they are:

$$P_{st}^{KF} = \sqrt{\frac{e(p^t, u_s) e_t}{e_s e(p^s, u_t)}}, \quad Q_{st}^{KF} = \sqrt{\frac{e(p^s, u_t) e_t}{e_s e(p^t, u_s)}}. \tag{20}$$

The values $e(p^t, u_s)$ and $e(p^s, u_t)$, define the pairs of Konüs indexes (20), and their calculating is reduced to two LP problem for calculating function $e(\cdot,\cdot)$ with parameters (p^t, u_s) and (p^s, u_t).

5.2.2 Regularization and solution of the Afriat's inequalities

Any consistent system of inequalities has, as a rule, a set of solution, and wen inequalities are linear, the solution set is convex and polyhedral. But Afriat's theorem doesn't account for inevitable errors in trade statistics (14), and due to the errors and/or the inadequacy of the model (12) to the market under study the inequality system (17) can be inconsistent. Besides, the solution sets of these systems, considered as multi-valued solution mapping, can be unstable concerning data variations. This means that the system's solution set is not continuous (in *Hausdorff metric*) from the data. A multi-valued mapping that is locally continuous by Hausdorff is called *regular*. A sufficient regularity condition for the solution mappings of convex inequality systems is the consistency of the corresponding systems of strict inequalities, i.e., the Slater's condition.

Thus, the problem of Afriat's inequalities solution is ill-posed, and its regularization requires solving two problems: 1– ensuring regularity of the system (17) solution set; 2 – constructing a criterion for choosing meaningful solutions that are continuously dependent on the data (14).

System (17) is regularized by the following transformations. This system is algebraically homogeneous and the *u*-numbers are included by differences $u_s - u_t$. Such a structure allows to impose two conditions on the desired solution, and it is productive to assign 'initial conditions' $\lambda_0 = 1, \quad u_0 = e_0$. Then, in the right sides of Afriat's inequalities, the relaxation parameter r is introduced with normalizing multipliers that ensure the independence of the order of the system's additive components. These techniques transform the system (17) to the system

$$\begin{cases} -u_t - \lambda_t a_{t0} \le -e_0 + re_t, \quad u_s \le e_{0s} + re_s, \\ u_s - u_t - \lambda_t a_{ts} \le r\sqrt{e_s e_t}, \quad s,t = \overline{1,T} \wedge s \ne t, \end{cases} \tag{21}$$

that is regular in the region of positive variables $\{\lambda_1,...,\lambda_T, u_1,...,u_T, r\}$.

To answer the question of the principal rationalizability of statistics (14) in the class of non-decreasing nonsatiated utility functions, it is enough to solve the problem *of minimal relaxation*

$$r_{\lambda u} = \arg\min\{r : (21), \lambda \ge 0, u \ge 0\}.$$

If the value $r_{\lambda u}$ does not exceed a sufficiently small positive number $r^*_{\lambda u}$, limiting the discrepancies of inequalities (21) under possible variations of data (14)[10], then the MDTh verification for this statistics is being solved positively. However, system (21) with $r = r_{\lambda u}$ does not satisfies the Slater's condition and may be irregular. To ensure the regularity, it is necessary to add a small number ρ>0 to $r_{\lambda u}$ and put in (21) $r = \max\{r_{\lambda u} + \rho, 0\}$ **.** Any nondecreasing unsatiated utility function $u(\cdot)$ whose Afriat's numbers are the positive solution to the regularized system (21) is a rationalizing function.

The last problem of Afriat's inequality solution is the choice a unique meaningful solution or the regular system (21). In our cited articles (2019, 2020, 2022) we have elaborated methods of choosing solutions that can be characterized as *optimistic, pessimistic* and *objective*. These characteristics have

[10] The value of $r^*_{\lambda u}$ should be assessed by experts or by numerical experimentation.

determined through the base-final KF indexes (20) relative to the study period $\{0,T\}$, that is, indexes $\left(P_{0T}^{KF}, Q_{0T}^{KF}\right)$ determined, in turn, through the system (21) solution. 'Optimistic' is the solution that provides the minimal value of the price index P_{0T}^{KF} and the maximal value of the quantity index Q_{0T}^{KF}; and 'pessimistic' is the solution that provides the opposite extremes of these indexes. It turns out (2020, 2022) that each of these two-criteria problems is resolved by one LP. The first one maximizes u_T and the second one minimizes u_T.

Finally, the 'objective' solution of (21) is the solution that provides the closest (in the root-mean-square metric) proximity of KF indexes $\left\{P_{0t}^{KF}, Q_{0t}^{KF} : t = \overline{1,T}\right\}$ to the usual Fisher formula indexes

$$P_{0t}^{F} = \sqrt{\frac{e_{t0}e_t}{e_0 e_{0t}}}, \quad Q_{0t}^{F} = \sqrt{\frac{e_{0t}e_t}{e_0 e_{t0}}}, \quad t = \overline{1,T},$$

which are the best relative to the Fisher criteria (CPIM, 2004, Ch. 16).

Some results of successful verifications of the presented MDTh by the nonparametric method on real consumption data in Russia are presented in our cited articles of (2019, 2020, 2022). In the latter article the official trade statistics of the 468 items of goods and services for the period 2012-2017 were analysed with constructing all three types of KF indexes: optimistic, pessimistic and objective. The indexes were constructed both for the initial statistics and for the main groups of goods: food products, non-food products, services.

6. Conclusion

The article has presented a way out of the reductionist impasse of the neoclassical demand theory, where it turned out to be impossible to create a market demand theory and an equilibrium theory, adequate to reality and of practical interest. The presented way out is a revision of the IDTh within the framework of the scientific holistic methodology while retaining the formal existing demand theory. This revision is in line with the desire of the founders of neoclassical Economics, Jevons and Walras, to create it on general scientific principles, and the holistic approach does not reject the basic neoclassical principles, which are optimality in the behaviour of economic agents and equilibrium in their rational interactions. Individuals can be thought as boundedly rational agents in accordance with Simon (1972), and the transfer of rationality to the SEC of the studied market is consistent with early Jevons (1866).

The meaningful economic and general scientific arguments for the revision are given, and the verification problem for the presented MDTh is considered in two variants: in parametric classes of smooth utility functions and in the general (nonparametric) class of nonsatiated utility functions. The parametric verification of the MDTh is factually, though implicitly, have fulfilled by the applied demand analysis of Stone-Deaton in a lot of positive implementations. In addition, this analysis can be freed from the problem of aggregation over consumers and thus expand its capabilities.

The presented nonparametric method for MDTh verification takes into account the typical lack of information about errors in standard trade statistics, and it is elaborated within the framework of the theory and methods for ill-posed problems. It is also a variative method for calculating the economic indexes of market demand. It allows, within the framework of the admissible arbitrariness

of constructing the utility function, to adjust the indexes depending on the goals of economic analysis of various socio-political groups.

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