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Abstract

This study develops a Malthusian model for the evolution of human society from hunting-gathering to agriculture and from agriculture to industrial production. Human society evolves across these stages as the population grows. However, under endogenous population growth, the population may stop growing at any stage. If it fails to reach the first threshold, the population remains as hunter-gatherers. If it reaches the first threshold, an agricultural society emerges. Then, if the population fails to reach the industrial threshold, it remains in an agricultural Malthusian trap without experiencing industrialization. Interestingly, high agricultural productivity not only triggers the Neolithic Revolution but also the subsequent industrialization. Using cross-country data to test this result, we employ an index of prehistoric biogeographic conditions that affect agricultural productivity as an instrument for the timing of transitions to agriculture and find that an earlier transition to agriculture has a positive effect on industrialization in the modern era.

JEL classification: O13, O14, J11
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1 Introduction

Archaeological evidence suggests that *Homo sapiens* emerged in Africa as early as about 300,000 years ago.[1] For most of its history, humans were hunter-gatherers. Then, the Neolithic Revolution (the transition from hunting-gathering to agriculture) occurred in the Fertile Crescent about 12,000 years ago and then in other parts of the world.[2] In the late 18th and early 19th century, the Industrial Revolution (the transition from agriculture to the manufacturing of goods) took place in Britain and then in continental Europe and the United States.[3] Are these transitions in the economic evolution of human society inevitable? If not, what are the different conditions that could have potentially made the transitions more or less likely to occur?

This study develops a Malthusian model that captures the economic evolution of human society from hunting-gathering to agriculture and then from agriculture to an industrial economy. In our model, human society evolves across these stages as the size of the population grows. However, under endogenous population growth determined by the fertility decisions of optimizing agents in our microfounded model, the population may stop growing at any stage and never reach the next threshold. If it fails to reach the first threshold, then the population remains in a hunting-gathering Malthusian trap. If the population size reaches the first threshold, then an agricultural society emerges; therefore, both the Boserupian and Malthusian forces are present in our model.[4] The Neolithic Revolution occurs under the following conditions: a high level of agricultural productivity, a low cost of fertility, and a strong preference for fertility. In the main text, we discuss the intuition of these results and their relation to existing hypotheses. We also provide empirical evidence that high agricultural productivity leads to an earlier transition to agriculture.

After an agricultural society emerges, the economy eventually becomes completely agricultural until it reaches the next threshold. If it fails to reach the next threshold, the economy remains in an agricultural Malthusian trap and does not experience industrialization. Industrialization is influenced by the same conditions as the Neolithic Revolution (namely, a high level of agricultural productivity, a low cost of fertility, and a strong preference for fertility) and also other conditions: a high level of industrial productivity, and a low fixed cost of industrial production. Therefore, the conditions (e.g., a high level of agricultural productivity) that trigger the Neolithic Revolution also trigger the subsequent industrialization, but not necessarily vice versa. Here the importance of the population size on industrialization is due to its increasing returns to scale (i.e., having a large enough market to cover the fixed costs associated with industrial production) as in Murphy *et al.* (1989), whereas the importance of the population size on the Neolithic Revolution is due to the decreasing returns to scale in hunting-gathering as in North and Thomas (1977). If the population reaches the industrial threshold, then a modern economy emerges and exhibits positive steady-state population growth.

[1] See Hublin *et al.* (2017) and Richter *et al.* (2017).

[2] See Barker (2006) for a detailed discussion of the archaeological evidence on the origins of agriculture and Larson *et al.* (2014) for an interdisciplinary approach to the understanding of the roles of the domestification of plants and animals on the transition to agriculture.

[3] See Madsen *et al.* (2010) and Madsen and Murtin (2017) for interesting empirical studies on the Industrial Revolution in Britain.

[4] Boserup (1965) argues that agricultural methods depend on the population size. Her idea has been extended to the case in which the transition to agriculture depends also on the population size; see Cohen (1977).

Empirically, we use cross-country data to examine our key theoretical result that high agricultural productivity not only triggers the Neolithic Revolution but also the subsequent industrialization. We follow previous empirical studies, such as Olsson and Hibbs (2005), Ashraf and Galor (2011) and Ang (2015), to consider an index of prehistoric biogeographic conditions that affect agricultural productivity and explore how it affects the transition to agriculture. Specifically, we use the index of biogeographic conditions as an instrument for the timing of transitions to agriculture and find that an earlier transition to agriculture has a positive effect on the degree of industrialization in the modern era.

This study relates to the literature on the economic modelling of the transition from hunting-gathering to agriculture; see Smith (1975) and North and Thomas (1977) for early studies and Weisdorf (2005) for an excellent review of this literature.[5] A subsequent study by Locay (1989) develops a dynamic general equilibrium model with endogenous fertility to explore the transition of the human population from nomadic hunter-gatherers to a sedentary agricultural society; see also the interesting studies by Olsson (2001) and Weisdorf (2003).[6] Baker (2008) estimates an extended version of the Locay model using historical data on the incidence of agriculture and finds empirical support for the model. Weisdorf (2011) explores the case in which the agricultural transition is caused by an exogenous discovery of agricultural technology.[7] Recent studies by Dow *et al.* (2009) and Dow and Reed (2015, 2022) consider climate change as a cause of the transition to agriculture, whereas Bowles and Choi (2019) explore the origins of private property as a reason for adopting agriculture.

Our model is based on Locay (1989) and Baker (2008) with the introduction of an industrial economy as the third stage of the economic evolutionary process, without which the population remains either in a hunting-gathering or an agricultural Malthusian trap in the long run. An important finding is that the transition from hunting-gathering to agriculture and the transition from agriculture to industrial production are both endogenous and share a similar set of determinants. For example, high agricultural productivity not only triggers the Neolithic Revolution but also the subsequent industrialization.[8] Ashraf and Galor (2011) provide empirical evidence that agricultural productivity has a significant positive effect on population density in the preindustrial era. Olsson and Hibbs (2005) also provide evidence to show that prehistoric biogeographic conditions that are favorable to agriculture can trigger the Neolithic Revolution and the subsequent development in the industrial era. Using an index of biogeographic conditions as instruments, Ang (2015) finds that the timing of transitions to agriculture has a significant effect on technology adoption from 1000 BC to 1500 AD. Our empirical analysis follows the footsteps of these studies but instead examines the effects of agricultural productivity on the timing of transitions to agriculture and the degree of industrialization in the modern era.

[5] Weisdorf (2005) also reviews the related archaeological and anthropological literature.

[6] Olsson (2001) develops a model that allows for four potential explanations for the agricultural transition: environmental condtions, population pressure, cultural influence, and external factors. Weisdorf (2003) develops a model in which an agricultural society allows for non-food-producing specialists who supply non-food goods.

[7] The approach in Locay (1989) and Baker (2008) implicitly assumes that the discovery of technology occurs before agents have incentives to adopt it; for example, Tudge (1999) discusses evidence that proto-farming existed much earlier than the Neolithic Revolution.

[8] See also Chu *et al.* (2021) who introduce an agricultural sector to the Schumpeterian model with endogenous takeoff in Peretto (2015) and show that high agricultural productivity triggers industrialization.

This study also relates to the literature on unified growth theory; see Galor and Weil (2000) for the seminal study and Galor (2005, 2011) for a comprehensive review. Studies in this literature explore the endogenous transition of an agricultural economy in a Malthusian trap to a modern industrial economy with technological progress and long-run economic growth. This study complements the interesting studies in this literature by developing a simple unified model that captures both the first transition from hunting-gathering to agriculture and the second transition from agriculture to a modern industrial economy. In the spirit of Diamond (1997), Olsson and Hibbs (2005) also model both of these important transitions in human history using a theoretical framework that focuses on the causal relationship between initial biogeographic conditions and the subsequent development of the economy. Specifically, they assume that a better biogeographic endowment causes a higher growth rate of productive knowledge, which in turn triggers the transitions once productive knowledge reaches certain exogenous thresholds. We take a more microfounded approach in which population growth is endogenously determined by optimizing agents and the transitions occur only when population size crosses thresholds that are also endogenously determined within the model. As a result, the transition from hunting-gathering to agriculture and the transition from agriculture to industry may not always occur depending on parameter conditions in our model.

The rest of this study is organized as follows. Section 2 presents the static model with exogenous population. Section 3 develops the dynamic model with endogenous population. Section 4 presents empirical evidence. Section 5 concludes.

2 A static model of economic evolution

Our model is based on Locay (1989) and Baker (2008). We extend the Locay model to introduce an industrial economy as the third stage of economic evolution. In the first stage, the population engages in hunting-gathering. In the second stage, an agricultural society emerges. In the third stage, an industrial economy emerges. The population consists of N identical agents. Each agent is endowed with l units of labor, which can be allocated to hunting-gathering l_H, farming l_F or industrial production l_Y. Therefore, the labor constraint faced by each agent is

$$l_H + l_F + l_Y = l. \tag{1}$$

In the pre-industrial era, industrial production does not yet exist, and hence, the constraint simplifies to $l_H + l_F = l$. There is also a fixed amount of land denoted as Z, which can be used for hunting-gathering or farming.

2.1 Hunting-gathering

Hunting-gathering takes place in available land that is not occupied for farming. We use $\bar{l}_H$ to denote the average amount of labor endowment devoted to hunting-gathering. Then, total food production from hunting-gathering is given by

$$H = \theta(\bar{l}_H N)^{\gamma}(Z_H)^{1-\gamma}, \tag{2}$$

where $\bar{l}_H N$ and $Z_H \leq Z$ are respectively the total amount of labor and land devoted to hunting-gathering. The parameters $\theta > 0$ and $\gamma \in (0, 1)$ measure respectively the productivity and labor intensity of the hunting-gathering process. An agent, who contributes l_H units of labor to hunting-gathering, receives h units of food production given by

$$h = \frac{l_H}{\bar{l}_H N} \theta (\bar{l}_H N)^{\gamma} (Z_H)^{1-\gamma}, \tag{3}$$

in which the agent takes $\bar{l}_H$ and Z_H as given.

2.2 Agriculture

Farming also requires both labor and land. The farming production of an agent, who devotes l_F units of labor to farming, is

$$f = \varphi (l_F)^{\alpha} z^{1-\alpha}, \tag{4}$$

where the parameters $\varphi > 0$ and $\alpha \in (0, 1)$ measure respectively the productivity and labor intensity in agriculture. z is the amount of land used by the agent. We follow Baker (2008) to assume a fixed ratio ρ of land to farming labor given by

$$z = \rho l_F \tag{5}$$

when agricultural land is not scarce (i.e., $\rho \bar{l}_F N < Z$); in this case, $f = \varphi \rho^{1-\alpha} l_F$. Weisdorf (2005) argues that the temporary constant returns to farming labor, which is also present in the analysis of North and Thomas (1977), is a reasonable assumption when there is abundant agricultural land. When agricultural land becomes scarce, it is equally divided between agents; i.e.,

$$z = Z/N. \tag{6}$$

In this case, there is no more land available for hunting-gathering (i.e., $Z_H = 0$); see North and Thomas (1977) for a discussion that with communal property rights on agricultural land, farmers have better access to land than hunter-gatherers.

2.3 Industrial production

As in Murphy *et al.* (1989), the operation of modern industrial production requires a fixed cost $\delta > 0$ under which total industrial output is given by[9]

$$Y = A(\bar{l}_Y N - \delta), \tag{7}$$

where $\bar{l}_Y N$ is the total amount of labor devoted to industrial production, and the parameter $A > 0$ determines the level of industrial productivity. The fixed cost is shared by all agents

[9] In Appendix A, we present an extension of our model in which the fixed cost δ is interpreted as a fixed cost of operation in a monopolistic industrial market. Specifically, one can think of the reduced-form production function in (7) as capturing a modern monopolistic market with firm-level production functions $Y = \{\int_0^1 [Y(i)]^{\varepsilon} di\}^{1/\varepsilon}$ and $Y(i) = A[l_Y(i) - \delta]$; see Appendix A for this analysis.

when the industrial economy operates. Then, the output of industrial production received by an agent, who devotes l_Y units of labor, is

$$y = A(l_Y - \delta/N). \tag{8}$$

Due to the fixed cost δ, the industrial market would not operate unless the population size N is sufficiently large.

2.4 From Neolithic Revolution to industrialization

In this section, we explore the evolution of the economy and impose the following parameter assumption: $A > \varphi\rho^{1-\alpha} > \theta\rho^{1-\gamma}$. The population begins as hunter-gatherers and evolves into an agricultural society before an industrial economy emerges. We will impose parameter restrictions to ensure the realistic scenario in which industrialization takes place only after the complete transition from hunting-gathering to agriculture.

We begin by assuming that each agent maximizes consumption given by

$$c = x + y = h + f + y. \tag{9}$$

Here we make a simplifying assumption that there is perfect substitutability between food production x and industrial production y in the consumption of agents. This assumption helps to keep our analysis tractable and is not entirely unrealistic because industrial production includes modern methods of food production that requires fixed investment.

In the initial stage, there is no industrial production, so we have $l_Y = 0$. An agent's decision is to choose labor allocation between hunting-gathering l_H and farming l_F to maximize food production x given by

$$x = h + f = \frac{l_H}{\bar{l}_H N}\theta(\bar{l}_H N)^{\gamma}(Z_H)^{1-\gamma} + \varphi(l_F)^{\alpha}z^{1-\alpha} = (l - l_F)\,\theta\left(\frac{Z_H}{\bar{l}_H N}\right)^{1-\gamma} + \varphi\rho^{1-\alpha}l_F, \tag{10}$$

where we have used the resource constraint on labor $l_H + l_F = l$ and the fixed ratio of land to farming labor $z = \rho l_F$. The first-order condition is given by

$$\frac{\partial x}{\partial l_F} = -\theta\left(\frac{Z_H}{\bar{l}_H N}\right)^{1-\gamma} + \varphi\rho^{1-\alpha} = -\theta\left[\frac{Z - \rho l_F N}{(l - l_F)N}\right]^{1-\gamma} + \varphi\rho^{1-\alpha}, \tag{11}$$

where we have invoked symmetry $\{l_H, l_F\} = \{\bar{l}_H, \bar{l}_F\}$ and also used the resource constraint on land $Z_H = Z - \rho l_F N$. In (11), $\varphi\rho^{1-\alpha}$ is the marginal product of farming labor l_F whereas $\theta\left[\frac{Z-\rho l_F N}{(l-l_F)N}\right]^{1-\gamma}$ is the average product of hunting labor $l_H = l - l_F$. In the following subsections, we first compare these two objects under different population levels.

2.4.1 Stage 1: Hunting-gathering

Equation (11) implies that if the following inequality holds:

$$N < \left(\frac{\theta}{\varphi\rho^{1-\alpha}}\right)^{1/(1-\gamma)} \frac{Z}{l}, \tag{12}$$

then $\partial x/\partial l_F < 0$ even at $l_F = 0$. In this case, all labor is allocated to hunting-gathering $l_H = l$ and the per capita output of food production is given by

$$x = h = \theta l^\gamma \left(\frac{Z}{N}\right)^{1-\gamma}, \tag{13}$$

which is increasing in hunting productivity θ, labor supply l and the amount of land Z but decreasing in the population size N due to the decreasing returns to labor in hunting-gathering.

2.4.2 Stage 2: From hunting-gathering to agriculture

Equation (11) and $\rho l_F N < Z$ imply that if the following inequalities hold:

$$\left(\frac{\theta}{\varphi\rho^{1-\alpha}}\right)^{1/(1-\gamma)} \frac{Z}{l} < N < \frac{Z}{\rho l}, \tag{14}$$

then $\partial x/\partial l_F = 0$ at some interior values of $\{l_F, l_H\} \in (0, l)$. In this case, the transition from hunting-gathering to agriculture begins. The first inequality shows that a reduction in hunting productivity θ or an increase in population size N could trigger this transition. In our static model, the reduction in hunting productivity θ can capture the extinction of large herding animals analyzed in Smith (1975),[10] whereas an exogenous increase in population size N can capture the population pressure theory discussed in Cohen (1977). However, as we will show, these results would be quite different in our dynamic model with endogenous population growth.

During the gradual transition from hunting-gathering to agriculture, the per capita output of food production is given by

$$x = h + f = (l - l_F)\,\theta \left(\frac{Z_H}{\bar{l}_H N}\right)^{1-\gamma} + \varphi\rho^{1-\alpha} l_F = \varphi\rho^{1-\alpha} l, \tag{15}$$

which uses $\theta \left[Z_H/(\bar{l}_H N)\right]^{1-\gamma} = \varphi\rho^{1-\alpha}$ from (11). Equation (15) shows that x is increasing in labor supply l and agricultural productivity $\varphi\rho^{1-\alpha}$.

[10] Smith (1975) considers a dynamic model of replenishable common resources in which animal extinction is caused by excessive hunting.

2.4.3 Stage 3: Complete transition to agriculture

When $N > Z/(\rho l)$, the transition from hunting-gathering to agriculture is complete (i.e., $l_F = l$) because $Z_H = 0$. At this stage of the economy, an industrial market still does not emerge if the population size is insufficient to cover the fixed cost δ. This threshold value of N is implicitly determined by the following equality:

$$\varphi l^{\alpha} \left(\frac{Z}{N} \right)^{1-\alpha} = A \left(l - \frac{\delta}{N} \right), \tag{16}$$

in which the left-hand side is farming output per capita when $l_F = l$ and decreasing in N whereas the right-hand side is the industrial output per capita when $l_Y = l$ and increasing in N. A simple graphical analysis would confirm that there exists a unique cutoff value of N for the emergence of an industrial economy, which is denoted as N_I and has the following comparative statics:

$$N_I(\underset{+}{\varphi}, \underset{+}{Z}, \underset{+}{\delta}, \underset{-}{A}, \underset{-}{l}). \tag{17}$$

This implies that by making agriculture more productive, higher agricultural productivity φ delays industrialization, which contradicts the evidence discussed in Nurkse (1953).[11] As we will show, this counterfactual result will be overturned under endogenous population growth.

In summary, if the following inequality holds:[12]

$$\frac{Z}{\rho l} < N < N_I, \tag{18}$$

then the agents would be better off allocating all their labor to farming (i.e., $l_F = l$). In this case, the level of output per capita is given by

$$x = f = \varphi l^{\alpha} \left(\frac{Z}{N} \right)^{1-\alpha}, \tag{19}$$

which is increasing in agricultural productivity φ, labor supply l and the amount of land Z but decreasing in the population size N due to the decreasing returns to labor in farming when agricultural land is scarce.

2.4.4 Stage 4: Industrial economy

If $N > N_I$, then the transition from agriculture to an industrial economy occurs. In this case, the level of output per capita is given by

$$y = A \left(l - \frac{\delta}{N} \right), \tag{20}$$

which is increasing in industrial productivity A, labor supply l and population size N but decreasing in the fixed cost δ of industrial production. Equation (20) is obtained by setting

[11] According to Nurkse (1953), technological improvements that raised agricultural productivity helped to release labor from agriculture to industrial production and were crucial for the Industrial Revolution.

[12] From (17), a sufficiently large δ would suffice to ensure $N_I > Z/(\rho l)$.

$l_Y = l$ in (8). When the population size is sufficiently large, the agents would immediately allocate all their labor to industrial production because the marginal product of industrial labor is greater than the marginal product of agricultural labor;[13] i.e.,

$$A > \varphi\rho^{1-\alpha} > \varphi(l_F)^{\alpha-1}\left(\frac{Z}{N}\right)^{1-\alpha} > \alpha\varphi(l_F)^{\alpha-1}\left(\frac{Z}{N}\right)^{1-\alpha}$$

for $l_F > Z/(\rho N)$.[14] Naturally, we assume that it is infeasible for humans to return to hunting-gathering at this stage.[15]

2.4.5 Summary

In this section, we summarize the level of consumption per capita at different levels of population as follows:

$$c = x + y = \begin{cases} h = \theta l^{\gamma}\left(\frac{Z}{N}\right)^{1-\gamma} & \text{for } N < \left(\frac{\theta}{\varphi\rho^{1-\alpha}}\right)^{1/(1-\gamma)}\frac{Z}{l} \\ h + f = \varphi\rho^{1-\alpha}l & \text{for } \left(\frac{\theta}{\varphi\rho^{1-\alpha}}\right)^{1/(1-\gamma)}\frac{Z}{l} < N < \frac{Z}{\rho l} \\ f = \varphi l^{\alpha}\left(\frac{Z}{N}\right)^{1-\alpha} & \text{for } \frac{Z}{\rho l} < N < N_I \\ y = A\left(l - \frac{\delta}{N}\right) & \text{for } N > N_I \end{cases}. \tag{21}$$

Equation (21) presents the level of per capita consumption c as population N increases. In summary, c is initially falling due to the decreasing returns to labor in hunting-gathering. Then, c reaches to a stationary level (from above) when the gradual transition from hunting-gathering to agriculture begins. Therefore, before the transition to agriculture, hunter-gatherers enjoy a higher level of consumption than the later farmers, which is consistent with archaeological evidence; see for example Cohen and Armelagos (1984). However, our model implies that the hunter-gatherers would have experienced a subsequent fall in consumption if they didn't adopt farming due to the decreasing returns to labor in hunting-gathering. When the transition from hunting-gathering to agriculture is complete, c becomes falling again due to the decreasing returns to labor in farming when agricultural land is scarce. When the industrial economy emerges, c becomes rising due to the increasing returns to scale in the presence of a fixed cost of industrial production and converges towards a steady-state level given by $y^* = Al$ as $N \to \infty$.

3 A dynamic model with endogenous population growth

The previous section presents a static model with an exogenous level of population. This section extends the model into a dynamic setting with endogenous population growth. We follow Locay

[13] In the case of a monopolistic market, the industrial transition may become gradual because the wage of industrial labor is less than A; see Appendix A for this analysis.

[14] For $l_F < Z/(\rho N)$, the marginal product of agricultural labor is simply $\varphi\rho^{1-\alpha} < A$.

[15] This is despite the availability of land Z_H for hunting-gathering. However, under a monopolistic market in Appendix A, land may still be occupied for agriculture even in the industrial era.

(1989) and Baker (2008) to consider overlapping generations of agents. Each agent lives for two periods. Each adult agent at time t has the following utility function:

$$u_t = (1 - \sigma) \ln c_t + \sigma \ln n_{t+1}, \tag{22}$$

where the parameter $\sigma \in (0, 1)$ measures the preference for fertility and n_{t+1} is the agent's number of children, who then become adults at time $t + 1$. Raising children is costly, and the level of consumption net of the fertility cost is given by

$$c_t = x_t + y_t - \beta n_{t+1}, \tag{23}$$

where the parameter $\beta > 0$ determines the cost of fertility. Substituting (23) into (22), we derive the utility-maximizing level of fertility n_{t+1} as

$$n_{t+1} = \frac{\sigma}{\beta}(x_t + y_t) \tag{24}$$

and $c_t = (1-\sigma)(x_t + y_t)$ in which the agent maximizes $x_t + y_t$ as in Section 2. Each adult agent has n_{t+1} children, and the number of adult agents at time t is N_t. Therefore, the law of motion for the adult population size (i.e., the labor force) is given by

$$N_{t+1} = n_{t+1} N_t = \frac{\sigma}{\beta}(x_t + y_t) N_t, \tag{25}$$

and the adult population growth rate at time t is

$$\frac{\Delta N_t}{N_t} \equiv \frac{N_{t+1} - N_t}{N_t} = \frac{\sigma}{\beta}(x_t + y_t) - 1, \tag{26}$$

which will be simply referred to as the population growth rate. In the following subsection, we will use the information from Section 2 to derive the population dynamics.

3.1 Stage 1: Hunting-gathering

Given an initial level of population:

$$N_0 < \left(\frac{\theta}{\varphi \rho^{1-\alpha}}\right)^{1/(1-\gamma)} \frac{Z}{l}, \tag{27}$$

the human population engages in hunting-gathering only. Substituting (13) into (26) yields the growth rate of population as

$$\frac{\Delta N_t}{N_t} = \frac{\sigma}{\beta} \theta l^\gamma \left(\frac{Z}{N_t}\right)^{1-\gamma} - 1, \tag{28}$$

which yields the following steady-state level of population in the hunting-gathering era:

$$N_H^* = \left(\frac{\sigma}{\beta} \theta l^\gamma\right)^{1/(1-\gamma)} Z. \tag{29}$$

If the following inequality holds:

$$N_H^* < \left(\frac{\theta}{\varphi\rho^{1-\alpha}}\right)^{1/(1-\gamma)} \frac{Z}{l} \Leftrightarrow \frac{\sigma}{\beta}\varphi\rho^{1-\alpha}l < 1, \tag{30}$$

then the human population would remain as hunter-gatherers indefinitely. Substituting (29) into (13) yields $x^* = \beta/\sigma$, which is increasing in fertility cost β and decreasing in the degree σ of fertility preference but independent of hunting productivity θ and land Z. In other words, the population is in a hunting-gathering Malthusian trap, in which higher hunting productivity θ and more land Z increase the level of population N_H^* but not the level of income x^*.

Alternatively, if $\sigma\varphi\rho^{1-\alpha}l > \beta$, then an agricultural society would emerge. Therefore, the transition from hunting-gathering to agriculture occurs under the following conditions: a low fertility cost β, a strong fertility preference σ, a high level of agricultural productivity $\varphi\rho^{1-\alpha}$, and a high level of labor supply l. A strong fertility preference σ and a low fertility cost β give rise to a higher level of population and make it more likely to cross the population threshold for the emergence of agriculture in a Boserupian manner, but they also reduce income $x^* = \beta/\sigma$ in case the population remains in a hunting-gathering Malthusian trap. Although a higher level of hunting productivity θ and a larger amount of land Z also increase population, they increase the endogenous threshold for agriculture as well by making hunting-gathering more attractive. These opposite effects cancel each other, and hence, hunting productivity θ and the amount of land Z do not affect the transition to agriculture, which stands in stark contrast to the case of exogenous population.

Finally, high agricultural productivity $\varphi\rho^{1-\alpha}$ reduces the endogenous threshold by making agriculture more attractive, and hence, a higher level of agricultural productivity $\varphi\rho^{1-\alpha}$ can trigger the Neolithic Revolution. This finding is consistent with the empirical evidence in Olsson and Hibbs (2005), who find that favorable biogeographic conditions can trigger the transition to agriculture. Olsson (2001) examines the archeological evidence in the Jordan Valley and concludes that the abundance of species suitable for agriculture was one of the key reasons for the transition to agriculture. This abundance of agricultural species corresponds to a high level of agricultural productivity in our model. Furthermore, our analysis implies that climate change that affects agricultural productivity would also affect the transition to agriculture; for example, Richerson *et al.* (2017) argue that the climatic conditions during the most recent Ice Age were hostile to agriculture and made the transition to agriculture impossible at that time.

3.2 Stage 2: From hunting-gathering to agriculture

Suppose the population size N_t crosses the first threshold; i.e.,

$$\left(\frac{\theta}{\varphi\rho^{1-\alpha}}\right)^{1/(1-\gamma)} \frac{Z}{l} < N_t < \frac{Z}{\rho l}. \tag{31}$$

Then, the transition from hunting-gathering to agriculture begins. We can substitute (15) into (26) to derive the population growth rate as

$$\frac{\Delta N_t}{N_t} = \frac{\sigma}{\beta}\varphi\rho^{1-\alpha}l - 1 > 0, \tag{32}$$

which is positive if and only if the transition to agriculture occurs (i.e., $\sigma\varphi\rho^{1-\alpha}l > \beta$) and implies that population N_t increases over time during the gradual transition from hunting-gathering to agriculture.

3.3 Stage 3: Complete transition to agriculture

Given (32), the level of population N_t eventually crosses the second threshold; i.e.,

$$\frac{Z}{\rho l} < N_t < N_I, \tag{33}$$

where N_I is implicitly given in (16) and (17). At this stage, we can substitute (19) into (26) to derive the growth rate of population as

$$\frac{\Delta N_t}{N_t} = \frac{\sigma}{\beta}\varphi l^\alpha \left(\frac{Z}{N_t}\right)^{1-\alpha} - 1, \tag{34}$$

which yields a steady-state level of population in agriculture as

$$N_A^* = \left(\frac{\sigma}{\beta}\varphi l^\alpha\right)^{1/(1-\alpha)} Z. \tag{35}$$

If N_t reaches N_A^* before reaching N_I, then the economy would remain as an agricultural society indefinitely. Substituting (35) into (19) yields $x^* = \beta/\sigma$, which is once again increasing in fertility cost β and decreasing in the degree σ of fertility preference but independent of agricultural productivity φ and land Z. In other words, the population is now in an agricultural Malthusian trap, in which higher agricultural productivity φ and more land Z increase the level of population N_A^* but not the level of income x^*.

3.4 Stage 4: Industrial economy

If the level of population N_t manages to cross the third threshold N_I, then an industrial economy emerges. In this case, we can substitute (20) into (26) to derive the population growth rate as

$$\frac{\Delta N_t}{N_t} = \frac{\sigma A}{\beta}\left(l - \frac{\delta}{N_t}\right) - 1, \tag{36}$$

which is increasing in N_t. Setting $\Delta N_t/N_t = 0$ yields the following level:

$$N_I^* = \frac{\delta}{l - \beta/(\sigma A)}, \tag{37}$$

above which the population grows over time during the industrial era.

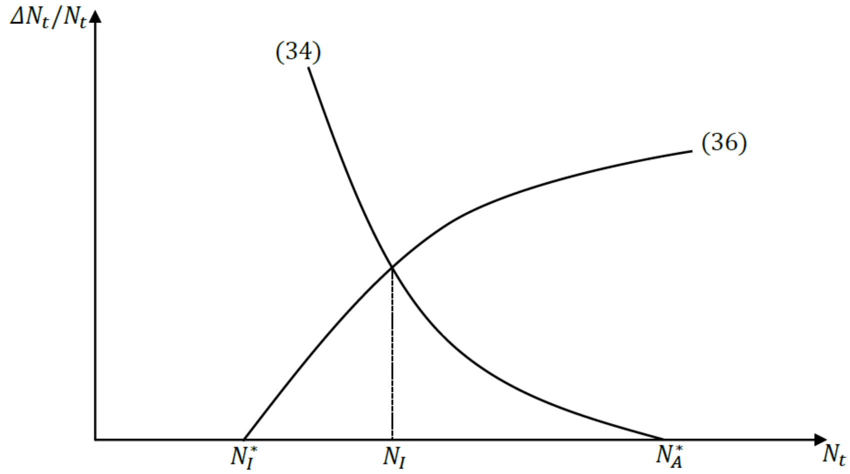

Figure 1: Industrial threshold

Figure 1 plots the population growth rate in (34) and (36) and shows that the economy switches from agriculture to industrial production when population crosses the threshold N_I.[16]

Figure 1 shows that if and only if $N_A^* > N_I^*$, then N_t would reach the third threshold N_I and trigger the emergence of an industrial economy. When $N_t > N_I$, the output level $x_t + y_t$ is higher under industrial production than under agricultural production. From (35) and (37), the inequality $N_A^* > N_I^*$ is equivalent to

$$\left(l - \frac{\beta}{\sigma A}\right)\left(\frac{\sigma}{\beta}\varphi l^{\alpha}\right)^{1/(1-\alpha)}\frac{Z}{\delta} > 1. \tag{38}$$

Therefore, the transition from an agricultural economy to an industrial economy occurs under the following conditions: a low fertility cost β, a strong fertility preference σ, a high level of agricultural productivity φ, a high level of labor supply l, a large amount of land Z, a high level of industrial productivity A, and a low fixed cost δ for operating industrial firms.

As before, a strong fertility preference σ and a low fertility cost β give rise to a higher level of population and make it more likely to cross the population threshold N_I for the emergence of an industrial economy, but they also reduce income $x^* = \beta/\sigma$ in case the population remains in an agricultural Malthusian trap. Interestingly, unlike the case of exogenous population, a high level of agricultural productivity φ can now trigger industrialization by raising the level of population. This result is consistent with the early work of Nurkse (1953) and Murphy *et al.* (1989) and also supported by the empirical evidence in Olsson and Hibbs (2005) and Ang (2015), who find that favorable initial biogeographic conditions can contribute to the subsequent development in the industrial era and technology adoption in as late as 1500 AD, in addition to the Neolithic Revolution.

Furthermore, a high level of industrial productivity A and a low fixed cost δ of industrial production reduce the endogenous threshold by making industrial production more attractive

[16] In Figure 1, (34) and (36) are determined by the left-hand side and right-hand side of (16), respectively.

and can also trigger industrialization. Finally, if the population size reaches the industrial threshold, then a modern economy emerges and the population growth rate rises towards a steady-state value given by $\Delta N/N = \frac{\sigma}{\beta}Al - 1$ in the long run.[17]

3.5 Summary

We summarize all the above results in the following proposition:

Proposition 1 *Under exogenous population growth, human society evolves from hunting-gathering to agriculture and then an industrial economy. Under endogenous population growth, the population may stop growing in a hunting-gathering society; in this case, the population remains as hunter-gatherers. The Neolithic Revolution occurs under a low fertility cost, strong fertility preference, high agricultural productivity, and high labor supply. The population may also stop growing in an agricultural society; in this case, the economy remains in an agricultural Malthusian trap. Industrialization occurs under a low fertility cost, strong fertility preference, high agricultural productivity, high labor supply, a large amount of agricultural land, high industrial productivity, and a low fixed cost of industrial production.*

Proof. The population growth rate is summarized in (39). From (30), if $\sigma\varphi\rho^{1-\alpha}l > \beta$, then N_t reaches the agricultural threshold before the hunting-gathering steady state N_H^*. If (38) holds, then N_t reaches the industrial threshold N_I before the agricultural steady state N_A^*. ■

If the population manages to evolve from hunting-gathering to agriculture and then activate the emergence of an industrial economy, the dynamics of the population growth rate can be summarized as follows:

$$\frac{\Delta N_t}{N_t} = \frac{\sigma}{\beta}(x_t + y_t) - 1 = \begin{cases} \frac{\sigma}{\beta}\theta l^\gamma \left(\frac{Z}{N_t}\right)^{1-\gamma} - 1 & \text{for } N_t < \left(\frac{\theta}{\varphi\rho^{1-\alpha}}\right)^{1/(1-\gamma)} \frac{Z}{l} \\ \frac{\sigma}{\beta}\varphi\rho^{1-\alpha}l - 1 & \text{for } \left(\frac{\theta}{\varphi\rho^{1-\alpha}}\right)^{1/(1-\gamma)} \frac{Z}{l} < N_t < \frac{Z}{\rho l} \\ \frac{\sigma}{\beta}\varphi l^\alpha \left(\frac{Z}{N_t}\right)^{1-\alpha} - 1 & \text{for } \frac{Z}{\rho l} < N_t < N_I \\ \frac{\sigma}{\beta}A\left(l - \frac{\delta}{N_t}\right) - 1 & \text{for } N_t > N_I \end{cases}. \quad (39)$$

Figure 2 plots the population growth rate $\Delta N_t/N_t$ for the following three scenarios: (a) the population converges to a hunting-gathering Malthusian trap as discussed in Section 3.1; (b) the population converges to an agricultural Malthusian trap as discussed in Section 3.3; and (c) the population achieves long-run growth as discussed in Section 3.4.

[17] Peretto (2021) also finds that the endogenous fertility rate rises towards a steady state in a Schumpeterian growth model with endogenous takeoff.

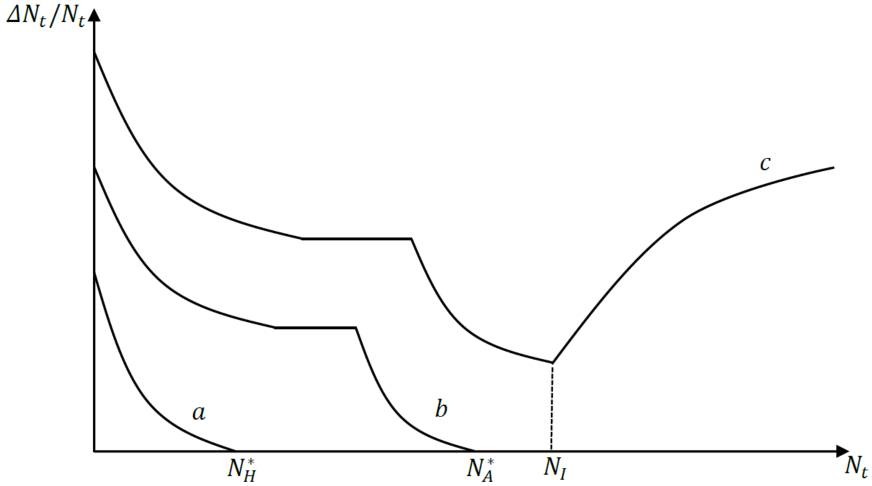

Figure 2: Dynamics of population growth

Figure 2 plots the following three scenarios: case (a) plots the population growth rate given by (28); case (b) plots the population growth rate given by (28), (32) and then (34); and case (c) plots the population growth rate given by (28), (32), (34) and then (36).

4 Empirical evidence

In this section, we use cross-country data to evaluate the effects of agricultural productivity on the transition to agriculture and the subsequent industrialization. Specifically, we follow Ashraf and Galor (2011) and Ang (2015) to employ a two-stage least squares regression:

$$\tau_j = \kappa_1 \varphi_j + \Phi_j + \epsilon_{1,j} \tag{40a}$$

$$y_j = \kappa_2 \widehat{\tau}_j + \Phi_j + \epsilon_{2,j} \tag{40b}$$

where (40a) describes the first-stage regression and (40b) describes the second-stage regression. In (40a), φ_j denotes the prehistoric level of agricultural productivity in present-day country j, for which we use an index of prehistoric biogeographic endowments as a proxy. We use data from Olsson and Hibbs (2005) on domesticable wild animals and plants known to exist in prehistory (in location that corresponds to present-day country j).[18] Then, we follow Ang (2015) to combine the two dimensions (animals and plants) into a single index by computing their first principal component.

In (40a), τ_j denotes the timing of agricultural transition, measured by the number of years before 2000 CE, whereas $\widehat{\tau}_j$ in (40b) denotes the predicted value of τ_j from the first-stage regression. y_j denotes the degree of industrialization, measured by the share of non-agricultural

[18]Domesticable plants refer to the number (33 species in total) of annual or perennial wild plants (such as wheat, rice, and millet, etc.) with a mean kernel weight of more than 10 mg. Domesticable animals refer to the number (14 species in total) of prehistoric mammals (such as goat, sheep, and pig, etc.) weighing over 45 kg.

employment in 1991. We use data in 1991 because data on employment shares is only available from 1991 for many countries, so using earlier data would lead to a sharp reduction in the sample size. Our underlying assumption is that countries experiencing earlier industrialization should have higher shares of non-agricultural employment in the modern era.

Our theory predicts that κ_1 and κ_2 are significantly positive. κ_1 being significantly positive implies that a higher level of agricultural productivity triggers an earlier transition to agriculture. κ_2 being significantly positive implies that an earlier transition to agriculture (triggered by a higher level of agricultural productivity) also causes a higher degree of industrialization in the modern era.

Φ is a set of control variables, including the constant term. We follow Nunn and Puga (2012) to control for a terrain ruggedness index, total land area, the ratio of fertile soil to land area, and the percentage of land area that is within 100 km of the nearest ice-free coast. We also consider continent fixed effects. Finally, $\epsilon_{1,j}$ and $\epsilon_{2,j}$ are the error terms. Table 1 presents the summary statistics and data sources.

Table 1: Summary statistics

	n	mean	sd	min	max
non-agricultural employment share (%)	94	64.09	26.65	7.87	99.72
timing of agricultural transition	94	4475.32	2357.30	400.00	10500.00
prehistoric biogeographic endowment index	94	0.08	1.40	-1.33	1.91
terrain ruggedness index	94	1.22	1.13	0.02	6.20
log level of land area	94	9.88	2.07	3.47	13.75
ratio of fertile soil (%)	94	35.77	22.50	0.00	100.00
ratio of area within 100 km of ice-free coast (%)	94	38.65	37.43	0.00	100.00

Data sources: World Bank Data for the share of non-agricultural employment; Putterman and Trainor (2006) for the timing of agricultural transition; Nunn and Puga (2012) for other variables.

Table 2 presents our regression results. Columns (1)-(2) do not include control variables except for the constant term and continent fixed effects, whereas columns (3)-(4) include control variables. The odd columns show the first-stage regression, whereas the even columns show the second-stage regression. The estimation results show that κ_1 and κ_2 are positive and statistically significant at least at the 5% level. Using the estimates in columns (3)-(4), we find that increasing the index of prehistoric biogeographic endowments by one (recall that this index ranges from -1.3 to 1.9) causes an earlier agricultural transition by about one millennium (1040 years) and a higher degree of industrialization reflected by a larger share of non-agricultural employment of 8.3% ($= 1040 \times 0.008\%$) in 1991.

Table 2: Effects of agricultural productivity on agricultural transition and industralization

	(1) first stage	(2) second stage	(3) first stage	(4) second stage
dependent variable:	*transition*	*industrialization*	*transition*	*industrialization*
productivity	1007.589***		1040.010***	
	(222.444)		(220.875)	
transition		0.009**		0.008***
		(0.004)		(0.003)
rugged			-3.811	-3.510
			(121.511)	(2.329)
landarea			21.883	-1.193
			(81.984)	(1.309)
soil			8.911	-0.254***
			(6.598)	(0.093)
nearcoast			-3.729	0.155*
			(5.386)	(0.079)
continent fixed effects	✓	✓	✓	✓
F-stat on instrument	33.465***		33.533***	
observations	94	94	94	94
R^2	0.719	0.353	0.728	0.507

Notes: Robust standard errors in parentheses. $* \ p < 0.1, ** \ p < 0.05, *** \ p < 0.01$. *In Table 2, productivity refers to the index of prehistoric biogeographic endowments; transition refers to the timing of agricultural transition; and industrialization refers to the non-agricultural employment share in 1991. The IV regressions employ the index of prehistoric biogeographic endowments as an instrument for the timing of agricultural transition.*

To ensure the robustness of our results, we explore other proxies for industrialization in the modern era. First, we consider the non-agricultural share of GDP as a dependent variable in the second-stage regression.[19] In this case, we can use earlier data in 1980 and still retain a sample size of 62.[20] Second, we consider the log of GDP per capita as a dependent variable in the second-stage regression.[21] In this case, we can use even earlier data in 1950 and retain a sample size of 88.[22] Table 3 presents the regression results. The estimated coefficients of κ_1 and κ_2 remain positive and statistically significant at the 1% level. These estimates imply that increasing the index of prehistoric biogeographic endowments by one causes an earlier agricultural transition by about one millennium as before and gives rise to a larger share of non-agricultural GDP of 7.2% (= 1031 × 0.007%) in 1980 and also an increase in GDP per

[19] Data source: World Bank Data.

[20] Our results (available upon request) are robust if we use data in 1970 or 1990. However, considering earlier data in 1960 would drastically reduce the sample size to 29.

[21] Data source: Maddison Project Database.

[22] Considering earlier data in 1940 would reduce the sample size to 36.

capita by 21.5% (= 1077 × 0.0002 × 100%) in 1950.[23]

Table 3: Robustness tests

	(1) first stage	(2) second stage	(3) first stage	(4) second stage
dependent variable:	*transition*	*GDP share*	*transition*	*GDP per capita*
productivity	1030.978***		1077.342***	
	(263.145)		(212.819)	
transition		0.007***		0.0002**
		(0.002)		(0.0001)
control variables	✓	✓	✓	✓
continent fixed effects	✓	✓	✓	✓
F-stat on instrument	23.961***		39.073***	
observations	62	62	88	88
R^2	0.804	0.169	0.765	0.536

Notes: Robust standard errors in parentheses. $*p < 0.1, **\ p < 0.05, ***p < 0.01$. *In Table 3, productivity refers to the index of prehistoric biogeographic endowments; transition refers to the timing of agricultural transition; GDP share refers to the non-agricultural share of GDP in 1980; and GDP per capita refers to the log of GDP per capita in 1950. The IV regressions employ the index of prehistoric biogeographic endowments as an instrument for the timing of agricultural transition.*

5 Conclusion

In this study, we have developed a simple Malthusian model that captures the economic evolution of human society across the three stages of hunting-gathering, agriculture and industrial production. We find that under endogenous population growth, the evolution to the next stage is not inevitable. If the population fails to reach the agricultural threshold, then the human population remains as hunter-gatherers. If the population fails to reach the industrial threshold, then the human population remains as agriculturalists. Our model identifies several potential causes for the Neolithic Revolution: a high level of agricultural productivity, a low cost of fertility, a strong preference for fertility, and a high level of labor supply. An implication is that the transitions to agriculture in different parts of the world (such as Central Mexico, China, the Middle East, and Sub-Saharan Africa) at different time periods could have been triggered by different reasons. Furthermore, the above conditions that trigger the Neolithic Revolution can also trigger the subsequent industrialization, but not necessarily vice versa because other conditions (such as a high level of industrial productivity and a low fixed cost of industrial production) may also trigger industrialization. Although our simple model is unlikely to capture all possible causes for the Neolithic Revolution and the subsequent industrialization, we find

[23] Recall that the dependent variable in column (4) of Table 3 is the log of GDP per capita.

empirical support for agricultural productivity as a determinant for the timing of transitions to agriculture and the degree of industrialization in the modern era.

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Appendix A: Monopolistic market in the industrial era

In this appendix, we replace the reduced-form industrial production function in (7) by a modern monopolistic market with a standard CES aggregator:[24]

$$Y = \left\{ \int_0^1 [Y(i)]^{\varepsilon} di \right\}^{1/\varepsilon}, \tag{A1}$$

where $\varepsilon \in (0, 1)$ determines the elasticity of substitution $1/(1-\varepsilon)$ between differentiated products $Y(i)$ for $i \in [0, 1]$. Profit maximization yields the conditional demand function:

$$Y(i) = \left[\frac{p}{p(i)} \right]^{1/(1-\varepsilon)} Y \Leftrightarrow p(i) = p \left[\frac{Y}{Y(i)} \right]^{1-\varepsilon}, \tag{A2}$$

where p and $p(i)$ are respectively the prices of Y and $Y(i)$ for $i \in [0, 1]$.

As in Krugman (1979), operating an industrial firm requires a fixed cost $\delta > 0$ under which the output of $Y(i)$ is

$$Y(i) = A[l_Y(i) - \delta], \tag{A3}$$

where $l_Y(i)$ is labor devoted to the production of $Y(i)$. The profit function for firm i is

$$\pi(i) = p(i)Y(i) - wl_Y(i) = pY^{1-\varepsilon}[Y(i)]^{\varepsilon} - w \left[\frac{Y(i)}{A} + \delta \right], \tag{A4}$$

where w is the wage rate of industrial labor. Profit maximization yields markup pricing:

$$p(i) = \frac{1}{\varepsilon} \frac{w}{A} > \frac{w}{A}, \tag{A5}$$

where w/A is the marginal cost of producing $Y(i)$. The amount of monopolistic profit is

$$\pi(i) = p(i)A[l_Y(i) - \delta] - wl_Y(i) = \frac{1-\varepsilon}{\varepsilon} w \left[l_Y(i) - \frac{\delta}{1-\varepsilon} \right], \tag{A6}$$

which is positive if and only if $l_Y(i) = \bar{l}_Y N > \delta/(1-\varepsilon)$ for all $i \in [0, 1]$. As before, due to the fixed cost δ, the industrial market would not operate unless population N is sufficiently large.

To be consistent with our baseline model, we assume that agents produce their own food output x for their own consumption and raising children. However, they need to purchase industrial output y (when available) using their industrial labor income wl_Y. Therefore, in the industrial era, each agent maximizes $x + y = f + y$ subject to farming production in (4), labor constraint $l_F + l_Y = l$ and the following budget constraint:

$$py = wl_Y + \frac{1}{N} \int_0^1 \pi(i) di, \tag{A7}$$

[24] One can endogenize the mass of varieties as $m \geq 1$, in which case growth in population N would expand varieties m and increase industrial output Y as in Romer (1990); see Section 4.3 in Chu (2022) for this analysis.

where profits $\pi(i) \geq 0$ are redistributed to all N agents equally. The first-order condition is

$$\frac{\partial(x+y)}{\partial l_F} = \underbrace{\alpha\varphi(l_F)^{\alpha-1}\left(\frac{Z}{N}\right)^{1-\alpha}}_{\equiv MPL_F} - \frac{w}{p}, \tag{A8}$$

where $w/p = w/p(i) = \varepsilon A$ from symmetry and markup pricing in (A5). Figure 3 plots (A8) and shows that there are two scenarios: (a) interior solution (i.e., $\varphi\rho^{1-\alpha} > \varepsilon A$) and (b) corner solution (i.e., $\varphi\rho^{1-\alpha} < \varepsilon A$). Recall that we have only assumed $\varphi\rho^{1-\alpha} < A$ but $\varepsilon < 1$.

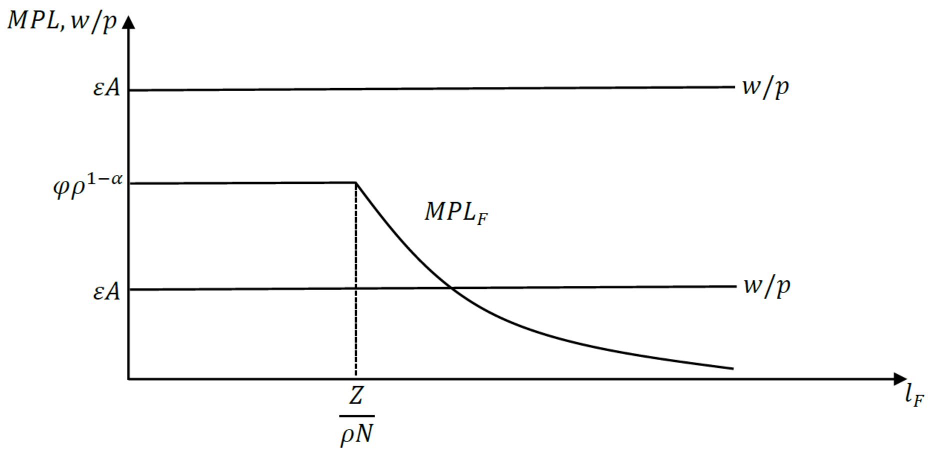

Figure 3: Labor market

Figure 3 plots (A8) and shows that there can be a gradual transition (i.e., $l_F > 0$) or an immediate transition (i.e., $l_F = 0$) from agriculture to industrial production.

Interior solution: If $\varphi\rho^{1-\alpha} > \varepsilon A$, then the equilibrium level of agricultural labor l_F from (A8) is

$$l_F = \left(\frac{\alpha\varphi}{\varepsilon A}\right)^{1/(1-\alpha)} \frac{Z}{N}, \tag{A9}$$

which implies that the equilibrium level of industrial labor is

$$l_Y = l - l_F = l - \left(\frac{\alpha\varphi}{\varepsilon A}\right)^{1/(1-\alpha)} \frac{Z}{N}. \tag{A10}$$

An industrial market would only emerge if N is sufficiently large to cover the fixed cost δ such that $l_Y N \geq \delta/(1-\varepsilon)$, which is required for nonnegative profit $\pi(i) \geq 0$. Then, (A10) yields

$$N \geq \frac{1}{l}\left[\left(\frac{\alpha\varphi}{\varepsilon A}\right)^{1/(1-\alpha)} Z + \frac{\delta}{1-\varepsilon}\right] \equiv N_I(\underset{+}{\varphi}, \underset{+}{Z}, \underset{+}{\delta}, \underset{-}{A}, \underset{-}{l}), \tag{A11}$$

which is now given by a closed-form solution and has the same comparative statics as (17).

Before the emergence of industrial production, the population growth rate $\Delta N_t/N_t$ and the steady-state population level N_A^* in the agricultural era are given by (34) and (35) in Section

3.3. If N_t reaches N_A^* before reaching N_I, then the economy would remain as an agricultural society indefinitely. From (35) and (A11), the inequality $N_A^* > N_I$ is equivalent to

$$\left(\frac{\sigma l}{\beta}\right)^{1/(1-\alpha)} > \left(\frac{\alpha}{\varepsilon A}\right)^{1/(1-\alpha)} + \frac{\delta}{(1-\varepsilon)\varphi^{1/(1-\alpha)} Z}, \tag{A12}$$

which shows that the gradual transition from an agricultural economy to an industrial economy begins under the following conditions: a low fertility cost β, a strong fertility preference σ, a high level of agricultural productivity φ, a high level of labor supply l, a large amount of land Z, a high level of industrial productivity A, and a low fixed cost δ for operating industrial firms. These conditions are the same as in Section 3.4, except that the transition in this case is gradual (i.e., $l_F > 0$) until $N_t \to \infty$.

Under the interior solution, the level of output per capita in the industrial era is given by

$$x + y = f + y = \varphi(l_F)^\alpha \left(\frac{Z}{N_t}\right)^{1-\alpha} + A\left(l_Y - \frac{\delta}{N_t}\right) = \varphi(l_F)^\alpha \left(\frac{Z}{N_t}\right)^{1-\alpha} - Al_F + A\left(l - \frac{\delta}{N_t}\right), \tag{A13}$$

which is decreasing in l_F because $\alpha\varphi(l_F)^{\alpha-1}(Z/N)^{1-\alpha} = w/p = \varepsilon A < A$. Then, (48) shows that l_F is decreasing in N. Substituting (A9) and (A13) into (26) yields the population growth rate, which as before converges towards the same steady state $\Delta N/N = \frac{\sigma}{\beta}Al - 1$ as $N_t \to \infty$.

Corner solution: If $\varphi\rho^{1-\alpha} < \varepsilon A$, then the level of industrial labor l_Y increases sharply from 0 to l when N_t crosses the threshold $N_I \equiv \delta/[(1-\varepsilon)l]$. In this case, the inequality $N_A^* > N_I$ is equivalent to

$$(1-\varepsilon)\left(\frac{\sigma}{\beta}\varphi l\right)^{1/(1-\alpha)} \frac{Z}{\delta} > 1, \tag{A14}$$

which uses (35) and has the same comparative statics for $\{\beta, \sigma, \varphi, l, Z, \delta\}$ as in Section 3.4. The only exception is industrial productivity A; however, a larger A makes the corner solution more likely to apply in which case industrialization could be triggered as a result because the threshold N_I decreases from (A11) to $N_I \equiv \delta/[(1-\varepsilon)l]$.

It is useful to note that although the industrial transition is immediate in this case, $N_I \equiv \delta/[(1-\varepsilon)l]$ is not the same as N_I in (16)-(17) and that there exists a unique interior value of $\varepsilon \in (0,1)$ above which $\delta/[(1-\varepsilon)l]$ is greater than N_I in (16)-(17) in which case industrialization occurs later because the markup ratio $1/\varepsilon$ is too small to cover the fixed cost δ. Finally, under the corner solution, the level of output per capita and the population growth rate in the industrial era are the same as (20) and (36), respectively. In the long run, the population growth rate rises towards the same steady state $\Delta N/N = \frac{\sigma}{\beta}Al - 1$ as $N_t \to \infty$.