A probabilistic analysis of governmental forms

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Abstract

This work presents a probabilistic analysis of governmental forms classically catalogued. Its especial findings are that, regardless of human concupiscence, (i) tyrannies are the most probable (evil) forms of classical government as well as the easiest ones to vanquish and (ii) monarchies are the most probable good forms of classical government, being preferable to all others both quantitatively and qualitatively, that is, probabilistically and ethico-metaphysically.

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1. Introduction

This work's contribution is the presentation of a probabilistic analysis of governmental forms, namely, it sets forth an assessment of the forms of government classically catalogued by means of the theory of probability. In order to achieve so it however first presents a study of the nature of governmental forms (Section 2). The probabilistic analysis of governmental forms classically catalogued is found in Section 3.

Section 4 concludes as follows. Aristocracies, monarchies and timocracies, which are the three classical forms of government, are indifferent inasmuch as natural law *(lex naturalis)* and eternal law *(lex aeterna)* may be upheld. Aristocratic rule is government by the aptest and is thereby timocratic, admitting of aristocratic monarchies and of aristocratic timocracies, strengthening such an indifference.

Monarchic rule is substantially ubiquitous to aristocracies and to timocracies as well, profiling them as accidental to it. Irrespective of human concupiscence, tyrannies are the most probable (evil) forms of classical government as well as the easiest ones to vanquish and monarchies are the most probable good forms of classical government. In view of such all aristocratic monarchies are to be ultimately preferred, polymathically, philosophically and religiously.

2. Nature of governmental forms

2.1 Monarchy, aristocracy and timocracy. Following Saint Thomas Aquinas [1], the forms of government classically catalogued are monarchies, aristocracies and timocracies. The etymological and common meaning of a monarchy *(monarkhia)* is a form of "government by one individual".

The etymological meaning of an aristocracy *(aristokratia)* is a form of "power by the best individual", while the common meaning of it is a form of "power by the best individuals", in the plural, historically identified with the rich, wealthy, noble or well born members of society (i.e. nobility).

The aristocratic aptness canon can be ultimately understood as being that of a virtuous man, who knows in light of the Remote Cause and judges straightly, ordaining everything to his end, especially his life, thereby living virtuously.

The etymological meaning of a timocracy *(timokratia)* according to the Platonic school is a form of "power by an honourable individual", the etymological meaning of a timocracy according to the Aristotelian

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school is a form of “power by an individual supplied with an endowment”, while the common meaning of it is a form of “power by honourable individuals”, in the plural.

Platonic timocracies can be functionally equated with polities. The etymological meaning of a polity *(politeia)* is “citizenship” and the common, pertinent meaning of it is a form of “civil government”. The customary deviations of the said three forms of government, substantially oppressive, are respectively catalogued as tyrannies, oligarchies and democracies.

2.2 Tyranny, oligarchy and democracy. The etymological and common meaning of a tyranny *(tyrannia)* is a form of “rule by absolutely one individual”. Tyrannies can be functionally equated with despotism, whose etymological and common meaning is a form of “rule by a sovereign” *(despotes),* and are often misidentified with dictatorships, autocracies, monocracies and sometimes even with totalitarianism, whose etymologies one can ignore absent loss of generality.

While dictatorships, autocracies and monocracies be merely synonymous with monarchies, not being oppressive of themselves, totalitarianism speaks to the coerced subservience of the individual to the state. According to Scholastic tradition, a monarch can degenerate into two types of tyrant: a legally selected monarch who subsequently becomes a tyrant; an illegally selected monarch who is thereby a tyrant (i.e. usurper).

The etymological and common meaning of an oligarchy *(oligarkhia)* is a form of “government by few individuals”. The etymological and common meaning of a democracy *(demokratia)* is a form of “power by the people”. Democracies can be functionally equated with demagogies.

The etymological meaning of a demagogy is a form of “leadership proper to the people” *(agogos demos),* while the common meaning of it is a form of “sway by individuals through the use of empty rhetoric”. Democracies often mask plutocracies, whose etymological and common meaning is a form of “power by wealthy individuals”. Oligarchies and democracies can be consequently said to be tyrannies of the few and of the people, respectively.

2.3 Monarchic substance. Non-monarchies, oppressive or not, arise under the form of republics. The etymological meaning of a republic *(res publica)* is “the thing proper to the people”, while the common meaning of it is a form of “government representative of the people”.

Republics are governmental forms antithetical to monarchies in that aristocracies and timocracies cannot arise under a monarchy by definition, being thereby encompassed by republics. Strictly speaking, however, both aristocracies and timocracies are substantially monarchic, inasmuch as decisions at the margin be taken by one individual, be it through the use of reason or without it: $\Box A_M$ and $\Box T_M$. In fact, decisions are taken by a single entity even if they be not at the margin, be it an individual or a group, absolutely majoritarian or relatively majoritarian (i.e. minoritarian).

Additionally, both monarchies and timocracies can be aristocratic: $\Diamond M_A$ and $\Diamond T_A$. By contrast, while an aristocracy be timocratic in itself, honour being guaranteed by aptness, but not the converse, monarchies can, but need not, be timocratic: $\Box A_T$, but $\Diamond M_T$.

In a word, monarchies are ubiquitous to all three classical forms of government and the aristocratic canon, inherently timocratic, is to perfect whichever form were chosen: $\Box M_M$, $\Box A_{M,\ A\vdash T}$ and $\Box T_{M,\ T}$; $\Diamond M_{A\vdash T}$ and $\Diamond T_{A\vdash T}$. Consequently, even republics are substantially monarchic.

It is thus the accidental and customary differentiation of the three which permits one to oppose monarchies to republics. More specifically, republics are necessary, but insufficient conditions for aristocracies and timocracies, since republics can be literally monarchic too: $A,\ T \longrightarrow R$, but $R \not\longrightarrow A,\ T$, since $(R \wedge \neg A,\ \neg T) = (R \wedge M)$. In other words, aristocracies and timocracies can be said to be republican, whereas republics can be said to admit of aristocracies and of timocracies and to literally admit even of monarchies.

Under such a light single party states are (i) monarchic insofar as one individual proper to the one and only party present in the nation possess all powers of the state and (ii) republican insofar as more than one individual proper to the one and only party present in the nation possess all powers of the state, namely, insofar as the powers of the state be distributed amongst more than one individual proper to the one and only party present in the nation. There can additionally exist federal monarchies as there can exist federal republics (i.e. federations, federacies, confederations, confederacies).

On account of the etymological meaning of a republic, republican power is often misunderstood as coming from below, rather than above, namely, from the people rather than from God, by which it would be passed down through aptness or inheritance.

Such need not be the case, although, because the accidental and customary differentiation of the three classical forms of governments allows aristocracies, timocracies and monarchies, all admitted by republics, to admit the procession of republican power from above.

3. Probabilistic analysis

3.1 Best form of government. The departure from natural and eternal law can be metaphysically said to be the defining characteristic of the deviations from the three classical forms of government. Monarchies, aristocracies and timocracies instead anchor the election of their social norms to natural and eternal law.

Unanimous election of social norms is therefore necessary, but insufficient for the three classical forms of government to be in place, for social norms must be unanimously elected in accord with natural and eternal law. It follows that as long as there be unanimity on social norm election and as long as social norms be anchored to natural and eternal law, so that positive law participate thereof, governmental forms are normatively indifferent, as taught by Pope Leo XIII [2, 3, 4].

Scholastic tradition, amongst which figures Saint Thomas Aquinas [1] in particular, positively maintains that the best form of government be a mixed one, combining monarchies with aristocracies or timocracies, even though the noblest one be normatively recognised as being monarchy, on account of its ubiquity.

Such is argued on the grounds of human concupiscence, by which man is prone to operating evil, so that consent to the concupiscence of one be limited by the resistance to the concupiscence of more than one. Not for nothing, Saint Thomas Aquinas [1] held tyrannies to be worse than oligarchies and oligarchies to be worse than democracies in turn, forasmuch as being respectively situated farther from the ethical pursuit of social welfare, by which individual welfare cannot precede it, spiritually and temporally. Yet, the probability that consent to the concupiscence of one be limited by the resistance to the concupiscence of more than one seems low, perhaps lower than that of a monarchy.

3.2 Formalisation. As seen, monarchies and timocracies do not inexorably concern the best, as do aristocracies, but merely unicity and honour, yet, the aristocratic aptness canon can be applied to monarchies and to timocracies as well. Additionally, while honour need not imply aptness, aptness does imply some degree of honour, if only owing to aptness itself.

Unicity is then ubiquitous to all three classical forms of government, for by natural and eternal law there cannot fail to exist a head, however enfeebled it may be (e.g, irrational, multiply minoritarian). On the other hand, because all men are inclined to evil and most men practise evil deviations from monarchies (i.e. tyrannies) appear to be the most probable.

More specifically, while a monarchy may be easier to establish than an aristocracy or a timocracy, a tyranny is expected to arise more easily arise than both (i) a monarchy, an aristocracy and a timocracy and (ii) a perfect oligarchy or a perfect democracy; in fact, it is expected to arise more easily than an imperfect oligarchy or an imperfect democracy as well.

It must be stressed that perfection signifies the practice of evil on the part of all constituting members of an oligarchy or a democracy, while imperfection signifies the practice of evil on the part of only some constituting members of an oligarchy or a democracy. As a consequence, a probabilistic analysis which may formalise such expectations is in order.

Let good be synonymous with the observance of natural and eternal law and let evil be synonymous with its antonym. Let one refer to the probability of being good or evil as the probability of concupiscence. In principle probability density function p transforms the set of positive naturals $\mathbb{N}_+$, representing citizens or members of society, into a semi-open, real interval between zero and one, representing the probability of concupiscence: $p : \mathbb{N}_+ \to (0,\ 1] \subset \mathbb{R}_+$ such that, $\forall i \in \mathbb{N}_+,\ p(i) = p_i \in (0,\ 1] \subset \mathbb{R}_+$ and $\sum_{i=1}^{n} p_i = 1$.

From such information there emerges probability space $(\Omega,\ \mathbb{N}_+,\ p)$, by which Ω is the sample space, $\mathbb{N}_+ \subset \mathcal{P}(\Omega)$ is the σ-algebra and p is the probability measure originating the probability density function.

The real interval between zero and one is semi-open because the probability of being neither good nor evil, $p(i) = 0$, is excluded by free will, which is inexorably exercised. In fact, free will is not even required to exclude the probability of being neither good nor evil, for very preordination to either good or evil suffices to exclude it. Strictly speaking, probability function p is a probability mass function, owing to the discrete nature of domain $\mathbb{N}_+$.

Net of baptismal reception, by which it is therefore ignorable, let there then exist an equal probability of inclination towards good across citizens inferior to that towards evil *(ex ante),* being all thereby strictly concupiscent; such formally signifies that a tyranny is more probable than a monarchy: $p_T \equiv q_i = q(i) = 1 - p(i) > p(i) = p_i \equiv p_M$.

PROPOSITION 3.2.1 (Monarchy) *The probability of a monarchy lies in a closed, real interval between zero and one half. Formally:*

$$p_M \equiv p(i) = p_i \in (0,\ 0.5) \subset \mathbb{R}_{++}. \tag{1}$$

Proof. The probability of a tyranny exceeds the probability of a monarchy by assumption: $(p_T \equiv q_i = q(i) = 1 - p(i) > p(i) = p_i \equiv p_M) \longrightarrow 1 > 2p(i) \longrightarrow 0.5 > p(i)$. In addition, by free will or preordination the probability of a monarchy is positive: $p(i) > 0$. *QED*

PROPOSITION 3.2.2 (Good non-monarchy) *The probability of a good non-monarchy (i.e. aristocracy, timocracy) is the product of the probabilities of concupiscence of its constituting members, which are thereby monarchically identical. Formally:*

$$p_{\neg M_G} \equiv \prod_{i=1}^{n} pr_i = \prod_{i=1}^{n} pr(i) = pr(1)pr(2)\cdots pr(n) = p(1)p(2)\cdots p(n) = p^n(i). \tag{2}$$

Proof. A non-monarchy is characterised by more than one member, whose probability of concupiscence is independent of that of the others: $p_{\neg M} \equiv \prod_{i=1}^{n} pr_i = \prod_{i=1}^{n} pr(i) = pr(1)pr(2)\cdots pr(n)$. All constituting members of a good non-monarchy thus happen to be monarchic, that is, good: $pr(i) = pr(\neg i) = p(i) \longrightarrow p_{\neg M_G} \equiv \prod_{i=1}^{n} pr_i = \prod_{i=1}^{n} pr(i) = \prod_{i=1}^{n} p(i) = p^n(i)$. *QED*

PROPOSITION 3.2.3 (Tyranny) *The probability of a tyranny lies in a closed, real interval between one half and one. Formally:*

$$p_T \equiv [1 - p(i)] = q(i) = q_i \in (0.5,\ 1) \subset \mathbb{R}_{++}. \tag{3}$$

Proof. The probability of a monarchy is smaller than one half: $0.5 > p(i) \longrightarrow 0.5 - 1 > p(i) - 1 \longrightarrow -(1 - 0.5) > -[1 - p(i)] \longrightarrow (1 - 0.5) < [1 - p(i)] \longrightarrow 0.5 < q(i)$. In addition, the probability of a monarchy is positive: $p(i) > 0 \longrightarrow p(i) - 1 > -1 \longrightarrow -[1 - p(i)] > -1 \longrightarrow [1 - p(i)] < 1 \longrightarrow q(i) < 1$. *QED*

PROPOSITION 3.2.4 (Evil, perfect non-tyranny) *The probability of an evil, perfect non-tyranny (i.e. perfect oligarchy, perfect democracy) is the product of the probabilities of concupiscence of its constituting members, which are thereby tyrannically identical. Formally:*

$$\begin{aligned} p_{\neg T_{EP}} &\equiv \prod_{i=1}^{n} pr_i = \prod_{i=1}^{n} pr(i) = pr(1)pr(2)\cdots pr(n) = \prod_{i=1}^{n} q(i) = \\ &= \prod_{i=1}^{n}[1 - p(i)] = [1 - p(1)][1 - p(2)]\cdots[1 - p(n)] = [1 - p(i)]^n. \end{aligned} \tag{4}$$

Proof. A non-tyranny is characterised by more than one member, whose probability of concupiscence is independent of that of the others: $p_{\neg T} \equiv \prod_{i=1}^{n} pr_i = \prod_{i=1}^{n} pr(i) = pr(1)pr(2)\cdots pr(n)$. All constituting members of an evil, perfect non-tyranny thus happen to be tyrannical, that is, evil: $pr(i) = pr(\neg i) = q(i) = [1 - p(i)] \longrightarrow p_{\neg T_{EP}} \equiv \prod_{i=1}^{n} pr_i = \prod_{i=1}^{n} pr(i) = \prod_{i=1}^{n} q(i) = \prod_{i=1}^{n}[1 - p(i)] = [1 - p(i)]^n$. *QED*

PROPOSITION 3.2.5 (Evil, imperfect non-tyranny) *The probability of an evil, imperfect non-tyranny (i.e. imperfect oligarchy, imperfect democracy) is the product of the probabilities of concupiscence of its constituting members, which are thereby not identical. Formally:*

$$p_{\neg T_{EI}} \equiv \prod_{i=1}^{n} pr_i = \prod_{i=1}^{n} pr(i) = pr(1)pr(2)\cdots pr(n), \tag{5}$$

where probability $pr(i) \neq pr(\neg i)$ such that there exists at least one probability $pr(i) = p(i)$ and at least one probability $pr(\neg i) = q(i) = [1 - p(i)]$.

Proof. A non-tyranny is characterised by more than one member, whose probability of concupiscence is independent of that of the others: $p_{\neg T} \equiv \prod_{i=1}^{n} pr_i = \prod_{i=1}^{n} pr(i) = pr(1)pr(2)\cdots pr(n)$. Not all constituting members of an evil, imperfect non-tyranny thus happen to be tyrannical, nor monarchic: $pr(i) \neq pr(\neg i)$ such that $\exists pr(i) = p(i)$ and $pr(\neg i) = q(i) = [1-p(i)]$, thus, $p_{\neg T_{EI}} \equiv \prod_{i=1}^{n} pr_i = \prod_{i=1}^{n} pr(i) = \prod_{i=1}^{n} pr(1)pr(2)\cdots pr(n)$ and no more. *QED*

It follows that probability density function effectively transforms the set of positive naturals $\mathbb{N}_+$ into a closed real interval between zero and one: $p : \mathbb{N}_+ \to (0,\ 1) \subset \mathbb{R}_{++}$ such that, $\forall i \in \mathbb{N}_+,\ p(i) = p_i \in (0,\ 1) \subset \mathbb{R}_{++}$ and $\sum_{i=1}^{n} p_i = 1$.

3.3 Probability combinations. Granted two or more ruling citizens, total probability combinations would be, $\forall n \in [2,\ \infty) \subset \mathbb{N}_+,\ 2 + n(n-1) = 2 + n^2 - n$, where 2 represents the probability combinations of good non-monarchy and evil, perfect non-tyranny and probability combinations $n^2 - n$ represent those of evil, imperfect non-tyranny.

Case 1. Evil, perfect non-tyranny probability $\neg T_{EP} = q^2(i)$, good non-monarchy probability $\neg M_G = p^2(i)$ and evil, imperfect non-tyranny probability $\neg T_{EI} = [q(i)p(i)]_2$, by which ruling citizens $n = 2$, give rise to total probability combinations $2 + 2(2-1) = 2 + 2 = 4$.

Case 2. Evil, perfect non-tyranny probability $\neg T_{EP} = q^3(i)$, good non-monarchy probability $\neg M_G = p^3(i)$, evil, imperfect non-tyranny probability one $\neg T_{EI1} = [q(i)q(i)p(i)]_3$ and evil, imperfect non-tyranny probability two $\neg T_{EI2} = [q(i)p(i)p(i)]_3$, by which ruling citizens $n = 3$, give rise to total probability combinations $2 + 3(3-1) = 2 + 6 = 8$.

Case n-1. Evil, perfect non-tyranny probability $\neg T_{EP} = q^n(i)$, good non-monarchy probability $\neg M_G = p^n(i)$, evil, imperfect non-tyranny probability one $\neg T_{EI1} = [q^{n-1}(i)p(i)]_n$, evil, imperfect non-tyranny probability two $\neg T_{EI2} = [q^{n-2}(i)p^2(i)]_n$, $\ldots$ and evil, imperfect non-tyranny probability $n-1$ $\neg T_{EIn-1} = [q(i)p^{n-1}(i)]_n$, by which ruling citizens n, give rise to total probability combinations $2 + n(n-1)$.

3.4 Cardinal order. There consequently emerge propositions relative to the cardinal order of the three classical forms of government and of their deviations, perfect and imperfect.

Proposition 3.4.1 (Cardinal suborder one) *The probability of a tyranny is greater than the probability of a monarchy, which is itself greater than that of a good non-monarchy. Formally:*

$$\forall n \in [2,\ \infty) \subset \mathbb{N}_+,\ (p_T > p_M > p_{\neg M_G}) \equiv [1 - p(i) > p(i) > p^n(i)]. \tag{6}$$

Proof. The probability of a tyranny exceeds that of a monarchy by assumption: $q(i) = 1 - p(i) > p(i)$. The probability of a monarchy then exceeds that of a good non-monarchy by construction: $\forall n \in [2,\ \infty) \subset \mathbb{N}_+$ and $p(i) \in (0,\ 0.5) \subset \mathbb{R}_{++},\ p(i) > p^n(i)$. In addition, the probability of a monarchy is contained in a proper superset of the set containing the probability of a good non-monarchy: $\forall n \in [2,\ \infty) \subset \mathbb{N}_+,\ p(i) = p_i \in (0,\ 0.5) \subset \mathbb{R}_{++} \longrightarrow p_i^n \in (0,\ 0.5^n) \subset (0,\ 0.5) \subset \mathbb{R}_{++}$. *QED*

Proposition 3.4.2 (Cardinal suborder two) *The probability of a tyranny is greater than the probability of an evil, perfect non-tyranny, which is itself greater than that of a good non-monarchy. Formally:*

$$\forall n \in [2,\ \infty) \subset \mathbb{N}_+,\ (p_T > p_{\neg T_{EP}} > p_{\neg M_G}) \equiv \{1 - p(i) > [1 - p(i)]^n > p^n(i)\}. \tag{7}$$

Proof. The probability of a tyranny exceeds that of an evil, perfect non-tyranny by construction: $\forall n \in [2,\ \infty) \subset \mathbb{N}_+$ and $q(i) \in (0.5,\ 1) \subset \mathbb{R}_{++},\ q(i) = 1 - p(i) > [1 - p(i)]^n = q^n(i)$. In addition, the probability of an evil, perfect non-tyranny is contained in a proper superset of the set containing the probability of a tyranny: $\forall n \in [2,\ \infty) \subset \mathbb{N}_+,\ 1 - p(i) = q(i) = q_i \in (0.5,\ 1) \subset \mathbb{R}_{++} \longrightarrow q_i^n \in (0.5^n,\ 1) \subset \mathbb{R}_{++} \longrightarrow q_i \in (0.5,\ 1) \subset (0.5^n,\ 1) \subset \mathbb{R}_{++}$. The probability of an evil, perfect non-tyranny exceeds that of a good non-monarchy likewise by construction: $\forall n \in [2,\ \infty) \subset \mathbb{N}_+,\ p(i) \in (0,\ 0.5) \subset \mathbb{R}_{++}$ and $q(i) \in (0.5,\ 1) \subset \mathbb{R}_{++},\ q(i) = 1 - p(i) > p(i) \longrightarrow q^n(i) = [1 - p(i)]^n > p^n(i)$, by which $q_i^n \in (0.5^n,\ 1) \subset \mathbb{R}_{++}$ and $p_i^n \in (0,\ 0.5^n) \subset (0,\ 0.5) \subset \mathbb{R}_{++}$. *QED*

The cardinal order of the three classical forms of government and of their perfect deviations thus far emerges as being the following: $p_T > p_M\ R\ p_{\neg T_{EP}} > p_{\neg M_G}$. More specifically, the relation between the probability of a monarchy and the probability of an evil, perfect non-tyranny appears to be unknown: $\forall n \in [2,\ \infty) \subset \mathbb{N}_+,\ p(i) \in (0,\ 0.5) \subset \mathbb{R}_{++}$ and $q(i) \in (0.5,\ 1) \subset \mathbb{R}_{++},\ \{p(i)\ R\ [1-p(i)]^n\} \equiv (p_M\ R\ p_{\neg T_{EP}})$.

Such is because the probability of a monarchy and the probability of an evil, perfect non-tyranny overlap: $p_i \in (0,\ 0.5) \subset \mathbb{R}_{++}$ and $q_i^n \in (0.5^n,\ 1) \subset \mathbb{R}_{++}$, by which $\lim_{n\to\infty}(0.5^n,\ 1) = (0,\ 1) \subset \mathbb{R}_{++} \longrightarrow (0,\ 0.5) \subset \lim_{n\to\infty}(0.5^n,\ 1) = (0,\ 1) \subset \mathbb{R}_{++}$, thus, $p_i \in (0,\ 0.5) \subset \lim_{n\to\infty}(0.5^n,\ 1) = (0,\ 1) \subset \mathbb{R}_{++}$ and $\lim_{n\to\infty} q_i^n \in (0,\ 1) \subset \mathbb{R}_{++}$.

Given multiple ruling citizens, if the probability of a monarchy were treated as being endogenous and if the closed, real interval in which it lies were treated as a constraint one could then ask when each polynomial relation of that in question may apply, solving for the said probability in turns: $\forall n \in [2, \infty) \subset \mathbb{N}_+,\ p_i \in (0,\ 0.5) \subset \mathbb{R}_{++}$ and $q(i) \in (0.5,\ 1) \subset \mathbb{R}_{++},\ p(i)\ R\ [1-p(i)]^n \longleftrightarrow p(i) \gtreqless [1-p(i)]^n \longrightarrow p^{\frac{1}{n}}(i) \gtreqless [1-p(i)] \longrightarrow p^{\frac{1}{n}}(i) + p(i) \gtreqless 1 \longrightarrow p^n(i) + p(i) \gtreqless 1 \longrightarrow p_i^n + p_i \gtreqless 1$.

Case 1. Let $n = 2$. It follows that $p_i^n + p_i = 1 \longrightarrow p_i^2 + p_i = 1 \longrightarrow p_{i_{1,\ 2}} = \frac{-b\pm\sqrt{b^2-4ac}}{2a} = \frac{-1\pm\sqrt{1+4}}{2} = \frac{-1\pm\sqrt{5}}{2} \longrightarrow \Re_{++}(p_i) = 0.62$ (2 *d.p.*), but $p_i \in (0,\ 0.5) \subset \mathbb{R}_{++}$, therefore, $p_i^2 + p_i \neq 1$. Likewise, $p_i^2 + p_i > 1 \longrightarrow \Re_{++}(p_i) \in (0.62,\ \infty) \subset \mathbb{R}_{++}$, but $p_i \in (0,\ 0.5) \subset \mathbb{R}_{++}$, therefore, $p_i^2 + p_i \ngtr 1$. Finally, $p_i^2 + p_i < 1 \longrightarrow \Re_{++}(p_i) \in (0,\ 0.62) \subset \mathbb{R}_{++}$, which satisfies $p_i \in (0,\ 0.5) \subset \mathbb{R}_{++}$, therefore, $p(i) < [1-p(i)]^n \longleftrightarrow p_M < p_{\neg T_{EP}}$.

Case 2. Let $n = 3$. It follows that $p_i^n + p_i = 1 \longrightarrow p_i^3 + p_i = 1 \longrightarrow \Re_{++}(p_i) = 0.68$ (2 *d.p.*), but $p_i \in (0,\ 0.5) \subset \mathbb{R}_{++}$, therefore, $p_i^3 + p_i \neq 1$. Likewise, $p_i^3 + p_i > 1 \longrightarrow \Re_{++}(p_i) \in (0.68,\ \infty) \subset \mathbb{R}_{++}$, but $p_i \in (0,\ 0.5) \subset \mathbb{R}_{++}$, therefore, $p_i^3 + p_i \ngtr 1$. Finally, $p_i^3 + p_i < 1 \longrightarrow \Re_{++}(p_i) \in (0,\ 0.68) \subset \mathbb{R}_{++}$, which satisfies $p_i \in (0,\ 0.5) \subset \mathbb{R}_{++}$, therefore, $p(i) < [1-p(i)]^n \longleftrightarrow p_M < p_{\neg T_{EP}}$.

Case 3. Let $n = 70$. It follows that $p_i^n + p_i = 1 \longrightarrow p_i^{70} + p_i = 1 \longrightarrow \Re_{++}(p_i) = 0.96$ (2 *d.p.*), but $p_i \in (0,\ 0.5) \subset \mathbb{R}_{++}$, therefore, $p_i^{70} + p_i \neq 1$. Likewise, $p_i^{70} + p_i > 1 \longrightarrow \Re_{++}(p_i) \in (0.96,\ \infty) \subset \mathbb{R}_{++}$, but $p_i \in (0,\ 0.5) \subset \mathbb{R}_{++}$, therefore, $p_i^{70} + p_i \ngtr 1$. Finally, $p_i^{70} + p_i < 1 \longrightarrow \Re_{++}(p_i) \in (0,\ 0.96) \subset \mathbb{R}_{++}$, which satisfies $p_i \in (0,\ 0.5) \subset \mathbb{R}_{++}$, therefore, $p(i) < [1-p(i)]^n \longleftrightarrow p_M < p_{\neg T_{EP}}$.

Such three cases, especially the third, which admits of no analytical solution (see https://en.wikipedia.org), were studied with the aid of https://www.wolframalpha.com. For a large enough quantity of ruling citizens the relation between the probability of a monarchy and the probability of an evil, perfect non-tyranny should emerge in favour of the latter: $\forall 2 \ll n < \infty,\ p_i \in (0,\ 0.5) \subset \mathbb{R}_{++}$ and $q(i) \in (0.5,\ 1) \subset \mathbb{R}_{++},\ p(i) < [1-p(i)]^n \longleftrightarrow p_M < p_{\neg T_{EP}}$? Moreover, even though the collocation of the probability of an evil, imperfect non-tyranny in the cardinal order of the three classical forms of government and of their deviations similarly appear to be unknown a closer inspection enables its identification.

Proposition 3.4.3 (Cardinal suborder three) *The probability of a tyranny is greater than the probability of an evil, imperfect non-tyranny. Formally:*

$$p_T > p_{\neg T_{EI}}. \tag{8}$$

Proof. The relation between the probability of a tyranny and the probability of an evil, imperfect non-tyranny appears to be unknown: $\forall n \in [2,\ \infty) \subset \mathbb{N}_+$, *ceteris paribus,* $p_T \gtreqless p_{\neg T_{EI}} \longleftrightarrow q(i) \gtreqless \{[q^{n-1}(i)p(i)] \veebar [q^{n-2}(i)p^2(i)] \veebar \ldots \veebar [q(i)p^{n-1}(i)]\} \longrightarrow \{[1 \gtreqless q^{n-2}(i)p(i)] \veebar [1 \gtreqless q^{n-3}(i)p^2(i)] \veebar \ldots \veebar [1 \gtreqless p^{n-1}(i)]\}$. Since probabilities $q_i^{(n)} \in (0.5^{(n)},\ 1) \subset \mathbb{R}_{++}$ and probabilities $p_i^{(n)} \in (0,\ 0.5^{(n)}) \subset \mathbb{R}_{++}$ the relation in question is discerned to be a strict inequality in favour of the probability of a tyranny: $\{[1 > q^{n-2}(i)p(i)] \veebar [1 > q^{n-3}(i)p^2(i)] \veebar \ldots \veebar [1 > p^{n-1}(i)]\} \longleftrightarrow \{[1 > (1-p(i))^{n-2}p(i)] \veebar [1 > (1-p(i))^{n-3}p^2(i)] \veebar \ldots \veebar [1 > p^{n-1}(i)]\} \longrightarrow p_T > p_{\neg T_{EI}}$. *QED*

Proposition 3.4.4 (Cardinal suborder four) *The probability of a monarchy is greater than the probability of an evil, imperfect non-tyranny. Formally:*

$$p_M > p_{\neg T_{EI}}. \tag{9}$$

Proof. The relation between the probability of a monarchy and the probability of an evil, imperfect non-tyranny appears to be unknown: $\forall n \in [2,\ \infty) \subset \mathbb{N}_+$, *ceteris paribus,* $p_M \gtreqless p_{\neg T_{EI}} \longleftrightarrow p(i) \gtreqless \{[q^{n-1}(i)p(i)] \veebar [q^{n-2}(i)p^2(i)] \veebar \ldots \veebar [q(i)p^{n-1}(i)]\} \longrightarrow \{[1 \gtreqless q^{n-1}(i)] \veebar [1 \gtreqless q^{n-2}(i)p(i)] \veebar \ldots \veebar [1 \gtreqless q(i)p^{n-2}(i)]\}$. Since probabilities $q_i^{(n)} \in (0.5^{(n)},\ 1) \subset \mathbb{R}_{++}$ and probabilities $p^{(n)}(i) \in (0,\ 0.5^{(n)}) \subset \mathbb{R}_{++}$

the relation in question is discerned to be a strict inequality in favour of the probability of a monarchy: $\{[1 > q^{n-1}(i)] \veebar [1 > q^{n-2}(i)p(i)] \veebar \ldots \veebar [1 > q(i)p^{n-2}(i)]\} \longleftrightarrow \{[1 > (1-p(i))^{n-1}] \veebar [1 > (1-p(i))^{n-2}p(i)] \veebar \ldots \veebar [1 > (1-p(i))p^{n-2}(i)]\} \longrightarrow p_M > p_{\neg T_{EI}}$. *QED*

Proposition 3.4.5 (Cardinal suborder five) *The probability of an evil, perfect non-tyranny is greater than the probability of an evil, imperfect non-tyranny. Formally:*

$$p_{\neg T_{EP}} > p_{\neg T_{EI}}. \tag{10}$$

Proof. The relation between the probability of an evil, perfect non-tyranny and the probability of an evil, imperfect non-tyranny appears to be unknown: $\forall n \in [2, \infty) \subset \mathbb{N}_+$, *ceteris paribus,* $p_{\neg T_{EP}} \gtreqless p_{\neg T_{EI}} \longleftrightarrow [1-p(i)]^n \gtreqless \{[q^{n-1}(i)p(i)] \veebar [q^{n-2}(i)p^2(i)] \veebar \ldots \veebar [q(i)p^{n-1}(i)]\} \longleftrightarrow q(i)^n \gtreqless \{[q^{n-1}(i)p(i)] \veebar [q^{n-2}(i)p^2(i)] \veebar \ldots \veebar [q(i)p^{n-1}(i)]\} \longrightarrow \{[q(i) \gtreqless p(i)] \veebar [q^2(i) \gtreqless p^2(i)] \veebar \ldots \veebar [q^{n-1}(i) \gtreqless p^{n-1}(i)]\} \longrightarrow \{[q(i) \gtreqless p(i)] \veebar [q(i) \gtreqless p(i)] \veebar \ldots \veebar [q(i) \gtreqless p(i)]\} \longleftrightarrow \{[1-p(i) \gtreqless p(i)] \veebar [1-p(i) \gtreqless p(i)] \veebar \ldots \veebar [1-p(i) \gtreqless p(i)]\}$. However, the probability of a tyranny exceeds that of a monarchy by assumption, thus, the relation in question is discerned to be a strict inequality in favour of the probability of an evil, perfect non-tyranny: $\{[1-p(i) > p(i)] \veebar [1-p(i) > p(i)] \veebar \ldots \veebar [1-p(i) > p(i)]\} \longrightarrow p_{\neg T_{EP}} > p_{\neg T_{EI}}$. *QED*

Proposition 3.4.6 (Cardinal suborder six) *The probability of an evil, imperfect non-tyranny is greater than the probability of a good non-monarchy. Formally:*

$$p_{\neg T_{EI}} > p_{\neg M_G}. \tag{11}$$

Proof. The relation between the probability of a good non-monarchy and the probability of an evil, imperfect non-tyranny appears to be unknown: $\forall n \in [2, \infty) \subset \mathbb{N}_+$, *ceteris paribus,* $p_{\neg M_G} \gtreqless p_{\neg T_{EI}} \longleftrightarrow p^n(i) \gtreqless \{[q^{n-1}(i)p(i)] \veebar [q^{n-2}(i)p^2(i)] \veebar \ldots \veebar [q(i)p^{n-1}(i)]\} \longrightarrow \{[p^{n-1}(i) \gtreqless q^{n-1}(i)] \veebar [p^{n-2}(i) \gtreqless q^{n-2}(i)] \veebar \ldots \veebar [p(i) \gtreqless q(i)]\} \longleftrightarrow \{[p^{n-1}(i) \gtreqless (1-p(i))^{n-1}] \veebar [p^{n-2}(i) \gtreqless (1-p(i))^{n-2}] \veebar \ldots \veebar [p(i) \gtreqless 1-p(i)]\} \longrightarrow \{[p(i) \gtreqless 1-p(i)] \veebar [p(i) \gtreqless 1-p(i)] \veebar \ldots \veebar [p(i) \gtreqless 1-p(i)]\}$. However, the probability of a tyranny exceeds that of a monarchy by assumption, thus, the relation in question is discerned to be a strict inequality in favour of the probability of an evil, imperfect non-tyranny: $\{[p(i) < (1-p(i))] \veebar [p(i) < (1-p(i))] \veebar \ldots \veebar [p(i) < 1-p(i)]\} \longrightarrow p_{\neg M_G} < p_{\neg T_{EI}}$. *QED*

As a consequence, the cardinal order of the three classical forms of government and of their deviations, perfect and imperfect, conclusively emerges as being the following: $p_T > p_M \; R \; p_{\neg T_{EP}} > p_{\neg T_{EI}} > p_{\neg M_G}$, by which, $\forall 2 \ll n < \infty$, *ceteris paribus,* $p_T > p_{\neg T_{EP}} > p_M > p_{\neg T_{EI}} > p_{\neg M_G}$?

As intuitable, such signifies that if citizens are all equally strictly concupiscent *(ex ante)* then: tyrannies are the most probable (evil) forms of classical government; monarchies and evil, perfect non-tyrannies are more probable than evil, imperfect non-tyrannies, which are themselves more probable than good non-monarchies; the probabilistic relation between monarchies and evil, perfect non-tyrannies depends on the quantity of ruling citizens, but seems to lies in favour of the latter.

3.5 Monarchic preference. Although the consideration of a tyranny as the worst deviation from the three classical forms of government be qualitatively rooted in its greater distance from the ethical pursuit of social welfare, by which individual welfare cannot precede it, spiritually and temporally, such a cardinal order quantitatively reinforces it.

Said cardinal order is moreover sufficient to declare that, unless individual probabilities of concupiscence be updated, a monarchy is preferable to an aristocracy or to a timocracy, forasmuch as simpler to achieve in view of its greater probability: $M \succ \neg M_G$.

Political philosophy would subjoin that a monarchy is preferable to an aristocracy or to a timocracy, all perfected by the aristocratic canon, on account of its ubiquity: $M_A \succ \neg M_{G_A}$. In addition, by the logical application of such a cardinal suborder to the concept of defeat a tyranny is discerned to be more easily vanquished than an oligarchy or a democracy, perfect or imperfect that it be: $(p_M > p_{\neg M_G}) \equiv [p_{AT} > (p_{AO} \veebar p_{AD})]$, where $AT \equiv$ anti-tyranny, $AO \equiv$ anti-oligarchy and $AD \equiv$ anti-democracy.

In other words, the superiority of the probability of a monarchy over that of a good non-monarchy can be even understood as a greater probability of success in substituting a tyrant with a monarch and thereby attaining to a monarch than in substituting multiple tyrants with multiple monarchs and thereby attaining to multiple monarchs.

The Scholastic consideration of tyrannies as worse than oligarchies and democracies *en bloc* is quantitively reinforced by the greater probability of a tyranny relative to that of an evil, perfect non-tyranny. On the other hand, the Scholastic estimation by which consent to the concupiscence of one be limited by the resistance to the concupiscence of more than one (i.e. evil, imperfect non-tyranny) is quantitatively contradicted, so that the positively best form of government be quantitatively discerned to be a monarchy and not a mixed one.

If the Scholastic idea of a mixed form of government as the positively best one by contrast entails the elusion of a tyrant by means of suitable positive law to weigh upon a monarch, as appears to be more probable, then it befits a monarchy as hereby modelled, by which the monarch himself is to abide by that positive law whose role is the specification of natural and eternal law. If it alternatively spoke to aristocracies or timocracies outright then it would be quantitatively contradicted once again.

3.6 Probability of non-concupiscence. The hypothetical existence of an equal probability of inclination towards evil across citizens inferior to that towards good *(ex ante)* banally alters the cardinal order of the three classical forms of government and of their deviations, perfect and imperfect, in the following way: $p_M > p_T \; R \; p_{\neg M_G} > p_{\neg T_{EI}} > p_{\neg T_{EP}}$.

As intuitable, such would signify that if citizens were all equally non-concupiscent *(ex ante)* then: monarchies would be the most probable (good) forms of classical government; tyrannies and good non-monarchies would be more probable than evil, imperfect non-tyrannies, which would be themselves more probable than evil, perfect non-tyrannies; the probabilistic relation between tyrannies and good non-monarchies would depend on the quantity of ruling citizens, but would seem to lie in favour of the latter.

Such an altered cardinal order would confirm both the quantitative consideration of a tyranny as the worst deviation from the three classical forms of government and the global optimality of a monarchy, effectively reinforcing it.

In detail, a monarchy would be the most probable form of government and thereby the simplest to achieve, accordingly rendering the vanquishment of a tyranny the most probable vanquishment of all deviations from the three classical forms of government.

3.7 Probability of semi-concupiscence. The hypothetical existence of an equal probability of inclination towards evil across citizens correspondent to that towards good *(ex ante)* alters the cardinal order of the three classical forms of government and of their deviations, perfect and imperfect, just as banally, if not more: $p_T = p_M > p_{\neg T_{EP}} = p_{\neg T_{EI}} = p_{\neg M_G}$.

As intuitable, such would signify that if citizens were all equally semi-concupiscent *(ex ante)* then: tyrannies would be as probable as monarchies and the two would be the most probable forms of classical government; evil, perfect non-tyrannies would be as probable as evil, imperfect non-tyrannies and as good non-monarchies.

Such an altered cardinal order would accordingly confirm both the quantitative consideration of a tyranny as the worst deviation from the three classical forms of government and the global optimality of a monarchy, albeit reinforcing it to a smaller degree.

In detail, a monarchy would be the most probable form of government and thereby the simplest to achieve, but so would tyranny; however, the vanquishment of a tyranny would still be the most probable vanquishment of all deviations from the three classical forms of government.

3.8 Further comments. The update of individual probabilities of concupiscence is such that the probability of inclination towards evil across citizens relative to that towards good is specific to each subject and thereby potentially unequal *(ex post).*

Such an update would although not alter the cardinal order of the three classical forms of government and of their deviations, perfect and imperfect, based on equal strict concupiscence across citizens *(ex ante),* for strict concupiscence would ever determine a greater probability of inclination towards evil than towards good across any citizen, even for saints in the making, as it were, that is, pious men, living godly or holy lives.

At worst individual probabilities of concupiscence would decrease towards one half across all citizens (i.e. quasi-pneumatic society), but they would never fall below it, being thereby captured by the above analysis. The only difference would be that the greater probabilities of monarchies and tyrannies would be specific to each subject, rather than equal across citizens, thereby reinforcing the need for an aristocratic monarchy; in fact, even if the update of individual probabilities of concupiscence were studied in the case of non-concupiscence the relevant cardinal order and its conclusions would not change.

By Catholic teaching, non-baptised citizens would be prone to operating evil even more than baptised citizens, consequently, the implicit assumption by which all citizens be unbaptised, as in a perfectly non-Catholic society, fortifies the above derivations.

Otherwise put, since all men are concupiscent even after the reception of Holy Baptism the implicit non-consideration of Holy Baptism is such that the probability of concupiscence can only worsen and confirm the relevant cardinal order. To be sure, the above analysis took no formal position on baptismal reception, but it can be comparably understood as having regarded almost all members of society as baptised (i.e. moral certainty).

For scopes of ratiocinative strength the definition of good and evil can be alternatively determined in a subjectivistic fashion, either by society (i.e. societal relativism, nihilism) or directly by individuals (i.e. individual relativism, structuralism), thereby excluding preordination. In other words, it need not be one abiding by natural and eternal law, on which polemics do not subjectivistically lack in fact; forsooth, although it be one abiding by natural and eternal law their definition can be subjected to societal or individual election, that is, to opinionism.

The nihilistic case would probably invoke semi-concupiscence, as opposed to strict concupiscence, and the relativistic and structuralistic cases would probably invoke non-concupiscence in terms of individual probabilities. The respective reasons are that: nihilism eliminates both objectivity and ethics, thereby emptying the probabilities of good and of evil of all of their signification, as captured by semi-concupiscence; relativism eliminates objectivity, thereby leaning towards a greater probability of inclination towards good across citizens, even if it be individual, as captured by non-concupiscence; structuralism eliminates objectivity, ethics and subjectivity, thereby paradoxically leaning towards a greater probability of inclination towards good across citizens as well, despite being individual, as captured by non-concupiscence.

In the nihilistic case such analyses would hold from a societal stance, while in the structuralistic case they would hold from an individual one (i.e. each would conduct his own), being both opposed to those holding from the stance of the Remote Cause perused above; in the relativistic cases they could hold from either one.

4. Conclusion

The following points can be put forth in summary.

Point 1. The three classical forms of government, which are monarchies, aristocracies and timocracies, are indifferent inasmuch as natural and eternal law may be upheld.

Point 2. Aristocratic rule is government by the aptest and is thereby timocratic, admitting of aristocratic monarchies and of aristocratic timocracies, strengthening such an indifference.

Point 3. Monarchic rule is substantially ubiquitous to all of the three classical forms of government, profiling them as accidental to it.

Point 4. Irrespective of human concupiscence, tyrannies are the most probable (evil) forms of classical government as well as the easiest ones to vanquish and monarchies are the most probable good forms of classical government.

Point 5. In view of such all aristocratic monarchies are to be ultimately preferred, polymathically, philosophically, religiously.

References

[1] Aquinas T. S. (1273) "Summa theologiae", https://www.newadvent.org
[2] Leo XIII P. (1881) "Diuturnum", https://www.vatican.va
[3] Leo XIII P. (1885) "Immortale Dei", https://www.vatican.va
[4] Leo XIII P. (1888) "Libertas", https://www.vatican.va