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Abstract

Paper proposes a model of retirement duration maximization based on retiree's ex-ante intended retirement age and subjective survival time estimate. The optimum result needs that retirement age is an increasing convex function of survival time estimate supporting postponed retirement age with longer intended retirement duration. As a result, the average actual observed retirement duration is less than the intended duration. The result is valid irrespectively of biasedness of subjective survival estimates.

JEL classification: J14, C41, D81

Keywords. Optimal retirement age, intended retirement duration, subjective survival time estimates, convex function

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1. Introduction

In economics, timing of retirement is typically modelled with life-cycle approach where labour supply, incomes, health, and wealth are the main forcing variables in retirement decisions (for partial literature reviews, see e.g. Blundell et al (2016), Laitner and Sonnega (2012), Blake (2006)). In the dynamic life-cycle models focusing on retirement period, the representative agent takes *expected lifetime* or some estimate of it as a given parameter in his/her "optimal" retirement date decision (see e.g. Wolfe (1983), Chang (1991), Bloom et al. (2004), Sheshinski (2006), Kalemli-Ozcan and Weil 2010). Surprisingly, the papers that focus explicitly on how retirement timing and age depend on death hazard or survival probability are almost non-existing. Sheshinski (2008) is an interesting deviant here as his focus is on annuities (see also Milevsky (2006)).

However, the literature on subjective survival probabilities have shown that persons derive serious estimates concerning their survival probabilities and let these also affect their retirement decisions (see, e.g. O'Donnell et al. (2008), Palloni and Novak (2016), Wu et al. (2015)). van Solinge and Henkens (2009) focus on subjective life expectations, retirement *intentions*, and actual retirement times with Dutch retirement data in two waves from years 2001 and 2007. They show that employees who *expect* to live longer, *intend* to retire later than those who expect a shorter life span. In addition, the results suggest that particularly employees with a high perceived life expectancy and an intention to work longer don't succeed in carrying their intentions into effect. Khan et al. (2014) show with the US data that survey respondents who are more optimistic about their survival to age 75 or 85 also expect to work five months longer on average. They report also that actual retirement increases with subjective life expectancy. In sum, the scant empirical evidence on the role of (subjective) death hazard rate and survival time estimates in the formation of retirement decision is mixed and inconclusive. This is a natural outcome as the retirement timing decision is a complex process where pre- and post-retirement factors play contrasting roles. Main difficulty here is the fact that retirement period is typically the last stage before death and the expectations over it are hard – even unpleasant – to do. Large subjective biases and over-estimates with respect to death age are often observed.

In the following we try to model some salient aspects of retirement process with the intended retirement spell approach where the subjective survival time estimates have an important role.

We show that maximization of *intended* retirement spell can happen if the *intended* retirement age is an *increasing convex* function of subjective survival time estimate.

2. Intended retirement duration

The importance of expected survival in the retirement time decision is evident when we observe that retiree must pay attention to the elementary counting relation $RETd_i \equiv \chi^*_{D,i} - T_{R,i} \geq 0$, where $RETd_i$ is his/her actualized retirement duration, $\chi^*_{D,i}$ is age of the death, and $T_{R,i}$ is the retirement age. However, at retirement age $T_{R,I}$, the remaining lifetime is a (unknown) random variable $\chi^*_{T,i}$. Retiree has some conjecture or expectation over it, i.e. $\chi_{T,i}$. Typically, $\chi_{T,i}$ is person's *subjective* estimate of his/her survival time to some specific age. This estimate of remaining lifetime can differ greatly from retiree's actual years to death. Note that, if subjective survival age estimate is important for the person in determining his/her retirement age, then we should write the above relation in the form of ex-ante *intended* or *planned* retirement duration equation like $RETd_i^I(\chi_{T,i}) = \chi_{T,i} - T_{R,i}(\chi_{T,i})$ where $\chi_{T,i}$ determines also the age of retirement.

Our benchmark hypothesis, not conflicting the present-day facts on increased longevity and retirement ages, is that if person has a high subjective estimate for his/her survival time he/she will postpone retirement age in a proportion to this estimate. Argument for the hypothesis has two parts. First, the intended retirement spell is always non-negative, i.e. in the equation $RET_i^I(\chi_{T,i}) = \chi_{T,i} - T_{R,i}(\chi_{T,i}) \geq 0$ the last term $T_{R,i}(\chi_{T,i})$ can't be larger than $\chi_{T,i}$. Secondly, the maximization of retirement duration can happen only when $T_{R,i}(\chi_{T,i})$ is *convex* in $\chi_{T,i}$. Now, the intended retirement duration increases when $0 < T_{R,i}\,'(\chi_{T,i}) \leq 1$ [1)], i.e. retirement age is postponed but proportionally less than the estimate of survival time increases.

We agree that many other factors – not analysed here – affect both subjective survival estimate and retirement age. Typically, pre-retirement income level compared to pension level delays the retirement age. Health and wealth condition retirement decision like preferences for leisure and work.

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[1] Note that $\underset{\{\chi_T\}}{Max}\{RETd^I(\chi_T) = \chi_T - T_R(\chi_T)\}$ gives $RETd^I\,'(\chi_T^{MAX}) = 1 - T_R\,'(\chi_T^{MAX}) = 0.$ Now $T_R\,'(\chi_T^{MAX}) = 1,$ and $RETd^I\,''(\chi_T^{MAX}) = -T_R\,''(\chi_T^{MAX}) < 0$ if $T_R\,''(\chi_T) > 0$. Under these assumptions function $RETd^I(\chi_T)$ is *concave* in χ_T with maximum taking place at χ_T^{MAX}.

3. Over-optimism of intended retirement duration

Next, we analyse what are the implications of above proposed retirement age model. Note that the actual ex-ante *expected retirement duration* for person is determined with his/her random remaining life age $\chi^*_{T,i}$

$$(1) \qquad E[RETd_i(\chi^*_{T,i})] = E[\chi^*_{T,i}] - E[T_{R,i}(\chi^*_{T,i})] \geq 0.$$

The last term in equation needs some clarification. It says that expected retirement age depends also on the random remaining lifetime. In other words, if the retiree could know in advance the random draw value of his/her lifetime length, the retiree would calculate his/her retirement age according to it. With a low value of true death risk, he/she postpones retirement age compared to case when the objective risk is high. However, in practice this approach is seldom operational for retiree's ex-ante derivation of the optimal length of intended retirement duration. Instead, the retiree forms the subjective estimate for his/her remaining life length with the age that maximizes his/her intended retirement duration: $\chi^{MAX}_{T,i}$ (see above, footnote 1). Now, under the assumption of unbiased estimate, $\chi^{MAX}_{T,i} = E[\chi^*_{T,i}]$, the actual expected retirement duration differs from the intended retirement duration as

$$(2) \qquad E[\chi^*_{T,i}] - E[T_{R,i}(\chi^*_{T,i})] \neq \chi^{MAX}_{T,i} - T_{R,i}(\chi^{MAX}_{T,i}) = E[\chi^*_{T,i}] - T_{R,i}(E[\chi^*_{T,i}]) .$$

The inequality comes from the result that $E[T_{R,i}(\chi^*_{T,i})] \neq T_{R,i}(E[\chi^*_{T,i}])$ because the retirement age is not a linear function of remaining life length.

Next, we analyse how big the inequality is and what is its substance. First, we approximate $RETd_i(\chi^*_{T,i})$ with the optimal survival time estimate $\chi^{MAX}_{T,i}$. The 2nd order Taylor approximation with $\chi^{MAX}_{T,i}$ for $RETd_i(\chi^*_{T,i})$ is

$$(3) \qquad \begin{aligned} RETd_i(\chi^*_{T,i})] = \chi^*_{T,i} - T_{R,i}(\chi^*_{T,i}) \approx \chi^{MAX}_{T,i} - T_{R,i}(\chi^{MAX}_{T,i}) + [1 - T_{R,i}\,'(\chi^{MAX}_{T,i})](\chi^*_{T,i} - \chi^{MAX}_{T,i}) \\ - \tfrac{1}{2} T_{R,i}\,''(\chi^{MAX}_{T,i})(\chi^*_{T,i} - \chi^{MAX}_{T,i})^2. \end{aligned}$$

Second, by taking expectation of this gives

$$E[RETd_i(\chi^*_{T,i})] \approx \chi^{MAX}_{T,i} - T_{R,i}(\chi^{MAX}_{T,i}) + [1 - T_{R,i}'(\chi^{MAX}_{T,i})]E(\chi^*_{T,i} - \chi^{MAX}_{T,i}) \tag{4}$$

$$- \tfrac{1}{2} T_{R,i}''(\chi^{MAX}_{T,i}) E(\chi^*_{T,i} - \chi^{MAX}_{T,i})^2$$

$$= RETd^I_i(\chi^{MAX}_{T,i}) + [1 - T_{R,i}'(\chi^{MAX}_{T,i})]E(\chi^*_{T,i} - \chi^{MAX}_{T,i}) - \tfrac{1}{2} T_{R,i}''(\chi^{MAX}_{T,i})\sigma^2_\chi.$$

The result reduces with $E(\chi^*_{T,i} - \chi^{MAX}_{T,i})^2 = \sigma^2_\chi > 0$, and with assumptions of $E(\chi^*_{T,i} - \chi^{MAX}_{T,i}) = 0$ and convexity of $T_{R,i}(\chi_{T,i})$ to

$$E[RETd_i(\chi^*_{T,i})] - RETd^I_i(\chi^{MAX}_{T,i}) \approx -\tfrac{1}{2} T_{R,i}''(\chi^{MAX}_{T,i})\sigma^2_\chi < 0. \tag{5}$$

We get an interesting result that the intended retirement duration is longer than the actual expected duration although the size of the prediction error of subjective life length estimate $E[\chi^*_{T,i} - \chi^{MAX}_{T,i}]$ is zero. Note that the recent empirical evidence does not support result that person's subjective estimate of his/her survival time is unbiased predictor of person's actual life length (see, e.g. Wu et al. 2015). When the subjective estimates are biased, $E[\chi^*_{T,i} - \chi^{MAX}_{T,i}] \neq 0$, the above over optimism result is still valid because the intended retirement duration maximization happens when $1 - T_{R,i}'(\chi^{MAX}_{T,i}) = 0$. In sum, irrespectively of the quality of subjective life length estimate, the retiree has too optimistic plan for his/her intended retirement length when compared to the actual expected retirement duration.

Alternatively, in terms of risk taking, we can say that the retiree, when not able to evaluate his/her expected retirement duration $E[RETd_i(\chi^*_{T,i})]$ in the correct way, behaves like a risk taker. The retiree takes the stake against the law of human mortality with maximising his/her intended retirement duration with subjective remaining life estimate. Now the retirement duration is over-estimated. This is a direct result of convex function $T_{R,i}(\chi_{T,i})$ that is supporting the optimal planned retirement duration.

4. Conclusions

The target of paper was to overcome some evident problems in derivation of optimal length of retirement duration. Main obstacles with this difficult problem were solved with intended retirement age approach that was based on subjective survival time estimates. We argued that, if people make (subjective) expectations on their life lengths in their retirement age decision, they do something like we have proposed with our model. They form subjective estimates on their survival times and adjust their planned retirement age according to it with is increasing convex function. We obtained an interesting result that the intended retirement duration is longer than the actual expected duration irrespectively of the size of prediction error of retiree's subjective survival time estimate.

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