The mercantile dilemma: formalisations and historical conclusions

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Abstract

The following contributions are hereby worked: one mathematically formalises Mundell's Impossible trio and Rodrik's Globalisation paradox, supplying the latter with a taxonomy in terms of the current account; by means of Kaldor's price endogeneity in output, one proves that external real money market disparity and trade generate external output mismatches and lead to autarky unless offset, using topology and dynamical systems; one characterises transfers and federalism and shows that all unitary states are federal polities and can merge into confederations; one demonstrates that the said external output mismatches can be only eluded via autarky or neutralisation, irrespective of federalism; one discerns (i) artificial currency areas guaranteeing inter-regional external output growth equality and (ii) modern protectionism as two Nash equilibria, to wit, the mercantile dilemma, especially rationalising the nexus between the Gold standard, the Industrial revolution and the Great divergence therethrough.

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Contents

1. Introduction

1.1 International trilemmas. In this article one studies two famous international macroeconomics trilemmas: Robert Mundell's [9] Impossible trio and Dani Rodrik's [13] Globalisation paradox. Mundell's trilemma stipulates that out of fixed exchange rates, financial openness and monetary policy independence at most two can be selected. Rodrik's trilemma spells that out of the nation state, deep economic integration and democratic politics at most two can be achieved.

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Mundell's trilemma is an established tenet of international macroeconomics and it rewarded Mundell with a Nobel prize; Rodrik's is not and is considered heterodox. To one's mind, such is so for two reasons: Mundell's trilemma was diagrammatically presented and it concerns the capital account; Rodrik's trilemma is still verbal and it concerns the current account. Formal reasoning is ever superior to informal reasoning, for it fleshes out relations, and diagrams are thereby better than words. Current account dynamics are the true divide of economics, between protectionists and advocates of free trade: business cycle stabilisation and wealth redistribution are orthodox tenets, all in all, and they treat the closed economy; protectionism treats the open one and it is outrage.

1.2 Contributions. One herewith mathematically formalises both trilemmas and through their resulting interaction one shows that Rodrik's trilemma didactically outshines that of Mundell, managing to coherently explain (i) autarkic mercantilism, (ii) the Gold standard, the Industrial revolution and the Great divergence and (iii) globalisation. One's contributions are therefore at once methodological and notional. Via Nicholas Kaldor's [4] price endogeneity in output, one firstly shows that the nation state and deep economic integration permanently implode: one does so both by means of topology and through a stylised dynamical system, whereby one calculates parametric conditions for instability. One then defines federalism and establishes that regions always feature it internally and by choice even externally. One thirdly proves that said implosion is avoidable through either autarky or neutralisation, regardless of federalism. Finally, one (historically) intersects both trilemmas' combinations through game theory and presents two Nash equilibria, to wit, the mercantile dilemma: (i) artificial currency areas guaranteeing inter-regional external output growth equality and (ii) modern protectionism.

2. Capital account: Impossible trio

2.1 Real and nominal variables. Foreign variables are identified by means of an asterisk superscript; whenever foreign variables be not defined the characterisations of their domestic counterparts are assumed to apply to them and whenever they may be defined as specular to domestic variables it is to be for clarity purposes. All variables except domestic and foreign nominal money supply M_S and M_S^* and domestic and foreign nominal exchange rate E and E^* are economically real, that is, they are nominal variables divided by price: $x = \frac{X}{p}$, $\forall X \in \mathbb{R}$ and $p \in \mathbb{R}\backslash\{0\}$; economically real variables admit supply dynamics on the part of firms and government via prices p (i.e. capital, labour, technology, subsidies, taxation); one employs the adjectives "nominal" and "real" whenever clarity may be at risk.

2.2 Output and money market. The adjectives "external" and "internal" respectively relate to the open and closed economy, in turn connected to tradable and non-tradable commodities. Domestic and foreign output x and x^*, domestic and foreign money demand m_D and m_D^* and domestic and foreign prices p and p^* are external and, for simplicity, unless clarity may otherwise warrant, one does not employ such an adjective: they would be otherwise denoted as x_E, x_E^*, m_{ED}, m_{ED}^*, p_E and p_E^*, respectively.

Under a single currency domestic and foreign nominal money supply M_S and M_S^* are internal and external; under a double currency they are subdivided into domestic and foreign internal currency M_I and M_I^* and domestic and foreign external currency M_E and M_E^* : internal currency is the currency circulating within a region's borders (i.e. those of a political region or nation); external currency is the currency circulating on the foreign exchange market.

Domestic real money supply m_S is defined as domestic nominal money supply M_S over domestic prices p : $m_S = \frac{M_S}{p}$. Foreign real money supply m_S^* is defined as foreign nominal money supply M_S^* over foreign prices p^* : $m_S^* = \frac{M_S^*}{p^*}$. Both are external and should be respectively denoted as m_{ES} and m_{ES}^*, but, for simplicity, one likewise avoids the adjective.

2.3 Prices and money demand. Domestic prices p increase in domestic autonomous wages aw, domestic autonomous capital return ark, domestic supply taxation t_S, changed domestic nominal money supply $\dot{M}_S$, changed domestic autonomous output supply $\dot{as}_x$, changed domestic autonomous long run supply $\dot{as}_L$, changed domestic output demand $\dot{d}_x$ and they decrease in domestic autonomous output supply as_x (i.e. external technology), domestic autonomous long run supply as_L (i.e. external and internal technology), changed domestic autonomous wages $\dot{aw}$, changed domestic autonomous capital return $\dot{ark}$, changed domestic supply taxation $\dot{t}_S$, changed domestic demand taxation $\dot{t}_D$: $p =$

$f(\overset{+}{a\dot{w}},\ \overset{+}{a\dot{r}k},\ \overset{+}{\dot{t}_S},\ \overset{+}{\dot{M}_S},\ \overset{+}{a\dot{s}_x},\ \overset{+}{a\dot{s}_L},\ \overset{+}{\dot{d}_x},\ \overset{-}{a\dot{s}_x},\ \overset{-}{a\dot{s}_L},\ \overset{-}{a\dot{w}},\ \overset{-}{a\dot{r}k},\ \overset{-}{\dot{t}_S},\ \overset{-}{\dot{t}_D})$. All variables are external, domestic autonomous long run supply as_L included, being both external and internal: the adjective is again avoided.

Domestic autonomous prices ap group domestic autonomous wages aw, domestic autonomous capital return ark, domestic autonomous output supply as_x, domestic autonomous long run supply as_L and domestic supply taxation t_S; changed domestic autonomous prices $a\dot{p}$ group changed domestic autonomous wages $a\dot{w}$, changed domestic autonomous capital return $a\dot{r}k$, changed domestic autonomous output supply $a\dot{s}_x$, changed domestic autonomous long run supply $a\dot{s}_L$ and changed domestic supply taxation $\dot{t}_S$; changed domestic autonomous money demand $a\dot{m}_D$ groups changed domestic output demand $\dot{d}_x$ and changed domestic demand taxation $\dot{t}_D$. Domestic prices p can be thus re-expressed according to the said collections, increasing in domestic autonomous prices ap, changed domestic nominal money supply $\dot{M}_S$ and changed domestic autonomous money demand $a\dot{m}_D$ and decreasing in changed domestic autonomous prices $a\dot{p}$: $p = f(\overset{+}{ap},\ \overset{+}{\dot{M}_S},\ \overset{+}{a\dot{m}_D},\ \overset{-}{a\dot{p}})$.

Domestic money demand m_D increases in domestic autonomous output supply as_x, domestic autonomous long run supply as_L and domestic output demand d_x and it decreases in domestic autonomous wages aw, domestic autonomous capital return ark, domestic supply taxation t_S and domestic demand taxation t_D : $m_D = f(\overset{+}{as_x},\ \overset{+}{as_L},\ \overset{+}{d_x},\ \overset{-}{aw},\ \overset{-}{ark},\ \overset{-}{t_S},\ \overset{-}{t_D})$. Domestic money demand m_D increases in domestic autonomous output supply as_x and domestic autonomous long run supply as_L because increments therein are assumed to be permanent, decreasing domestic prices p and thereby increasing domestic money demand m_D. Correspondingly, permanent increments in domestic autonomous wages aw, domestic autonomous capital return ark and domestic supply taxation t_S increase domestic prices p and thereby decrease domestic money demand m_D.

Domestic autonomous money demand am_D obviously groups domestic output demand d_x and domestic demand taxation t_D; domestic money demand m_D can be thus re-expressed as increasing in domestic autonomous money demand am_D and decreasing in domestic autonomous prices ap : $m_D = f(\overset{+}{am_D},\ \overset{-}{ap})$.

Domestic output demand d_x increases in domestic autonomous output demand ad_x (i.e. protectionistic consumption) and foreign autonomous output demand ad_x^* and it decreases in foreign tariffs tf^* : $d_x = f(\overset{+}{ad},\ \overset{+}{ad^*},\ \overset{-}{tf^*})$. Domestic supply and demand taxation t_D and t_S respectively increase in domestic autonomous supply and demand taxation at_S and at_D and they respectively decrease in domestic supply and demand subsidies s_S and s_D : $t_S = f(\overset{+}{at_S},\ \overset{-}{s_S})$ and $t_D = f(\overset{+}{at_D},\ \overset{-}{s_D})$. All variables are again external and the adjective is eluded anew.

2.4 Fiscal depreciations. Domestic supply taxation t_S is the instrument through which fiscal depreciations arise. In detail, fiscal depreciations are effected by decreasing domestic supply taxation t_S on domestic output x, which decreases domestic prices p and thereby domestic real interest rate r, in turn increasing domestic real exchange rate e (see subsection 2.6); fiscal appreciations are effected accordingly.

The decrease in domestic supply taxation t_S on domestic output x to the end of a fiscal depreciation also increases domestic money demand m_D, for domestic output x, consequently, if the fiscal depreciation is aimed at offsetting a prevenient decrease in domestic output x, domestic demand taxation t_D on domestic output x must be increased in order to neutralise the excess increase in domestic output x otherwise generated.

In the absence of domestic savings sv fiscal depreciation is financed by decreasing domestic structural spending g_S.

2.5 Monetary fiscality. Domestic structural spending g_S is normally financed through domestic income and savings taxation T, but monetary fiscality seems more efficient, namely, domestic lump sum taxation T on domestic nominal money supply M_S : tax evasion would be reduced to consumption taxation (i.e. domestic autonomous demand taxation at_D and domestic autonomous closed taxation at_C), which would remain in place for domestic cyclical spending $g_C = f(\overset{+}{s_S},\ \overset{+}{s_D},\ \overset{-}{as_S},\ \overset{-}{ad_S})$, external and internal.

Monetary fiscality in proportion to projected category use would hardly be feasible inasmuch as the categories benefiting from the services provided by the specific domestic structural spending g_S may hardly afford their prospected usage thereof.

In short, monetary fiscality would have the advantage of eliminating domestic income tax evasion and minimising domestic savings sv accumulation abroad.

2.6 Exchange rates. Domestic nominal exchange rate E is an increasing function of domestic nominal money supply M_S and of foreign money demand m_D^* and a decreasing function of foreign nominal money supply M_S^* and of domestic money demand m_D : $E = f(\overset{+}{M_S},\ \overset{+}{m_D^*},\ \overset{-}{M_S^*},\ \overset{-}{m_D})$. Domestic real exchange rate e is an increasing function of domestic real money supply m_S and of foreign money demand m_D^* and a decreasing function of foreign real money supply m_S^* and of domestic money demand m_D : $e = f(\overset{+}{m_S},\ \overset{+}{m_D^*},\ \overset{-}{m_S^*},\ \overset{-}{m_D})$.

Foreign nominal exchange rate E^* is an increasing function of foreign nominal money supply M_S^* and of domestic money demand m_D and a decreasing function of domestic nominal money supply M_S and of foreign money demand m_D^* : $E^* = f(\overset{+}{M_S^*},\ \overset{+}{m_D},\ \overset{-}{M_S},\ \overset{-}{m_D^*})$. Foreign real exchange rate e^* is an increasing function of foreign real money supply m_S^* and of domestic money demand m_D and a decreasing function of domestic real money supply m_S and of foreign money demand m_D^* : $e^* = f(\overset{+}{m_S^*},\ \overset{+}{m_D},\ \overset{-}{m_S},\ \overset{-}{m_D^*})$.

Strictly speaking, domestic nominal exchange rate E is domestic real exchange rate e multiplied by domestic prices p and domestic real exchange rate e is an increasing function of foreign real interest rate r^* and of domestic expected real interest rate er and a decreasing function of domestic real interest rate r and of foreign expected real interest rate er^* : $E = ep$ and $e = f(\overset{+}{r^*},\ \overset{+}{er},\ \overset{-}{r},\ \overset{-}{er^*})$. In turn, domestic real interest rate r is an increasing function of domestic money demand m_D and a decreasing function of domestic real money supply m_S; comparably, domestic expected real interest rate er is a decreasing function of domestic real interest rate r, accounting for over and undershooting: $r = f(\overset{+}{m_D},\ \overset{-}{m_S})$; $er = f(\overset{-}{r})$. Analogous definitions ultimately apply to foreign nominal exchange rate E^*, foreign real exchange rate e^*, foreign real interest rate r^* and foreign expected real interest rate er^*. For simplicity, such deeper definitions are eschewed.

2.7 Fixations and floatations. For constant foreign nominal money supply M_S^* and foreign money demand m_D^*, domestic nominal exchange rate E fixations and managed floats (or crawling pegs) are respectively arbitrary and semi-arbitrary changes in domestic nominal money supply M_S and domestic money demand m_D, matching a predetermined value (i.e. real, metallic, monetary). For constant foreign real money supply m_S^* and foreign money demand m_D^*, domestic real exchange rate e fixations and managed floats (or crawling pegs) are respectively arbitrary and semi-arbitrary changes in domestic real money supply m_S and domestic money demand m_D, matching a predetermined value (i.e. real, metallic, monetary).

Domestic nominal exchange rate E floatations or free floats are non-arbitrary changes in domestic nominal money supply M_S and domestic money demand m_D, irrespective of foreign nominal money supply M_S^* and foreign money demand m_D^*. Domestic real exchange rate e floatations or free floats are non-arbitrary changes in domestic real money supply m_S and domestic money demand m_D, irrespective of foreign real money supply m_S^* and foreign money demand m_D^*.

Definition 2.8 (Rigidity, openness and independence) *Let X and Y be respective subsets of the real hyperplane and let their elements be growth rates: $X = \left\{x \in \mathbb{R}^n :\ x = \frac{\dot{x}_l}{x_l}\right\},\ \forall n \geq 1$, and $Y = \left\{y \in \mathbb{R} :\ y = \frac{\dot{y}_l}{y_l}\right\}$, wherein subscript l signifies levels. Let f be a real, twice continuously differentiable function: $f : X \to Y$ and $f \in C^2$.*

1. Rigidity is the fixed equality of the domestic real exchange rate to a number within a non-negative real interval. 2. Openness is the increase of the domestic real exchange rate in the domestic real money supply and in foreign money demand and the decrease of the domestic real exchange rate in the foreign real money supply and in domestic money demand. 3. Independence is the variable equality of the domestic nominal money supply to a non-negative real number. Formally:

1. $R:\ e = \tilde{e} \in [\underline{e},\ \overline{e}] \subset \mathbb{R}_+$ (rigidity);

2. $O:\ e = f(\overset{+}{m_S},\ \overset{+}{m^*_D},\ \overset{-}{m^*_S},\ \overset{-}{m_D})$ (openness);

3. $I:\ M_S \in \mathbb{R}_+$ (independence).

One works in growth rates, rather than levels, because one wishes to eventually model full employment domestic output x levels despite level disparities in domestic and foreign real money supply m_S and m^*_S and in domestic and foreign money demand m_D and m^*_D (see section 3).

2.9 Rigidity and flexibility. Rigidity R means that domestic real exchange rate e is fixed to value $\tilde{e}$ in some non-negative real interval $[\underline{e},\ \overline{e}]$: under rigidity R domestic real exchange rate e equals a specific non-negative real value $\tilde{e}$. Such models Mundell's fixed exchange rates, whereby domestic and foreign real money supply m_S and m^*_S and domestic and foreign money demand m_D and m^*_D are constant.

Flexibility F is non-rigidity $\neg R$ and it can be written as $f(\overset{+}{m_S},\ \overset{+}{m^*_D},\ \overset{-}{m^*_S},\ \overset{-}{m_D}) = e \in \mathbb{R}_+$ such that domestic real exchange rate e (with domestic nominal exchange rate E thereby) floats in the non-negative real number line $\mathbb{R}_+$; indeed, domestic real exchange rate e must be a function of domestic and foreign real money supply m_S and m^*_S and of domestic and foreign money demand m_D and m^*_D and domestic nominal exchange rate E must be a function of domestic and foreign nominal money supply M_S and M^*_S and of domestic and foreign money demand m_D and m^*_D, otherwise both would in effect be arbitrarily fixed to a non-negative real number, contradictorily tracing rigidity R.

Specifically, flexibility F is not the fixed equality of domestic real exchange rate e to a number within a non-negative real interval: $e \neq \tilde{e} \in [\underline{e},\ \overline{e}] \subset \mathbb{R}_+$.

2.10 Mundell's exchange rates. In more detail, Mundell's fixed exchange rates can be defined as fixations in domestic nominal exchange rate E or, more broadly, in domestic real exchange rate e. In the case of fixed domestic real exchange rate e one's above definition of rigidity R models Mundell's fixed exchange rates by tracing domestic real money supply m_S and domestic money demand m_D fixations.

In the case of fixed domestic nominal exchange rate E movements in domestic and foreign prices p and p^* are not by definition considered, therefore, one's above characterisation of rigidity R models Mundell's fixed exchange rates as well, meeting domestic nominal money supply M_S and domestic money demand m_D fixations and residually leaving domestic and foreign prices p and p^* unvaried.

Correspondingly, Mundell's floating exchange rates would be modelled as flexibility F, also defined above, whereby domestic real exchange rate e (with nominal exchange rate E thereby) is allowed to float in the non-negative real number line $\mathbb{R}_+$.

2.11 Openness and closure. Openness O means that domestic real exchange rate e increases in domestic real money supply m_S and in foreign money demand m^*_D and that it decreases in foreign real money supply m^*_S and in domestic money demand m_D. Such models Mundell's financial openness (i.e. capital account convertibility).

Closure C is non-openness $\neg O$ and it can be written as $e \neq f(\overset{+}{m_S},\ \overset{+}{m^*_D},\ \overset{-}{m^*_S},\ \overset{-}{m_D})$ such that domestic real exchange rate e is not a function of domestic and foreign real money supply m_S and m^*_S and of domestic and foreign money demand m_D and m^*_D. Closure implies that domestic nominal exchange rate E is not a function of domestic and foreign nominal money supply M_S and M^*_S and of domestic and foreign money demand m_D and $m^*_D:\ E \neq f(\overset{+}{M_S},\ \overset{+}{m^*_D},\ \overset{-}{M^*_S},\ \overset{-}{m_D})$.

Under closure C foreign real exchange rate e^* is likewise not a function of foreign and domestic real money supply m^*_S and m_S and of foreign and domestic money demand m^*_D and $m_D:\ e^* \neq f(\overset{+}{m^*_S},\ \overset{+}{m_D},\ \overset{-}{m_S},\ \overset{-}{m^*_D})$. Such implies that foreign nominal exchange rate E^* is not a function of foreign and domestic nominal money supply M^*_S and M_S and of foreign and domestic money demand m^*_D and $m_D:\ E^* \neq f(\overset{+}{M^*_S},\ \overset{+}{m_D},\ \overset{-}{M_S},\ \overset{-}{m^*_D})$.

Under closure C domestic nominal money supply M_S cannot be exchanged for any other currency and can thus be priced only relative to other domestic assets.

2.12 Single and double currency. Under openness O there is a single currency; under closure C there is either a single or a double currency. Under closure C and a single currency there is rigidity R, because if flexibility F were present then domestic real exchange rate e would be a function of domestic and foreign real money supply m_S and m_S^* and of domestic and foreign money demand m_D and m_D^*, violating closure C.

Under closure C and a double currency domestic non-negative real external currency $M_E \in \mathbb{R}_+$, on the foreign exchange market or not, is either permanently retained for imports (exchanged for foreign non-negative real external currency $M_E^* \in \mathbb{R}_+$ or foreign nominal money supply M_S^* therefor) or eventually converted into domestic positive real internal currency $M_I \in \mathbb{R}_{++}$ at predetermined conversion rate $\frac{M_E}{M_I}$.

Under a single currency domestic nominal money supply M_S equals domestic internal currency M_I : $M_S = M_I$.

2.13 Independence and dependence. Independence I means that domestic nominal money supply M_S floats in the non-negative real number line $\mathbb{R}_+$: under independence I domestic nominal money supply M_S equals any non-negative real number. Specifically, independence I is not the fixed equality of domestic nominal money M_S supply to a non-negative real number: $M_S \neq \tilde{M} \in \mathbb{R}_+$. Such models Mundell's monetary policy independence.

Dependence D is non-independence $\neg I$ and it can be written as $M_S = \tilde{M} \in \left[\underline{M},\ \overline{M}\right] \subset \mathbb{R}_+$ such that domestic nominal money supply M_S is fixed to value $\tilde{M}$ is some non-negative real interval $\left[\underline{M},\ \overline{M}\right]$.

2.14 Currency substitution. Domestic real exchange rate e and domestic nominal money supply M_S are non-negative because they admit reduction to zero, the latter being purchased through foreign currency or reserves at a given rate in currency substitution (e.g. dollarisation).

Both under openness O and closure C and a single currency domestic nominal money supply M_S is substituted for foreign nominal money supply M_S^* or foreign external currency M_E^* : $M_S \sim M_S^* \veebar M_E^*$.

Under closure C and a double currency only domestic external currency M_E is substituted, for foreign nominal money supply M_S^* or foreign external currency M_E^* on the foreign exchange market: $M_E \sim M_S^* \veebar M_E^*$.

2.15 Monetary and foreign exchange policy. Under independence I the real non-negative value of domestic nominal money supply M_S is ideally determined by anti-cyclical monetary policy (i.e. neutralisation of internal temporary supply shocks and, sub-optimally, also of demand).

Under rigidity and openness $R \wedge O$ non-negative real interval $[\underline{e},\ \overline{e}]$ specified for domestic real exchange rate e is determined by foreign exchange policy (i.e. managed floats or crawling pegs) such that there exists a compromise between monetary policy and foreign exchange policy; thus, under openness O monetary policy is fully enjoyed only through independence and flexibility $I \wedge F$.

One stresses that non-negative real interval $[\underline{e},\ \overline{e}]$ models managed floats and crawling pegs and that fixation and floatation or free floats are respectively $e = \tilde{e} \in \mathbb{R}_+$ and $f(\overset{+}{m_S},\ \overset{+}{m_D^*},\ \overset{-}{m_S^*},\ \overset{-}{m_D}) = e \in \mathbb{R}_+$ (i.e. predetermined and random). Consequently, under rigidity and independence $R \wedge I$ domestic nominal money supply M_S's real non-negative value is determined by monetary policy; non-negative real interval $[\underline{e},\ \overline{e}]$ is by contrast thereby absent under a foreign exchange policy of fixation and respectively present under one of managed floatation.

PROPOSITION 2.16 (Impossible trio) *Rigidity, openness and independence cannot be simultaneously selected: rigidity and openness imply dependence; rigidity and independence imply closure; openness and independence imply flexibility. Formally:*

1. $R \wedge O \longrightarrow D$ (s.q.n. dependence);
2. $R \wedge I \longrightarrow C$ (s.q.n. closure);
3. $O \wedge I \longrightarrow F$ (s.q.n. flexibility).

Proof. Lemma 2.16.1 (s.q.n. dependence; direct) Rigidity R means that $e = \tilde{e} \in [\underline{e},\ \overline{e}] \subset \mathbb{R}_+$ and openness O means that $e = f(\overset{+}{m_S},\ \overset{+}{m_D^*},\ \overset{-}{m_S^*},\ \overset{-}{m_D})$; therefore, $m_S,\ m_S^*,\ m_D$ and m_D^* are constant and

$M_S = \tilde{M} \in \mathbb{R}_+$ specifically: $e = \tilde{e} \in [\underline{e},\ \overline{e}] \subset \mathbb{R}_+$ and $e = f(\overset{+}{m_S},\ \overset{+}{m^*_D},\ \overset{-}{m^*_S},\ \overset{-}{m_D}) \vdash m_S = \tilde{m}_S,\ m_D = \tilde{m}_D,\ \bar{m}^*_S$ and $\bar{m}^*_D \vdash M_S = \tilde{M} \in \mathbb{R}_+$. Thus, $R \wedge O \longrightarrow D$.

Lemma 2.16.2 (s.q.n. closure; contradiction) Assume openness O, which means that $e = f(\overset{+}{m_S},\ \overset{+}{m^*_D},\ \overset{-}{m^*_S},\ \overset{-}{m_D})$; rigidity R means that $e = \tilde{e} \in [\underline{e},\ \overline{e}] \subset \mathbb{R}_+$, therefore, $m_S,\ m^*_S,\ m_D$ and m^*_D are constant and $M_S = \tilde{M} \in \mathbb{R}_+$ specifically, contradicting independence I, which means that $M_S \neq \tilde{M} \in \mathbb{R}_+ :\ e = f(\overset{+}{m_S},\ \overset{+}{m^*_D},\ \overset{-}{m^*_S},\ \overset{-}{m_D})$ and $e = \tilde{e} \in [\underline{e},\ \overline{e}] \subset \mathbb{R}_+ \vdash m_S = \tilde{m}_S,\ m_D = \tilde{m}_D,\ \bar{m}^*_S$ and $\bar{m}^*_D \vdash M_S = \tilde{M} \in \mathbb{R}_+$, but $M_S \neq \tilde{M} \in \mathbb{R}_+ \vdash e \neq f(\overset{+}{m_S},\ \overset{+}{m^*_D},\ \overset{-}{m^*_S},\ \overset{-}{m_D})$. Thus, $R \wedge I \longrightarrow C$.

Lemma 2.16.3 (s.q.n. flexibility; contradiction) Assume rigidity R, which means that $e = \tilde{e} \in [\underline{e},\ \overline{e}] \subset \mathbb{R}_+$; openness O means that $e = f(\overset{+}{m_S},\ \overset{+}{m^*_D},\ \overset{-}{m^*_S},\ \overset{-}{m_D})$, therefore, $m_S,\ m^*_S,\ m_D$ and m^*_D are constant and $M_S = \tilde{M} \in \mathbb{R}_+$ specifically, contradicting independence I, which means that $M_S \neq \tilde{M} \in \mathbb{R}_+ :\ e = \tilde{e} \in [\underline{e},\ \overline{e}] \subset \mathbb{R}_+$ and $e = f(\overset{+}{m_S},\ \overset{+}{m^*_D},\ \overset{-}{m^*_S},\ \overset{-}{m_D}) \vdash m_S = \tilde{m}_S,\ m_D = \tilde{m}_D,\ \bar{m}^*_S$ and $\bar{m}^*_D \vdash M_S = \tilde{M} \in \mathbb{R}_+$, but $M_S \neq \tilde{M} \in \mathbb{R}_+ \vdash e \neq \tilde{e} \in [\underline{e},\ \overline{e}] \subset \mathbb{R}_+$. Thus, $O \wedge I \longrightarrow F$. *QED*

Out of rigidity R, openness O and independence I at most two can be selected. Rigidity and openness $R \wedge O$ mean that domestic real exchange rate e is fixed to non-negative real value $\tilde{e}$, that it is a function of domestic and foreign real money supply m_S and m^*_S and of domestic and foreign money demand m_D and m^*_D and that domestic nominal money supply M_S is thus fixed to value $\tilde{M}$, not floating and violating independence I : rigidity and openness $R \wedge O$ are sufficient for dependence D.

Rigidity and independence $R \wedge I$ mean that if openness O were active then domestic real exchange rate e would be a function of domestic and foreign real money supply m_S and m^*_S and of domestic and foreign money demand m_D and m^*_D, that it would be fixed to non-negative real value $\tilde{e}$ and that domestic nominal money supply M_S would thus be fixed to value $\tilde{M}$, not floating and violating independence I, being a contradiction: rigidity and independence $R \wedge I$ are sufficient for closure C.

Openness and independence $O \wedge I$ mean that if rigidity R were active then domestic real exchange rate e would be fixed to non-negative real value $\tilde{e}$, that it would be a function of domestic and foreign real money supply m_S and m^*_S and of domestic and foreign money demand m_D and m^*_D and that domestic nominal money supply M_S would thus be fixed to value $\tilde{M}$, not floating and violating independence I, being a contradiction: openness and independence $O \wedge I$ are sufficient for flexibility F.

2.17 Capital account combinations. More predicate calculus can reveal that of eight capital account combinations (i.e. of rigidity R, openness O and independence I) the feasible ones are the following.

Beforehand, one remarks that OMOs is the acronym of open market operations, whereby a region's central bank conducts monetary or foreign exchange policy on the bourse, through government or corporate bonds, foreign reserves or other securities. The case of decree fixation is abbreviated to decree and it is such that: balance of payments transactions necessarily pass by the central bank (i.e. externally); domestic nominal money supply M_S is not exchanged on the bourse (i.e. internally). Currency duality (i.e. single or double currency) is also specified, together with the corresponding orientation (i.e. fixation or floatation) and the attendant mode of realisation (i.e. OMOs, decree or sole domestic money demand and prices):

1. $O \wedge D \wedge R$ [single currency, fixed (OMOs)];
2. $O \wedge I \wedge F$ [single currency, floating (OMOs)];
3. $C \wedge I \wedge R$ [single currency, floating (OMOs)-fixed (decree);
double currency, floating (OMOs)-fixed (decree or OMOs)];
4. $O \wedge D \wedge F$ [single currency, floating (sole domestic money demand and prices)];
5. $C \wedge D \wedge F$ [double currency, fixed (decree or OMOs)-floating (OMOs)];
6. $C \wedge I \wedge F$ [double currency, floating (OMOs)-floating (OMOs)];
7. $C \wedge D \wedge R$ [single currency, fixed (decree or OMOs)-fixed (decree);
double currency, fixed (decree or OMOs)-fixed (decree or OMOs)];

the infeasible combination is $R \wedge O \wedge I$, as seen above. The first three are those of the Impossible trio, which is ultimately a deduction and as such a law, not a mere theory: by confounding the Impossible trio combinations with the remaining feasible ones most instead improperly regard it as a theory (see Thomas Palley [12], for instance). The three converses of the Impossible trio are additionally disproven thereby: $D \longrightarrow R \wedge O = \neg(D \wedge F \vee C)$, admitting $D \wedge F \wedge C$, $D \wedge F \wedge O$ and $D \wedge R \wedge C$, which are all true, whence $D \not\longrightarrow R \wedge O$; $C \longrightarrow R \wedge I = \neg(C \wedge F \vee D)$, admitting $C \wedge F \wedge D$, $C \wedge F \wedge I$ and $C \wedge R \wedge D$, which are all true, whence $C \not\longrightarrow R \wedge I$; $F \longrightarrow O \wedge I = \neg(F \wedge C \vee D)$, admitting $F \wedge C \wedge D$, $F \wedge C \wedge I$ and $F \wedge O \wedge D$, which are all true, whence $F \not\longrightarrow O \wedge I$.

2.18 Actual cases. The first three mark actual cases. Openness, dependence and rigidity $O \wedge D \wedge R$ present a single, fixed currency, enacted via OMOs. Examples are customary fixed exchange rates, managed floats or crawling pegs, currency unions (e.g. European monetary system, European monetary union, African financial community), currency boards (e.g. 1990s Argentina dollarisation) and currency substitutions (e.g. contemporary Panama and Ecuador dollarisation).

Openness, independence and flexibility $O \wedge I \wedge F$ present a single, floating currency, whereby monetary policy is enacted via OMOs (i.e. foreign exchange policy is absent). Examples are customary floating exchange rates and reserve currency acquisitions (i.e. accumulation).

Closure, independence and rigidity $C \wedge I \wedge R$ present either a single or a double currency, floating internally and fixed externally in both events: under a single currency internal floatation is enacted via OMOs and external fixation is enacted via decree; under a double currency internal floatation is enacted via OMOs and external fixation is enacted either via decree or via OMOs. A single currency example is early the 2000s Chinese Yuan Renminbi in relation to the American Dollar.

2.19 Other cases. Openness, dependence and flexibility $O \wedge D \wedge F$ present a single, floating currency, whereby changes in domestic real exchange rate e take place only through changes in domestic money demand m_D and domestic prices p (i.e. besides monetary policy, foreign exchange policy is additionally absent).

Closure, dependence and flexibility $C \wedge D \wedge F$ present a double currency, internally fixed, enacted via decree or OMOs, and externally floating, enacted via OMOs; the combination is such that changes in domestic real exchange rate e take place only through changes in domestic money demand m_D and domestic prices p (i.e. besides monetary policy, foreign exchange policy is additionally absent).

Closure, independence and flexibility $C \wedge I \wedge F$ present a double currency, internally and externally floating, enacted via OMOs; the combination is such that changes in domestic real exchange rate e take place only through changes in domestic money demand m_D and domestic prices p (i.e. foreign exchange policy is absent).

Closure, dependence and rigidity $C \wedge D \wedge R$ present either a single or a double currency, internally and externally fixed: under a single currency internal fixation is enacted either via decree or via OMOs and external fixation is enacted via decree; under a double currency internal and external fixations are enacted either via decree or via OMOs. The combination is such that monetary policy is absent.

2.20 Great divergence. A historic instance of openness, rigidity and dependence $O \wedge R \wedge D$ is the Gold standard. The United kingdom adopted the Gold standard in 1821, the Second industrial revolution occurred in the United kingdom around 1870 and the Great divergence of the West followed thereafter. Mundell's trilemma cannot explain such a concomitance because it does not contemplate the impact of the nominal money supply, of prices and of money demand on external output; Rodrik's trilemma can.

3. Current account: Globalisation paradox

Definition 3.1 (Sovereignty, trade and efficiency) *Let X and Y be respective subsets of the real hyperplane and let their elements be growth rates: $X = \left\{x \in \mathbb{R}^n : \ x = \frac{\dot{x}_l}{x_l}\right\}$, $\forall n \geq 1$, and $Y = \left\{y \in \mathbb{R} : \ y = \frac{\dot{y}_l}{y_l}\right\}$, wherein subscript l signifies levels. Let f be a real, twice continuously differentiable function: $f : X \to Y$ and $f \in C^2$.*

1. Sovereignty is the inequality between the domestic and foreign real money supply or between domestic and foreign money demand. 2. Trade is the increase of domestic output in the domestic real money supply

and in domestic money demand and the decrease of domestic output in the foreign real money supply and in foreign money demand. 3. Efficiency is the equality of domestic output to foreign output. Formally:

$$\begin{aligned}&1.\ S:\ m_{S,\,D} \neq m^*_{S,\,D}\ \text{(sovereignty)};\\ &2.\ T:\ x = f(\overset{+}{m_S},\ \overset{+}{m_D},\ \overset{-}{m^*_S},\ \overset{-}{m^*_D})\ \text{(trade)};\\ &3.\ E:\ x = x^*\ \text{(efficiency)}.\end{aligned}$$

3.2 Sovereignty and subjection. Sovereignty S means that domestic real money supply m_S does not equal foreign real money supply m^*_S or that domestic money demand m_D does not equal foreign money demand m^*_D : under sovereignty S domestic real money supply m_S does not correspond to foreign real money supply m^*_S or domestic money demand m_D does not correspond to foreign money demand m^*_D.

Such models Rodrik's nation state: it is called sovereignty because a region allows domestic real money supply m_S or domestic money demand m_D not to equal the others'; sovereignty S in terms of price or money demand disparity would especially reflect the institutional differences discussed by Rodrik (i.e. through firms and government).

The inequality between domestic and foreign real money supply m_S and m^*_S or between domestic and foreign money demand m_D and m^*_D is such that the disjunction is inclusive, namely, there can be a simultaneous inequality in the real money supplies and money demands, however, it is such that the effects on domestic and foreign output x and x^* do not neutralise each other: $m_{S,\,D} \neq m^*_{S,\,D}$ is such that (i) $m_S \neq m^*_S$, (ii) $m_D \neq m^*_D$ or (iii) $m_S \neq m^*_S$ and $m_D \neq m^*_D$ such that $x_{m_S} \neq x_{m_D},\ x_{m^*_S} \neq x_{m^*_D},\ x_{m_{S,\,D}} \neq -x_{m^*_{S,\,D}},\ x^*_{m^*_S} \neq x^*_{m^*_D},\ x^*_{m_S} \neq x^*_{m_D}$ and $x^*_{m^*_{S,\,D}} \neq -x^*_{m_{S,\,D}}$ (because $x_{m_S} - x_{m_D} \neq 0,\ x_{m^*_S} - x_{m^*_D} \neq 0,\ x_{m_{S,\,D}} + x_{m^*_{S,\,D}} \neq 0$ and *vice versa).*

Subjection SB is non-sovereignty $\neg S$ and it can be written as $m_{S,\,D} = m^*_{S,\,D}$ such that domestic real money supply m_S equals foreign real money supply m^*_S and such that domestic money demand m_D equals foreign money demand m^*_D : $\neg(m_S \neq m^*_S \vee m_D \neq m^*_D) = (m_S = m^*_S \wedge m_D = m^*_D)$.

3.3 Trade and autarky. Trade T means that domestic output x increases in domestic real money supply m_S and in domestic money demand m_D and that it decreases in foreign real money supply m^*_S and in foreign money demand m^*_D. Trade T in terms of foreign output x^* means that it increases in foreign real money supply m^*_S and in foreign money demand m^*_D and that it decreases in domestic real money supply m_S and in domestic money demand m_D : $x^* = f(\overset{+}{m^*_S},\ \overset{+}{m^*_D},\ \overset{-}{m_S},\ \overset{-}{m_D})$.

Such models Rodrik's deep economic integration. Under subjection SB a change in domestic real money supply m_S or domestic money demand m_D can be offset by (reciprocal) opposing changes therein or by a change in foreign real money supply m^*_S or foreign money demand m^*_D from abroad and *vice versa.*

Autarky A is non-trade $\neg T$ and it can be bilaterally written as $x \neq f(\overset{+}{m_S},\ \overset{+}{m_D},\ \overset{-}{m^*_S},\ \overset{-}{m^*_D})$ and $x^* \neq f(\overset{+}{m^*_S},\ \overset{+}{m^*_D},\ \overset{-}{m_S},\ \overset{-}{m_D})$ such that domestic output x and foreign output x^* are not functions of domestic real money supply m_S and domestic money demand m_D and of foreign real money supply m^*_S and foreign money demand m^*_D. Specifically, domestic output x and foreign output x^* are zero because the external output sector disappears: $x = x^* = 0$.

3.4 Efficiency and inefficiency. Efficiency E means that domestic output x equals foreign output x^* : under efficiency E domestic output x corresponds to foreign output x^*.

Such models Rodrik's democratic politics: it is called efficiency E because a region maintains full employment or at least symmetric unemployment (envisaged in growth rates).

Inefficiency I is non-efficiency $\neg E$ and it can be written as $x \neq x^*$ such that domestic output x does not equal foreign output x^*.

3.5 Exports and imports. The reason for which domestic output x decreases in foreign real money supply m^*_S and in foreign money demand m^*_D is that changes therein increase foreign output x^*, foreign exports ex^* and therefore domestic imports im, also decreasing domestic exports ex, and *vice versa.* Indeed, domestic output x is also the difference between domestic exports ex and domestic imports im, valued at

domestic real exchange rate e in real terms, and *vice versa:* $\frac{px}{p} = \frac{pex}{p} - \left(\frac{Ep^*}{p}\right) im \longrightarrow x = ex - e \cdot im$ and $\frac{p^*x^*}{p^*} = \frac{p^*ex^*}{p^*} - \left(\frac{E^*p}{p^*}\right) im^* \longrightarrow x^* = ex^* - e^* \cdot im^*$.

Such is so because of the following, whereby all variables are domestic. Demand D is the sum of private consumption c, public consumption g (i.e. government spending) and firm consumption i (i.e. investment); supply S is the sum of non-tradable (i.e. internal) output y_{NTR} and tradable output y_{TR}, which is in turn the sum of output x and imports im, weighted at real exchange rate e, net of exports ex : $D \equiv c + i + g = y_{NTR} + y_{TR} = y_{NTR} + (x + e \cdot im - ex) \equiv S$. Since demand D and non-tradable output y_{NTR} hereby equal zero (i.e. there is no internal economy) external output x is derived to equal said difference: $D = y_{NTR} = 0 \longrightarrow x = ex - e \cdot im$.

In more detail, domestic exports ex increase in domestic real money supply m_S and in domestic money demand m_D and decrease in foreign real money supply m_S^* and in foreign money demand m_D^* and foreign exports ex^* increase in foreign real money supply m_S^* and in foreign money demand m_D^* and decrease in domestic real money supply m_S and in domestic money demand m_D : $ex = (\overset{+}{m_S},\ \overset{+}{m_D},\ \overset{-}{m_S^*},\ \overset{-}{m_D^*})$ and $ex^* = (\overset{+}{m_S^*},\ \overset{+}{m_D^*},\ \overset{-}{m_S},\ \overset{-}{m_D})$.

Likewise, domestic imports im decrease in domestic real money supply m_S and in domestic money demand m_D and increase in foreign real money supply m_S^* and in foreign money demand m_D^* and foreign imports im^* decrease in foreign real money supply m_S^* and in foreign money demand m_D^* and increase in domestic real money supply m_S and in domestic money demand m_D : $im = (\overset{-}{m_S},\ \overset{-}{m_D},\ \overset{+}{m_S^*},\ \overset{+}{m_D^*})$ and $im^* = (\overset{-}{m_S^*},\ \overset{-}{m_D^*},\ \overset{+}{m_S},\ \overset{+}{m_D})$.

In the absence of an internal economy and in the presence of autarky A domestic output x and foreign output x^* equal zero and so do domestic exports ex, domestic imports im, foreign exports ex^* and foreign imports im^* : $D = y_{NTR} = 0$ and A are such that $x = x^* = ex = im = ex^* = im^* = 0$. All else equal, the equation of domestic output x and foreign output x^* to zero need not although imply that of domestic exports ex, domestic imports im, foreign exports ex^* and foreign imports im^*, which under trade T could merely annul domestic output x and foreign output x^* : provided $D = y_{NTR} = 0$, $x = x^* = 0 \not\longrightarrow ex = im = ex^* = im^* = 0$, since $(x = x^* = 0) \wedge (ex,\ im,\ ex^*,\ im^* \in \mathbb{R}_{++})$, namely, under T, $\forall ex,\ im,\ ex^*,\ im^* \in \mathbb{R}_{++}$, $ex = e \cdot im \longrightarrow x = 0$ and $ex^* = e^* \cdot im^* \longrightarrow x^* = 0$.

3.6 Marshall Lerner conditions. The increase of domestic output x in domestic real money supply m_S is the satisfaction of a Marshall Lerner condition applied to equation $x = ex - e \cdot im$ and its variables' respective functions, dictating an increase in domestic output x given an increase in domestic real money supply m_S if and only if the sum of domestic export elasticity η_{ex} and of the modulus elasticity of domestic imports η_{im} net of the elasticity of the domestic real exchange rate η_e is positive: $x_{m_S} > 0 \longleftrightarrow \eta_{ex} + |\eta_{im}| - \eta_e > 0$; more specifically, $x_{m_S} = ex_{m_S} - (e_{m_S} im + e \cdot im_{m_S}) \longrightarrow \frac{x_{m_S} m_S}{ex} = \eta_{ex} - \frac{m_S}{e \cdot im}(e_{m_S} im + e \cdot im_{m_S}) = \eta_{ex} - \eta_e - \eta_{im}$, since $ex = e \cdot im$ at equilibrium, thus, $x_{m_S} > 0 \longleftrightarrow \eta_{ex} - \eta_e - \eta_{im} > 0$ and since $\eta_{im} < 0$, domestic imports im being demand in domestic real money supply m_S, $|\eta_{im}| = -\eta_{im}$; for completeness, $\eta_{ex},\ \eta_e > 0$, domestic exports ex and domestic real exchange rate e being supply in domestic real money supply m_S. Analogously, a decrease in domestic output x given an increase in foreign real money supply m_S^* takes place if and only if the sum of domestic export elasticity η_{ex} and of the modulus elasticity of the domestic real exchange rate η_e net of the elasticity of domestic imports η_{im} is negative: $x_{m_S^*} < 0 \longleftrightarrow \eta_{ex} + |\eta_e| - \eta_{im} < 0$, since $\eta_{im} > 0$ and $\eta_{ex},\ \eta_e < 0$ such that $|\eta_e| = -\eta_e$ (i.e. domestic imports im are supply in foreign real money supply m_S^* and domestic exports ex and domestic real exchange rate e are demand in foreign real money supply m_S^*).

The increase of domestic output x in domestic money demand m_D is the satisfaction of another Marshall Lerner condition applied to equation $x = ex - e \cdot im$ and its variables' respective functions, dictating an increase in domestic output x given an increase in domestic money demand m_D if and only if the sum of domestic export elasticity η_{ex}, of the modulus elasticity of the domestic real exchange rate η_e and of the modulus elasticity of domestic imports η_{im} is positive: $x_{m_D} > 0 \longleftrightarrow \eta_{ex} + |\eta_e| + |\eta_{im}| > 0$; more specifically, $x_{m_D} = ex_{m_D} - (e_{m_D} im + e \cdot im_{m_D}) \longrightarrow \frac{x_{m_D} m_D}{ex} = \eta_{ex} - \frac{m_D}{e \cdot im}(e_{m_D} im + e \cdot im_{m_D}) = \eta_{ex} - \eta_e - \eta_{im}$, since $ex = e \cdot im$ at equilibrium, thus, $x_{m_D} > 0 \longleftrightarrow \eta_{ex} - \eta_e - \eta_{im} > 0$ and since $\eta_e,\ \eta_{im} < 0$, domestic real exchange rate e and domestic imports im being demand in domestic money demand m_D, $|\eta_e| = -\eta_e$ and

$|\eta_{im}| = -\eta_{im}$; for completeness, $\eta_{ex} > 0$ domestic exports ex being supply in domestic money demand m_D. Analogously, a decrease in domestic output x given an increase in foreign money demand m^*_D takes place if and only if domestic export elasticity η_{ex} net of the elasticity of the domestic real exchange rate η_e and of the elasticity of domestic imports η_{im} is negative: $x_{m^*_D} < 0 \longleftrightarrow \eta_{ex} - \eta_e - \eta_{im} < 0$, since $\eta_{ex} < 0$ and $\eta_{im},\ \eta_e > 0$ (i.e. domestic exports ex are demand in foreign money demand m^*_D and domestic imports im and domestic real exchange rate e are supply in foreign money demand m^*_D).

PROPOSITION 3.7 (Globalisation paradox) *Sovereignty, trade and efficiency cannot be simultaneously selected: sovereignty and trade imply inefficiency; sovereignty and efficiency imply autarky; trade and efficiency imply subjection. Formally:*

1. $S \wedge T \longrightarrow I$ (s.q.n. inefficiency);
2. $S \wedge E \longrightarrow A$ (s.q.n. autarky);
3. $T \wedge E \longrightarrow SB$ (s.q.n. subjection).

Proof. Lemma 3.7.1 (s.q.n inefficiency; direct) Sovereignty S means that $m_{S,\,D} \neq m^*_{S,\,D}$ and trade T means that $x = f(\overset{+}{m_S},\ \overset{+}{m_D},\ \overset{-}{m^*_S},\ \overset{-}{m^*_D})$ and $x^* = f(\overset{+}{m^*_S},\ \overset{+}{m^*_D},\ \overset{-}{m_S},\ \overset{-}{m_D})$, therefore, x and x^* are unequal: $m_{S,\,D} \neq m^*_{S,\,D}$ such that $x_{m_S} \neq x_{m_D},\ x_{m^*_S} \neq x_{m^*_D},\ x_{m_{S,\,D}} \neq -x_{m^*_{S,\,D}},\ x^*_{m^*_S} \neq x^*_{m^*_D},\ x^*_{m_S} \neq x^*_{m_D}$ and $x^*_{m^*_{S,\,D}} \neq -x^*_{m_{S,\,D}}$; $x = f(\overset{+}{m_S},\ \overset{+}{m_D},\ \overset{-}{m^*_S},\ \overset{-}{m^*_D})$ and $x^* = f(\overset{+}{m^*_S},\ \overset{+}{m^*_D},\ \overset{-}{m_S},\ \overset{-}{m_D}) \vdash x \neq x^*$. Thus, $S \wedge T \longrightarrow I$.

Lemma 3.7.2 (s.q.n. autarky; contradiction) Assume trade T, which means that $x = f(\overset{+}{m_S},\ \overset{+}{m_D},\ \overset{-}{m^*_S},\ \overset{-}{m^*_D})$ and $x^* = f(\overset{+}{m^*_S},\ \overset{+}{m^*_D},\ \overset{-}{m_S},\ \overset{-}{m_D})$; sovereignty S means that $m_{S,\,D} \neq m^*_{S,\,D}$, therefore, x and x^* are unequal, contradicting efficiency E, which means that $x = x^* :\ x = f(\overset{+}{m_S},\ \overset{+}{m_D},\ \overset{-}{m^*_S},\ \overset{-}{m^*_D})$ and $x^* = f(\overset{+}{m^*_S},\ \overset{+}{m^*_D},\ \overset{-}{m_S},\ \overset{-}{m_D})$; $m_{S,\,D} \neq m^*_{S,\,D}$ such that $x_{m_S} \neq x_{m_D},\ x_{m^*_S} \neq x_{m^*_D},\ x_{m_{S,\,D}} \neq -x_{m^*_{S,\,D}},\ x^*_{m^*_S} \neq x^*_{m^*_D},\ x^*_{m_S} \neq x^*_{m_D}$ and $x^*_{m^*_{S,\,D}} \neq -x^*_{m_{S,\,D}} \vdash x \neq x^*$, but $x = x^* \vdash x \neq f(\overset{+}{m_S},\ \overset{+}{m_D},\ \overset{-}{m^*_S},\ \overset{-}{m^*_D})$ and $x^* \neq f(\overset{+}{m^*_S},\ \overset{+}{m^*_D},\ \overset{-}{m_S},\ \overset{-}{m_D})$ such that $x = x^* = 0$. Thus, $S \wedge E \longrightarrow A$.

Lemma 3.7.3 (s.q.n. subjection; contradiction) Assume sovereignty S, which means that $m_{S,\,D} \neq m^*_{S,\,D}$; trade T means that $x = f(\overset{+}{m_S},\ \overset{+}{m_D},\ \overset{-}{m^*_S},\ \overset{-}{m^*_D})$ and $x^* = f(\overset{+}{m^*_S},\ \overset{+}{m^*_D},\ \overset{-}{m_S},\ \overset{-}{m_D})$, therefore, x and x^* are unequal, contradicting efficiency E, which means that $x = x^* :\ m_{S,\,D} \neq m^*_{S,\,D}$ such that $x_{m_S} \neq x_{m_D},\ x_{m^*_S} \neq x_{m^*_D},\ x_{m_{S,\,D}} \neq -x_{m^*_{S,\,D}},\ x^*_{m^*_S} \neq x^*_{m^*_D},\ x^*_{m_S} \neq x^*_{m_D}$ and $x^*_{m^*_{S,\,D}} \neq -x^*_{m_{S,\,D}}$; $x = f(\overset{+}{m_S},\ \overset{+}{m_D},\ \overset{-}{m^*_S},\ \overset{-}{m^*_D})$ and $x^* = f(\overset{+}{m^*_S},\ \overset{+}{m^*_D},\ \overset{-}{m_S},\ \overset{-}{m_D}) \vdash x \neq x^*$, but $x = x^* \vdash m_{S,\,D} = m^*_{S,\,D}$. Thus, $T \wedge E \longrightarrow SB$. *QED*

Out of sovereignty S, trade T and efficiency E at most two can be selected. Sovereignty and trade $S \wedge T$ mean that domestic real money supply m_S does not equal foreign real money supply m^*_S or that domestic money demand m_D does not equal foreign money demand m^*_D, whereby the effects on domestic and foreign output x and x^* do not neutralise each other, that domestic and foreign output x and x^* are functions of domestic and foreign real money supply m_S and m^*_S and of domestic and foreign money demand m_D and m^*_D and that domestic output x does not thus equal foreign output x^* : sovereignty and trade $S \wedge T$ are sufficient for inefficiency I.

Sovereignty and efficiency $S \wedge E$ mean that if trade T were active then domestic and foreign output x and x^* would be functions of domestic and foreign real money supply m_S and m^*_S and of domestic and foreign money demand m_D and m^*_D, that domestic real money supply m_S would not equal foreign real money supply m^*_S or that domestic money demand m_D would not equal foreign money demand m^*_D, whereby the effects on domestic and foreign output x and x^* would not neutralise each other, and that domestic output x would not thus equal foreign output x^* : sovereignty and efficiency $S \wedge E$ are sufficient for autarky A.

Trade and efficiency $T \wedge E$ mean that if sovereignty S were active then domestic real money supply m_S

would not equal foreign real money supply m_S^* or domestic money demand m_D would not equal foreign money demand m_D^*, whereby the effects on domestic and foreign output x and x^* would not neutralise each other, that domestic and foreign output x and x^* would be functions of domestic and foreign real money supply m_S and m_S^* and of domestic and foreign money demand m_D and m_D^* and that domestic output x would not thus equal foreign output x^* : trade and efficiency $T \wedge E$ are sufficient for subjection SB.

3.8 Current account combinations. More predicate calculus can reveal that of eight current account combinations (i.e. of sovereignty S, trade T and efficiency E) the feasible ones are the following:

1. $S \wedge A \wedge E$ (efficient sovereign autarky);
2. $S \wedge T \wedge I$ (inefficient sovereign trade);
3. $SB \wedge T \wedge E$ (efficient subjected trade);
4. $SB \wedge A \wedge E$ (efficient subjected autarky);

the infeasible combinations are $S \wedge T \wedge E$ (efficient sovereign trade), as seen above, $S \wedge A \wedge I$ (inefficient sovereign autarky), $SB \wedge T \wedge I$ (inefficient subjected trade) and $SB \wedge A \wedge I$ (inefficient subjected autarky). Combinations $S \wedge A \wedge I$ and $SB \wedge A \wedge I$ are infeasible because autarky A and inefficiency I are contradictory, inter-regional output equalling zero by the definition of autarky A and thereby meeting the definition of efficiency E : (i) $m_{S,\,D} \neq m^*_{S,\,D}$, (ii) $m_{S,\,D} = m^*_{S,\,D}$ and $x = x^* = 0 \vdash x = x^*$. Combination $SB \wedge T \wedge I$ is infeasible because under trade T subjection SB preserves inter-regional real money supply and money demand equality, thereby meeting the definition of efficiency E : $m_{S,\,D} = m^*_{S,\,D},\ x = f(\overset{+}{m_S},\ \overset{+}{m_D},\ \overset{-}{m_S^*},\ \overset{-}{m_D^*})$ and $x^* = f(\overset{+}{m_S^*},\ \overset{+}{m_D^*},\ \overset{-}{m_S},\ \overset{-}{m_D}) \vdash x = x^*$.

The first converse of the Globalisation paradox is additionally proven thereby: $I \longrightarrow S \wedge T = \neg(I \wedge SB \vee A)$, admitting $I \wedge SB \wedge A,\ I \wedge SB \wedge T$ and $I \wedge S \wedge A$, which are all false. The last two converses of the Globalisation paradox are similarly disproven thereby: $A \longrightarrow S \wedge E = \neg(A \wedge SB \vee I)$, admitting $A \wedge SB \wedge I,\ A \wedge S \wedge I$ and $A \wedge SB \wedge E$, which is true, thus, $A \not\longrightarrow S \wedge E;\ SB \longrightarrow T \wedge E = \neg(SB \wedge A \vee I)$, admitting $SB \wedge A \wedge I,\ SB \wedge T \wedge I$ and $SB \wedge A \wedge E$, which is true, thus, $SB \not\longrightarrow T \wedge E$.

Proposition 3.9 (Sovereignty and trade implosion) *Let domestic and foreign output be a real sequence:* $\{x,\ x^*\} \subset \mathbb{R}$. *Assume the existence of a non-negative lower bound on inter-regional output at which autarky happens:* $x,\ x^* > \tilde{x} \geq 0$: *A. Sovereignty and trade imply that the sequence of output converges to the output lower bound. Formally:*

$$S \wedge T \longrightarrow \lim_{m_{S,\,D} \to \infty} x^*_{m_{S,\,D}} = \tilde{x}.$$

Proof. By sovereignty S domestic real money supply m_S or domestic money demand m_D can respectively exceed foreign real money supply m_S^* or foreign money demand m_D^* and by trade T domestic output x can consequently exceed foreign output x^* : $m_{S,\,D} > m^*_{S,\,D},\ x = f(\overset{+}{m_S},\ \overset{+}{m_D},\ \overset{-}{m_S^*},\ \overset{-}{m_D^*})$ and $x^* = f(\overset{+}{m_S^*},\ \overset{+}{m_D^*},\ \overset{-}{m_S},\ \overset{-}{m_D}) \vdash x > x^*$. More clearly, a change in domestic output x given a change in domestic real money supply m_S or domestic money demand m_D is positive and a change in foreign output x^* given a change in domestic real money supply m_S or domestic money demand m_D is negative: $x_{m_{S,\,D}} > 0$ and $x^*_{m_{S,\,D}} < 0$. Thus, for some large enough domestic real money supply m_S or domestic money demand m_D foreign output x^* is arbitrarily close to output lower bound $\tilde{x}$: $||x^*_{m_{S,\,D}} - \tilde{x}|| < ||x_{m_{S,\,D}} - \tilde{x}|| = \varepsilon > 0,\ \exists m_{S,\,D} = \bar{m} \in \mathbb{R}_{++}$. Such means that $\lim_{m_{S,\,D} \to \infty} x^*_{m_{S,\,D}} = \tilde{x}$. *QED*

Domestic output x and foreign output x^* form a real sequence. There also exists a non-negative output lower bound at which autarky A happens, $\tilde{x}$. Sovereignty and trade $S \wedge T$ mean that domestic real money supply m_S or domestic money demand m_D can be respectively greater than foreign real money supply m_S^* or foreign money demand m_D^*, that domestic output x and foreign output x^* respectively increase and decrease therein, that domestic output x can thus be greater than foreign output x^* and that, for some large enough domestic real money supply m_S or domestic money demand m_D, the normed distance between foreign output x^* and output lower bound $\tilde{x}$ is therefore arbitrarily small, that is, the normed

distance between foreign output x^* and output lower bound $\tilde{x}$ is therefore smaller than the normed distance between domestic output x and output lower bound $\tilde{x}$: $d(x^*_{m_{S,\,D}},\ \tilde{x}) \to 0$ as $m_{S,\,D} \to \infty$ or $x^*_{m_{S,\,D}} \to \tilde{x}$ as $m_{S,\,D} \to \infty$.

Given a change in foreign real money supply m^*_S or foreign money demand m^*_D the proposition can be alternatively formulated in terms of domestic output x diminution and foreign output x^* augmentation.

Under autarky A domestic output x and foreign output x^* equal zero, but output lower bound $\tilde{x}$, at which autarky A happens, does not have to be zero because the implosion caused by sovereignty and trade $S \wedge T$ can curtail trade T before, discretionally. Indeed, trade T is necessarily curtailed (i.e. output lower bound $\tilde{x}$, whereat autarky A occurs, is attained to) once domestic capital k, labour l and savings sv have been depleted and foreign credit is no longer available: $w,\ rk = f(\bar{\dot{x}},\ \ldots)$ and $sv = f(\bar{\dot{x}^*},\ \ldots)$ such that $0 = f(0,\ \ldots)$ (see subsection 3.11 for price endogeneity in output).

3.10 Money market and prices. Under trade T an increment in domestic output x is caused by an increment in domestic real money supply m_S or in domestic money demand m_D : the increment in nominal real money supply M_S or the decrement in domestic prices p increases domestic real money supply m_S, decreases domestic real interest rate r, increases domestic capital k demand and increases domestic output x; the increment in domestic money demand m_D increases domestic labour l demand and increases domestic output x.

However, the respective increments in demand for domestic capital k and for domestic labour l increase domestic prices p, decrease domestic real money supply m_S and decrease domestic output x. The initial decrement in domestic prices p happens through a decrement in domestic autonomous prices ap and the initial increment in domestic money demand m_D happens through an increment in domestic autonomous money demand am_D. A permanent decrement in domestic autonomous prices ap also increases domestic money demand m_D, increases demand for domestic labour l (which increases domestic prices p, decreases domestic real money supply m_S and decreases domestic output x) and increases domestic output x.

Domestic prices p therefore increase in domestic autonomous prices ap, changed domestic nominal money supply $\dot{M}_S$ and changed domestic autonomous money demand $a\dot{m}_D$ and they decrease in changed domestic autonomous prices $a\dot{p}$, anew: $p = f(\overset{+}{a\dot{p}},\ \overset{+}{\dot{M}_S},\ \overset{+}{a\dot{m}_D},\ \overset{-}{a\dot{p}})$. Domestic money demand m_D correspondingly increases in domestic autonomous money demand am_D and it decreases in domestic autonomous prices ap, anew: $m_D = f(\overset{+}{am_D},\ \overset{-}{ap})$.

If domestic prices p adjust actively (i.e. they are flexible) then domestic output x does not change, save for temporary and permanent changes in domestic autonomous prices ap : $x_{m_S} m_{S_{M_S}} + x_p p_{\dot{M}_S} = 0$ and $x_{m_D} m_{D_{am_D}} + x_p p_{a\dot{m}_D} = 0$ (i.e. demand shocks, to the nominal money supply and money demand, temporary or permanent); $x_p p_{ap} + x_p p_{a\dot{p}} < 0$ and $x_p p_{ap} + x_{m_D} m_{D_{ap}} + x_p p_{a\dot{p}} < 0$ (i.e. supply shock, to prices, temporary and permanent). If domestic prices p adjust sluggishly (i.e. they are rigid) then domestic output x changes: $x_{m_S} m_{S_{M_S}} + x_p p_{\dot{M}_S} > 0$ and $x_{m_D} m_{D_{am_D}} + x_p p_{a\dot{m}_D} > 0$ (i.e. demand shocks, to the nominal money supply and money demand, temporary or permanent, to which supply eventually adjusts); $x_p p_{ap} + x_p p_{a\dot{p}} < 0$ and $x_p p_{ap} + x_{m_D} m_{D_{ap}} + x_p p_{a\dot{p}} < 0$ (i.e. supply shock, to prices, temporary and permanent). Clearly, in the face of both rigid and flexible adjustments in domestic prices p a permanent change in domestic autonomous prices ap ultimately generates a lower domestic output x than a temporary change in domestic autonomous prices ap : $(x_p p_{ap} + x_{m_D} m_{D_{ap}} + x_p p_{a\dot{p}}) < (x_p p_{ap} + x_p p_{a\dot{p}}) < 0$.

3.11 Endogenous prices. Be that as it may, the insight of the Gold standard, the Second industrial revolution and the Great divergence is that domestic output x increases permanently: Kaldor conjectured that such might happen through domestic prices p endogeneity in domestic output x, that is, in domestic supply s (i.e. technology) or domestic wages w : domestic productivity increases or foreign labour l^* supply immigrates into domestic labour l supply (i.e. galvanisation or immigration; see Petrus Johannes Verdoorn [18] and Anthony Thirlwall [16] for the first).

Regardless of whether one work in growth rates or levels, under non-linearity endogeneity of domestic prices p in domestic output x can determine increasing inter-regional output growth differences and under linearity it can determine constant inter-regional output growth differences: the condition for such an instability under non-linearity is the decrease and convexity of domestic prices p in changed domestic output $\dot{x}$ and under linearity it is the decrease of domestic prices p in changed domestic output $\dot{x}$. Thus,

in order for domestic output x to continue growing domestic prices p must additionally be a decreasing and convex function of changed domestic output $\dot{x}$ (i.e. increasing returns): $p = f(\overset{+}{\dot{ap}},\ \overset{+}{\dot{M}_S},\ \overset{+}{\dot{am}_D},\ \overset{-}{\dot{ap}},\ \overset{-+}{\dot{x}})$; under linearity convexity is preserved, although unnecessarily, since a linear function is both convex and concave.

Irrespective of the curvature in inter-regional output growth (i.e. increasing or constant), endogeneity of domestic prices p in domestic output x driven by domestic supply s or domestic wages w determines bilateral inter-regional output growth differences. Unilateral ones would be displayed by endogeneity of domestic prices p in domestic output x driven by domestic supply s in the absence of domestic real money supply m_S and domestic money demand m_D effects on foreign output x^* and *vice versa;* endogeneity of domestic prices p in domestic output x driven by domestic wages w would nonetheless still determine bilateral inter-regional output growth differences, for the inter-regional labour supply would yet be redistributed. Indeed, if the modulus decrease of domestic prices p in changed domestic output $\dot{x}$, through domestic wages w, is exceeded by the increase of foreign prices p^* in changed foreign output $\dot{x}^*$, through foreign wages w^*, then, as part of the foreign labour l^* force relocates domestically, part of the inter-regional labour supply is diluted (i.e. found idle), modelling structural unemployment: $|p_{\dot{x}}| < p^*_{-\dot{x}^*}$; cyclical unemployment by contrast concerns internal output dynamics.

Domestic prices p endogeneity in domestic output x thus causes permanent increments in domestic output x through temporary and permanent changes in domestic nominal money supply M_S and domestic autonomous money demand am_D in the face of sluggish domestic prices p and through temporary and permanent changes in domestic autonomous prices ap in the face of sluggish and active domestic prices $p:\ x_{M_S}+x_p p_{\dot{M}_S}+x_p p_{\dot{x}} > 0$ and $x_{m_D} m_{D_{am_D}}+x_p p_{\dot{am}_D}+x_p p_{\dot{x}} > 0$ (i.e. demand shocks, to the nominal money supply and money demand, temporary or permanent, under sluggish prices); $x_p p_{ap} + x_p p_{\dot{ap}} + x_p p_{\dot{x}} < 0$ (i.e. temporary supply shock, to prices, under sluggish and active prices); $x_p p_{ap} + x_{m_D} m_{D_{ap}} + x_p p_{\dot{ap}} + x_p p_{\dot{x}} < 0$ (i.e. permanent supply shock, to prices, under sluggish and active prices).

3.12 Neutralisation. Neutralisation is the equality between a change in cumulative domestic output x given a change in domestic nominal money supply M_S and a change in domestic output x given (i) a change in domestic autonomous money demand am_D or (ii) a negative change in domestic autonomous prices $ap:\ x_{M_S} + x_p p_{\dot{M}_S} + x_p p_{\dot{x}} = x_{m_D} m_{D_{am_D}}$ or $x_{M_S} + x_p p_{\dot{M}_S} + x_p p_{\dot{x}} = -x_p p_{ap}$. Indeed, neutralisation is a situation of trade and efficiency $T \wedge E$ and therefore of subjection SB; in more detail, (i) the change in domestic autonomous prices ap offsets that in domestic nominal money supply M_S and thereby leaves domestic real money supply m_S unchanged and equal to foreign real money supply m^*_S, inter-regional money demand remaining constant, fulfilling subjection SB, (ii) the change in domestic autonomous money demand am_D offsets that in domestic nominal money supply M_S and thereby violates sovereignty S in that the effects of domestic real money supply m_S and domestic money demand m_D on domestic output x neutralise each other: (i) $x_{M_S} + x_p p_{\dot{M}_S} + x_p p_{\dot{x}} = -x_p p_{ap}$ such that $m_S = m^*_S$, provided $\bar{m}_D = \bar{m}^*_D$; (ii) $x_{M_S} + x_p p_{\dot{M}_S} + x_p p_{\dot{x}} = x_{m_D} m_{D_{am_D}}$.

An offsetting change in domestic autonomous money demand am_D resembles autarky A because it would be caused by an offsetting change in domestic output demand d_x or domestic demand taxation t_D : domestic autonomous output demand ad_x, foreign autonomous output demand ad^*_x or domestic supply or demand subsidy s_S or s_D would increase or domestic autonomous supply or demand taxation at_S or at_D or foreign tariffs tf^* would decrease, nevertheless, trade T would still be in theoretical place.

Sovereignty and trade $S \wedge T$ imply inefficiency I, thus, trade and efficiency $T \wedge E$ can thence be achieved (if and) only if the total change in domestic output x given a change in domestic nominal money supply M_S, that is, the change in domestic output x passing through domestic prices p readjustment and endogeneity, is offset by a negative change in domestic autonomous money demand am_D or a change in domestic autonomous prices ap. In the face of trade T an opposite change in domestic nominal money supply M_S would neutralise inefficiency I through subjection SB, for domestic real money supply m_S would then again equal foreign real money supply m^*_S, all else equal, thus, trade and efficiency $T \wedge E$ would lack sovereignty S. In detail, domestic prices p readjustment and endogeneity would be reduced to zero: $x_{M_S} + x_p p_{\dot{M}_S} + x_p p_{\dot{x}} = x_{M_S} \longleftrightarrow x_p p_{\dot{M}_S} + x_p p_{\dot{x}} = 0$.

Neutralisation can also be defined such that the initial change in domestic output x is driven, not by domestic nominal money supply M_S, but by domestic autonomous prices ap or domestic autonomous

money demand am_D and offset accordingly.

4. Inefficiency model

4.1 Dynamical system. One now models a permanent augmentation in domestic output x through a dynamical system for which one determines an instability condition (with regard to domestic output x).

The dynamical system is a six equation model of the above functions: the first equation is for domestic output x, which omits foreign real money supply m_S^* and foreign money demand m_D^*, studying sole domestic shocks; the second equation is for domestic money demand m_D; the third equation is for domestic prices p; the fourth, fifth and sixth are for domestic nominal money supply M_S, domestic autonomous money demand am_D and domestic autonomous prices ap. It is the following:

$$
\begin{aligned}
&x = \alpha_x M_S - \beta_x p + \gamma_x m_D \text{ (domestic output)};\\
&m_D = \alpha_{m_D} am_D - \beta_{m_D} ap \text{ (domestic money demand)};\\
&p = \alpha_p ap + \beta_p \dot{M}_S + \gamma_p \dot{am}_D - \delta_p \dot{ap} - \epsilon_p \dot{x} \text{ (domestic prices)};\\
&M_S = \sigma_{M_S} \varepsilon_{M_S} t \text{ (domestic nominal money supply)};\\
&am_D = \sigma_{am_D} \varepsilon_{am_D} t \text{ (domestic autonomous money demand)};\\
&ap = \mu_{ap} t + \sigma_{ap} \varepsilon_{ap} t \text{ (domestic autonomous prices)},
\end{aligned}
$$

where domestic prices arithmetic mean μ_{ap} models the drift of the mnemonic process (i.e. random walk) of domestic prices p and where generic white noise $\varepsilon \sim \mathcal{N}(0,\ \sigma^2)$, that is, normally distributed with a mean of zero and a finite variance, models the shock of the mnemonic process. One notices that changed domestic nominal money supply $\dot{M}_S = \sigma_{M_S} \varepsilon_{M_S}$, changed domestic autonomous money demand $\dot{am}_D = \sigma_{am_D} \varepsilon_{am_D}$ and changed domestic autonomous prices $\dot{ap} = \mu_{ap} + \sigma_{ap} \varepsilon_{ap}$.

4.2 Parametrisation. The usage of growth rates in relation to endogenous variables allows to treat parameters as elasticity measures.

Generic parameter set $\{\alpha,\ \beta,\ \gamma,\ \delta\} = \Theta \subset \mathbb{R}_+$. In the first equation the sum of parameters α_x, β_x and γ_x amounts to one. In the second equation the sum of parameters α_{m_D} and β_{m_D} exceeds one. In the third equation the sum of parameters α_p, β_p, γ_p and δ_p exceeds one; parameter $\epsilon_p \in (1,\ \infty)$, modelling domestic prices p decrease and convexity (which is linearly irrelevant) in domestic output x. The first equation's parametrisation is coherently based on a constant elasticity of substitution function of imperfect complements (i.e. Cobb Douglas), that is, equally weighting money demand and supply and the nominal money supply and prices therein; the parametrisation of the second and of the third equation arises from the parametric restrictions proceeding from the treated economic theory (see subsections 3.10-12) in conjunction with the parametrisation of the first equation.

4.3 Inefficiency. The present dynamical system can be consequently termed an inefficiency model. The inefficiency model tautologically features trade T in the non-zero modelling of domestic output x and it features sovereignty S in view of the exogenous shocks to domestic nominal money supply M_S, domestic autonomous money demand am_D or domestic autonomous prices ap. The inefficiency model additionally features rigidity R, because it models domestic nominal money supply M_S as a mnemonic process, as opposed to an amnesic one (i.e. autoregressive process of order one), whereof managed floats or crawling pegs would be examples (flexibility F would instead feature a mnemonic process absent an effected shock); it does not speak to openness O and independence I because it does not model the internal sector.

Since domestic nominal money supply M_S, domestic autonomous money demand am_D and domestic autonomous prices ap are exogenous and one does not momentarily carry out policy analysis, that is, one only determines a condition for instability at present, one can omit the last three equations; one can then substitute the equations for domestic money demand m_D and for domestic prices p in the equation for domestic output x, solve the homogeneous differential equation for domestic output x, by finding the roots and by treating the rest of the equation as the heterogeneous component, and determine the instability condition for domestic output x, by noting that the eigenvalues are the inverse of the roots:

$$x = \alpha_x M_S - \beta_x[\alpha_p ap + \beta_p \dot{M}_S + \gamma_p a\dot{m}_D - \delta_p a\dot{p} - \epsilon_p \dot{x}] + \gamma_x[\alpha_{m_D} am_D - \beta_{m_D} ap] =$$
$$= \beta_x \epsilon_p \dot{x} + \kappa \text{ (domestic output heterogeneous differential equation)}$$
$$\longrightarrow x = \beta_x \epsilon_p \dot{x} \text{ (homogeneous component)}$$
$$\longrightarrow e^{zt} - z\beta_x \epsilon_p e^{zt} = 0 \longrightarrow 1 - z\beta_x \epsilon_p = 0 \longrightarrow z = \frac{1}{\beta_x \epsilon_p} \text{ and } \lambda = \frac{1}{z} = \beta_x \epsilon_p,$$

where $\{z_i\}_{i=1}^n \subset \mathbb{C}$. In continuous time, $t \in \mathbb{R}$, the condition for instability in domestic output x is $\lambda = \beta_x \epsilon_p > 0$. Since data are discrete the dynamical system and the condition for instability in domestic output x can be contemplated in discrete time, $t \in \mathbb{Z}$, where endogenous variables $v \mapsto v_t$ and changed endogenous variables $\dot{v} \mapsto v_{t-1}$, where variance weighted white noise processes become $v_t = v_{t-1} + \sigma_v \varepsilon_{vt}$ and where the mnemonic process with drift becomes $v_t = \mu_v + v_{t-1} + \sigma_v \varepsilon_{vt} : |\lambda| = \beta_x \epsilon_p > 1$. Endogenous variable transformations $v \mapsto v_t$ and $\dot{v} \mapsto v_{t-1}$ are certainly not according to a discretisation of the attendant continuous time state space, but they do satisfy the treated economic theory and thereby justify a discrete time state space of its own. In detail, the relationship between endogenous variables and changed endogenous variables in continuous time can be hereby understood, without loss of generality, as that between present endogenous variables and past endogenous variables in discrete time, whereby the change has already occurred. Consequently, because parameter $\beta_x \in [0,\ 1]$ (by the Cobb Douglas parametrisation) one notes the following restrictions: if $\beta_x = 0$ then $\frac{1}{\beta_x}$ is a contradiction and if $\beta_x = 1$ then $\epsilon_p > \frac{1}{\beta_x} = 1$, thus, $\beta_x \in (0,\ 1]$ and $\epsilon_p \in (1,\ \infty)$.

Cumulative domestic output x changes given changes in domestic nominal money supply M_S, domestic autonomous money demand m_D and autonomous prices ap determine the following parametric restrictions: $x_{M_S} + x_p p_{\dot{M}_S} + x_p p_{\dot{x}} = \alpha_x - \beta_x \beta_p + \beta_x \epsilon_p > 0$ and $x_{m_D} m_{D_{am_D}} + x_p p_{a\dot{m}_D} + x_p p_{\dot{x}} = \gamma_x \alpha_{m_D} - \beta_x \gamma_p + \beta_x \epsilon_p > 0$ (i.e. demand shocks, to the nominal money supply and money demand, temporary or permanent, under sluggish prices); $x_p p_{ap} + x_p p_{a\dot{p}} + x_p p_{\dot{x}} = -\beta_x \alpha_p + \beta_x \delta_p + \beta_x \epsilon_p < 0$ (i.e. temporary supply shock, to prices, under sluggish and active prices); $x_p p_{ap} + x_{m_D} m_{D_{ap}} + x_p p_{a\dot{p}} + x_p p_{\dot{x}} = -\beta_x \alpha_p - \gamma_x \beta_{m_D} + \beta_x \delta_p + \beta_x \epsilon_p < 0$ (i.e. permanent supply shock, to prices, under sluggish and active prices). Neutralisation determines the ulterior parametric restriction $x_{M_S} + x_p p_{\dot{M}_S} + x_p p_{\dot{x}} = \alpha_x - \beta_x \beta_p + \beta_x \epsilon_p = x_{m_D} m_{D_{am_D}} = \gamma_x \alpha_{m_D}$ or $x_{M_S} + x_p p_{\dot{M}_S} + x_p p_{\dot{x}} = \alpha_x - \beta_x \beta_p + \beta_x \epsilon_p = -x_p p_{ap} = \beta_x \alpha_p$.

4.4 Parameters. In the first equation domestic nominal money supply elasticity of domestic output α_x and domestic price elasticity of domestic output β_x both equal 0.25 and domestic money demand elasticity of domestic output γ_x equals 0.5. In the second equation domestic autonomous money demand elasticity of domestic money demand α_{m_D} and domestic autonomous price elasticity of domestic money demand β_{m_D} both equal 2.0025. In the third equation domestic autonomous price elasticity of domestic prices α_p equals 4.01, changed domestic nominal money supply elasticity of domestic prices β_p equals 1, changed domestic autonomous money demand elasticity of domestic prices γ_p equals 4.005; changed domestic autonomous price elasticity of domestic prices δ_p equals 8.0149 and changed domestic output elasticity of domestic prices ϵ_p equals 4.01. In the sixth equation drift μ_{ap} equals 0.005, quarterly tracing a two per cent annual technology growth rate (i.e. $0.005 \cdot 4 = 0.02$); in the last three equations white noise standard deviations σ equal 0.01, suggesting a one per cent quarterly standard deviation shock.

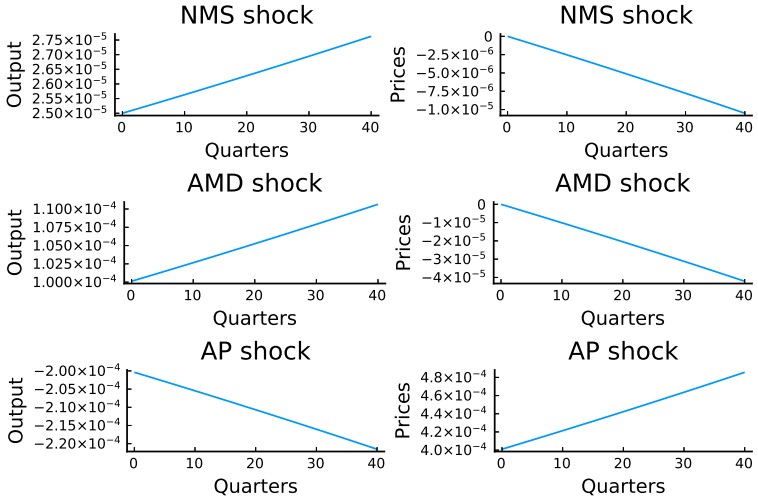
The first equation's parametrisation assigns equal shares to domestic real money supply m_S and domestic money demand m_D and to domestic nominal money supply M_S and domestic prices p within domestic real money supply m_S. Changed domestic output elasticity of domestic prices ϵ_p equals 4.01 because it minimally exceeds $\frac{1}{\beta_x} = \frac{1}{0.25} = 4$. All other parameters are calculated on account of trade and efficiency $T \wedge E$, relying on the notions of readjustment and neutralisation via the above parametric restrictions. Changed domestic nominal money supply elasticity of domestic prices β_p is thus derived from the readjustment condition $x_{M_S} + x_p p_{\dot{M}_S} = \alpha_x - \beta_x \beta_p = 0.25 - 0.25\beta_p = 0$. Domestic autonomous money demand elasticity of domestic money demand α_{m_D} is derived from the neutralisation condition $x_{M_S} + x_p p_{\dot{M}_S} + x_p p_{\dot{x}} - x_{m_D} m_{D_{am_D}} = \alpha_x - \beta_x \beta_p + \beta_x \epsilon_p - \gamma_x \alpha_{m_D} = 0.25 + 0.25(4.01 - 1) - 0.5\alpha_{m_D} = 0$. Domestic autonomous price elasticity of domestic money demand β_{m_D} is set in symmetry to domestic autonomous money demand elasticity of domestic money demand α_{m_D}. Domestic autonomous price elasticity of domestic

prices α_p is derived from the neutralisation condition $x_{M_S}+x_p p_{\dot{M}_S}+x_p p_{\dot{x}}+x_p p_{ap} = \alpha_x-\beta_x\beta_p+\beta_x\epsilon_p-\beta_x\alpha_p = 0.25+0.25(4.01-1-\alpha_p) = 0$. Changed domestic autonomous money demand elasticity of domestic prices γ_p is derived from the readjustment condition $x_{m_D} m_{D_{am_D}}+x_p p_{\dot{am}_D} = \gamma_x\alpha_{m_D}-\beta_x\gamma_p = 0.5\cdot 2.0025-0.25\gamma_p = 0$. Changed domestic autonomous price elasticity of domestic prices δ_p is derived from the readjustment condition $x_p p_{ap}+x_{m_D} m_{D_{ap}}+x_p p_{\dot{ap}} = -\beta_x\alpha_p-\gamma_x\beta_{m_D}+\beta_x\delta_p = 0.25(\delta_p-4.01)-0.5\cdot 2.0025 < 0$, namely, δ_p is minimally exceeded by 8.015.

4.5 State space. The discretised and parametrised dynamical system can therefore be vectorially written as $x_t = \mu+Ax_{t-1}+B\varepsilon_t$, so that endogenous variable vector $x_t \in \mathbb{R}^{6\times 1}$ and x_{-1} be not contemplated, drift intercept vector $\mu \in \mathbb{R}^{6\times 1}$, companion matrix $A \in \mathbb{R}^{6\times 6}$ and exogenous shock matrix $B \in \mathbb{R}^{6\times 3}$. Abstracting from drifts, period zero's impulse response function matrix to some exogenous shock is therefore $x_0 = B\varepsilon_0$, where $\varepsilon_0 = \sigma$ is to rescale; consequently, the jth period impulse response function matrix to the same exogenous shock is $x_j = A^j B\varepsilon_0 = A^j B\sigma$.

In view of the above parametrisation below is a plot of domestic output x and domestic prices p impulse response functions to exogenous shocks in domestic nominal money supply M_S, domestic autonomous money demand am_D and domestic autonomous prices ap at a forty quarter period horizon.

Figure 1: Impulse response functions



Note. Impulse response functions of domestic output x and domestic prices p to 0.01 standard deviation shocks in domestic nominal money supply M_S, domestic autonomous money demand am_D and domestic autonomous prices ap at a horizon of forty quarters. Sovereignty and trade $S \wedge T$ implosion is envisaged in all three cases: in the third case domestic output x implodes and in the first two cases it explodes.

The impulse response functions envisage sovereignty and trade $S \wedge T$ implosion in all cases: in the first two cases domestic output x increases indefinitely and domestic prices p decrease indefinitely, until the upper bound on inter-regional output $\tilde{x}$ at which autarky happens be reached; in the third case domestic output x decreases indefinitely and domestic prices p increase indefinitely, until the lower bound on inter-regional output $\tilde{x}$ at which autarky happens be reached.

4.6 Industrial revolution. The Gold standard preceded the Second industrial revolution and the Second industrial revolution was such that England became the workshop of the world; shortly after there began the Great divergence, whereby output growth in the West definitively overtook output growth in the East. The inefficiency model justifies the advantage gained by English domestic output x through the fixed superiority of domestic nominal money supply M_S in the Gold standard or through the inferiority in domestic prices p and the lasting rise in English domestic output x through endogeneity of domestic prices p therein, all other domestic and foreign factors equal. In a word, it rationalises the rise of industry and modern sustained growth as inter-regional growth, whereby one region technologically grows at the stagnation or decline of the other, not as intra-regional growth, whereby a region technologically grows irrespective of the others and all regions technologically grow together in potency: the first is a restatement

of mercantilistic growth, of long run demand sustainability, and the second a restatement of libertarian growth, of long run supply sustainability, which are not mutually exclusive.

5. Federalism

Definition 5.1 (Transfers, output with transfers and inter-regional output) *Domestic or foreign transfers equal the arithmetic mean of output across the two regions minus domestic or foreign output, respectively. Domestic or foreign output with transfers equals domestic or foreign output plus domestic or foreign transfers, respectively. Inter-regional output equals domestic output with transfers plus foreign output with transfers: inter-regional output is equal to the sum of regional outputs. Formally:*

$$\begin{aligned}
&tr = \frac{x + x^*}{2} - x,\ tr^* = \frac{x + x^*}{2} - x^*;\\
&\hat{x} = x + tr,\ \hat{x}^* = x^* + tr^*;\\
&\overline{x} = \hat{x} + \hat{x}^* = x + tr + x^* + tr^* = x + x^*.
\end{aligned}$$

Domestic transfers tr are the difference between an average of domestic output x and of foreign output x^* and domestic output x. Foreign transfers tr^* are the difference between an average of domestic output x and of foreign output x^* and foreign output x^*. Domestic output with transfers $\hat{x}$ is the sum of domestic output x and of domestic transfers tr. Foreign output with transfers $\hat{x}^*$ is the sum of foreign output x^* and of foreign transfers tr^*. Inter-regional output $\overline{x}$ is the sum of domestic output with transfers $\hat{x}$ and of foreign output with transfers $\hat{x}^*$: inter-regional output $\overline{x}$ is the sum of domestic output x and of foreign output x^*, for $tr + tr^* = \frac{x+x^*}{2} - x + \frac{x+x^*}{2} - x^* = 0$.

5.2 Federalism and trade. The scope of transfers TR is to calculate an average of domestic and foreign output x and x^* in order to distribute it to each region in a fiscal union or confederation. Trade T is to stop even under federalism F, nonetheless, because even though transfers TR may turn inter-regional trade T into intra-regional trade T the implosion of sovereignty and trade $S \wedge T$ continues at most until the net importer have reached depletion of domestic labour l supply or capital k, barred access to domestic savings sv or foreign credit, and at least until either region decide to stop trade T before.

In period t the net importer would finance domestic imports im through domestic external currency M_E exchanged for foreign external currency M_E^* at domestic nominal exchange rate E, obtained back in part for domestic exports ex : domestic external currency M_E would have been either saved in period $t-1$, privately borrowed from the net exporter (at a risk premium) in period t or duly printed then. Indeed, in the event the net importer did not export to the net exporter in period t the former would have to finance domestic imports im in period $t+1$ either through domestic external currency M_E privately borrowed from the net exporter or through domestic external currency M_E saved in period $t-1$ or printed then: in the absence of either trade T would stop.

More broadly, commodity i is exchanged at the new domestic terms of trade $tot = \frac{Ep_i^*}{p_i}$ in each period t until cessation: trade T stops whenever domestic capital k, supply of domestic labour l and domestic savings sv have been depleted and whenever inter-regional credit be no longer available. Under trade T domestic output x is an imperfect substitute of foreign output x^* at the minimum and a perfect substitute of foreign output x^* at the maximum: such means that, net of foreign credit and savings sv, imports im are financed only through exports ex and that under implosion of sovereignty and trade $S \wedge T$ net importers will ultimately stop importing, all else equal.

Federalism F can be fully understood as follows: a confederation is a fiscal union of nations; a nation is a fiscal union of corporations; a corporation is a fiscal union of enterprises. Therefore, the Globalisation paradox applies to nations as much as it may apply to corporations and it applies to corporations as much as it may apply to enterprises, which means that corporations and enterprises can face sovereignty and trade $S \wedge T$ implosion as much as nations: if a nation features internal inefficient sovereign trade $S \wedge T \wedge I$ then its society faces a trade T imbalance.

5.3 Federalism or autarky. Given the feasible current account combinations, federalism F or autarky A are inevitable. More clearly, any feasible current account combination featuring no trade T features

autarky A by definition; any feasible current account combination featuring inter-regional (i.e. external) transfers TR features external federalism F_E by definition; any feasible current account combination featuring intra-regional (i.e. internal) transfers TR features internal federalism F_I by definition, that is, unitary regions, regardless of autarky A or external federalism F_E. Thus, internal federalism F_I and thereby federalism F are ubiquitous; furthermore, if external federalism F_E were present then federalism F would be present all the more, with or without autarky $A:\ S \wedge A \wedge E \vdash F_I$ and potentially F_E; $S \wedge T \wedge I \vdash F_I$ and potentially F_E; $SB \wedge T \wedge E \vdash F_I$ and potentially F_E; $SB \wedge A \wedge E \vdash F_I$ and potentially F_E. In other words, all feasible current account combinations are internally federal and can additionally be externally federal or autarkic.

The case of inefficient sovereign trade $S \wedge T \wedge I$ is of interest because it is actual. Sovereignty and trade $S \wedge T$ implode and therefrom patently yield either of the following combinations. The first is efficient sovereign autarky $S \wedge A \wedge E$, which features autarky A and internal federalism F_I, with or without external federalism $F_E:\ S \wedge A \wedge E \vdash F_I$ and potentially F_E such that $(S \wedge A \wedge E)_{TR}$. The second is efficient subjected trade $SB \wedge T \wedge E$, which features internal federalism F_I, with or without external federalism $F_E:\ SB \wedge T \wedge E \vdash F_I$ and potentially F_E such that $(SB \wedge T \wedge E)_{TR}$. The third is efficient subjected autarky $SB \wedge A \wedge E$, which features autarky A and internal federalism F_I, with or without external federalism $F_E:\ SB \wedge A \wedge E \vdash F_I$ and potentially F_E such that $(SB \wedge A \wedge E)_{TR}$. Before the implosion of sovereignty and trade $S \wedge T$ the two regions could already enter into a confederation, possibly diverting attention from the said implosion, being there both internal and external federalism F_I and $F_E:\ S \wedge T \wedge I \vdash F_I$ and potentially F_E such that $(S \wedge T \wedge I)_{TR}$.

5.4 Historical combinations. Four feasible current account combinations stand out historically, howbeit: autarkic mercantilism; the Gold standard; globalisation; world socialisation. Autarkic mercantilism is efficient sovereign autarky $S \wedge A \wedge E$ because autarky A and therefore efficiency E are embraced at sovereignty S's inter-regional disparity in the external real money market under closure C (e.g. Navigation acts, 2000s USA and tariffs on Chinese imports).

The Gold standard is inefficient sovereign trade $S \wedge T \wedge I$ because sovereignty S's inter-regional disparity in the external real money market and trade T without neutralisations cause an age of implosion, often gradual, whereby there develops a philia for the net exporter (e.g. Second industrial revolution and English workshop of the world, European monetary union and European Germanisation).

Globalisation is efficient subjected trade $SB \wedge T \wedge E$ because the initial inefficient sovereign trade $S \wedge T \wedge I$, normally started by inter-regional disparity in external currencies and aggravated by inter-regional disparity in autonomous prices, is downwardly nullified by neutralisation's offsetting, eroding changes in domestic autonomous prices *ap* (e.g. European monetary union and labour market precariat, Libertarianism and golden straightjackets). It could be argued that globalisation could also theoretically promote upward nullifications of inter-regional disparities in the external real money market, that is, offsetting, blossoming changes in inter-regional real money supply or money demand, not eroding ones, but contemporary history permits one to record the phenomenon as thus described.

World socialisation is efficient subjected trade $SB \wedge T \wedge E$ as well, absent the prevenient adoption of inefficient sovereign trade $S \wedge T \wedge I$, because regions practise trade T at inter-regional parity in external currency and proscribe almost all changes in inter-regional autonomous prices and autonomous money demand, so that fiscal depreciations, inhuman working conditions, disproportionate territorial expansions, tariffs and protectionistic measures may be conscientiously eluded and innovations in technology altogether mutualised (equitably; more below), in a veritable socialisation of the world.

The most desirable combination after world socialisation is blatantly autarkic mercantilism, but game theory is to ascertain so.

6. Nash equilibria

6.1 Players, strategies and payoffs. A non-cooperative static game is an *n-uple* composed of a set of players, sets of player strategies and a player payoff function. In the present case the set of players has two elements, which are domestic central bank CB and domestic treasury $T:\ I = \{CB,\ T\}$. Domestic central bank CB's strategy set contains Impossible trio combinations rigidity and openness $R \wedge O$, rigidity and independence $R \wedge I$ and openness and independence $O \wedge I$, at positive disparity D_+ or parity P in the external

real money market: $S_{CB} = \{(R \wedge O)_{D_+},\ (R \wedge O)_P,\ (R \wedge I)_{D_+},\ (R \wedge I)_P,\ (O \wedge I)_{D_+},\ (O \wedge I)_P\}$. Domestic treasury T's strategy set contains Globalisation paradox combinations sovereignty and trade $S \wedge T$, sovereignty and efficiency $S \wedge E$ and trade and efficiency $T \wedge E$, at no transfers NTR or transfers TR, in agreement with the foreign region: $S_T = \{(S \wedge T)_{NTR},\ (S \wedge T)_{TR},\ (S \wedge E)_{NTR},\ (S \wedge E)_{TR},\ (T \wedge E)_{NTR},\ (T \wedge E)_{TR}\}$. The strategy profile set is the product of the player strategy sets: $(s_{CB},\ s_T) \in (S_{CB} \times S_T)$. The player payoff function is a bijection of the strategy profile set into the real number set or line: $\pi : (S_{CB} \times S_T) \to \mathbb{R}$.

6.2 Information. In game theory: symmetric information means that payoffs are not subjective; certain information means that player types do not change; complete information means that player types are known to all players; imperfect information means that players cannot observe other player actions. Static games are imperfect information games, because they are simultaneous. One has not specified player types; the game one has constructed consequently presents asymmetric, certain, complete and imperfect information.

6.3 Game equilibria. John Nash [10] used the Kakutani fixed point theorem to prove that every game with finite players and mixed strategies presents an equilibrium. Mixed strategies are continuous probability assignments to pure strategies, consequently, mixed strategies are uncountably infinite and their sets are thereby compact and convex, meeting Nash's use of the Kakutani fixed point theorem for equilibrium existence: $\forall i \in I,\ p : S_i \to [0,\ 1] \subset \mathbb{R}_+$, where p is a probability density function, such that, $\forall j \in [1,\ n] \subset \mathbb{N}_+,\ p(s_{ij}) = p_{ij} \in [0,\ 1] \subset \mathbb{R}_+$ and $\sum_{j=1}^{n} p_{ij} = 1$; $\forall i \in I,\ f : S_i \times [0,\ 1] \to \Sigma_i \subseteq \mathbb{R}_+$, where f is a probability assignment function, such that, $\forall j \in [1,\ n] \subset \mathbb{N}_+,\ f(s_{ij} p_{ij}) = \sigma_{ij} \in \Sigma_i \subseteq \mathbb{R}_+$ and $\sum_{j=1}^{n} s_{ij} p_{ij} = \sigma_i$. Mixed strategies are understood as randomisations over pure strategies. Alternatively, pure strategies are understood as mixed strategies wherein particular pure strategies are played with a probability of one. The mixed strategy sets of domestic central bank CB and domestic treasury T are respectively denoted Σ_{CB} and Σ_T. The mixed strategy game is thus the quadruple $\Gamma_{MX} = \{I,\ \{\Sigma_i\}_{i=CB}^{T},\ \pi\} = \{I,\ \Sigma_{CB},\ \Sigma_T,\ \pi\}$.

A best response function is a bijection of other players $\neg i$'s mixed strategy set into player i's mixed strategy set such that player i's mixed strategy is the best mixed strategy given other players $\neg i$'s mixed strategies, that is, a best response: $\forall i \in I,\ \rho_i : \Sigma_{\neg i} \to \Sigma_i$ such that $\sigma_i^* = \rho_i(\sigma_{\neg i}) = \sum_{j=1}^{n} s_{ij} p_{ij}^*$. A Nash equilibrium is a strategy profile such that its payoff features player i's best response given other players $\neg i$'s best responses; it is thus a strategy profile of matching best responses: $\forall i \in I,\ NE := (\sigma_i^*,\ \sigma_{\neg i}^*)$ such that $\pi(\sigma_i^*,\ \sigma_{\neg i}^*)$. A weak Nash equilibrium is a Nash equilibrium in which player i's best response is one or more: $\forall i \in I,\ NE_{WK} := (\sigma_i^*,\ \sigma_{\neg i}^*)$ such that $\pi_i(\sigma_i^*,\ \sigma_{\neg i}^*) \geq \pi_i(\sigma_i,\ \sigma_{\neg i}^*)$, whereby $\sigma_i^* \neq \sigma_i$ or $\sigma_i^* = \sigma_i$. A strict Nash equilibrium is a Nash equilibrium in which player i's best response is one: $\forall i \in I,\ NE_{ST} := (\sigma_i^*,\ \sigma_{\neg i}^*)$ such that $\pi_i(\sigma_i^*,\ \sigma_{\neg i}^*) > \pi_i(\sigma_i,\ \sigma_{\neg i}^*)$, whereby $\sigma_i^* \neq \sigma_i$.

6.4 Central bank payoffs. One gives rise to domestic central bank CB payoffs as follows. Domestic central bank CB prefers such strategies in a decreasing order: rigidity and independence at parity $(R \wedge I)_P$; openness and independence at parity $(O \wedge I)_P$; openness and independence at positive disparity $(O \wedge I)_{D_+}$; rigidity and independence at positive disparity $(R \wedge I)_{D_+}$; rigidity and openness at parity $(R \wedge O)_P$; rigidity and openness at positive disparity $(R \wedge O)_{D_+}$.

It firstly prefers rigidity and independence $R \wedge I$ to openness and independence $O \wedge I$ under parity P because it desires to inhibit gains in competitiveness through fixed parity in external currency. It then prefers openness and independence $O \wedge I$ to rigidity and independence $R \wedge I$ at positive disparity D_+ because it desires to eliminate gains in competitiveness, only through flexible disparity in external currency, since flexible parity in external currency has already been ranked. It finally prefers rigidity and openness $R \wedge O$ under parity P to rigidity and openness $R \wedge O$ under positive disparity D_+ because it desires to export monetary policy without gains in competitiveness.

Overall, domestic central bank CB prefers to avoid and to eliminate gains in competitiveness to exporting monetary policy. Such implies the cardinal transformation $(R \wedge I)_P \succ (O \wedge I)_P \succ (O \wedge I)_{D_+} \succ (R \wedge I)_{D_+} \succ (R \wedge O)_P \succ (R \wedge O)_{D_+} \mapsto 6 > 5 > 4 > 3 > 2 > 1$.

6.5 Treasury payoffs. One gives rise to domestic treasury T payoffs as follows. Under parity P sovereignty and trade $S \wedge T$ and sovereignty and efficiency $S \wedge E$ are excluded and under positive disparity D_+ trade and efficiency $T \wedge E$ is excluded, thus, domestic treasury T cannot thereby interact.

Under positive disparity D_+ domestic treasury T prefers such strategies in a decreasing order: sovereignty and efficiency at no transfers $(S \wedge E)_{NTR}$; sovereignty and efficiency at transfers $(S \wedge E)_{TR}$; sovereignty and

trade at transfers $(S \wedge T)_{TR}$; sovereignty and trade at no transfers $(S \wedge T)_{NTR}$. It first prefers sovereignty and efficiency at no transfers $(S \wedge E)_{NTR}$ to sovereignty and efficiency at transfers $(S \wedge E)_{TR}$ because it desires to protect domestic enterprises from sovereignty and trade $S \wedge T$ without a fiscal union. It then prefers sovereignty and trade at transfers $(S \wedge T)_{TR}$ to sovereignty and trade at no transfers $(S \wedge T)_{NTR}$ because it desires to mitigate sovereignty and trade $S \wedge T$ implosion through a fiscal union. Such implies the cardinal transformation $(S \wedge E)_{NTR} \succ (S \wedge E)_{TR} \succ (S \wedge T)_{TR} \succ (S \wedge T)_{NTR} \mapsto 4 > 3 > 2 > 1$.

Under parity P domestic treasury T prefers such strategies in a decreasing order: trade and efficiency at no transfers $(T \wedge E)_{NTR}$; trade and efficiency at transfers $(T \wedge E)_{TR}$. It prefers trade and efficiency at no transfers $(T \wedge E)_{NTR}$ to trade and efficiency at transfers $(T \wedge E)_{TR}$ because it desires to avoid a fiscal union. Such implies the cardinal transformation $(T \wedge E)_{NTR} \succ (T \wedge E)_{TR} \mapsto 2 > 1$.

PROPOSITION 6.6 (Pure strategy Nash equilibria) *The game has two pure strategy Nash equilibria: rigidity and independence at parity and trade and efficiency at no transfers* $((R \wedge I)_P,\ (T \wedge E)_{NTR})$; *openness and independence at positive disparity and sovereignty and efficiency at no transfers* $((O \wedge I)_{D_+},\ (S \wedge E)_{NTR})$.

Proof. Domestic central bank CB's best responses are the following. If domestic treasury T selects sovereignty and trade $S \wedge T$ or sovereignty and efficiency $S \wedge E$ at no transfers NTR or transfers TR then domestic central bank CB responds with openness and independence $O \wedge I$ at positive disparity $D_+ :\ s_T = (S \wedge T)_{NTR},\ (S \wedge T)_{TR},\ (S \wedge E)_{NTR},\ (S \wedge E)_{TR} \longrightarrow \rho_{CB}(s_T) = (O \wedge I)_{D_+}$. If domestic treasury T selects trade and efficiency $T \wedge E$ at no transfers NTR or transfers TR then domestic central bank CB responds with rigidity and independence $R \wedge I$ at parity $P :\ s_T = (T \wedge E)_{NTR},\ (T \wedge E)_{TR} \longrightarrow \rho_{CB}(s_T) = (R \wedge I)_P$.

Domestic treasury T's best responses are the following. If domestic central bank CB selects rigidity and openness $R \wedge O$, rigidity and independence $R \wedge I$ or openness and independence $O \wedge I$ at positive disparity D_+ then domestic treasury T responds with sovereignty and efficiency $S \wedge E$ at no transfers $NTR :\ s_{CB} = (R \wedge O)_{D_+},\ (R \wedge I)_{D_+},\ (O \wedge I)_{D_+} \longrightarrow \rho_T(s_{CB}) = (S \wedge E)_{NTR}$. If domestic central bank CB selects rigidity and openness $R \wedge O$, rigidity and independence $R \wedge I$ or openness and independence $O \wedge I$ at parity P then domestic treasury T responds with trade and efficiency $T \wedge E$ at no transfers $NTR :\ s_{CB} = (R \wedge O)_P,\ (R \wedge I)_P,\ (O \wedge I)_P \longrightarrow \rho_T(s_{CB}) = (T \wedge E)_{NTR}$.

The intersection of the best responses is strategy profile: rigidity and independence at parity and trade and efficiency at no transfers $((R \wedge I)_P,\ (T \wedge E)_{NTR})$; openness and independence at positive disparity and sovereignty and efficiency at no transfers $((O \wedge I)_{D_+},\ (S \wedge E)_{NTR})$. *QED*

Table 1: Mercantile dilemma

Central bank		Treasury					
		$S \wedge T$		$S \wedge E$		$T \wedge E$	
		NTR	TR	NTR	TR	NTR	TR
$R \wedge O$	D_+	Gold standard (1, 1)	United states of America (1, 2)	(1, 4)	(1, 3)		
	P					(2, 2)	International clearing union (full) (2, 1)
$R \wedge I$	D_+	Bretton woods (3, 1)	(3, 2)	Navigation acts (3, 4)	(3, 3)		
	P					Artificial currency area (6, 2)*	(6, 1)
$O \wedge I$	D_+	Post-Bretton woods (4, 1)	International clearing union (partial) (4, 2)	Modern protectionism (4, 4)*	(4, 3)		
	P					(5, 2)	Optimal currency area (5, 1)

Note. The mercantile dilemma, evidenced by means of asterisks, arises from the intersection of the capital and current account trichotomy, that is, of Mundell's and Rodrik's trilemma (i.e. the Impossible trio and the Globalisation paradox). For domestic central bank CB combination: $R \wedge O$ is rigidity and openness; $R \wedge I$ is rigidity and independence; $O \wedge I$ is openness and independence; D_+ is positive disparity and P is parity in the external real money market. For domestic treasury T combination: $S \wedge T$ is sovereignty and trade; $S \wedge E$ is sovereignty and efficiency; $T \wedge E$ is trade and efficiency; NTR are no transfers and TR are transfers. Feasible player payoffs are in round brackets; historical cases of certain strategy profiles are displayed. Strategy profile $((R \wedge O)_{D_+},\ (S \wedge T)_{NTR})$ is also the European monetary union and dollarisation; strategy profile $((R \wedge I)_{D_+},\ (S \wedge T)_{NTR})$ is also contemporary China; strategy profiles $((R \wedge O)_P,\ (T \wedge E)_{NTR})$ and $((O \wedge I)_P,\ (T \wedge E)_{NTR})$ are autonomous international clearing unions, in growth, or globalisation, in decline; all strategy profiles presenting $(O \wedge I)_{D_+}$ admit reserve currency acquisitions (for under rigidity at positive disparity R_{D_+} the peg prohibits them); all strategy profiles presenting rigidity R admit managed floats or currency substitutions. The pure strategy Nash equilibria are the Artificial currency area and Modern protectionism, to wit, the mercantile dilemma.

6.7 International clearing union. In 1944 John Maynard Keynes [5] advocated the creation of the International clearing union and of the Bancor currency: the Bancor currency would have been the

currency in which all international trade T would have been conducted and flexible external currencies under openness and independence $O \wedge I$ would have fluctuated against the Bancor; the International clearing union would have transferred the balance of payment excesses from the net exporters to the net importers in *de facto* fiscal unions, through transfers TR. Keynes' International clearing union would have moreover forced external currency realignments, in managed floats, under rigidity and openness $R \wedge O$; it would have thus reached efficiency E through trade and subjection $T \wedge SB$.

In terms of the game one constructed Keynes' proposal without forced realignments in external currency is strategy profile openness and independence at positive disparity and sovereignty and trade at transfers $((O \wedge I)_{D_+},\ (S \wedge T)_{TR})$ and Keynes' proposal with forced realignments in external currency is strategy profile rigidity and openness at parity and trade and efficiency at transfers $((R \wedge O)_P,\ (T \wedge E)_{TR})$, which were both rejected.

6.8 World socialisation and autarkic mercantilism. Keynes' full proposal was a form of *ex post* clearance; the Nash equilibrium rigidity and independence at parity and trade and efficiency at no transfers $((R \wedge I)_P,\ (T \wedge E)_{NTR})$ is a form of *ex ante* clearance, because it anticipates the balance of payment mismatches through fixed parity in external currencies, by means of inter-regional external real money market equality and technology communism (suitably covering the opportunity costs of research and development, conducted centrally, in principle), under rigidity R, independence I trade T and efficiency E, without a fiscal union: world socialisation, a most optimal currency area.

The Nash equilibrium openness and independence at positive disparity and sovereignty and efficiency at no transfers $((O \wedge I)_{D_+},\ (S \wedge E)_{NTR})$ finally shows that autarkic mercantilism is a second best to world socialisation, whereby the balance of payment mismatches leading to sovereignty and trade $S \wedge T$ implosion are avoided through trade T curtailment.

The mercantile dilemma is therefore the dichotomy between the two pure strategy Nash equilibria, artificial currency areas and modern protectionism, excluding globalisation indeed.

7. Literature critique

7.1 Inconsistent quartet. Before closing six remarks are in order. Firstly, Tommaso Padoa-Schioppa's [11] Inconsistent quartet is Mundell's trilemma (i.e. Impossible trio) augmented to include trade T : it is imprecise because if trade T is excluded then two categories out of the capital account trichotomy can at most be selected, not three (as the epithet by contrast suggests); furthermore, trade T can be selected alongside any two categories of the capital account trichotomy, because trade T pertains to the current account trichotomy, that is, Rodrik's trilemma (i.e. Globalisation paradox), which intersects Mundell's trilemma to a deeper extent.

7.2 Quadrilemma. Secondly, Joshua Aizenman's [1] Quadrilemma is the extension of Mundell's trilemma to reserve currency acquisitions: it is imprecise because the accumulation of reserve currency, which is the opposite of currency substitution, can happen both under financial openness and monetary policy independence (i.e. openness and independence $O \wedge I$) and under fixed exchange rates and monetary policy independence (i.e. rigidity and independence $R \wedge I$), with a double currency and external OMOs whilst preserving rigidity R's peg (i.e. nullifying potential revaluations), that is, fully within Mundell's trilemma.

It can also happen under any other closure combination (i.e. $C \wedge D \wedge F,\ C \wedge I \wedge F,\ C \wedge D \wedge R$): under combinations $C \wedge D \wedge F$ and $C \wedge I \wedge F$ reserve currency can be accumulated through OMOs; under combination $C \wedge D \wedge R$ reserve currency can be indirectly accumulated with a double currency through external OMOs whilst preserving rigidity R's peg (i.e. nullifying potential revaluations), otherwise abandoning it.

Strictly speaking, reserve currency accumulation arises even under combinations $O \wedge D \wedge R$ and $O \wedge D \wedge F$ and under combinations $C \wedge I \wedge R$ and $C \wedge D \wedge R$ with a single currency, although passively, whereby it is acquired as a mere byproduct of foreign external currency M_E^* obtainment from the importer in exchange for domestic external currency M_E to the end of domestic exports ex.

7.3 Dilemma. To Hélène Rey [14], thirdly, monetary policy independence is a necessary and sufficient condition for financial closure, given fixed or floating exchange rates. The reason she conveys is that

financial openness contemporarily triggers private or public debt crises, so that monetary policy yield to debt crises corrections, to the detriment of other closed or open economy anti-cyclical interventions, seemingly losing monetary policy independence.

One objects that such is altogether inconsistent on the part of necessity (i.e. monetary policy dependence need not imply financial openness, as combinations $C \wedge D \wedge F$ and $C \wedge D \wedge R$ bear out) and specious on the part of sufficiency, owing to a semantic blunder, for Mundell's monetary policy independence amidst financial openness is in relation to exchange rate floatation or fixation, not other anti-cyclical interventions: the question is "Can one do it?", not "What can one do?", it is "Can one make use of monetary policy at all?", not "What is most constraining one's use of anti-cyclical monetary policy?"; Mundell's issue is a normative one, whilst Rey treats of the positive.

In practice, if the exchange rate is fixed the peg is then abandoned in order to correct the debt crisis and if it is floating correction is then all the more immediate, in financial openness and monetary policy independence either way. The punchline is that monetary policy independence and financial openness are still sufficient for exchange rate floatation and Mundell's trilemma is especially applied.

7.4 National democracy. Palley [12] fourthly describes Rodrik's trilemma as analytically erroneous, for globalisation is not incompatible with national democracy, he highlights: many a democracy have partaken in globalisation to this day, XIX century golden straightjacket United kingdom included, and so have autocracies, as contemporary Singapore. Rodrik on the other hand assumes that national policies driven by democracy exclude globalisation, democracy being not thereby residual, which to Palley is consequently flawed.

Yet, could Rodrik's scrutiny of the contemplated issues have actually been deeper? Could Palley himself not have fully understood Rodrik after all? Incidentally, Palley does stress the power of language in economics, but struggles to rigorously frame his usage of the categories "globalisation" and "national democracy" in Rodrik's trilemma, against whose categories his criticism is ultimately raised, partly also owing to Rodrik's presentation of his deep economic integration as hyper-globalisation. Tangentially, Palley also fails to represent the Impossible trio, presenting it as a trilemma between floating exchange rates, financial openness and monetary policy independence, the first category thereof being truly fixed exchange rates (fundamental to the derivation of the trilemma and to the latter's differentiation from the other capital account combinations).

Palley appears to understand (i) Rodrik's deep economic integration or hyper-globalisation, herewith formalised as trade T, as globalisation and (ii) national democracy as Rodrik's nation state and democratic politics at once, respectively formalised as sovereignty S and efficiency E herewith. As a result, in terms of the present formalisation of the Globalisation paradox, he deduces a compromise between trade T on the one hand and sovereignty and efficiency $S \wedge E$ (i.e. Bretton woods compromise) on the other, therefrom working his remark whereby globalisation and national democracy have coexisted. In other words, according to Palley's nominal comprehension of the Globalisation paradox, Rodrik's theorisation whereby sovereignty and efficiency $S \wedge E$ exclude trade T in autarky A is shown to be incorrect (i.e. national democracy does not exclude globalisation). Palley thus proceeds to formulate a dilemma between globalisation and national politics, democratic or autocratic that they may be.

One first of all observes that in Rodrik's trichotomy globalisation is better understood, not as deep economic integration, but as the combination (a side, not an angle) of deep economic integration and democratic politics. In other words, as hereby formalised, globalisation is properly understood as trade and efficiency $T \wedge E$ (i.e. global federalism) in subjection SB (i.e. no nation state), whereby the autonomy in giving rise to disparities in the external real money market is forgone towards the inter-regional objective of preserving exchange and equal growth as a (pejorative) result of sovereignty and trade $S \wedge T$ (i.e. golden straightjacket) implosion, not to begin with in the aspired progress of world socialisation. Yet, even if globalisation were understood as deep economic integration, Palley's dilemma is reconciled with Rodrik's by observing that Rodrik himself conceded democratic politics as not necessarily democratic, existing there an identical compromise between (i) globalisation and national politics or (ii) globalisation, nation states and politics at large. Indeed, Rodrik's globalisation trichotomy becomes a dichotomy inasmuch as Rodrik's nation state and democratic politics be conflated into national democracy, as Palley operated, perhaps though at the cost of having misconstrued his real articulation and conclusions.

Rodrik's insight, to one's mind, is therefore merely such: if globalisation is global federalism then

it collides with nation states, not national democracies, as Palley would argue; if globalisation is deep economic integration then it collides with national politics, as Palley does argue.

7.5 Macro-foundations. Fifthly, 1970s, old macro-econometrics was flawed not because implicitly optimising or not rationally expecting, but because estimating parameters for policy analysis, as opposed to axiomatically calibrating them: the only message of the Lucas critique is that policy robust models are time independent, that parameters cannot be thus determined by the data, but by axioms, and that the data account for forecasting at a given policy. Therefore, because axiomatic foundations are not limited to optimisation problems with rational expectations, macro-foundations as hereby employed feature legitimacy.

Indeed, the present macro-foundations relinquish the modelling system of equation triplets for supply, demand and market clearing with regard to commodities in favour of the direct approach of one equation per variable, whereby each equation captures the dynamics of all variables on implicit account of the canonical law of supply and demand (i.e. respectively increasing and decreasing in prices). Now, the Sonnenschein Mantel Debreu or "Anything goes" theorem articulates that the excess agent demands of individual markets (e.g. output, money, labour, capital) aggregate into many excess demands for the given market, preserving the sole properties of continuity, homogeneity of degree zero and market clearing: at heart, it conveys that if the canonical law of supply and demand stems from microeconomic constituents then it need not apply to the macroeconomy, especially if modelled by means of representative agents (i.e. households, firms, intermediaries, government). By considering the economy as a whole and directly applying the canonical law of supply and demand to it, thereby ignoring its underlying components, the present macro-foundations are immune to the theorem, unlike optimisation problems with rational expectations of representative agents.

Lastly, statistical significance is no more than a sufficient condition for theoretical model validation, that is, it is certainly unnecessary, because of misspecification, and the ultimate validation theoretical models must receive from micro-econometrics is that from data statistics, not regression analysis. Potential criticisms therefrom are therefore likewise wanting in definitiveness.

7.6 Extensions. In the sixth place, section 4 did not model domestic autonomous prices ap and domestic autonomous money demand am_D, by means of wages, capital return, technology, output demand and taxation, because of modelling parsimony: the model's complexity is to augment in the research questions number and the results of its preceding versions must remain unvaried. In future work the external sector should explicitly present domestic autonomous prices ap and domestic autonomous money demand am_D as well as domestic real rates of interest r, expected interest er and exchange e; future work should moreover envision the internal sector, modelled in accord with a corporativistic outlook, by which firms give rise to guilds and guilds give rise to polities via fiscal unions, both similarly segmented into tradable and non-tradable sectors.

8. Conclusion

8.1 Trilemmas and implosion. The present study has formalised Mundell's Impossible trio and Rodrik's Globalisation paradox. In Mundell's trilemma a region cannot simultaneously fix its real exchange rate, identify it as a function of domestic and foreign external real money supply and money demand and maintain its internal nominal money supply floating. In Rodrik's trilemma a region cannot simultaneously discriminate its external real money supply and money demand from their foreign counterparts, maintain its external output as a function of domestic and foreign real money supply and money demand and identify it with foreign external output.

By means of Kaldor's price endogeneity in output (i.e. long run demand sustainability) the present study has then shown that in the framework of Rodrik's trilemma external real money market disparity and trade generate inter-regional external output mismatches, which unless neutralised by changes in the inter-regional external real money market ultimately dismantle trade.

8.2 Federalism and Nash equilibria. One has thirdly defined transfers such that confederations are the sums of inter-regional external outputs achieved via transfers, demonstrating that all unitary regions are federal and that all regions can form a confederation, regardless of the trade between them: by extension, one has upheld corporativism, for one has demonstrated that polities are fiscal unions of guilds and that

guilds are fiscal unions of firms and that the ultimate bout of sovereignty and trade implosion is at the firm level, speaking to Marc Mélitz's [8] new new trade theory.

According to Thirlwall [17], Kaldor attributed price decrease and convexity in output to the manufacturing sector, consequently, firms (and thereby guilds and polities) specialising in the secondary sector should be the ones to benefit from the inefficiency advantage of the Rodrik trilemma, eclipsing the primary and tertiary sectors until trade cessation or neutralisation. Indeed, one has also proven that the said inter-regional external output mismatches can be offset only through autarky or changes in the inter-regional external real money market.

One has finally derived two Nash equilibria, corresponding to (i) an artificial currency union guaranteeing efficiency in the face of trade and to (ii) autarky in the presence of flexible disparity in external currency, to wit, the mercantile dilemma.

8.3 Historical relevance. In the process one believes to have historically borne out the following causal relationships: 1600s Dutch foreign competitiveness and the English Navigation acts (see Jonathan Israel [3]); the Gold standard, the Second industrial revolution and the Great divergence (see Micheal Bordo and Harold James [2]); South Korea's rise and Japan's lost *trentennium;* the 1990s Argentinian dollarisation and crisis; the European monetary union, European Germanisation and European precariat, German included (see James Meade [7]); the 2000s Dollar Yuan fixation, the imperial American trade deficit and the resurgence of (American) protectionism - at least until "COVID-19".

More broadly, one believes the present study to have rationalised the deindustrialisation of the United kingdom, of Western Europe, of Japan and of North America to the gain of the East and the frenzy of libertarianism, the golden straightjacket and globalisation.

In line with Keynes' International clearing union and Bancor proposal, one is lastly convinced of having re-established the following three tenets: external real money supply or money demand disparity together with price endogeneity in output supplants the theory of comparative advantages, speaking to Paul Krugman's [6] new trade theory (see Thirlwall [16]); long run economic growth is not only determined by productivity and technology shocks, but also by demand for external output and a ratio weakly greater than one half of external output to total output is its ultimate discriminant (i.e. $\frac{x}{y} \geq \frac{1}{2}$); autarkic mercantilism (see Antonio Serra [15]), not globalisation, is the best alternative to world socialisation.

In a word, the demonisation of the revived autarkic waves, subsided throughout "COVID-19", but now somewhat back on the rise, seemed to precisely characterise the sentinel of globalisation's actual intent: real wage compression and living standard erosion of developed nations - furthered by "COVID-19", the ongoing conflict between the North Atlantic Treaty Organisation (NATO) and Russia and that which yet is to ensue.

References

[1] Aizenman J. (2013) "The impossible trinity - from the policy trilemma to the policy quadrilemma", Global journal of economics.

[2] Bordo M. and James H. (2013) "The European crisis in the context of the history of previous financial crises", Journal of macroeconomics.

[3] Israel J. (1997) "England's mercantilist response to Dutch world trade primacy, 1647-74", Conflicts of empires: Spain, the Low Countries and the struggle for world supremacy (1585-1713), Bloomsbury.

[4] Kaldor N. (1970) "The case for regional policies", Scottish journal of political economy.

[5] Keynes J. M. (1980) "The collected writings", Volume 25: activities, 1940-44: shaping the post-war world: the Clearing union, Cambridge university press.

[6] Krugman P. (1979) "Increasing returns, monopolistic competition, and international trade", Journal of international economics.

[7] Meade J. (1957) "The balance of payments problems of a European free trade area", Economic journal.

[8] Mélitz M. (2003) "The impact of trade on intra-industry reallocations and aggregate industry productivity", Econometrica.

[9] Mundell R. (1963) "Capital mobility and stabilization policy under fixed and flexible exchange rates", Canadian journal of economics.

[10] Nash J. (1950) “Equilibrium points in n-person games”, Proceedings of the national academy of sciences.
[11] Padoa-Schioppa T. (1988) “The European monetary system: a long-term view”, Cambridge university press.
[12] Palley T. (2017) “The fallacy of the globalization trilemma: reframing the political economy of globalization and implications for democracy”, FMM working paper.
[13] Rodrik D. (2011) “The globalization paradox: democracy and the future of the world economy”, William Warder Norton and company.
[14] Rey H. (2015) “Dilemma not trilemma: the global financial cycle and monetary policy independence”, NBER working paper.
[15] Serra A. (1613) “Breve trattato delle cause che possono far abbondare li regni d’oro e d’argento dove non sono miniere”, G. G. Destefanis (1803).
[16] Thirlwall A. (2014) “Kaldor’s 1970 regional growth model revisited”, Scottish journal of political economy.
[17] Thirlwall A. (2002) “The nature of economic growth”, Edward Elgar publishing.
[18] Verdoorn P. J. (1949) “Fattori che regolano lo sviluppo della produttività del lavoro”, L’industria.

Appendix

Such is the Julia code generating the impulse response functions graph (wherein # is to replace %).

```
using LinearAlgebra, Plots, Statistics

%Linear system parameters
alpha_x=0.25;
beta_x=0.25;
gamma_x=0.5;

alpha_md=2.0025;
beta_md=2.0025;

alpha_p=4.01;
beta_p=1;
gamma_p=4.005;
Δ_p=8.0149;
epsilon_p=4.01;

sigma_MS=0.01;
sigma_amd=0.01;
sigma_ap=0.01;

%State equation matrices
P=[1 -gamma_x beta_x -alpha_x 0 0;
    0 1 0 0 -alpha_md beta_md;
    0 0 1 0 0 -alpha_p;
    0 0 0 1 0 0;
    0 0 0 0 1 0;
    0 0 0 0 0 1];
Q=[zeros(2, 6);
    -epsilon_p 0 0 beta_p gamma_p -Δ_p;
    0 0 0 1 0 0;
    0 0 0 0 1 0;
    0 0 0 0 0 1];
R=[zeros(3, 3);
    sigma_MS 0 0;
    0 sigma_amd 0;
    0 0 sigma_ap];

A=P\Q; %A=inv(P)*Q
B=P\R; %B=inv(P)*R

%System instability check
```

```
evalA, evecA=eigen(A);
if (any(abs.(evalA).>1))
    println("The system is unstable");
else %(any(abs.(evalA).≤1))
    println("The system is not unstable");
end

%Impulse response functions
H=41;
Xirf_nms=zeros(6, H);
Xirf_amd=zeros(6, H);
Xirf_ap=zeros(6, H);

%a)nominal money supply
Xirf_nms[:, 1]=B[:, 1]*sigma_MS;
for j=2:H
    Xirf_nms[:, j]=A*Xirf_nms[:, j-1];
end

%b)autonomous money demand
Xirf_amd[:, 1]=B[:, 2]*sigma_amd;
for j=2:H
    Xirf_amd[:, j]=A*Xirf_amd[:, j-1];
end

%c)autonomous prices
Xirf_ap[:, 1]=B[:, 3]*sigma_ap;
for j=2:H
    Xirf_ap[:, j]=A*Xirf_ap[:, j-1];
end

%d)graphs
t=0:40;

pyplot();
p1=plot(t, Xirf_nms[1, :], xlabel="Quarters", ylabel="Ouput", label=false, grid=false);
p2=plot(t, Xirf_nms[3, :], xlabel="Quarters", ylabel="Prices", label=false, grid=false);
p1s=plot(p1, p2, layout=(1, 2), title="NMS shock");

p3=plot(t, Xirf_amd[1, :], xlabel="Quarters", ylabel="Ouput", label=false, grid=false);
p4=plot(t, Xirf_amd[3, :], xlabel="Quarters", ylabel="Prices", label=false, grid=false);
p2s=plot(p3, p4, layout=(1, 2), title="AMD shock");

p5=plot(t, Xirf_ap[1, :], xlabel="Quarters", ylabel="Ouput", label=false, grid=false);
p6=plot(t, Xirf_ap[3, :], xlabel="Quarters", ylabel="Prices", label=false, grid=false);
p3s=plot(p5, p6, layout=(1, 2), title="AP shock");

plot(p1s, p2s, p3s, layout=(3, 1))
savefig("IRFs.eps")
```