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Abstract

Indian economy is going through underlying changes in post-pandemic recovery process. Effect of policies, monetary or fiscal on macroeconomy needs a thorough analysis in these recessionary times. In this context, this study develops a closed-economy DSGE model to see the impact of monetary policy in the Indian economy. The model includes price rigidities, and parameters are calibrated using the data on Indian economy. The model includes two sectors - production and consumption, an inflation targeting regime following the Taylor rule. Model is simulated for a positive productivity shock and an expansionary monetary policy shock. Results show that positive productivity shock improves economic activity and an expansionary monetary policy shock increases output for the short-term only.

Keywords: DSGE models, New-Keynesian, monetary policy, general equilibrium, Indian economy, calibration.

JEL Classification: C32, E32, E37, E52.

1 Introduction

One of the main objective of macroeconomics is to learn how the overall economy works, and to analyse how certain changes in one sector affect others and the economy as a whole. Like in natural sciences, economists are also interested in carrying out experiments to study the impacts of specific changes and disruptions on the economy. But unlike economics, researchers in other fields like Physics or Chemistry have the luxury to conduct experiments in laboratories where they can replicate the real world conditions and complete their experiments. The Dynamic Stochastic General Equilibrium or popular as DSGE models provide macroeconomists such laboratories where they also conduct experiments to know in advance the effects of certain policy changes, disruptions and anticipated or unanticipated shocks at aggregate level. We can call these models macroeconomic laboratories because they are grounded in microeconomic theory and construct a model economy in such a way that allows for more structural analysis and evolution of business cycles. Though, prone to criticisms, but DSGE approach is the core of present day macroeconomic modelling.

Kydland & Prescott (1982) were the first proponents of DSGE modelling. They originated out of real business cycle (RBC) theory which argue that exogenous shocks can help in explaining the fluctuations in the economy. RBC models held unrealistic assumptions like perfect competition, absence of asymmetric assumptions etc. These assumptions have been relaxed in later New-Keynesian versions of DSGE models which include nominal rigidities in pricing, investment adjustment costs and include different shocks to bring the models closer to the real economy as close as possible. More refined models by Christiano et al. (2005), Fernández-Villaverde & Rubio-Ramírez (2006),Smets & Wouters (2007) are some examples of DSGE models with New-Keynesian flavour.

There is a plethora of DSGE literature dedicated to explain the business cycles in developed economies. But, it is sparse on emerging market economies. These economies have different characteristics than developed ones. Developing nations economies face different frictions and distortions. In Indian context, Peiris et al. (2010) and V. Gabriel et al. (2012) build DSGE models including financial frictions and find that such models fit the data very well. Banerjee et al. (2020) highlights that informal sector affects the monetary transmission in the Indian economy. To trace the post-pandemic recovery, Sharma & Behera (2022) analyse output gap using the DSGE model and find it to be superior than the traditional HP filter method.

The present study build and calibrate a closed economy New-Keynesian DSGE model for the Indian economy. The Model presented in later section is at early stage and includes only two sectors - consumption and production along with a monetary authority which sets the interest rates. Model parameters are calibrated following the literature based on Indian quarterly data. Model is tested for productivity and monetary policy shocks. Results of our calibrated model are in line with the economic theory and existing literature on India. Impulse responses generated after productivity shock shows increase in consumption and investment. An expansionary monetary shock increases output as well as inflation but decreases the demand for government bonds. The paper is arranged in three sections. Next section two lays out the basic framework for the Model with separate subsections dedicated to representative household, firm and monetary authority. Section 3 discusses the simulation results.

2 Model

In this section, we attempt to build a closed economy New-Keynesian DSGE model following Rudebusch & Swanson (2012) and Costa (2018). Our model is in early stage and consists only two agents - households and firms. Model is discussed in the following sections.

2.1 Households

There is a closed economy with no government sector. The economy is populated by a continuum of households indexed by $j\epsilon[0,1]$. The household maximizes his utility function which is additively separable in consumption and labour. Household consume goods and supply labour to the firms. Household maximizes the following utility function:

$$\max_{C_{j,t},L_{j,t},B_t,K_{j,t+1}} u_t = E_t \sum_{t=0}^{\infty} \beta^t \left[\frac{C_{j,t}^{1-\sigma}}{1-\sigma} - \frac{L_{j,t}^{1+\psi}}{1+\psi} \right] \tag{1}$$

where,

E_t - Expectation operator,

C - Consumption,

L - Labour supplied by household (in number of hours)

β - discount factor,

σ - coefficient of relative risk-aversion (reciprocal to the elasticity of substitution of consumption),

ψ - marginal disutility of labour (reciprocal to the elasticity of substitution of labour supply)

In line with the RBC models,the utility function chosen is a constant relative risk-aversion (CRRA)[1] utility function. CRRA utility functions are widely used in DSGE models because they are compatible with balanced growth[2] along with the optimal steady-state. It is concave utility function with the properties of $u_C > 0, u_L < 0$ and $u_{CC}, u_{LL} < 0$. $u_L < 0$ means that labour has negative effect on the utility i.e. the more labour household supplies, less satisfaction he drives.

Household maximizes the utility function subject to the budget constraint:

$$P_t(C_{j,t} + I_{j,t}) + \frac{B_{j,t+1}}{r_t} = W_t L_{j,t} + R_t K_{j,t} + B_{j,t} + \Pi_t \tag{2}$$

where,

P - General Price level

I - Investment

W - Wages

K - Capital Stock

B_t - One maturity bond issued by the government

R - return on capital

r - Interest rate set by the central bank

Π - dividends to households by firms

In budget constraint equation (2), $\left(\frac{1}{r_t}\right)$ is the price of the government bond.[3] Our household derives his income from three sources - supplying labour, renting capital and holding the government bond. In the economy, capital is accumulated following the rule-

$$K_{j,t+1} = (1 - \delta)K_{j,t} + I_{j,t} \tag{3}$$

[1]Following King et al. (1988); Clarida et al. (2000); Gali & Monacelli (2008) and Gertler & Karadi (2011) among others

[2]In balanced growth, growth rate is constant at steady-state.

[3]Bond price is inversely related to the interest paid on holding the bond.

where δ is the depreciation rate of capital.

Next, we form the Lagrangian to solve the above maximization problem. After substituting $I_{j,t} = K_{j,t+1} - (1-\delta)K_{j,t}$ from equation (3) in the budget constraint, Lagrangian is -

$$\mathcal{L} = E_t \sum_{t=0}^{\infty} \beta^t \left[\frac{C_{j,t}^{1-\sigma}}{1-\sigma} - \frac{L_{j,t}^{1+\psi}}{1+\psi} \right] - \lambda_{j,t}[P_t C_{j,t} + P_t K_{j,t+1} - P_t(1-\delta)K_{j,t} + \frac{B_{j,t+1}}{r_t} - W_t L_{j,t} - R_t K_{j,t} - B_{j,t} - \Pi_t] \tag{4}$$

$\lambda_{j,t}$ is the Lagrangian multiplier. First order conditions for consumption, labour, capital and bond are:

$$\frac{\partial \mathcal{L}}{\partial C_{j,t}} = C_{j,t}^{-\sigma} - \lambda_{j,t} P_t = 0 \tag{5}$$

$$\frac{\partial \mathcal{L}}{\partial L_{j,t}} = -L_{j,t}^{\psi} + \lambda_{j,t} W_t = 0 \tag{6}$$

$$\frac{\partial \mathcal{L}}{\partial K_{j,t+1}} = -\lambda_{j,t} P_t + \beta E_t \lambda_{j,t+1}[(1-\delta)E_t P_{t+1} + E_t R_{t+1}] = 0 \tag{7}$$

$$\frac{\partial \mathcal{L}}{\partial B_{j,t+1}} = -\frac{\lambda_{j,t}}{r_t} + \beta E_t \lambda_{j,t+1} = 0 \tag{8}$$

from equation (5) and (6), solving for λ gives the following equation:

$$C_{j,t}^{\sigma} L_{j,t}^{\psi} = \frac{W_t}{P_t} \tag{9}$$

Equation (9) can be interpreted as the labour supply equation because it equates the marginal rate of substitution between consumption and leisure on the left hand side to their relative prices on the right hand side. In next step, we try to find the inter-temporal consumption/saving Euler equation. To get the Euler equation, we substitute the value of Lagrangian multiplier ($\lambda_{j,t}$) from equation (5) in equation (7). After some algebraic manipulation we get the Euler equation:

$$\left(\frac{E_t, C_{j,t+1}}{C_{j,t}} \right)^{\sigma} = \beta \left[(1-\delta) + E_t \left(\frac{R_{t+1}}{P_{t+1}} \right) \right] \tag{10}$$

As we see, above equation is an inter-temporal condition which can be interpreted that households must be indifferent between consuming one more unit today (in period t) and saving that unit, earning some interest on it, and then consuming it in the next period (t+1). From equation (8), we can get Euler equation for government bond:

$$\frac{\lambda_{j,t}}{r_t} = \beta E_t \lambda_{j,t+1} \tag{11}$$

2.2 Firms

Proposed model is a New-Keynesian model. It features imperfect competition in the production sector. In NK models, prices are temporarily rigid and adjusts with a lag. We assume price stickiness in the model. In production sector, there are two types of firms - final goods producing firms and intermediate goods producing firms.

2.3 Final Goods Firms

Final goods producing firms operate in a perfectly competitive market. Firms follow Dixit-Stiglitz (Dixit & Stiglitz (1977)) aggregator function:

$$Y_t = \left(\int_0^1 Y_{j,t}^{\frac{\xi-1}{\xi}} dj \right)^{\frac{\xi}{\xi-1}} \tag{12}$$

$Y_{j,t}$ is intermediate good and Y_t is the final good after aggregating the intermediate goods. $\xi > 1$ is the elasticity of substitution between intermediate goods. If P_t is the nominal price of final goods and $P_{j,t}$ is the price of intermediate goods, then profit maximizing problem of final goods firms is:

$$\max_{Y_{j,t}} P_t Y_t - \int_0^1 P_{j,t} Y_{j,t} dj \tag{13}$$

Substituting the expression for Y_t from production function in equation (12) yields:

$$\max_{Y_{j,t}} P_t \left(\int_0^1 Y_{j,t}^{\frac{\xi-1}{\xi}} dj \right)^{\frac{\xi}{\xi-1}} - P_{j,t} \int_0^1 Y_{j,t} dj \tag{14}$$

F.O.C. for the above problem leads to:

$$P_t \left(\int_0^1 Y_{j,t}^{\frac{\xi-1}{\xi}} dj \right)^{\frac{1}{\xi-1}} Y_{j,t}^{\frac{-1}{\xi}} - P_{j,t} = 0 \tag{15}$$

rearranging the aggregator function in equation (12) gives the following expression for Y_t:

$$Y_t^{\frac{1}{\xi}} = \left(\int_0^1 Y_{j,t}^{\frac{\xi-1}{\xi}} dj \right)^{\frac{1}{\xi-1}} \tag{16}$$

substituting the R.H.S. of this equation in equation (15) and after doing some algebraic manipulation gives the demand function for intermediate goods:

$$Y_{j,t} = Y_t \left(\frac{P_t}{P_{j,t}} \right)^{\xi} \tag{17}$$

This demand function is directly proportional to aggregate demand (Y_t) and indirectly proportional to the relative price level. Now, substituting this expression for $Y_{j,t}$ back in the aggregator in equation (12):

$$Y_t = \left[\int_0^1 \left\{ Y_t \left(\frac{P_t}{P_{j,t}} \right)^{\xi} \right\}^{\frac{\xi-1}{\xi}} dj \right]^{\frac{\xi}{\xi-1}} \tag{18}$$

again after some algebraic manipulation,

$$P_t = \left[\int_0^1 P_{j,t}^{1-\xi} dj \right]^{\frac{1}{1-\xi}} \tag{19}$$

This equation gives the expression for aggregate price level.

2.3.1 Intermediate Goods Firms

Firms in this sector produce differentiated intermediate goods and sell them to final goods producing firms. Due to the differentiated nature of their products, they enjoy some degree of market power, therefore there is monopolistic competition in this market structure. In first stage, intermediate firm determines the amount of labour and capital to minimize it's production cost. Firms use both labour and physical capital and follow the Cobb-Douglas production function:

$$Y_{j,t} = A_t K_{j,t}^{\alpha} L_{j,t}^{1-\alpha} \tag{20}$$

where A_t is the technology and follows an AR(1) process:

$$\log A_t = (1 - \phi_A) \log \bar{A} + \phi_A \log A_{t-1} + \epsilon_t \tag{21}$$

where, $\bar{A}$ is the productivity at steady-state, ϕ_A, is the autoregressive parameter ϵ_t is the productivity shock with $\epsilon_t \sim N(0, \sigma_a)$.

Cobb-Douglas production function has some properties - It is strictly increasing and concave function, which means $F_L, F_K > 0$ and $F_{LL}, F_{KK} < 0$. Production function gives constant returns to scale and follow Inada[4] conditions.

The problem of the firm is to minimise the production cost subject to the production function in equation (20):

$$\min_{L_{j,t},K_{j,t}} W_t L_{j,t} + R_t K_{j,t} \tag{22}$$

subject to,

$$Y_{j,t} = A_t K_{j,t}^{\alpha} L_{j,t}^{1-\alpha} \tag{23}$$

The Lagrangian for this problem is:

$$\mathcal{L} = W_t L_{j,t} + R_t K_{j,t} + \nu_{j,t} \left(Y_{j,t} - A_t K_{j,t}^{\alpha} L_{j,t}^{1-\alpha} \right) \tag{24}$$

where, $\nu_{j,t}$ is the Lagrangian multiplier. First order conditions w.r.t. labour and capital are:

$$\frac{\partial \mathcal{L}}{\partial L_{j,t}} = W_t - (1-\alpha)\nu_{j,t} A_t K_{j,t}^{\alpha} L_{j,t}^{\alpha} = 0 \tag{25}$$

$$\frac{\partial \mathcal{L}}{\partial K_{j,t}} = R_t - \alpha \nu_{j,t} A_t K_{j,t}^{\alpha-1} L_{j,t}^{1-\alpha} = 0 \tag{26}$$

Here, the Lagrange multiplier $\nu_{j,t}$ shows the shadow prices of change in the ratio of capital and labour used. Therefore, we can consider the Lagrangian multiplier as the marginal cost ($mc_{j,t}$). Now the above equations are:

$$L_{j,t} = (1-\alpha) mc_{j,t} \frac{Y_{j,t}}{W_t} \tag{27}$$

and

$$K_{j,t} = \alpha mc_{j,t} \frac{Y_{j,t}}{R_t} \tag{28}$$

Since total cost for the firm j is:

$$TC_{j,t} = W_t L_{j,t} + R_t K_{j,t} \tag{29}$$

[4] $\lim_{L\to 0} F_L = \infty; \lim_{L\to\infty} F_L = 0$ and $\lim_{K\to 0} F_K = \infty; \lim_{K\to\infty} F_K = 0$

substituting equation (27) and (28) in total cost function and dividing by output [5], we get the expression for the marginal cost:

$$mc_{j,t} = \frac{1}{A_t}\left(\frac{W_t}{1-\alpha}\right)^{1-\alpha}\left(\frac{R_t}{\alpha}\right)^{\alpha} \tag{30}$$

2.3.2 Calvo Pricing

In the next stage, firm defines the prices of intermediate goods. In our model, we assume that firm decides the prices following the Calvo rule (Calvo, 1983). Under this rule, in a period, only a fraction of total firms selected are allowed to change the prices when they receive the random signal. Remaining firms define their prices following the stickiness rule, like maintaining the previous period's price or updating the price based on previous period's inflation rate. We follow the previous period's price rule ($P_{j,t} = P_{t-1}$) to introduce stickiness.

Following the Calvo pricing rule, we assume that there is a φ probability that a firm keeps its price fixed in the next period and a $(1-\varphi)$ probability that it receives the random signal and reset the prices. For the firm which reset its prices, there is φ probability that the price remain fixed in time t+1 and φ^2 probability to remain fixed in t+2 and so on. The maximization problem of the firm which reset its prices can be defined by subtracting total costs from the total revenue in the following way:

$$\max_{P^*_{j,t}} E_t \sum_{i=0}^{\infty} (\beta\varphi)^i \left(P^*_{j,t} Y_{j,t+i} - TC_{j,t+i}\right) \tag{31}$$

where $P^*_{j,t}$ is the optimal price. Substituting the expression for $Y_{j,t}$ from equation(17) in equation (31) and replacing the $TC_{j,t+i} = mc_{j,t+i} \times Y_{j,t+i}$ gives:

$$\max_{P^*_{j,t}} E_t \sum_{i=0}^{\infty} (\beta\varphi)^i \left[P^*_{j,t} Y_{t+i}\left(\frac{P_{t+i}}{P^*_{j,t}}\right)^{\xi} - Y_{t+i}\left(\frac{P_{t+i}}{P^*_{j,t}}\right)^{\xi} mc_{j,t+i}\right] \tag{32}$$

taking the first derivative and solving for $P^*_{j,t}$ gives:

$$P^*_{j,t} = \left(\frac{\xi}{\xi-1}\right) E_t \sum_{i=0}^{\infty} (\beta\varphi)^i mc_{j,t+i} \tag{33}$$

Since all the firms which reset the prices face the same marginal cost. Therefore, $P^*_{j,t}$ is the

[5] Because of the assumptions of zero fixed cost and constant returns to scale.

same price for all $(1-\varphi)$ price resetting firms. Now, from equation (19), the expression can also be written as: $P_t^{1-\xi} = \left[\int_0^1 P_{j,t}^{1-\xi} dj\right]$ and the equation for the aggregate price level can be solved in the following way:

$$P_t^{1-\xi} = \int_0^{\varphi} P_{t-1}^{1-\xi} dj + \int_{\varphi}^{1} P_t^{*1-\xi} dj \tag{34}$$

solving the equation gives the expression for general price level:

$$P_t = \left[\varphi P_{t-1}^{1-\xi} + (1-\varphi) P_t^{*1-\xi}\right]^{\frac{1}{1-\xi}} \tag{35}$$

2.4 Central Bank

Now, we introduce a monetary policy authority, typically a central bank in any economy which sets the interest rates. We assume that central bank follows a simple Taylor rule (Taylor, 1993) and sets the interest rate keeping in mind two broad objectives - price stability and economic growth. We follow the Taylor rule defined in Costa (2018) and Banerjee et al. (2020)as:

$$\frac{r_t}{\bar{r}} = \left(\frac{r_{t-1}}{\bar{r}}\right)^{\gamma_r} \left[\left(\frac{\pi_t}{\bar{\pi}}\right)^{\gamma_\pi} \left(\frac{Y_t}{\bar{Y}}\right)^{\gamma_Y}\right]^{(1-\gamma_r)} s_t^m \tag{36}$$

where, γ_r - smoothing parameter

γ_Y - interest rate sensitivity of output

γ_π - interest rate sensitivity of inflation

s_t^m - monetary policy shock which follows AR(1) process:

$$\log s_t^m = (1-\rho_m)\log \bar{s}^m + \rho_m \log \bar{s}_{t-1}^m + \epsilon_{m,t} \tag{37}$$

Since there is symmetry in the preferences of both households and firms, so they are represented by representative agents. So, we can remove the j subscript from the equations. Model equations removing the j subscript can be written as:

$C_{j,t}^{\sigma} L_{j,t}^{\psi} = \frac{W_t}{P_t}$: Labour Supply

$\left(\frac{E_t, C_{t+1}}{C_t}\right)^{\sigma} = \beta\left[(1-\delta) + E_t\left(\frac{R_{t+1}}{P_{t+1}}\right)\right]$: Euler Equation

$K_{t+1} = (1-\delta)K_t + I_t$: Law of Capital Accumulation

$Y_t = A_t K_t^{\alpha} L_t^{1-\alpha}$: Production function

$mc_t = \frac{1}{A_t}\left(\frac{W_t}{1-\alpha}\right)^{1-\alpha}\left(\frac{R_t}{\alpha}\right)^{\alpha}$: Marginal Cost

$L_t = (1-\alpha)mc_t\frac{Y_t}{W_t}$: Labour Demand

$K_t = \alpha mc_t\frac{Y_t}{R_t}$: Capital Demand

$P_t^* = \left(\frac{\xi}{\xi-1}\right)E_t\sum_{i=0}^{\infty}(\beta\varphi)^i mc_{t+i}$: Optimal Price level

$\pi_t = \frac{P_t}{P_{t-1}}$: Inflation rate

$Y_t = C_t + I_t$: Equilibrium Condition

$\log A_t = (1-\phi_A)\log\bar{A} + \phi_A\log A_{t-1} + \epsilon_t$: Productivity Shock

2.5 Steady State

Next step in solving model is to define the steady-state [6]values. We remove time subscript and solve above equations for steady-state for households:

$$\bar{C}^{\sigma}\bar{L}^{\psi} = \frac{\bar{W}}{\bar{P}} \tag{38}$$

$$1 = \beta\left[(1-\delta) + \left(\frac{\bar{R}}{\bar{P}}\right)\right] \tag{39}$$

$$\delta\bar{K} = \bar{I} \tag{40}$$

For firms:

$$\bar{Y} = \bar{K}^{\alpha}\bar{L}^{1-\alpha} \tag{41}$$

$$\bar{L} = (1-\alpha)\bar{mc}\frac{\bar{Y}}{\bar{W}} \tag{42}$$

$$\bar{K} = (\alpha)\bar{mc}\frac{\bar{Y}}{\bar{R}} \tag{43}$$

$$\bar{mc} = \left(\frac{\bar{W}}{1-\alpha}\right)^{1-\alpha}\left(\frac{\bar{R}}{\alpha}\right)^{\alpha} \tag{44}$$

[6]A variable is said to be in steady-state, if it's value does't change over time, i.e., $E_t z_{t+1} = z_t = z_{t-1} = \bar{z}$

Solving from equation (33), $E_t \sum_{i=0}^{\infty} (\beta\varphi)^i = \frac{1}{1-\beta\varphi}$. Substituting this value in the optimal pricing equation and defining steady-state:

$$\bar{P} = \left(\frac{\xi}{\xi - 1}\right)\left(\frac{1}{1 - \beta\varphi}\right)\bar{mc} \tag{45}$$

and equilibrium condition

$$\bar{Y} = \bar{C} + \bar{I} \tag{46}$$

Where, a bar over variable shows it's steady-state. For some variables, it is easy to get the steady state values analytically, but for most variables, it's not possible. The standard practice is to solve for steady states numerically for such variables. Like in equation (21), it's difficult to solve for steady state value of productivity. In literature, $\bar{A}$ is given the value 1. Following the literature, we also assign unit value to the productivity steady-state. We also normalise general price level to 1 ($\bar{P} = 1$) which is again a standard practice in literature to simplify the model.

With these equations ready, we try to solve for steady-states for our variables of interest. From equation (11), steady -state value for interest rate simply is

$$\bar{r} = \frac{1}{\beta} \tag{47}$$

Next we start with $\bar{R}$, as in equation (36), $\bar{R}$ depends only on parameter values and $\bar{P} = 1$. Rearranging equation (39):

$$\bar{R} = \bar{P}\left[\left(\frac{1}{\beta}\right) - (1 - \delta)\right] \tag{48}$$

It's easy to find steady-state values for $\bar{R}$ by putting calibrated values of parameters. Next, we can find the steady-state values for $\bar{mc}$ with the help of $\bar{R}$. So, from equation (45):

$$\bar{mc} = \left(\frac{\xi - 1}{\xi}\right)(1 - \beta\varphi)\bar{P} \tag{49}$$

Next we solve for $\bar{W}$, from equation (44), the expression for $\bar{W}$ can be written as:

$$\bar{W} = (1 - \alpha)\bar{mc}^{\frac{1}{1-\alpha}}\left(\frac{\alpha}{\bar{R}}\right)^{\frac{\alpha}{1-\alpha}} \tag{50}$$

So far, we get he steady-state values for the capital, labour and general prices. Now, we can aim for consumption and investment demands. $\bar{I}$ ca be obtained by substituting equation (43) into equation (40):

$$\bar{I} = \left(\frac{\delta\alpha\bar{mc}}{\bar{R}}\right)\bar{Y} \tag{51}$$

and solving for $\bar{C}$ and $\bar{Y}$, we get:

$$\bar{C} = \frac{1}{\bar{Y}^{\frac{\psi}{\sigma}}}\left[\frac{\bar{W}}{\bar{P}}\left(\frac{\bar{W}}{(1-\alpha)\bar{mc}}\right)^{\psi}\right]^{\frac{1}{\sigma}} \tag{52}$$

$$\bar{Y} = \left(\frac{\bar{R}}{\bar{R}-\alpha\delta\bar{mc}}\right)^{\frac{\sigma}{\sigma+\psi}}\left[\frac{\bar{W}}{\bar{P}}\left(\frac{\bar{W}}{(1-\alpha)\bar{mc}}\right)^{\psi}\right]^{\frac{1}{\sigma+\psi}} \tag{53}$$

2.6 Calibration

To get the steady-state values of the variables and to solve model numerically, we need to assign values to the parameters. There are two methods in the literature - calibration and estimation of the parameters. Though estimation is the most recommended method, but we restrict to the calibration method for the study. Calibration is a popular method in DSGE literature. In this method, parameters are given values based on the standard literature which are observed from the data. We also follow the existing literature in assigning the parameter values. Value of depreciation rate for capital (δ) is taken as 0.025 which means around 10% capital depreciation per annum is broadly in line with literature (V. J. Gabriel et al., 2011; Das & Nath, 2019). Discount factor (β) is set to 0.98 following V. J. Gabriel et al. (2011) (literature broadly defines the value of discount factor between 0.97 to 0.99). Share of capital in production (α) is 0.30 taken from Banerjee et al. (2020). Value of coefficient of relative risk-aversion (σ) is in line with Indian case from V. J. Gabriel et al. (2011). Smets & Wouters (2007) take the price stickiness parameter (φ) value 0.75 which is around the estimated value using Indian data by V. J. Gabriel et al. (2011) and Sharma & Behera (2022). The value of Frisch elasticity of labour supply (inverse of ψ) is contested in literature and is in the range between 0.25 to 1.28[7]. We take the value 2.7 following Indian studies (Anand & Prasad, 2010; Sharma & Behera, 2022).

[7] Christiano et al. (1996); Rotemberg & Woodford (1997)

The Table 3 below gives the description of calibration of the structural parameters.

Table 1: Calibration of Parameters

Parameter	Meaning	Value	Source
σ	Coefficient of Relative Risk Aversion	1.50	Gabriel et al., 2011; Das & Nath, 2019
ψ	Inverse of Frisch elasticity of labour supply	2.7	Anand & Das, 2010; Sharma & Behera, 2022
β	Discount Factor	0.98	Gabriel et al., 2011
δ	Depreciation rate	0.025	Banerjee & Basu, 2017
α	Share of capital	0.30	Banerjee & Basu, 2017; Banerjee et al., 2020
ξ	Elasticity of substitution between intermediate goods	7.02	Gabriel, 2016
φ	Price stickiness parameter	0.75	Smets & Wouter, 2007
γ_r	Interest rate smoothening parameter	0.80	Banerjee et al., 2018
γ_Y	Interest rate sensitivity of output	0.50	Banerjee et al., 2017
γ_π	Interest rate sensitivity of inflation	1.20	Gabriel et al., 2011
ρ_A	Productivity shock autoregressive parameter	0.95	
ρ_m	MP shock autoregressive parameter	0.95	

2.7 Log-linearisation of the model

After getting the steady-states, next step is to log-linearise the model around the steady-state. Solving the linear model is relatively simple compared to non-linear models. To get the intuition of linear model is often easier than the non-linear version of the model. Therefore, it is a standard practice in literature to solve the model with log-linear approximations[8]. In log-linearisation process, we replace all the necessary equations in the model by approximations, which are linear in the log-deviation form. We use Uhlig's method (Uhlig (1999)) for our log-linearisation. In this method, a variable is replaced in this way: a variable Z_t is replaced by $\bar{Z}e^{\tilde{Z}_t}$, where $\tilde{Z} = \log Z - \log \bar{Z}$. Uhlig method gives the following set of tools to solve for more than one variable:

$$e^{(\tilde{Y}_t + b\tilde{Z}_t)} \approx 1 + \tilde{Y}_t + b\tilde{Z} \tag{54}$$

$$\tilde{Y}_t \tilde{Z}_t \approx 0 \tag{55}$$

$$E_t\left[be^{\tilde{Z}_{t+1}}\right] \approx b + bE_t[\tilde{Z}_{t+1}] \tag{56}$$

[8]King et al. (1988); Campbell (1994) are among the firsts to solve RBC models through log-linearisation.

Using the Uhlig method and applying these tools, we log-linearise our model equations. First solving for the labour supply equation

$$C_{j,t}^{\sigma} L_{j,t}^{\psi} = \frac{W_t}{P_t}$$

replacing C_t by $\bar{C}e^{\tilde{C}_t}$ and following the same for other variables, we get

$$\bar{C}^{\sigma} \bar{L}^{\psi} e^{(\sigma \tilde{C}_t + \psi \tilde{L}_t)} = \frac{\bar{W}}{\bar{P}} e^{(\tilde{W}_t - \tilde{P}_t)}$$

using the rule in equation (54), above equation can be transformed into,

$$\bar{C}^{\sigma} \bar{L}^{\psi} (1 + \sigma \tilde{C}_t + \psi \tilde{L}_t) = \frac{\bar{W}}{\bar{P}} (1 + \tilde{W}_t - \tilde{P}_t)$$

since at steady-state, $\bar{C}^{\sigma} \bar{L}^{\psi} = \frac{\bar{W}}{\bar{P}}$ (equation (38)), we get the final log-linearised form of the labour supply equation:

$$\sigma \tilde{C} + \psi \tilde{L} = \tilde{W} - \tilde{P} \tag{57}$$

Calculating in similar ways, we get the log-linearised form of other model equations in the following way;

Euler equation:

$$\frac{\sigma}{\beta}(E_t \tilde{C}_{t+1} - \tilde{C}_t) = \frac{\bar{R}}{\bar{P}} E_t(\tilde{R}_{t+1} - \tilde{P}_{t+1}) \tag{58}$$

Euler equation for government bonds:

$$\tilde{\lambda}_t - \tilde{r}_t = \tilde{\lambda}_{t+1} \tag{59}$$

Marginal cost:

$$\tilde{mc}_t = (1 - \alpha)\tilde{W}_t + \alpha \tilde{R}_t - \tilde{A}_t \tag{60}$$

Capital demand:

$$\tilde{K}_t = \tilde{mc}_t + \tilde{Y}_t - \tilde{R}_t \tag{61}$$

Labour demand:

$$\tilde{L}_t = \tilde{mc}_t + \tilde{Y}_t - \tilde{W}_t \tag{62}$$

Production function:

$$\tilde{Y}_t = \tilde{A}_t + \alpha\tilde{K}_t + (1-\alpha)\tilde{L}_t \tag{63}$$

Law of capital motion:

$$\tilde{K}_{t+1} = (1-\delta)\tilde{K}_t + \delta\tilde{I}_t \tag{64}$$

Optimal price level:

$$\tilde{P}_t^* = (1-\beta\varphi)E_t\sum_{i=0}^{\infty}(\beta\varphi)^i\tilde{mc}_{t+i} \tag{65}$$

General price level:

$$\tilde{P}_t = \varphi\tilde{P}_{t-1} + (1-\varphi)(1-\beta\varphi)E_t\sum_{i=0}^{\infty}(\beta\varphi)^i\tilde{mc}_{t+i} \tag{66}$$

Monetary policy rule:

$$\tilde{r}_t = \gamma_r\tilde{r}_{t-1} + (1-\gamma_r)(\gamma_\pi\tilde{\pi} + \gamma_Y\tilde{Y}_t) + \tilde{s}_t^m \tag{67}$$

Inflation rate:

$$\tilde{\pi} = \tilde{P}_t - \tilde{P}_{t-1} \tag{68}$$

Equilibrium condition:

$$\bar{Y}\tilde{Y}_t = \bar{C}\tilde{C}_t + \bar{I}\tilde{I}_t \tag{69}$$

Productivity shock:

$$\tilde{A}_t = \rho_A\tilde{A}_{t-1} + \epsilon_t \tag{70}$$

Monetary Policy Shock:

$$\tilde{s}_t^m = \rho_m\tilde{s}_{t-1}^m + \epsilon_{m,t} \tag{71}$$

3 Results

After transforming the model equations in log-linearisation form, we simulate the model for monetary policy and productivity shocks. Model is simulated using the Dynare 5.1 in Matlab. In this section, we discuss the results of impulse response functions to one std. deviation to the productivity shock and monetary policy shock. Impulse responses are simulated for 40 periods.

3.1 Productivity Shock

Figure 1 and Figure 2 show the effects of a positive productivity shock on the variables. A positive productive shock makes means of production more efficient, i.e. increases marginal productivities of labour and capital. This leads to increase in the demand of labour and capital by firms, leading to a spike in the prices of labour and capital i.e. wages and rent. Higher wages and rent on capital increases household's income resulting in higher inflation. Higher inflation binds central bank to increase the interest rate. Increased level of wage rates induce households to consume more leisure due to income effect, thereby supplying less labour. Resulting higher aggregate supply in the economy due to the productivity growth increases the investment. In summary, we see that a positive productivity shock increases spending variables like investment and consumption as well as input prices.

3.2 Monetary Policy Shock

Figure 3 and Figure 4 plot the simulated impulse responses of an expansionary monetary policy. An expansionary monetary policy increases the money supply base and lowers the short-term interest rates. As we see in the graphs, expansionary shock triggers a positive response to output and prices, so high inflation. Taylor rule specified in the model works as an automatic stabilizer because higher inflation increases the policy rates via the Taylor rule and hence keeping the inflation in a defined band. The lower short-term interest rate increases the price of the government bonds ($P_t^B = \frac{1}{r_t}$), which leads to a decrease in the demand of these bonds by households. Now, households purchase less bonds and save more by investing in physical capital. While lower interest rate incentivise more private investment, on the other hands, it lowers opportunity for government bonds.

To summarise, the study presents a basic set-up of a New-Keynesian dynamic stochastic general equilibrium (DSGE) model. DSGE models are state of the art models in macroeconomic analysis. These models are based on theoretical foundations, hence are more capable in providing structural analysis compared to their counterpart reduced form vector autoregressive (VAR) models. The present study builds a two-sector closed DSGE model with nominal price rigidities. In the model, two economic agents interact among each other and a monetary authority sets the interest rate. Model features imperfect competition in intermediate firms production sector where firms follow the Calvo rule to set their prices. To make model equations more intuitive and to take the model to computational techniques, it is transformed in the log-linearised form. Model is solved numerically by assigning parameters values. Simulation of the model presents interesting results which are in line with the economic theory. A positive productivity shock improves economic activity and an expansionary monetary policy shock increases output for the short-term but decreases the demand for government bonds.

The current model can be extended by including the government sector which will enable to analyse fiscal policy shocks along with monetary shocks. Certain improvements like inclusion of habit persistence in the utility function, integration of term-structure dynamics can improve the performance of the model by making it more competitive for policy analysis and forecasting.

Figures

Figure 1: IRF of a positive productivity shock (part1)

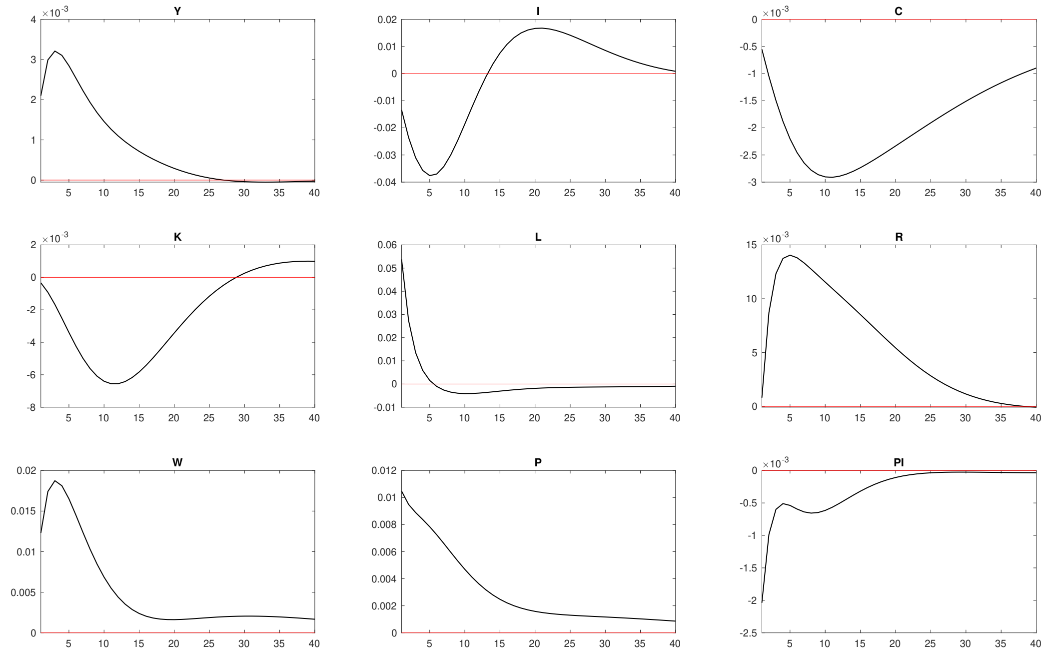


Figure 2: IRF of a positive productivity shock (part2)

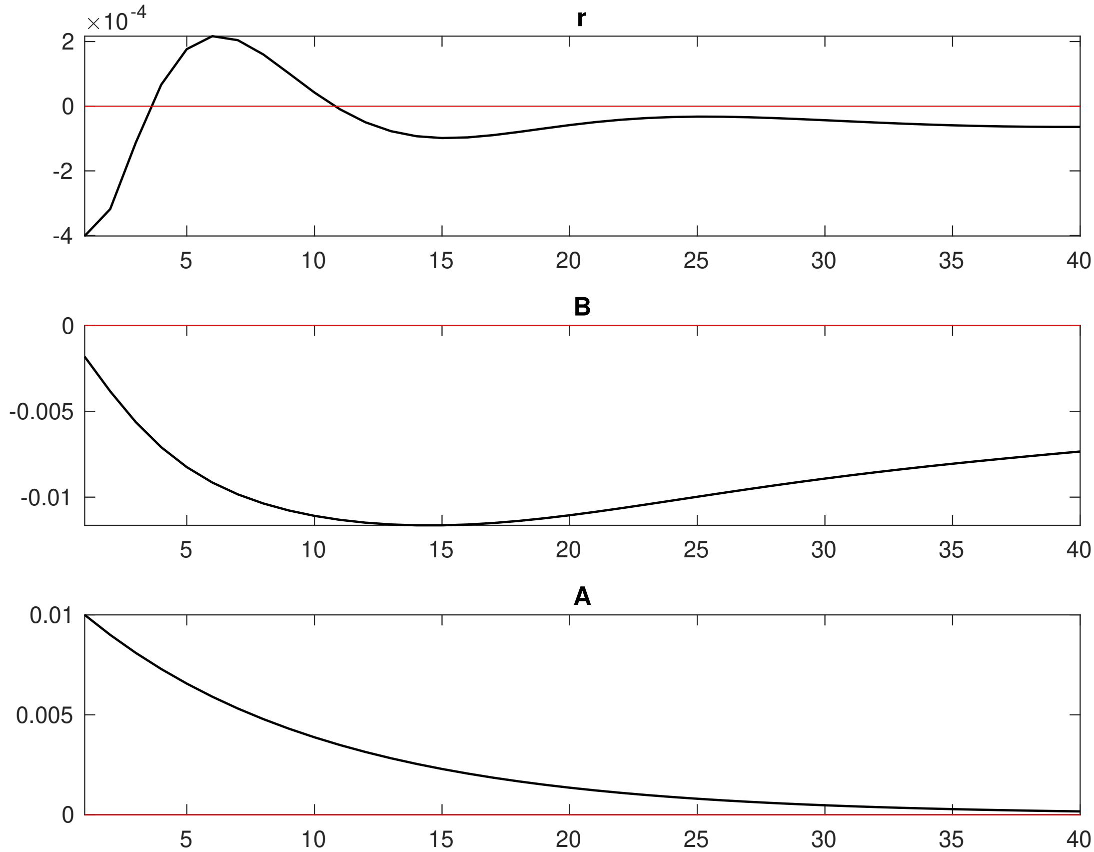


Figure 3: IRF of an expansionary monetary policy shock (part1)

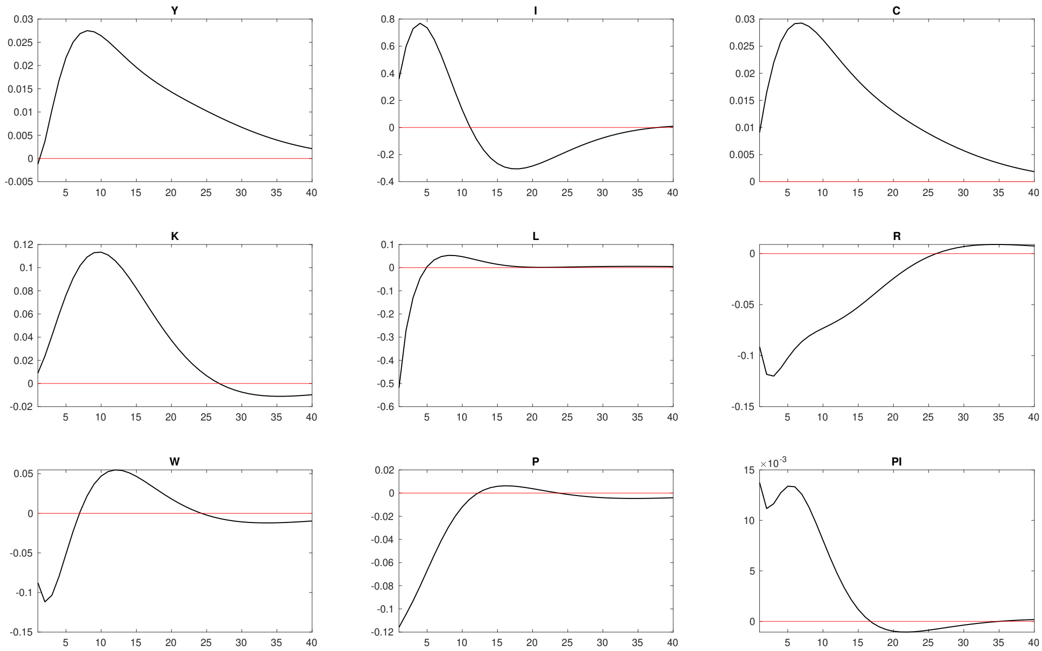
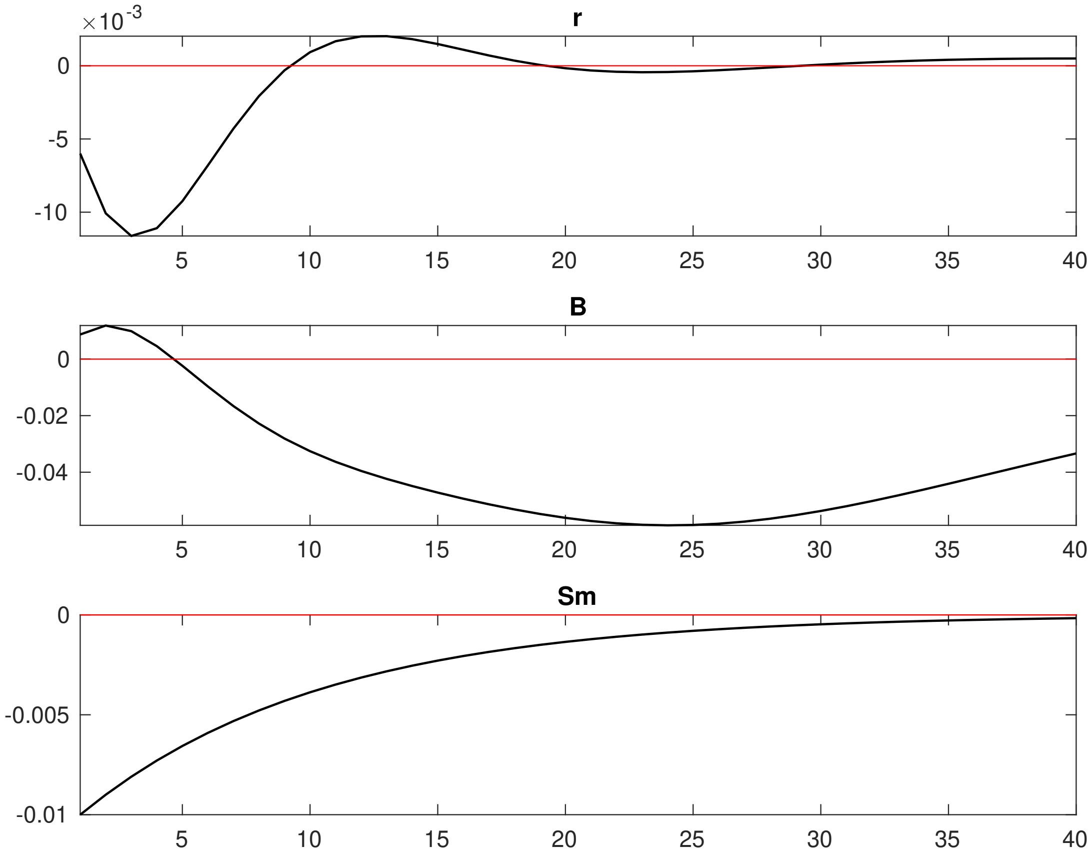


Figure 4: IRF of an expansionary monetary policy shock (part2)



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