Importance of Total Coupon in Utility Maximization: A Sensitivity Analysis

Mohajan, Devajit and Mohajan, Haradhan

Department of Civil Engineering, Chittagong University of Engineering Technology, Chittagong, Bangladesh, Department of Mathematics, Premier University, Chittagong, Bangladesh

7 September 2022

Importance of Total Coupon in Utility Maximization: A Sensitivity Analysis

Devajit Mohajan

Department of Civil Engineering, Chittagong University of Engineering & Technology, Chittagong, Bangladesh

Email: devajit1402@gmail.com

Mobile: +8801866207021

Haradhan Kumar Mohajan

Department of Mathematics, Premier University, Chittagong, Bangladesh

Email: haradhan1971@gmail.com

Mobile: +8801716397232

Abstract

This study takes an attempt to discuss utility maximization policies. The property of a commodity that enables it to satisfy human wants is called utility. The sensitivity analysis is included in the operation to show optimal policy of an organization. This study deals with four commodities and two constraints, such as budget constraint, and coupon constraint. The economic predictions of future production are necessary for the sustainable production. In the study Lagrange multipliers technique is applied to operate 6×6 bordered Hessian and 6×10 Jacobian appropriately. The sensitivity analysis between commodity and total budget are discussed with detail mathematical analysis.

Keywords: Lagrange multipliers, sensitivity analysis, total coupon, utility maximization

1. Introduction

In the 21st century, global economy becomes challenging, and mathematical economics plays a crucial role for the development of economic structures; especially for developing mathematical models (Carter, 2001). In mathematical economics, utility is an important portion for the organizations, and production of commodities depends on the satisfaction of the consumers (Fishburn, 1970). In the society, consumers always expect to gain maximum satisfaction from the consumption of their purchased goods (Stigler, 1950). Utility shows that individuals in the societies seek to obtain the highest level of satisfaction from their purchasing goods (Kirsh, 2017).

The concept of utility was developed in the late 18^{th} century by the English moral philosopher, jurist, and social reformer Jeremy Bentham (1748-1832) and English philosopher, political economist, Member of Parliament (MP) and civil servant John Stuart Mill (1806-1873) (Bentham, 1780). Since economy sees its benefits and also sees welfare of human; therefore, utility maximization is a blessing both for humankind and the organization (Eaton & Lipsey, 1975). The organizations always try to maximize their profits; and they must be conscious about the utility maximization of the consumers (Dixit, 1990).

The method of Lagrange multipliers is a very useful and powerful technique in multivariate calculus, which transforms a constrained problem to a higher dimensional unconstrained problem (Islam et al., 2011; Mohajan, 2022). In this study, we consider a utility maximization problem subject to two constraints; namely, budget constraint and coupon constraint. We have stressed on the sensitivity analysis between commodity and total coupon (Mohajan & Mohajan, 2022c,d). We have used both 6×6 bordered Hessian and 6×10 Jacobian for developing utility maximization (Mohajan & Mohajan, 2022a,e).

2. Literature Review

The literature review section is an introductory region of research, which shows the works of previous researchers in the same field within the existing knowledge (Polit & Hungler, 2013). Two US scholars, mathematician John V. Baxley and economist John C. Moorhouse have discussed the utility maximization method through the mathematical formulation (Baxley & Moorhouse, 1984). Eminent mathematician Jamal Nazrul Islam and his coauthors have discussed utility maximization and some other optimization problems. In these studies they have considered reasonable interpretation of the Lagrange multipliers (Islam et al., 2010, 2011). Pahlaj Moolio and his coauthors have also taken attempts to develop and solve optimization problems (Moolio et al., 2009).

Jannatul Ferdous and Haradhan Kumar Mohajan have taken attempts to calculate profit maximization problems (Ferdous & Mohajan, 2022). Devajit Mohajan and Haradhan Kumar Mohajan have discussed on profit maximization by using four variable inputs, such as capital, labor, principal raw materials, and other inputs in an industry (Mohajan & Mohajan, 2022a). Haradhan Kumar Mohajan has also used three inputs, such as capital, labor and other inputs for the sustainable production of a factory of Bangladesh (Mohajan, 2021b).

3. Methodology of the Study

Researchers often write a methodology section with details of the research analysis. It is considered as a way of explaining how a research work is carried out. Therefore, it is the organized and meaningful procedural works that follow scientific methods efficiently (Kothari, 2008). Research methodology shows the ways to the researchers for organizing, planning, designing and conducting a good research (Legesse, 2014). It helps to identify research areas and projects within these areas (Blessing et al., 1998). In this article we have used both qualitative and quantitative research procedures (Mohajan, 2018b, 2020). In this study we have considered four commodity variables: Y_1, Y_2, Y_3, and Y_4; and two Lagrange multipliers λ_1 and λ_2. We have used 6×6 bordered Hessian, and later 6×10 Jacobian to discuss sensitivity analysis (Mohajan & Mohajan, 2022e).

Both reliability and validity are powerful tools for doing a seminal research work. In this article we have tried to maintain them as far as possible (Mohajan, 2017b). To prepare this article we have depended on the optimization related mathematical secondary data sources (Mohajan, 2011, 2018a; Islam et al., 2012a,b). We have consulted with the published research papers, books of well-known authors, handbooks of expert researchers, research reports, internet, websites, etc. (Mohajan, 2012; Chowdhury et al., 2013; Mohajan & Mohajan, 2022f).

4. Objective of the Study

Main objective of this study is to discuss the usefulness of total coupon during the utility maximization, where sensitivity analysis is investigated. The other subsidiary objectives are as follows:

- to use the bordered Hessian and Jacobian for optimization,
- to explain the results properly, and
- to show the mathematical calculations in some details.

5. An Economic Model

In this study we consider four commodities: Y_1, Y_2, Y_3, and Y_4 (Moolio et al., 2009; Mohajan & Mohajan, 2022b). Let a consumer wants to purchase y_1, y_2, y_3, and y_4 amounts from these four commodities Y_1, Y_2, Y_3, and Y_4, respectively. The utility function can be written as (Roy et al., 2021; Mohajan & Mohajan, 2022b),

$$u(y_1, y_2, y_3, y_4) = y_1 y_2 y_3 y_4 . \tag{1}$$

The budget constraint of the consumers is,

$$B(y_1, y_2, y_3, y_4) = p_1 y_1 + p_2 y_2 + p_3 y_3 + p_4 y_4 \tag{2}$$

where p_1, p_2, p_3, and p_4 are the prices of per unit of commodities y_1, y_2, y_3, and y_4, respectively. Now the coupon constraint is,

$$M(y_1, y_2, y_3, y_4) = m_1 y_1 + m_2 y_2 + m_3 y_3 + m_4 y_4 , \tag{3}$$

where m_1, m_2, m_3, and m_4 are the coupons necessary to purchase a unit of commodity of y_1, y_2, y_3, and y_4, respectively.

Using (1), (2), and (3) we can express Lagrangian function $V(y_1, y_2, y_3, y_4, \lambda_1, \lambda_2)$ as (Baxley & Moorhouse, 1984; Ferdous & Mohajan, 2022; Mohajan & Mohajan, 2022b),

$$\begin{aligned} V(y_1, y_2, y_3, y_4, \lambda_1, \lambda_2) &= y_1 y_2 y_3 y_4 + \lambda_1 (B - p_1 y_1 - p_2 y_2 - p_3 y_3 - p_4 y_4) \\ &\quad + \lambda_2 (M - m_1 y_1 - m_2 y_2 - m_3 y_3 - m_4 y_4). \end{aligned} \tag{4}$$

Lagrangian function (4) is a 6-dimensional unconstrained problem that maximizes utility functions; where λ_1 and λ_2 are two Lagrange multipliers.

Now we consider the bordered Hessian (Mohajan, 2021a; Mohajan & Mohajan, 2022c),

$$|H| = \begin{vmatrix} 0 & 0 & -B_1 & -B_2 & -B_3 & -B_4 \\ 0 & 0 & -M_1 & -M_2 & -M_3 & -M_4 \\ -B_1 & -M_1 & V_{11} & V_{12} & V_{13} & V_{14} \\ -B_2 & -M_2 & V_{21} & V_{22} & V_{23} & V_{24} \\ -B_3 & -M_3 & V_{31} & V_{32} & V_{33} & V_{34} \\ -B_4 & -M_4 & V_{41} & V_{42} & V_{43} & V_{44} \end{vmatrix} . \tag{5}$$

Now taking first and second order and cross-partial derivatives in (4) we obtain (Islam et al. 2009a,b; Mohajan & Mohajan, 2022d);

$$B_1 = p_1, \qquad B_2 = p_2, \qquad B_3 = p_3, \qquad B_4 = p_4 .$$

$$M_1 = m_1, \qquad M_2 = m_2, \qquad M_3 = m_3, \qquad M_4 = m_4 . \tag{6}$$

$V_{11}=0$, $\quad V_{12}=V_{21}=y_3y_4$, $\quad V_{13}=V_{31}=y_2y_4$,

$V_{14}=V_{41}=y_2y_3$, $\quad V_{22}=0$, $\quad V_{23}=V_{32}=y_1y_4$,

$V_{24}=V_{42}=y_1y_3$, $\quad V_{33}=0$, $\quad V_{34}=V_{43}=y_1y_2$, $\quad V_{44}=0$. (7)

We use $p_3=p_1$ and $p_4=p_2$, i.e., a pair of prices are same, and $m_3=m_1$ and $m_4=m_2$, i.e., a pair of coupon numbers are same. Now we consider that in the expansion of (5) every term contains $p_1p_2m_1m_2$, then from (5) we can derive (Mohajan & Mohajan, 2022e);

$$|H|=-2p_1p_2m_1m_2<0. \tag{8}$$

For y_1, y_2, y_3, y_4, λ_1, and λ_2 in terms of p_1, p_2, p_3, p_4, m_1, m_2, m_3, m_4, B, and M we can calculate sixty partial derivatives, such as $\frac{\partial\lambda_1}{\partial p_1}$, $\frac{\partial\lambda_2}{\partial p_1}$, …, $\frac{\partial\lambda_1}{\partial m_1}$, $\frac{\partial\lambda_2}{\partial m_1}$, …, $\frac{\partial y_1}{\partial p_1}$, …, $\frac{\partial y_1}{\partial m_1}$, …, $\frac{\partial\lambda_1}{\partial B}$, …, $\frac{\partial\lambda_1}{\partial M}$, etc., (Islam et al., 2010; Mohajan, 2021c). Now we consider 6×6 Hessian and Jacobian matrix as (Mohajan & Mohajan, 2022a; Mohajan, 2021b)

$$J=H=\begin{bmatrix} 0 & 0 & -B_1 & -B_2 & -B_3 & -B_4 \\ 0 & 0 & -M_1 & -M_2 & -M_3 & -M_4 \\ -B_1 & -M_1 & V_{11} & V_{12} & V_{13} & V_{14} \\ -B_2 & -M_2 & V_{21} & V_{22} & V_{23} & V_{24} \\ -B_3 & -M_3 & V_{31} & V_{32} & V_{33} & V_{34} \\ -B_4 & -M_4 & V_{41} & V_{42} & V_{43} & V_{44} \end{bmatrix} \tag{9}$$

which is non-singular at the optimum point $\left(y_1^*, y_2^*, y_3^*, y_4^*, \lambda_1^*, \lambda_2^*\right)$. Since the second order conditions have been satisfied, so the determinant of (9) does not vanish at the optimum, i.e., $|J|=|H|$; and we apply the implicit-function theorem. We have total 16 variables in our study, such as λ_1, λ_2, y_1, y_2, y_3, y_4 p_1, p_2, p_3, p_4, m_1, m_2, m_3, m_4, B, and M. By the implicit function theorem, we can write (Moolio et al., 2009; Islam et al., 2011; Mohajan, 2021c);

$$\begin{bmatrix} \lambda_1 \\ \lambda_2 \\ y_1 \\ y_2 \\ y_3 \\ y_4 \end{bmatrix}=\mathbf{G}\left(p_1, p_2, p_3, p_4, m_1, m_2, m_3, m_4, B, M\right). \tag{10}$$

Now the 6×10 Jacobian matrix for $\mathbf{G}$, regarded as J_G is given by (Mohajan et al., 2013; Mohajan, 2021a),

$$J_G = \begin{bmatrix} \frac{\partial \lambda_1}{\partial p_1} & \frac{\partial \lambda_1}{\partial p_2} & \frac{\partial \lambda_1}{\partial p_3} & \frac{\partial \lambda_1}{\partial p_4} & \frac{\partial \lambda_1}{\partial m_1} & \frac{\partial \lambda_1}{\partial m_2} & \frac{\partial \lambda_1}{\partial m_3} & \frac{\partial \lambda_1}{\partial m_4} & \frac{\partial \lambda_1}{\partial B} & \frac{\partial \lambda_1}{\partial M} \\ \frac{\partial \lambda_2}{\partial p_1} & \frac{\partial \lambda_2}{\partial p_2} & \frac{\partial \lambda_2}{\partial p_3} & \frac{\partial \lambda_2}{\partial p_4} & \frac{\partial \lambda_2}{\partial m_1} & \frac{\partial \lambda_2}{\partial m_2} & \frac{\partial \lambda_2}{\partial m_3} & \frac{\partial \lambda_2}{\partial m_4} & \frac{\partial \lambda_2}{\partial B} & \frac{\partial \lambda_2}{\partial M} \\ \frac{\partial y_1}{\partial p_1} & \frac{\partial y_1}{\partial p_2} & \frac{\partial y_1}{\partial p_3} & \frac{\partial y_1}{\partial p_4} & \frac{\partial y_1}{\partial m_1} & \frac{\partial y_1}{\partial m_2} & \frac{\partial y_1}{\partial m_3} & \frac{\partial y_1}{\partial m_4} & \frac{\partial y_1}{\partial B} & \frac{\partial y_1}{\partial M} \\ \frac{\partial y_2}{\partial p_1} & \frac{\partial y_2}{\partial p_2} & \frac{\partial y_2}{\partial p_3} & \frac{\partial y_2}{\partial p_4} & \frac{\partial y_2}{\partial m_1} & \frac{\partial y_2}{\partial m_2} & \frac{\partial y_2}{\partial m_3} & \frac{\partial y_2}{\partial m_4} & \frac{\partial y_2}{\partial B} & \frac{\partial y_2}{\partial M} \\ \frac{\partial y_3}{\partial p_1} & \frac{\partial y_3}{\partial p_2} & \frac{\partial y_3}{\partial p_3} & \frac{\partial y_3}{\partial p_4} & \frac{\partial y_3}{\partial m_1} & \frac{\partial y_3}{\partial m_2} & \frac{\partial y_3}{\partial m_3} & \frac{\partial y_3}{\partial m_4} & \frac{\partial y_3}{\partial B} & \frac{\partial y_3}{\partial M} \\ \frac{\partial y_4}{\partial p_1} & \frac{\partial y_4}{\partial p_2} & \frac{\partial y_4}{\partial p_3} & \frac{\partial y_4}{\partial p_4} & \frac{\partial y_4}{\partial m_1} & \frac{\partial y_4}{\partial m_2} & \frac{\partial y_4}{\partial m_3} & \frac{\partial y_4}{\partial m_4} & \frac{\partial y_4}{\partial B} & \frac{\partial y_4}{\partial M} \end{bmatrix}. \tag{11}$$

$$= -J^{-1} \begin{bmatrix} -y_1 & -y_2 & -y_3 & -y_4 & 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & -y_1 & -y_2 & -y_3 & -y_4 & 0 & 1 \\ -\lambda_1 & 0 & 0 & 0 & -\lambda_2 & 0 & 0 & 0 & 0 & 0 \\ 0 & -\lambda_1 & 0 & 0 & 0 & -\lambda_2 & 0 & 0 & 0 & 0 \\ 0 & 0 & -\lambda_1 & 0 & 0 & 0 & -\lambda_2 & 0 & 0 & 0 \\ 0 & 0 & 0 & -\lambda_1 & 0 & 0 & 0 & -\lambda_2 & 0 & 0 \end{bmatrix}. \tag{12}$$

The inverse of Jacobian is, $J^{-1} = \frac{1}{|J|} C^T$, where $C = \left(C_{ij}\right)$, the matrix of cofactors of J, and T indicates transpose, then (12) becomes (Mohajan, 2017a; Islam et al., 2009b, 2011),

$$J_G = -\frac{1}{|J|} C^T \begin{bmatrix} -y_1 & -y_2 & -y_3 & -y_4 & 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & -y_1 & -y_2 & -y_3 & -y_4 & 0 & 1 \\ -\lambda_1 & 0 & 0 & 0 & -\lambda_2 & 0 & 0 & 0 & 0 & 0 \\ 0 & -\lambda_1 & 0 & 0 & 0 & -\lambda_2 & 0 & 0 & 0 & 0 \\ 0 & 0 & -\lambda_1 & 0 & 0 & 0 & -\lambda_2 & 0 & 0 & 0 \\ 0 & 0 & 0 & -\lambda_1 & 0 & 0 & 0 & -\lambda_2 & 0 & 0 \end{bmatrix}. \tag{13}$$

Now 6×6 transpose matrix C^T can be represented by,

$$C^T = \begin{bmatrix} C_{11} & C_{21} & C_{31} & C_{41} & C_{51} & C_{61} \\ C_{12} & C_{22} & C_{32} & C_{42} & C_{52} & C_{62} \\ C_{13} & C_{23} & C_{33} & C_{43} & C_{53} & C_{63} \\ C_{14} & C_{24} & C_{34} & C_{44} & C_{54} & C_{64} \\ C_{15} & C_{25} & C_{35} & C_{45} & C_{55} & C_{65} \\ C_{16} & C_{26} & C_{36} & C_{46} & C_{56} & C_{66} \end{bmatrix}. \tag{14}$$

Using (14) we can write (11) as a 6×10 Jacobian matrix (Mohajan & Mohajan, 2022b);

$$J_G=-\frac{1}{|J|}\begin{bmatrix} -y_1C_{11}-\lambda_1C_{31} & -y_2C_{11}-\lambda_1C_{41} & -y_3C_{11}-\lambda_1C_{51} & -y_4C_{11}-\lambda_1C_{61} & -y_1C_{21}-\lambda_2C_{31} \\ -y_1C_{12}-\lambda_1C_{32} & -y_2C_{12}-\lambda_1C_{42} & -y_3C_{12}-\lambda_1C_{52} & -y_4C_{12}-\lambda_1C_{62} & -y_1C_{22}-\lambda_2C_{32} \\ -y_1C_{13}-\lambda_1C_{33} & -y_2C_{13}-\lambda_1C_{43} & -y_3C_{13}-\lambda_1C_{53} & -y_4C_{13}-\lambda_1C_{63} & -y_1C_{23}-\lambda_2C_{33} \\ -y_1C_{14}-\lambda_1C_{34} & -y_2C_{14}-\lambda_1C_{44} & -y_3C_{14}-\lambda_1C_{54} & -y_4C_{14}-\lambda_1C_{64} & -y_1C_{24}-\lambda_2C_{34} \\ -y_1C_{15}-\lambda_1C_{35} & -y_2C_{15}-\lambda_1C_{45} & -y_3C_{15}-\lambda_1C_{55} & -y_4C_{15}-\lambda_1C_{65} & -y_1C_{25}-\lambda_2C_{35} \\ -y_1C_{16}-\lambda_1C_{36} & -y_2C_{16}-\lambda_1C_{46} & -y_3C_{16}-\lambda_1C_{56} & -y_4C_{16}-\lambda_1C_{66} & -y_1C_{26}-\lambda_2C_{36} \end{matrix}$$

$$\begin{matrix} -y_2C_{21}-\lambda_2C_{41} & -y_3C_{21}-\lambda_2C_{51} & -y_4C_{21}-\lambda_2C_{61} & C_{11} & C_{21} \\ -y_2C_{22}-\lambda_2C_{42} & -y_3C_{22}-\lambda_2C_{52} & -y_4C_{22}-\lambda_2C_{62} & C_{12} & C_{22} \\ -y_2C_{23}-\lambda_2C_{43} & -y_3C_{23}-\lambda_2C_{53} & -y_4C_{23}-\lambda_2C_{63} & C_{13} & C_{23} \\ -y_2C_{24}-\lambda_2C_{44} & -y_3C_{24}-\lambda_2C_{54} & -y_4C_{24}-\lambda_2C_{64} & C_{14} & C_{24} \\ -y_2C_{25}-\lambda_2C_{45} & -y_3C_{25}-\lambda_2C_{55} & -y_4C_{25}-\lambda_2C_{65} & C_{15} & C_{25} \\ -y_2C_{26}-\lambda_2C_{46} & -y_3C_{26}-\lambda_2C_{56} & -y_4C_{26}-\lambda_2C_{66} & C_{16} & C_{26} \end{bmatrix}. \qquad (15)$$

Now we analyze the nature of consumption of commodity y_1 when total coupon M increases. Taking $T_{3(10)}$, (i.e., term of 3rd row and 10th column) from both sides of (15) we get (Islam et al., 2011; Mohajan & Mohajan, 2022e),

$$\frac{\partial y_1}{\partial M}=-\frac{1}{|J|}\left[C_{23}\right]$$

$$=-\frac{1}{|J|}\text{Cofactor of } C_{23}$$

$$=\frac{1}{|J|}\begin{vmatrix} 0 & 0 & -B_2 & -B_3 & -B_4 \\ -B_1 & -M_1 & V_{12} & V_{13} & V_{14} \\ -B_2 & -M_2 & V_{22} & V_{23} & V_{24} \\ -B_3 & -M_3 & V_{32} & V_{33} & V_{34} \\ -B_4 & -M_4 & V_{42} & V_{43} & V_{44} \end{vmatrix}$$

$$=\frac{1}{|J|}\left\{-B_2\begin{vmatrix} -B_1 & -M_1 & V_{13} & V_{14} \\ -B_2 & -M_2 & V_{23} & V_{24} \\ -B_3 & -M_3 & V_{33} & V_{34} \\ -B_4 & -M_4 & V_{43} & V_{44} \end{vmatrix}+B_3\begin{vmatrix} -B_1 & -M_1 & V_{12} & V_{14} \\ -B_2 & -M_2 & V_{22} & V_{24} \\ -B_3 & -M_3 & V_{32} & V_{34} \\ -B_4 & -M_4 & V_{42} & V_{44} \end{vmatrix}-B_4\begin{vmatrix} -B_1 & -M_1 & V_{12} & V_{13} \\ -B_2 & -M_2 & V_{22} & V_{23} \\ -B_3 & -M_3 & V_{32} & V_{33} \\ -B_4 & -M_4 & V_{42} & V_{43} \end{vmatrix}\right\}$$

$$=\frac{1}{|J|}\left[-B_2\left\{-B_1\begin{vmatrix} -M_2 & V_{23} & V_{24} \\ -M_3 & V_{33} & V_{34} \\ -M_4 & V_{43} & V_{44} \end{vmatrix}+M_1\begin{vmatrix} -B_2 & V_{23} & V_{24} \\ -B_3 & V_{33} & V_{34} \\ -B_4 & V_{43} & V_{44} \end{vmatrix}+V_{13}\begin{vmatrix} -B_2 & -M_2 & V_{24} \\ -B_3 & -M_3 & V_{34} \\ -B_4 & -M_4 & V_{44} \end{vmatrix}-V_{14}\begin{vmatrix} -B_2 & -M_2 & V_{23} \\ -B_3 & -M_3 & V_{33} \\ -B_4 & -M_4 & V_{43} \end{vmatrix}\right\}\right.$$

$$+B_3\left\{-B_1\begin{vmatrix} -M_2 & V_{22} & V_{24} \\ -M_3 & V_{32} & V_{34} \\ -M_4 & V_{42} & V_{44} \end{vmatrix}+M_1\begin{vmatrix} -B_2 & V_{22} & V_{24} \\ -B_3 & V_{32} & V_{34} \\ -B_4 & V_{42} & V_{44} \end{vmatrix}+V_{12}\begin{vmatrix} -B_2 & -M_2 & V_{24} \\ -B_3 & -M_3 & V_{34} \\ -B_4 & -M_4 & V_{44} \end{vmatrix}-V_{14}\begin{vmatrix} -B_2 & -M_2 & V_{22} \\ -B_3 & -M_3 & V_{32} \\ -B_4 & -M_4 & V_{42} \end{vmatrix}\right\}$$

$$-B_4\left\{-B_1\begin{vmatrix}-M_2 & V_{22} & V_{23}\\ -M_3 & V_{32} & V_{33}\\ -M_4 & V_{42} & V_{43}\end{vmatrix}+M_1\begin{vmatrix}-B_2 & V_{22} & V_{23}\\ -B_3 & V_{32} & V_{33}\\ -B_4 & V_{42} & V_{43}\end{vmatrix}+V_{12}\begin{vmatrix}-B_2 & -M_2 & V_{23}\\ -B_3 & -M_3 & V_{33}\\ -B_4 & -M_4 & V_{43}\end{vmatrix}-V_{13}\begin{vmatrix}-B_2 & -M_2 & V_{22}\\ -B_3 & -M_3 & V_{32}\\ -B_4 & -M_4 & V_{42}\end{vmatrix}\right\}\Bigg]$$

$$\begin{aligned}
=\frac{1}{|J|}\Big[& B_1B_2M_2V_{34}^2 - B_1B_2M_4V_{23}V_{34} - B_1B_2M_3V_{24}V_{34} - B_2^2M_1V_{34}^2 + B_2B_4M_1V_{23}V_{34} + B_2B_3M_1V_{24}V_{34} \\
& + B_2^2M_4V_{13}V_{34} - B_2B_4M_2V_{13}V_{34} - B_2B_3M_4V_{13}V_{24} + B_2B_4M_3V_{13}V_{24} - B_2^2M_3V_{14}V_{34} - B_2B_3M_2V_{14}V_{34} \\
& + B_2B_3M_4V_{14}V_{23} - B_2B_4M_3V_{14}V_{23} - B_1B_3M_2V_{24}V_{34} + B_1B_3M_3V_{24}^2 - B_1B_3M_4V_{23}V_{24} + B_2B_3M_1V_{24}V_{34} \\
& - B_3^2M_1V_{24}^2 + B_3B_4M_1V_{23}V_{24} - B_2B_3M_4V_{12}V_{34} - B_3B_4M_3V_{12}V_{24} + B_3^2M_4V_{12}V_{24} - B_3B_4M_3V_{12}V_{24} \\
& - B_2B_3M_3V_{14}V_{24} + B_2B_3M_4V_{14}V_{23} + B_3^2M_2V_{14}V_{24} - B_3B_4M_2V_{14}V_{23} - B_1B_4M_2V_{23}V_{34} - B_1B_4M_3V_{23}V_{24} \\
& + B_1B_4M_4V_{23}^2 + B_2B_4M_1V_{23}V_{34} + B_4^2M_1V_{23}^2 - B_2B_4M_3V_{12}V_{34} - B_3B_4M_2V_{12}V_{34} - B_3B_4M_4V_{12}V_{23} \\
& + B_4^2M_3V_{12}V_{23} + B_2B_4M_3V_{13}V_{24} - B_2B_4M_4V_{13}V_{23} - B_3B_4M_2V_{13}V_{24} + B_4^2M_2V_{13}V_{23}\Big]
\end{aligned}$$

$$\begin{aligned}
=\frac{1}{|J|}\Big[& p_1p_2m_2y_1^2y_2^2 - p_1p_2m_4y_1^2y_2y_4 - p_1p_2m_3y_1^2y_2y_3 - p_2^2m_1y_1^2y_2^2 + p_2p_4m_1y_1^2y_2y_4 + p_2p_3m_1y_1^2y_2y_3 \\
& + p_2^2m_4y_1y_2^2y_4 - p_2p_4m_2y_1y_2^2y_4 - p_2p_3m_4y_1y_2y_3y_4 + p_2p_4m_3y_1y_2y_3y_4 - p_2^2m_3y_1y_2^2y_3 - p_2p_4m_2y_1y_2^2y_3 \\
& + p_2p_3m_4y_1y_2y_3y_4 - p_2p_4m_3y_1y_2y_3y_4 - p_1p_3m_3y_1^2y_2y_3 + p_1p_3m_3y_1^2y_3^2 - p_1p_3m_4y_1^2y_3y_4 \\
& + p_2p_3m_1y_1^2y_2y_3 - p_3^2m_1y_1^2y_3^2 + p_3p_4m_1y_1^2y_3y_4 - p_2p_3m_4y_1y_2y_3y_4 - p_3p_4m_3y_1y_3^2y_4 + p_3^2m_4y_1y_3^2y_4 \\
& - p_3p_4m_3y_1y_3^2y_4 - p_2p_3m_3y_1y_2y_3^2 + p_2p_3m_4y_1y_2y_3y_4 + p_3^2m_2y_1y_2y_3^2 - p_3p_4m_2y_1y_2y_3y_4 \\
& - p_1p_4m_2y_1^2y_2y_4 - p_1p_4m_3y_1^2y_3y_4 + p_1p_4m_3y_1^2y_4^2 - p_2p_4m_1y_1^2y_2y_4 + p_3p_4m_1y_1^2y_3y_4 + p_4^2m_1y_1^2y_4^2 \\
& - p_2p_4m_3y_1y_2y_3y_4 - p_3p_4m_2y_1y_2y_3y_4 - p_3p_4m_4y_1y_3y_4^2 + p_4^2m_3y_1y_3y_4^2 + p_2p_4m_3y_1y_2y_3y_4 \\
& - p_2p_4m_4y_1y_2y_4^2 - p_3p_4m_2y_1y_2y_3y_4 + p_4^2m_2y_1y_2y_4^2\Big].
\end{aligned} \tag{16}$$

$$\begin{aligned}
\frac{\partial y_1}{\partial M}=\frac{1}{|J|}\Big[& \left(p_1p_2m_2-p_2^2m_1\right)y_1^2y_2^2+\left(p_1p_3m_3-p_3^2m_1\right)y_1^2y_3^2+\left(p_1p_4m_3+p_4^2m_1\right)y_1^2y_4^2 \\
& +\left(-p_1p_2m_4-p_1p_4y_2\right)y_1^2y_2y_4+\left(p_2p_3m_1-p_1p_2m_3-p_1p_3m_3+p_2p_3m_1\right)y_1^2y_2y_3 \\
& +\left(p_2^2m_4-p_2p_4m_2\right)y_1y_2^2y_4+\left(-p_2^2m_3-p_2p_4m_2\right)y_1y_2^2y_3+\left(p_3^2m_2-p_2p_3m_3\right)y_1y_2y_3^2 \\
& +\left(p_3^2m_4-p_3p_4m_3-p_3p_4m_3\right)y_1y_3^2y_4+\left(2p_3p_4m_1-p_1p_3m_4-p_1p_4m_3\right)y_1^2y_3y_4 \\
& -3p_3p_4m_2y_1y_2y_3y_4+\left(p_4^2m_2-p_2p_4m_4\right)y_1y_2y_4^2\Big].
\end{aligned} \tag{17}$$

Using $y_1=y_2=y_3=y_4=1$ in (17) we get,

$$\begin{aligned}
\frac{\partial y_1}{\partial M}=\frac{1}{|J|}\Big[& \left(p_1p_2m_2+p_4^2m_1+2p_2p_3m_1+p_2^2m_4+p_3^2m_2+2p_3p_4m_1+p_4^2m_2\right)-\left(p_3^2m_1+p_2^2m_1+3p_3p_4m_2\right. \\
& +p_1p_2m_4+p_1p_4m_2+p_1p_2m_3+2p_2p_4m_2+p_2^2m_3+p_2p_3m_3+p_3^2m_4+2p_3p_4m_3 \\
& \left.+p_1p_3m_4+p_2p_4m_4\right)\Big].
\end{aligned} \tag{18}$$

We consider, $p_1=p_2=p_3=p_4=p$, and $m_1=m_2=m_3=m_4=m$, then $|J|=-2p^2m^2$; and (18) gives,

$$\frac{\partial y_1}{\partial M}=-\frac{4}{m}<0, \tag{19}$$

where $m > 0$.

Inequality (19) indicates that when the quantity of total coupon to purchase commodity y_1 increases, the level of consumption of the commodity y_1 decreases. This situation indicates that commodity y_1 is an inferior good.

We consider $p_3 = p_1$, and $p_4 = p_2$; and $m_3 = m_1$, and $m_4 = m_2$, then $|J| = |H| = -2p_1p_2m_1m_2$, and from (16) we get,

$$\frac{\partial m_1}{\partial M} = \frac{p_1^2m_1 + p_2^2m_1 + 4p_1p_2m_2 + p_1^2m_2 + p_2^2m_2}{2p_1p_2m_1m_2} > 0, \tag{20}$$

where $p_1, p_2, m_1, m_2 > 0$.

Inequality (20) indicates that if the total coupon of individual/community increases, the level of consumption of commodity y_1 will also increases. We believe that commodity y_1 is not an inferior good; it may be a superior good, and it has no supplementary goods (Islam et al., 2010; Mohajan, 2021b; Mohajan & Mohajan, 2022c).

Now we analyze the nature of consumption of commodity y_2 when the total coupon M increases. Taking $T_{4(10)}$, (i.e., term of 4th row and 10th column) from both sides of (11) and (15) we get (Islam et al., 2010; Mohajan & Mohajan, 2022c,e),

$$\frac{\partial y_2}{\partial M} = -\frac{1}{|J|}[C_{24}]$$

$$= -\frac{1}{|J|}\text{Cofactor of } C_{24}$$

$$= -\frac{1}{|J|}\begin{vmatrix} 0 & 0 & -B_1 & -B_3 & -B_4 \\ -B_1 & -M_1 & V_{11} & V_{13} & V_{14} \\ -B_2 & -M_2 & V_{21} & V_{23} & V_{24} \\ -B_3 & -M_3 & V_{31} & V_{33} & V_{34} \\ -B_4 & -M_4 & V_{41} & V_{43} & V_{44} \end{vmatrix}$$

$$= -\frac{1}{|J|}\left\{-B_1\begin{vmatrix} -B_1 & -M_1 & V_{13} & V_{14} \\ -B_2 & -M_2 & V_{23} & V_{24} \\ -B_3 & -M_3 & V_{33} & V_{34} \\ -B_4 & -M_4 & V_{43} & V_{44} \end{vmatrix} + B_3\begin{vmatrix} -B_1 & -M_1 & V_{11} & V_{14} \\ -B_2 & -M_2 & V_{21} & V_{24} \\ -B_3 & -M_3 & V_{31} & V_{34} \\ -B_4 & -M_4 & V_{41} & V_{44} \end{vmatrix} - B_4\begin{vmatrix} -B_1 & -M_1 & V_{11} & V_{13} \\ -B_2 & -M_2 & V_{21} & V_{23} \\ -B_3 & -M_3 & V_{31} & V_{33} \\ -B_4 & -M_4 & V_{41} & V_{43} \end{vmatrix}\right\}$$

$$= -\frac{1}{|J|}\left[-B_1\left\{-B_1\begin{vmatrix} -M_2 & V_{23} & V_{24} \\ -M_3 & V_{33} & V_{34} \\ -M_4 & V_{43} & V_{44} \end{vmatrix} + M_1\begin{vmatrix} -B_2 & V_{23} & V_{24} \\ -B_3 & V_{33} & V_{34} \\ -B_4 & V_{43} & V_{44} \end{vmatrix} + V_{13}\begin{vmatrix} -B_2 & -M_2 & V_{24} \\ -B_3 & -M_3 & V_{34} \\ -B_4 & -M_4 & V_{44} \end{vmatrix} - V_{14}\begin{vmatrix} -B_2 & -M_2 & V_{23} \\ -B_3 & -M_3 & V_{33} \\ -B_4 & -M_4 & V_{43} \end{vmatrix}\right\}\right.$$

$$+B_3\left\{-B_1\begin{vmatrix}-M_2 & V_{21} & V_{24}\\ -M_3 & V_{31} & V_{34}\\ -M_4 & V_{41} & V_{44}\end{vmatrix}+M_1\begin{vmatrix}-B_2 & V_{21} & V_{24}\\ -B_3 & V_{31} & V_{34}\\ -B_4 & V_{41} & V_{44}\end{vmatrix}-V_{14}\begin{vmatrix}-B_2 & -M_2 & V_{21}\\ -B_3 & -M_3 & V_{31}\\ -B_4 & -M_4 & V_{41}\end{vmatrix}\right\}-B_4\left\{-B_1\begin{vmatrix}-M_2 & V_{21} & V_{23}\\ -M_3 & V_{31} & V_{33}\\ -M_4 & V_{41} & V_{43}\end{vmatrix}\right.$$

$$\left.+M_1\begin{vmatrix}-B_2 & V_{21} & V_{23}\\ -B_3 & V_{31} & V_{33}\\ -B_4 & V_{41} & V_{43}\end{vmatrix}-V_{13}\begin{vmatrix}-B_2 & -M_2 & V_{21}\\ -B_3 & -M_3 & V_{31}\\ -B_4 & -M_4 & V_{41}\end{vmatrix}\right\}$$

$$=-\frac{1}{|J|}\Big[B_1^2M_2V_{34}^2 - B_1^2M_4V_{23}V_{34} - B_1^2M_3V_{24}V_{34} - B_1B_2M_1V_{34}^2 + B_1B_4M_1V_{23}V_{34} + B_1B_3M_1V_{23}V_{34}$$
$$- B_1B_2M_4V_{13}V_{34} - B_1B_4M_2V_{13}V_{34} + B_1B_3M_4V_{13}V_{24} - B_1B_4M_3V_{13}V_{24} + B_1B_2M_3V_{14}V_{34} - B_1B_3M_2V_{14}V_{34}$$
$$+ B_1B_3M_4V_{14}V_{23} + B_1B_4M_3V_{14}V_{23} - B_1B_3M_2V_{14}V_{34} + B_1B_3M_4V_{12}V_{34} + B_1B_3M_3V_{14}V_{24} - B_1B_3M_4V_{13}V_{24}$$
$$+ B_2B_3M_1V_{14}V_{34} - B_3B_4M_1V_{12}V_{34} - B_3^2M_1V_{14}V_{24} - B_1B_4M_1V_{13}V_{24} - B_2B_3M_3V_{14}^2 + B_2B_3M_4V_{13}V_{14}$$
$$+ B_3^2M_2V_{14}^2 - B_3B_4M_2V_{13}V_{14} - B_1B_4M_2V_{13}V_{34} + B_1B_4M_3V_{12}V_{34} - B_1B_4M_3V_{14}V_{23} + B_1B_4M_4V_{13}V_{23}$$
$$+ B_2B_4M_1V_{13}V_{34} - B_3B_4M_1V_{12}V_{34} + B_3B_4M_1V_{14}V_{23} - B_4^2M_1V_{13}V_{23} + B_2B_4M_3V_{13}V_{14} - B_2B_4M_4V_{13}^2$$
$$- B_3B_4M_2V_{13}V_{14} + B_4^2M_2V_{13}^2 + B_3B_4M_4V_{12}V_{13} - B_4^2M_3V_{12}V_{13}\Big]$$

$$=-\frac{1}{|J|}\Big[p_1^2m_2y_1^2y_2^2 - p_1^2m_4y_1^2y_2y_4 - p_1p_2m_1y_1^2y_2^2 + p_1p_4m_1y_1^2y_2y_4 + p_1p_3m_1y_1^2y_2y_4 - p_1p_2m_4y_1y_2^2y_4$$
$$- p_1p_4m_2y_1y_2^2y_4 + p_1p_3m_4y_1^2y_2y_3 - p_1p_4m_3y_1y_2y_3y_4 + p_1p_2m_3y_1y_2^2y_3 + p_1p_3k_2y_1y_2^2y_3$$
$$+ p_1p_3m_4y_1y_2y_3y_4 - p_1p_3m_2y_1y_2^2y_3 + p_1p_4m_3y_1y_2y_3y_4 + p_1p_3m_4y_1y_2y_3y_4 + p_1p_3m_3y_1y_2y_3^2$$
$$- p_1p_3m_4y_1y_2y_3y_4 + p_2p_3m_1y_1y_2^2y_3 - p_3p_4m_1y_1y_2y_3y_4 - p_3^2m_1y_1y_2y_3^2 - p_1p_4m_1y_1y_2y_3y_4 - p_2p_3m_3y_2^2y_3^2$$
$$+ p_2p_3m_4y_2^2y_3y_4 + p_3^2m_2y_2^2y_3^2 - p_3p_4m_2y_2^2y_3y_4 - p_1p_4m_2y_1y_2y_3y_4 + p_1p_4m_3y_1y_2y_3y_4$$
$$- p_1p_4m_3y_1y_2y_3y_4 + p_1p_4m_4y_1y_2y_4^2 + p_2p_4m_1y_1y_2^2y_4 - p_3p_4m_1y_1y_2y_3y_4 + p_3p_4m_1y_1y_2y_3y_4$$
$$- p_4^2m_1y_1y_2y_4^2 + p_2p_4m_3y_2^2y_3y_4 - p_2p_4m_4y_2^2y_4^2 - p_3p_4m_2y_2^2y_3y_4 + p_4^2m_2y_2^2y_4^2 + p_3p_4m_4y_2y_3y_4^2$$
$$- p_4^2m_3y_2y_3y_4^2\Big]$$

$$=-\frac{1}{|J|}\Big[\left(p_1^2m_2 - p_1p_2m_1\right)y_1^2y_2^2 + \left(p_4^2m_2 - p_2p_4m_4\right)y_2^2y_4^2 + \left(p_1p_3m_4 - p_1^2m_3 - p_1^2m_4\right)y_1^2y_2y_3$$
$$+\left(p_1p_3m_1 + p_1p_4m_1\right)y_1^2y_2y_4 + \left(p_3^2m_2 - p_2p_3m_3\right)y_2^2y_3^2 + \left(p_1p_3m_4 - p_3p_4m_1 - p_1p_4m_2 - p_1p_4m_1\right)y_1y_2y_3y_4$$
$$+\left(p_2p_4m_1 - p_1p_4m_2 - p_1p_2m_4\right)y_1y_2^2y_4 + \left(p_1p_2m_3 + p_2p_3m_1\right)y_1y_2^2y_3 + \left(p_1p_3m_3 - p_3^2m_1\right)y_1y_2y_3^2$$
$$+\left(p_2p_3m_4 - p_3p_4m_2\right)y_2^2y_3y_4 + \left(p_1p_4m_4 - p_4^2m_1\right)y_1y_2y_4^2 + \left(p_2p_4m_3 - p_3p_4m_2\right)y_2^2y_3y_4$$
$$+\left(p_3p_4m_4 - p_4^2m_3\right)y_2y_3y_4^2\Big]. \tag{21}$$

Now we use $p_3 = p_1$, and $p_4 = p_2$ where pair of prices are same, and $m_3 = m_1$, and $m_4 = m_2$, i.e., two types of coupon numbers are same, $|J| = |H| = -2p_1p_2m_1m_2$. We put $y_1 = y_2 = y_3 = y_4 = 1$ then (21) becomes (Mohajan & Mohajan, 2022b);

$$\frac{\partial y_2}{\partial M} = \frac{3p_1^2m_2 + p_2^2m_1 - p_1p_2m_1 - 2p_1p_2m_2}{2p_1p_2m_1m_2}. \tag{22}$$

Now we use, $m_1 = m_2 = m$ in (22), and then we get,

$$\frac{\partial y_2}{\partial M} = \frac{(p_1 - p_2)^2 + p_1(2p_1 - p_2)}{2mp_1 p_2}, \tag{23}$$

where $p_1, p_2, m > 0$.

Now if $2p_1 \geq p_2$ in (23) we get,

$$\frac{\partial y_2}{\partial M} > 0. \tag{24}$$

Inequality (24) indicates that if the total coupon of individual/community increases, the level of consumption of commodity y_2 will also increases. We believe that commodity y_2 is not an inferior good; it may be a superior good, and it has no supplementary goods (Islam et al., 2010; Mohajan, 2021b; Mohajan & Mohajan, 2022c).

For $\frac{\partial y_2}{\partial M} < 0$ we have;

$$(p_1 - p_2)^2 + p_1(2p_1 - p_2) < 0. \tag{25}$$

Now let, $\frac{p_2}{p_1} = x > 0$ then from (25) we get,

$$x^2 - 3x + 3 < 0, \tag{26}$$

where there is no real root. Therefore, $\frac{\partial y_2}{\partial M} \not< 0$. From this study we have realized that $\frac{\partial y_2}{\partial M} \neq 0$, so that, we face one possible situation as in (24), i.e., $\frac{\partial y_2}{\partial M} > 0$.

6. Conclusions

In this study we have tried to discuss sensitivity analysis between commodity and total coupon during utility maximization analysis. Thinking for the novel researchers we have stressed on detail mathematical calculations. We have used two constraints: budget constraint and coupon constraint to perform the research efficiently. We have used four commodity variables to operate the mathematical formulation properly. We have observed that the method of Lagrange multipliers is a very useful and powerful technique in multivariate calculus, and we have applied this device to investigate the optimization problems.

References

Baxley, J. V., & Moorhouse, J. C. (1984). Lagrange Multiplier Problems in Economics. *The American Mathematical Monthly*, 91(7), 404-412.

Bentham, J. (1780). *An Introduction to the Principles of Morals and Legislation*. CreateSpace Independent Publishing Platform.

Blessing, L. T. M., Chakrabarti, A., & Wallace, K. M. (1998). An Overview of Descriptive Studies in Relation to a General Design Research Methodology. In E. Frankenberger, P. Badke-Schaub & H. Birkhofer (Eds.), *Designers: The Key to Successful product Development*. Berlin: Springer Verlag.

Carter, M. (2001). *Foundations of Mathematical Economics*. MIT Press, Cambridge, London.

Chowdhury, T. U., Datta, R., & Mohajan, H. K. (2013). Green Finance is Essential for Economic Development and Sustainability. *International Journal of Research in Commerce, Economics & Management*, 3(10), 104-108.

Dixit, A. K. (1990). *Optimization in Economic Theory* (2nd Ed.). Oxford University Press, Oxford.

Eaton, B., & Lipsey, R. (1975). The Principle of Minimum Differentiation Reconsidered: Some New Developments in the Theory of Spatial Competition. *Review of Economic Studies*, 42(1), 27-49.

Ferdous, J., & Mohajan, H. K. (2022). Maximum Profit Ensured for Industry Sustainability. *Annals of Spiru Haret University. Economic Series*, 22(3), 317-337.

Fishburn, P. C. (1970). *Utility Theory for Decision Making*. Huntington, NY: Robert E. Krieger.

Islam, J. N., Mohajan, H. K., & Moolio, P. (2009a). Preference of Social Choice in Mathematical Economics. *Indus Journal of Management & Social Sciences*, 3(1), 17-38.

Islam, J. N., Mohajan, H. K., & Moolio, P. (2009b). Political Economy and Social Welfare with Voting Procedure. *KASBIT Business Journal*, 2(1), 42-66.

Islam, J. N., Mohajan, H. K., & Moolio, P. (2010). Utility Maximization Subject to Multiple Constraints. *Indus Journal of Management & Social Sciences*, 4(1), 15-29.

Islam, J. N., Mohajan, H. K., & Moolio, P. (2011). Output Maximization Subject to a Nonlinear Constraint. *KASBIT Business Journal*, 4(1), 116-128.

Islam, J. N., Mohajan, H. K., & Datta, R. (2012a). Stress Management Policy Analysis: A Preventative Approach. *International Journal of Economics and Research*, 3(6), 1-17.

Islam, J. N., Mohajan, H. K., & Datta, R. (2012b). Aspects of Microfinance of Grameen Bank of Bangladesh. *International Journal of Economics and Research*, 3(4), 76-96.

Kirsh, Y. (2017). Utility and Happiness in a Prosperous Society. *Working Paper Series*, No. 37-2017, Institute for Policy Analysis, The Open University of Israel.

Kothari, C. R. (2008). *Research Methodology: Methods and Techniques* (2nd Ed.). New Delhi: New Age International (P) Ltd.

Legesse, B. (2014). *Research Methods in Agribusiness and Value Chains*. School of Agricultural Economics and Agribusiness, Haramaya University.

Mohajan, D., & Mohajan, H. K. (2022a). Profit Maximization Strategy in an Industry: A Sustainable Procedure. *Law and Economy*, 1(3), 17-43. https://doi:10.56397/LE.2022.10.02

Mohajan, D., & Mohajan, H. K. (2022b). Utility Maximization Analysis of an Organization: A Mathematical Economic Procedure. *Law and Economy*, Manuscript Submitted.

Mohajan, D., & Mohajan, H. K. (2022c). Utility Maximization Investigation: A Bordered Hessian Method. *Annals of Spiru Haret University. Economic Series*, Manuscript Submitted.

Mohajan, D. & Mohajan, H. K. (2022d). Sensitivity Analysis among Commodities and Prices: Utility Maximization Perspectives (Unpublished Manuscript).

Mohajan, D. & Mohajan, H. K. (2022e). Sensitivity Analysis among Commodities and Coupons during Utility Maximization (Unpublished Manuscript).

Mohajan, D., & Mohajan, H. K. (2022f). Memo Writing Procedures in Grounded Theory Research Methodology. *Studies in Social Science & Humanities*, 1(4), 10-18.

Mohajan, H. K. (2011). Greenhouse Gas Emissions Increase Global Warming. *International Journal of Economic and Political Integration*, 1(2), 21-34.

Mohajan, H. K. (2012). Green Marketing is a Sustainable Marketing System in the Twenty First Century. *International Journal of Management and Transformation*, 6(2), 23-39.

Mohajan, H. K. (2017a). Optimization Models in Mathematical Economics. *Journal of Scientific Achievements*, 2(5), 30-42.

Mohajan, H. K. (2017b). Two Criteria for Good Measurements in Research: Validity and Reliability. *Annals of Spiru Haret University. Economic Series*, 17(3), 58-82.

Mohajan, H. K. (2018a). *Aspects of Mathematical Economics, Social Choice and Game Theory*. PhD Dissertation. University of Chittagong, Chittagong, Bangladesh.

Mohajan, H. K. (2018b). Qualitative Research Methodology in Social Sciences and Related Subjects. *Journal of Economic Development, Environment and People*, 7(1), 23-48.

Mohajan, H. K. (2020). Quantitative Research: A Successful Investigation in Natural and Social Sciences. *Journal of Economic Development, Environment and People*, 9(4), 52-79.

Mohajan, H. K. (2021a). Utility Maximization of Bangladeshi Consumers within Their Budget: A Mathematical Procedure. *Journal of Economic Development, Environment and People*, 10(3), 60-85.

Mohajan, H. K. (2021b). Product Maximization Techniques of a Factory of Bangladesh: A Sustainable Procedure. *American Journal of Economics, Finance and Management*, 5(2), 23-44.

Mohajan, H. K. (2021c). Estimation of Cost Minimization of Garments Sector by Cobb-Douglass Production Function: Bangladesh Perspective. *Annals of Spiru Haret University. Economic Series*, 21(2), 267-299.

Mohajan, H. K. (2022). Cost Minimization Analysis of a Running Firm with Economic Policy. *Annals of Spiru Haret University. Economic Series*, 22(3), 171-181.

Mohajan, H. K., Islam, J. N., & Moolio, P. (2013). *Optimization and Social Welfare in Economics*. Lambert Academic Publishing, Germany.

Moolio, P., Islam, J. N., & Mohajan, H. K. (2009). Output Maximization of an Agency. *Indus Journal of Management and Social Sciences*, 3(1), 39-51.

Polit, D. F., & Hungler, B. P. (2013). *Essentials of Nursing Research: Methods, Appraisal, and Utilization* (8th Ed.). Philadelphia: Wolters Kluwer/Lippincott Williams and Wilkins.

Roy, L., Molla, R., & Mohajan, H. K. (2021). Cost Minimization is Essential for the Sustainable Development of an Industry: A Mathematical Economic Model Approach. *Annals of Spiru Haret University. Economic Series*, 21(1), 37-69.

Stigler, G. J. (1950). The Development of Utility Theory. I. *Journal of Political Economy*, 58(4), 307-327.