Borrowing to finance public investment: a politico-economic analysis of fiscal rules

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22 August 2021

Borrowing to Finance Public Investment: A Politico-Economic Analysis of Fiscal Rules *

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December 31, 2022

Abstract

This study focuses on the golden rule of public finance, which distinguishes public investment from consumption spending when borrowing and permits only debt-financed public investment, in an overlapping-generations model with physical and human capital accumulation. In this model, the rule and the associated fiscal policy are endogenous, chosen in each period by a short-lived government representing existing generations. We calibrate the model to Germany, Japan, and the United Kingdom, where the rule has been in place, and show that Germany follows the rule while Japan and the United Kingdom break it, which is consistent with current literature. Subsequently, we evaluate the government's choice and the resulting political distortions of physical and human capital accumulation from the perspective of future generations. We compute the optimal proportion of debt-financed public investment in terms of minimizing the political distortions and find that in each country, the optimal proportion is lower than the one determined by the short-lived government.

- Keywords: Fiscal Rule; Golden Rule of Public Finance; Probabilistic Voting; Overlapping Generations; Political Distortions
- JEL Classification: D70, E62, H63

*The authors gratefully acknowledge financial support from Grant-in-Aid for Early-Career Scientists from the Japan Society for the Promotion of Science (No. 18K12802, Uchida) and Grant-in-Aid for Scientific Research (C) from the Japan Society for the Promotion of Science (No. 21K01539, Ono).

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1 Introduction

Many developed countries have incurred large budget deficits, thereby accumulating debt in the aftermath of the global financial crisis of 2008/9 (OECD, 2021). On the one hand, budget deficits enable governments to spend more than they receive in tax revenue, thereby providing citizens with higher levels of public goods and services in the short run. On the other hand, budget deficits might reduce economic growth by inhibiting capital accumulation, thus lowering the provision of public goods and services in the long run. In addition, fiscal deficits raise the issue of intergenerational equity in fiscal burdens. This is because they imply a transfer of the burden of public spending from the current generation to future generations.

To cope with large budget deficits and their associated problems, many developed countries have implemented various types of fiscal rules (Budina et al., 2012; Schaechter et al., 2012; Wyplosz, 2013; Lledó et al., 2017; Caselli et al., 2018). A well-known fiscal rule is the balanced budget rule (BB). BB is a constitutional requirement that ensures that tax revenues must be sufficient to cover expenditures and interest payments on debt, implying zero or negative deficits (Azzimonti et al., 2016). BB has the advantage of maintaining fiscal discipline each year but presents a disadvantage in terms of efficiency and intergenerational equity. In terms of efficiency, BB does not tax rates to be smoothed over time (Stockman, 2001, 2004). In terms of intergenerational equity, BB requires current generations to finance the entire public investment burden, even though its benefits would be received only by future generations (Bom and Ligthart, 2014).

The golden rule of public finance (GR) is a fiscal rule that addresses the abovementioned problems of BB (Buiter et al., 1993; Corsetti and Roubini, 1996; Robinson, 1998). GR, which allows budget deficits to finance only public investment but not current expenditure, has been in place in Germany, Japan, and the United Kingdom (Kumar et al., 2009). Following the development of this alternative rule, many researchers analyzed GR in growth models with public capital. They showed that GR could gradually improve growth and welfare across generations (e.g., Greiner and Semmler, 2000; Ghosh and Mourmouras, 2004a,b; Greiner, 2008; Yakita, 2008; Minea and Villieu, 2009; Agénor and Yilmaz, 2017; Ueshina, 2018). These studies consider tax rates and/or expenditure as given, and thus, their conclusions rely on the assumption that fiscal policy instruments are independent of changes in the fiscal rule. However, the fiscal rule greatly influences the governments' choice of fiscal policy (e.g., Fatás and Mihov, 2006). Therefore, considering the endogenous response of fiscal policies to the fiscal rule would provide a new perspective on the consequences of the fiscal rule for growth and welfare.

Several approaches have been attempted to investigate the effects of fiscal rules on fiscal policy formation and the resulting impacts on growth and welfare. They include Barseghyan and Battaglini (2016), Arai et al. (2018), Andersen (2019), and Uchida and Ono (2021).[1] The

[1] In addition, several studies investigate the effects of fiscal rules on fiscal policy formation. These include Bisin et al. (2015), Halac and Yared (2018), Coate and Milton (2019), Bouton et al. (2020), Dovis and Kirpalani (2020,

analysis of Arai et al. (2018) is based on a model without public capital, and thus, their analysis does not focus on GR. Barseghyan and Battaglini (2016), Andersen (2019) and Uchida and Ono (2021) presented models with public investment in human capital or infrastructure that works as productive capital, but their main focus was on the austerity program or debt ceiling.[2] A notable exception is Bassetto and Sargent (2006), who presented a multi-period overlapping-generations model with durable public goods. They calibrated the model to the US economy and show that GR may approximate the Pareto-efficient allocation.

Our study differs from Bassetto and Sargent (2006) in the following two aspects. First, fiscal rules are endogenously determined for each period through voting. Fiscal rules are generally set by law or constitution in many countries and cannot be easily changed. However, exemptions from the rules are granted in Japan and the United Kingdom with the approval of the Diet and Parliament, respectively. We approximate the political mechanism of such rule exemptions by modeling the determination of the fiscal rule via voting patterns in each period and call it a politically preferred fiscal rule. We show that the politically preferred fiscal rule is affected by structural parameters representing preferences for public goods and the elasticity of human capital with respect to public investment. The differences in these parameters might explain why some countries follow the GR while others break it.

Second, we assume a logarithmic utility function. Under this assumption, income and substitution effects offset each other, but a general equilibrium effect of policy choices through the interest rate still remains. This general equilibrium effect creates a distortion in which the level of physical capital deviates from the efficient level. This type of distortion is abstracted in Bassetto and Sargent (2006), who adopted a quasilinear utility function but assumed no general equilibrium effect despite its importance being well documented in political economy literature (e.g., Gonzalez-Eiras and Niepelt, 2008, 2012; Song et al., 2012). Our analysis shows that this type of distortion makes the efficient fiscal rule more stringent.

To execute our analysis, we employ a three-period overlapping-generations model with physical and human capital. Each generation comprises many identical individuals who live over three periods: the young who benefit from public investment in human capital, the middle-aged who work, and older adults who are retired. Public investment and parental human capital are inputs in the human capital formation process, contributing to the human capital formation of children, and thus productivity in output per worker. Governments, as elected representatives, finance public investment in human capital and unproductive public goods provision through taxes on capital and labor income as well as through public debt issuance. When expenditure is constrained by fiscal rules, public goods provision must be financed solely by tax revenues, while a certain proportion of the public investment, denoted by $\phi \geq 0$, can be financed by public debt

2021), and Piguillem and Riboni (2020). They rely on models without physical and/or public capital, and hence, they do not focus on the growth and welfare effects of fiscal rules over time and across generations.

[2] Andersen (2019) mentioned GR as an alternative rule, but their analysis is limited to a brief sketch as a possible extension.

issuance. In particular, the rule does not allow for deficit when $\phi = 0$; all public investments in human capital are allowed to be financed with public debt issuance when $\phi = 1$; and all public investments plus a part of current expenditures are debt-financed and GR is broken when $\phi > 1$.

Under this framework, we begin our analysis by considering the politics of fiscal policy formation for a given fiscal rule, ϕ. In particular, we assume probabilistic voting to demonstrate the extent to which generations face conflicts over such policies. In each period, middle-aged and older adult individuals vote on candidates.[3] The government, represented by elected politicians, maximizes the political objective function of the weighted sum of utilities of the middle-aged and older adult populations. In this voting environment, we show that the current policy choice affects the decision on future policy via physical and human capital accumulation. This intertemporal effect creates the three driving forces that shape fiscal policy, namely, a general equilibrium effect through the interest rate, a disciplining effect through the capital income tax rate in the next period, and a disciplining effect through public goods provision in the next period. The three effects induce the government to finance part of its expenditure via public debt issuance.

We calibrate the model economy to Germany, Japan, and the United Kingdom, where the GR has been in place (Kumar et al., 2009). We study how changes in the fiscal rule affect the government's choice of fiscal policies and the resulting allocation of physical and human capital. We show that a higher ϕ, implying a larger share of public investment financed by public debt issuance, lowers physical capital accumulation through the crowding-out effect. This in turn raises the marginal cost of the labor income tax, and thus induces the government to choose a lower labor income tax rate. We also show that a higher ϕ has two conflicting effects on the choice of public investment: a negative effect through the crowding-out effect on physical capital, and a positive effect through a reduced tax burden. Given these conflicting effects, there is a threshold value of ϕ at which the two effects are balanced, and an increase in ϕ has an inverse U-shaped effect on the ratio of public investment to GDP around the threshold value of ϕ.

We then move on to the analysis of the politically preferred ϕ, that is, the debt-financed proportion of public investment that realizes the maximization of the political objective function. We show that the politically preferred ϕ depends on the two structural parameters, θ, representing the preferences for public goods, and η, representing the elasticity of human capital with respect to public investment. A higher θ yields a stronger incentive for the government to cut public debt issuance, and a higher η implies a higher marginal benefit of public investment, incentivizing the government to lower the debt-financed proportion of public investment. Given this property, we compute the politically preferred ϕ for the three countries and show that it is less than one in Germany that has the highest θ and η. We also show that the politically

[3] The young may also have an incentive to vote since they would benefit from public investment financed by taxing capital and labor income. However, for the tractability of analysis, we assume that politicians do not care about their preferences following Saint-Paul and Verdier (1993), Bernasconi and Profeta (2012), and Lancia and Russo (2016). This assumption is supported in part by the fact that a significant portion of the young are below the voting age.

preferred ϕ is greater than one in Japan and the United Kingdom, which implies that these two countries break the GR. These model predictions are consistent with the evidence; a waiver of the GR has been approved by the Diet almost every year since 1975 in Japan (Kumar et al., 2009), and the GR adopted in 1997 was met only for the first few years in the United Kingdom (Wyplosz, 2013).

The abovementioned politically preferred fiscal rule is a choice that only considers the generations existing at the time of voting. As public investment has a long-lasting effect, it is necessary to evaluate the rule from a generational perspective. For this aim, we describe the optimal allocation chosen by a benevolent, long-lived planner who can commit to all its choices at the beginning of a period. Assuming such a hypothetical planner, we focus on the deviations between the political equilibrium allocation and the allocation of the planner in terms of physical and human capital accumulation and refer to the total deviations as political distortions.

We explore the fiscal rule that minimizes political distortions and find that in the three countries, the human capital in the political equilibrium is always lower than that in the planner's allocation, whereas the physical capital in the political equilibrium is higher (lower) than that in the planner's allocation when ϕ, representing the fiscal rule, is low (high). This property indicates that the political distortions exhibit a hump-shape pattern in response to an increase in ϕ, and thus the distortions are minimized at the top of the hump. In addition, the distortion-minimizing ϕ is lower than the politically preferred ϕ for the three countries, suggesting that the consideration of future generations calls for the legislature to lower the proportion of debt-finance in public investment.

The remainder of this paper is organized as follows. Section 2 introduces the model and characterizes an economic equilibrium that describes the behavior of households and firms for a given set of fiscal policies. Section 3 presents the politics of fiscal policy formation, characterizes a political equilibrium for a given fiscal rule, and investigates the effects of the fiscal rule on fiscal policy formation. It also demonstrates the political determination of the fiscal rule and investigates its property. Section 4 characterizes the planner's allocation and evaluates the politically preferred fiscal rule from the viewpoint of the planner. It also presents the fiscal rule that minimizes the deviation between the political equilibrium and the planner's allocation. Section 5 provides the concluding remarks. All proofs are presented in the appendix.

2 Model

The discrete time economy starts in period 0 and consists of overlapping generations. Individuals are identical within a generation and live for three periods: youth, middle age, and older adult age. Each middle-aged individual has $1+n$ children. The middle-aged population for period t is N_t and the population grows at a constant rate of $n(>-1) : N_{t+1} = (1+n)N_t$.

2.1 Individuals

Individuals display the following economic behavior over their life cycles. During youth, they make no economic decisions and receive public investment in human capital financed by the government. In middle age, individuals work, receive market wages, and pay taxes. They use after-tax income for consumption and savings. Individuals retire in their older adult years and receive and consume returns from savings.

Consider an individual born in period $t-1$. In period t, the individual is middle-aged and endowed with h_t units of human capital inherited from his or her parents. The individual supplies these units inelastically in the labor market and obtains labor income $w_t h_t$, where w_t is the wage rate per efficient unit of labor in period t. After paying tax $\tau_t w_t h_t$, where $\tau_t \in (0,1)$ is the period t labor income tax rate, the individual distributes the after-tax income between consumption c_t and savings invested in physical capital s_t. Therefore, the period t budget constraint for the middle-aged becomes $c_t + s_t \leq (1-\tau_t) w_t h_t$.

The period $t+1$ budget constraint in older adult age is $d_{t+1} \leq \left(1-\tau^k_{t+1}\right) R_{t+1} s_t$, where d_{t+1} is consumption, τ^k_{t+1} is the period $t+1$ capital income tax rate, $R_{t+1}(>0)$ is the gross return from investment in physical capital, and $R_{t+1} s_t$ is the return from savings. The results are qualitatively unchanged if the capital income tax is on the net return from savings rather than the gross return from savings.

The human capital of children in period $t+1$, h_{t+1}, is a function of government spending on public investment, x_t, and the human capital of their parents, h_t. In particular, h_{t+1} is formulated using the following equation:

$$h_{t+1} = D\left(x_t\right)^{\eta}\left(h_t\right)^{1-\eta}, \tag{1}$$

where $D(>0)$ is a scale factor and $\eta \in (0,1)$ denotes the elasticity of human capital with respect to public investment.[4] Private investment in human capital is abstracted away from the analysis as the focus of this study is on the financing method of public investment.[5]

The preferences of the middle-aged in period t are specified by the following expected utility function in the logarithmic form, $U^M_t = \ln c_t + \theta \ln g_t + \beta\left(\ln d_{t+1} + \theta \ln g_{t+1}\right)$, where g is per capita public goods provision, $\beta \in (0,1)$ is the discount factor, and $\theta(>0)$ is the degree of preferences for public goods. This represents the preferences of the young in period $t-1$, U^Y_{t-1}, because they make no economic decision, and their consumption is included in that of their parents. The preferences of older adults in period t are given by $U^O_t = \ln d_t + \theta \ln g_t$.

[4] Our modeling follows Boldrin and Montes (2005), Docquier et al. (2007), Gonzalez-Eiras and Niepelt (2012), Kunze (2014), and Andersen and Bhattacharya (2020).

[5] The private investment of the parents may also contribute to human capital formation. For example, parents' time (Glomm and Ravikumar, 1996, 2001, 2003; Glomm and Kaganovich, 2008) or spending (Glomm, 2004; Lambrecht et al., 2005; Kunze, 2014) may complement public investment. However, in the present model, parents have no incentive to invest in human capital privately because they exhibit no altruism toward their children. By contrast, children (i.e., the young) may have an opportunity to invest in their own human capital formation by borrowing from their parents in their youth and repaying the loan in middle age. Uchida and Ono (2021) show that this possibility is ruled out if private investment is a perfect substitute for public investment (Lancia and Russo, 2016; Bishnu and Wang, 2017)

We substitute the budget constraints into the utility function of the middle-aged, U_t^M, to form the following unconstrained maximization problem:

$$\max_{\{s_t\}} \ln\left[(1-\tau_t)w_t h_t - s_t\right] + \theta \ln g_t + \beta\left[\ln\left(1-\tau_{t+1}^k\right) R_{t+1}s_t + \theta \ln g_{t+1}\right], \tag{2}$$

where g_t and g_{t+1} are taken as given. By solving the problem in (2), we obtain the following savings and consumption functions:

$$s_t = \frac{\beta}{1+\beta}\cdot(1-\tau_t)w_t h_t, \tag{3}$$

$$c_t = \frac{1}{1+\beta}\cdot(1-\tau_t)w_t h_t, \tag{4}$$

$$d_{t+1} = \left(1-\tau_{t+1}^k\right) R_{t+1}\cdot\frac{\beta}{1+\beta}\cdot(1-\tau_t)w_t h_t. \tag{5}$$

2.2 Firms

Each period contains a continuum of identical firms that are perfectly competitive profit maximizers. According to Cobb–Douglas technology, the firms produce a final good Y_t using two inputs: aggregate physical capital K_t and aggregate human capital $H_t \equiv N_t h_t$. Aggregate output is given by $Y_t = A\left(K_t\right)^{\alpha}\left(H_t\right)^{1-\alpha}$, where $A(>0)$ is a scale parameter and $\alpha \in (0,1)$ denotes the capital share. The production function in intensive form is $y_t = y\left(k_t, h_t\right) \equiv A\left(k_t\right)^{\alpha}\left(h_t\right)^{1-\alpha}$ where $y_t \equiv Y_t/N_t$, $k_t \equiv K_t/N_t$, and $h_t \equiv H_t/N_t$ denote per capita output, physical capital, and human capital, respectively.

The first-order conditions for profit maximization with respect to H_t and K_t are

$$w_t = w\left(k_t, h_t\right) \equiv (1-\alpha)A\left(k_t\right)^{\alpha}\left(h_t\right)^{-\alpha}, \tag{6}$$

$$R_t = R\left(k_t, h_t\right) \equiv \alpha A\left(k_t\right)^{\alpha-1}\left(h_t\right)^{1-\alpha}, \tag{7}$$

where w_t and R_t are labor wages and the gross return on physical capital, respectively. The conditions state that firms hire human and physical capital until the marginal products are equal to the factor prices. Capital is assumed to depreciate fully within each period.

2.3 Government Budget Constraint and Fiscal Rule

Government expenditure items are a public investment in human capital and expenditure on (unproductive) public goods provision. They are financed by capital and labor taxes, as well as public debt issues. Let B_t denote the aggregate inherited debt. The aggregate government budget constraint in period t is

$$B_{t+1} + \tau_t^k R_t s_{t-1} N_{t-1} + \tau_t w_t h_t N_t = N_{t+1}x_t + G_t + R_t B_t, \tag{8}$$

where B_{t+1} is newly issued public debt; $\tau_t^k R_t s_{t-1} N_{t-1}$ is the aggregate capital tax revenue; $\tau_t w_t h_t N_t$ is the aggregate labor tax revenue; $N_{t+1}x_t$ is the aggregate public investment; G_t is aggregate public goods provision, and $R_t B_t$ is debt repayment. We assume a one-period debt structure to derive analytical solutions from the model. We also assume that the government in each period is committed to not repudiating the debt.

Consider a situation in which government expenditures are constrained by fiscal rules. In particular, following the literature on GR, we focus on fiscal rules that impose constraints on how to finance public investment. The fiscal rule is given by $B_{t+1} = \phi N_{t+1} x_t$, or:

$$(1+n)b_{t+1} = \phi(1+n)x_t, \tag{9}$$

where $b_{t+1} \equiv B_{t+1}/N_{t+1}$ is per-capita public debt and $\phi\,(\geq 0)$ represents the fiscal rule that determines the percentage of public investment to be financed by public debt issuance.

Substitution of (9) into (8) leads to the associated per capita form of the government budget constraint:

$$\tau_t^k R_t \frac{s_{t-1}}{1+n} + \tau_t w_t h_t = (1-\phi)(1+n)x_t + \frac{2+n}{1+n} g_t + R_t b_t. \tag{10}$$

When $\phi = 0$, the rule does not allow for a deficit, and requires a balanced budget. All government expenditures must be financed using tax revenues.[6] When $\phi = 1$, all public investments can be financed by public debt issuance. This rule allows a budget deficit only to finance public investment, and prohibits the financing of current expenditures (i.e., public goods provision) by budget deficit. When $\phi > 1$, all public investments plus a part of public goods provision are financed by public debt issuance. As ϕ increases, the fiscal burden of public investment is increasingly deferred to future generations.

2.4 Economic Equilibrium

Public debt is traded in the domestic capital market. The market clearing condition for capital is $B_{t+1} + K_{t+1} = N_t s_t$, which expresses the equality of total savings by the middle-aged population in period t, $N_t s_t$ to the sum of the stocks of aggregate public debt and aggregate physical capital at the beginning of period $t+1$, $B_{t+1} + K_{t+1}$. Using $k_{t+1} \equiv K_{t+1}/N_{t+1}$, $h_{t+1} = H_{t+1}/N_{t+1}$, the profit-maximization condition in (6), and the savings function in (3), we can rewrite the abovementioned condition as

$$(1+n)\left(k_{t+1} + b_{t+1}\right) = s\left(\tau_t; k_t, h_t\right) \equiv \frac{\beta}{1+\beta}(1-\tau_t) w\left(k_t, h_t\right) h_t. \tag{11}$$

The following defines the economic equilibrium in the present model.

Definition 1 Given a sequence of policies, $\left\{\tau_t^k, \tau_t, x_t, g_t\right\}_{t=0}^{\infty}$, an *economic equilibrium* is a sequence of allocations $\{c_t, d_t, s_t, k_{t+1}, b_{t+1}, h_{t+1}\}_{t=0}^{\infty}$ and prices $\{\rho_t, w_t, R_t\}_{t=0}^{\infty}$ with initial conditions $k_0 (> 0), b_0 (\geq 0)$ and $h_0 (> 0)$, such that (i) given $\left(w_t, R_{t+1}, \tau_{t+1}^k, \tau_t, x_t, g_t, g_{t+1}\right)$, (c_t, d_{t+1}, s_t) solves the utility-maximization problem; (ii) given (w_t, ρ_t), k_t solves the firm's profit maximization problem; (iii) given (w_t, h_t, R_t, b_t), $\left(\tau_t^k, \tau_t, x_t, b_{t+1}\right)$ satisfies the government budget constraint; and (iv) the capital market clears: $(1+n)\left(k_{t+1} + b_{t+1}\right) = s_t$.

[6] Specifically, the government has a balanced budget if the third item of the expenditure in (10) is the net repayment of debt, $(R_t - 1)b_t$, rather than the gross repayment of debt, $R_t b_t$; the fiscal rule in (9) is modified to $(1+n)b_{t+1} = \phi(1+n)x_t + b_t$. However, this modification makes it unable to obtain a closed-form solution. Given this limitation, we employ the rule in (9) in the present study.

Definition 1 allows us to reduce the economic equilibrium conditions to a system of difference equations that characterizes the motion of (k_t, b_t, h_t). The system includes the capital market clearing condition in (11), the human capital formation function in (1), and the government budget constraint in (10) with the optimality conditions of firms in (6) and (7). The government budget constraint in (10) is reformulated as

$$TR^K\left(\tau_t^k; k_t, h_t\right) + TR\left(\tau_t; k_t, h_t\right) = (1-\phi)(1+n)x_t + \frac{2+n}{1+n}g_t + R\left(k_t, h_t\right)b_t, \tag{12}$$

where $TR^K\left(\tau_t^k; k_t, h_t\right)$ and $TR\left(\tau_t; k_t, h_t\right)$, representing the tax revenues from capital and labor income, respectively, are defined as follows:

$$\begin{aligned} TR^K\left(\tau_t^k; k_t, h_t\right) &\equiv \tau_t^k R\left(k_t, h_t\right)\left(k_t + b_t\right), \\ TR\left(\tau_t; k_t, h_t\right) &\equiv \tau_t w\left(k_t, h_t\right) h_t. \end{aligned}$$

In economic equilibrium, the indirect utility of the middle-aged population in period t, V_t^M, and that of the older adult population in period t, V_t^O, can be expressed as functions of fiscal policy, physical and human capital, and public debt as follows:

$$\begin{aligned} V_t^M &= V^M\left(\tau_t, x_t, g_t, \tau_{t+1}^k, g_{t+1}; k_{t+1}, h_{t+1}, b_{t+1}, k_t, h_t\right) \\ &\equiv \ln c\left(\tau_t, k_t, h_t\right) + \theta \ln g_t + \beta\left[\ln d\left(\tau_{t+1}^k; k_{t+1}, h_{t+1}, b_{t+1}\right) + \theta \ln g_{t+1}\right], \end{aligned} \tag{13}$$

$$V_t^O = V^O\left(\tau_t^k, g_t; k_t, b_t, h_t\right) \equiv \ln d\left(\tau_t^k; k_t, h_t, b_t\right) + \theta \ln g_t, \tag{14}$$

where $c\left(\tau_t, k_t, h_t\right)$ and $d\left(\tau_t^k; k_t, h_t, b_t\right)$, representing consumption in middle and older adult ages, respectively, are defined as follows:

$$\begin{aligned} c\left(\tau_t, k_t, h_t\right) &\equiv \frac{1}{1+\beta}\left(1-\tau_t\right) w\left(k_t, h_t\right) h_t, \\ d\left(\tau_t^k; k_t, h_t, b_t\right) &\equiv \left(1-\tau_t^k\right) R\left(k_t, h_t\right)(1+n)\left(k_t + b_t\right). \end{aligned}$$

3 Politics

Based on the characterization of the economic equilibrium in Subsection 2.4, we consider the politics of fiscal policy formation. In particular, we employ probabilistic voting à la Lindbeck and Weibull (1987). In this voting scheme, there is electoral competition between two office-seeking candidates. Each candidate announces a set of fiscal policies subject to the government budget constraint. As demonstrated by Persson and Tabellini (2000), the platforms of the two candidates converge in the equilibrium to the same fiscal policy that maximizes the weighted average utility of voters (i.e., the middle-aged and older adults).

In the present framework, the young, the middle-aged, and older adults have the incentive to vote. While the young may benefit from public investment through human capital accumulation, we assume that their preferences are not considered by politicians. We impose this assumption, which is often used in the literature (e.g., Saint-Paul and Verdier, 1993; Bernasconi and Profeta, 2012; Lancia and Russo, 2016), for tractability reasons. However, this assumption could be supported in part by the fact that a significant proportion of the young are below the voting age.

The political objective is defined as the weighted sum of the utility of the middle-aged and older adults, given by $\tilde{\Omega}_t \equiv \omega V_t^O + (1+n)(1-\omega)V_t^M$, where $\omega \in (0,1)$ and $1-\omega$ are the political weights placed on older adults and middle-aged, respectively. The weight on the middle-aged is adjusted by the gross population growth rate, $(1+n)$, to reflect their share of the population. We divide $\tilde{\Omega}_t$ by $(1+n)(1-\omega)$ and redefine the objective function as follows:

$$\Omega_t = \frac{\omega}{(1+n)(1-\omega)} V_t^O + V_t^M, \tag{15}$$

where the coefficient $\omega/(1+n)(1-\omega)$ of V_t^O represents the relative political weight on older adults.

The political objective function suggests that the current policy choice affects the decision on future policy via physical and human capital accumulation. In particular, the period t choices of τ_t^k, x_t, g_t, and b_{t+1} affect the formation of physical and human capital in period $t+1$. This, in turn, influences the decision making on the period-$t+1$ fiscal policy. To demonstrate such an intertemporal effect, we employ the concept of a Markov-perfect equilibrium under which fiscal policy in the present period depends on the current payoff-relevant state variables.

In our framework, the payoff-relevant state variables are physical capital k_t, public debt b_t, and human capital h_t. Thus, the expected rate of capital income tax for the next period, τ_{t+1}^k, is given by the function of the period-$t+1$ state variables, $\tau_{t+1}^k = T^k(k_{t+1}, b_{t+1}, h_{t+1})$. We denote the arbitrary lower limits of τ and τ^k by $-\underline{\tau}(<0)$ and $-\underline{\tau}^k(<0)$, respectively. By using recursive notation with z' denoting the next period z, we can define a Markov-perfect political equilibrium in the present framework as follows.

Definition 2 A *Markov-perfect political equilibrium* is a set of functions, $\langle \hat{T}, \hat{T}^k, \hat{X}, \hat{G}, \hat{B} \rangle$, where $\hat{T} : \Re_+^3 \to (-\underline{\tau}, 1)$ is the labor income tax rule, $\tau = \hat{T}(k,b,h)$, $\hat{T}^k : \Re_+^3 \to \left(-\underline{\tau}^k, 1\right)$ is a capital income tax rule, $\tau^k = \hat{T}^k(k,b,h)$, $\hat{X} : \Re_+^3 \to \Re_+$ is a public investment rule, $x = \hat{X}(k,b,h)$, and $\hat{G} : \Re_+^3 \to \Re_+$ is a public goods provision rule, so that (i) given k, b, and h, $\langle \hat{T}(k,b,h), \hat{T}^k(k,b,h), \hat{X}(k,b,h), \hat{G}(k,b,h) \rangle$ is a solution to the problem of maximizing Ω in (15), subject to the human capital formation function in (1), the capital market clearing condition in (11), and the government budget constraint in (12); and (ii) $\hat{B} : \Re_+^3 \to \Re_+$ is a public debt rule, $b' = \hat{B}(k,b,h)$, that follows the fiscal rule in (9).[7]

[7]The state variables do not line up in compact sets because they grow across periods. To define the equilibrium more precisely, we need to redefine the equilibrium as a mapping from a compact set to a compact set by introducing the following notations: $\hat{x}_t \equiv x_t/y(k_t, h_t)$, $\hat{g}_t \equiv g_t/y(k_t, h_t)$, and $\hat{b}_{t+1} \equiv b_{t+1}/y(k_t, h_t)$. However, for simplicity, we define the equilibrium as in Definition 2.

3.1 Political Equilibrium

We derive the political equilibrium policy functions and the associated sequence of per capita physical and human capital. To obtain the set of policy functions, we conjecture the capital income tax rate and public goods provision in the next period as follows:

$$\tau^{k\prime} = 1 - T^k \cdot \frac{1}{\alpha\left(1+\frac{b'}{k'}\right)}, \tag{16}$$

$$g' = G \cdot y\left(k', h'\right), \tag{17}$$

where $T^k(>0)$ and $G(>0)$ are constant. The conjectures in (16) and (17) suggest that at the aggregate level, the after-tax capital income and the government expenditure on public goods provision are linearly related to GDP.

Given these conjectures, we consider the optimization problem in Definition 2, and obtain the following first-order derivatives:

$$\tau^k : \tfrac{\omega}{(1+n)(1-\omega)}\frac{d_{\tau^k}}{d} + \lambda TR^K_{\tau^k} = 0, \tag{18}$$

$$\tau : \frac{c_\tau}{c} + \beta\left(\frac{d'_\tau}{d'} + \theta\frac{g'_\tau}{g'}\right) + \lambda TR_\tau = 0, \tag{19}$$

$$g : \left(\frac{\omega}{(1+n)(1-\omega)} + 1\right)\frac{\theta}{g} - \lambda\frac{2+n}{1+n} = 0, \tag{20}$$

$$x : \beta\left(\frac{d'_x}{d'} + \theta\frac{g'_x}{g'}\right) - \lambda\left(1-\phi\right)\left(1+n\right) = 0, \tag{21}$$

where $\lambda\,(\geq 0)$ is the Lagrangian multiplier associated with the government budget constraint in (12). The choice of x determines the level of public debt issuance according to the rule in (9). The impact of debt issuance decisions through such a rule is contained in the term $\left(\frac{d'_\tau}{d'} + \theta\frac{g'_\tau}{g'}\right)$ of (19) and the terms $\beta\left(\frac{d'_x}{d'} + \theta\frac{g'_x}{g'}\right)$ and $\lambda\left(1-\phi\right)\left(1+n\right)$ of (21).

The first-order conditions in (18)–(21) are summarized as follows, focusing on the marginal benefit of public goods provision appearing on the left-hand side of (20):

$$\left(\frac{\omega}{(1+n)(1-\omega)} + 1\right)\frac{\theta}{g} = -\frac{2+n}{1+n}\cdot\frac{\frac{\omega}{(1+n)(1-\omega)}\frac{d_{\tau^k}}{d}}{TR^K_{\tau^k}}, \tag{22}$$

$$\left(\frac{\omega}{(1+n)(1-\omega)} + 1\right)\frac{\theta}{g} = -\frac{2+n}{1+n}\cdot\frac{\frac{c_\tau}{c} + \beta\left(\frac{d'_\tau}{d'} + \theta\frac{g'_\tau}{g'}\right)}{TR_\tau}, \tag{23}$$

$$\left(\frac{\omega}{(1+n)(1-\omega)} + 1\right)\frac{\theta}{g} = \frac{2+n}{1+n}\cdot\frac{\beta\left(\frac{d'_x}{d'} + \theta\frac{g'_x}{g'}\right)}{1+n}. \tag{24}$$

According to the expressions in (22)–(24), the government chooses policies to equate the marginal benefits of public goods provision that appear on the left-hand side with the marginal benefits of capital income tax cut, marginal net benefits of the labor income tax cut, and marginal benefits of public investment that appear on the right-hand side, respectively.

A detailed interpretation of these conditions is as follows. The intuition for the marginal benefit of public goods provision appearing on the left-hand side of each equation is straightforward. An increase in public goods leads to an increase in the marginal utility of public goods for older adults and for the middle-aged. The right-hand side of (22) represents the marginal benefit of capital income tax cut. A decrease in the capital income tax rate raises the consumption of older adults, and thus makes them better off. We evaluate this effect based on the change in tax revenue through capital income taxation represented by the term $TR^K_{\tau^k}$ in the denominator. The right-hand sides of (23) and (24) include the following three inter-temporal effects: the general equilibrium effect of capital through the interest rate, $R^{'}$; the disciplining effect through the capital income tax rate, $\tau^{k\prime}$; and the disciplining effect through public goods provision, g'. These effects play crucial roles in shaping fiscal policy, which are explained in turn below.

First, consider the right-hand side of (23), showing the marginal benefit of the labor income tax cut. A cut of the labor income tax rate causes disposable income to rise, thereby increasing the consumption of the middle-aged, as represented by the term c_τ/c. The term $\beta\left(d'_\tau/d' + \theta g'_\tau/g'\right)$ includes the marginal costs and benefits of the labor income tax cut that the current middle-aged population are expected to receive when they are older adults. The cut of the labor income tax rate increases the disposable income of the middle-aged, and thus expands their savings, which in turn raises their consumption when they are older adults. In addition, the increase in savings works to lower the return from savings, $R^{'}$, and thus reduces their consumption when they are older adults. Simultaneously, the increase in savings works to lower the capital income tax rate in the next period, $\tau^{k\prime}$, which raises their consumption when they are older adults. These effects are represented by the term d'_τ/d'. The term $\theta g'_\tau/g'$ shows that the labor income tax cut stimulates savings and capital accumulation, and thus promotes public goods provision in the next period. The right-hand side evaluates these effects based on the change in the tax revenue through the labor income tax cut, as represented by the term TR_τ in the denominator.

Next, consider the right-hand side of (24), showing the marginal net benefit of public investment. The investment promotes human capital accumulation and thus, raises the return from savings, $R^{'}$. This leads to an increase in consumption for the older adults. Simultaneously, the increase in the human capital level leads to an increase in public goods provision in the next period. In addition, under the fiscal rule in (9), an increase in public investment expands public debt issuance, which slows down physical capital accumulation. This raises the interest rate R', and increases consumption in older adult age. Simultaneously, impeded physical capital accumulation works to raise the capital income tax rate in the next period, $\tau^{k\prime}$, and thus reduces the consumption of older adults. Furthermore, impeded physical capital accumulation lowers public goods provision in the next period, g'. The right–hand side of (24) includes these five marginal costs and benefits, which affect the middle-aged when they are older adults.

In deriving the policy functions using the first-order conditions in (18)–(21), alongside the fiscal rule in (9) and the government budget constraint in (12), we further conjecture that the policy functions of τ and x are given by

$$\tau = T, \tag{25}$$

$$(1+n)x = X \cdot y(k,h), \tag{26}$$

where T and X are constant. We can verify the conjectures in (16), (17), (25), and (26) and obtain the following result:

Proposition 1 *Given a fiscal rule in (9), there is a Markov-perfect political equilibrium such that the policy functions of τ^k, g, x, τ, and b' are given by*

$$\begin{aligned}
1-\tau^k &= T^k \cdot \frac{1}{\alpha \frac{k+b}{k}}, \\
\frac{2+n}{1+n} g &= G \cdot y(k,h), \\
(1+n)x &= X \cdot y(k,h), \\
\tau &= T, \\
(1+n)b' &= \phi X \cdot y(k,h),
\end{aligned}$$

where T^k, G, X, and T are defined by:

$$\begin{aligned}
T^k &\equiv \frac{\omega}{(1+n)(1-\omega)}(1-\alpha)\left[\frac{1}{1-T} + \frac{\alpha\beta(1+\theta)\frac{\beta}{1+\beta}(1-\alpha)}{\frac{\beta}{1+\beta}(1-\alpha)(1-T)-\phi X}\right]^{-1}, \\
G &\equiv \left(\frac{\omega}{(1+n)(1-\omega)}+1\right)\theta\left[\frac{1}{1-T} + \frac{\alpha\beta(1+\theta)\frac{\beta}{1+\beta}(1-\alpha)}{\frac{\beta}{1+\beta}(1-\alpha)(1-T)-\phi X}\right]^{-1}(1-\alpha), \\
X &\equiv \frac{I(\phi)}{(1-\phi)\phi}\left\{1+\left[\frac{\omega}{(1+n)(1-\omega)}(1+\theta)+\theta\right]\frac{\frac{\beta}{1+\beta}-\frac{I(\phi)}{1-\phi}}{\frac{\beta}{1+\beta}\left[1+\alpha\beta(1+\theta)\right]-\frac{I(\phi)}{1-\phi}}+\frac{I(\phi)}{\phi}\right\}^{-1}, \\
T &\equiv 1-\frac{1}{1-\alpha}\left\{1+\left[\frac{\omega}{(1+n)(1-\omega)}(1+\theta)+\theta\right]\frac{\frac{\beta}{1+\beta}-\frac{I(\phi)}{1-\phi}}{\frac{\beta}{1+\beta}\left[1+\alpha\beta(1+\theta)\right]-\frac{I(\phi)}{1-\phi}}+\frac{I(\phi)}{\phi}\right\}^{-1},
\end{aligned}$$

where $I(\phi)$ appeared in the expressions of X and T is

$$I(\phi) \equiv \frac{H_1(\phi) - \sqrt{(H_1(\phi))^2 - 4H_2(\phi)}}{2},$$

and $H_1(\phi)$ and $H_2(\phi)$ are

$$\begin{aligned}
H_1(\phi) &\equiv \beta(1+\theta)(\alpha+\eta(1-\alpha))\phi + \frac{\beta}{1+\beta}\left[1+\alpha\beta(1+\theta)\right](1-\phi), \\
H_2(\phi) &\equiv (1-\phi)\phi\beta(1+\theta)\eta(1-\alpha)\frac{\beta}{1+\beta}.
\end{aligned}$$

Proof. See Appendix A.1.

Proposition 1 implies that the policy functions have the following features. First, the capital income tax rate is increasing in public debt but decreasing in physical capital. A higher level of public debt increases the burden of debt repayment. The government responds to the increased burden by raising the capital income tax rate. By contrast, a higher level of physical capital lowers the interest rate and thus reduces the burden of debt repayment. This enables the government to lower the capital income tax rate. Second, the labor income tax rate is independent of the state variables and is constant across periods, whereas the levels of public goods provision and public investment are linear functions of output, y. Given the fiscal rule in (9), this property, with the first one, is necessary for the government budget constraint in (12) to be satisfied in each period.

Having established the policy functions, we are now ready to present physical and human capital accumulation. We substitute the policy functions presented in Proposition 1 into the capital market clearing condition in (11) and the human capital formation function in (1), and obtain

$$\frac{k'}{k} = \frac{1}{1+n}\left(\frac{\beta}{1+\beta}(1-T)(1-\alpha) - \phi X\right) A \left(\frac{h}{k}\right)^{1-\alpha}, \tag{27}$$

$$\frac{h'}{h} = D\left[\frac{1}{1+n} X A \left(\frac{k}{h}\right)^{\alpha}\right]^{\eta}. \tag{28}$$

Given the initial condition, $\{k_0, h_0\}$, the sequence $\{k_t, h_t\}$ is characterized by the above two equations in (27) and (28). A *steady state* is defined as a political equilibrium with $h'/k' = h/k$. In other words, the ratio of human to physical capital is constant across periods. Eqs. (27) and (28) lead to:

$$\frac{h'}{k'} = \Psi\left(\frac{h}{k}\right) \equiv \frac{D\left[\frac{1}{1+n} X A\right]^{\eta}}{\frac{1}{1+n}\left(\frac{\beta}{1+\beta}(1-T)(1-\alpha) - \phi X\right) A} \left(\frac{h}{k}\right)^{\alpha(1-\eta)},$$

with the property of $\Psi'(\cdot) > 0$ and $\Psi''(\cdot) < 0$. This property shows that there is a unique stable steady state for the path of $\{h/k\}$.

3.2 Effects of Fiscal Rules

The result in Subsection 3.1 indicates that the fiscal rule ϕ, taken as an institutional parameter at this stage, has a decisive effect on the formation of fiscal policies. In particular, the rule has direct impacts on τ (the labor income tax rate) via the term $\beta\left(d'_\tau/d'/ + \theta g'_\tau/g'\right)$ in (19) and on x (the public investment) via the terms $\beta\left(d'_x/d' + \theta g'_x/g'\right)$ and $\lambda(1-\phi)(1+n)$ in (21). The rule has no such direct impact on τ^k (the capital income tax rate) and g (public goods provision) although they are affected by the rule via the choice of τ and x. Thus, we focus on the first-order conditions with respect to τ in (19) and x in (21) to investigate the direct effects of the fiscal rule. For this aim, we reformulate (19) and (21) as follows:

$$\tau : \underbrace{\frac{-1}{1-\tau}}_{\equiv \frac{c_\tau}{c}} + \underbrace{\frac{-\alpha\beta(1+\theta)\frac{\beta}{1+\beta}w(k,h)h}{\frac{\beta}{1+\beta}(1-\tau)w(k,h)h - \phi(1+n)x}}_{\equiv \beta\left(\frac{d'_\tau}{d'}+\theta\frac{g'_\tau}{g'}\right)} + \underbrace{\lambda w(k,h)h}_{\equiv \lambda TR_\tau} = 0, \tag{29}$$

$$x : \underbrace{\frac{-\alpha\beta(1+\theta)\phi(1+n)}{\frac{\beta}{1+\beta}(1-\tau)w(k,h)h - \phi(1+n)x} + \frac{\beta\eta(1+\theta)(1-\alpha)}{x}}_{\equiv \beta\left(\frac{d'_x}{d'}+\theta\frac{g'_x}{g'}\right)} - \lambda(1-\phi)(1+n) = 0. \tag{30}$$

Equations (29) and (30) show that the fiscal rule, represented by ϕ, affects the formation of the labor income tax rate and the ratio of public investment to GDP, respectively. Given the difficulty of showing its qualitative impacts, we clarify the effects quantitatively based on numerical methods. In particular, we calibrate the model economy such that the steady-state political equilibrium allocation and policies match some key statistics of each sample country. The sample countries are Germany, Japan, and the United Kingdom, which have adopted GR (golden rule of public finance) as a fiscal rule over the past several decades (Kumar et al., 2009).

Our strategy is to calibrate the model economy in such a manner that for each sample country, the political equilibrium demonstrated in Proposition 1 matches some key statistics over the time period 1995–2016. We assume that each period of the present model lasts 30 years; this assumption is standard in quantitative analyses of two- or three-period overlapping-generations models (e.g., Gonzalez-Eiras and Niepelt, 2008; Lancia and Russo, 2016). We assume the share of capital, α, is common to the three countries and fix $\alpha = 1/3$, in line with Song et al. (2012) and Lancia and Russo (2016).

We calibrate the country-specific parameters β, ω, η, and θ to simultaneously match the average statistics of the labor income tax rate (τ), the ratios of general government final consumption expenditure (G/Y), public investment ($N'X/Y$), and government deficit (B'/Y) for each country during the period 1995–2016. Table 1 reports the average statistics and the estimated parameter values for each sample country. Appendix A.2 provides details on calibration.

Figure 1 illustrates the numerical result of the effects of an increased ϕ on the labor income tax rate, T (Panel (a)), and the ratio of public investment to GDP, X (Panel (b)), for the three countries. On the basis of the result in Figure 1, we can interpret Eqs. (29) and (30) as follows. First, Eq. (29) shows that the fiscal rule, represented by ϕ, affects the formation of the labor income tax rate. The greater ϕ is, the greater the share of public investment financed by public debt issuance. Debt issuance lowers physical capital accumulation through crowding-out effects, reduces the next-period public goods provision, and generates the following two opposing effects on older adult consumption: a positive effect via an increase in the interest rate, and a negative effect via an increase in the next-period capital income tax rate. These effects are observed in the second term of the left-hand side in (29). In the present framework, the latter effect dominates the former one; thus, an increase in ϕ leads to an increase in the marginal cost of

Country	POP	TAX	GOV	INV	DEF
Germany	1.0004	0.3925	0.1919	0.0223	0.0210
Japan	1.0006	0.2236	0.1786	0.0456	0.0588
United Kingdom	1.0059	0.2270	0.1892	0.0230	0.0384
Country	n	β	ω	η	θ
Germany	0.0117	$0.0976(\approx (0.9808)^{120})$	0.5242	0.7426	0.2489
Japan	0.0178	$0.2409(\approx (0.9882)^{120})$	0.4361	0.5480	0.2433
United Kingdom	0.1921	$0.1463(\approx (0.9841)^{120})$	0.4518	0.4202	0.2488

Table 1: Data and calibrated parameters for Germany, Japan, and the United Kingdom. The first five columns are the annual gross population growth rate (POP), the labor income tax rate (TAX), the ratio of general government consumption expenditure to GDP (GOV), the ratio of public investment to GDP (INV), and the deficit-to-GDP ratio (DEF), during 1995–2016. The last five columns are the calibrated parameter values of n, β, ω, η, and θ.
Note: In the table, the quarterly values of β are presented for reference, based on the values of β obtained from the calibration. For example, the quarterly β for Germany is about 0.9808.
Sources: World Development Indicators (`https://databank.worldbank.org/source/world-development-indicators`, accessed on September 9, 2022) for data on POP and GOV; the data archive of Professor McDaniel (`https://www.caramcdaniel.com/research`, accessed on April 6, 2021) for data on TAX; OECD gross domestic product (GDP) indicator (`doi:10.1787/dc2f7aec-en`, accessed on April 12, 2022), OECD investment ($GFCF$) indicator (`doi:10.1787/b6793677-en`, accessed on April 12, 2022), and OECD investment by sector indicator (`doi:10.1787/abd72f11-en`, accessed on April 12, 2022) for data on INV; OECD library, General government deficit indicator (`https://doi.org/10.1787/cc9669ed-en`, accessed on February 25, 2021) for data on DEF.

τ. To offset this increase in the marginal cost, the government has the incentive to lower the labor income tax rate as ϕ increases. Therefore, an increase in ϕ leads to a decrease in the labor income tax rate, as we can see from Panel (a) of Figure 1.

Second, Eq. (30) shows that the fiscal rule, represented by ϕ, has two conflicting effects on the choice of public investment, x. The first is a negative effect on x observed in the first term on the left-hand side; the effect is qualitatively similar to the negative effect on τ, as mentioned previously. The second is a positive effect on x, as observed in the third term on the left-hand side. This effect arises because the tax burden associated with public investment, x, decreases as the debt-financed proportion of public investment, ϕ, increases. Given these conflicting effects, there is a threshold value of ϕ at which the two effects are balanced, and an increase in ϕ has an inverse U-shaped effect on the ratio of public investment to GDP around the threshold value, as illustrated in Panel (b) of Figure 1.

3.3 The Preferences of the Voters for Fiscal Rules

Thus far, we have considered fiscal rules as institutionally given and investigated the impact of the rules on fiscal policy decisions. However, fiscal rules themselves are generally introduced after deliberation and voting in parliaments. Thus, it is natural to assume that they are also determined through the political process of voting. In this subsection, we introduce such a

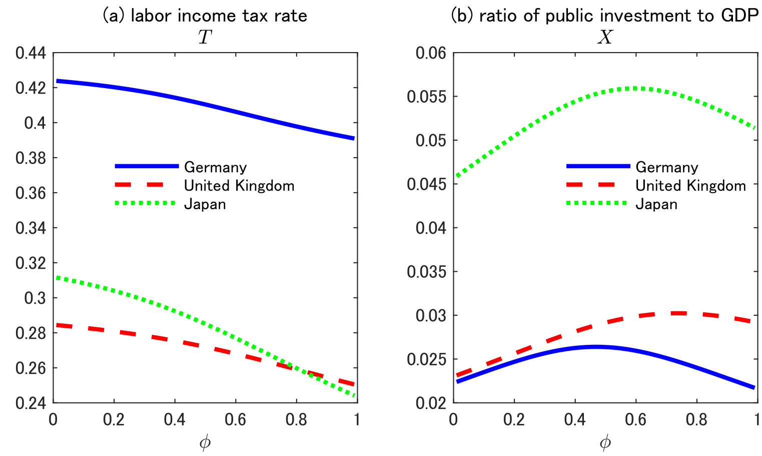

Figure 1: Effects of an increased ϕ on the labor income tax rate, T (Panel (a)), and the ratio of public investment to GDP, X (Panel (b)), for Germany, Japan, and the United Kingdom.

process into the model and show how the process and the resulting fiscal rules are affected by the structural parameters of the model economy.

The timing of the fiscal rule and fiscal policy decisions is as follows. (i) First, the office-seeking two candidates, say O^A and O^B, simultaneously and non-cooperatively, announce their fiscal rules, denoted by ϕ^A and ϕ^B, respectively. (ii) Given ϕ^A and ϕ^B, the two candidates, simultaneously and non-cooperatively, announce their policy platforms, $\left(\tau^A, \tau^{kA}, x^A, g^A\right)$ and $\left(\tau^B, \tau^{kB}, x^B, g^B\right)$, respectively, as described at the beginning of this section. The elected candidate implements their announced fiscal rule and policy platform. This two-stage game is solved by backward induction. Thus, the fiscal rule determined through voting, called a *politically preferred fiscal rule*, is defined as ϕ, which maximizes the political objective function subject to the policy functions presented in Proposition 1.

In writing down the political objective function at the first stage, recall the indirect utility functions of the middle-aged and older adults derived in Section 2. Using the policy function obtained in Proposition 1 and the equilibrium condition of the capital market in (11), the political objective function in (15) at the first stage is written as follows

$$\begin{aligned}\Omega = &\frac{\omega}{(1+n)(1-\omega)}\left[\ln d\left(\tau^k; k, h, n\right) + \theta \ln g\right] \\ &+ \ln c\left(\tau; k, h\right) + \theta \ln g + \beta\left[\ln d'\left(\tau, x, b'; k, h\right) + \theta \ln g'\left(\tau, x, b'; k, h\right)\right], \qquad (31)\end{aligned}$$

where the derivation of (31) is given in Appendix A.3. Thus, the problem of the government at the first stage is to choose ϕ to maximize Ω in (31) subject to the fiscal rule in (9), the government budget constraint in (12), and the policy functions derived in Proposition 1. The solution to the problem, denoted by $\phi_{political}$, is provided in the following proposition.

Proposition 2 *A politically preferred fiscal rule, $\phi_{political}$, along the Markov-perfect political equilibrium characterized in Proposition 1 is*

$$\phi_{political} = \frac{1-\alpha(1+\theta)}{(1+\theta)\eta(1-\alpha)}. \qquad (32)$$

Country	$\phi_{political,i}$	$\phi_{planner,i}$
Germany	0.944	0.327
Japan	1.289	0.420
United Kingdom	1.669	0.564

Table 2: The politically preferred fiscal rule, $\phi_{political}$, in the second column and the distortion minimizing fiscal rule, $\phi_{planner}$, in the third column.

Proof. See Appendix A.3.

The result demonstrated in Proposition 2 suggests that θ, representing the preferences for public goods, and η, representing the elasticity of human capital with respect to public investment, are crucial to the determination of the politically preferred fiscal rule. A greater θ implies that voters attach a larger weight to public goods expenditure, and thus provide a stronger incentive for the government to cut public debt issuance. A greater η implies a greater marginal benefit of public investment, thus, incentivizing the government to lower the debt-financed proportion of public investment, ϕ, from the viewpoint of balancing the marginal costs and benefits of public investment, as we can see in Eq. (30). Thus, the politically preferred fiscal rule, $\phi_{political}$, decreases as θ and η increase.

Based on the result in Proposition 2 with the calibration reported in Table 1, we compute the politically preferred fiscal rule for the three countries, as reported in the second column of Table 2. Germany obtains the lowest politically preferred ϕ because it attains the highest θ and η among the three countries. As for the politically preferred ϕ of Japan and the United Kingdom, the relative degree of the effects of θ and η is important because θ is higher but η is lower in the United Kingdom than in Japan. Table 2 reports that the politically preferred ϕ of Japan is lower than that of the United Kingdom, suggesting that the effect of η overcomes the effect of θ in determining the politically preferred ϕ of these two countries.

The result in Table 2 also reports that among the three countries, the politically preferred ϕ is less than one in Germany, whereas it is greater than one in Japan and the United Kingdom. This suggests that the latter two countries broke GR during the period covered. This model prediction is consistent with the evidence. Wyplosz (2013) reports that in the United Kingdom, GR, which was adopted in 1997, was met only for the first few years. In Japan, Article 4 of Public Finance Law 1947 stipulates: "Expenditures of the State shall be financed by revenue other than public bonds or borrowings."[8] However, the following proviso states: "However, public bonds may be issued, or borrowings may be made to finance public works expenditure, capital contributions, and loans, within the amount approved by the Diet." Based on this proviso, a waiver of this rule has been approved by the Diet every year since 1975, except for the period 1990-1993 (Kumar et al., 2009).

We should note that the politically preferred fiscal rule in (32) is the same as the ratio of

[8] Source: Ministry of Internal Affairs and Communications, e-Gov Japan (Hourei Kensaku, in Japanese): https://elaws.e-gov.go.jp/document?lawid=322AC0000000034 (accessed on October 12, 2022).

public debt to public investment, b'/x in the absence of fiscal rules (see Appendix A.3 for the proof of this statement). In the absence of fiscal rules, the government is free to choose public debt issuance and public investment, b' and x, from the viewpoint of maximizing its objective as long as the choice meets the government budget constraint. In the presence of fiscal rules, the government is constrained by the rule at the timing of its policy choice, but it can manage the rule in advance at the timing of the fiscal rule choice. Therefore, when the fiscal rule is chosen via voting, the ratio of public debt issuance to public investment is the same regardless of whether the fiscal rule is in place or not.

4 Political Distortions

In Subsection 3.3, we have demonstrated the politically preferred fiscal rule, $\phi_{political}$, set through voting. We have also shown that the rule is in accordance with GR in Germany, but not in Japan or the United Kingdom. This contrasting result is due to the differences in structural parameters across countries. Furthermore, even in Germany, where GR is abided by, the debt-financed proportion of public investment, reported in Table 2, is high and close to one. These results may be because the politically preferred fiscal rule is a choice that considers only the generations existing at the time of voting and does not take into account future generations.

To evaluate the politically preferred fiscal rule from a generational perspective, we begin by assuming a benevolent planner who can commit to its choices at the start of the economy (i.e., period 0). Assuming such a planner, we characterize the planner's allocation that maximizes an infinite discounted sum of generational utilities for an arbitrary social discount factor in Subsection 4.1. In Subsection 4.2, we evaluate the political equilibrium characterized in Subsection 3.1 by comparing it with the planner's allocation. We focus on the deviations between the political equilibrium allocation and the planner's allocation, in terms of physical and human capital accumulation, and call the total deviations *political distortions*. We explore the fiscal rule that minimizes political distortions. This exploration should provide us with some insight into the setting of fiscal rules that take future generations into account.

4.1 Planner's Allocation

The planner is assumed to value the welfare of all generations. In particular, its objective is to maximize a discounted sum of the lifecycle utility of all current and future generations, $SW = \gamma^{-1}U_0^O + \sum_{t=0}^{\infty}\gamma^t U_t^M$, $0 < \gamma < 1$, under the human capital formation function in (1) and the resource constraint, $N_t c_t + N_{t-1}d_t + K_{t+1} + N_{t+1}x_t + (N_t + N_{t-1})\,g_t = A\,(K_t)^{\alpha}\,(H_t)^{1-\alpha}$, or,

$$c_t + \frac{d_t}{1+n} + (1+n)k_{t+1} + (1+n)x_t + \frac{2+n}{1+n}g_t = A\,(k_t)^{\alpha}\,(h_t)^{1-\alpha}\,, \tag{33}$$

where k_0 and h_0 are given. The parameter $\gamma \in (0,1)$ is the the discount factor of the planner. Reverse discounting, $1/\gamma(> 1)$, must be applied to U_0^O (i.e., the utility of the older adult population in period 0) to preserve dynamic consistency.

Solving the problem leads to the following characterization of the planner's allocation.

Proposition 3 *Given k_0 and h_0, a sequence of the planner's allocation, $\{c_t, d_t, x_t, g_t, k_{t+1}, h_{t+1}\}_{t=0}^{\infty}$, satisfies the human capital formation function in (1), the resource constraint in (33), and the following:*

$$c_t = \frac{\gamma(1-\gamma)[1-\alpha\gamma(1-\eta)]}{(1+\theta)(\gamma+\beta)[1-\gamma(1-\eta)]} A(k_t)^{\alpha}(h_t)^{1-\alpha}, \tag{34}$$

$$\frac{d_t}{1+n} = \frac{\beta(1-\gamma)[1-\alpha\gamma(1-\eta)]}{(1+\theta)(\gamma+\beta)[1-\gamma(1-\eta)]} A(k_t)^{\alpha}(h_t)^{1-\alpha}, \tag{35}$$

$$(1+n)x_t = \frac{(1-\alpha)\gamma\eta}{1-\gamma(1-\eta)} A(k_t)^{\alpha}(h_t)^{1-\alpha}, \tag{36}$$

$$\frac{2+n}{1+n} g_t = \frac{\theta(1-\gamma)[1-\alpha\gamma(1-\eta)]}{(1+\theta)[1-\gamma(1-\eta)]} A(k_t)^{\alpha}(h_t)^{1-\alpha}. \tag{37}$$

Proof. See Appendix A.4.

Based on the result in Proposition 3, we demonstrate physical and human capital accumulation in the planner's allocation as follows:

$$\frac{k'}{k} = \frac{\alpha\gamma}{1+n} A\left(\frac{h}{k}\right)^{1-\alpha}, \tag{38}$$

$$\frac{h'}{h} = D\left[\frac{1}{1+n}\frac{(1-\alpha)\gamma\eta}{1-\gamma(1-\eta)} A\left(\frac{k}{h}\right)^{-\alpha}\right]^{\eta}. \tag{39}$$

The derivation of (38) and (39) is provided in Appendix A.4. Equations (38) and (39) are used as benchmarks to assess the efficiency of the political equilibrium path of $\{k_t, h_t\}$.

4.2 Minimizing Political Distortions

The planner's allocation is characterized by the sequence of $\{k_t, h_t\}$ that satisfies (38) and (39). The political equilibrium allocation for a given fiscal rule is characterized by the sequence of $\{k_t, h_t\}$ that satisfies (27) and (28). Subsequently, given the same initial condition, $\{k_0, h_0\}$, the political equilibrium allocation coincides with the planner's allocation if (27) and (28) coincide with (38) and (39), respectively. In general, the two allocations do not coincide because the means of adjusting the political equilibrium allocation is limited to a single fiscal rule, while it is necessary to match the two sequences of physical and human capital.

Given this limitation, we consider the choice of a fiscal rule, ϕ, that minimizes the deviations of physical and human capital sequences in the political equilibrium allocation from those in the planner's allocation. The choice aims to minimize the political distortions specified by the following cost function, denoted by $C(\cdot)$:

$$C(\phi) \equiv \left(\frac{\beta}{1+\beta}(1-T)(1-\alpha) - \phi X - \alpha\gamma\right)^2 + \left(X - \frac{(1-\alpha)\gamma\eta}{1-\gamma(1-\eta)}\right)^2,$$

where the first and second terms show the deviations of physical and human capital in the political equilibrium allocation from those in the planner's allocation, respectively, for a given pair of (k, h). The deviations are assumed to be evaluated by quadratic functions to formulate the cost-minimization problem. By choosing a fiscal rule that minimizes this cost, we can bring the political equilibrium allocation closest to the planner's allocation. We should note that the cost is assessed each period for a given level of physical and human capital; therefore, the minimization problem is static in nature. We assume $\gamma = \beta = 0.99^{120}$ in the following numerical analysis.

Figure 2 plots the impact of the fiscal rule on human capital distortions, physical capital distortions, and political distortions, for Germany, Japan, and the United Kingdom. The distortions in human and physical capital are plotted as the linear difference between the level in the political equilibrium and the level in the planner's allocation. In so doing, we can observe whether the levels of human and physical capital in the political equilibrium are lower or higher than those in the planner's allocation. As observed in Figure 2, in the three countries, the human capital in the political equilibrium allocations is always lower than that in the planner's allocation. However, the physical capital in the political equilibrium is higher (lower) than that in the planner's allocation when ϕ is low (high).

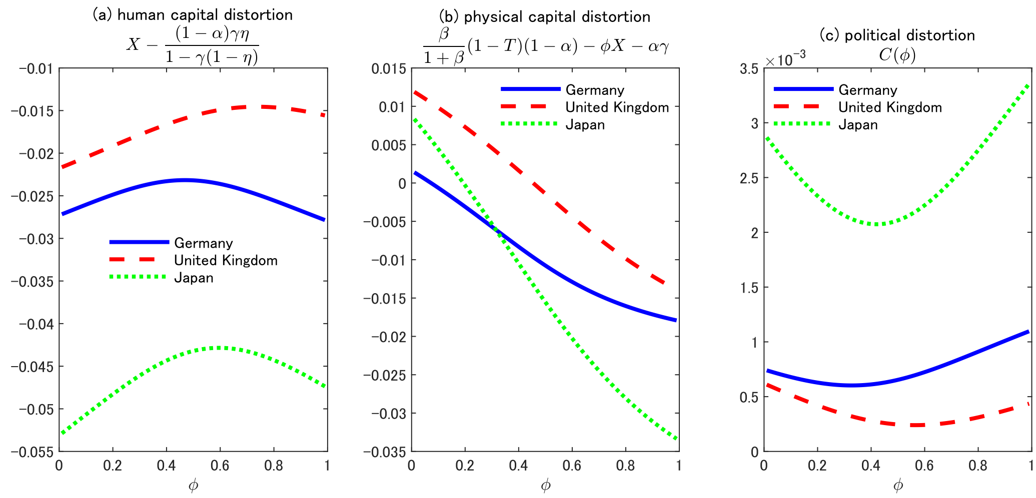

Figure 2: Effects of an increased ϕ on human capital distortion (Panel (a)), physical capital distortion (Panel (b)), and political distortions (Panel (c)) for Germany, Japan, and the United Kingdom.

The interpretation of the result observed from Figure 2 is as follows. First, we focus on the distortion of human capital, $X - (1-\alpha)\gamma\eta/[1-\gamma(1-\eta)](<0)$, depicted in Panel (a). To understand how the choice of the fiscal rule ϕ corrects the distortion, recall (30) that expresses the first-order condition with respect to public investment in the political equilibrium:

$$\frac{-\alpha \overbrace{\beta}^{*2a} \overbrace{(1+\theta)}^{*1a}\phi \overbrace{(1+n)}^{*3a}}{\underbrace{\frac{\beta}{1+\beta}}_{*2b}(1-\tau)w(k,h)h - \phi(\underbrace{1+n}_{*3b})x} + \frac{\beta\eta(1+\theta)(1-\alpha)}{x} - \lambda\left(1-\phi\right)\left(\underbrace{1+n}_{*3c}\right) = 0. \tag{40}$$

Equation (40) shows that the fiscal rule ϕ has two opposing effects on the marginal cost of public investment and thus the ratio of public investment to GDP, X, in the political equilibrium. A higher ϕ leads to a higher marginal cost of public investment as observed in the first term; it also leads to a lower marginal cost as observed in the third term. In the present framework, the positive effect through the third term outweighs (is outweighed by) the negative effect through the first term when the initial ϕ is low (high). Thus, an increase in ϕ produces an initial increase followed by a decrease in the ratio of public investment to GDP, X, that is, a hump-shape pattern of X in the political equilibrium.

The hump-shape pattern indicates that there is a critical value of ϕ that attains a maximized value of X and thus a minimized distortion of human capital in the political equilibrium. As the marginal costs of public investment depend on the three country-specific parameters, θ, β and n, these parameters also affect the determination of the X-maximizing ϕ. Their effects are expressed by the terms $*1a$, $*2a$, $*2b$, $*3a$, $*3b$, and $*3c$ in (40). The term $*1a$ says that the crowding-out effect of public debt issuance to finance public investment becomes stronger as θ increases. This works to decrease the X-maximizing ϕ. The terms $*2a$, $*3a$, and $*3b$ also represent the crowding-out effect and so work in the same direction as the term $*1a$. The term $*2b$ indicates that the higher β, the higher the savings rate, which in turn weakens the crowding-out effect of public debt. The term $*2b$ thus has an opposite effect to the term $*2a$ on the X-maximizing ϕ. The term $*3c$ indicates that the marginal cost of public investment increases as the population growth rate, n, increases. This works to raise the X-maximizing ϕ. In summary, β and n each has opposing effects on the X-maximizing ϕ, whereas the effect of θ is decisive and negative.

Among the three countries, Germany is characterized by the highest θ, followed by the United Kingdom and Japan (see Table 1). Thus, as we can see from Figure 1, Germany realizes the lowest X-maximizing ϕ among the three countries. However, the X-maximizing ϕ is higher in the United Kingdom than in Japan, although the calibrated θ is higher in the United Kingdom than in Japan. This suggests that the other two parameters, β and n, have crucial roles in shaping the order of the X-maximizing ϕ of these two countries. In particular, the United Kingdom is characterized by a lower β and a higher n than Japan, indicating that the effects through the terms $*2a$ and $*3c$ are decisive and realize a higher X-maximizing ϕ of the United Kingdom.[9]

Next, we focus on the distortion of physical capital, $\beta (1-T)(1-\alpha)/(1+\beta) - \phi X - \alpha\gamma$, as shown in Panel (b) of Figure 2. In addition to the hump-shape effect through the term X,

[9]The country-specific parameter η, observed in the second term of (40), might also affect the determination of the X-maximizing ϕ. A higher η implies a higher marginal benefit of public investment, thereby working to lower the X-maximizing ϕ from the viewpoint of balancing the marginal costs and benefits of public investment. Among the three sample countries, Germany has the highest η, followed by Japan and the United Kingdom (see Table 1). Therefore, other things being equal, X-maximizing ϕ is inferred to be lowest in Germany, followed by Japan and the United Kingdom. This prediction is consistent with the numerical result reported in Panel (b) of Figure 1. However, we focus on θ, β and n rather than η because the fiscal rule ϕ has no direct effect on the marginal benefit of public investment.

there are two other effects of the fiscal rule on physical capital accumulation. The first is a negative effect on physical capital through the crowding-out effect. This effect is observed in the term ϕ of $\phi \cdot X$. The second is a positive effect through the term T. An increase in ϕ shifts the burden of government spending from taxes to public debt. The resulting decrease in the tax burden, T, leads to an increase in saving, and thus, an increase in physical capital. When the initial ϕ is low, the positive effects through the terms X and T outweigh the negative effect through the term ϕ; hence, the physical capital is over-accumulated from the viewpoint of the planner. As ϕ increases, the negative effect through the term ϕ exceeds the positive effect through the term T, thus eliminating the excess accumulation of physical capital in the political equilibrium. Above a certain threshold level of ϕ, the physical capital is under-accumulated in the political equilibrium. Furthermore, the distortion from the under-accumulation of physical capital increases with an increase in ϕ because of the additional effect of the reduction of human capital through the term X. Thus, the distortion of physical capital also initially decreases and then increases as ϕ increases.

Panel (c) of Figure 2 illustrates the effects of an increase in ϕ on the political distortion for the three countries, and the third column of Table 2 reports the ϕ that minimizes the political distortions, denoted by $\phi_{planner}$. Given the effects on human and physical capital distortions that we have observed from Panels (a) and (b), we find that political distortions also exhibit a hump-shape pattern in response to an increase in ϕ. In particular, the order of the ϕ that minimizes the political distortions among the three countries is the same as the order of the ϕ that minimizes the human capital distortions. In addition, the distortion-minimizing ϕ in the planner's allocation, $\phi_{planner}$, is lower than the politically preferred ϕ, $\phi_{political}$, for the three countries. These properties indicate that the structural parameters play important roles in determining fiscal rules that minimize political distortions in each country, and that the consideration of future generations calls for legislature to lower the proportion of debt-financed public investment. Our result suggests that we should carefully take into account the short-sightedness of successive governments as well as the values of country-specific structural parameters, such as preference for public goods, discount factor, and the population growth rate when we consider the design of fiscal rules from a generational perspective.

In closing this section, we should note that our result emphasizes the importance of considering the distortion of physical capital. If we discarded such distortion from the model, the ϕ minimizing the political distortion from a generational perspective would be higher than the one we demonstrate in Figure 2. This means that the result of Bassetto and Sargent (2006), who discarded the distortion of physical capital by assuming a quasi-linear utility function, may have overestimated the optimal debt-financed proportion of public investment. In addition, if policy-makers with a generational perspective set the proportion while ignoring the capital distortions, the resulting budget deficit of the economy could become excessive. Our result, therefore, suggests the importance of taking into account distortions in physical capital accumulation when we seek to finance public investment with public debt.

5 Conclusion

In this study, we consider the golden rule of public finance (GR), which states that budget deficit is allowed only to finance public investment but not current expenditure. We investigate what proportion of public investment should be financed by public debt issuance from a generational perspective and what proportion is politically preferred by successive short-sighted governments. To address these questions, we develop a politico-economic overlapping-generations model where fiscal policy is determined for each period via probabilistic voting and calibrate the model economy to Germany, Japan, and the United Kingdom, where GR has been in place. We show that in politics, Germany sets the proportion that follows the GR while Japan and the United Kingdom set the proportions that break the GR, which are consistent with the literature. We subsequently compute the optimal proportion taking into account the long-lasting effects of public investment on future generations and show that it is lower than the politically preferred one in each country.

The novelty of the present study is twofold. First, we quantitatively show whether each country follows the GR depends on structural parameters such as preferences for public goods and the elasticity of human capital with respect to public investment. Differences in structural parameters successfully explain differences in the responses of the countries to fiscal rule. Second, we point out that ignoring the distortion of physical capital may lead to an overestimation of the optimal debt-financed proportion of public investment. The effect of policy through physical capital accumulation cannot be ignored in the choice of fiscal rules. Our results with these two characteristics are expected to provide important implications for policy makers in designing fiscal rules from a generational perspective.

Our model can be expanded in several directions. For instance, it would be straightforward to incorporate other fiscal rules such as revenue and expenditure rules, both of which have been widely introduced in developed countries. This would enable us to compare and evaluate several types of fiscal rules in terms of minimizing political distortions. Additionally, our model can be used to explore a wide variety of policy questions: to what extent do changes in fiscal rules affect the fiscal burden on each generation? What fiscal rules are optimal from the perspective of maximizing economic growth? Which fiscal rule is most efficient from the perspective of minimizing political distortions? Addressing these questions would provide policymakers with richer information.

A Proofs and Supplementary Explanations

A.1 Proof of Proposition 1

Given the conjecture in (25) and (26), we can reformulate the first-order condition with respect to x in (21) as follows:

$$\lambda = \frac{\beta (1+\theta)}{(1-\phi)\, y(k,h)} \left[(-1) \frac{\alpha\phi (1+\theta,)}{\frac{\beta}{1+\beta} (1-\alpha)(1-T) - \phi X} + \frac{\eta (1-\alpha)}{X} \right]. \tag{A.1}$$

We can also reformulate the first-order condition with respect to τ in (19) as follows:

$$\lambda = \frac{1}{(1-\alpha)\, y(k,h)} \left[\frac{1}{1-T} + \frac{\alpha\beta (1+\theta) \frac{\beta}{1+\beta} (1-\alpha)}{\frac{\beta}{1+\beta} (1-\alpha)(1-T) - \phi X} \right]. \tag{A.2}$$

With (A.1) and (A.2), we obtain

$$\frac{\beta (1+\theta)\, \eta (1-\alpha)}{(1-\phi)} \cdot \frac{1}{X} = \frac{1}{(1-\alpha)(1-T)} + \frac{\alpha\beta (1+\theta) \left[\frac{\beta}{1+\beta} + \frac{\phi}{1-\phi} \right]}{\frac{\beta}{1+\beta} (1-\alpha)(1-T) - \phi X}, \tag{A.3}$$

where (A.3) includes two undetermined constants, T and X. Denote the left-hand (right-hand) side of (A.3) by $LHS^{(A.3)}$ ($RHS^{(A.3)}$). They have the following properties: $\partial LHS^{(A.3)}/\partial X < 0$, $\lim_{X\to 0} LHS^{(A.3)} = +\infty$, $\lim_{X\to +\infty} LHS^{(A.3)} = 0$, $\partial RHS^{(A.3)}/\partial X > 0$, $\lim_{X\to 0} RHS^{(A.3)} \in (0, +\infty)$, and $\lim_{X\to \frac{\beta}{1+\beta}(1-\alpha)(1-T)/\phi} RHS^{(A.3)} = +\infty$. These properties imply that given T, there is a unique X that satisfies (A.3): $X = X(T)$.

Given T and X, the first-order condition with respect to τ^k in (18) is reformulated as

$$1 - \tau^k = \frac{\omega}{(1+n)(1-\omega)} (1-\alpha) \left[\frac{1}{1-T} + \frac{\alpha\beta (1+\theta) \frac{\beta}{1+\beta} (1-\alpha)}{\frac{\beta}{1+\beta} (1-\alpha)(1-T) - \phi X} \right]^{-1} \frac{1}{\alpha^{\frac{k+b}{k}}}. \tag{A.4}$$

Eq. (A.4) shows that the conjecture of τ^k in (16) is correct as long as T and X are constant.

Next, given T and X, the first-order condition with respect to g in (20) is reformulated as

$$\frac{2+n}{1+n} g = \left(\frac{\omega}{(1+n)(1-\omega)} + 1 \right) \theta \left[\frac{1}{1-T} + \frac{\alpha\beta (1+\theta) \frac{\beta}{1+\beta} (1-\alpha)}{\frac{\beta}{1+\beta} (1-\alpha)(1-T) - \phi X} \right]^{-1} (1-\alpha)\, y(k,h). \tag{A.5}$$

Eq. (A.5) shows that the conjecture of g in (17) is correct as long as T and X are constant.

The remaining task is to show that the conjectures of T in (25) and X in (26) are correct. Consider the government budget constraint in (12). We substitute the policy functions derived thus far into the constraint and rearrange the terms to obtain

$$\begin{aligned} &\alpha - \frac{\omega}{(1+n)(1-\omega)} \left[\frac{1}{1-T} + \frac{\alpha\beta (1+\theta) \frac{\beta}{1+\beta} (1-\alpha)}{\frac{\beta}{1+\beta} (1-\alpha)(1-T) - \phi X} \right]^{-1} (1-\alpha) + T(1-\alpha) \\ &= \left(\frac{\omega}{(1+n)(1-\omega)} + 1 \right) \theta \left[\frac{1}{1-T} + \frac{\alpha\beta (1+\theta) \frac{\beta}{1+\beta} (1-\alpha)}{\frac{\beta}{1+\beta} (1-\alpha)(1-T) - \phi X} \right]^{-1} (1-\alpha) + (1-\phi) X. \end{aligned} \tag{A.6}$$

The expression in (A.6) is independent of the state variables, k, h, and b, and times. Thus, we can verify that the two unknown parameters, X and T, are constant and solved for using (A.3) and (A.6).

■

A.2 Calibration

A.2.1 Calibration on n, β, ω, η, and θ

Country specific parameters, n, β, ω, η, and θ, are calibrated in the following way. The population growth rate, n, is obtained from the average of each sample country during the 1995—2016 period. Let POP_j denote the annual gross population growth rate of country j. The net population growth rate for 30 years is $(POP_j)^{30} - 1$.

To determine the remaining four parameters, β, ω, η, and θ, we focus on the labor income tax rate, τ, the ratio of the public goods provision to GDP, G/Y, the ratio of public investment to GDP, $N'x/Y$, and the ratio of public debt to GDP, B'/Y. These four variables are given by

$$\tau_j = T_j \equiv TAX_j, \tag{A.7}$$
$$\left.\frac{G}{Y}\right|_j = G_j \equiv GOV_j, \tag{A.8}$$
$$\left.\frac{N'x}{Y}\right|_j = X_j \equiv INV_j, \tag{A.9}$$
$$\left.\frac{B'}{Y}\right|_j = \phi X_j \equiv DFF_j, \tag{A.10}$$

where the subscript j is the country code. We use the data of TAX_j, GOV_j, INV_j, and DFF_j for each sample country during 1995–2016 and solve the four equations in (A.7) – (A.10) for β_j, ω_j, η_j, and θ_j. The result is presented in Table 1.

The capital income tax rate is not a target for the calibration. Using the values of the parameters estimated based on the calibration, we need to compute the capital income tax rate and check whether it is consistent with the data of the three sample countries. Table 3 reports the capital income tax rates we compute on the basis of the calibration and those obtained from Professor McDaniel's data archive.[10] The result in Table 3 shows that the computed capital income tax rates are higher than the actual rates for the three countries. This suggests that there is room for improvement in the calibration, but the computed capital income tax rates fall within the range $(0, 1)$. We, therefore, use the calibration result in Table 1 to carry out the analysis in the main text.

A.2.2 Data on Public Investment

To estimate the ratio of public investment to GDP, we use the following three sorts of the data: the gross domestic product (GDP), gross fixed capital formation ($GFCG$), and the ratio

[10] https://www.caramcdaniel.com/ (accessed on February 17, 2022).

Country	estimation	data
Germany	0.4981	0.1826
Japan	0.6056	0.2235
United Kingdom	0.6118	0.2771

Table 3: Estimated and actual capital income tax rates of the sample three countries.
Source: Actual capital income tax rates are obtained from Professor McDaniel's data archive: `https://www.caramcdaniel.com/` (accessed on February 17, 2022).

of the investment by sector, general government ($ISGG$). Using this data, we first compute the ratio of public investment to GDP of country i in year t, denoted by $INV_{i,t}$, as follows:

$$INV_{i,t} = \frac{(ISGG_{i,t}/100) \cdot GFCF_{i,t}}{GDP_{i,t}}.$$

Then, INV_i, representing the average ratio of public investment to GDP of country i during 1995–2016, is computed based on the following equation:

$$INV_i = \frac{1}{22} \sum_{t=1995}^{2016} INV_{i,t}.$$

■

A.3 Supplement to Subsection 3.3

A.3.1 Derivation of Ω in (31)

Recall the indirect utility function of the middle in (13), which is reformulated using the recursive notation as follows:

$$V^M\left(\tau, x, g, \tau^{k\prime}, g'; k, h, k', h', b'\right) = \ln c\,(\tau; k, h) + \theta \ln g + \beta \left[\ln d'\left(\tau^{k\prime}; k', h', b'\right) + \theta \ln g'\right]. \tag{A.11}$$

The term d' in (A.11) is reformulated as follows:

$$\begin{aligned} d'(\cdot) &= \left(1 - \tau^{k\prime}\right) R\left(k', h'\right)(1+n)\left(k' + b'\right) \\ &= T^k \frac{k'}{\alpha\,(k' + b')} \alpha A \left(k'\right)^{\alpha - 1} \left(h'\right)^{1-\alpha} (1+n)\left(k' + b'\right) \\ &= (1+n) T^k \cdot y\left(k', h'\right), \end{aligned} \tag{A.12}$$

where the equality in the first line comes from the budget constraint for older adults, $d' = \left(1 - \tau^{k\prime}\right) R's$ and the capital market clearing condition, $(1+n)(k'+b') = s$; the equality in the second line comes from the conjecture of $\tau^{k\prime}$ in (16) and the first-order condition with respect to capital in (7), and the equality in the third line comes from the production function, $y(k,h) = A\,(k)^{\alpha}\,(h)^{1-\alpha}$. The term $g'(\cdot)$ is reformulated, using the conjecture of g' in (17), as

$$g'(\cdot) = G \cdot y\left(k', h'\right). \tag{A.13}$$

The term $y(k', h')$, which appeared in (A.12) and (A.13), is further reformulated as follows:

$$
\begin{aligned}
y\left(k', h'\right) &= A\left[\frac{1}{1+n}\frac{\beta}{1+\beta}(1-\tau)(1-\alpha)\, y(k,h) - b'\right]^{\alpha}\left[D\,(h)^{1-\eta}\,(x)^{\eta}\right]^{1-\alpha} \\
&= y\left(\tau, x, b'; k, h\right) \qquad \text{(A.14)}
\end{aligned}
$$

where we use the capital market clearing condition in (11) and the human capital formation function in (1) in deriving (A.14). Thus, the terms $d'(\cdot)$ in (A.12) and $g'(\cdot)$ in (A.13) are now given by

$$
\begin{aligned}
d' &= d'\left(\tau, x, b'; k, h\right) \equiv (1+n)T^{k} \cdot y\left(\tau, x, b'; k, h\right), \\
g' &= g'\left(\tau, x, b'; k, h\right) \equiv G \cdot y\left(\tau, x, b'; k, h\right),
\end{aligned}
$$

respectively, and the indirect utility function of the middle-aged in (A.11) is reformulated as follows:

$$
\begin{aligned}
V^{M}\left(\tau, x, g, b'; k, h\right) &= \ln c\,(\tau; k, h) + \theta \ln g \\
&\quad + \beta\left[\ln d'\left(\tau, x, g, b'; k, h\right) + \theta \ln g'\left(\tau, x, g, b'; k, h\right)\right]. \qquad \text{(A.15)}
\end{aligned}
$$

We substitute the indirect utility function of older adults in (14) and that of the middle-aged in (A.15) into the political objective function in (15) and then obtain (31).

A.3.2 Proof of Proposition 2

The problem of the government is to choose ϕ to maximize Ω in (31) subject to the government budget constraint in (12), the fiscal rule in (9), and the policy functions presented in Proposition 1. Substituting (12) and (9) into (31) and rearranging the terms, we have

$$
\begin{aligned}
\Omega \approx\ & \frac{\omega}{(1+n)(1-\omega)} \ln d\left(\tau^{k}; k, h, n\right) + \ln c\,(\tau; k, h) \\
& + \left(\frac{\omega}{(1+n)(1-\omega)} + 1\right)\theta \ln \underbrace{\frac{1+n}{2+n}\left[TR^{K}\left(\tau^{k}; k, h\right) + TR\,(\tau; k, h) - (1-\phi)(1+n)x - R\,(k,h)\, b\right]}_{=g} \\
& + \beta\,(1+\theta)\,\alpha \ln \underbrace{\left[\frac{\beta}{1+\beta}(1-\tau)(1-\alpha)\, y(k,h) - \phi(1+n)x\right]}_{=(1+n)k'} + \beta\,(1+\theta)\,\eta\,(1-\alpha)\ln x, \qquad \text{(A.16)}
\end{aligned}
$$

where the politically unrelated terms are omitted from the expression in (A.16).

We write the policy functions of τ^k, τ, and x derived in Proposition 1 in the following implicit form:

$$
\begin{aligned}
\tau^{k} &= \tau^{k}(\phi, k, h), \\
\tau &= \tau(\phi), \\
x &= x(\phi, k, h).
\end{aligned}
$$

We substitute these policy functions into (A.16) and obtain, by using the envelope theorem, the first-order condition with respect to ϕ as follows:

$$\frac{\partial \Omega}{\partial \phi} = 0 \Leftrightarrow \left(\frac{\omega}{(1+n)(1-\omega)} + 1 \right) \theta \frac{\frac{1+n}{2+n}}{g} = \frac{\beta (1+\theta) \alpha}{(1+n)k'}. \tag{A.17}$$

To solve (A.17) for ϕ, recall the policy functions of g presented in Proposition 1,

$$\frac{2+n}{1+n} g = \left(\frac{\omega}{(1+n)(1-\omega)} + 1 \right) \theta \left[\frac{1}{1-T} + \frac{\alpha\beta (1+\theta) \frac{\beta}{1+\beta} (1-\alpha)}{\frac{\beta}{1+\beta} (1-\alpha)(1-T) - \phi X} \right]^{-1} (1-\alpha)\, y(k,h), \tag{A.18}$$

and the capital market clearing condition in (11) with the fiscal rule in (9),

$$(1+n)k' = \frac{\beta}{1+\beta} (1-\tau)(1-\alpha)\, y(k,h) - \phi X y(k,h). \tag{A.19}$$

We substitute (A.18) and (A.19) into (A.17) and rearrange the terms to obtain:

$$\frac{1}{1-T} = \frac{\alpha (1+\theta) \frac{\beta}{1+\beta} (1-\alpha)}{\frac{\beta}{1+\beta} (1-\alpha)(1-T) - \phi X},$$

or,

$$\frac{\beta}{1+\beta} (1 - \alpha (1+\theta)) = \frac{I(\phi)}{1-\phi}, \tag{A.20}$$

where we use the definition of T and X in Proposition 1 in deriving (A.20).

Substituting the definition of $I(\phi)$ into (A.20), we obtain

$$\frac{\beta}{1+\beta} (1 - \alpha (1+\theta)) = \frac{H_1(\phi) - \sqrt{(H_1(\phi))^2 - 4H_2(\phi)}}{2(1-\phi)},$$

or,

$$-H_2(\phi) = -\frac{\beta}{1+\beta} (1 - \alpha(1+\theta))(1-\phi) H_1(\phi) + \left[\frac{\beta}{1+\beta} (1 - \alpha(1+\theta))(1-\phi) \right]^2. \tag{A.21}$$

With the use of the definition of $H_1(\phi)$ and $H_2(\phi)$, (A.21) is further reformulated as in (32).

A.3.3 Ratio of Public Debt to Public Investment in the Absence of Fiscal Rules

In the absence of fiscal rules, the problem of the government is to choose $\left(\tau^k, \tau, g, x, b'\right)$ to maximize Ω subject to the human capital formation function in (1), the capital market clearing condition in (11), and the government budget constraint in (12) with $\phi = 0$. Based on the conjecture of the policy functions in (16) and (17), we can write the first-order conditions with respect to τ^k, τ, g, x, and b' as follows:

$$\tau^k : (-1)\tfrac{\omega}{(1+n)(1-\omega)}\tfrac{1}{1-\tau^k} + \lambda R(k,h)(k+b) = 0, \tag{A.22}$$

$$\tau : (-1)\frac{1}{1-\tau} + (-1)\frac{\alpha\beta\,(1+\theta)\,\frac{\beta}{1+\beta}w(k,h)h}{\frac{\beta}{1+\beta}\,(1-\tau)\,w(k,h)h-(1+n)b'} + \lambda w(k,h)h = 0, \tag{A.23}$$

$$g : \left(\frac{\omega}{(1+n)(1-\omega)}+1\right)\frac{\theta}{g} - \lambda\frac{2+n}{1+n} = 0, \tag{A.24}$$

$$x : \frac{\beta\eta\,(1+\theta)\,(1-\alpha)}{,}x - \lambda(1+n) = 0, \tag{A.25}$$

$$b' : (-1)\alpha\beta\,(1+\theta)\,\frac{1+n}{\frac{\beta}{1+\beta}\,(1-\tau)\,w(k,h)h-(1+n)b'} + \lambda(1+n) = 0. \tag{A.26}$$

With (A.23) and (A.26), we obtain

$$(1+n)b' = \frac{\beta}{1+\beta}\left[1-\alpha\,(1+\theta)\right](1-\tau)\,w(k,h)h. \tag{A.27}$$

We substitute (A.27) into (A.23) and obtain

$$\lambda = \frac{1+\beta}{(1-\tau)\,w(k,h)h}. \tag{A.28}$$

Using (A.28), we can reformulate (A.22), (A.24), and (A.25) as follows:

$$1-\tau^k = \frac{\omega}{(1+n)(1-\omega)}\cdot\frac{(1-\tau)\,w(k,h)h}{1+\beta}\cdot\frac{1}{R(k,h)(k+b)}, \tag{A.29}$$

$$\frac{2+n}{1+n}g = \left(\frac{\omega}{(1+n)(1-\omega)}+1\right)\theta\frac{1-\tau}{1+\beta}w(k,h)h, \tag{A.30}$$

$$(1+n)x = \frac{\beta}{1+\beta}\eta\,(1+\theta)\,(1-\alpha)\,(1-\tau)\,w(k,h)h. \tag{A.31}$$

We substitute (A.27), (A.29), (A.30), and (A.31) into the government budget constraint in (12) and rearrange the terms to obtain the labor income tax rate:

$$1-\tau = \frac{1}{\Lambda}\cdot\frac{1+\beta}{1-\alpha}. \tag{A.32}$$

We also obtain τ^k, g, x, and b' by substituting (A.32) into (A.29), (A.30), (A.31), and (A.27), respectively:

$$1-\tau^k = \frac{1}{\Lambda}\cdot\frac{\omega}{(1+n)(1-\omega)}\cdot\frac{1}{\alpha\frac{k*b}{k}},$$

$$\frac{2+n}{1+n}g = \left(\frac{\omega}{(1+n)(1-\omega)}+1\right)\frac{\theta}{\Lambda}y(k,h),$$

$$(1+n)x = \frac{\beta\eta(1-\alpha)\,(1+\theta)}{\Lambda}y(k,h),$$

$$(1+n)b' = \frac{\beta\left[1-\alpha\,(1+\theta)\right]}{\Lambda}y(k,h),$$

where Λ is defined by

$$\Lambda \equiv (1+\theta)\left[1+\beta\,(\alpha+\eta(1-\alpha))+\frac{\omega}{(1+n)(1-\omega)}\right].$$

The expressions in the first and second lines verify the conjectures in (16) and (17), respectively. The ratio of $(1+n)b'$ to $(1+n)x$ is $\left(1-\alpha\left(1+\theta\right)\right)/\left(1+\theta\right)\eta\left(1-\alpha\right)$, which is the same as in (32).

■

A.4 Proof of Proposition 3

In the present framework, the state variable h_t does not line up in a compact set because it continues to grow along an optimal path. To reformulate the planner's problem into one in which the state variable lines up in a compact set, we undertake the following normalization:

$$\tilde{c}_t \equiv c_t/h_t,\ \tilde{d}_t \equiv d_t/h_t,\ \tilde{x}_t \equiv x_t/h_t,\ \tilde{k}_t \equiv k_t/h_t,\ \tilde{g}_t \equiv g_t/h_t.$$

Then, with the use of $h_{t+1} = D\left(h_t\right)^{1-\eta}\left(x_t\right)^{\eta}$, the resource constraint in (33) is reformulated as

$$\tilde{c}_t + \frac{\tilde{d}_t}{1+n} + (1+n)\tilde{k}_{t+1}D\left(\tilde{x}_t\right)^{\eta} + (1+n)\tilde{x}_t + \frac{2+n}{1+n}\tilde{g}_t = A\left(\tilde{k}_t\right)^{\alpha}. \tag{A.33}$$

Using the variables normalized by h, we can reformulate the utility functions of the period-0 older adult and the middle-aged agents as follows:

$$\begin{aligned}
U_0^O &= \beta\left[\ln\tilde{d}_0 + \theta\ln\tilde{g}_0 + (1+\theta)\ln h_0\right],\\
U_0^M &= \ln\tilde{c}_0 + \theta\ln\tilde{g}_0 + (1+\theta)\ln h_0 + \beta\left[\ln\tilde{d}_1 + \theta\ln\tilde{g}_1 + (1+\theta)\ln D\left(\tilde{x}_0\right)^{\eta}h_0\right],\\
U_1^M &= \ln\tilde{c}_1 + \theta\ln\tilde{g}_1 + (1+\theta)\ln D\left(\tilde{x}_0\right)^{\eta}h_0 + \beta\left[\ln\tilde{d}_2 + \theta\ln\tilde{g}_2 + (1+\theta)\ln D^2\left(\tilde{x}_0\right)^{\eta}\left(\tilde{x}_1\right)^{\eta}h_0\right],\\
&\ \ \vdots\\
U_t^M &= \ln\tilde{c}_t + \theta\ln\tilde{g}_t + (1+\theta)\ln\left(D\right)^{t}\left(\tilde{x}_{t-1}\right)^{\eta}\left(\tilde{x}_{t-2}\right)^{\eta}\cdot\cdots\cdot\left(\tilde{x}_0\right)^{\eta}h_0\\
&\quad + \beta\left[\ln\tilde{d}_{t+1} + \theta\ln\tilde{g}_{t+1} + (1+\theta)\ln\left(D\right)^{t+1}\left(\tilde{x}_t\right)^{\eta}\left(\tilde{x}_{t-1}\right)^{\eta}\cdot\cdots\cdot\left(\tilde{x}_0\right)^{\eta}h_0\right].\\
&\ \ \vdots
\end{aligned}$$

In particular, the utility of the period-t for the middle-aged is rewritten as

$$\begin{aligned}
U_t^M &= \ln\tilde{c}_t + \theta\ln\tilde{g}_t + \beta\ln\tilde{d}_{t+1} + \beta\theta\ln\tilde{g}_{t+1}\\
&\quad + \eta\left(1+\beta\right)\left(1+\theta\right)\sum_{j=0}^{t-1}\ln\tilde{x}_j + \eta\beta\ln\tilde{x}_t + \left(1+\beta\right)\left(1+\theta\right)\ln h_0 + \left[t+\beta\left(t+1\right)\right]\left(1+\theta\right)\ln D.
\end{aligned}$$

Thus, by removing the terms that are irrelevant for determining the allocation, the social welfare function, denoted by SW, becomes

$$\begin{aligned}
SW \simeq\ & \frac{\beta}{\gamma}\left(\ln\tilde{d}_0 + \theta\ln\tilde{g}_0\right)\\
&+ \ln\tilde{c}_0 + \theta\ln\tilde{g}_0 + \beta\ln\tilde{d}_1 + \beta\theta\ln\tilde{g}_1 + \eta\beta(1+\theta)\ln\tilde{x}_0\\
&+ \gamma\cdot\left[\ln\tilde{c}_1 + \theta\ln\tilde{g}_1 + \beta\ln\tilde{d}_2 + \beta\theta\ln\tilde{g}_2 + \eta\left(1+\beta\right)\left(1+\theta\right)\ln\tilde{x}_0 + \eta\beta(1+\theta)\ln\tilde{x}_1\right]\\
&+ \gamma^2\cdot\left[\ln\tilde{c}_2 + \theta\ln\tilde{g}_2 + \beta\ln\tilde{d}_3 + \beta\theta\ln\tilde{g}_3 + \eta\left(1+\beta\right)\left(1+\theta\right)\left(\ln\tilde{x}_0 + \ln\tilde{x}_1\right) + \eta\beta(1+\theta)\ln\tilde{x}_2\right]\\
&+ \cdots,
\end{aligned}$$

that is,

$$SW \simeq \sum_{t=0}^{\infty} \gamma^t \cdot \left\{ \ln \tilde{c}_t + \frac{\beta}{\gamma} \ln \tilde{d}_t + \theta \left(1 + \frac{\beta}{\gamma}\right) \ln \tilde{g}_t + \eta(1+\theta) \left[\beta + \frac{\gamma(1+\beta)}{1-\gamma}\right] \ln \tilde{x}_t \right\}. \tag{A.34}$$

By plugging (A.33) into (A.34), the planner's problem becomes

$$\max \sum_{t=0}^{\infty} \gamma^t \cdot \left\{ \ln \left[A\left(\tilde{k}_t\right)^{\alpha} - \frac{\tilde{d}_t}{1+n} - (1+n)\tilde{k}_{t+1} D\left(\tilde{x}_t\right)^{\eta} - (1+n)\tilde{x}_t - \frac{2+n}{1+n}\tilde{g}_t \right] \right.$$
$$\left. + \frac{\beta}{\gamma} \ln \tilde{d}_t + \theta \left(1 + \frac{\beta}{\gamma}\right) \ln \tilde{g}_t + \eta(1+\theta)\left[\beta + \frac{\gamma(1+\beta)}{1-\gamma}\right] \ln \tilde{x}_t \right\}$$

given $\tilde{k}_0$.

We can express the Bellman equation for the problem as follows:

$$V(\tilde{k}) = \max_{\{\tilde{d},\tilde{x},\tilde{k}'\}} \left\{ \ln \left[A\left(\tilde{k}\right)^{\alpha}, - \frac{\tilde{d}}{1+n} - (1+n)\tilde{k}' D\left(\tilde{x}\right)^{\eta} - (1+n)\tilde{x} - \frac{2+n}{1+n}\tilde{g} \right] \right.$$
$$\left. + \frac{\beta}{\gamma} \ln \tilde{d} + \theta\left(1 + \frac{\beta}{\gamma}\right) \ln \tilde{g} + \eta(1+\theta)\left[\beta + \frac{\gamma(1+\beta)}{1-\gamma}\right] \ln \tilde{x} + \gamma V(\tilde{k}') \right\}. \tag{A.35}$$

We guess that $V(\tilde{k}') = z_0 + z_1 \ln \tilde{k}'$, where z_0 and z_1 are undetermined coefficients. For this guess, (A.35) becomes

$$V(\tilde{k}) = \max_{\{\tilde{d},\tilde{x},\tilde{g},\tilde{k}'\}} \left\{ \ln \left[A\left(\tilde{k}\right),^{\alpha} - \frac{\tilde{d}}{1+n} - (1+n)\tilde{k}' D\left(\tilde{x}\right)^{\eta} - (1+n)\tilde{x} - \frac{2+n}{1+n}\tilde{g} \right] \right.$$
$$\left. + \frac{\beta}{\gamma} \ln \tilde{d} + \theta\left(1 + \frac{\beta}{\gamma}\right) \ln \tilde{g} + \eta(1+\theta)\left[\beta + \frac{\gamma(1+\beta)}{1-\gamma}\right] \ln \tilde{x} + \gamma \left[z_0 + z_1 \ln \tilde{k}'\right] \right\}. \tag{A.36}$$

The first-order conditions with respect to $\tilde{d}$, $\tilde{x}$, $\tilde{g}$, and $\tilde{k}'$ are

$$\tilde{d}: \frac{-\frac{1}{1+n}}{A\left(\tilde{k}\right)^{\alpha} - \frac{\tilde{d}}{1+n} - (1+n)\tilde{k}' D\left(\tilde{x}\right)^{\eta} - (1+n)\tilde{x} - \frac{2+n}{1+n}\tilde{g}} + \frac{\beta}{\gamma}, \cdot \frac{1}{\tilde{d}} = 0, \tag{A.37}$$

$$\tilde{x}: \frac{-(1+n)\left[\eta \tilde{k}' D\left(\tilde{x}\right)^{\eta-1} + 1\right]}{A\left(\tilde{k}\right)^{\alpha}, - \frac{\tilde{d}}{1+n} - (1+n)\tilde{k}' D\left(\tilde{x}\right)^{\eta} - (1+n)\tilde{x} - \frac{2+n}{1+n}\tilde{g}} + \frac{\eta(1+\theta)\left[\beta + \frac{\gamma(1+\beta)}{1-\gamma}\right]}{\tilde{x}} = 0, \tag{A.38}$$

$$\tilde{g}: \frac{-\frac{2+n}{1+n}}{A\left(\tilde{k}\right)^{\alpha} - \frac{\tilde{d}}{1+n} - (1+n)\tilde{k}' D\left(\tilde{x}\right)^{\eta} - (1+n)\tilde{x} - \frac{2+n}{1+n}\tilde{g}} + \theta\left(1 + \frac{\beta}{\gamma}\right) \cdot \frac{1}{\tilde{g}} = 0, \tag{A.39}$$

$$\tilde{k}': \frac{-(1+n) D\left(\tilde{x}\right)^{\eta}}{A\left(\tilde{k}\right)^{\alpha} - \frac{\tilde{d}}{1+n} - (1+n)\tilde{k}' D\left(\tilde{x}\right)^{\eta} - (1+n)\tilde{x} - \frac{2+n}{1+n}\tilde{g}} + \frac{\gamma z_1}{\tilde{k}'} = 0. \tag{A.40}$$

Eqs. (A.37) and (A.40) lead to

$$\frac{\tilde{d}}{1+n} = \frac{\beta}{\gamma} \cdot \frac{\tilde{k}'(1+n) D\left(\tilde{x}\right)^{\eta}}{\gamma z_1}, \tag{A.41}$$

Eqs. (A.38) and (A.40) lead to

$$\gamma z_1 = \eta \tilde{k}' D\left(\tilde{x}\right)^{\eta-1} \left\{ (1+\theta)\left[\beta + \frac{\gamma(1+\beta)}{1-\gamma}\right] - \gamma z_1 \right\}, \tag{A.42}$$

and Eqs. (A.39) and (A.42) lead to

$$\frac{2+n}{1+n}\tilde{g} = (1+n)D\left(\tilde{x}\right)^{\eta}\theta\left(1+\frac{\beta}{\gamma}\right)\frac{\tilde{k}'}{\gamma z_1}. \tag{A.43}$$

By substituting (A.41) and (A.43) into (A.40) and rearranging the terms, we obtain

$$\gamma z_1(1+n)\tilde{x} = \gamma z_1 A\left(\tilde{k}\right)^{\alpha} - \tilde{k}'(1+n)D\left(\tilde{x}\right)^{\eta}\left[(1+\theta)\left(1+\frac{\beta}{\gamma}\right)+\gamma z_1\right]. \tag{A.44}$$

We multiply both sides of (A.42) by $(1+n)\tilde{x}$ and rearrange the terms to obtain

$$(1+n)\tilde{k}'D\left(\tilde{x}\right)^{\eta} = \frac{\gamma z_1/\eta}{(1+\theta)\left[\beta+\frac{\gamma(1+\beta)}{1-\gamma}\right]-\gamma z_1}(1+n)\tilde{x}. \tag{A.45}$$

Using (A.41) – (A.45), we obtain

$$\frac{\tilde{d}}{1+n} = \frac{\frac{\beta}{\gamma\eta}}{(1+\theta)\left[\beta+\frac{\gamma(1+\beta)}{1-\gamma}+\frac{1}{\eta}\left(1+\frac{\beta}{\gamma}\right)\right]-\gamma z_1\left(1-\frac{1}{\eta}\right)}A\left(\tilde{k}\right)^{\alpha}, \tag{A.46}$$

$$(1+n)\tilde{x} = \frac{(1+\theta)\left[\beta+\frac{\gamma(1+\beta)}{1-\gamma}\right]-\gamma z_1}{(1+\theta)\left[\beta+\frac{\gamma(1+\beta)}{1-\gamma}+\frac{1}{\eta}\left(1+\frac{\beta}{\gamma}\right)\right]-\gamma z_1\left(1-\frac{1}{\eta}\right)}A\left(\tilde{k}\right)^{\alpha}, \tag{A.47}$$

$$\frac{2+n}{1+n}\tilde{g} = \frac{\frac{\theta}{\eta}\left(1+\frac{\beta}{\gamma}\right)}{(1+\theta)\left[\beta+\frac{\gamma(1+\beta)}{1-\gamma}+\frac{1}{\eta}\left(1+\frac{\beta}{\gamma}\right)\right]-\gamma z_1\left(1-\frac{1}{\eta}\right)}A\left(\tilde{k}\right)^{\alpha}, \tag{A.48}$$

$$(1+n)\tilde{k}'D\left(\tilde{x}\right)^{\eta} = \frac{\frac{\gamma z_1}{\eta}}{(1+\theta)\left[\beta+\frac{\gamma(1+\beta)}{1-\gamma}+\frac{1}{\eta}\left(1+\frac{\beta}{\gamma}\right)\right]-\gamma z_1\left(1-\frac{1}{\eta}\right)}A\left(\tilde{k}\right)^{\alpha}. \tag{A.49}$$

We substitute (A.46)–(A.49) into the Bellman equation in (A.36) and obtain

$$V(\tilde{k}) = \left\{\alpha(1+\theta)\left[\left(1+\frac{\beta}{\gamma}\right)+\eta\left[\beta+\frac{\gamma\left(1+\beta\right)}{1-\gamma}\right]\right]+\alpha\gamma z_1\left(1-\eta\right)\right\}\ln\tilde{k}+C\left(z_0,z_1\right),$$

where $C\left(z_0,z_1\right)$ is the collective notation for constant terms. The guess is verified if $z_0 = C\left(z_0,z_1\right)$ and

$$z_1 = \alpha(1+\theta)\left[\left(1+\frac{\beta}{\gamma}\right)+\eta\left[\beta+\frac{\gamma\left(1+\beta\right)}{1-\gamma}\right]\right]+\alpha\gamma z_1\left(1-\eta\right).$$

Therefore, z_1 is given by

$$z_1 = \frac{\alpha(1+\theta)\left\{\left(1+\frac{\beta}{\gamma}\right)+\eta\left[\beta+\frac{\gamma(1+\beta)}{1-\gamma}\right]\right\}}{1-\alpha\gamma\left(1-\eta\right)},$$

and the corresponding policy functions are obtained as expressed in Proposition 3.

Using the policy functions presented in Proposition 3, we demonstrate the accumulation of physical and human capital. The physical capital accumulation, k'/k, is computed using the resource constraint in (33) and the policy functions in Proposition 3 as follows:

$$\frac{k'}{k} = \frac{\alpha\gamma}{1+n}A\left(\frac{h}{k}\right)^{1-\alpha}. \tag{A.50}$$

The human capital accumulation, h'/h, is computed using the human capital formation function in (1) and the policy function of x in Proposition 3 as follows:

$$\frac{h'}{h} = D\left[\frac{1}{1+n} \cdot \frac{(1-\alpha)\,\gamma\eta}{1-\gamma\,(1-\eta)} A \left(\frac{k}{h}\right)^{\alpha}\right]^{\eta}. \tag{A.51}$$

■

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