CBDC as imperfect substitute to bank deposits: a macroeconomic perspective

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CBDC as Imperfect Substitute to Bank Deposits: A Macroeconomic Perspective *

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Abstract

The impact of Central Bank Digital Currency (CBDC) is analyzed in a closed-economy model with monopolistic competition in banking and where CBDC is an imperfect substitute with bank deposits. The design of CBDC is characterized by its interest rate, its substitutability with bank deposits, and its relative liquidity. We examine how interest-bearing CBDC would affect the banking sector, public finance, GDP and welfare. Welfare may improve through three channels: seigniorage; a lower opportunity cost of money; and a redistribution away from bank owners. In our numerical analysis we find a maximum welfare improvement of 60 bps in consumption terms.

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1 Introduction

As our economies are becoming increasingly digital, central banks around the world are exploring the possibility of issuing central bank digital currency (CBDC). Since there are various ways to implement CBDCs, it important to understand its implications. For example, CBDC could mainly substitute cash, which would have little impact on financial intermediation. Alternatively, it could substitute checking deposits and could lead to banking disintermediation. Although a growing literature is exploring the macroeconomic implications of CBDC, our understanding is still limited.[1] Under some conditions, CBDC leaves economic outcomes unchanged, as shown in Brunnermeier and Niepelt (2019). In contrast, other studies show that the disintermediation implied by CBDCs would reduce bank loans and possibly output (see Keister and Sanches (2022), or Chiu et al. (2022)), while Barrdear and Kumhof (2021) predict a large increase in output. Results depend in particular on how easily banks can substitute checking deposits by other types of funding and how substitutable are checking deposits with CBDC. The interest rate on CBDC and the competitive structure of the banking sector may also play significant roles.

The purpose of this paper is to shed light on these issues and give quantitative estimates on the potential benefits of CBDC in a model with monopolistic competition in banking, where CBDC and bank deposits are imperfect substitutes. We model imperfect substitutability by assuming that CBDC and bank deposits contribute to the formation of a composite liquid asset, which is useful to households as it reduces the transaction cost of acquiring goods for consumption.[2] Given the interest paid by each type of money, households' demand for each reflects the optimal trade-off between maximizing interest

[1] E.g., see Anhert et al. (2022), Auer et al. (2021), Infante et al. (2022), and Niepelt (2021) for recent surveys of the literature.

[2] This framework extends the idea present in Feenstra (1986), Rebelo and Vegh (1996) and Schmitt-Grohé and Uribe (2004) that money is demanded as it reduces a transaction or liquidity cost. Barrdear and Kumhof (2021) adopt a similar approach. Imperfect substitutability is also modeled by introducing CBDC in the utility function (e.g., Agur et al. (2021), or Ferrari et al. (2022)) or in search models, where CBDC is used for different transactions (e.g., Assenmacher et al. (2021)). However, several papers in the literature assume perfect substitutability between CBDC and bank deposits or focus on the interaction between cash and CBDC (e.g., Davoodalhosseini (2022)).

collection and minimizing the transaction cost, given the imperfect substitutability between the different monies.

CBDC design involves three dimensions in our model: the interest rate it pays; its liquidity relative to bank deposits – which, in the model, is the weight of CBDC in the formation of the composite liquid asset – and its degree of substitutability with bank deposits. In practice, liquidity may be related to technological aspects of the design, such as the rapidity of payments, or to any fee structure. Substitutability might involve the interoperability between CBDC and bank deposits (see Brunnermeier and Landau (2019) for discussions on this issue), or some characteristics that might differentiate the two monies and make one more suitable than the other in certain circumstances. For example CBDC might be in the form of token, might grant more or less privacy than bank deposits, might be more secure than bank deposits or might for example offer better conditions for international transactions.[3]

We analyze the welfare impact of CBDC in the steady state. We identify three channels through which CBDC may improve welfare. First, through CBDC the central bank may increase its seigniorage revenue, which, everything else equal, would allow the government to reduce income taxes. Second, if households can earn higher interest on their money (CBDC and/or deposit) holdings, they optimally choose to increase their money holdings and thus pay a lower transaction cost on consumption. Third, the introduction of CBDC may lead to a reallocation of banks' rents to the general population, whether in the form of tax reduction (first channel) or in the form of higher interest payment (second channel). If bank rents are collected by a wealthier fraction of the population, this shift implies that CBDC induces some degree of reduction of inequality.

Seigniorage is an important endogenous variable in the model. Its magnitude depends on all three dimensions of CBDC (interest rate, liquidity, substitutability). In particular, seigniorage is non-monotonic in the interest rate paid by CBDC, as a higher interest rate decreases seigniorage per unit CBDC issued, but increases its demand.

The optimal interest rate on CBDC is the one that reaches the best compromise between raising higher seigniorage to lower tax distortions or paying higher interest to

[3]Since there may be technical constraints in the choice of liquidity and substitutability, in our quantitative welfare analysis we will only consider the interest rate as a policy variable.

lower the opportunity cost of holding money. The optimal interest rate depends on how high are existing tax rates, as the higher the tax rate, the higher the distortion they bring to the economy. Thus, with a higher tax rate the potential benefit of the first channel – collecting seigniorage and lower taxes – is higher, hence the optimal interest rate on CBDC is lower. This is relevant since, as reported e.g. by Trabandt and Uhlig (2011), the amount of labor taxation differs enormously between different countries: it is around 25% in the United States and it averages more than 40% in the EU-14 countries.

However, the quantitative analysis shows that these two channels would bring only a modest welfare improvement: at the optimum they would bring an increase of only 9 basis points in consumption terms for countries with a labor tax rate of 25%, and of 20 basis points for countries with a tax rate of 45%.

The third channel we consider is the reallocation of banks' rents that may lead to a reduction of inequality. In one parameterization of the model we consider the limit case in which a zero-size set of "bankers" own the banks and receive all the profits.[4] CBDC allows non-bankers to take over part of the rents associated to deposits, whether in the form of tax reductions or in the form of interest on CBDC holdings. Taking into account this channel, together with the previous two, we find that the welfare of non-bankers, which coincides with general welfare if the set of non-bankers has zero size, increases by 54 basis points in countries with 25% labor tax rate and by 59 basis points in countries with 45% labor tax rate.

We also emphasize that these benefits require historically normal interest rates (our baseline rate is 4%). At interest rates close to zero, all three of our channels lose their efficacy: seigniorage clearly is also close to zero, the opportunity cost of holding any form of money is close to zero without the need of introducing CBDC, and banks collect zero rents from deposits, implying that there are no rents that CBDC can redistribute to the public.

Most of the literature on CBDC assumes perfect competition in banking or does

[4]This parameterization could represent the situation in which the government's welfare function assigns a much higher weight to a fraction of the population that receives a negligible share of the profits. In the United States, for example, households in the bottom 90% of the wealth distribution own only 10% of the stock. See for example "How America's 1% came to dominate equity ownership", https://www.ft.com/content/2501e154-4789-11ea-aeb3-955839e06441

not model banks explicitly. Exceptions are Andolfatto (2021) who assumes a one bank monopoly and Chiu et al. (2022) who assume Cournot competition with smaller number of banks. In these frameworks, the interest rate on CBDC affects the optimal deposit interest rate and can affect welfare through this channel. With monopolistic competition, however, individual banks take the average deposit rate as given so that the deposit rate is unaffected by the CBDC interest rate.[5]

While our approach share some features with Barrdear and Kumhof (2021), our paper estimates a significantly lower welfare benefit of CBDC. Their estimate of a 3% GDP increase is due in large part to the following channel, absent from our model. When issuing CBDC, the central bank buys public debt from private investors. In their model this is assumed to result in a lower interest rate on government bonds, which brings savings to the government and general welfare improvements. Chiu and Davoodalhosseini (2021) consider a general equilibrium model where cash and deposits play different roles for payments. They find that an interest-bearing cash-like CBDC improves welfare because the main impact is the reduction in the opportunity cost of money holdings. A similar effect is also present in our framework.

Since we focus on the steady state, we do not examine the cyclical issues associated with CBDC. Using a DSGE model, Burlon et al. (2022) find a positive cyclical impact of CBDC as the increased seigniorage is transferred to households and increases their consumption. Piazzesi et al. (2022) consider varying the interest rate on CBDC for monetary policy objectives.

There are various potential channels through which CBDC could affect bank lending,[6] but there is uncertainty about the sign and the magnitude of this effect.[7] We abstract from these channels and an important feature of our model is that the two main functions

[5]Empirical evidence for monopolistic competition in the banking sector is provided e.g., by Drechsler, Savov and Schnabl (2017). Gerali et al. (2010) introduce monopolistic competitive banks in a DSGE model. Kurlat (2019) assumes a finite number of banks with entry.

[6]For example, because of reduced profits as in Burlon et al. (2022); because deposits, but not CBDC, are associated with credit lines, as in Piazzesi and Schneider (2022); or because of an increased cost of wholesale funding, as in Whited et al. (2022).

[7]Andolfatto (2021) and Chiu et al. (2022) show that CBDC might increase lending. Also, the evidence in the literature is mixed about whether or not an increase in bank competition has adverse consequences on banks' optimal lending choices (see e.g., De Nicolo and Boyd (2005) for a review of this literature).

of banks, deposit taking and credit provision, do not interact. This is because the financial markets provide an alternative source of financing, although at a higher interest rate.[8] This reduces bank profits on deposits, but it does not affect the marginal cost of funding for banks, which is always equal to the risk-free rate, and for this reason it does not affect credit extension in the steady state of our model.

The rest of the paper is organized as follows: Section 2 presents the model and Section 3 describes the steady state equilibrium. Section 4 discusses the calibration and Section 5 outlines the numerical results, in terms of the relative demand for CBDC and bank deposits, seigniorage collected by the government, the optimal choice of the interest rate on CBDC and the welfare implications. Section 6 concludes.

2 A Model with CBDC

We consider a closed economy with two types of agents – households and bank owners – firms, banks, and finally the government and the central bank. This economy is similar in many respects to the classical monetary economy in Gali (2015) and to the economy in Del Negro and Sims (2015). As in the continuous-time model of Del Negro and Sims (2015) there is an explicit role for money, as the latter mitigates the transaction cost of consumption. However, as described in detail in the next subsection, our model features multiple types of money, which are imperfect substitutes.

2.1 Demand for Bank Deposits and CBDC

Our model comprises two types of agents, households and bank owners, described in detail in Section 2.2. All the action in the model is on the part of households, which in particular generate money demand. Households decide how to allocate savings between the following assets: a nominal asset a (e.g. government bonds), paying the nominal

[8]Alternatively, banks could borrow from the central bank as in Brunnermeier and Niepelt (2019): when the central bank expands its liabilities by issuing CBDC, it might acquire claims vis-à-vis the banking sector, thus providing substitute funding for banks. In Brunnermeier and Niepelt (2019) economic outcomes are unchanged if central bank funding is provided at the same conditions as deposit funding, and if the central bank pays the same interest on CBDC as banks do on deposits. In our model, the interest on substitute bank funding would be equal to the risk-free interest rate.

risk-free rate r_t^*, bank deposits $d^b(j)$ for each bank j, paying a nominal interest $r_t^b(j)$, and CBDC d^c, paying nominal interest r_t^c. Both bank deposits and CBDC reduce transactions costs, but they are imperfect substitutes.

Bank deposits are issued by a continuum of banks of size 1 in monopolistic competition. The equilibrium interest rate on bank deposits is typically lower than the safe rate r^* due to the costs of managing deposits and to banks' market power, as discussed in section 2.4.

As in Schmitt-Grohé and Uribe (2004) and Del Negro and Sims (2015), we assume that households incur transactions costs $c_t s_t$ to consume c_t. These costs can be reduced by holding bank deposits and CDBC. More precisely, s_t is a function of money velocity $x_t \equiv p_t c_t / d_t$, where p_t is the price level and d_t is a composite of the deposits of all banks and CBDC. This composite captures the imperfect substitutability among deposits. We assume a CES structure:

$$d_t = \left(\alpha_c (d_t^c)^{\frac{\epsilon_{cb}-1}{\epsilon_{cb}}} + \alpha_b (d_t^b)^{\frac{\epsilon_{cb}-1}{\epsilon_{cb}}} \right)^{\frac{\epsilon_{cb}}{\epsilon_{cb}-1}} \tag{1}$$

d_t^b is a composite of all bank deposits:

$$d_t^b \equiv \left(\int (d_t^b(j))^{1-\frac{1}{\epsilon^b}} dj \right)^{\frac{\epsilon^b}{\epsilon^b-1}} \tag{2}$$

where ϵ^b is the elasticity of substitution between deposits at different banks. $\frac{\alpha_c}{\alpha_b}$ can be interpreted as the relative liquidity of CDBC with respect to bank deposits, and ϵ_{cb} is the elasticity of substitution between bank deposits and CBDC.

The interest rate on CBDC r_t^c is set by the central bank. The relative liquidity and the elasticity of substitution between bank deposits and CBDC can be a design choice of the government. We assume that

$$\alpha_c^{\epsilon_{cb}} + \alpha_b^{\epsilon_{cb}} = 1 \tag{3}$$

as in this case one unit of the numeraire good results at most in one unit of the composite d_t (when $\alpha_c^{\epsilon_{cb}}$ is allocated in CBDC and $\alpha_b^{\epsilon_{cb}}$ is allocated in bank deposits). The world without CBDC is one where $\alpha_c = 0$ and $\alpha_b = 1$.[9]

[9] If the introduction of CBDC implied $\alpha_c^{\epsilon_{cb}} + \alpha_b^{\epsilon_{cb}} > 1$, it could improve the overall efficiency of the payment system since fewer resources would be needed to alleviate the transaction cost. However, we abstract from this effect to concentrate on the effect of the competition between bank deposits and CBDC.

While we do not introduce cash in our baseline model, the Appendix includes an extension of this framework featuring cash alongside bank deposits and CBDC.

2.2 Households and Bank Owners

Households are a measure-one set of agents who work in firms, consume, and save. In addition, they own a fraction of firms and banks. They derive utility from consumption and disutility from working. We assume separable CRRA preferences so that the household's periodic flow utility is given by

$$u(c_t, h_t) = log(c_t) - \frac{h_t^{1+\gamma}}{1+\gamma} \qquad \gamma \geq 0$$

where c_t is consumption and h_t denotes labor supply. The household's expected lifetime utility is:

$$U = \sum_{t=0}^{\infty} \beta^t u(c_t, h_t) \tag{4}$$

The household's budget constraint is

$$\begin{aligned} & (1-\tau_h)w_t h_t + (1+r^*_{t-1})a_{t-1} + \int (1+r^b_{t-1}(j))d^b_{t-1}(j)dj \\ + \quad & (1+r^c_{t-1})d^c_{t-1} + \zeta(1-\tau_b)\Pi^b_t = p_t c_t (1+s_t) + \int d^b_t(j)dj + d^c_t + a_t + p_t t_t \end{aligned} \tag{5}$$

where w_t is the (nominal) wage, a_t are holdings of the risk-free bond, and Π^b_t are bank dividends, t_t are lump-sum taxes. τ_h and τ_b are labor income and dividend tax rates. ζ is the fraction of banks that is owned by households.[10]

The remaining fraction $1-\zeta$ belongs to the second type of agent in the model, bank owners. This is a set of agents of size ν, who do not work and, importantly, are not subject to the transaction cost. Hence their wealth, w^{bo}, is invested in the risk-free asset, and their budget constraint is simply

$$p_t c^{bo}_t + w^{bo}_t = \frac{1-\zeta}{\nu}\Pi^b_t + (1+r^*_{t-1})w^{bo}_{t-1} \tag{6}$$

where c^{bo} is consumption per unit-size bank owner.

While ζ can take any value between 0 and 1, in our numerical analysis we will consider the two extreme cases $\zeta = 1$ and $\zeta = 0$. In the first case, the banking sector is irrelevant

[10]The firm sector is perfectly competitive. Hence, firm profits are zero and it is not important to specify the firm ownership.

and we fall into the representative-agent model, in which households own the banks and equally share all bank profits. In the second extreme, banks are not held by households and bank owners collect all the profits. We will consider the case where $\nu \to 0$ so that bank owners do not matter for welfare.

Households maximize their utility subject to (5). First-order conditions are standard and are described in the Appendix. Below we will assume a specific form for the transactions cost, similar to Schmitt-Grohé and Uribe (2004). This cost is a function of money velocity $x_t = \frac{p_t c_t}{d_t}$

$$s(x_t) = Ax_t + \frac{B}{x_t} - 2\sqrt{AB} \tag{7}$$

where A and B are constant parameters.

The demand equation for the deposits of each bank j is

$$d_t^b(j) = \left(\frac{r_t^* - r_t^b(j)}{r_t^* - r_t^b} \right)^{-\epsilon^b} d_t^b \tag{8}$$

where

$$r_t^* - r_t^b \equiv \left(\int (r_t^* - r_t^b(j))^{1-\epsilon^b} dj \right)^{\frac{1}{1-\epsilon^b}} \tag{9}$$

In equilibrium, all banks offers the same deposit rate r_t^b, as we see in more detail in Section 2.4. From the Euler equations, we obtain the relationship between bank deposits holdings and CBDC holdings:

$$d_t^b = \left(\frac{\alpha_b}{\alpha_c} \times \frac{r_t^* - r_t^c}{r_t^* - r_t^b} \right)^{\epsilon_{cb}} d_t^c \tag{10}$$

so that there is a simple relationship between holdings of bank deposits and the composite liquid asset

$$d_t = f_t d_t^b \tag{11}$$

with the proportionality factor f_t given by

$$f_t = \left(\frac{r_t^* - r_t^b}{\alpha_b} \right)^{\epsilon_{cb}} \left(\alpha_c^{\epsilon_{cb}} (r_t^* - r_t^c)^{1-\epsilon_{cb}} + \alpha_b^{\epsilon_{cb}} (r_t^* - r_t^b)^{1-\epsilon_{cb}} \right)^{\frac{\epsilon_{cb}}{\epsilon_{cb}-1}} \tag{12}$$

(Notice that without CBDC, i.e., with $\alpha_c = 0$, $\alpha_b = 1$, we have $f_t = 1$ and $d_t = d_t^b$). Defining the "composite interest rate" r^{comp} such that

$$(r_t^* - r^{comp}) \equiv \left(\alpha_c^{\epsilon_{cb}} (r_t^* - r_t^c)^{1-\epsilon_{cb}} + \alpha_b^{\epsilon_{cb}} (r_t^* - r_t^b)^{1-\epsilon_{cb}} \right)^{\frac{1}{1-\epsilon_{cb}}} \tag{13}$$

(12) can be written as

$$f_t = \left(\frac{r_t^* - r_t^b}{\alpha_b (r_t^* - r_t^{comp})} \right)^{\epsilon_{cb}} \tag{14}$$

Comparing the Euler equation for the bond with that for bank deposits, money velocity is

$$x_t = \sqrt{\frac{r_t^* - r_t^{comp} + B(1 + r_t^*)}{(1 + r_t^*)A}} \tag{15}$$

so that the demand for bank deposits is

$$d_t^b = \frac{p_t c_t}{f_t} \sqrt{\frac{(1 + r_t^*)A}{r_t^* - r_t^{comp} + B(1 + r_t^*)}} \tag{16}$$

The demand for CBDC can be easily obtained by combining (10) and (16).

Finally, with simple algebra we obtain that the total cost (in terms of lost interest) paid by households to acquire money instruments and thus reduce the transaction cost satisfies the equilibrium relationship

$$d_t^b(r_t^* - r_t^b) + d_t^c(r_t^* - r_t^c) = d_t(r^* - r_t^{comp}) \tag{17}$$

where d_t is the composite money instrument defined in (1).

The interest semi-elasticity of money demand, defined as the percentage change in the demand for money instruments for a one percentage change in the *spread* between the interest paid by money and the risk-free rate, is essentially determined by the parameter B.

$$\iota = -\frac{1}{2} \times \frac{1}{B(1 + r^*) + (r^* - r^{comp})} \tag{18}$$

In the Appendix we show an extension of the model adding cash as a third money instrument, paying zero interest. Specifically, we have a nested CES structure in which cash and CBDC are imperfect substitutes; the composite of cash and CBDC, in turn, is an imperfect substitute of bank deposits. We show that if the "composite interest" (defined similarly as in (13)) of cash and CBDC is equal to the value of r^c in the two-instrument model of this section, economic outcomes are unchanged: household holdings of the three instruments are such that in equilibrium households pay the same transaction cost of consumption, and the cost of holding money is also unchanged, given by (17).

2.3 Firms

There is a representative firm with Cobb-Douglas production function

$$y_t = z k_t^{\alpha} h_t^{1-\alpha} \tag{19}$$

where k_t is capital, installed in period $t-1$. A fraction φ of capital can only be financed by banks (e.g., for the financing of working capital), so that $\varphi p_{t-1}k_t = l_{t-1}$, where l_{t-1} are the loans that the firm obtains from the bank in period $t-1$, to be repaid at t. The remaining fraction $1-\varphi$ is financed by issuing bonds at interest rate r^*_{t-1}.

We assume monopolistic competition in the loan market, so that, similarly to deposits, loans are a bundle of loans from different banks[11]

$$l_t \equiv \left(\int (l_t(i))^{1-\frac{1}{\epsilon^l}} di \right)^{\frac{\epsilon^l}{\epsilon^l-1}} \tag{20}$$

where ϵ^l is the elasticity of substitution for loans from different banks and the index i denotes a bank. The working capital constraint can be rewritten as

$$k_t = \frac{\left(\int (l_{t-1}(i))^{1-\frac{1}{\epsilon^l}} di \right)^{\frac{\epsilon^l}{\epsilon^l-1}}}{p_{t-1}\varphi} \tag{21}$$

Firms choose loans, capitals and labor to maximize profits, which, taking into account the working capital constraint, can be written as

$$\Pi_t = p_t z \left(\frac{\left(\int (l_{t-1}(i))^{1-\frac{1}{\epsilon^l}} di \right)^{\frac{\epsilon^l}{\epsilon^l-1}}}{\phi p_{t-1}} \right)^{\alpha} h_t^{1-\alpha} - w_t h_t - \int l_{t-1}(i) r^l_{t-1}(i) di - (1-\varphi)\, r^*_{t-1} \frac{\left(\int (l_{t-1}(i))^{1-\frac{1}{\epsilon^l}} di \right)^{\frac{\epsilon^l}{\epsilon^l-1}}}{\varphi} \tag{22}$$

We obtain that firms' loan demand is

$$l_t(i) = \left(\frac{r^l_t(i)}{r^l_t} \right)^{-\epsilon^l} l_t \tag{23}$$

where $r^l_t(i)$ is the loan interest rate charged by bank i and the "market loan rate" r^l_t is

$$r^l_t = \left(\int (r^l_t(i))^{1-\epsilon^l} di \right)^{\frac{1}{1-\epsilon^l}} \tag{24}$$

In equilibrium all banks choose the same rate r^l_t. The capital/labor ratio chosen by firms is

$$\frac{k_t}{h_t} = \left(\frac{z\alpha}{\hat{r}^K_{t-1}} \right)^{\frac{1}{1-\alpha}} \tag{25}$$

[11]Paravisini, Rappoport and Schnabl (2015) provide empirical evidence of specialization in bank lending, which supports the idea of monopolistic competition in the lending market.

where $\hat{r}^K_{t-1} = \frac{p_{t-1}}{p_t}(\varphi r^l_{t-1} + (1-\varphi) r^*_{t-1})$ is the real cost of a unit of capital (we will denote real interest rates with "hatted" symbols . Finally, with competitive labor markets,

$$\frac{w_t}{p_t} = (1-\alpha) z \left(\frac{z\alpha}{\hat{r}^K_{t-1}} \right)^{\frac{\alpha}{1-\alpha}} \tag{26}$$

2.4 Banks

We assume that there is a size-one continuum of banks in monopolistic competition in the deposit market and in the loan market. The aggregate bank balance sheet is

$$l_t + b^b_t + m_t = d^b_t + a^b_t + e^b_t \tag{27}$$

where on the asset side (LHS) we have bonds held by the banks b^b_t, required reserves m_t and loans l_t, and on the liability side (RHS) we have bank deposits d^b_t, other bank liabilities (such as bonds) a^b_t, and bank equity e^b_t.

Bonds on the asset and liability side, b^b_t and a^b_t, yield an interest rate r^*_t, whereas reserves yield an interest rate r^m_t determined by the central bank. Required reserves are a fraction ϕ of deposits: $m_t = \phi d^b_t$.

Loans are provided with cost c^l at the nominal interest rate $r^l_t(j)$ for bank j. Deposits are provided with cost c^b at the (nominal) rate $r^b_t(j)$. For now, we assume that costs c^l and c^b are constant. Profits of bank j are

$$\Pi^b_t(j) = (1+r^l_{t-1}(j) - c^l) l_{t-1}(j) + (1+r^*_{t-1})(b^b_{t-1}(j) - a^b_{t-1}(j)) + (1+r^m_{t-1}) m_{t-1}(j) - (1+r^b_{t-1}(j) + c^b) d^b_{t-1}(j) \tag{28}$$

Using the bank balance sheet and the reserve ratio, this can be rewritten as:

$$\Pi^b_t(j) = [(1-\phi) r^*_{t-1} + \phi r^m_{t-1} - (r^b_{t-1}(j) + c^b)] d^b_{t-1}(j) + [r^l_{t-1}(j) - c^l - r^*_{t-1}] l_{t-1}(j) \tag{29}$$

In equilibrium all profit-maximizing banks choose the same deposit rate. This rate is

$$r^b_t(j) = r^b_t = r^* - (c^b + \phi(r^*_t - r^m_t)) \frac{\epsilon^b}{\epsilon^b - 1} \tag{30}$$

Notice, however, that if there is a zero-lower-bound on the nominal interest rate, the above expression should be modified as

$$r^b_t(j) = r^b_t = max \left(0, r^* - (c^b + \phi(r^*_t - r^m_t)) \frac{\epsilon^b}{\epsilon^b - 1} \right) \tag{31}$$

and loan rate

$$r_t^l(j) = \frac{\epsilon^l}{\epsilon^l - 1}(r^* + c_l) \tag{32}$$

The result in (30)-(31) allows us to formulate the following Proposition:

Lemma 1: With banks in monopolistic competition, the choice of the deposit rate by each bank is not affected by CBDC. The deposit rate does not change in reaction to a change in the relative liquidity between CBDC and bank deposits, or in reaction to a change in the interest rate paid by CBDC.

The intuition behind this somewhat surprising result is that each bank competes with other banks for deposits, but perceives the aggregate demand for bank deposits (and of CBDC) as fixed, not internalizing how the relative demand for the two monies depends on the interest paid in aggregate by the banking system. However, competition with CBDC implies lower overall demand for bank deposits, so that in equilibrium each bank relies less on deposits and more on other liabilities, such as bank bonds and/or equity.

The loan rate is unaffected by deposits or CBDC altogether. All banks choose therefore the same value (32) of the loan rate, with or without CBDC. The quantity of loans is not affected by CBDC because banks can replace deposits by borrowing from the market at interest rate r^*.

2.5 Government

The government needs to fund a constant exogenous real expenditure g. The government receives central bank profits (seigniorage) $\mathcal{S}$, levies taxes on labor income at rate τ_h and on bank profits at rate τ_b (firm profits are 0 due to perfect competition in the goods markets). It pays interest r_{t-1}^* on the debt contracted in the previous period b_{t-1}^g.[12] The government budget constraint is:

$$\tau_h w_t h_t + \tau_b \Pi_t^b + \mathcal{S}_t + b_t^g + t_t = g + (1 + r_{t-1}^*) b_{t-1}^g \tag{33}$$

The presence of CBDC may increase seigniorage collected by the central bank, in which case the government may be able to finance its expenditure by levying lower taxes. In

[12]Government bonds were not explicitly mentioned as an investment choice for households, since they are assumed to be perfect substitutes of other risk-free bonds.

particular, we will assume that with higher seigniorage the government decides to lower the most distortionary tax, i.e., the tax on labor.

2.6 Central Bank

The central bank issues the monetary base m_t, consisting in bank reserves, as well as CBDC d_t^c, and holds assets a_t^c bearing the risk-free interest rate. Assuming zero equity at the beginning of each period, its balance sheet is $m_t + d_t^c = a_t^c$.

The central bank sets three interest rates: the nominal risk-free rate r_t^*, which determines inflation, the interest on reserves r_t^m, and the interest on CBDC r_t^c. For the risk-free rate we assume a Taylor rule

$$r_t^* = \rho + \phi_\pi(\pi_t - \pi^*) \tag{34}$$

with $\phi_\pi > 1$. Here $\rho \equiv \beta^{-1} - 1$ and π^* is the inflation target. As in the classical monetary economy of Gali (2015), the Taylor rule (34) implies that inflation is uniquely determined as

$$\pi_t = \Sigma_{s+0}^{\infty} \phi_\pi^{-(s+1)}(\hat{r}_{t+s} - \rho) \tag{35}$$

and the real interest rate $\hat{r}_t \equiv r^* + \pi_{t+1}$ is determined by the consumption process

$$(1 + \hat{r}_t)^{-1} = \beta \frac{U'(c_{t+1})}{U'(c_t)} \tag{36}$$

which implies that in steady state $\hat{r} = \rho$ and inflation is at target.

The growth in monetary base is determined by the inflation target and money market equilibrium is simply given by $m_t = \phi d_t^b$. Central bank profits are given by seigniorage

$$\mathcal{S}_t = (r_{t-1}^* - r_{t-1}^m)m_{t-1} + (r_{t-1}^* - r_{t-1}^c - c^c)d_{t-1}^c \tag{37}$$

where c^c is the cost of managing CBDC, and are distributed each period to the government.

3 Steady State Equilibrium

Since there is no shock, the equilibrium is a steady state characterized by the following conditions. Given the wage paid by firms, the interest paid by the risk-free asset, by bank deposits and by CBDC, the tax rates chosen by the government, households make decisions about labor, consumption, savings in the risk-free asset, bank deposits and CBDC to maximize utility. Given the cost of capital (determined by the risk-free rate and the loan rate chosen by banks) and the cost of labor (wage), firms choose capital and labor to maximize profits. Given deposits demand (which also depends on the rate offered by CBDC) and loan demand, banks choose the rate on deposits and on loans to maximize their profits. The wage is such that labor markets clear. All the equations determining steady state real variables are summarized in the Appendix.

Our purpose is twofold. First, we want to analyze the effect of the introduction of CBDC on the steady state equilibrium, as well as the effect of different CBDC design choices, such as the relative liquidity between CBDC and bank deposits (as measured by the ratio α_c/α_b) and of the elasticity of substitution between the two monies. Second, we want to find the optimal choices of the government. This is discussed in the next subsection.

A few more words on the impact of the inflation target π^*. We see from equations (50)-(57) that π^* does not affect real variables, as long as the interest rate spreads $r^* - r^b$, $r^* - r^c$, $r^* - r^m$, $r^l - r^*$ are unaffected by inflation. However, one situation in which may not be the case is if the real rate (given by $\hat{r} = \beta^{-1} - 1$ in steady state) is so low as to be smaller than the optimal spread (30) chosen by banks, and nominal rates are subject to a zero lower bound. In this case, if inflation, and hence the nominal rate r^*, is not high enough, the deposit spread, given by (31) would be strictly smaller than (30), which would affect the steady state variables. As discussed in Section 4, in this case the benefit of introducing CBDC would also be smaller.

3.1 Optimal Government Choices

The government chooses the interest rate on CBDC, its liquidity and substitutability relative to bank deposits in order to maximize welfare. Welfare is defined from the point of view of the households, ignoring the bankers: $W = log(c) - h^{1+\gamma}/(1+\gamma)$. This has two possible interpretations: either the government cares more about households than about bankers, or the share λ of bankers is very small, so that despite their high (per unit-size) consumption bankers' contribution to general welfare is negligible.

The first channel available to the government to improve welfare is seigniorage: as stated in Section 2.5, we assume that government expenditure is exogenous, and higher seigniorage allows the government to lower the (distortionary) labor tax rate. Indeed, the steady state equations (58), (50) and (53) in Appendix B show that the labor tax is reduced by seigniorage, and that a lower labor tax increases labor and consumption.

Seigniorage depends in particular on the liquidity parameters of deposits and CBDC, α_b and α_c, and on the substitutability parameter ϵ_{cb}. The two propositions below (proved in the online Appendix) shed some light on the optimal choices in order to maximize the impact of this channel:

Proposition 1:

If $\alpha_b^{\epsilon_{cb}} \epsilon_{cb} > 1$ and the marginal cost of managing deposits c^c is negligible:

- **The interest rate r^c that maximizes seigniorage is larger than the interest rate on deposits r^b.**
- **If, in addition, $\epsilon_{cb} > 1.5$, the optimal value of r^c is decreasing in the CBDC liquidity parameter α_c.**
- **The peak value of seigniorage in the r^c dimension ($max_{r^c}\ \mathcal{S}$) is increasing in the CBDC liquidity parameter α_c and in the substitutability parameter ϵ_{cb}.**

The condition $\alpha_b^{\epsilon_{cb}} \epsilon_{cb} > 1$ excludes the region of the parameter space in which the elasticity of substitution between bank deposits and CBDC is very small and/or the liquidity of bank deposits is much lower than the liquidity of CBDC. Intuitively, in the latter region the central bank can almost act as a monopolist and collect high seigniorage by choosing $r^c < r^b$. Technically, it may be a difficult task to design a CBDC with these properties, so that the condition $\alpha_b^{\epsilon_{cb}} \epsilon_{cb} > 1$ seems more realistic. It seems also reasonable to assume a negligible marginal cost of managing CBDC, given that most costs faced by banks, such as branch openings and marketing, would likely be much smaller for CBDC.

Proposition 2:

Under the conditions of Proposition 1 the maximum value of seigniorage is achieved in the limit $\epsilon_{cb} \to \infty$ (so that the two monies are perfect substitutes)

and r^c is infinitesimally higher than r^b.

Proposition 2 tells us that, in the region of the parameter space defined by $\alpha_b^{\epsilon_{cb}} \epsilon_{cb} > 1$ (intuitively, unless the central bank is able to design a CBDC with very low substitutability with – or much more liquid than – bank deposits) it is optimal, from the point of view of maximizing seigniorage, to design CBDC as a perfect substitute of bank deposits and outcompete the latter by setting the interest on CBDC just infinitesimally higher than the interest on bank deposits.

Seigniorage is however not the only channel available to the government to improve welfare. By paying high interest on CBDC, the government/central bank can lower the opportunity cost of holding money. In this case households would hold a higher amount of liquid assets, with the effect of lowering the transaction cost. A lower transaction cost actually stimulates labor supply and increases consumption (see (50) and (53) in Appendix B). To maximize the impact of this channel, the government/central bank should set the interest rate on CBDC equal to the risk-free rate, so that households would stop holding deposits altogether and hold instead enough CBDC to reduce the transaction cost to zero. The latter would also be the scenario in which the biggest amount of bank profits would flow to households, which is optimal for households' welfare if household do not (fully) own banks.

The above discussion makes it clear that there is a tradeoff between reducing taxes, which would require setting the interest on CBDC low enough to collect significant seigniorage, or reducing the opportunity cost of holding money. This tradeoff crucially depends on the level of taxation. For example in some European countries, in which the level of labor taxation is of the order or 45%, reducing taxes would bring a bigger benefit than in countries such as the US in which the level of labor taxation is of the order of 25%. The tradeoff also depends on the parameter the share of banks ζ owned by households, since redistribution is maximal for $\zeta = 0$. This discussion can be summarized by the following Proposition (see the online Appendix for a formal proof):

Proposition 3: The interest rate on CBDC that maximizes consumption and welfare is decreasing in the labor tax rate and is decreasing in the share of banks held by households.

4 Calibration

Table 1: Model Parameters	
Parameter	Description
$r^* = 4\%$	risk-free rate
$A = 0.0111$	Transaction cost parameter
$B = 0.07524$	Transaction cost parameter
$\gamma = 1$	Inverse Frisch elasticity
$\phi = 0.08$	reserve ratio
$\tau_b = 25\%$	tax rate on bank profits
$\varphi = 0.2$	working capital requirement
$r^m = 0$	interest rate on bank reserves
$c^b = 0.25\%$	managing cost of bank deposits
$c^l = 0.5\%$	managing cost of loans
c^c=0.25%	managing cost of CBDC
$\alpha = \frac{1}{3}$	Cobb-Douglas capital share
$\epsilon^b = 1.40$	Elasticity of substitution of bank deposits
$\epsilon^l = 6.67$	Elasticity of substitution of bank loans
$wealth/c = 4$	wealth over consumption ratio

Table 1 summarizes our parameter choices. The parameters that are most important for our experiment are those affecting money demand and the banking system. In our baseline case we use the values for parameters A and B of the transaction cost estimated for the US economy by Schmitt-Grohe and Uribe (2004), which imply, according to (18), an interest semi-elasticity of money demand equal to -0.05. This is consistent with the estimation on the long-run money demand by Ball (2001), and also with the more recent estimates by Drechsler, Savov and Schnabl (2017), that, similarly to us, focus on the demand for deposits as a function of the deposit spread.[13]

[13]Drechsler, Savov and Schnabl (2017) find that a percentage point increase in the risk-free rate corresponds on average to a 60 bps increase in the deposit spread, and a 4% decrease in the demand for deposits. Hence,

For the banking system, the parameters ϵ^b (elasticity of substitution between deposits of different banks) is calibrated so that the deposit spread (difference between the deposit rate and the risk-free rate) is 2%, an historical average in the US and Europe alike.[14] The parameter ϵ^l (elasticity of substitution between loans of different banks) is calibrated so that the loan spread –difference between the loan rate and the risk-free rate – is 1%. This value is appropriate for the US but is low for other countries; however our results are not sensitive to this parameter, as the loan extension activity by banks is not affected by the introduction of CBDC.

Only indirect data is available to estimate the banks' cost of managing deposits and loans. According to call report data from the Federal Financial institution Examination Council,[15] total operating costs for US banks amount to around 2% of the value of bank assets, and fee income is around 1% of bank assets. If operating costs (net of fees) are equally distributed across assets and liabilities, then we could take 50 bps as an estimate of the cost of operating deposits and loans. However it is likely that operating costs, whose biggest component is given by employee salaries, are much higher on the investment side than on deposits. We therefore use 25 bps as baseline value of the cost of operating deposits (net of fees), but also consider (in Section 5.3) a scenario with the alternative value of 50 bps. We use 50 bps as the operational cost of loans.

The required reserve ratio ϕ is now zero in the United States but was 10% until 2020. It can be much higher in less advanced economies (for example, it is around 40% in Argentina). Our baseline value is 5%, and we consider alternative values in Section 5.3.

Another important parameter for our analysis is the inverse Frisch elasticity γ, which affects the extent to which labor taxation is distortionary. We use a standard value equal to 1 in our baseline scenario, but later consider a range of values from 0.25 to 4. Household wealth, given in our model by the sum of the households investment in the risk-free asset, in bank deposits and in CBDC, is set to 4 times annual consumption, similar to the ratio in the US (see e.g., Piketty and Zucman (2014)). Finally, the value

a 1% increase in the deposit spread corresponds to a 5% decrease in the demand for deposits.

[14]As pointed out by Drechsler, Savov and Schnabl (2017), the deposit spread in the US is increasing in the risk-free rate. However, a spread around 2% is an historical average. Data on deposit rates in several European countries from the World Bank open database confirm that this is the case also in Europe.

[15]Downloadable at *https://cdr.ffiec.gov.*

of productivity (expressed by the variable z) is irrelevant to our experiment as it does not affect the *percentage change* in consumption, labor and welfare induced by CBDC, so it can be normalized to 1.

We will consider different scenarios for the new parameters associated with CBDC, in particular the relative liquidity between CBDC and bank deposits, and their elasticity of substitution.

5 Results

Given our parameter calibration, in this section we outline our numerical results, in terms of the relative demand for bank deposits and CBDC, seigniorage collected by the government, the optimal choice of the interest rate on CBDC and welfare implications.

5.1 CBDC Demand and Seigniorage

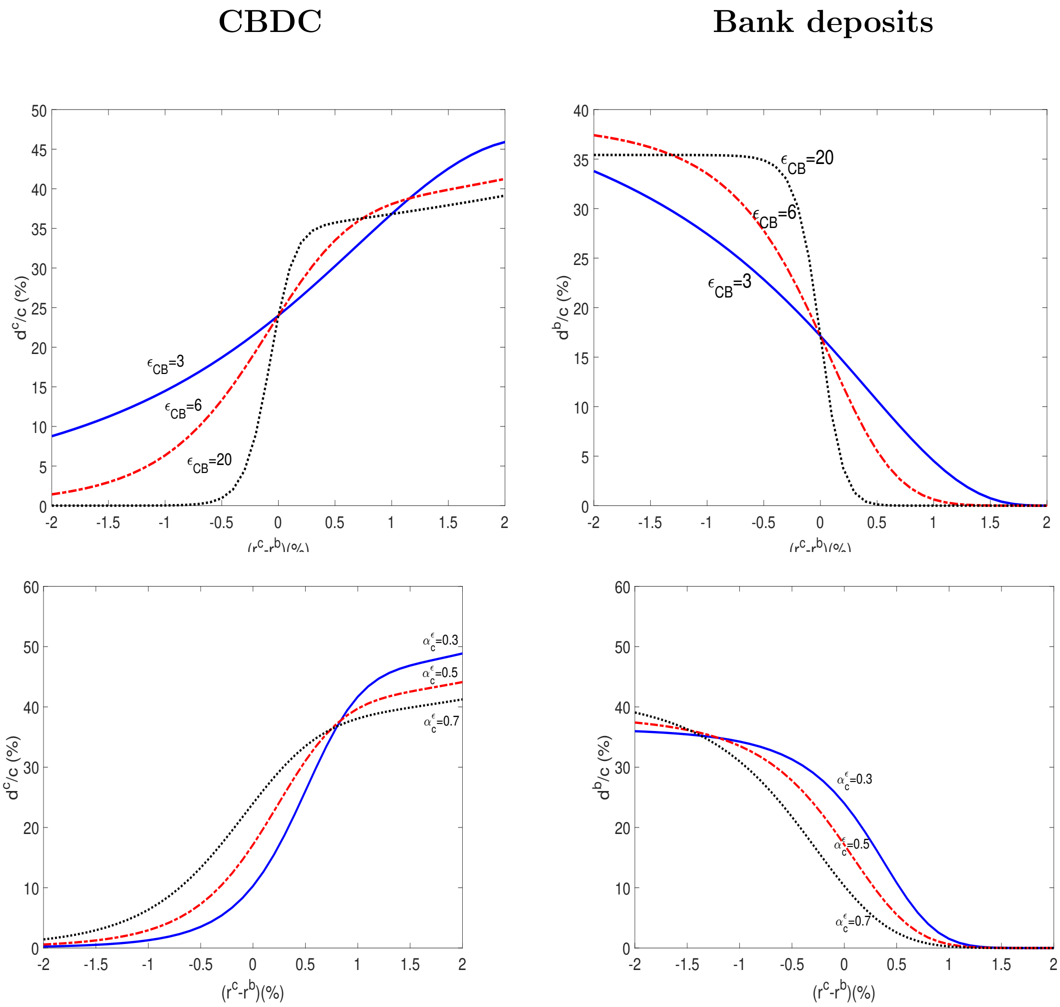
We start by examining the impact of the CDBC interest rate r_t^c on the demand for CBDC and bank deposits for different levels of substitutability and relative liquidity of CBDC. An increase in r^c tends to increase the demand for CBDC and decrease the demand for bank deposits. However, the demand for both instruments is non-monotonic in their elasticity of substitution ϵ_{cb} and in their relative liquidity.

The four panels of Figure 1 show the demand for CBDC (in the two left panels) and bank deposits (in the two right panels) when r^c is within 2 percentage points higher or lower than the interest paid by bank deposits, r^b, i.e., in a range of 4 percentage points below the risk free rate in our calibration.

In the top panels we set $\alpha_b = \alpha_c = 0.5^{\frac{1}{\epsilon_{cb}}}$ (meaning that CBDC and bank deposits are equivalent from the point of view of liquidity, so that if they paid the same interest, households would allocate the same amount of resources on the two), and show demand curves for three values of ϵ_{cb}: $\epsilon_{cb} = 3$, which we take as a representative case of "low substitutability" between bank deposits and CBDC; $\epsilon_{cb} = 6$ (medium substitutability) and $\epsilon_{cb} = 20$ (high substitutability).

In the bottom panels we set $\epsilon_{cb} = 6$ (the medium substitutability case) and show the results for three different values of α_c (α_b and α_c are related by (3)). These three values are such that $\alpha_c = 0.3^{\frac{1}{\epsilon_{cb}}}$, $\alpha_c = 0.5^{\frac{1}{\epsilon_{cb}}}$, and $\alpha_c = 0.7^{\frac{1}{\epsilon_{cb}}}$, implying that, of the resources

Figure 1: Demand for CBDC and bank deposits

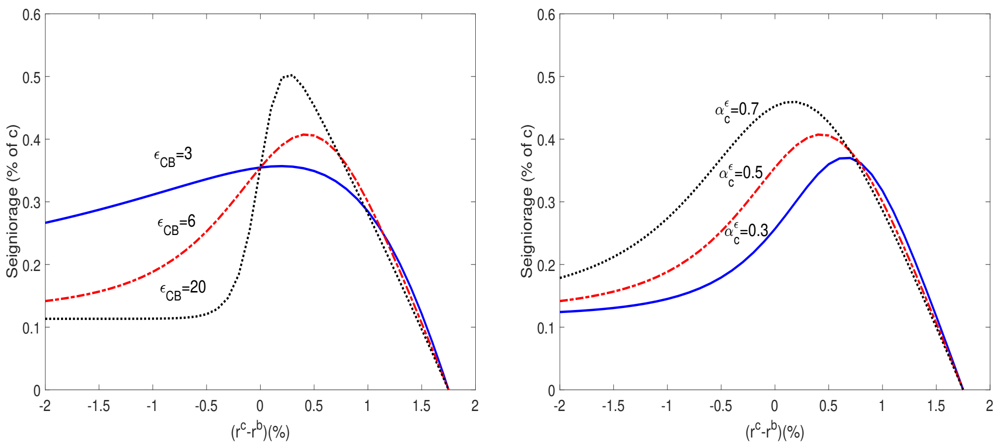


allocated in liquid assets (bank deposits or CBDC), households would choose to allocate 30%, 50% and 70%, respectively, in CBDC if the two paid the same interest.

The main takeaways from the two top panels are the following

- When the interest paid by CBDC is below the interest paid by bank deposits, demand for CBDC is decreasing in the elasticity of substitution ϵ_{cb}.
  The intuition is that the more the two instruments are substitutable, the less households are willing to hold the more costly one, i.e., CBDC.
- When the interest paid by CBDC is higher than the one paid by bank deposits, but the spread $r^c - r^b$ is not too large, the same effect persists, in the other direction:

Figure 2: Seigniorage Revenues



the more substitutable the two instruments, the less households are willing to hold bank deposits.

- When interest paid by CBDC $r^c - r^b$ is large enough, so that r^c is close to the risk-free rate, holdings of CBDC become *decreasing* in ϵ_{cb}. The intuition is that when the two instruments are less substitutable, it takes a higher amount of one to substitute for the other. This may be worthwhile if one instrument (CBDC in this case) is almost costless.

Next, in Figure 2 we examine how seigniorage is affected by the interest rate choice. On the left panel we set $\alpha_c = \alpha_b = 0.5^{\frac{1}{\epsilon_{cb}}}$ (equal liquidity properties for CBDC and bank deposits) and show the three curves of seigniorages as a function of $r^c - r^b$ for the three values of the elasticity of substitution previously considered: $\epsilon_{cb} = 3$, $\epsilon_{cb} = 6$ and $\epsilon_{cb} = 20$. On the right panel we fix $\epsilon_{cb} = 6$ and show the same curves for different values of α_c.

Seigniorage revenues are non-monotonic in r^c, interest paid on CBDC, as the demand for CBDC is increasing and the central bank profit per unit of CBDC is decreasing in r^c. As seen in Figure 2, the location of the interior maximum depends both on the elasticity of substitution between bank deposits and CBDC, and their relative liquidity. The main results emerging from Figure 2 are consistent with Propositions 1 and 2: in all the cases we analyze the peak of seigniorage occurs for $r^c < r^b$; however, as the liquidity of CBDC increases, the value of r^c that maximizes seigniorage gets closer and closer to r^b. Finally, the value of seigniorage at the peak is increasing in both the liquidity of CBDC and the elasticity of substitution.

5.2 Optimal Policy and Welfare gains

The main numerical results about the impact on CBDC on consumption, labor, seigniorage and welfare are summarized in Table 2, along with the optimal choice of interest rate. Specifically, the numbers in Table 2 refer to four key environments: when (pre-CBDC) labor tax rate is 25% and 45%, in "case a" (when households fully own banks and equally share bank profits, so that the parameter ζ in (5) is equal to 1) and "case b" (when $\zeta = 0$ and "bankers" receive all bank profits).[16] To obtain these numbers, we set the liquidity of CBDC equal to that of bank deposits ($\alpha_c = 0.5^{\frac{1}{\epsilon_{cb}}}$), and the elasticity of substitution at the high level, $\epsilon_{cb} = 20$.

Table 2: CBDC-induced changes in the economy

τ_h=**25%**	"case a"	"case b"	τ_h=**45%**	"case a"	"case b"
Consumption	+27 bps	+54 bps	Consumption	+41 bps	+62 bps
Labor	+22 bps	0	Labor	+26 bps	+4 bps
Labor tax rate	-0.14%	-0.12%	Labor tax rate	-0.30%	-0.27%
Optimal $(r^* - r^c)$	-0.96%	-0.85%	Optimal $(r^* - r^c)$	-1.54%	-1.42%
Seigniorage	+26 bps	+22 bps	Seigniorage	+45 bps	+ 41 bps
Welfare	+9 bps	+54 bps	Welfare	+20 bps	+59 bps

The plots in Figure 3 show the optimal interest rate as a function of the labor tax rate, in "case a" and "case b". Consistently with Proposition 3, we see that the optimal interest rate on CBDC is increasing in the tax rate, and it is higher in "case b" than in "case a": everything else equal, a higher interest rate reduces the demand for bank deposits and increases the demand for CBDC, and this allows households to take over a higher share of the rents associated to deposits, which were previously held by bankers.

Figure 4 shows the welfare gain when r^c is at the optimal level, for different values of the substitutability parameter. The left panel shows the welfare gain as a function of the labor tax rate, in "case a" and "case b", when $\alpha_c = 0.5^{\frac{1}{\epsilon_{cb}}}$ and $\epsilon_{cb} = 6$. The right

[16] Assuming the share of bankers ν is zero, bankers are irrelevant for welfare.

panel shows the same, when setting $\epsilon_{cb} = 20$. We see that the welfare gain in this case is increasing in the elasticity of substitution, although very mildly. In "case a", the welfare gain ranges from a modest 7-8 bps when the labor tax rate is 20% to a more significant 18-20 bps when the labor tax rate is 45%. On the other hand, in "case b" the welfare gain ranges between 52-53 bps (when $\tau_h = 20\%$) to 58-60 bps (when $\tau_h = 45\%$).

Figure 3: Optimal CBDC rate

$\epsilon_{cb} = 6$ $\epsilon_{cb} = 20$

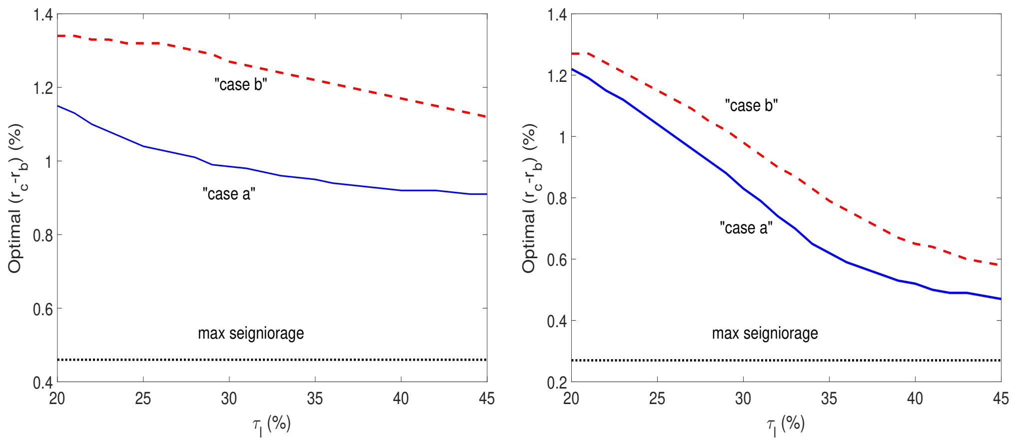


Figure 4: Welfare gain

$\epsilon_{cb} = 6$ $\epsilon_{cb} = 20$

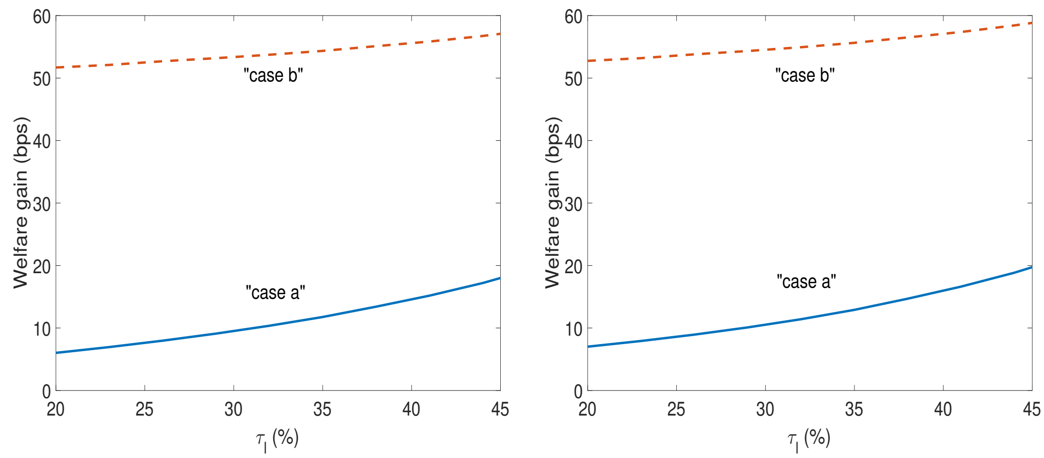


Figure 5 shows how the welfare gain at the optimal interest level r^c changes with liquidity and substitutability between CBDC and bank deposits. We observe that, if the liquidity of CBDC is low relative to that of bank deposits, the welfare gain is quite sensitive to the elasticity of substitution between CBDC and bank deposits. Intuitively, if CBDC is significantly less liquid than bank deposits, to make CBDC attractive we need to set the interest paid by CBDC significantly higher than the interest paid by bank deposits; but the substitutability between the two is low, demand for bank deposits continues to be high unless r^c is very close to the risk-free rate. This means that the seigniorage the central bank can collect is necessarily low, which lowers the welfare gain, especially when labor taxes are at the high end of the spectrum. The figure also shows that, everything else equal, welfare increases with CBDC liquidity and substitutability. However, if the two monies are very substitutable and CBDC is at least as liquid as bank deposits, no big gains can be achieved by further increasing the liquidity of CBDC. This may be relevant since – although disregarded in this model – it seems likely that increasing the liquidity of CBDC might involve higher costs for the central bank.

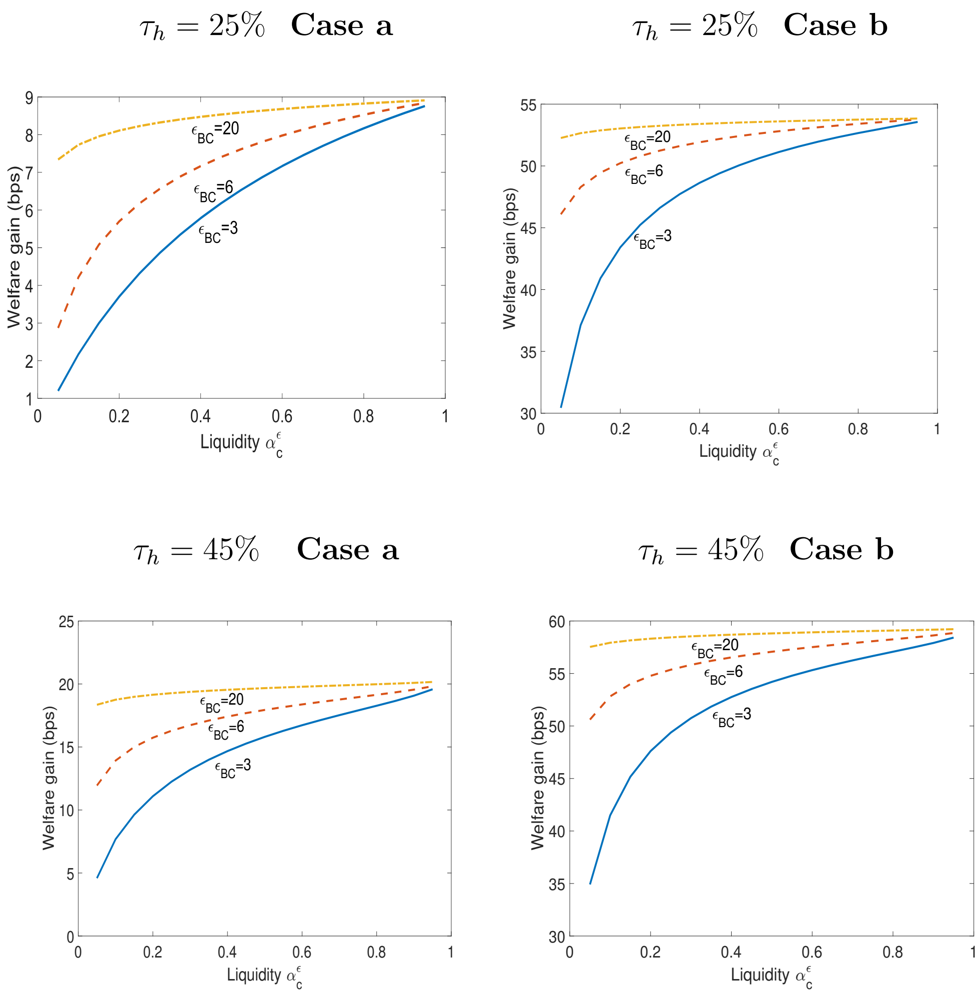
Finally, Figure 6 shows how consumption, welfare and banks' profits depend on the choice of the interest rate on CBDC.[17]

5.3 Alternative Parameter Choices

Table 3 shows the welfare improvement brought by CBDC with some alternative parameter choices. One quantity that has a significant impact on results is the Frisch elasticity of substitution, i.e., the inverse of the parameter γ, which is equal to 1 in the baseline case. We consider here two alternative values:$\gamma = 0.25$ (corresponding to Frisch elasticity equal to 4, among the highest values considered in the literature) and $\gamma = 4$ (Frisch elasticity equal to 0.25, in the low range of estimated "micro-elasticities"). As is intuitive, CBDC has the potential to bring higher welfare improvement when the elasticity is high, i.e., when taxation has a stronger distortionary effect on labor. Welfare improvements in "case a" are indeed higher when $\gamma = 0.25$ (and lower when $\gamma = 4$). However, in "case b", the welfare improvement is essentially independent of the Frisch elasticity: in this case, to maximize the redistribution from bankers to non-bankers it is optimal to set the rate on CBDC close to the risk-free rate. However, this involves small

[17]Our simplified model for banks identifies banks profits with net interest income (NII), abstracting from all other costs. As the figure shows, the order of magnitude for banks' profits in the model is around 1.5-2% of consumption, comparable with banks' NII in the United States, but much higher than actual banks' profits.

Figure 5: Welfare gain vs Liquidity and Elasticity of Substitution

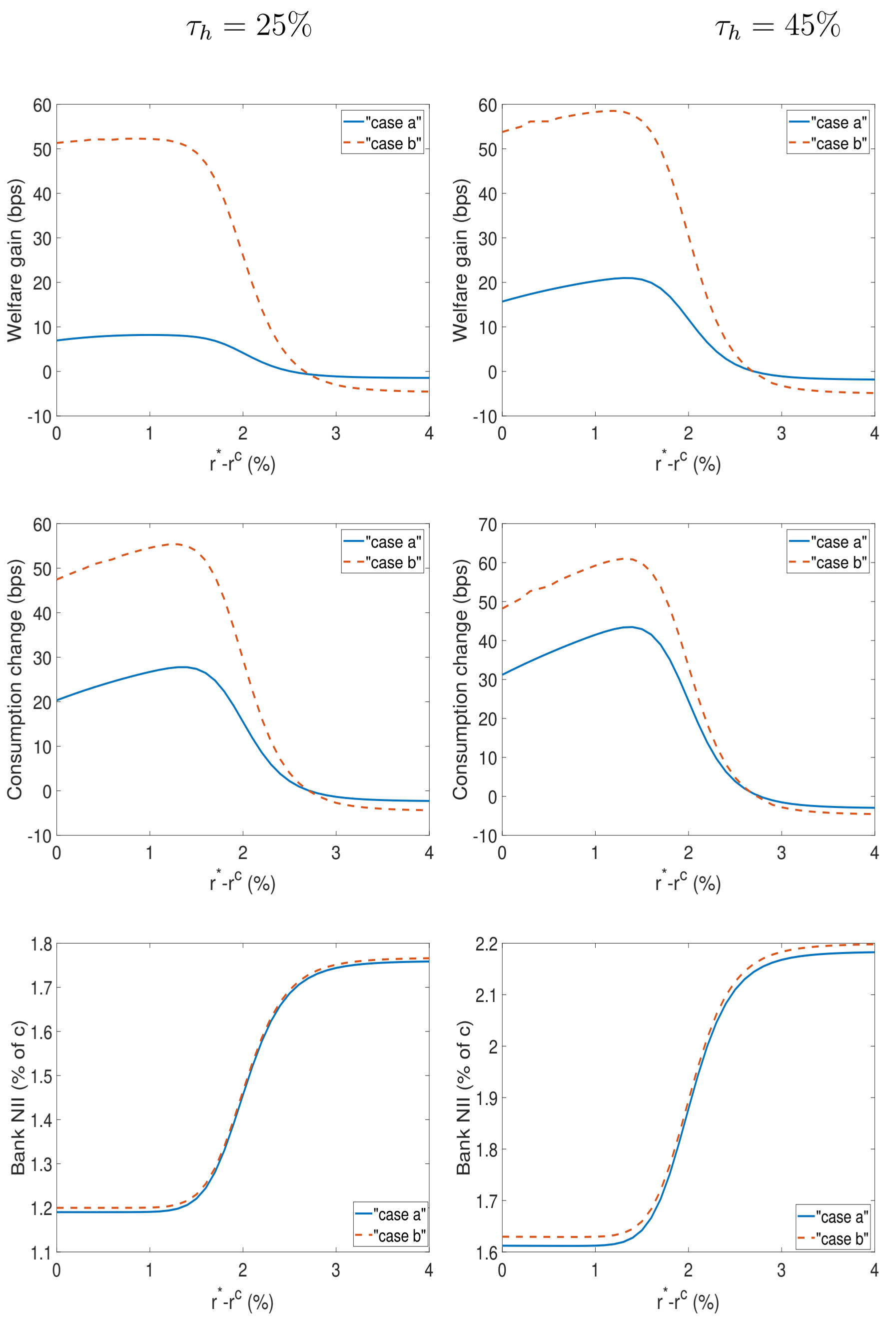


seigniorage collection, hence small tax reduction.[18]

The parameter that has the biggest impact on results is the interest semi-elasticity of money demand, governed by the parameter B of the transaction cost (see (18)). A higher semi-elasticity means that the distortion associated with the low interest on money has stronger effects on the economy, so CBDC, by paying interest close to the risk-free rate, has the potential to bring bigger welfare improvements. We consider here a value of the

[18]Similarly, if we abstract from distortionary taxes and assume that all taxes are lump-sum, we obtain a lower welfare improvement in "case a", since the channel through which seigniorage can improve welfare is inactive, but essentially unchanged welfare improvement in "case b".

Figure 6



semi-elasticity equal to -0.12.[19] As labor taxes are high (45%) and at the same time the interest semi-elasticity is high, the welfare gains induced by CBDC reach 35 bps in "case a" and 85 bps in "case b".

Moreover, we show results obtained with alternative values of the cost of managing

[19]For example, this is the value of the semi-elasticity estimated by Benati et al. (2021) for Switzerland.

deposits and loans, reserve requirement, the corporate tax rate (used in our model as the tax rate on bank profits), banks' degree of competition in the loan market, the working capital requirement for firms –affecting the extent to which firms are dependent on bank loans– and households' wealth as a fraction of annual consumption.

We see that the impact of these parameters is not large. However, parameters affecting deposits have some impact on our results. In general, with parameter values implying that banks' rent collection on deposits is high (low reserve ratio, low cost of managing deposits) the introduction of CBDC has a stronger welfare impact. Similarly, if the corporate tax rate is low, implying a stronger degree of inequality between households and bankers, the introduction of CBDC has a higher potential of smoothing such inequality and improving welfare.

Instead, results are essentially unaffected by a change in the parameters related to loans (the loan spread, the cost of managing loans, the working capital requirement, which affects the extent to which firms need to rely on bank loans), as the loan-extension function of banks is essentially unaffected by the introduction of CBDC. Household wealth has also no impact on results.

Finally, we report the welfare improvement in a world with no distortionary taxes. In this case the optimal CBDC rate would be equal to the risk-free rate, since there would be no scope for seigniorage to reduce taxes.

One last case that we want to address, also discussed in Section 3, is when the nominal risk-free rate is so low that the deposit spread set by banks is constrained by the zero lower bound and thus lower than the desired value (30), equal to 2% in our calibration. In the limit of zero nominal rate, the deposit rate must also be zero and all three channels analyzed in this paper lose their effectiveness. The plots in Figure 7 show that the welfare improvement brought by CBDC depends essentially linearly on the bank deposit spread $r^* - r^b$, and is zero when this spread (minus the cost of managing deposits) is zero.

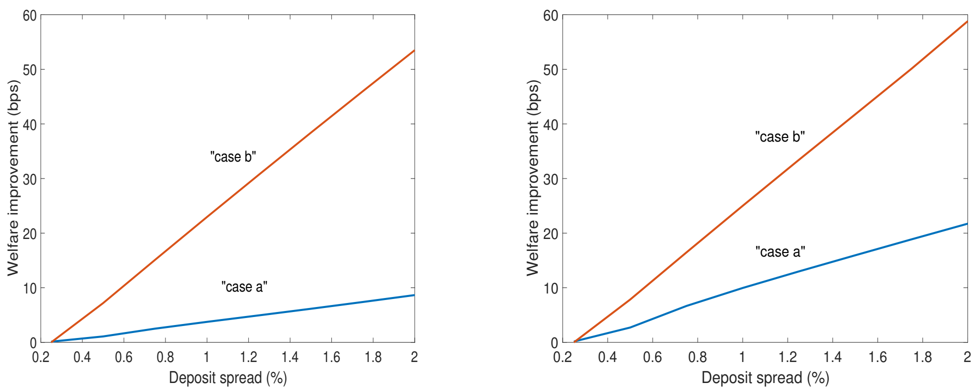
6 Conclusion

There is an intense discussion in policy circles about the potential introduction of a broad retail CBDC. While there are various microeconomic aspects related to its implementa-

Table 3: CBDC-induced welfare changes with alternative parameter values				
	$\tau_h = 25\%$ case a	$\tau_h = 25\%$ case b	$\tau_h = 45\%$ case a	$\tau_h = 45\%$ case b
Baseline	+9 bps	+54 bps	+20 bps	+59 bps
$\gamma = 0.25$	+28 bps	+55 bps	+31 bps	+60 bps
$\gamma = 4$	+2 bps	+54 bps	+15 bps	+58 bps
$\iota = -0.12$	+21 bps	+82 bps	+35 bps	+ 85bps
$c_d = 0.005$	+7 bps	+45 bps	+16 bps	+47 bps
Reserve ratio = 0	+11 bps	+58 bps	+22 bps	+63 bps
Reserve ratio = 10%	+8 bps	+ 49 bps	+17 bps	+54 bps
$\tau_b = 35\%$	+8 bps	+ 48 bps	+17 bps	+ 53 bps
$\tau_b = 15\%$	+10 bps	+ 60 bps	+23 bps	+65 bps
$\epsilon^l = 4$	+9 bps	+54 bps	+20 bps	+59 bps
$\varphi = 0.3$	+9 bps	+54 bps	+20 bps	+59 bps
$wealth/c = 2$	+9 bps	+54 bps	+20 bps	+59 bps
	case a		case b	
Lump-sum taxes	+3bps		+49bps	

Figure 7: Welfare gain as a function of deposit spread

$\tau_h = 25\%$ $\tau_h = 45\%$



tion, in this paper we consider its macroeconomic implications. Most likely, CBDC will not be a perfect substitute of cash or bank deposits. This imperfect substitutability is a key element in our analysis and we show the impact of CBDC under various degrees of substitutability. In our welfare analysis, we find that CBDC could be an instrument to mitigate two distortions in the economy: distortionary taxation and the opportunity cost of holding money, which is much higher than the cost of providing money. Clearly this benefit would be higher, the higher the extent of the distortions. In our benchmark case, we find that the benefits of CBDC in reducing distortions would be modest: even in economies with high labor taxes (around 45%), welfare would improve at most by 20 bps in consumption terms. Instead, we found higher welfare gains from the redistribution of rents associated to deposits from bankers to non-bankers. The welfare improvement to non-bankers (and to the whole population in the limit in which bankers are a negligible minority) could reach about 60 bps when taking into account this channel. The welfare gains might be higher in countries in which the Frisch elasticity and/or the interest semi-elasticity of money demand is very high. Indeed, these are the cases in which the two distortions mentioned above have stronger effect on the economy.

Appendix

A. Household FOCs

FOC with respect to consumption

$$\frac{1}{c_t} = \lambda_t(1 + s(x_t) + x_t s'(x_t)) \tag{38}$$

Specialized to the case of the transaction cost in the form (7), (A.10) becomes

$$\frac{1}{c_t} = \lambda_t(1 + 2Ax_t - 2\sqrt{AB}) \tag{39}$$

FOC with respect to hours worked

$$h_t^\gamma = \lambda_t W_t (1 - \tau_h) \tag{40}$$

FOC with respect to bank deposits d_t^b

$$\lambda_t \left(1 - (Ax_t^2 - B)\alpha_b \left(\frac{d}{d^b}\right)^{\frac{1}{\epsilon_{cb}}}\right) = \lambda_{t+1}(1 + r_t^b) \tag{41}$$

FOC with respect to CBDC d_t^c

$$\lambda_t\left(1-(Ax_t^2-B)\alpha_c\left(\frac{d}{d_c}\right)^{\frac{1}{\epsilon_{cb}}}\right)=\lambda_{t+1}(1+r_t^c) \tag{42}$$

FOC with respect to the risk-free asset a_t

$$\lambda_t=\lambda_{t+1}(1+r^*) \tag{43}$$

(39), (40), (41), (42) and (43) imply the three Euler equations:

$$\frac{1}{c_t(1+2Ax_t-2\sqrt{AB})}\left(1-(Ax_t^2-B)\alpha_b\left(\frac{d}{d_b}\right)^{\frac{1}{\epsilon_{cb}}}\right)=\beta(1+r_t^b)\frac{1}{c_{t+1}(1+2Ax_{t+1}-2\sqrt{AB})} \tag{44}$$

$$\frac{1}{c_t(1+2Ax_t-2\sqrt{AB})}\left(1-(Ax_t^2-B)\alpha_c\left(\frac{d}{d_c}\right)^{\frac{1}{\epsilon_{cb}}}\right)=\beta(1+r_t^c)\frac{1}{c_{t+1}(1+2Ax_{t+1}-2\sqrt{AB})} \tag{45}$$

$$\frac{1}{c_t(1+2Ax_t-2\sqrt{AB})}=\beta(1+r^*)\frac{1}{c_{t+1}(1+2Ax_{t+1}-2\sqrt{AB})} \tag{46}$$

and the labor/leisure tradeoff condition

$$h_t^{\gamma}=\frac{W_t(1-\tau_h)}{c_t(1+2Ax_t-2\sqrt{AB})} \tag{47}$$

B. Steady state equations

In steady state, the real interest rate is $\hat{r}=\beta^{-1}-1$ and the nominal risk-free rate is r^* such that $\frac{1+r^*}{1+\pi^*}=1+\hat{r}$; the central bank pays a constant rate r^m on bank reserves and r^c on CBDC; banks pay a constant rate r^b on deposits, related to the rate on reserves and to model parameters by (30) (or (31) if there is a zero-lower-bound on nominal rates), and demand a constant loan rate r^l given by (32). The unit cost of capital is $\hat{r}^k=\varphi r^l+(1-\varphi)r^*-\pi^*$. Given these rates, households choose a constant money velocity

$$x=\sqrt{\frac{r^*-r^{comp}+B(1+r^*)}{(1+r^*)A}} \tag{48}$$

with

$$f=\left(\frac{r^*-r^b}{\alpha_b(r^*-r^{comp})}\right)^{\epsilon_{cb}} \tag{49}$$

The other relevant variables of the model, consumption c, labor h, capital k, real wages $\hat{w}\equiv\frac{w}{p}$, real loans $\hat{l}$,real bank deposits $\hat{d}^b$, CBDC $\hat{d}^c$, real bank profits $\hat{\Pi}$ (we use the hatted symbols for these variables to distinguish them from their nominal counterparts) are determined by the following equations

$$
\begin{aligned}
c(1+s(x)) &= (1-\tau_h)\hat{w}h + wealth\ \hat{r} - \hat{d}^b(r^* - r^b) - \hat{d}^c(r^* - r^c) + \zeta(1-\tau_b)\hat{\Pi}^b - \hat{t} \\
&= (1-\tau_h)\hat{w}h + wealth\ \hat{r} - \hat{d}(r^* - r^{comp}) + \zeta(1-\tau_b)\hat{\Pi}^b - \hat{t} && (50) \\
\hat{d}^b &= \frac{c}{fx} && (51) \\
\hat{d}^c &= \left(\frac{\alpha_c}{\alpha_b} \times \frac{r^* - r^b}{r^* - r^c}\right)^{\epsilon_{cb}} \hat{d}^b && (52) \\
h^\gamma &= \frac{\hat{w}(1-\tau_h)}{c(1+s(x)+xs'(x))} && (53) \\
k &= \left(\frac{z\alpha}{\hat{r}^K}\right)^{\frac{1}{1-\alpha}} h && (54) \\
\hat{w} &= (1-\alpha)z\left(\frac{z\alpha}{\hat{r}^K}\right)^{\frac{\alpha}{1-\alpha}} && (55) \\
\hat{l} &= \varphi k && (56) \\
\hat{\Pi} &= ((r^* - r^b - c^b) - \phi(r^* - r^m))\hat{d}^b + (r_l - r^* - c^l)\hat{l} && (57)
\end{aligned}
$$

Finally, we assume that the government, in order to finance an exogenous expenditure g, chooses an exogenous tax on profits τ_b and sets the labor tax τ_h endogenously, so that

$$\tau_h = \frac{g - \tau_b\hat{\Pi}^b - (r^* - r^m_{t-1})\hat{m} - (r^* - r^c - c^c)\hat{d}^c}{\hat{w}h} \quad (58)$$

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CBDC as Imperfect Substitute to Bank Deposits: a Macroeconomic Perspective.

Online Appendix

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A Model with Cash

We now present a model including three types of money: bank deposits, cash and CBDC. In our modeling, central-bank-issued money d^c is a composite of cash and CBDC

$$d_t^c = (\alpha_{cash}(cash_t)^{\frac{\varepsilon-1}{\varepsilon}} + \alpha_{cbdc}(cbdc_t)^{\frac{\varepsilon-1}{\varepsilon}})^{\frac{\varepsilon}{\varepsilon-1}} \tag{A.1}$$

with

$$\alpha_{cash}^{\varepsilon} + \alpha_{cbdc}^{\varepsilon} = 1 \tag{A.2}$$

The transaction cost of consumption $s(x_t)$ is still a function of money velocity $x = pc/d$ and the composite money instrument d_t is still given by

$$d_t = \left(\alpha_c(d_t^c)^{\frac{\epsilon_{cb}-1}{\epsilon_{cb}}} + \alpha_b(d_t^b)^{\frac{\epsilon_{cb}-1}{\epsilon_{cb}}}\right)^{\frac{\epsilon_{cb}}{\epsilon_{cb}-1}} \tag{A.3}$$

as in (1). However d_t^c is now reinterpreted as the composite (A.1), while d_t^b is still the composite of bank deposits (2). Cash pays zero interest and CBDC pays interest r_t^{cbdc}.

The household budget constraint is now

$$\begin{aligned}(1-\tau_h)w_t h_t + (1+r_t^*)a_{t-1} + \int (1+r_{t-1}^b(j))d_{t-1}^b(j)dj + (1+r_{t-1}^{cbdc})cbdc_{t-1} + cash_{t-1} \\ + \zeta(1-\tau_b)\Pi_t^b = p_t c_t(1+s_t) + \int d_t^b(j)dj + cbdc_t + a_t + cash_t + p_t t_t\end{aligned} \tag{A.4}$$

First-order conditions (39),(40),(41),(43) are unchanged, however the FOC with respect to the central-bank-issued money (42) needs to be replaced with two conditions, with respect to cash and CBDC, respectively

$$\lambda_t\left(1-(Ax_t^2-B)\alpha_c\alpha_{cash}\left(\frac{d}{d_c}\right)^{\frac{1}{\epsilon_{cb}}}\left(\frac{d_c}{cash}\right)^{\frac{1}{\varepsilon}}\right) = \frac{\lambda_{t+1}}{c_{t+1}(1+2Ax_{t+1}-2\sqrt{AB})} \tag{A.5}$$

$$\lambda_t\left(1-(Ax_t^2-B)\alpha_c\alpha_{cbdc}\left(\frac{d}{d_c}\right)^{\frac{1}{\epsilon_{cb}}}\left(\frac{d_c}{cbdc}\right)^{\frac{1}{\varepsilon}}\right) = \frac{\lambda_{t+1}(1+r^{cbdc})}{c_{t+1}(1+2Ax_{t+1}-2\sqrt{AB})} \tag{A.6}$$

(A.5)and (A.6), together with (43), imply that the optimal cash and CBDC holdings satisfy

$$\frac{cash_t}{cbdc_t} = \left(\frac{\alpha_{cash}}{\alpha_{cbdc}}\frac{r_t^* - r_t^{cbdc}}{r_t^*}\right)^{\varepsilon} \tag{A.7}$$

and imply the equilibrium relationship

$$cash_t r_t^* + cbdc_t(r_t^* - r_t^{cbdc}) = d_t^c(r^* - r_t^c) \tag{A.8}$$

where r_t^c is now defined via the relationship

$$(r^* - r_t^c) = \left((r^*)^{1-\varepsilon} \alpha_{cash}^{\varepsilon} + (r_t^* - r_t^{cbdc})^{1-\varepsilon} \alpha_{cbdc}^{\varepsilon} \right)^{\frac{1}{1-\varepsilon}} \tag{A.9}$$

Furthermore, from the Euler equations (41) and (A.6) we have

$$\frac{d_t^c}{d_t^b} = \left(\frac{\alpha_c}{\alpha_b} \alpha_{cbdc} \left(\frac{d_t^c}{cbdc_t} \right)^{\frac{1}{\varepsilon}} \frac{r_t^* - r_t^b}{r_t^* - r_t^{cbdc}} \right)^{\varepsilon} \tag{A.10}$$

By combining (A.7) and (A.1)

$$d_t^c = \frac{cbdc_t}{\alpha_b^{\varepsilon}} \left(\frac{r_t^* - r_t^{cbdc}}{r - r^c} \right)^{\varepsilon} \tag{A.11}$$

Inserting (A.11) in (A.10) we re-obtain the relationship

$$d_t^b = \left(\frac{\alpha_b}{\alpha_c} \times \frac{r_t^* - r_t^c}{r_t^* - r_t^b} \right)^{\epsilon_{cb}} d_t^c \tag{A.12}$$

showing that in equilibrium resources are split between bank deposits and the "basket" of cash and CBDC the same way that they were split between bank deposits and CBDC in the model with two instruments. (A.7) also implies that he total opportunity cost of holding money is

$$d_t^b(r^* - r_t^b) + cash_t r_t^* + cbdc_t(r_t^* - r_t^{cbdc}) = d_t^b(r^* - r_t^b) + d_t^c(r^* - r_t^c) \tag{A.13}$$

This analysis shows that with cash as a third instrument, economic outcomes may be unchanged. Cash pays zero interest by construction. If the composite interest defined by (A.9), that can be interpreted as the interest paid by the "basket" of cash and CBDC, equals the interest paid by CBDC in the model with only two instruments, all outcomes are identical: households allocate the same resources in money instruments – implying that they incur the same transaction cost of consumption – and pay the same opportunity cost (A.13) of holding money.

B Proof of Proposition 1

If $\alpha_b^{\epsilon_{cb}} \epsilon_{cb} > 1$ and the marginal cost of managing deposits is negligible:

- a) *The interest rate r^c that maximizes seigniorage is larger than the interest rate on deposits r^b,* Neglecting c^c, the component of seigniorage due to CBDC is $\mathcal{S}^{cbdc} = (r^* - r^c)d^c$. Given equations (10), (16) and (12), demand for CBDC can be written as

$$\frac{d^c}{pc} = \alpha_c^{\epsilon_{cb}}(r^* - r^c)^{-\epsilon_{cb}}(r^* - r^{comp})^{\epsilon_{cb}}\sqrt{\frac{A(1+r^*)}{r^* - r^{comp} + B(1+r^*)}} \tag{B.1}$$

so that

$$\mathcal{S}^{cbdc} = \alpha_c^{\epsilon_{cb}} pc(r^* - r^c)^{1-\epsilon_{cb}}(r^* - r^{comp})^{\epsilon_{cb}}\sqrt{\frac{A(1+r^*)}{r^* - r^{comp} + B(1+r^*)}} \tag{B.2}$$

with $(r^* - r^{comp})$ given by (13). Define

$$x \equiv \left(\alpha_b^{\epsilon_{cb}} + \alpha_c^{\epsilon_{cb}}\frac{(r - r^c)^{1-\epsilon_{cb}}}{(r - r^b)^{1-\epsilon_{cb}}}\right)^{\frac{1}{1-\epsilon_{cb}}} \tag{B.3}$$

and remember that $r^* - r^b$ is given.

We can write

$$\mathcal{S}^{cbdc} = \tilde{b}\ (x^{1-\epsilon_{cb}} - \alpha_b^{\epsilon_{cb}})x^{\epsilon_{cb}}(a + bx)^{-1/2} \tag{B.4}$$

with $a \equiv B(1+r^*)$, $b \equiv (r^* - r^b)$ and $\tilde{b} \equiv pc(r^* - r^b)$. Hence

$$\mathcal{S}^{cbdc} = \tilde{b}\ (x - \alpha_b^{\epsilon_{cb}}x^{\epsilon_{cb}})(a + bx)^{-1/2} \tag{B.5}$$

The variable x is an increasing function of $(r^* - r^c)$ and the FOC of the problem can be obtained by differentiating with respect to x. The FOC wrt x reads

$$\left(1 - \alpha_b^{\epsilon_{cb}}\epsilon_{cb}x^{\epsilon_{cb}-1} - \frac{1}{2}(x - \alpha_b^{\epsilon_{cb}}x^{\epsilon_{cb}})\frac{b}{(a+bx)}\right) = 0 \tag{B.6}$$

Clearly seigniorage (B.5) and the factor $(x - \alpha_b^{\epsilon_{cb}}x^{\epsilon_{cb}})$ are positive for any value of $r^c < r^*$. Hence for x to be an interior maximum of seigniorage it must be that

$$\alpha_b^{\epsilon_{cb}}\epsilon_{cb}x^{\epsilon_{cb}-1} < 1 \tag{B.7}$$

So if $\alpha_b^{\epsilon_{cb}}\epsilon_{cb} > 1$, then $x^{\epsilon_{cb}-1} < 1$. Hence, since $\epsilon_{cb} > 1$, it must be that $x < 1$. From the definition of x (B.3) and the relationship $\alpha_b^{\epsilon_{cb}} + \alpha_b^{\epsilon_{cb}} = 1$, it follows that, in order for $x < 1$, we need $(r^* - r^c) < (r^* - r^b)$, or $r^c > r^b$.

- b) *The optimal value of r^c is decreasing in the CBDC liquidity parameter α_c, if $\epsilon_{cb} > 1.5$*

  Define $\tilde{\alpha}_b \equiv \alpha_b^{\epsilon_{cb}}$, and

$$\mu(x, \tilde{\alpha}_b) = \left(1 - \tilde{\alpha}_b\epsilon_{cb}x^{\epsilon_{cb}-1} - \frac{1}{2}(x - \tilde{\alpha}_b x^{\epsilon_{cb}})\frac{b}{(a+bx)}\right) \tag{B.8}$$

where x is defined in (B.3). For each value of $\tilde{\alpha}_b$ the value of x maximizing seigniorage we have $\mu(x(\tilde{\alpha}_b), \tilde{\alpha}_b) = 0$.

The implicit value theorem tells us how the seigniorage-maximizing value of x varies with $\tilde{\alpha}_b$:

$$\frac{\partial x}{\partial \tilde{\alpha}_b} = -\frac{\frac{\partial \mu}{\partial \tilde{\alpha}_b}}{\frac{\partial \mu}{\partial x}} \tag{B.9}$$

and we have

$$\frac{\partial \mu}{\partial \tilde{\alpha}_b} = -x^{\epsilon_{cb}-1}\left(\epsilon_{cb} - \frac{1}{2}\frac{bx}{(a+bx)}\right) < 0 \tag{B.10}$$

$$\begin{aligned}\frac{\partial \mu}{\partial x} = \quad & \tilde{\alpha}_b x^{\epsilon_{cb}-2}\left(-(\epsilon_{cb}-1)\left(\epsilon_{cb} - \frac{1}{2}\frac{bx}{a+bx}\right) + \frac{1}{2}\frac{bx}{a+bx} - \frac{1}{2}\frac{(bx)^2}{(a+bx)^2}\right) \\ = \quad & -\epsilon_{cb}\left((\epsilon_{cb}-1) - \frac{1}{2}\frac{bx}{a+bx}\right) - \frac{1}{2}\frac{(bx)^2}{(a+bx)^2}\end{aligned} \tag{B.11}$$

Since $a > 0$, $b > 0$ and hence $\frac{bx}{a+bx}$, the first term on the RHS of (B.11) is negative for $\epsilon_{cb} > 1.5$. Thus, we have $\frac{\partial \mu}{\partial x} < 0$. Hence, for $\epsilon_{cb} > 1.5$ we have

$$\frac{\partial x}{\partial \tilde{\alpha}_b} = -\frac{\frac{\partial \mu}{\partial \tilde{\alpha}_b}}{\frac{\partial \mu}{\partial x}} < 0 \tag{B.12}$$

Since $\alpha_c^{\epsilon_{cb}} = 1 - \alpha_b^{\epsilon_{cb}}$, we deduce that the seigniorage-maximizing value of x is increasing in α_c, hence, from the definition of x (B.3), the seigniorage-maximizing CBDC spread $r^* - r^c$ is increasing in α_c, or equivalently, the seigniorage-maximizing r^c is decreasing in α_c.

- c) *The peak value of seigniorage in the r^c dimension is increasing in the CBDC liquidity parameter α_c and is increasing in the substitutability parameter ϵ_{cb}.*

  (B.5) implies that the component of seigniorage due to CBDC can be written as

$$\mathcal{S}^{cbdc} = \tilde{b}\,(x - (1-\tilde{\alpha}_c)x^{\epsilon_{cb}})(a+bx)^{-1/2} \tag{B.13}$$

The seigniorage-maximizing value of x is a function of $\tilde{\alpha}_c$ and ϵ_{cb}, hence $\mathcal{S}^* = \mathcal{S}^*(x(\tilde{\alpha}_c, \epsilon_{cb}), \tilde{\alpha}_c, \epsilon_{cb})$ where $\mathcal{S}^* = max_{r^c}\mathcal{S}$. By the envelope theorem peak seigniorage satisfies

$$\frac{d\mathcal{S}^*}{d\tilde{\alpha}_c} = \frac{\partial \mathcal{S}^*}{\partial \tilde{\alpha}_c} \tag{B.14}$$

$$\frac{d\mathcal{S}^*}{d\epsilon_{cb}} = \frac{\partial \mathcal{S}^*}{\partial \epsilon_{cb}} \tag{B.15}$$

It is easy to see from (B.13) that $\frac{\partial \mathcal{S}^*}{\partial \tilde{\alpha}_c} > 0$ and that $\frac{\partial \mathcal{S}^*}{\partial \epsilon_{cb}} > 0$ when x is smaller than 1. Since, as proved in point a), the seigniorage-maximizing value of x is smaller than 1, we have indeed that $\frac{\partial \mathcal{S}^*}{\partial \tilde{\alpha}_c} > 0$ and $\frac{\partial \mathcal{S}^*}{\partial \epsilon_{cb}} > 0$, i.e. the peak value of seigniorage in the r^c dimension is increasing in $\tilde{\alpha}_c$ and ϵ_{cb}.

C Proof of Proposition 2

Under the conditions of Proposition 1 the maximum of seigniorage is achieved in the limit $\epsilon_{cb} \to \infty$ (so that the two monies are perfect substitutes) and r^c infinitesimally higher than r^b.

As per point c) in Proposition 1) the peak value of seigniorage is increasing in the parameter ϵ_{cb}. So, in the region $\alpha_b^{\epsilon_{cb}} \epsilon_{cb} > 1$, the maximum value of seigniorage is achieved in the limit $\epsilon_{cb} \to \infty$. We can write

$$(r^* - r^{comp}) = \alpha_c^{\frac{\epsilon_{cb}}{1-\epsilon_{cb}}} (r^* - r^c) \left(1 + \frac{\alpha_b^{\epsilon_{cb}}}{\alpha_c^{\epsilon_{cb}}} \frac{(r^* - r^b))^{1-\epsilon_{cb}}}{(r^* - r^c))^{1-\epsilon_{cb}}} \right)^{\frac{1}{1-\epsilon_{cb}}} \tag{C.1}$$

In the limit $\epsilon_{cb} \to \infty$, since $0 < \alpha_b^{\epsilon_{cb}} < 1$ and $0 < \alpha_c^{\epsilon_{cb}} < 1$, $\alpha_c^{\frac{\epsilon_{cb}}{1-\epsilon_{cb}}} = 1$, and, since by Proposition 1 $(r^* - r^c) < (r^* - r^b)$, $(r^* - r^{comp}) \to (r^* - r^c)$. Intuitively, in the limit $\epsilon_{cb} \to \infty$, bank deposits and CBDC are perfect substitutes, and as long as $(r^* - r^c) < (r^* - r^b)$, CBDC completely outcompetes bank deposits.

The component of seigniorage due to CBDC (B.2) can then be written as

$$\mathcal{S}^{cbdc} = pc\ \alpha_c^{\epsilon_{cb}} (r^* - r^c) \sqrt{\frac{A(1 + r^*)}{r^* - r^c + B(1 + r^*)}} \tag{C.2}$$

This is analogous to the seigniorage achieved by a monopolist bank, but only under the constraint $(r^* - r^c) < (r^* - r^b)$. Without this constraint, the seigniorage-maximizing r^c would surely be below r^b since the latter is the optimal interest chosen by banks in monopolistic competition. Hence the constraint $(r^* - r^c) < (r^* - r^b)$ is binding, and the optimal r^c is just infinitesimally above r^b. This holds independently of the value of α_c.

D Proof of Proposition 3

The interest rate on CBDC that maximizes consumption and welfare is decreasing in the labor tax rate and is decreasing in the share of banks held by households.

For each value of r^c, or equivalently of $r^* - r^c$, households optimally choose h, d^b, d^c (labor, bank deposits and CBDC) so that $\frac{\partial W}{\partial h} = \frac{\partial W}{\partial d^b} = \frac{\partial W}{\partial d^c} = 0$ where W is welfare. The optimal value of $r^* - r^c$ chosen by the government/central bank is such that $\frac{dW}{d(r^*-r^c)} = 0$. The consumption-maximizing value of $r^* - r^c$ is such that $\frac{dc}{d(r^*-r^c)} = 0$. We'll show that the same value of $(r^* - r^c)$ optimizes both welfare and consumption.

We have

$$\begin{aligned} \frac{dW}{d(r^* - r^c)} &= \frac{\partial W}{\partial h}\frac{\partial h}{\partial (r^* - r^c)} + \frac{\partial W}{\partial d^b}\frac{\partial d^b}{\partial (r^* - r^c)} + \frac{\partial W}{\partial d^c}\frac{\partial d^c}{\partial (r^* - r^c)} \\ &+ \frac{\partial W}{\partial \tau_h}\frac{\partial \tau_h}{\partial (r^* - r^c)} + \frac{\partial W}{\partial (r^* - r^c)} \end{aligned} \tag{D.1}$$

Since the first three terms on the RHS of (D.1) are zero due to the households FOCs we have (envelope theorem)

$$\frac{dW}{d(r^* - r^c)} = \frac{\partial W}{\partial \tau_h}\frac{\partial \tau_h}{\partial (r^* - r^c)} + \frac{\partial W}{\partial (r^* - r^c)} \tag{D.2}$$

and similarly

$$\frac{dc}{d(r^* - r^c)} = \frac{\partial c}{\partial \tau_h}\frac{\partial \tau_h}{\partial (r^* - r^c)} + \frac{\partial c}{\partial (r^* - r^c)} \tag{D.3}$$

Given the utility function (4), steady state welfare is $W = \frac{1}{1-\beta}\left(log(c) - \frac{h^{1+\gamma}}{1+\gamma}\right)$, with steady state consumption c given by (50). We have

$$\frac{\partial W}{\partial \tau_h} = \frac{1}{(1-\beta)c}\,\frac{\partial c}{\partial \tau_h} \tag{D.4}$$

$$\frac{\partial W}{\partial (r^* - r^c)} = \frac{1}{(1-\beta)c}\,\frac{\partial c}{\partial (r^* - r^c)} \tag{D.5}$$

The latter two equations imply that the same r^c maximizes welfare and consumption. We can therefore focus on analyzing the condition $\frac{dc}{d(r^*-r^c)} = 0$. To this effect, the first thing to notice is that finding the partial derivatives $\frac{\partial c}{\partial \tau_h}$, $\frac{\partial c}{\partial (r^*-r^c)}$ is complicated by the fact that we don't have an explicit solution for c, rather c is the solution of (50). We define the function

$$\mu(c,\tau_h,(r^*-r^c)) = -c(1+s(x)) + (1-\tau_h)wh$$
$$+ \quad wealth\ \hat{r} - d^b\frac{r^*-r^b}{1+\pi^*} - d^c\frac{r^*-r^b}{1+\pi^*} + \zeta(1-\tau_b)\Pi^b - t \qquad \text{(D.6)}$$

Notice that c appears in different places in this function, including the transaction cost $s(x)$. By the implicit function theorem we then have

$$\frac{\partial c}{\partial \tau_h} = -\frac{\frac{\partial \mu}{\partial \tau_h}}{\frac{\partial \mu}{\partial c}}$$
$$\frac{\partial c}{\partial (r^*-r^c)} = -\frac{\frac{\partial \mu}{\partial (r^*-r^c)}}{\frac{\partial \mu}{\partial c}} \qquad \text{(D.7)}$$

Given (53) and (58) τ_h solves the equation

$$\tau_h(1-\tau_h)^{\frac{1}{\gamma}} = \frac{(g-\tau_b\Pi_b-\mathcal{S})}{w^{1+\frac{1}{\gamma}}}c^{\frac{1}{\gamma}}(1+s(x)+xs'(x))^{\frac{1}{\gamma}} \qquad \text{(D.8)}$$

With some algebra we obtain

$$\frac{\partial \tau_h}{\partial(r^*-r^c)} = -\frac{1}{wh}\frac{\left(\tau_b\frac{\partial \Pi}{\partial(r^*-r^c)}+\frac{\partial \mathcal{S}}{\partial(r^*-r^c)}\right)}{1-\frac{1}{\gamma}\frac{\tau_h}{1-\tau_h}} + \frac{1}{\gamma}\frac{1}{wh}\frac{\partial ln(s(x)+xs'(x))}{\partial(r^*-r^c)} \qquad \text{(D.9)}$$

and

$$\frac{dc}{d(r^*-r^c)} = -\frac{1}{\frac{\partial\mu}{\partial c}}\left(-d^c - \frac{1}{\gamma}\frac{\partial ln(s(x)+xs'(x))}{\partial(r^*-r^c)}\right.$$
$$\left. + \quad \zeta(1-\tau_b)\frac{\partial \Pi}{\partial(r^*-r^c)} + \frac{\left(\tau_b\frac{\partial \Pi}{\partial(r^*-r^c)}+\frac{\partial \mathcal{S}}{\partial(r^*-r^c)}\right)}{1-\frac{1}{\gamma}\frac{\tau_h}{1-\tau_h}}\right) \qquad \text{(D.10)}$$

We notice that the factor $-\frac{1}{\frac{\partial\mu}{\partial c}}$ is positive. The first two terms inside the parenthesis on the RHS of (D.10) are negative for all values of r^*-r^c, thus tending to push the optimal value of (r^*-r^c) toward zero (or equivalently r^c toward r^*). The last terms on the right-hand side of (D.10) are positive (the term proportional to $\frac{\partial \mathcal{S}}{\partial(r^*-r^c)}$, conveying the seigniorage channel, is positive for values of r^c higher than the seigniorage-maximixing value), thus tending to push the optimal (r^*-r^c) toward an interior value. Now:

- As the share ζ of banks held by households decreases, the positive terms on the RHS (D.10) decrease and the negative terms stay constant, resulting in a lower value of the consumption-maximizing (and seigniorage-maximizing) $(r^* - r^c)$, or equivalently in a higher value of r^c.

- As τ_h increases, the positive terms on the RHS (D.10) increase, and the negative terms stay constant, again resulting in a a higher value of the optimal r^c.