A micro-macro model of foreign direct investment: knowledge-based gravity forces, self-selection and third-country effects

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Abstract:

The paper develops a stand-alone and testable gravity model to explain international patterns of foreign direct investment (FDI). The core model is based on knowledge-based gravitational forces that are directly or indirectly linked to a country's economic mass (GDP). The micro-economic part of the model explains the bilateral extensive FDI margin, i.e. the Firms self-select into FDI if their productivity is high enough to overcome the fixed costs of setting up costs a foreign subsidiary, using its proprietary knowledge assets as crystallization kernel. Aggregated at country level, the model explains the occurrence of zero FDI flows between countries. The bilateral part of the model accounts for direct FDI friction costs. The model is generalized to a n-country world by also accounting for the relative FDI friction costs of all countries, quantified via FDI-based multilateral resistance terms. The paper derives testable predictions from the model. The model implications have high potential policy relevance.

JEL codes: D23, D25, F23 , G32, L1, O34

Keywords: foreign direct investment, firm behavior, decision model, structural gravity, zero FDI flows, policy implications

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1. Introduction

This paper proposes a new stand-alone model for explaining bilateral foreign direct investment (FDI) patterns. The model is based on decisions of individual firms, takes into account knowledge-capital conditions in origin countries, and FDI-affecting policies in destination countries. The model supports a structural gravity model of FDI patterns, but it also explains behavior at the extensive margin of foreign direct investment, including zero FDI flows.

A much-cited review article on the determinants of FDI concludes that a gravity model specification tends to provide a reasonably good fit for actual cross-country FDI data (Blonigen, 2005). However, the same author complains that: "*Ideally, the FDI literature would have an established model and empirical specification that lays out the primary long-run determinants of FDI location. [...] However, there is no [...] paper that lays out a tractable model that specifically identifies gravity variables as the sole determinants of FDI patterns*". The latter is exactly what this paper aims at.

The gravity model has often been used in empirical studies of FDI, but without a theory that explains the gravity-conform outcomes.[2] The other side of the spectrum is formed by complex and large models that integrate FDI, trade, differentiated FDI motives, consumer behavior, economic growth, and innovation.[3] While such papers may be interesting and offer inspiring perspectives, the models are too complex and too data-demanding to be falsifiable by empirical testing with current data (cf. Lewbel, 2019; Markusen, 2021). Their authors often resort to calibrated numerical exercises (sometimes called 'guestimating' or 'estibrating') in order to demonstrate the plausibility of the model. Such numerical exercises are always inferior to a real empirical test, if only because there are too many possibilities to overfit the data to obtain expected or desired outcomes.

This paper proposes a testable and falsifiable theory of bilateral FDI patterns. It builds on behavior of individual firms setting up foreign subsidiaries and changing the latter's activity levels in response to changes in their competitive environment. This process feeds and shapes FDI patterns at country level. That is where economic mass starts to count as the basis for the patterns that gravity models invariably find. Economic mass (GDP) is important as attractor of FDI, because it creates larger markets where more foreign product varieties in larger volumes can find their ways to consumers. But economic mass is also important for outward FDI pressure, because larger countries tend to have more firms, increasing the absolute probability that some of these firms develop unique knowledge assets that can also be exploited in other countries via FDI. Moreover, larger economic mass (country size)

[2] Examples are: Kleinert and Toubal (2010); Davies and Kristjandottir (2010); Brainard (1997), Braconier *et al.* (2005); Egger and Pfaffermayr (2004).
[3] Cf. Bergstrand and Egger (2007); Anderson *et al*. (2019); Ramondo and Rodríguez-Clare (2013); Allen *et al.* (2014); Arkolakis *et al*. (2018).

makes it possible spend more on public knowledge creation that can be encapsulated and converted to firm-specific knowledge assets that domestic firms use as basis for engaging in foreign FDI. The demand pull for inward FDI depends on market size and the relative knowledge gap of countries. Analogue with Newton's gravity model, we distinguish a number of FDI friction factors that may countervail the attractive forces of economic mass. Friction factors may stem from man-made policies (that facilitate or discourage bilateral FDI), cultural differences, language, and physical factors such as spatial distance. We have one problem less than Newton had, because we know that firm behavior carries the operation of FDI gravity, while Newton just accepted the physical gravity force without knowing what 'carried' it.[4]

The present paper expands on earlier work by offering a better micro foundation for firm-level and country-level FDI drivers. The model includes the insights from recent work on testing the Markusen's knowledge-capital model of FDI (Kox, 2022).

The paper makes several contributions to the literature. The first contribution is a new reading of the knowledge-capital model (Markusen, 2002) that makes it possible to derive knowledge-based gravity forces. Secondly, the model distinguishes different policy-related economic friction factors that weaken or countervail the economic gravity forces. The model allows for friction forces that are not symmetric in both directions (inward, outward FDI). Thirdly, the paper explains the existence of zero bilateral FDI flows using a micro-model, which explains the self-selection process of firms into becoming a multinational. Only the most productive firms can absorb the FDI friction costs. This yields a result that is comparable to the Melitz (2003) paper on self-selection into export activity. Fourthly, the *n*-country version of the model derives FDI-specific multilateral resistance terms. They reflect relative FDI friction costs for all countries, weighted by the economic scales of the origin and destination country. Fifthly, we discuss the methodology for testing our stand-alone FDI gravity model using state-of-the-art econometric insights. Finally, the paper is policy-relevant by showing that the FDI-specific multilateral resistance terms form a rich quantitative information source for evaluating the impact of unilateral, bilateral or multilateral policies on FDI patterns.

The structure of the paper is as follows. Section 2 opens with a few stylized empirical facts on the role of domestic knowledge capital for outward FDI. In Sections 3-7 we stepwise develop the gravity-based FDI model at the country-pair level. Section 3 models the country-level push factors of outward FDI, based on a new version of the knowledge-capital interpretation of FDI. Section 4 models the country-level pull factors of inward FDI in host countries and how these, together with the push factors in FDI origin countries, create the basic drivers for bilateral FDI in a world without FDI frictions. Section 5 introduces two types of FDI friction factors for bilateral FDI. Section 6 provides a micro-economic foundation for the decision of firms to engage in bilateral FDI. It shows that bilateral FDI frictions increase the fixed costs for firms with FDI ambitions. The micro model quantifies the cut-off productivity level that is necessary to absorb the up-front fixed costs of bilateral FDI. Section 7 expands the model to an *n*-country world, by taking on board the effect that

[4] Despite Einstein's general relativity theory, gravity still has many mysteries. A graviton as physical 'carrier' of the gravity force still has not been identified and gravity's interaction with other fundamental forces is still object of research and debate (cf. Panek, 2019; Zee, 2018; Wilczek, 2015, 2021; Bernhard Cohen, 1981).

developments in third countries may have on relative FDI friction costs. The latter affect the selection of FDI destination countries (viewed from the multinational firm's perspective). Section 8 summarizes the main results in a small, 5-equation core model that brings together the knowledge-capital based gravity forces, the self-selection process among firms, and the third-country effects. Section 9 evaluates the micro-macro interaction in the model and derives testable predictions from the model, together with some ideas for testing the model. Section 10 summarizes the findings and concludes.

2. Knowledge capital and FDI: some stylized facts

This section calls attention to the asymmetric role of knowledge assets for outward and inward FDI of countries. It shows some stylized facts that inspire the FDI model that this paper will specify.

Economic scale pervades all macroeconomic statistics. A country's GDP affects both the magnitude of inward and outward FDI as well as the magnitude a country's combined knowledge assets. A meaningful international comparison of country's knowledge assets must therefore correct for economic size (GDP). We do so by expressing FDI and aggregate knowledge assets per unit of GDP. The annual inward and outward FDI stocks of a country are normalized by its annual GDP.[5] For knowledge assets, we use a well-documented indicator that is annually produced by a consortium of Cornell University, INSEAD, and the World Intellectual Property Organization (WIPO), covering a large set of countries.[6] It builds on about 80 innovation and knowledge-creation indicators per country, which are aggregated in a consistent and uniform way. That results in their aggregate input-based Global Innovation Index (GII). It is a dimensionless number that we normalize by annual GDP (PPP) weights to allow an unbiased international comparison.

To get a generalized picture per country, we take each country's mean score over the period 2000-2019, both for the FDI indicators and for the GII indicators. A further data-cleaning step is that we delete ten countries from the sample that are most active in policy practices that facilitate tax-sheltering and tax avoidance.[7] This step is taken to avoid that the emerging picture is blurred by fiscally motivated FDI flows. The resulting data set is used for ranking all country scores with respect to FDI and with respect to GII scores. The scatter plot of Figure 1 displays how narrow the country rankings for GDP-corrected knowledge assets correlate with the country scores for outward FDI stocks. Figure 2 depicts the same for inward FDI scores, which are apparently only weakly related with domestic stocks of knowledge assets.

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[5] Based on FDI stocks data from IMF (CSID), OECD, Eurostat and UNCTAD.

[6] E.g. Dutta (2020). We use their input-based innovation indicator, because that comes closest to our later approach in this paper.

[7] Cf. Damgaard *et al*. (2019). The removed countries are: Netherlands Antilles, American Samoa, Bahamas, Bermuda, Switzerland, Cayman Islands, Cyprus, Iceland, Liberia, Luxembourg, Marshall Islands, Malta, Mauritius, Netherlands, Panama, Seychelles, British Virgin Islands, US Virgin Islands.

Figure 1 Rank correlation between outward FDI stocks per unit of GDP and Global Innovation Index, 114 countries, mean scores over period 2000-2019

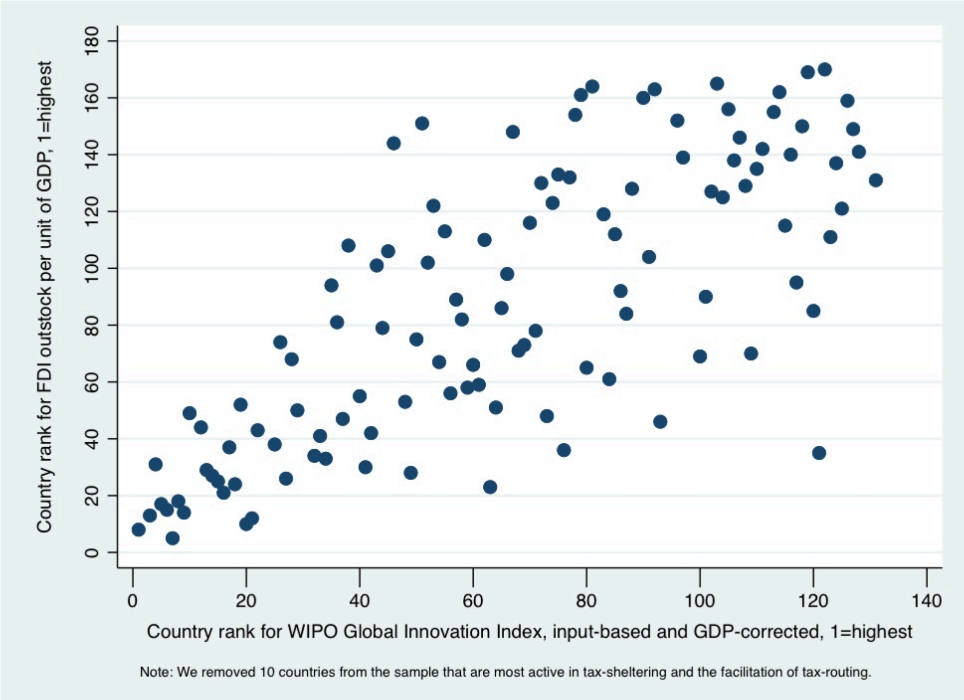
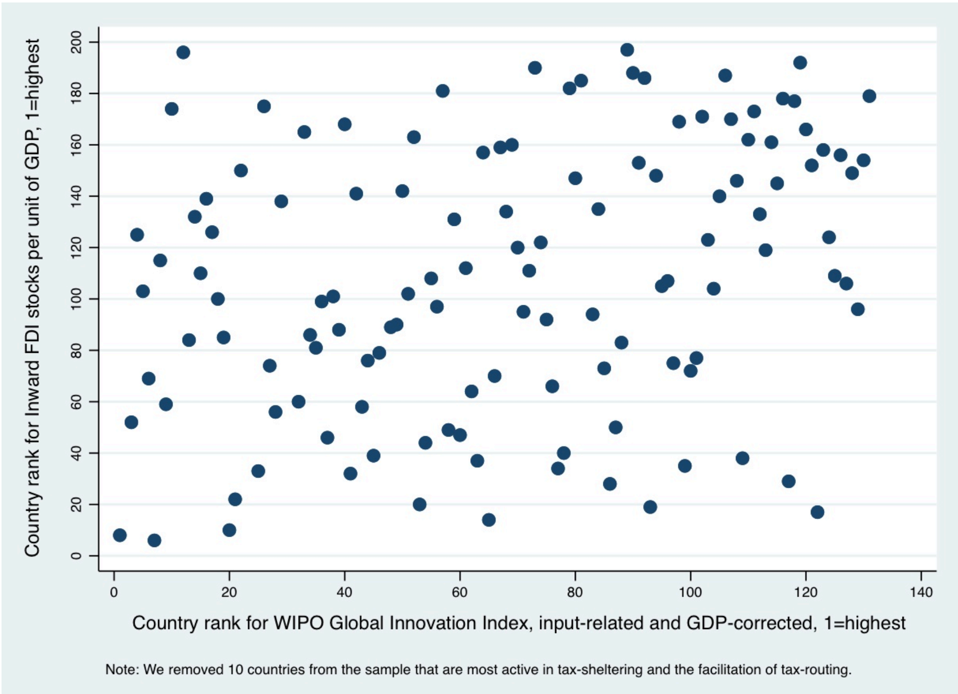


Figure 2 Rank correlation between inward FDI stocks per unit of GDP and Global Innovation Index, 123 countries, mean scores over period 2000-2019



These data suggest that domestic knowledge assets are important for outward FDI, but not - or much less so- for inward FDI. This pattern is confirmed by Table 1 which offers the regression results for the same data. Panel A displays the results of the OLS regression for explaining the outward FDI rank of countries; the estimated coefficient for the explanatory

GII variable is almost one with a just a small standard error. Panel B shows the corresponding regression results for explaining the inward FDI rank of countries; here the estimated coefficient for the explanatory GII variable is less than 0.50.

Table 1 OLS Regression results: how does the Global Innovation Index (rank) explain a country's rank with respect to outward and inward FDI stocks, 2000-2019

	estimated coefficient	White-robust S.E.	t-value	P>|t|	95% confid. interval	
					low	high
Panel A, Dependent variable: country rank with respect to outward FDI stocks per unit of GDP						
* rank_GII_gdpcorrected	0.9721	0.0705	13.79	0.000	0.8325	1.1117
* constant	23.169	4.6056	5.03	0.000	14.043	32.294
No. of obs = 114 ; F(1, 112) = 190.3; Prob > F = 0.0000; R^2 = 0.560						
Panel B, Dependent variable: country rank with respect to inward FDI stocks per unit of GDP						
* rank_GII_gdpcorrected	0.4161	0.1198	3.47	0.001	0.1790	0.6532
* constant	79.903	9.4718	8.44	0.000	61.152	98.656
No. of obs = 123 ; F(1, 112)= 12.07; Prob > F = 0.0007 ; R^2 = 0.089						

The results imply that a country's rank score for the WIPO Global Innovativeness Index almost fully explained a country's rank score for outward FDI stocks over the period 2000-2019. A country's rank score for inward FDI stocks has a much smaller relation with the domestic innovativeness. These findings were found to be robust after taking subsamples of the full country set. We have extended this investigation area by taking about 50 more the disaggregated indicators of the knowledge-creation activities of each country's public and private sector. The outcomes strongly support the results that were presented here (Kox, 2022).

The knowledge-capital model of FDI by Markusen and others[8] provides a plausible theory that could fit our stylized facts with regard outward FDI flows. It assumes that multinational firms own footloose proprietary knowledge assets, not tied to their home-country location and non-rival in their intra-firm use. Using the latter in a foreign subsidiaries increases the returns to these assets. This mechanism generates firm-level scale effects. The knowledge-capital model has not yet been adequately tested, mainly due to data issues. In the own words of Davies and Markusen (2021): "*The importance of intangible assets to understanding multinationals is acknowledged but remains a conceptual and theoretical curiosity due to the difficulties in observing and measuring the existence and contribution of these assets*".

3. Firm-level knowledge-capital, economic mass and outward FDI

The knowledge-capital approach stands for a stripped, but clear-cut version of the FDI process. Broadly speaking, FDI is a bundled concept, which includes aspects of *ownership* (economic control, international management hierarchy, equity versus debt financing), *location* (international capital flows, greenfield investment, mergers or acquisitions, tax

[8] E.g. Markusen, 2002; Carr *et al.*, 2001; Markusen and Maskus, 2003; McGrattan and Prescott, 2009, 2010; Holmes, *et al.*, 2011; Burstein and Monge-Naranjo (2009).

routing, market seeking, resource seeking, production networks, global value chains), and *internalization of transactions* (headquarter services, intra-firm knowledge assets, transaction costs, intra-company finance flows).[9]

In applying Ockham's razor, we adopt the knowledge-capital (KC) interpretation as a lean and stylized perspective on the multinational firm. It assumes that firms own unique knowledge assets (patents, in-house know-how, blueprints, procedures, technology networks, reputations and trademarks). These assets may be applied elsewhere without affecting their original value, motivated by an increase of the returns to such assets. The KC model regards this process as the core of foreign direct investment process. If the knowledge-capital approach is correct, then countries with high outward FDI should have a relative abundance of proprietary knowledge assets.

We propose a re-interpretation of the KC model that focuses on the interaction between public and firm-level knowledge-creation activities. Though firms are the main commercializing agents of national knowledge capital, their competitive edge partly rests on the knowledge products from the public and semi-public sector. Public institutions like universities, national institutions for technology transfer, and scientific publications form a constant source of new ideas. Effectively, most knowledge products from the public and semi-public sector can be characterized as non-proprietary and outside the market domain. It is rare that the public sector itself commercially exploits public-held patents.[10] Often, such patents are, before expiration, given away to national firms, or are sold via auctions.[11] Hence, the public knowledge sector is generous with its products and forms an important source of free knowledge externalities for firms. Public knowledge products generate, share and disseminate innovations and discoveries via publications, congresses, staff mobility, intermediary supplier networks, and education-related activities.[12] By contrast, firms work almost exclusively on the basis of proprietary knowledge capital. Firms absorb free knowledge products of the public and semi-public sector, encapsulate public inputs and recombine them with firm-specific knowledge, thus creating privately owned knowledge assets that form the basis for marketable products, technologies, brands, and even new business models.

Model set-up. The model will be developed stepwise, starting with a setting of heterogeneous firms. Firms only differ by their productivity characteristics. Productivity affects both the creation of proprietary knowledge assets, and the firm's capacity to absorb the up-front fixed costs of becoming a multinational corporation. From there, we aggregate at country level to obtain the push and pull factors that shape gravity-like bilateral FDI forces. After a 2-country case without economic frictions, we add FDI frictions and show how this affects the firms' self-selection process into becoming a multinational company. Finally, we extend the model to a n-country scale, in which not only direct bilateral frictions, but also third-country effects affect the FDI decision.

Heterogeneous firms. Following Jovanovic (1982), each firm obtains an unobservable,

9 Cf. Dunning, 2001; Feenstra, 2004; Head and Ries, 2008.
10 Cf. Agrawal and Henderson, 2002; Calderini *et al*., 2007; Perkmann *et al*., 2013.
11 Cf. Mazzucato, 2014; Arundel *et al*., 2013; Escalona Reynoso, 2010; Maskus and Reichman, 2004; Boyle, 2003a, 2003b; Carlsson and Fridh, 2002; Cohen *et al*., 2000; Henderson *et al*., 1998.
12 Cf. van Elk *et al*., 2019; Gerbin and Drnovsek, 2016; Audretsch and Stephan, 1996, 1999; Arundel *et al*., 2013; Breschi and Catalini, 2010; Toole and Czarnitzki, 2010; Verhoogen, 2021; Keller, 2004.

random draw (λ_{si}) from a domestic productivity distribution. Each firm s in country i is uniquely identified by its productivity λ_{si}. The minimum productivity level is $\lambda_m > 0$ can be used for normalization of all productivity performances. With a productivity level $\lambda_{si} < \lambda_m$, a firm drops out. Firms with productivity levels $\lambda_{si} > \lambda_m$ are able to make a profit in the domestic market.[13] However, to keep the model applicable across countries, we make – without loss of generality– two simplifying assumptions. Firstly, λ_m is identical for all countries. Secondly, all firms with productivities $\lambda_{si} > \lambda_m$ are linearly ordered by productivity level $\lambda(s)$. If the total number of firms in a country i is $\boldsymbol{S_i}$, one gets the following productivity distribution:[14]

$$G(\lambda_{si}) = b \int_1^{\boldsymbol{S_i}} \lambda(s)\, ds \qquad with\, s \in [1,2,\ldots,\boldsymbol{S_i}]$$

in which b is a dimensional constant. These assumptions will allow to keep the model transparent and tractable.

Public knowledge creation. We regard the public knowledge system as an input-output process. Its input side accounts for dedicated human and material resources that are used for creating new knowledge and reactivating 'older' knowledge. It has a throughput and processing phase where efficiency, concentration, and incentives for creativity matter. And it has an output side where knowledge products, educated persons, technologies, and a learning-oriented institutional environment 'pop out'. Let M_{pit} be the active public KC stock of country i in year t; it is a product of current and past efforts. "Active" emphasizes that knowledge from the past forms a perishable 'good', requiring constant refreshment, re-education, reappropriation, re-transfer, documentation and dissemination actions by the current generation, otherwise it decays and becomes dead knowledge. Public knowledge creation is supposed to be fully financed from tax receipts, and hence depends on GDP size. We take the present and past distribution of country's GDP size as given. Knowledge is a multi-dimensioned entity, but can be defined in terms of costs as a GDP fraction. At any time t, M_{pio} defines the older, path-dependent vintages of publicly created knowledge. We use the following production function for M_{pit}:

$$M_{pit} = v_{it}\left[\varepsilon_{it}\, M_{pio} + \beta_{it}\right] Y_{it} \tag{1}$$

in which $\beta_{it} > 0$ is country i's coefficient for knowledge development activities in year t, expressed as a fraction of GDP. We assume that β_{it} also includes the costs for attracting new foreign knowledge products. Similarly, $\varepsilon_{it} > 0$ is the GDP fraction spent for keeping 'old' knowledge stock M_{io} fresh and active. Parameter $v_{it} > 0$ measures national throughput efficiency, representing a mix of overall labor productivity and the efficiency of knowledge circulation (associated with qualities of connectivity, creativity incentives, legal and

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[13] That minimum condition is not enough for becoming a profitable multinational, to be shown later on.

[14] Many empirical studies (Aoyama *et al*., 2008; Axtell 2001, 2006) find a Pareto distribution of firm productivities. See also studies on trade by firms with heterogeneous productivity (Chaney, 2022, 2018; Helpman *et al*., 2004; Melitz and Redding, 2014; Bekkers and Francois, 2018; Kleinert and Toubal, 2010). However, for a theoretical model the assumption of a Pareto distribution is not strictly necessary. The linear distribution has, like the Pareto distribution, the property that all subsets of the distribution have the same fractal structure.

institutional framework).[15] Countries may differ considerably as to the knowledge throughput aspects.

The lagged knowledge component M_{pio} has a vintage structure, in which $\beta_{i,t-\theta}\ Y_{i,t-\theta}$ represents the newly create knowledge in year $t-\theta$, and $0 < \delta_{it-\theta} \leq 1$ is the depreciation rate with which the knowledge vintage is annually depreciated until year $t-\Theta$ in which a one-shot discarding of the oldest knowledge cohort follows.[16] So, the aggregation is restricted to only $\Theta - 1$ years. The lagged component M_{pio} thus aggregates over a finite time horizon:

$$M_{pio} = \sum_{\theta=1}^{\Theta-1} \frac{\beta_{i,t-\theta}}{(1-\delta_{i,t-\theta})^{\theta}}\ Y_{i,t-\theta} \quad with\ 0 < \delta_{i,t-\theta} < 1\ ;\ \delta_{i,t-\Theta} = 1 \qquad (2)$$

M_{pio} is only related to GDP of preceding years. The ratio $\beta_{it}/(1-\delta_{it})$ thus determines the speed of annual knowledge-rejuvenation. Competition and creative destruction increase the depreciation rate, and shorten the renewal cycle. Higher renewal rate gives a younger stock of knowledge capital.[17]

Creation of proprietary knowledge by firms. For private knowledge creation by firms we set up a similar production function, but with two important differences. Firstly, firm-level knowledge production partly depends on the public knowledge inputs in their home country. A second difference is that individual firms are heterogeneous with respect to their innovativeness, management capabilities and overall efficiency. Let M_{sit} be the proprietary knowledge stock of firm s ($s \in 1,..,S$) in country *i* at time *t*. The production of M_{sit} depends on three activities. The first two are similar to the public sector: internal creation of new private knowledge assets, and re-activation of a firm's older private knowledge stocks. But the third activity is the absorption and encapsulation of public knowledge inputs. Again, all firm-level costs of knowledge creation are expressed as fraction of country *i*'s GDP to obtain a uniform value dimension:

$$M_{sit} = (v_{it})^{z_s}\ [\alpha_{sit} + \omega_{sit}.M_{pi,t-1} + \varepsilon_{sit}.M_{sio}]\ .Y_{it} \qquad with\ s = 1,..,S \qquad (3)$$

in which parameter $\alpha_{sit} > 0$ represents the firm's costs of own knowledge-creation activities (hiring of in-house or outside specialists, firm-level R&D, development of new product varieties, marketing concepts or business models). Parameter $\varepsilon_{sit} > 0$ represents firm-level costs for keeping the firm's 'old' knowledge stocks M_{sio} fresh and active.[18] New is parameter $\omega_{sit} > 0$, which stands for the costs of absorbing public knowledge products through networking activity, setting up internal learning projects, or the hiring of specialists in order to access new knowledge areas.[19] ω_{sit} also includes the costs of recombining the public knowledge inputs with firm-internal knowledge, and the costs of turning the mixed

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[15] The throughput efficiency is assumed to be a dimensionless scalar, implying that the value of the knowledge outputs is a function of its input costs. For model simplicity and transparency, we assume v_{it} to be identical for all sub-processes of a national knowledge system.

[16] The annual cohorts of country *i*'s stock of knowledge capital can be consistently aggregated by the constant elasticity of substitution (CES) vintage aggregator proposed by Benhabib (2019): $M_{io} = [a_1 M_{it-1}^{1-\varepsilon} + a_2 M_{it-2}^{1-\varepsilon} + \ldots + a_n M_{it-(\Theta-1)}^{1-\varepsilon}]^{1/1-\varepsilon}$, in which $\varepsilon > 1$ is the elasticity of substitution and $a_1 \ldots a_{\Theta-1}$ the size shares of the annual knowledge stock cohorts, which sum to 1.

[17] Note that having a relative young public knowledge stock may form a quality-ladder asset for FDI partner countries.

[18] This relates to issues like internal courses for new employees, writing protocols and procedure, documentation, company 'how to ...' handbooks, internal refreshment courses, and HR management.

[19] Cf. Lind and Ramondo, 2022.

knowledge products into excludable private assets, e.g. through patenting or secrecy measures.[20] Note that firms use not the very latest public knowledge, but a lagged knowledge cohort ($M_{pi,t-1}$) as input.[21] The heterogeneity of firms enters in equation (3) via the expression $(v_{it})^{z_s}$ before the square brackets. It is written in this way to stress the conformity with the public knowledge-creation system, but the fixed effect z_s ($s \in 1,..,S_i$) distributes firm productivities around the average national throughput efficiency v_{it}. Firm productivity affects not only its standard operations, but also its knowledge-related activities.

For completeness, we show the vintage structure of the 'old' knowledge cohorts of each firm M_{sio} that is the same as for the public sector:

$$M_{sio} = \sum_{\theta=1}^{\Theta-1} \frac{\alpha_{si,t-\theta}}{(1-\delta_{si,t-\theta})^{\theta}}\, Y_{i,t-\theta} \qquad with\ \ 0 < \delta_{si,t-\theta} < 1\ ;\ \delta_{si,t-\Theta} = 1 \qquad (4)$$

Total proprietary knowledge assets of all firms in country *i* can be approximated by $M_{fit} = \sum_s M_{sit}$. This must be considered as a gross total, because knowledge creation is here calculated by input costs (GDP fraction). It is possible and even likely that the individual knowledge-creation efforts of firms result in duplications that still have to prove their value in the market.[22]

The equations (1-4) offer a stylized description of knowledge flows between the public and private sector, as dimensioned by economic mass. The equations may generate a rich array of dynamics. Private M_{fit} and public M_{pit} contain lagged components, that depend on past decisions and on the time variance of GDP; these elements create a path dependency that affects current knowledge stocks. For tractability of the model, we clarify the basic time dynamics by assuming that public and private sector have the same depreciation method (Θ, δ) and by removing the time variance of the main behavioral parameters (ε_i , β_i , v_i , ε_{si} , α_{si} and ω_{si}). This allows to obtain a reduced-form expression that is only defined by ratio parameters and national economic mass (GDP). Firm-specific productivity $(v_i)^{z_s}$ is abbreviated as λ_{si}.

Proposition 1: *If behavioral parameters are time invariant, the combined proprietary knowledge stock of firms M_{fit} (expressed as GDP share) has the following time dynamics*:[23]

$$M_{fit} = \frac{Y_{it}}{v_i} \sum_{s=1}^{S_i} \lambda_{si} \left\{ \alpha_{si} + \omega_{si}\, \beta_i\, [\varepsilon_i\, A_i + Y_{it-1}] + \frac{\varepsilon_{si}}{v_i} B_i \right\} \qquad (5)$$

in which two complex, lagged and GDP-related terms are abbreviated for shortness as $A_i = \sum_{\theta=1}^{\Theta-1} (1-\delta)^{\theta-1}\, Y_{i,t-(\theta-1)}$ and $B_i = \sum_{\theta=1}^{\Theta-1}\, \alpha_{i,t-\theta}\, Y_{i,t-\theta} (1-\delta)^{\theta}$. Note that all

[20] Cf. Crouset et al., 2022; Ding *et al*. 2022.
[21] This is plausible and prevents endogeneity loops within the model.
[22] Firms synchronously use the same inputs from public knowledge creation. This increases the probability of duplication (being new to the firm is quite different from being new to the market). In public-sector knowledge creation duplication also happens, but because these efforts are more open, the risks of duplication are probably lower than holds for firm-level innovation, where innovation efforts are commonly more subject to strategical secrecy measures.
[23] The Annex provides the proof of Proposition 1.

explanatory terms of the equation are dimensioned in relation to the national economic mass (GDP).

A country's stock of firm-owned knowledge assets (M_{fit}) serves as basis for potential outward FDI. Intuitively, this links to Figure 1. However, whether this potential indeed materializes depends on two conditions that both must be complied with. The first condition is technical and requires that knowledge assets are footloose, meaning that the application of the knowledge asset is separable from its original production location. If the technical condition is fulfilled, its use in a new foreign affiliate increases the returns to such knowledge assets. The second condition is economical, and requires that the application of the firm's proprietary knowledge in a foreign subsidiary does not diminish the overall value of the knowledge asset (non-rivalness). The expected profits of the foreign subsidiary should outweigh the fixed costs of setting up such a subsidiary and the potential loss of export sales from the original production location.[24] If these conditions are satisfied, the firms' willingness to supply outward FDI is potentially unbounded and the potential outward FDI supply constitutes a gravity 'push' force. FDI_{ijt}^{sup} expresses a firm's potential KC supply for setting up foreign subsidiaries:[25]

$$FDI_{sijt}^{sup} = \begin{cases} q_{si}\,(M_{sit})^{h} & if \sum_{s=1}^{S} R_{sijt} \geq 0 \\ 0 & otherwise \end{cases} \qquad \forall \;\; i,j \tag{6}$$

in which $q_{si} = [0,1]$ is a firm-specific parameter that reflects separability of proprietary knowledge assets, and $h > 0$ is a parameter for supply elasticity. R_{sijt} is firm-level expected profitability of using M_{sit} for creating up or expanding a foreign subsidiary in country j. The conditionality of equation (6) will be formulated more precisely after dealing with the implications of self-selection by heterogeneous firms. Potential FDI supply depends via M_{sit} on economic mass (Y_i). Note that M_{sit} in equation (6) includes the main source of firm heterogeneity (labor productivity λ_{si}), so that FDI_{sijt}^{sup} is linked with the micro-economic foundation of our model (to be elaborated later). Aggregated to country level, the vector of potential total bilateral FDI supply amounts to:

$$\overrightarrow{FDI_{ijt}^{sup}} = \sum_{s=1}^{S_i} FDI_{sijt}^{sup} \qquad \forall \;\; i,j \tag{7}$$

Note the total number of firms in a country (S_i) could be a separate source of economic mass along with GDP size. We prevent this by imposing that S_i has a constant relation with GDP size. It implies that S_k of large country k may indeed be larger than the S_x of small country x, but this mass effect is already covered by the GDP sizes of both countries.

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24 Ding *et al.* (2022) reach a similar conclusion with regard to a firm's investment in proprietary intangible capital. Karmakar *et al.* (2022) find that intangibles investment are a good proxy for productivity-enhancing investment (positive effect on total factor productivity), irrespective of the way these investments are financed.

25 FDI activities could extend to all $j \in [1,2,..,N]$ potential destination countries, but whether this actually happens depends also on other factors (cf. section 5).

4. Inward FDI, economic mass and knowledge capital

Bilateral FDI is not only driven by a supply push, but also by a demand pull. The market potential of a country j is determined primarily by its economic mass. Subsidiaries in a foreign market where individual consumers have a preference for more product variety will always find product demand, proportional to its GDP size. Traditional gravity models of FDI mostly confirm this.[26]

There is a second demand pull factor that could be identified as a separate cause. If country j has a positive knowledge gap with country i, this may induce active policies from public authorities or merger bids from firms in country j to attract FDI from country i. Foreign FDI forms an access road to desired knowledge capital. Larger economies may absorb more foreign commercial technology than smaller countries, but that effect is already captured by overall market size. Quantifying the demand-pull potential of real knowledge gap therefore requires a correction for differences in GDP size of both countries. We propose the following procedure for calculating a bilateral real knowledge gap (ΔM^{d}_{fijt}), based on firm-owned private knowledge assets:[27]

$$\Delta M^{d}_{fijt} = \frac{1}{Y_{jt}} \left[\frac{M_{fit}}{Y_{it}/Y_{jt}} - M_{fjt} \right] \qquad \forall\, i, j \tag{8}$$

The values from M_{fit} and (with adaptated suffices) also M_{fjt} follow directly from equation (5), so we will henceforth only use the abbreviation ΔM^{d}_{fijt} for the real bilateral knowledge gap. In the presence of a positive real knowledge gap ($\Delta M^{d}_{fijt} > 0$), governments develop active investment-attracting policies. They wish to maximize FDI-related domestic learning externalities that raises labor productivity and organizational efficiency, and that may open new sectors in the domestic economy (e.g. Lu *et al.*, 2017; Tao and Wang, 1998; Amighini *et al.*, 2017; Vujanović *et al.*, 2022), or may increase domestic wage incomes (e.g. Setzler and Tintelnot, 2019).[28] For analytical clarity about demand-pull vector forces, we should identify the separate FDI-attracting role of a real knowledge gap. This being said, we must immediately add that the knowledge gap at the aggregate level is just a proxy. The forces of technological attraction are likely to differ by industry, so that real bilateral knowledge gaps for bilateral FDI traffic will in most cases play some role, rather than being a binary (on-off) force.[29] This is analogue to the role of product variety in bilateral trade between country: it explains why one will always find some intra-industry traffic between two countries. Hence, the role of the real bilateral knowledge gaps for bilateral FDI traffic will in most cases play some role, rather than being a binary (on-off) force. ΔM^{d}_{fijt} is related

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26 Cf. Tanaka, 2009; Kleinert and Toubal, 2010; Blonigen and Piger, 2014. In more recent structural gravity models of FDI (like Anderson *et a*l., 2019; Kox and Rojas, 2020), the time-variant GDP impact on inward FDI is fully absorbed in the set of estimation dummies (origin-time, host-time).

27 Earlier FDI gravity tests (cf. Blonigen and Piger, 2014) often used the bilateral GDP gap between two countries to explain inward FDI, but what they actually estimated was probably the effect of a mix of the scale-corrected bilateral knowledge gap $\Delta M^{d\,*}_{fijt}$ and the bilateral GDP gap strictu sensu.

28 Positive learning externalities may be channelled through the employment relation (learning by observing, staff mobility), through the channel of domestic intermediary suppliers (e.g. standards with regards to product quality, delivery and transport, packaging and labelling), and through the channel of new products and technologies (Ghodsi and Jovanovic, 2022; Verhoogen, 2021; Keller, 2004; Keller and Yeaple, 2013).

29 This could explain the limited impact of knowledge assets on inward FDI in Table 1 (panel B).

to economic mass of both countries, but in a complex and indirect way. Sub-national regions and individual industries have their own catching-up and falling-behind processes.

Based on the preceding analysis we propose that the pulling force on inward FDI has the following form at the bilateral level:

$$\overrightarrow{FDI_{jit}^{dem}} = \chi_j \left[Y_{jt}\right]^{\gamma} + \zeta_{jt}\, \Delta M_{fijt}^{\dashv} \tag{9}$$

in which $\chi_j > 0$ is a country-specific proportionality constant, $\gamma > 0$ is the elasticity of inward FDI with respect to market size,[30] and $\zeta_{jt} \geq 0$ captures country *j*'s average reaction parameter to a real bilateral knowledge gap.

Thus far, we identified the push and pull factors that shape bilateral FDI going from country *i* to country j. $\overrightarrow{FDI_{ijt}^{sup}}$ reflects the potential outward push forces of origin country *i* and $\overrightarrow{FDI_{jit}^{dem}}$ represents the potential inward pull forces from country j. Both mutually reinforce each other and they can be presented in a multiplicative way, as in Newton's gravity equation.[31] In a world without frictions and third-country influences, actual bilateral FDI amounts to:

$$FDI_{ijt}^{nofric} = \mathscr{g}_{ij} \left(\overrightarrow{FDI_{ijt}^{sup}} \,.\, \overrightarrow{FDI_{jit}^{dem}}\right) \tag{10}$$

in which $\mathscr{g}_{ij}$ is the proportionality factor between potential and actual bilateral FDI. The push and pull factors shape bilateral FDI that goes from country *i* to country j. It is important to note that this is not a *net* flow, but that there may synchronously also be a non-zero reverse flow (FDI_{jit}^{nofric}) that goes in opposite direction. This is due to the fact that countries are no homogeneous entities, but may have sectors, industries and even large multinational corporations whose strengths and weaknesses are differently distributed, also with respect to their knowledge-capital assets. The procedure for deriving the reverse flow FDI_{jit}^{nofric} is the same as the one described here, but with flipped country suffices.

5. Two types of FDI friction factors

In Newton's universal gravity law the force of gravitational attraction between two masses falls with the square of the physical distance between both masses. For FDI we propose two types of bilateral friction sources. The first category is most comparable to Newton's physical distance factor. It covers bilateral FDI obstacles ($\rho_{ij} > 0$) that are time-invariant and unrelated to current policies; it includes physical distance, time zone difference, having different legal systems, having a different language, not sharing a border, and not having a common cultural history and comparable institutions. Moreover, these friction sources are symmetric in both directions ($\rho_{ij} = \rho_{ji}$) and therefore also hamper bilateral FDI in a both directions.

[30] Empirical estimates demonstrate that the values of γ are often close to 1 (e.g. Chaney, 2018).
[31] Newton's formulated his universal gravity law as: $F = G.M.m/R^2$ in which F is the force of gravitational attraction between mass M and mass m. F equals the product of both masses ($M.m$) times a gravitational constant G, and divided by the square of the mutual distance *R* between both masses. Newton based this law on the assumption that each (circular) mass could be shrunken to a point mass in its centre. Then, by considering the gravity as the combined gravity effect exerted by an array of infinitely small slices of the circular mass, with most mass concentrated at its equator, it explains why he used the radius *R* of this circular mass as distance measure (e.g. Zee, 2018).

The second category of bilateral frictions (η_{ijt}) is policy-related and time variant.[32] Examples are statutory prohibitions of FDI in particular industries or regions, discriminatory treatment for specific foreign investors, economic needs tests for foreign investors, nationality requirements for management, or other specific additional administrative obstacles. The η_{ijt}-type frictions often have a composite 'gamut' nature, with different policy rules for different industries or regions. Typically η_{ijt}-type frictions are non-symmetric ($\eta_{ijt} \neq \eta_{jit}$) in both directions. This type of frictions may at times also have components with a negative sign, e.g. in case of liberalization policies, investment subsidies or other types of FDI facilitation. Bilateral or multilateral cooperation agreements like preferential trade and investment agreements often apply positive discrimination for member-state firms, thus lowering η_{ijt} symmetrically for member countries.

Important is that FDI frictions of type (η_{ijt}, η_{ijt}) can occur at two sides of a border. Domestic policies of origin country *i* may also hinder outgoing FDI going to country *j*, e.g. by restrictive policies for companies in military or strategic industries, supply chain restrictions, or by simple bureaucracy. The FDI gravity force can then be expressed in a Newtonian way, in which FDI_{ijt}^{nofric} is the product of both economic masses and with the both types of friction factors also operating multiplicatively:[33]

$$FDI_{ijt}^{fric} = \frac{\mathcal{G}_{ij} \; FDI_{ijt}^{nofric}}{(1 + \rho_{ij} + \eta_{ijt})(1 + \rho_{ji} + \eta_{jit})} \qquad \forall\, i, \forall j \tag{11}$$

in which $\mathcal{G}_{ij}$ is the friction-adapted proportionality constant. Higher friction factors lead to a squared decrease of bilateral FDI. Because both economic masses and at least part of the FDI frictions are variable over time, this equation should have time suffices. FDI_{ijt}^{fric} is the (potential) bilateral FDI from country *i* that would enters country *j* in a situation where no other country pairs are considered and where all firms are equally capable of running a foreign subsidiary. Note that FDI_{ijt}^{fric} will always be strictly positive if FDI_{ijt}^{nofric}>0. However, this property of equation (11) is a provisional result that will disappear, once the model is adapted to a *n*-country world and to the consequences of self-selection by firms.

6. Micro-foundation of outward FDI decisions

Our micro foundation is based on heterogeneous firms that only produce with labor, and that differ by productivity rate. Productivity rate λ_{si} has an impact on knowledge- creation activities and on overall firm performance. Operating profitably in the domestic market minimally requires a productivity rate $\lambda_{si} > \lambda_m$. However, a structurally higher productivity level is required for firms that aspire to become a multinational corporation.[34] This is caused

[32] For expository reasons, we leave out a third, also policy-related type of FDI frictions. Policy measures may domestically be non-discriminatory, but a high degree of policy heterogeneity between domestic regulations in two countries will nonetheless operate as an obstacle for bilateral FDI, because it leads to cost duplication and country-specific sunk entry costs, often in a non-symmetric way (cf. Nordås and Kox, 2008).

[33] Both ρ_{ij} and η_{ijt} are assumed to be numerically bounded [>0, 1].

[34] Multinational activities typically have a productivity that is higher than exporting firms, and substantially higher than firms that operate solely in the domestic market. Cf. Bernard et al., 2005, 2018, 2013; Wagner, 2012, 2017; Kox and Rojas, 2010; Bekes and Muraközy, 2016.

by two types of fixed costs that a firm must absorb up-front, in case of multinational operation. First, setting up and running a headquarter that monitors, communicates with, and supervises over foreign subsidiaries requires overhead labor tasks c_{ijo}. The second fixed-cost barrier is formed by a fixed one-off labour task f_{ijo} for setting up a new foreign subsidiary (e.g. preparation, market prospecting, organising financial start-up conditions, setting up logistics and local suppliers, learning to deal with foreign institutions, and complying with local regulations). The eventual wage costs of both overhead tasks depend on the firm's labor productivity level λ_{si}. With wage level W_i the costs of running a headquarter and setting up a foreign subsidiary become, respectively, $H_{sij} = W_i\ (c_{ijo}/\lambda_{si})$ and $F_{sij} = W_i\ (f_{ijo}/\lambda_{si})$. These fixed-cost expenses also hold in a situation that there are no bilateral frictions for FDI. The setup costs form an investment that firms want to recoup in τ years by equal annual amounts, so that the annualised fixed setup costs amount to H_{sij}/τ and F_{sij}/τ. A 'new' multinational has to absorb both amounts, while it is only F_{sij}/τ for a firm that just adds a new foreign subsidiary. So, H_{sij} may be the source of intra-company scale effects.

It is plausible that the bilateral FDI frictions increase the fixed-cost implications of becoming a multinational.[35] The combined friction term in equation (11) may be abbreviated as: $\varphi_{ijt} \equiv \left(1 + \rho_{ij} + \eta_{ijt}\right)\left(1 + \rho_{ji} + \eta_{jit}\right)$. The effective fixed annual setup costs for the headquarter and a foreign subsidiary in the case of bilateral FDI frictions become, respectively, $\varphi_{ijt}H_{sij}/\lambda_{si}\tau$ and $\varphi_{ijt}F_{sij}/\lambda_{si}\tau$. It is now possible to derive the cut-off productivity rate λ^*_{ijt} that is minimally required for a positive FDI decision by a firm that does not yet have an adequate headquarter and still has no foreign subsidiaries. This firm must absorb both types of fixed investment costs.

Let for firm *s* in country *i* the expected sales revenues a new subsidiary in country *j* ($\tilde{R}_{sijt}$) be proportional to the lagged GDP (Y_{jt-1}) of country *j*, using a self-assessed proportionality factor r_{sj} for overall market size, and a parameter $\mu_{sij} > 0$ that reflects the firm's self-assessment of its competitive strength within country *j*'s domestic market:

$$\tilde{R}_{sijt} = \mu_{sij}\, r_{sj}\;\, Y_{jt-1} \qquad\qquad \forall\, s, j, i \qquad (12)$$

Disregarding variable costs, a firm's expected gross profits (PR_{sijt}) from starting FDI activities in country *j* amount to:

$$PR_{sijt} = \tilde{R}_{sijt} - \varphi_{ijt}\left[\frac{H_{sij} + F_{sij}}{\tau}\right] \qquad\qquad \forall\, s, j, i \qquad (13)$$

and in reduced form:

$$PR_{sijt} = \mu_{sij}\, r_{sj}\, Y_{jt-1} - \frac{\varphi_{ijt}}{\tau\,\lambda_{si}}\,\left[\,W_i\left(c_{ijo} + f_{ijo}\right)\right] \qquad \forall\, s, j, i \qquad (14)$$

Using the first-order condition for profit maximalization to equation (14) yields the cut-off productivity at which a domestic firm with productivity rate λ^*_{ijt} breaks even if it becomes a

35 More differences in distance, language, cultural and legal systems or more discriminatory regulatory provisions tends to increase the fixed setup costs of a subsidiary and the headquarter costs. Formally, we assume that $(dc_{ijo}/d\eta_{ijt}) > 0$; $(df_{ijo}/d\eta_{ijt}) > 0$; $(dc_{ijo}/d\rho_{ijt}) > 0$; $(df_{ijo}/d\rho_{ijt}) > 0$.

multinational:

$$\lambda_{ijt}^{*} = \frac{\varphi_{ijt}}{\tau\,\mu_{sijt}\,r_{sj}} . \left[\frac{W_i\left(c_{ij0} + f_{ij0}\right)}{Y_{jt-1}}\right] \qquad (15)$$

Firms decide on a country-by-country basis to engage in bilateral FDI. Only firms with a productivity draw $\lambda_{si} > \lambda_{ijt}^{*}$ will engage in profitable FDI activities in country *j*. Note that the cut-off productivity rate increases with the friction term φ_{ijt}, but decreases with the market size (Y_{jt-1}) of the destination economy. So, a larger market may compensate for higher FDI friction costs. The cut-off productivity rate λ_{ijt}^{*} is specific per country pair, and it may change over time. Having said that, we apply Ockham's razor and generalize the conclusion based on the essentials by assuming that $\mu_{sijt} = \mu_{ij}$, $W_i = W$ and $r_{sj} = rj$, so that:

$$\lambda_{ijt}^{*} = \frac{\varphi_{ijt}}{\tau\,\mu_{ij}\,r_j}\left[\frac{W\left(c_{ij0} + f_{ij0}\right)}{Y_{jt-1}}\right] \qquad (16)$$

The truncated distribution of all domestic firms with $\lambda_{sit} > \lambda_{ijt}^{*}$ has a similar structure as the productivity distribution for the national universe. Given the linear distribution of firm productivities (Section 3) it is now possible to derive the integer rank number ($s \in [1,2,\ldots,S_i]$ of the firm s^{*} with the cut-off (break-even) productivity rate:

$$s_{ijt}^{*} = int\left(\frac{\lambda_{ijt}^{*}}{b}\right) \qquad (17)$$

The integration constant *b* and the integer operator in equation (17) ensure that a unique firm s_{ijt}^{*} is identified. Only the subset of firms $[s_{ijt}^{*} , S_i]$ is able to make a profit as a multinational firm. Equation (17) makes it possible to quantify the potentially profitable bilateral FDI supply :

$$\overrightarrow{FDI_{ijt}^{sup\,\blacksquare}} = \sum_{k=s_{ijt}^{*}}^{S_i} q_{ki}\,(M_{kit})^{h} \qquad with\ \neg\left[s_{ijt}^{*} , S_i\right] = \emptyset \qquad (18)$$

This effectively means that the subset $[s_{ijt}^{*} , S_i]$ must be empty in many cases. The knowledge-asset term M_{kit} is directly derived from equation (5), but aggregated over the firms subset $k \in [s_{ijt}^{*} , S_i]$:

$$M_{kit} = \frac{Y_{it}}{v_i}\sum_{k=s_{ijt}^{*}}^{S_i} \lambda_{ki}\left\{\alpha_{ki} + \omega_{ki}\,\beta_i\,[\varepsilon_i\,A_i + Y_{it-1}] + \frac{\varepsilon_{ki}}{v_i}B_i\right\} \qquad (5a)$$

The condition that the subset $[s_{ijt}^{*} , S_i]$ is not empty, is far from trivial. The statistics on the world pattern of bilateral FDI show however an overwhelming presence of zeros.[36] Within our model, the presence of zero FDI flows must be –given the fundamental willingness of firms subset *k* to use their proprietary knowledge assets (M_{kit}) for outward FDI– that *all* firm-level productivities in country *i* are insufficient to overcome the bilateral FDI friction

[36] Cf. Helpman et al. (2004).

costs, and that the size of destination country *j*'s market is not sufficient to compensate for that.

A useful count statistic is the share of all firms of country *i* that could profitably have direct investment in country *j* (also called the extensive margin of FDI). In the 2-country case it would amount to:

$$\mathbb{E}_{ijt} = \frac{1}{S_i}\sum_{k=s_{ijt}^*}^{S_i} 1$$

While equations (6, 7) would always predict a strictly positive bilateral FDI flow, this is with equation (18) no longer the case. If $\mathbb{E}_{ijt}$=0, potential bilateral FDI ($\overrightarrow{FDI_{ijt}^{sup}}$) will also be zero. In all other cases, potential bilateral FDI supply is now based on M_{kit}^* , i.e. the proprietary knowledge assets of only those firms that have a sufficiently high productivity to cope with the fixed FDI setup costs (augmented by the bilateral friction costs). At country level it means that the supply FDI vector changes: $\overrightarrow{FDI_{ijt}^{sup\,\blacksquare}} < \overrightarrow{FDI_{ijt}^{sup}}$. The micro-level model thus has macro repercussions at the country level. The self-selection of firms effectively internalizes the direct FDI friction costs (ρ_{ij}, η_{ijt}) that are associated with entering country *j* with a foreign subsidiary. The self-selection process lowers potential FDI supply.[37]

Equation (11) must now be sacrificed, because the concept of FDI_{ijt}^{nofric} is no longer relevant after oneof its two components ($\overrightarrow{FDI_{ijt}^{sup}}$) has changed, while also the friction factor $(1 + \rho_{ij} + \eta_{ijt})$ is now internalised in firm behaviour via the cut-off productivity rate. The self-selection of firms in country *i* has no impact on the FDI demand-pull vector ($\overrightarrow{FDI_{jit}^{dem}}$). If $\mathcal{G}_{ij}$ is the gravity proportionality constant, the eventual bilateral FDI (FDI_{ijt}^{BIL}) in the 2-country case becomes:

$$FDI_{ijt}^{BIL} = \frac{\mathcal{G}_{ij}\left(\overrightarrow{FDI_{ijt}^{sup\,\blacksquare}} . \overrightarrow{FDI_{jit}^{dem}}\right)}{\left(1 + \rho_{ji} * \eta_{jit}\right)} \qquad \forall\, i, \forall j \qquad (19)$$

The within-borders friction factors (ρ_{ji}, η_{jit}) in country *i* that hinder outgoing FDI have not changed. They are not internalized by the micro model and therefore have to be specified separately, because it remains relevant from the perspective of country *j*.

$\overrightarrow{FDI_{ijt}^{sup\,\blacksquare}}$ informs us about the *potential* maximum amount of bilateral FDI that firms of country *i* would be willing to invest in destination country *j* in year *t*, given the bilateral FDI frictions. However, there is more that plays a role. Firms evaluate also the opportunity costs of *not* investing in other destination countries than country *j*. Here we enter the domain of relative FDI friction costs and third-country effects.

[37] This section identified the impact of bilateral FDI frictions on the extensive margin of FDI. FDI self-selection by firms yields results that are fully compatible with the results by Melitz (2003) on firm exports in the presence of fixed entry costs for exporters.

7. Structural FDI gravity model in a *N*-country world

World FDI patterns form a closed system, if only because at least some multinational companies apply a global perspective in their investment-location decisions.[38] Strategic decisions are driven by expectations on market outlooks and relative FDI costs. An increase (or lowering) of the FDI friction costs in country *Z* could cause a cascade of location-substitution decisions that affect third countries. Changes in the FDI friction costs or the market perspectives for country Z may change the relative attractiveness of countries X and Y, without the latter having changed anything in their own friction-affecting policies. Policy changes in nearby large economies may send out stronger and more geographically extended 'ripples of change' than similar policies in remote small island states would do. This is the market-size factor.

Anderson and Van Wincoop (2003) were the first to explicitly model and quantify the role of third-country effects for international trade. Already a few years later, neglecting such general equilibrium effects in empirical studies was labelled the "gold medal error" by Baldwin and Taglioni (2006). A number of empirical papers show that ignoring third-party effects of trade policies leads to substantial biases in the results (cf. Head and Mayer, 2014; Fally, 2015; Yotov *et al*., 2016). The crux is that actual trade patterns are affected by both absolute and relative trade frictions. Relative trade costs reflect the opportunity costs of not directing bilateral trade to other countries. It is also a valid issue for FDI decisions. Structural FDI gravity analysis explicitly deals with the third-country effects.[39]

Anderson and Van Wincoop (2003) found a way to deal with third-party effects by expressing relative trade costs via what they call *multilateral resistance* (MR) terms; these terms aggregate relative trade costs in a consistent way. We apply their method and most[40] of their suggestions to model bilateral FDI patterns in an *N*-country world. A comprehensive FDI analysis should regard the full set of country pairs with their relative FDI friction costs and their relative market sizes.

The factor φ_{ijt} in equation (16) captures the direct FDI frictions between a country pair. The indirect or relative FDI friction costs are more complex, because they must be normalized by the average economic mass and the average friction costs of *all* country pairs. The MR terms do so by aggregating from two perspectives, for outgoing FDI (origin country perspective), and for incoming FDI (perspective of the host/destination country). The outward MR term P_{it} measures —from the perspective of origin country i— the relative attractiveness of each potential destination country. It is a combination of the direct friction costs, the relative friction costs for entering that market, and the country's potential FDI absorption capacity (proxied by relative GDP size):

[38] Most firms with multinational aspirations evaluate at least a small set of destination-country alternatives (Dunning, 2001; Cantwell and Narula, 2001). Only a few very large firms with experienced firm headquarters will be in the position to apply a really global perspective.

[39] Baltagi *et al*. (2007) and Bergstrand and Egger (2007, 2010) found that third-country effects also occur in relation to FDI. For structural gravity estimations on FDI, see Anderson, Larch and Yotov (2019); Kox and Rojas (2020, 2019).

[40] The assumption by Anderson and Van Wincoop (2003) that bilateral frictions between partner countries should be fully symmetric is redundant and overly simplifying. As shown in equation (11) our model of bilateral FDI friction also includes direction-specific friction costs ($\eta_{ijt} \neq \eta_{jit}$).

$$P_{it} = \left[\sum_{j=1}^{N} \left\{\frac{1+\rho_{ij}+\eta_{ijt}}{\Pi_{jt}}\right\}^{1-\sigma} \cdot \left(\frac{Y_{jt}}{Y_t}\right)\right]^{\frac{1}{1-\sigma}} \qquad \forall\, j, i \tag{20}$$

The first term between accolades measures the relative friction costs by normalizing the direct FDI friction costs of destination country j (i.e. η_{ijt}, ρ_{ij}) with the average inward MR terms of *all* potential partner countries (Π_{jt}). The second term measures the relative FDI absorption capacity of each potential partner country as proxied by its share in world GDP ($Y_t = \sum_j^N Y_{jt}$). All destination-country alternatives are made comparable with a constant elasticity of substitution ($\sigma > 1$) that is assumed to be equal for all countries.[41] Relative friction costs increase in the direct FDI friction costs in country j (η_{jit}, ρ_{ij}), but decrease in average inward friction costs elsewhere (Π_{jt}).

When the world FDI pattern is considered as an integrated system, friction costs on each side of the national border must be considered. This also holds for the frictions that hinder outgoing FDI of each origin country ($i \in 1,..,N$). Seen from the perspective of the FDI-receiving country, it will be easier to attract FDI from countries with low policy-related obstacles for outgoing FDI. The inward MR term Π_{jt} is consistently aggregated by evaluating different FDI origin alternatives, as the mirror image of equation (20):

$$\Pi_{jt} = \left[\sum_{i=1}^{N} \left\{\frac{1+\rho_{ji}+\eta_{jit}}{P_{it}}\right\}^{1-\sigma} \cdot \left(\frac{Y_{it}}{Y_t}\right)\right]^{\frac{1}{1-\sigma}} \qquad \forall\, i, j \tag{21}$$

The direct pairwise friction costs ($\rho_{ji} + \eta_{jit}$) that affect incoming FDI are now normalized with the average outward MR term P_{it} of each origin country, thus expressing relative friction costs that affect incoming FDI. The MR terms form an inseparable module of the *n*-country model. Mathematically, the fact that equations (20) and (21) refer to each other could hint at fixed-point problems and non-unique solutions. However, it has been proven that unique solutions exist for the Π_{jt} and P_{it} vectors.[42]

Together, both MR equations map worldwide relative FDI friction costs for all countries, weighted by the relative economic mass of all countries. These equations describe the economic gravity field in which firms shape their internationalization strategies. The field is a function of both economic masses and frictions, each fluctuating across time and place. Relative economic masses tend to fluctuate only lightly over time (relative GDP size tends to change slowly), but frictions may fluctuate more heavily (e.g. in case of economic sanctions or other radical policy changes). Local strength of the economic gravity field tends to be

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[41] The $\sigma > 1$ constant substitution elasticity also expresses that countries have a preference for variety with respect to foreign knowledge or technology capital in the form of FDI.

[42] Inputs outside the international economics literature are instructive. In 1772, Joseph-Louis Lagrange solved the *n*-body gravity problem for celestial objects by taking mass-weighted triangulation averages (root mean square of all relative distances, weighted by the total mass of all objects) of distances between all 3-tuple configurations (cf. Barbour, 2020). Dealing with an analogue problem in input-output analysis, Dietzenbach and Miller (2009) prove that the equivalent of the MR terms have unique solutions relative to a neutral normalising constant. Translating their results to FDI, it requires: (a) all countries are represented in the world FDI matrix; (b) all countries invest in their own economies (elements on main matrix diagonal are strictly positive); and (3) there is no group of countries that operates in FDI-autarky or in FDI--isolation (matrix can not be made block diagonal). Poissonnier (2019) even argues that it would suffice to satisfy condition (c) only. Anderson and Yotov (2012) and Anderson (2011) proceed along the lines of the Dietzenbach-Miller approach.

distributed quite unevenly across the world. The field strength can be very strong in the neighborhood of large economies (USA, EU, China), but quite weak in remote peripheral areas like the Pacific Ocean. All of these effects are embodied in the MR terms (Π_{jt} , P_{it}), because they are calculated separately per country. Localized effects from neighboring countries are accounted for, because physical distance is accounted for by the friction term ρ_{ji}. Similarly, element η_{jit} accounts for any hectic local time shocks in FDI frictions. The localized changes are normalized by the globally averaged MR terms (Π_{jt} , P_{it}). Overall, the MR terms are a perfect representation of the relative attractiveness of both FDI destination countries and FDI origin countries.

It is only now that we can present the final, *n*-country version of the testable FDI gravity equation. It uses the result of equation (1) and applies a correction for market-size-weighted relative friction costs:

$$FDI_{ijt} = \left[\frac{\Pi_{jt} \, . P_{it}}{\varphi_{ijt} - 1}\right] . \; FDI_{ijt}^{BIL} \qquad \forall \, i, j \qquad (22)$$

If the friction term (between square brackets) differs from one, the predicted actual bilateral FDI deviates from the potential bilateral FDI (FDI_{ijt}^{BIL}). This is in line with the micro-economic model. Recall that equation (12) has two assessment parameters for firm-specific revenue expectations in a foreign market. Equation (22) implies that individual firms may re-assess their initial expectations, once observing third-country alternatives.[43] The actual bilateral FDI will be higher than the strictly bilateral term FDI_{ijt}^{BIL}, if $(\Pi_{jt} \, . P_{it}) > \varphi_{ijt}$,. The adjustment will materialize via the average amount of outward FDI per firm (intensive margin of FDI), because the initial FDI_{ijt}^{BIL} underrated the relative profitability of investing in country *j*.[44] However, if $(\Pi_{jt} \, . P_{it}) < \varphi_{ijt}$, the actual bilateral FDI will be lower than FDI_{ijt}^{BIL}, also via the intensive FDI margin. Equation (22) is the core result of our stand-alone, gravity-based FDI model.

8. The core model

Much of the Sections 3-7 was focused on the development of the model, with all background references, definitions, and other conceptual development steps. However, the main results can be summed up in a small five-equation core model.[45]

Proposition 2: *The core model can be summarized in only five equations*.[46]

$$FDI_{ijt} = \mathcal{G}_{ij} \left[\frac{(\Pi_{jt} \, . P_{it}) \left\{ \chi_j \, [Y_{jt}]^{\gamma} + \zeta_{jt} \, \Delta M_{fijt}^{d} \right\}}{(\varphi_{ijt} - 1)(1 + \rho_{ji} . \eta_{jit})} \right] \sum_{k=s_{ijt}^{*}}^{S_i} q_{ki} \, (M_{kit})^{h} \qquad (I)$$

$$M_{kit} = \frac{Y_{it}}{v_i} \sum_{k=s_{ijt}^{*}}^{S_i} \lambda_{ki} \left\{ \alpha_{ki} + \omega_{ki} \, \beta_i \, [\varepsilon_i \, A_i + Y_{it-1}] + \frac{\varepsilon_{ki}}{v_i} B_i \right\} \qquad (II)$$

[43] The alternative destination countries offer a form of internal check on initial expectations (cf. eq. 14).
[44] The average intensive FDI-margin at firm level $\mathbb{I}_{ijt}$ is defined as $\mathbb{I}_{ijt} \equiv FDI_{ijt}/(S_i \, \mathbb{E}_{ijt})$.).
[45] All other equations were used only for the expository development of the model.
[46] The Annex provides the proof of Proposition 2.

$$s^*_{ijt} = int\left(\frac{\varphi_{ijt}\ w(c_{ij0}\ + f_{ij0}\ )}{b\ \tau\ \mu_{ij}\ r_j\ Y_{jt-1}}\right) \qquad (III)$$

$$\Pi_{jt} = \left[\sum_{i=1}^{N}\left\{\frac{1+\rho_{ji} * \eta_{jit}}{P_{it}}\right\}^{1-\sigma} \cdot \left(\frac{Y_{it}}{Y_t}\right)\right]^{\frac{1}{1-\sigma}} \qquad \forall\ i,j \qquad (IV)$$

$$P_{it} = \left[\sum_{j=1}^{N}\left\{\frac{1+\rho_{ij} * \eta_{ijt}}{\Pi_{jt}}\right\}^{1-\sigma} \cdot \left(\frac{Y_{jt}}{Y_t}\right)\right]^{\frac{1}{1-\sigma}} \qquad \forall\ j,i \qquad (V)$$

Equation (I) predicts the actual bilateral FDI in the *n*-country model. Equation (II) describes the proprietary knowledge assets supplied by the group of firms that has the capacity to profitably use these assets in foreign subsidiaries. Equation (III) provides the (integer) rank number of the firm that has the minimum required productivity for going multinational, which is the main result of the productivity-based self-selection process among firms in the origin country. The equations (IV) and (V) present the basis for the relative FDI friction costs, in the form of, respectively, the inward and outward MR terms.

9. Discussion: a stochastic model with testable predictions

Our new reading of Markusen's FDI knowledge-capital model makes it possible to link up with gravity analysis. The model is further related to the work by Anderson, Larch and Yotov (2019), who presented a FDI gravity equation as part of a larger general equilibrium model that also explains bilateral trade and overall capital accumulation. Their model setup prevents them to separately test their FDI sub-model. There are four important differences between their FDI model and ours. Firstly, Anderson *et al*. can only explain zero bilateral FDI flows from absolute restrictions on inward FDI by (potential) destination countries. However, this does not explain why many countries have no outward FDI whatsoever. We explain this stylized fact from the self-selection process of firms and the existence of fixed FDI costs. Secondly, they assume without motivation that the FDI-relevant MR-terms are identical to those for bilateral trade.[47] Our model derives FDI-specific MR terms. Thirdly, our model allows for non-symmetric FDI frictions between partner countries. Fourth and finally, our model offers an entirely new knowledge-based explanation for the basic FDI gravity forces.

Sections 3-7 propose a new stand-alone model that deterministically explains bilateral FDI patterns. It is deterministic in the sense that it predicts bilateral FDI flows if all decision makers had access to all relevant information. The micro-model explains the extensive FDI-margin at firm level, which provides the main conditionality for non-zero bilateral FDI. Worldwide, many bilateral FDI flows still are zero flows, so the contribution of the micro model is utterly relevant. After the self-selection process, the firm's amount of FDI per country forms the accommodating factor that accounts for the relative bilateral frictions. Even though it accounts for relative bilateral frictions, the model still is a partial equilibrium

[47] Bergstrand and Egger (2007, 2010) show that bilateral trade and FDI flows are driven by a similar process, but there is no reason to assume that the 'trade process' and the 'FDI process' have identical numerical parameters.

model, because it takes the current and past distribution of economic masses (GDP) as an exogenous input.[48]

This paper presents a model for explaining worldwide bilateral FDI patterns at the most general level. It means that we disregard idiosyncrasies, e.g. behavioral FDI elements that are peculiar to specific sectors or FDI types (vertical, export-platform, global value chains). For the intensive FDI margin ($\mathbb{I}_{ijt}$), such elements may be crucial. Some elements could be built in easily[49], but it leads away from the general gravity mechanism and it is non-essential from the perspective of Ockham's razor.

An important simplifying model assumption is that all decision makers have access to all relevant information, as is common in many economic models. However, at the micro-level, decision makers never have the full world information at their disposal. Therefore, equation (I) works only stochastically, through the law of mass action, through trial and error, across many cases of over-shooting or under-estimation errors. Firms take FDI decisions on a country-by-country basis, sometimes by comparing a limited set of potential destination countries. They assess their own FDI capabilities, based on private knowledge of their own labor productivity and the available knowledge on foreign FDI friction costs. The gravity model works through the self-selection behavior of firms, and when the model is correct, it will even beat those many firm-level selection errors or governmental policy errors that are driven by incomplete knowledge or wrong expectations.[50] The role of relative FDI friction costs (Π_{jt}, P_{it}) is most probably mostly driven by the choices of a limited number of large firms with a real global FDI horizon. Finally there is the role of measurement errors that may be important for FDI statistics.[51] Taken together, these factors imply that the model will at best be found stochastically correct after estimation.

Nonetheless, the core model is falsifiable by empirical estimation, because it yields a set of testable predictions. Parameters of the relative economic masses (GDP) of origin and destination country should have positive and significant signs.

- Parameters for estimated FDI friction costs (physical distance, lacking a common language, lacking a common border, lacking common institutions or history, having different regulations, policy-made obstacles to bilateral FDI) should have significant and negative signs.

[48] Olivero and Yotov (2012) elaborate how the economic mass of countries converges or diverges through changes in their policy-based friction costs and through endogenous changes in FDI and trade.
[49] An obvious option that is compatible with the Eaton and Kortum (2002) model, would be to link the host country's productivity and wage levels to incoming FDI. Brainard (1997) found that the presence of trade obstacles positively affects the choice for FDI as international expansion strategy. Also, vertical FDI motives in upstream or downstream foreign expansion activities remain outside the scope of this paper (e.g. Markusen, 2002; Carr *et al.*, 2004)).
[50] Baltagi *et al.* (2007) find evidence for US manufacturing FDI that the impact of FDI-facilitating policies by potential host countries on bilateral FDI patterns is undermined by the host country's remoteness from main consumer markets (and by weaknesses in local skilled-labour supply).
[51] The available bilateral FDI data are improving in the last few years, but statistical reporting of FDI is still insufficiently coordinated internationally, resulting in different reporting of the same bilateral flow by origin and destination country, partly reported origin or destination countries, and anomalies such as dimensional mistakes and negative FDI stocks. Because of statistical deficiencies, it may be necessary to define a rest-of-world category, specific for each origin and destination country, to capture unspecified or unreported bilateral FDI, based on mirror data of bilateral FDI partners.

- A negative correlation should exist between the incidence of zero outward FDI flows (either $i \rightarrow j$ or $j \rightarrow i$) and the relative productivity performance of a country.
- A positive correlation should exist between the relative magnitude of outward FDI flows and the parameters that measure a country's relative abundance of proprietary knowledge stocks (cf. equation 5).
- A negative sign is expected for estimated parameters that measure relative bilateral distance, 'language distance', 'cultural distance', 'distance between legal systems', and the incidence of relative inward 'FDI-closedness' policies.
- A positive sign is expected for being members of the same preferential trade agreements, and for being members of the same bilateral investment agreement, because the latter are expected to lower bilateral FDI frictions.

For empirical testing, we propose a stepwise identification strategy that starts from a very general specification of the regression model with almost only fixed effects for time, for individual countries and for country pairs. The fixed effects absorb all country- and country-pair differences, including the time variation in direct friction costs and economic mass (GDP), and also including non-modelled or unobservable data variation. After this comprehensive assessment, different strategies can be applied to 'peel off' more specific information from the general results. The crux in econometric testing of this model is whether it remains standing despite the non-modelled, non-observed, or even non-observable impacts on world bilateral FDI patterns. This gives a range of challenges that must be dealt with in the econometric testing of the model:

- impact of non-specified factors (sectors, fiscal motives, intra-firm trade-off between FDI and intra-company trade);[52]
- non-observed or even unknown impact factors that are relevant at the firm or/or country level, such as tax routing;
- time-related regional or worldwide shocks;
- any confounding impact factor that affect both the dependent variable (FDI_{ijt}) and some or all of the explanatory variables ($Y_{it}, Y_{jt}, \eta_{ijt}, \eta_{jit}$) and may lead to biased econometric results.

With regard to econometric testing of gravity models, Yotov *et al.* (2016) and Head and Mayer (2014) provide valuable toolkits and 'cookbooks'.[53] The recent literature suggests that the Pseudo-Poisson Maximum Likelihood (PPML) estimator is a good first start. It has the advantage that it effectively deals with zero bilateral FDI flows, and also with situations in which the variance of the error term depends on at least one of the explanatory variables (Santos Silva and Tenreyro, 2006, 2011; Correia et al., 2020). Both characteristics are important for the FDI data.

Zeros are overwhelmingly present in the world bilateral FDI matrices, and their incidence at lower aggregation levels (e.g. by sector) will even be stronger than our micro-model predicts. A sharp distinction has to be made between real zero FDI and missing (or non-reported, suppressed) data. Recent FDI data sets of IMF, OECD, Eurostat and UNCTAD identify non-reported, confidential FDI flows. This is valuable information that should be used in the

[52] E.g. Bergstrand and Egger 2007, 2010; Anderson *et al.*, 2019; Damgaard *et al.*, 2019;
[53] See also Kox and Rojas (2020) for a review of best practices in testing structural FDI gravity models.

testing procedure. Heteroskedastic error terms are caused by the large differences in relative size of countries. The structure of the data variables should therefore be a co-determinant for the choice of a particular estimator, while robustness tests with different estimators are advisable.[54]

Data consistency restrictions. For a consistent test of our model it is important to impose a squareness restriction on the world FDI matrix. This is something that must be done in the data preparation phase. It ensures that the world FDI system can be treated as a closed system. The following accounting condition must hold:

$$Z = \sum_{i=1}^{N} FDI_{ijt} - \sum_{j=1}^{n} FDI_{jit} = 0 \qquad \forall\, i,j \qquad (23)$$

This means not only that all origin and destination countries must be included, but also that the main diagonal of the matrix (i.e. FDI_{iit}, FDI_{jjt}) of domestic investments must be filled. For adequately capturing the impact of third-country externalities on bilateral FDI, it is important that accounting condition (23) is satisfied. This allows to capture the general-equilibrium effects caused by changes in market sizes (GDP) and in relative FDI friction costs, including intra-national friction costs for investment.[55]

The literature suggests that under-reporting of FDI stocks is a bigger problem than over-reporting. In that case, the use of mirror data from partner countries is a relatively easy first check towards achieving the *Z=0* condition of eq. (23).[56] While the quality of bilateral FDI stock data has improved considerably during last decennium, there still are several anomalies in the data. One of them is the occurrence of negative bilateral FDI stocks. The phenomenon can be explained by the vintage structure of bilateral FDI stocks and the way in which these past flows were financed.[57] Old vintages of FDI assets within a particular host country may be subject to local changes in accounting systems, local valuation changes and local changes in the structure of asset financing. These actions take place outside the explanatory scope of the bilateral FDI gravity model and they are not necessarily related to the proprietary knowledge capital of the origin country. Our model explains semi-positive current FDI stocks and FDI decisions by firms, and not the ex-post valuation changes in existing FDI stocks. So, reported negative stocks in the data are best set to zero.

Finally, the estimation process of the model generates also quantitative estimates for the multilateral resistance (MR) terms, both inwards (Π_{jt}) and outwards (P_{it}).[58] The inward MR terms form a rich data source that allows secondary analysis of the determinants of FDI

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54 PPML assumes a constant variance-to-mean ratio (dispersion index), whereas Gamma PML assumes a constant coefficient of variation. PPML puts more emphasis than Gamma PML on observations with large expected FDI. Head and Mayer (2014) and Martínez-Zarzoso (2013) recommend controlled tests with different types of Monte-Carlo generated databases. An often suggested alternative for PPML is the Gamma PML. Camarero *et al*. (2019) use the Negative Binomial PML, but according to Head & Mayer (2014) this estimator cannot perform correctly in the presence of many zeros. A test on the adequacy of the estimator is easy to apply.

55 Cf. Agnosteva *et al*. (2019), Anderson *et al*. (2019, 2020), Yotov *et al*. (2016), Olivero and Yotov (2012).

56 It may not be enough. If still holds that $Z > 0$, the difference might be added to a fictive ROW 'country' of the least-reporting world aggregate (either $\sum_{i=1}^{J} FDI_{it}$ or $\sum_{j=1}^{J} FDI_{jt}$), while adding a similar, but empty ROW 'country' to the other world aggregate.

57 Cf. equation (II) of the core model.

58 Cf. Correia et al., 2020; Weidner and Zylkin, 2021; Anderson and Yotov, 2012.

friction costs, like quantifying the impact of different types of domestic policies on a country's inward MR terms. Similarly, one may use the outward MR terms for analysis that quantifies the impact of domestic policies in the origin countries have on the magnitude of outgoing FDI flows. The FDI-related MR terms can also be used as quantitative indicator for assessing whether (and to what extent) bilateral or multilateral policies like preferential trade agreements, WTO, regional integration pacts, or bilateral investment agreements are effective between member countries (cf. Kox and Rojas, 2019).

10. Summary and conclusions

The paper develops a stand-alone gravity model to explain international patterns of foreign direct investment (FDI). Using Ockham's razor, we opt for the smallest possible model that could explain worldwide FDI patterns. The 5-equation core model is based on knowledge-based gravitational forces that are directly or indirectly linked to a country's economic mass (GDP). The micro-economic part of the model explains the bilateral extensive FDI margin, i.e. the aggregate result of firm decisions on setting up a new subsidiary in another country, with the firm's intangible knowledge assets as crystallization kernel. The model is generalized to a n-country world by also accounting for the externalities caused by third-country effects. The FDI-based multilateral resistance (MR) terms quantify the relative FDI friction costs of all countries, weighted by the size of their markets. The MR terms vary by year and country pair. They in fact describe the constantly fluctuating gravity field, in which firm-level FDI decisions are being taken. The model allows for different types of FDI friction costs, including policy-related country-specific or pair-specific costs. Physical and policy-made friction costs factors could explain why the bilateral FDI is small, but cannot explain why several countries have zero outward FDI flows with all partner countries. This stylized fact is explained by the firm-level self-selection mechanism that forms part of the core model.

The model provides several policy-relevant outcomes. It shows the huge impact of public and private knowledge-creation policies on outward FDI. The model also quantifies the impact of policy-made FDI frictions on expected bilateral FDI. As a secondary output the model yields quantitative indicators for the FDI-based MR terms. The latter form a splendid quantitative yardstick for evaluating the impact that national and international policies have had on bilateral and worldwide FDI flows.

Annex Mathematical proofs

Proposition 1. If public and private sector have the same depreciation method (Θ, δ) and we take out the time variance of the main parameters (ε_{it} , β_{it} , v_{it} , ε_{sit} , α_{sit} and ω_{sit}), the equations (1-4) reduce to:

$$M_{pit} = v_i \left[\varepsilon_i \, M_{pio} + \beta_i\right] Y_{it} \tag{1a}$$

$$M_{pio} = \sum_{\theta=1}^{\Theta-1} \beta_i \, Y_{i,t-\theta} \, (1-\delta)^{\theta} \qquad with \ 0 < \delta < 1; \ ; \ \delta_{t-\Theta} = 1 \tag{2a}$$

$$M_{sit} = (v_i)^{z_s} \left[\alpha_{si} + \omega_{si}. M_{pi,t-1} + \varepsilon_{si}. M_{sio}\right] . Y_{it} \qquad with \, s = 1,..,S \tag{3a}$$

$$M_{sio} = \sum_{\theta=1}^{\Theta-1} \alpha_{i,t-\theta}\, Y_{i,t-\theta}(1-\delta)^{\theta} \qquad with\ \ 0<\delta<1\,;\ \delta_{t-\Theta}=1 \qquad (4a)$$

Starting from aggregation of all firm-level knowledge creation effort, we get equation A1, in which:

$$M_{fit} = \sum_{s=1}^{S_i} (v_i)^{z_s} \left[\alpha_{si} + \omega_{si}.M_{pi,t-1} + \varepsilon_{si}.M_{sio}\right].Y_{it} \qquad (A1)$$

This rewrites as:

$$M_{fit} = Y_{it}.\sum_{s=1}^{S_i} (v_i)^{z_s} \left[\alpha_{si} + \omega_{si}.M_{pi,t-1} + \varepsilon_{si}.M_{sio}\right] \qquad (A2)$$

And using eq. (1a), $M_{pi,t-1}$ can be written as $M_{pi,t-1} = v_i\left[\varepsilon_i\, M_{pio,t-1} + \beta_i\right] Y_{it-1}$. After substituting this into (A2) and some rearrangement, this yields:

$$M_{fit} = Y_{it}\sum_{s=1}^{S_i} (v_i)^{z_s-1} \left\{\alpha_{si} + \omega_{si}\left[\varepsilon_i M_{pio,t-1} + \beta_i Y_{it-1}\right] + \frac{\varepsilon_{si}}{v_i}.M_{sio}\right\} \qquad (A3)$$

From equations (2a) and (4a) the lagged knowledge stocks M_{sio} and $M_{pio,t-1}$ may be substituted into (A3):

$$M_{fit} = Y_{it}.\sum_{s=1}^{S_i}(v_i)^{z_s-1}\left\{\alpha_{si} + \omega_{si}\,\varepsilon_i\,\beta_i\left[\sum_{\theta=1}^{\Theta-1} Y_{i,t-(\theta-1)}(1-\delta)^{\theta-1}\right] + \right.$$
$$\left. + \omega_{si}\,\beta_i\, Y_{it-1} + \frac{\varepsilon_{si}}{v_i}\left[\sum_{\theta=1}^{\Theta-1} \alpha_{i,t-\theta}\, Y_{i,t-\theta}(1-\delta)^{\theta}\right]\right\} \qquad (A4)$$

Firm-level productivity $(v_i)^{z_s}$ can be abbreviated as λ_{si}. And because $\lambda_{si} \equiv (v_i)^{z_s}$, the first term before the accolade can be decomposed to (λ_{si}/v_i), showing the ratio of firm productivity and the national productivity average. Since the latter $(1/v_i)$ is not firm-specific, it can be brought outside the summation operator. Further, the two complex GDP-related terms $\sum_{\theta=1}^{\Theta-1}(1-\delta)^{\theta-1}\, Y_{i,t-(\theta-1)}$ and $\sum_{\theta=1}^{\Theta-1} \alpha_{i,t-\theta}\, Y_{i,t-\theta}(1-\delta)^{\theta}$ can be abbreviated as, respectively, A_i and B_i, so that one obtains one obtains *Proposition 1*:

$$M_{fit} = \frac{Y_{it}}{v_i}.\sum_{s=1}^{S_i} \lambda_{si}\left\{\alpha_{si} + \omega_{si}\,\beta_i\,[\varepsilon_i\, A_i + Y_{it-1}] + \frac{\varepsilon_{si}}{v_i} B_i\right\} \qquad QED\ \blacksquare$$

Proposition 2

Deriving Proposition 2 only requires nine core equations of the model: (5a), (9), (16)-(22), while all other equations support only the expository development of the full model. Combining eqs. (18, 19, 22) with demand-pull equation (9) one obtains the magnitude of bilateral FDI in the *n*-country world:

$$FDI_{ijt} = \mathcal{G}_{ij}\left[\frac{\left(\Pi_{jt}\,.P_{it}\right)\left\{\chi_j\left[Y_{jt}\right]^{\gamma} + \zeta_{jt}\,\Delta M_{fijt}^{d}\right\}}{(\varphi_{ijt}-1)\left(1+\rho_{ji}.\eta_{jit}\right)}\right]\sum_{k=s_{ijt}^{*}}^{S_i} q_{ki}\,(M_{kit})^{h} \qquad (I)$$

$$M_{kit} = \frac{Y_{it}}{v_i} \sum_{k=s_{ijt}^*}^{s_i} \lambda_{ki} \left\{ \alpha_{ki} + \omega_{ki}\, \beta_i \,[\varepsilon_i\, A_i + Y_{it-1}] + \frac{\varepsilon_{ki}}{v_i} B_i \right\} \qquad (II)$$

Eqs. (16, 17) together identify the bottom productivity rank of firms in country *i* with outward FDI, as used in eq. (I) and (II):

$$s_{ijt}^* = int\left(\frac{\varphi_{ijt}\, w\left(c_{ij0} + f_{ij0}\right)}{b\, \tau\, \mu_{ij}\, r_j\, Y_{jt-1}} \right) \qquad (III)$$

And the relative FDI friction costs follow from eqs. (19, 20):

$$\Pi_{jt} = \left[\sum_{i=1}^{N} \left\{ \frac{1 + \rho_{ji} * \eta_{jit}}{P_{it}} \right\}^{1-\sigma} \cdot \left(\frac{Y_{it}}{Y_t} \right) \right]^{\frac{1}{1-\sigma}} \qquad \forall\, i, j \qquad (IV)$$

$$P_{it} = \left[\sum_{j=1}^{N} \left\{ \frac{1 + \rho_{ij} * \eta_{ijt}}{\Pi_{jt}} \right\}^{1-\sigma} \cdot \left(\frac{Y_{jt}}{Y_t} \right) \right]^{\frac{1}{1-\sigma}} \qquad \forall\, j, i \qquad (V)$$

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