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The role of investment wedges in the Carlstrom-Fuerst economy and business cycle accounting

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Abstract

Many researches that apply business cycle accounting (hereafter, BCA) to actual data conclude that models with investment frictions or investment wedges are not promising for modeling business cycle dynamics. In this paper, we apply BCA to artificial data generated by a variant model of Carlstrom and Fuerst (1997, *American Economic Review*), which is one of representative models with investment frictions. We find that BCA leads us to conclude that models of investment wedges are not promising according to the criteria of BCA, although the true model contains investment frictions.

Keywords: Business cycle accounting; investment wedge; investment friction; wedge decompsition

JEL Classification: C68, E13, E32

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1 Introduction

Chari, Kehoe, and McGrattan (2002, 2007) (hereafter, CKM) propose a simple method, *business cycle accounting* (hereafter, BCA), to investigate a promising class of frictional models. In BCA, the economy is assumed as a standard neoclassical prototype model with time-varying productivity, labor tax, investment tax, and government consumption. These are called *efficiency*, *labor*, *investment*, and *government wedges*, respectively. This assumption is justified by equivalence results. This prototype model with wedges covers a large class of frictional business cycle models. Wedges are measured so that the prototype model perfectly accounts for the observed data. After the measurement of wedges, the importance of each wedge is evaluated through counterfactual simulations under an alternative sequence of wedges: *wedge decompositions*. The importance of wedges is judged by the similarity of output prediction to actual data.

Many papers including CKM deny the importance of investment wedges. The models of Bernanke, Gertler, and Gilchrist (1999) and Carlstrom and Fuerst (1997) are often cited as representative models with investment frictions.

In this paper, we apply BCA to artificial data generated by a variant model of Carlstrom and Fuerst (1997). We find that BCA leads us to conclude that models with investment wedges are not promising according to the criteria of BCA, although the true model contains investment frictions. This is because BCA focuses only on determining whether or not investment wedges are the driving force of business cycles. In the Carlstrom-Fuerst economy, the role of investment wedges is to delay the propagation of technology shocks, which is consistent with empirical facts. BCA cannot capture such a role of investment wedges.

The remainder of this paper is as follows. In Section 2, we introduce our economy and the prototype model for BCA. Section 3 applies BCA to data generated by the Carlstrom-Fuerst economy. Section 4 draws some concluding remarks.

2 The model

2.1 Carlstrom-Fuerst economy

First, we provide a brief description about our Carlstrom-Fuerst economy.[1] There are two types of consumers: households and entrepreneurs. Households own capital stock and supply labor input and capital for the production of output to competitive firms. Entrepreneurs own capital stocks and supply capital to firms. Entrepreneurs also have investment technology, and they produce investment goods. Moreover, they have to borrow working capital to produce investment goods, however, the amount of borrowing are limited by their net worth because of the agency problem.

There are two modifications in our Carlstrom-Fuerst economy: utility function and the introduction of government consumption. We employ the constant relative risk aversion (CRRA) utility function $u(c_t, \ell_t) = \log(c_t) + \nu \log(1 - \ell_t)$, since the utility function of the prototype model is CRRA. We also introduce constant government consumption g.[2] The equilibrium system of our Carlstrom-Fuerst economy is summarized as follows:

$$\nu \frac{c_t}{1-\ell_t} = (1-\alpha) \cdot \frac{y_t}{\ell_t}, \tag{1}$$

$$\frac{q_t}{c_t} = \beta E_t \left[\frac{1}{c_{t+1}} \left\{ q_{t+1}(1-\delta) + \alpha \cdot \frac{y_{t+1}}{k_{t+1} + z_{t+1}} \right\} \right], \tag{2}$$

$$q_t = \beta \gamma E_t \left[\left\{ q_{t+1}(1-\delta) + \alpha \cdot \frac{y_{t+1}}{k_{t+1} + z_{t+1}} \right\} \frac{q_{t+1} f(\bar{\omega}_{t+1})}{1 - q_{t+1} g(\bar{\omega}_{t+1})} \right], \tag{3}$$

$$q_t = \left[1 - \Phi(\bar{\omega}_t)\mu + \phi(\bar{\omega}_t)\mu \frac{f(\bar{\omega}_t)}{f'(\bar{\omega}_t)} \right]^{-1}, \tag{4}$$

$$i_t^* = \frac{n_t}{1 - q_t g(\bar{\omega}_t)}, \tag{5}$$

$$n_t = z_t \left[q_t(1-\delta) + r_t \right], \tag{6}$$

$$y_t = A_t \left[k_t + z_t \right]^{\alpha} \ell_t^{1-\alpha}, \tag{7}$$

[1] See Carlstrom and Fuerst (1997, 1998) for details.

[2] We introduce government consumption to make the volatility of the government wedge in the associated prototype model small.

$$\left[k_{t+1} + z_{t+1}\right] = (1-\delta)\left[k_t + z_t\right] + i_t^*\left[1 - \Phi(\bar{\omega}_t)\mu\right], \tag{8}$$

$$e_t + q_t z_{t+1} = q_t i_t^* f(\bar{\omega}_t), \tag{9}$$

$$c_t + e_t + i_t^* + g = y_t, \tag{10}$$

$$\log(A_{t+1}) = \rho \log(A_t) + \varepsilon_{t+1}. \tag{11}$$

(1) is the intratemporal optimization condition; (2) is the Euler equation for households; (3) is the Euler equation for entrepreneurs; (4) and (5) are conditions for the optimal contract; (6) is the definition of net worth; (7) is the aggregate production function; (8) is the evolution of aggregate capital; (9) is the budget constraint of entrepreneurs; (10) is the resource constraint; and (11) is the evolution of aggregate productivity.

2.2 Prototype economy with wedges

The equilibrium system of the associated prototype economy is as follows:

$$\nu \frac{\tilde{c}_t}{1-\tilde{\ell}_t} = (1-\alpha)\cdot\frac{\tilde{y}_t}{\tilde{\ell}_t}, \tag{12}$$

$$\frac{1+\tau_{x,t}}{\tilde{c}_t} = \beta E_t\left[\frac{1}{\tilde{c}_{t+1}}\left\{(1+\tau_{x,t+1})(1-\delta) + \alpha\cdot\frac{\tilde{y}_{t+1}}{\tilde{k}_{t+1}}\right\}\right], \tag{13}$$

$$\tilde{y}_t = \tilde{A}_t \tilde{k}_t^{\alpha} \tilde{\ell}_t^{1-\alpha}, \tag{14}$$

$$\tilde{c}_t + \tilde{g}_t + \tilde{i}_t = \tilde{y}_t, \tag{15}$$

$$\tilde{k}_{t+1} = (1-\delta)\tilde{k}_t + \tilde{i}_t, \tag{16}$$

where $\tilde{A}_t$ is the efficiency wedge; $1/(1+\tau_{x,t})$ is the investment wedge; and $\tilde{g}_t$ is the government wedge.[3] The evolution of $\boldsymbol{s}_t \equiv [\tilde{A}_t, \tau_{x,t}, \tilde{g}_t]'$ is

$$\boldsymbol{s}_{t+1} = \boldsymbol{P}_0 + \boldsymbol{P}\boldsymbol{s}_t + \boldsymbol{\varepsilon}_{t+1}. \tag{17}$$

[3] We eliminate the labor wedge following Chari, Kehoe, and McGrattan (2002).

2.3 Equivalence result

If we interpret $i_t^*\left[1 - \Phi(\bar{\omega}_t)\mu\right]$ as investment i_t of the Carlstrom-Fuerst economy and $k_t + z_t$ as the total capital stock K_t, the following equivalence result holds.[4]

Proposition 1 (Equivalence result) *The equilibrium allocation of the Carlstrom-Fuerst economy* $\{c_t, \ell_t, i_t, K_{t+1}, y_t\}_{t=0}^{\infty}$ *coincides with that of the prototype model* $\{\tilde{c}_t, \tilde{\ell}_t, \tilde{i}_t, \tilde{k}_{t+1}, \tilde{y}_t\}_{t=0}^{\infty}$ *if*

$$A_t = \tilde{A}_t, \quad (18)$$

$$q_t = 1 + \tau_{x,t}, \quad (19)$$

$$e_t + i_t^* \Phi(\bar{\omega}_t)\mu + g = \tilde{g}_t. \quad (20)$$

The proof is simple. It is easily obtained in a straightforward manner by comparing two equilibrium systems (1) - (11) and (12) - (16). The remaining problem is the existence of the VAR(1) representation of wedges, as discussed in Nutahara and Inaba (2008). In this case, the VAR(1) representation of wedges exists under realistic parameter values since the number of wedges is three and the number of endogenous and exogenous variables in the Carlstrom-Fuerst economy is three. Moreover, Theorems 1 and 2 in Nutahara and Inaba (2008) are satisfied. This proposition states that the equilibrium allocation of the Carlstrom-Fuerst economy is consistent with that of the prototype model if the investment and government wedges fluctuate positively.

3 BCA in the Carlstrom-Fuerst economy

First, we generate artificial data from our Carlstrom-Fuerst economy. The parameter values of the Carlstrom-Fuerst economy are as follows. Most variables are taken from Carlstrom and Fuerst (1997, 1998). The discount rate β is .99; the depreciation rate of capital δ, .02; and the persistence of technology ρ, .95. The production share of capital

[4]This equivalence result is different from that in Chari, Kehoe, and McGrattan (2002). They construct a prototype economy with a type of adjustment costs of investment.

α is .36. The weight of leisure in utility ν is calibrated such that the steady-state labor supply equals .3. The discount rate of entrepreneurs $\gamma\beta$ is $.973\beta$; the standard deviation of idiosyncratic technology shock σ, .37; and the monitoring cost μ, .15. The steady-state ratio of government purchase to output, g/y, is .1. We approximate the equilibrium system by the log-linearization and employ the method of Uhlig (1999) to calculate the policy functions. We generate 1000 long-period artificial data.

Following the standard method of BCA, we measure efficiency, labor, investment, and government wedges so that the prototype model can perfectly account for data of consumption, investment, labor, and output. Then, we obtain wedge decompositions by providing only one wedge.[5] Figure 1 shows the actual output and output prediction with only one wedge.

[Insert Figure 1]

The contribution of investment wedge to output is rather small and negative. Hence, by the criteria of BCA, the investment wedge is not promising. However, our data-generating model is the Carlstrom-Fuerst economy, which is one of the representative models with investment frictions.

Why does BCA lead us to such a conclusion? One might consider that we mismeasured the investment wedge. Figure 2 shows the model and the measured investment wedges for the first 100 periods.[6] As shown in Figure 2, the investment wedge is measured almost correctly.[7]

[Insert Figure 2]

This is due to the role of investment wedges. Figure 3 shows the impulse responses to the one percent technology shock in the Carlstrom-Fuerst economy.

[Insert Figure 3]

[5] The procedure that we employ here will be available from authors upon request.

[6] The model investment wedge is obtained by solving the extended equilibrium system (1) - (11) and (13).

[7] Other wedges are also measured correctly since they are measured directly under intratemporal conditions.

When technology shocks hit the economy, output increases and the investment wedge $1/(1+\tau_{x,t})$ decreases, or the distortion in the investment process increases. The important feature of the Carlstrom-Fuerst economy is the hump-shaped impulse response of output to technology shock, which is consistent with the finding of Cogley and Nason (1995). The role of investment wedges is to delay the propagation of shocks. Then, the main driving force of output is the efficiency wedge, and the investment wedge explains deviations from the simple real business cycle model in the Carlstrom-Fuerst economy. BCA cannot capture such a role of investment wedges.

4 Concluding remarks

In this paper, we apply BCA to artificial data generated by the model with investment frictions and find that it leads us to conclude that models of investment wedges are not promising according to the criteria of BCA, although the true model contains investment frictions. This is because BCA only focuses on determining whether or not investment wedges are the driving force of business cycles. In the Carlstrom-Fuerst economy, the role of investment wedges is to delay the propagation of technology shocks, which is consistent with empirical facts. BCA cannot capture such a role of investment wedges. Therefore, we have to be careful while interpreting the results of wedge decomposition in BCA.

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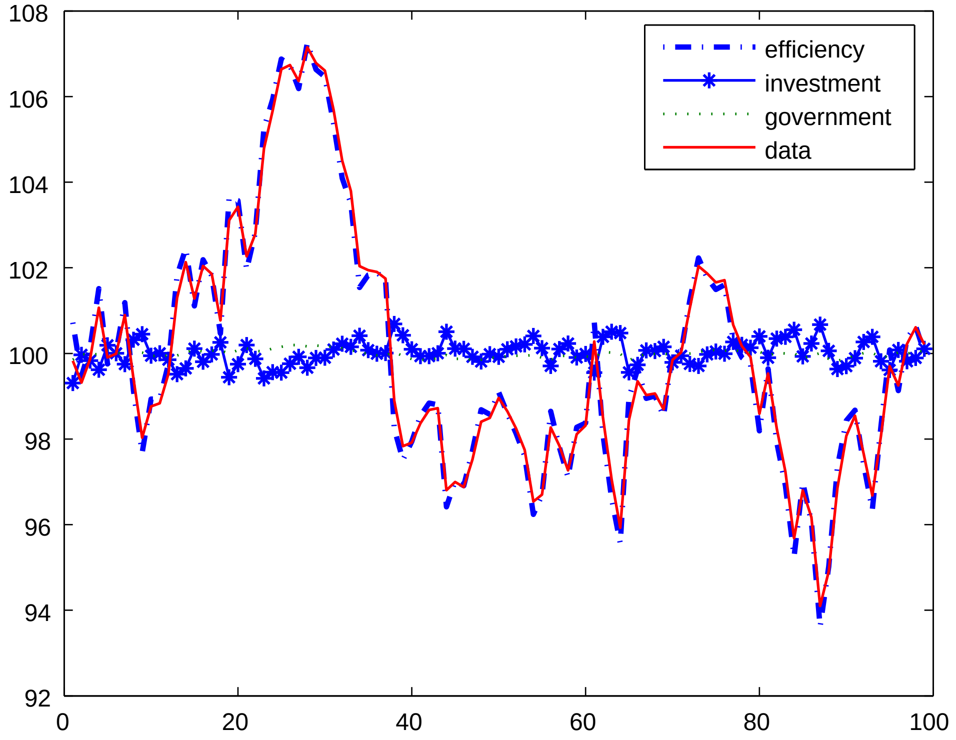

Figure 1: Output decomposition with one wedge

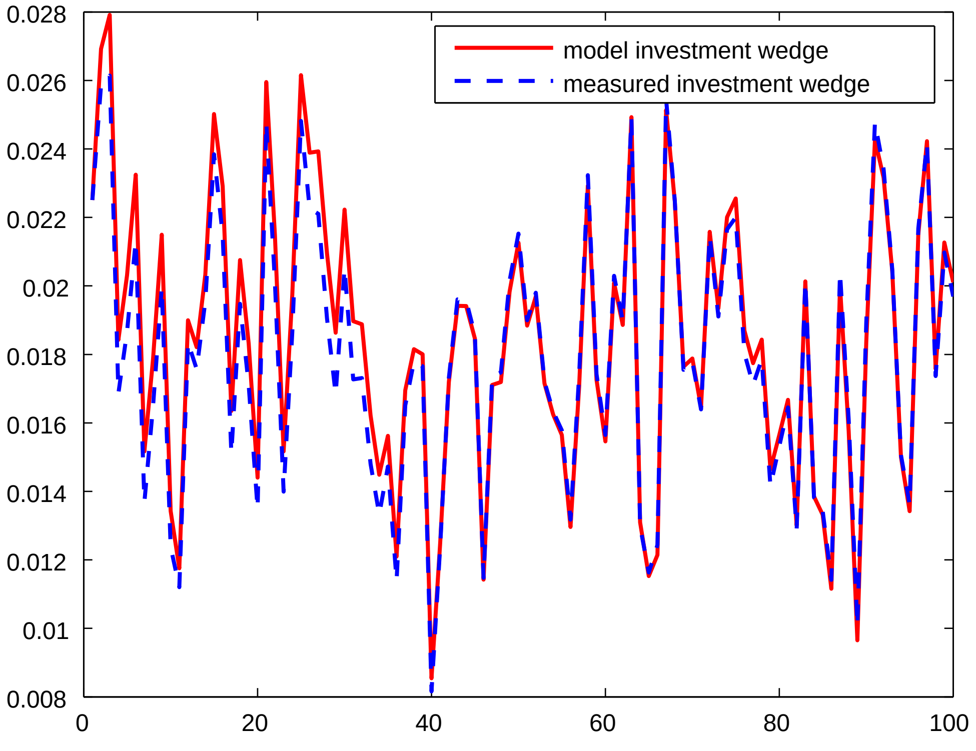

Figure 2: Model and measured investment wedges

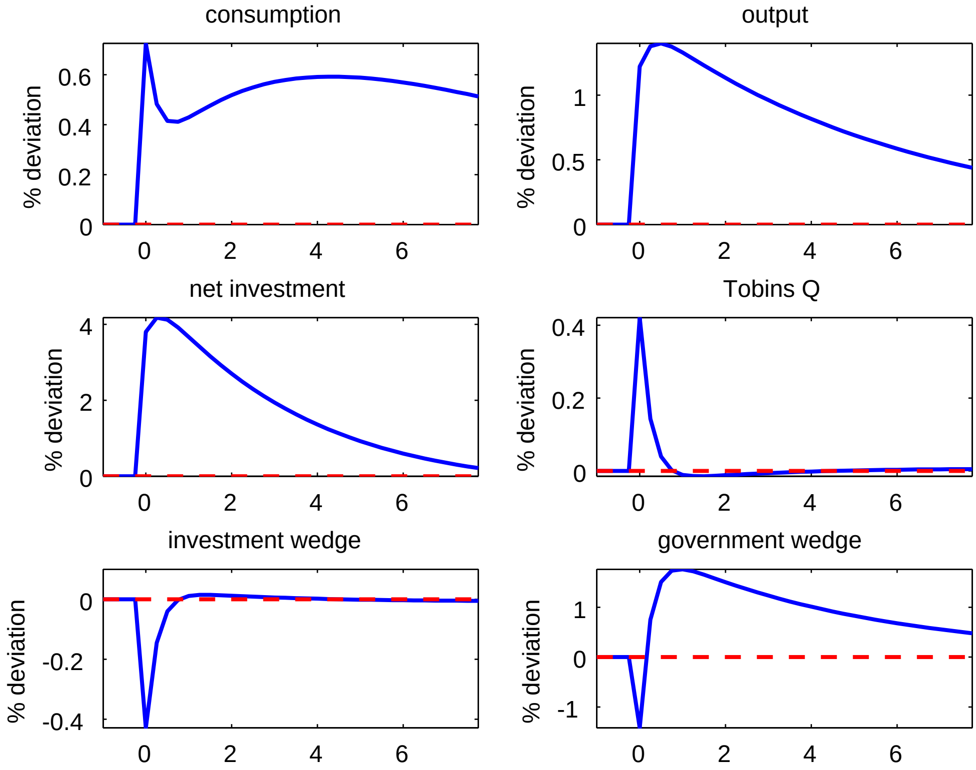

Figure 3: Impulse responses to one percent technology shock of the Carlstrom-Fuerst economy