Social Identity, Redistribution, and Development

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Abstract

Empirical works suggest that income redistribution promotes economic growth and development by reducing inequality and increasing educational investment of the poor. However, the scale of redistribution, to be precise, the inequality-reducing effect of taxes and transfers, is limited in many developing countries. Why is the scale of redistribution small, and how does it affect development? This paper focuses on the role of social identity, whose importance in redistribution and development is supported in existing empirical studies. Under what conditions is national identity realized, and how does it affect the economic outcomes?

To answer the questions, this paper develops a dynamic model of income redistribution and educational investment augmented with social identification and explores the interaction among identity, redistribution, and development theoretically.

Keywords: social identity, redistribution, nation-building policies, economic development

JEL classification numbers: D72, D74, O10, O20

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1 Introduction

Cross-country differences in economic growth and development are enormous. Empirical evidence shows that income and asset inequalities are negatively associated with these performances (Deininger and Squire, 1998; Easterly, 2007). This suggests that redistributive policies might stimulate growth and development by mitigating inequality. Indeed, the empirical study by Berg et al. (2018) indicates that income redistribution, unless very large-scale, increases economic growth by lowering income inequality. They also find that lower inequality is associated with higher human capital. Further, Hanushek and Woessmann (2012a) find that an increase in educational achievement, measured by cognitive skills, has a large effect on growth. These findings suggest that redistribution might promote growth by reducing inequality and thus increasing educational investment of the poor.

However, Berg et al. (2018, Figure 5) find that the scale of redistribution, to be precise, the inequality-reducing effect of taxes and transfers, in many developing countries is much smaller than in developed countries. Further, whereas the redistributive effect of the fiscal system is greater in countries with higher market income inequality among developed nations, such a tendency is not observed for developing nations. Goni, Lopez, and Serven (2011) find that, while market inequality is not very different between Latin American and Western European countries, after-tax after-transfer inequality is much higher in the former group of countries. This is also the case when public expenditures on education and health as well as cash transfers are considered.

Why is the scale of redistribution small in many developing countries, and how does it affect economic development? This paper focuses on the role of social identity. The lack of a shared national identity, that is, the dominance of subnational identities over national identity, is often blamed for poor economic performance of socially diverse countries (Collier, 2009; Michalopoulos and Papaioannou, 2015; Fukuyama, 2018). And empirical findings suggest that national identity has a positive effect on redistribution (Chen and Li, 2009; Transue, 2007; Singh, 2015). Under what conditions is national identity realized, and how does it affect redistribution and development?

Technological change, a major driving force of growth and development besides human and physical capital accumulation, becomes increasingly skill-biased. Technology available for present developing countries is much more skill-biased than the one developed countries used when they underwent the modernization and industrialization of the economies. How does skill-biased technical change (SBTC) affect identity, redistribution, and development?

This paper explores the interaction among identity, redistribution, and development theoretically. To do so, it develops a dynamic model of income redistribution and educational investment augmented with social identification, mainly drawing on the model by Shayo (2009).

Shayo (2009) develops a model that augments the standard political economy model of redistributive taxation (Meltzer and Richard, 1981) with socio-psychological factors. In the model, two classes, the poor and the rich, exist and the government imposes proportional tax on their incomes to provide lump-sum transfer, where the tax rate is determined by voting. What is different from the standard model is that individual utility depends not only on disposable income but also negatively on the *perceived distance* between oneself and the group one identifies with (their class or the nation) and positively on the group's *status*. In other words, one bears a large cognitive cost when they are very different from other members of the group in relevant aspects (e.g., income level), but takes pride in being a member when the group's status is high (e.g., when its average income is high). These socio-psychological components are major determinants of social identification and intergroup behaviors, according to influential theories in social psychology (Tajfel and Turner, 1986; Turner et al., 1987) and empirical evidence (Manning and Roy, 2010; Hett, Mechtel, and Kröll, 2020; Fouka, Mazumder, and Tabellini, 2022).[1,2] Importantly, because the components differ depending on which group one identifies with, the social identities of people influence the tax rate and thus their disposable incomes. Further, social identity is *endogenously determined*: one chooses the identity that brings them higher utility. Hence, identity and individual and aggregate outcomes interact with each other.

The present model differs from Shayo (2009) mainly in two respects.[3] First, pre-tax pre-transfer incomes are endogenously determined. The rich (poor) are skilled (unskilled) workers and their earnings depend on the proportion of skilled workers. Second, the model is dynamic and variables such as the proportion of skilled workers, earnings, social identity, and tax rate change endogenously over time. The dynamic part of the model is based on Galor and Zeira (1993) and Yuki (2007, 2008), in which individuals who are heterogeneous in wealth received from their parents decide on educational spending that must be self-financed and are needed to become skilled workers. In this type of models, the proportion of those who have enough wealth for education in the initial period is the critical variable determining the dynamics. That is, if the proportion of such individuals is high enough, the share of skilled workers and total output increase and the earnings differential between skilled and unskilled workers decreases over time, and the welfare level of everyone becomes equal in the long run; otherwise, the skilled workers' share and total output are low and the earnings and welfare disparities are high even in the long run, i.e., the society is in "poverty trap".

Based on such a model, the paper examines the dynamics and long-run outcomes of the skilled workers' share, identity, redistribution, and development. Main results are summarized as follows.

First, given the skilled workers' share, the rate of redistributive taxation is higher as the proportion of individuals identifying with the nation is higher. This is consistent with empirical findings on the relationship between national identity and redistribution (Chen and Li, 2009; Transue, 2007; Qari, Konrad and Geys, 2012; Singh, 2015).

Second, the dynamics and long-run outcomes are affected by the level of the exogenous component of the national status (in comparison to that of the class status) and inter-class differences in attributes representing culture, norms, and values or their salience in people's minds.[4] In the real world, the exogenous part of the national status would be high when the people believe that they share a glorious history, rich culture, or a "right" sense of values because they feel proud of belonging to such a nation.

In particular, when the exogenous component of the national status is higher or inter-class cultural differences are smaller or less salient in people's minds, the society is less likely to be in "poverty trap" in the sense that it can avoid the trap under the worse initial condition on wealth distribution, i.e., the lower share of those who can afford education. Further, when the society begins with a favorable initial condition, the proportion of skilled workers increases faster and the

[1] The concept of perceived distance is the basis of a major social psychological theory, self-categorization theory (Turner et al., 1987). Intergroup status differences comprise major factors affecting intergroup behaviors such as conflict and discrimination, according to a closely related theory—social identity theory (Tajfel and Turner, 1986).

[2] Evidence suggests that perceived distance and status affect identity. For example, Hett, Mechtel, and Kröll (2020), based on a lab experiment, find that participants in the experiment prefer groups to which they have a smaller social distance and which have a higher social status and their social identity preferences are related to their choices in dictator games.

[3] Other important differences from Shayo (2009) are the following. First, the perceived distance depends on the difference in disposable income between oneself and the group one identifies with, not the difference in pre-tax pre-transfer income. Second, the tax rate is determined by probabilistic voting (Lindbeck and Weibull, 1987), not by majority voting. These settings are the same as Ghiglino, Júarez-Lunam, and Müller (2021), but in their model, social identities of individuals are exogenous.

[4] The national (class) status also depends on the average disposable income of the nation (class) and thus is endogenous.

equality in welfare is realized sooner, when the national status is higher for exogenous reasons or the inter-class cultural distances are smaller or of less concern. This is because social identity and redistribution depend on these exogenous factors.[5] What is notable is the case where these factors are not very high or low. In this case, while the skilled workers' share is low, skilled workers identify with their class and unskilled workers identify with the nation and the dynamics do not depend on the exogenous factors. When the skilled share becomes sufficiently high, the society generally experiences the change in social identity, which has a significant effect on the subsequent dynamics. If the national status is relatively high for exogenous reasons or inter-class cultural differences are small or not very salient in people's minds, the society shifts to *universal national identity*, as a result, the rate of redistributive taxation increases and thus the upward mobility of the poor accelerates; otherwise, it shifts to *universal class identity*, thus the tax rate decreases and the upward mobility slows down or stops.

Third, skill-biased technical change (SBTC) has negative effects on the upward mobility and long-run outcomes by widening the inter-class wage differential and changing social identity and redistribution. As SBTC proceeds, the society becomes more likely to be in "poverty trap", and when it starts with a favorable initial condition, the skilled workers' share increases more slowly and the equality in welfare is realized later. Further, when SBTC continues, the society generally shifts to an equilibrium with a smaller portion of people identifying with the nation at some point. The shift lowers the rate of redistributive tax and makes the negative effects of SBTC stronger.

The result suggests that large cross-country differences in the level and speed of development might be due to differences in the exogenous component of the national status (relative to that of the class status) and in inter-class distances in culture, norms, and values or people's concerns with the distances, as well as differences in the initial distributions of wealth and access to advanced technology. In many developing countries, the belief that people share a glorious history, rich culture, or a “right” sense of values is weak and inter-class differences in culture, norms, and values are large or thought to be serious. According to the model, these make the national status low or the perceived distance to the other class large, the formation of common national identity difficult, and as a result, the scale of redistribution limited, the upward mobility of the poor through education and the pace of development slow. Various empirical studies (Blouin and Mukand, 2019; Chen, Lin, and Yang, 2020; Cáeres-Delpiano et al., 2021) indicate that *nation-building policies*, such as school education and government propaganda that emphasize common history, culture, and values and policies promoting between-group contact that might make inter-class distances in culture, norms, and values less salient in people's minds can effectively fortify national identity. Such policies could lift the national status or diminish (or deemphasize) the inter-class differences, thus would be critical in divided societies.

The result on SBTC shows that as the skill biasedness of technology is higher, inequality is higher, national identity and large-scale redistribution are harder to be realized, thus the upward mobility and the speed of development are slower. This may be another reason why the pace of development, particularly of the poor, in many developing countries are slower than when developed countries experienced the industrialization and modernization of the economies. The result might also explain the lack of increased demand for and scale of redistribution in advanced economies during the last several decades (Ashok et al., 2016; Piketty, Saez, and Stantcheva, 2014).

Finally, classic modernization theories in political science (Deutsch, 1953; Gellner, 1983; Weber, 1979), based on the past experience of Europe, argue that modernization (including industrializa-

[5] That is, the proportion of people identifying with the nation and the rate of redistributive taxation are higher, when the exogenous part of the national status is higher or inter-class cultural differences are smaller or of less concern.

tion and universal education) leads to widespread national identity at the expense of subnational identities (Robinson, 2014). The result of the present model suggests that these theories hold only when the national status is relatively high or inter-class distances in culture, norms, and values are relatively small or of little concern.

This paper contributes to the theoretical literature on the relation between social identity and redistribution (Shayo 2009; Lindqvist and Östling, 2013; Holm, 2016; Dhami, Manifold, and al-Nowaihi, 2021; and Ghiglino, Júarez-Lunam, and Müller, 2021). Besides Shayo (2009), some settings of the model follow Ghiglino, Júarez-Lunam, and Müller (2021) (footnote 3), who consider a society with two ethnicities and three income groups, but unlike their model, identities are endogenous. None of the works analyze the relation among identity, redistribution, and development.

More broadly, the paper adds to the theoretical literature on relations between identity and economic behaviors (Akerlof and Kranton, 2000; Shayo, 2009; Benabou and Tirole, 2011; Bisin et al., 2011; Sambanis and Shayo, 2013; Bernard, Hett, and Mechtel, 2016; Carvalho and Dippel, 2020; Grossman and Helpman, 2020; Bonomi, Gennaioli, and Tabellini, 2021; Yuki, 2021).[6] By generalizing the pioneering work of Akerlof and Kranton (2000), Shayo (2009) constructs the basic framework and applies it to analyze the political economy of income redistribution. Shayo's (2009) framework has been applied to various issues. The most closely related to the present work is Yuki (2021). Drawing on the seminal work on the interaction between social identity and ethnic conflict by Sambanis and Shayo (2013), Yuki (2021) examines how the shift of economic activities from ethnically-segregated traditional sectors to the integrated modern sector driven by the increased productivity of the latter sector affects identity, ethnic conflict, and development. Unlike the present model, his model does not consider educational investment and income redistribution. Another notable application is Grossman and Helpman (2020), who, motivated by a recent reversal of trade policies in some western countries seemingly influenced by rises of populism and ethnic tensions, constructs a political economy model of trade policy with social identification and examine how policies are affected by changes in the identification patterns triggered by events such as increased ethnic tensions.

The rest of the paper is organized as follows. Section 2 presents and Section 3 examines the static model. Section 4 presents and analyzes the dynamic model, and Section 5 presents and explains the main results. Section 6 concludes. Appendix A contains propositions used in Section 3, Appendix B provides supplementary analysis for Section 4, and Appendix C contains proofs.

2 Model

The society consists of skilled workers, unskilled workers, and a government. The government imposes a proportional tax on earnings to finance lump-sum transfer. The tax-transfer policy is determined by probabilistic voting. Utility depends not only on one's disposable income but also on socio-psychological components that depend on her social identity. Identity is determined endogenously: one chooses to identify with her economic class or the nation. Markets are competitive.

2.1 Economic environment

The total population is 1 and the number of skilled workers is H, which is endogenized in Section 4. Workers supply 1 unit of labor to receive earnings, pay the proportional tax on earnings, receive the lump-sum transfer, and spend disposable income on consumption. The disposable income of individual i is denoted by

[6] Besides the already mentioned works, recent empirical and experimental studies on identity include Dehdari and Gehring (2021) and Assouad (2021).

$$y_i = (1-\tau)w_i + T, \tag{1}$$

where w_i is her earnings, $\tau \in [0,1)$ is the tax rate, and T is the transfer.

The government uses tax revenue entirely for the lump-sum tranfer, but taxation involves deadweight loss. The deadweight loss is assumed to be quadratic, thus the governmental budget constraint equals

$$T = \left(\tau - \frac{1}{2}\tau^2\right)\overline{w}, \tag{2}$$

where $\overline{w}$ is average earnings of the population.

The final good is produced by using skilled labor and unskilled labor as inputs. The production function takes the following CES form:

$$Y(=\overline{w}) = \left\{\alpha(A_sH)^{\frac{\sigma-1}{\sigma}} + (1-\alpha)[A_u(1-H)]^{\frac{\sigma-1}{\sigma}}\right\}^{\frac{\sigma}{\sigma-1}}, \alpha \in (0,1),\ \sigma \in (1,3], \tag{3}$$

where A_s (A_u) is the level of skilled (unskilled) labor augmenting techonology, and σ is the elasticity of substitution between skilled and unskilled workers. $\sigma \in (1,3]$ is assumed following Autor, Goldin, and Katz (2020) who estimate $\sigma = 1.62$ using U.S. data and state that estimates in the literature typically fall in the 1 to 2.5 range.

From first-order conditions of the profit maximization problem of a representative firm, the wage of skilled workers and that of unskilled workers respectively equal

$$w_s = \left\{\alpha(A_sH)^{\frac{\sigma-1}{\sigma}} + (1-\alpha)[A_u(1-H)]^{\frac{\sigma-1}{\sigma}}\right\}^{\frac{\sigma}{\sigma-1}-1} \alpha(A_sH)^{\frac{\sigma-1}{\sigma}}\frac{1}{H}, \tag{4}$$

$$w_u = \left\{\alpha(A_sH)^{\frac{\sigma-1}{\sigma}} + (1-\alpha)[A_u(1-H)]^{\frac{\sigma-1}{\sigma}}\right\}^{\frac{\sigma}{\sigma-1}-1} (1-\alpha)[A_u(1-H)]^{\frac{\sigma-1}{\sigma}}\frac{1}{1-H}. \tag{5}$$

2.2 Preferences

As in Shayo (2009), individual utility depends not only on one's disposable income but also negatively on the *perceived distance* to a social group with which one identifies (either one's economic class—skilled/unskilled, i.e., the middle and upper classes/the working class—or the nation) and positively on the *status* of the group. In other words, one incurs a mental cost when one is different from others of the group in relevant features but takes pride in belonging to the group when its status is high. These socio-psychological components, based on influential theories in social psychology (Tajfel and Turner, 1986; Turner et al., 1987), are the major factors affecting social identification and intergroup behaviors (Manning and Roy, 2010; Hett, Mechtel, and Kröll, 2020; Fouka, Mazumder, and Tabellini, 2022).[7]

An individual perceives the distance or proximity to a social group (her class or the nation) based on the difference between her disposable income and the average disposable income of the

[7] For the United Kingdom, Manning and Roy (2010) find that the non-whites, whose perceived distance to the "average national" would be greater than that of the whites, are less likely to think of themselves as British. They also find that immigrants from poorer and less democratic (i.e., lower status) countries assimilate faster into a British identity. Hett, Mechtel, and Kröll (2020), based on a lab experiment, find that participants in the experiment prefer groups to which they have a smaller social distance and which have a higher social status and their social identity preferences are related to their choices in dictator games. Fouka, Mazumder, and Tabellini (2022) find that migrations of African Americans from the South to non-southern metropolitan areas stimulated assimilation of European immigrants, which is measured by outcomes such as marriages with native whites and residential integration, for the years 1910–30. Further, they provide evidence suggesting that the higher integration resulted from decreased perceived distance of native whites to European immigrants.

group. The *perceived distance* of an individual of class C ($C = S, U$; S [U] is for skilled [unskilled]) to group G ($G = C, N$; N is for the nation) is represented by[8]

$$\begin{aligned} d_{CG} &= |y_C - y_G| \\ &= (1-\tau)\,|w_C - w_G| \quad (\text{from}(1)), \end{aligned} \tag{6}$$

where y_G and w_G are respectively the average disposable income and earnings of the group and y_C and w_C are those of the class. The distance is measured in absolute value for analytical tractability, following Ghiglino, Júarez-Lunam, and Müller (2021). In the model of Shayo (2009), the perceived distance depends also on the difference in membership in each class, which may be interpreted as the difference in non-economic attributes that represent class-specific culture, norms of behavior, values etc. For ease of presentation, such a dependence is not modeled here, but is considered in Section 5.1.1.

The *status of the social group* G ($G = C, N$) one identifies with, S_G, depends on the exogenous component $\widetilde{S_G}$ and the average disposable income of the group:

$$S_G = \delta \widetilde{S_G} + y_G, \tag{7}$$

where δ is the weight on the exogenous component.[9] The exogenous component of the class status is assumed to be the same for both classes, i.e., $\widetilde{S_S} = \widetilde{S_U} \equiv \widetilde{S_C}$, to simplify analysis significantly; this would not affect main results qualitatively.[10]

The level of the exogenous component of the *national status* $\widetilde{S_N}$ would be high when the people of the nation believe that they share a glorious history, rich culture, or a "right" sense of values or when a nation records commendable performance in international sports competitions because the people would feel proud of belonging to such a nation.

Finally, the utility of a class C individual when she identifies with group G is given by

$$u_{CG} = y_C - \beta d_{CG} + \gamma S_G, \qquad \beta, \gamma > 0. \tag{8}$$

Thus, she cares about not only her own disposable income but also the difference in disposable income between the average individual of the group and her, the group's average disposable income, and the exogenous component of the group's status.

By substituting (1), (2), (6), and (7) into the above equation, the utility for each class-identity combination can be expressed as:

[8] The concept of perceived distance is developed in cognitive psychology in studying how a person categorizes information that comes in to her (stimuli) (Nosofsky, 1986). Turner et al. (1987) apply the concept to the categorization by a person of people, including herself, into social groups, in constructing an influential social psychological theory, self-categorization theory. The theory tries to explain psychological basis of social identification.

[9] Similar to works such as Grossman and Helpman (2021), status is an absolute measure. By contrast, in Shayo (2009), status is a relative measure and is defined as the difference from the reference group. The main results remain unchanged under the alternative specification.

[10] In Ghiglino, Júarez-Lunam, and Müller (2021), $\widetilde{S_N} = \widetilde{S_C} = 0$ is assumed. In Shayo (2009), $\widetilde{S_C}$ can be different between the poor and the rich but the rich's status does not affect the main result because, unlike the present paper, only the poor can influence the policy.

$$
\begin{aligned}
u_{SN} &= (1-\tau)w_s+\left(\tau-\frac{1}{2}\tau^2\right)\overline{w}-\beta(1-\tau)(w_s-\overline{w})+\gamma\left[\delta\widetilde{S_N}+(1-\tau)\overline{w}+\left(\tau-\frac{1}{2}\tau^2\right)\overline{w}\right] \\
&= (1+\gamma)\left(\tau-\frac{1}{2}\tau^2\right)\overline{w}+(1-\tau)w_s-\beta(1-\tau)(w_s-\overline{w})+\gamma\left[\delta\widetilde{S_N}+(1-\tau)\overline{w}\right], \quad (9) \\
u_{SS} &= (1+\gamma)\left(\tau-\frac{1}{2}\tau^2\right)\overline{w}+(1-\tau)w_s+\gamma\left[\delta\widetilde{S_C}+(1-\tau)w_s\right], \quad (10) \\
u_{UN} &= (1+\gamma)\left(\tau-\frac{1}{2}\tau^2\right)\overline{w}+(1-\tau)w_u-\beta(1-\tau)(\overline{w}-w_u)+\gamma\left[\delta\widetilde{S_N}+(1-\tau)\overline{w}\right], \quad (11) \\
u_{UU} &= (1+\gamma)\left(\tau-\frac{1}{2}\tau^2\right)\overline{w}+(1-\tau)w_u+\gamma\left[\delta\widetilde{S_C}+(1-\tau)w_u\right]. \quad (12)
\end{aligned}
$$

In the model, one's social identity, that is, the group one identifies with, is *not fixed*. Between the nation and their class, one "chooses" the group bringing higher utility because of a shorter perceived distance or higher status.[11] One's identity may change if levels of the variables affecting the utility directly or indirectly through choices of others change.

2.3 Political environment and Timing of decisions

As in Grossman and Helpman (2021) and Ghiglino, Júarez-Lunam, and Müller (2021), the political environment is based on the probabilistic voting model (Lindbeck and Weibull, 1987). Two parties, parties 1 and 2, that are different in the non-policy dimension ("ideology") compete for the public office by announcing their electoral platforms on the tax rate, τ_1 and τ_2. They propose platforms to maximize the probability of wining the majority election, expecting that individuals cast their votes sincerely. Voters care about both the policy platforms and the parties' "ideologies".

Individual i in class C who identifies with group G prefers party 1 if

$$
u_{CG}(\tau_1) \geq u_{CG}(\tau_2) + \eta_i + \mu, \quad (13)
$$

where $u_{CG}(\tau_j)$ $(j = 1, 2)$ is the "non-ideology" component of the utility given by (8) when party j implements the tax rate τ_j. η_i is an individual-specific parameter that measures the voter's "ideological" bias toward party 2 and has a uniform distribution on $\left[-\frac{1}{2\phi}, \frac{1}{2\phi}\right]$, where $\phi > 0$. A negative value of η_i implies that the individual has an "ideological" bias in favor of party 1. μ measures the average popularity of party 2 relative to party 1 in the population and has a uniform distribution on $\left[-\frac{1}{2\psi}, \frac{1}{2\psi}\right]$, where $\psi > 0$.

The timing of decisions is as follows. Workers choose their social identities; then, the parties announce their policy platforms noncooperatively; finally, workers vote for the party that offers higher utility and the winning party implements the proposed policy.

3 Analysis

3.1 Tax rate

Because the model can be solved by backward induction, the determination of the tax rate τ is examined first.

[11] By assumption, one does not identify with the nation and their class simultaneously. Conversely, in the model of Grossman and Helpman (2021), an individual identifies with her class always and with the nation also if the additional identity increases the utility, where the utility depends on the sum of the perceived distance to and the status of each group with which she identifies. The present paper does not adopt this specification owing to the complexities associated with the additional terms and the difficulties when analyzing the model.

From (13), a class C ($C = S, U$) individual identifying with group G ($G = C, N$) is indifferent between the two parties when η_i equals $\eta_{CG} \equiv u_{CG}(\tau_1) - u_{CG}(\tau_2) - \mu$. Thus, all individuals in this category with $\eta_i \leq \eta_{CG}$ prefer party 1. Hence, the share of votes party 1 gets equals

$$\begin{aligned} s_1 &= H\phi\left[p\left(\eta_{SN}+\frac{1}{2\phi}\right)+(1-p)\left(\eta_{SS}+\frac{1}{2\phi}\right)\right]+(1-H)\phi\left[q\left(\eta_{UN}+\frac{1}{2\phi}\right)+(1-q)\left(\eta_{UU}+\frac{1}{2\phi}\right)\right] \\ &= \frac{1}{2}-\phi\mu+\phi\begin{pmatrix} H\{p[u_{SN}(\tau_1)-u_{SN}(\tau_2)]+(1-p)[u_{SS}(\tau_1)-u_{SS}(\tau_2)]\} \\ +(1-H)\{q[u_{UN}(\tau_1)-u_{UN}(\tau_2)]+(1-q)[u_{UU}(\tau_1)-u_{UU}(\tau_2)]\} \end{pmatrix}, \end{aligned} \tag{14}$$

where $p \in [0,1]$ ($q \in [0,1]$) is the proportion of skilled (unskilled) workers identifying with the nation. s_1 depends on μ and thus is a random variable.

From the above equation, the probability that party 1 wins the election is

$$\begin{aligned} \Pr\left[s_1 \geq \frac{1}{2}\right] &= \Pr\left[\mu \leq \begin{pmatrix} H\{p[u_{SN}(\tau_1)-u_{SN}(\tau_2)]+(1-p)[u_{SS}(\tau_1)-u_{SS}(\tau_2)]\} \\ +(1-H)\{q[u_{UN}(\tau_1)-u_{UN}(\tau_2)]+(1-q)[u_{UU}(\tau_1)-u_{UU}(\tau_2)]\} \end{pmatrix}\right] \\ &= \frac{1}{2}+\psi\begin{pmatrix} H\{p[u_{SN}(\tau_1)-u_{SN}(\tau_2)]+(1-p)[u_{SS}(\tau_1)-u_{SS}(\tau_2)]\} \\ +(1-H)\{q[u_{UN}(\tau_1)-u_{UN}(\tau_2)]+(1-q)[u_{UU}(\tau_1)-u_{UU}(\tau_2)]\} \end{pmatrix}. \end{aligned} \tag{15}$$

Because the probability that party 2 wins the election equals $1 - \Pr\left[s_1 \geq \frac{1}{2}\right]$ and τ_1 and τ_2 enter symmetrically in the above equation, the unique equilibrium is such that the two parties propose the same tax rate, τ, that maximizes the utilitarian social welfare function:

$$\begin{aligned} &H\{pu_{SN}(\tau)+(1-p)u_{SS}(\tau)\} + (1-H)\{qu_{UN}(\tau)+(1-q)u_{UU}(\tau)\} \\ &= (1+\gamma)\left(\tau-\frac{1}{2}\tau^2\right)\overline{w}+(1-\tau)\begin{pmatrix} \overline{w}+\gamma\{H[p\overline{w}+(1-p)w_s]+(1-H)[q\overline{w}+(1-q)w_u]\} \\ -\beta[Hp(w_s-\overline{w})+(1-H)q(\overline{w}-w_u)] \end{pmatrix}+\text{constants}, \end{aligned} \tag{16}$$

where (9)–(12) are used to derive the last equation.

From the above equation, the proposed tax rate equals

$$\tau = 1-\frac{1}{(1+\gamma)\overline{w}}\begin{pmatrix} \overline{w}+\gamma\{H[p\overline{w}+(1-p)w_s]+(1-H)[q\overline{w}+(1-q)w_u]\} \\ -\beta\{Hp(w_s-\overline{w})+(1-H)q(\overline{w}-w_u)\} \end{pmatrix}, \tag{17}$$

if the right-hand side of the equation is positive, otherwise, $\tau = 0$. Note that when $\beta = \gamma = 0$, i.e., socio-psychological factors do not affect utility, $\tau = 0$ holds due to the linear utility, the utilitarian social welfare, and the cost of taxation.

The next lemma shows that (p, q) with p or $q \in (0, 1)$ cannot be a stable equilibrium, thus only $(p, q) = (0, 0), (1, 1), (0, 1), (1, 0)$ can be stable equilibria.

Lemma 1 *Only* $(p, q) = (0, 0), (1, 1), (0, 1), (1, 0)$ *can be stable equilibria.*

Proof. See Appendix C. ■

Based on this lemma, the following proposition summarizes how the tax rate depends on identity choices of the two groups and H.

Proposition 1 *(i) (a)* $\tau = 0$ *when* $p = q = 0$*, i.e, everyone identifies with their class.*

(b) $\tau = \frac{2\beta}{1+\gamma}(a(H) - H)$*, where* $a(H) \equiv \frac{\alpha(A_sH)^{\frac{\sigma-1}{\sigma}}}{\alpha(A_sH)^{\frac{\sigma-1}{\sigma}}+(1-\alpha)[A_u(1-H)]^{\frac{\sigma-1}{\sigma}}} \geq H$ *from* $w_s \geq w_u$*, when* $p = q = 1$*, i.e., everyone identifies with the nation.*

(c) When $p = 0$ *and* $q = 1$*, i.e., the skilled identify with their class and the unskilled identify with the nation,* $\tau = \frac{\beta-\gamma}{1+\gamma}(a(H)-H)$ *if* $\beta > \gamma$ *and* $\tau = 0$ *if* $\beta \leq \gamma$*.*

(d) $\tau = \frac{\beta+\gamma}{1+\gamma}(a(H)-H)$ *when* $p = 1$ *and* $q = 0$*, i.e., the skilled identify with the nation and the unskilled identify with their class.*

(ii) *Given* H*,* τ *is highest when* $p = q = 1$ *and if* $\beta > (\leq)\gamma$*, it is lowest if* $p = q = 0$ *(when* $p = q = 0$ *and* $p = 0,\ q = 1$*). (*$p = 1, q = 0$ *is not an equilibrium when* $\beta \leq \gamma$*.)*

(iii) *Given* p *and* q*, there exists* $H^+ \in (0, \overline{H})$*, where* $\overline{H}$ *is* H *satisfying* $H = a(H) \Leftrightarrow w_s = w_u$*, such that* $\frac{d\tau}{dH} > (<) 0$ *for* $H < (>) H^+$*.*

Proof. See Appendix C. ■

The tax rate is 0 when $p=q=0$, i.e, everyone identifies with their class, and if $\beta \leq \gamma$, when $p=0$ and $q=1$, i.e., the skilled identify with their class and the unskilled identify with the nation.[12] In other cases, τ equals a constant times $a(H)-H$, where $a(H) \equiv \frac{\alpha(A_s H)^{\frac{\sigma-1}{\sigma}}}{\alpha(A_s H)^{\frac{\sigma-1}{\sigma}} + (1-\alpha)[A_u(1-H)]^{\frac{\sigma-1}{\sigma}}}$. Given H, the tax rate is highest when $p=q=1$, i.e, everyone identifies with the nation, and if $\beta > (\leq)\gamma$, it is lowest when $p=q=0$ (when $p=q=0$ and $p=0$, $q=1$).[13] ($p = 1$, $q = 0$ is not an equilibrium when $\beta \leq \gamma$.)

Hence, given the proportion of skilled workers, the rate of redistributive taxation is higher as the proportion of individuals identifying with the nation is higher.[14,15] National identity has a positive effect on the tax rate for the following reasons. First, redistribution reduces the between-class disparity in disposable income and thus contributes to narrowing the perceived distance to the "average national". Second, skilled workers with a national identity accept a high tax rate that hurts their disposable income and the status of their class because they are concerned with the national status rather than the class status. While concern with the national status has a negative effect on the tax rate preferred by unskilled workers because tax lowers the average disposable income (due to the taxation cost) and the national status, this effect is dominated by the effect through the perceived distance or the effects associated with skilled workers.[16] The result is consistent with empirical findings on the relationship between national identity and redistributive

[12] Ghiglino, Júarez-Lunam, and Müller (2021) allow β and γ to be different between the classes. In this case, $\tau > 0$ holds even when $p=q=0$ if γ (the importance of status in the utility) of the unskilled is strictly greater than that of the skilled. In the present paper, β and γ are assumed to be common to the two classes to make the subsequent analysis of social identification and of the dynamics of social identity and economic outcomes tractable.

[13] Thus, if $\beta > \gamma$, the tax rate is at an intermediate level when $p = 0$, $q = 1$ and $p = 1$, $q = 0$. The proposition does not compare the tax rates of these cases because as shown later, the two cases cannot be an equilibrium at the same level of H.

[14] This is true for the model of Ghiglino, Júarez-Lunam, and Müller (2021) too, in which β and γ can be different between the classes, except that τ when $p=q=0$ is higher than when $p=0$, $q=1$ iff $\beta \leq \gamma$ for the unskilled, i.e., status is more important than perceived distance in their utility, *and* their γ is strictly greater than γ of the skilled.

[15] Shayo (2009) shows that the tax rate preferred by the poor is *lower* under national identity than under class identity. Because the poor determine the tax policy in his model, this is true for the implemented tax rate as well. By contrast, in the present model, τ preferred by the unskilled is higher (lower) under national identity iff $\beta > (<)\gamma$ and the implemented τ is *always* (weakly) *higher* when the proportion of those identifying with the nation is higher. The reason why τ preferred by the unskilled can be higher under national identity is that perceived distance depends on the difference in disposable income. If the perceived distance depends on the difference in *pre-tax pre-transfer* income as in the Shayo model, the preferred τ is lower under national identity. (By contrast, status does depend on disposable income in his model too.) The result on the implemented τ holds because not only the unskilled but also the skilled, whose preferred τ is *always* higher under national identity, influence the tax policy.

[16] To be precise, when $\beta < \gamma$ and $p = 0$, i.e., status is more important than perceived distance in one's utility and the skilled identify with their class, this effect dominates the effect through the perceived distance (no effects associated with the skilled) and thus the preferred tax rate is lower when $q = 1$ than when $q = 0$. However, since the implemented τ equals 0 when $p = q = 0$, $\tau = 0$ when $p = 0$, $q = 1$ as well.

policies (Chen and Li, 2009; Transue, 2007; Qari, Konrad and Geys, 2012; Singh, 2015).[17,18]

Unlike Shayo (2009) and Ghiglino, Júarez-Lunam, and Müller (2021), pre-tax pre-transfer incomes are endogenous and depend on H, thus the tax rate changes with H. The relation between the skilled workers' share and the tax rate is non-monotonic; namely, τ increases with H for relatively small H and decreases with H for relatively large H. The result can be explained as follows. The tax rate preferred by one with a national identity rises as the pre-tax pre-transfer perceived distance to the "average national" increases, because greater redistribution is needed to counteract the larger distance.[19] An increase in H lowers τ preferred by skilled workers with a national identity because it decreases $w_s - \overline{w} = (1-H)(w_s - w_u)$ and thus the distance to the "average national", while it raises (lowers) τ preferred by the unskilled for relatively small (large) H because it increases (decreases) $\overline{w} - w_u = H(w_s - w_u)$ and the distance. When H is small (large), the influence of unskilled (skilled) workers on the tax policy is stronger due to their population share. Hence, the implemented tax rate increases (decreases) with H for relatively small (large) H.[20]

To ensure that a higher tax rate always increases the disposable income of unskilled workers for the range of τ that can be realized, the following assumption is imposed.

Assumption 1 $\frac{2\beta}{1+\gamma} \leq 1$.

The following corollary shows how disposable incomes of skilled and unskilled workers and the inter-class inequality in disposable income depend on their social identities.

Corollary 1 (*i*) *Given H, the disposable income of skilled workers and the inter-class inequality in disposable income are lowest when $p = q = 1$ and if $\beta > (\leq)\gamma$, they are highest when $p = q = 0$ (when $p = q = 0$ and $p = 0$, $q = 1$).*

(*ii*) *Under Assumption 1, given H, the disposable income of unskilled workers is highest when $p = q = 1$ and if $\beta > (\leq)\gamma$, it is lowest when $p = q = 0$ (when $p = q = 0$ and $p = 0$, $q = 1$).*

Proof. See Appendix C. ■

[17] Chen and Li (2009) conduct lab experiments to examine effects of induced group identity on social preferences and find that participants are more averse to payoff differences to groups they identify with. Transue (2007), based on a survey experiment on American whites, finds that, compared to those who feel close to their racial group, those who feel close to the nation support more for a tax increase to improve educational opportunities of minorities and making American identity salient increases their support. Qari, Konrad and Geys (2012), based on data from OECD countries, find that the income tax burden of an above-middle income group is positively related to its strength of national pride (which, as shown below, fosters national identity according to the model), after controlling for the relative income position. Singh (2015), based on statistical analysis and comparative historical analysis of Indian states, show that states with a stronger sense of shared identity spend more on education and health.

[18] By contrast, Shayo (2009), based on survey data for democratic countries, finds that national identity is *negatively* related to support for redistribution in most developed countries, while the evidence is mixed for developing countries. Possible reasons for the different result are as follows. First, he includes dummies for the strength of national identity as independent variables in regressions, while to examine the result of Proposition 1, variables measuring the strength of national identity relative to class identity would be needed: it is possible that both the absolute strength of national identity and the relative strength of class identity are large simultaneously. Second, the national identity dummies, which are based on answers to the question "How proud are you to be [e.g., French] ?" in the World Values Survey, might capture the strength of national pride one has when comparing their country with *other countries*, not subnational groups (e.g., class).

[19] When one identifies with her class, the perceived distance is 0 and thus it does not affect the preferred tax rate.

[20] To be precise, when $p \neq q$, increased H affects τ through the status term as well, unless $p = 0, q = 1$ and $\beta \leq \gamma$, in which $\tau = 0$. When $p = 0, q = 1$ and $\beta > \gamma$, the effect through the status term and the one through the perceived distance term operate in the opposite directions, but the latter dominates, thus the relation between H and τ is as described in the main text. When $p = 1, q = 0$, the effect through the two terms operate in the same direction.

Given H, the disposable income of unskilled workers is highest, that of skilled workers and the inter-class disparity in disposable income are lowest when $p{=}q{=}1$, while if $\beta > (\leq)\gamma$, the unskilled income is lowest, the skilled income and the inter-class inequality are highest when $p{=}q{=}0$ (when $p = q = 0$ and $p = 0,\ q = 1$). That is, given the skilled workers' share, the disposable income of the unskilled (skilled) is higher (lower) and thus the inter-class income inequality is lower as the proportion of those identifying with the nation is higher.

3.2 Social identity

Next, what social identities workers choose are examined. The choice of social identity affects the perceived distance term and the status term of one's utility. Thus, one chooses the identity with the higher value of the sum of the two terms. Hence, from (9)−(12) and $H \leq \overline{H}$, i.e., $w_s \geq w_u$, the following conditions are obtained ($\Delta\widetilde{S_N} \equiv \widetilde{S_N} - \widetilde{S_C}$). Note that $H \geq \frac{\beta+\gamma}{2\beta}$ holds only when $\beta > \gamma$.

$$p = q = 1 \text{ iff } \gamma\delta\Delta\widetilde{S_N} \geq (1-\tau)(w_s - w_u)\max[(\beta+\gamma)(1-H), (\beta-\gamma)H]$$

$$\Leftrightarrow \gamma\delta\Delta\widetilde{S_N} \geq (1-\tau)(w_s - w_u)(\beta+\gamma)(1-H) \text{ for } H \leq \min\left\{\tfrac{\beta+\gamma}{2\beta}, \overline{H}\right\} \quad (18)$$

$$\text{and } \gamma\delta\Delta\widetilde{S_N} \geq (1-\tau)(w_s - w_u)(\beta-\gamma)H \text{ for } H \in \left[\tfrac{\beta+\gamma}{2\beta}, \overline{H}\right] \text{ when } \overline{H} > \tfrac{\beta+\gamma}{2\beta}, \quad (19)$$

where τ is the tax rate when $p = q = 1$, i.e., $\tau = \frac{2\beta}{1+\gamma}(a(H) - H)$ from Proposition 1 (i)(b).

$$p = q = 0 \text{ iff } \gamma\delta\Delta\widetilde{S_N} \leq (w_s - w_u)\min[(\beta+\gamma)(1-H), (\beta-\gamma)H]$$

$$\Leftrightarrow \gamma\delta\Delta\widetilde{S_N} \leq (w_s - w_u)(\beta-\gamma)H \text{ for } H \leq \min\left\{\tfrac{\beta+\gamma}{2\beta}, \overline{H}\right\} \quad (20)$$

$$\text{and } \gamma\delta\Delta\widetilde{S_N} \leq (w_s - w_u)(\beta+\gamma)(1-H) \text{ for } H \in \left[\tfrac{\beta+\gamma}{2\beta}, \overline{H}\right] \text{ when } \overline{H} > \tfrac{\beta+\gamma}{2\beta}, \quad (21)$$

because $\tau = 0$ from from Proposition 1 (i)(a).

$$p{=}0, q{=}1 \text{ iff } \gamma\delta\Delta\widetilde{S_N} \leq (\beta+\gamma)(1-\tau)(1-H)(w_s - w_u) \text{ and } \gamma\delta\Delta\widetilde{S_N} \geq (\beta-\gamma)(1-\tau)H(w_s - w_u), \quad (22)$$

where $\tau = \frac{\beta-\gamma}{1+\gamma}(a(H) - H)$ when $\beta > \gamma$ and $\tau = 0$ when $\beta \leq \gamma$ from from Proposition 1 (i)(c). This happens only for $H \leq \min\left\{\frac{\beta+\gamma}{2\beta}, \overline{H}\right\}$ because the RHS of the first condition must be greater than that of the second condition.

$$p{=}1, q{=}0 \text{ iff } \gamma\delta\Delta\widetilde{S_N} \geq (\beta+\gamma)(1-\tau)(1-H)(w_s - w_u) \text{ and } \gamma\delta\Delta\widetilde{S_N} \leq (\beta-\gamma)(1-\tau)H(w_s - w_u), \quad (23)$$

where $\tau = \frac{\beta+\gamma}{1+\gamma}(a(H) - H)$ from Proposition 1 (i)(d). This happens only for $H \in \left[\frac{\beta+\gamma}{2\beta}, \overline{H}\right]$.

Propositions in Appendix A examine combinations of H and $\Delta\widetilde{S_N}$ such that each of these equations hold. To prove the propositions, the following assumption is imposed.

Assumption 2 $\tau = \frac{2\beta}{1+\gamma}(a(H) - H) < \frac{1}{2}$ *at* $H = H^+$, *i.e.,* H *satisfying* $a'(H) - 1 = 0$.

This assumption states that the maximum possible tax rate is less than $\frac{1}{2}$. It is not restrictive because a part of the tax revenue used for redistribution (rather than non-redistributive governmental expenditure) is much less than half of aggregate labor income in the real economy. As with Assumption 1, $\frac{2\beta}{1+\gamma} \leq 1$, the assumption holds when $\frac{2\beta}{1+\gamma}$ is sufficiently small.

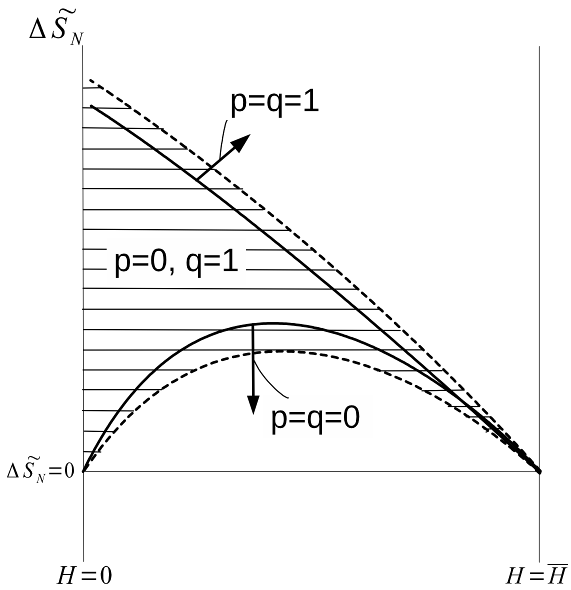

Figure 1: Equilibria when $\beta > \gamma$ and $\frac{A_s}{A_u}$ is relatively low

3.2.1 Result

The following analysis focuses on the more interesting case $\beta > \gamma$; the analysis when $\beta \leq \gamma$ is presented in Appendix A.[21] Based on Proposition A2 (i) in the appendix, Figure 1 shows combinations of H and $\Delta\widetilde{S_N} \equiv \widetilde{S_N} - \widetilde{S_C}$ (the difference in the exogenous component of the national status and of the class status) such that each equilibrium exists when $\frac{A_s}{A_u}$ is low enough that $\overline{H} \leq \frac{\beta+\gamma}{2\beta}$ holds. ($\overline{H}$ is H satisfying $H = a(H) \Leftrightarrow w_s = w_u$.) $p = q = 1$, i.e., universal national identity, is realized when H and $\Delta\widetilde{S_N}$ are in the region on or right of the solid downward-sloping curve; $p = q = 0$, i.e., universal class identity, is realized when H and $\Delta\widetilde{S_N}$ are in the region on or below the solid convex curve; and $p = 0$, $q = 1$, i.e., skilled workers identity with their class and unskilled workers identify with the nation, when they are in the region with horizontal lines.[22]

Given H, everyone identifies with the nation (their class) when $\Delta\widetilde{S_N}$ is relatively high (low), that is, when the people are very (not very) proud of the nation relative to their class for non-economic reasons, e.g., culture and history. When $\Delta\widetilde{S_N}$ is in the intermediate range, skilled (unskilled) workers identity with their class (the nation) because skilled worker are more likely to identify with their class than unskilled workers due to the higher status of their class, which is consistent with the empirical finding by Shayo (2009).[23]

Realized equilibria depend also on H. When H is small, $p = 0$, $q = 1$ (the skilled [unskilled] identify with their class [the nation]) is the only equilibrium for a wide range of $\Delta\widetilde{S_N}$, but as H increases, the region of $p = 0$, $q = 1$ shrinks and $p = q = 1$, i.e., universal national identity ($p = q = 0$,

[21] The other reason for the focus on the case $\beta > \gamma$ is that, when $\beta \leq \gamma$, the figure illustrating combinations of H and $\Delta\widetilde{S_N}$ such that each equilibrium exists (Figure 5 in Appendix A) changes greatly and becomes closer to the corresponding figure for $\beta > \gamma$, when, as assumed in Section 5.1.1 and Shayo (2009), the perceived distance depends also on differences in non-economic attributes that represent class-specific culture, norms of behavior, values, etc.

[22] The lower dividing line for $p = 0$, $q = 1$ increases (decreases) with H for small (large) H, but when H is intermediate, the relation with H is not analytically clear.

[23] Shayo (2009), based on survey data for developed countries and East European countries, finds that the relationship between national identity and household income is significantly negative in almost all the countries.

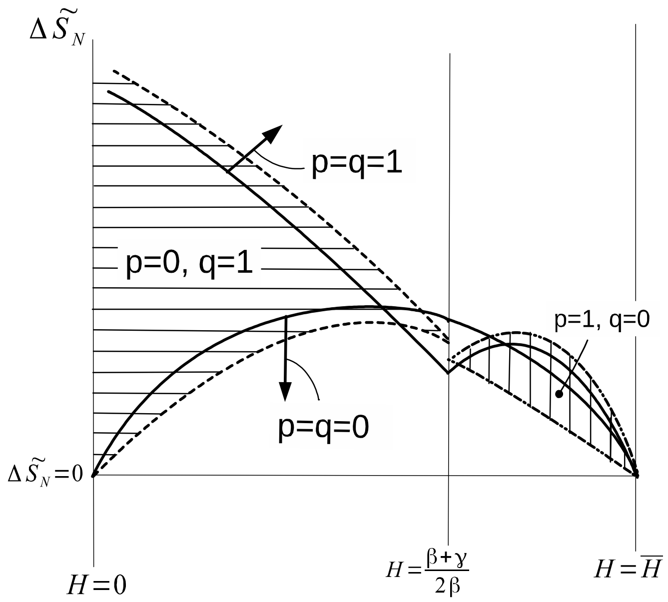

Figure 2: Equilibria when $\beta > \gamma$ and $\frac{A_s}{A_u}$ is relatively high

i.e., universal ethnic identity) becomes more dominant when $\Delta\widetilde{S_N}$ is relatively high (low).

How realized equilibria change with H can be understood mainly from the fact that $w_s - w_u$ decreases with H. When H is low, skilled workers identify with their class because the perceived distance to the "average national" is large compared to the distance to their class (which is 0) and the status of their class is high relative to that of the nation, both of which are due to a large wage differential and their small population share. By contrast, unskilled workers identify with the nation when H is low because their large population share makes the perceived distance to the "average national" small and the large wage disparity makes the national status high relative to the status of their class. As H increases, the wage disparity shrinks and the average income becomes closer to the skilled income; thus, the distance of skilled workers to the "average national" decreases and the national status relative to their class status rises. Hence, when $\Delta\widetilde{S_N}$ is relatively high, they switch from the class identity to the national identity when H becomes large enough. As for unskilled workers, their declining population share widens the distance to the "average national", while the falling inequality narrows the distance. When H is relatively small (large), the former (latter) effect dominates.[24] Hence, when $\Delta\widetilde{S_N}$ is relatively but not very low, they switch from the national identity to the class identity with an increase in H. (Then, they return to the national identity, although as will be clear in Section 5, this is not very relevant.)

The figure shows that there are regions with multiple equilibria. This is because two-way positive causations exist between national identity and tax rate: as the proportion of those identifying with the nation is higher, the tax rate is higher (Proposition 1), while as the tax rate is higher, the inter-class disparity in disposable income is lower and thus more people identity with the nation.

Figure 2 illustrates equilibria when $\frac{A_s}{A_u}$ is high enough that $\overline{H} > \frac{\beta+\gamma}{2\beta}$ holds, based on Proposition A2 (ii) and (iii) in Appendix A. The figure looks complicated but is similar to Figure 1 for $H \leq \frac{\beta+\gamma}{2\beta}$.

[24] As for the effect on the national status relative to the class status, when H is relatively small (large), an increase in H raises (lowers) the relative national status and thus the utility from identifying with the nation. However, the effect on the perceived distance dominates because of $\beta > \gamma$.

For $H > \frac{\beta+\gamma}{2\beta}$, differently from the previous case, $p=0$, $q=1$ is not an equilibrium and $p=1$, $q=0$ is realized when $\Delta\widetilde{S_N}$ is relatively but not extremely low, i.e., in the region with vertical lines.

3.3 Summary

The above results, by combining those on the tax rate and social identity, can be summarized as follows. First, given H, the rate of redistributive taxation is high (low) when $\Delta\widetilde{S_N}$ is relatively high (low) because the proportion of those having a national identity is high (low). Second, when $\Delta\widetilde{S_N}$ is in the intermediate range, identity and the tax rate change with the skilled workers' share. When H is low, skilled workers identify with their class, unskilled workers identify with the nation, and the tax rate is low. When H is high enough, for relatively high $\Delta\widetilde{S_N}$, everyone identifies with the nation and the tax rate is high, while for relatively low $\Delta\widetilde{S_N}$, everyone identifies with their class and the tax rate is at least as low as the rate when H is low. The results suggest the importance of the level of $\Delta\widetilde{S_N}$, which would be high when the people believe that they share a glorious history, rich culture, or a "right" sense of values, for national identity and redistributive taxation.[25]

The question is how H changes over time. The next section develops a dynamic version of the model in which H is determined endogenously.

4 Dynamics

The rest of the paper examines how social identity, redistribution, earnings, and inter-class inequalities change over time when H changes endogenously and A_s and A_u grow exogenously with rising $\frac{A_s}{A_u}$ (skill-biased technical change). This section presents a dynamic version of the model and examines the dynamics of H. The dynamic part of the model is based on Galor and Zeira (1993) and Yuki (2007, 2008), in which individuals who are heterogeneous in wealth received from their parents decide on educational spending needed to become skilled workers.

4.1 Model

A_s and A_u are fixed until the last part of Section 5.1. Consider a deterministic, discrete-time, and small-open OLG economy in which individuals live for two periods, first as a child, then as an adult. Each adult has a single child and thus the population is constant over time. The population of each generation is normalized to be 1. Lifetime of an individual is as follows.

In childhood, she receives a transfer b from her parent and spends it on assets a and educational expenditure e, which is required to become a skilled worker, to maximize the utility given by (24) below. She takes into account the effect of her investment decision not only on future income but also on the socio-psychological components of the utility. The educational investment is a discrete choice, i.e. takes education or not, costs $\overline{e}$, and brings a gross economic return of $w_s - w_u$. The investment must be self-financed due to the absence of credit markets. Thus, when $b < \overline{e}$, one does not expend on education, i.e., $e = 0$, and becomes an unskilled worker. Investment in assets is a continuous choice, and brings a gross rate of economic return of $1 + r$.

In adulthood, the individual obtains income from assets and work and spends it on consumption c and a transfer to her child b'. As in the model in Section 2, she also chooses a group with which she identifies and votes for a party that brings her higher utility. When she belongs to

[25] As mentioned before, Section 5.1.1 examines the model in which, like Shayo (2009), the perceived distance depends also on differences in non-economic attributes that represent class-specific culture, norms of behavior, values, etc. Analysis in the section shows that how much people care about inter-class differences in the attributes too are important for the results.

class C $(C = S, U)$ and identifies with group G $(G = C, N)$, she maximizes the following utility subject to the budget constraint:[26]

$$\max\ v_{CG} = \frac{1}{(\lambda)^{\lambda}(1-\lambda)^{1-\lambda}}(b')^{\lambda}(c)^{1-\lambda} - \beta d_{CG} + \gamma S_G,\ \ \lambda \in (0,1), \tag{24}$$

$$s.t.\quad c + b' = (1-\tau)w_C + T + (1+r)a, \tag{25}$$

where w_C is the wage for class C. By solving the maximization problem, the following consumption and transfer rules are obtained.

$$c = (1-\lambda)[(1-\tau)w_C + T + (1+r)a], \tag{26}$$

$$b' = \lambda[(1-\tau)w_C + T + (1+r)a]. \tag{27}$$

Results on identity choice and the tax rate are same as before because the indirect utlity function equals the utility function of the original model plus $(1 + r)a$.[27]

From the above setting, H is equal to the proportion of individuals who receive $b \geq \overline{e}$ *and* spend $e = \overline{e}$ in childhood. Let F be the proportion of those who receive $b \geq \overline{e}$. Then, if the utility gain from educational investment is non-negative *even when* everyone with $b \geq \overline{e}$ takes education, $H = F$ holds; this is the case when F is not large, as shown in Appendix B. If the utility gain is negative with $H = F$, which is true when F is sufficiently large, H is smaller than F and determined so that one is indifferent between making the educational investment and not. Denote such H by $H^* \in (0, \overline{H})$, whose determination is formally explained in Appendix B.

4.2 Dynamics of F and H

Given the distribution of b over the population in the initial period (thus the initial value of F), the dynamics of H when $F < H^*$, which equal the dynamics of F, are determined by the dynamics of b of each lineage. (When $F \geq H^*$, $H = H^*$ is time-invariant when exogenous variables are constant.)

Suppose $F_t < H^*$ and consider an individual who is born in period $t - 1$ and spends her adulthood in period t. Her investment decisions depend on the received transfer (henceforth, subscript t represents variables for those *born* in period t):

$$\text{If}\quad b_t < \overline{e},\quad a_t = b_t,\ e_t = 0, \tag{28}$$

$$\text{If}\quad b_t \geq \overline{e},\quad a_t = b_t - \overline{e},\ e_t = \overline{e}. \tag{29}$$

By substituting (28) into (27), the dynamic equation linking the received transfer b_t to the transfer to her child b_{t+1} when she received $b_t < \overline{e}$ and thus is an unskilled worker equals

$$b_{t+1} = \lambda\left[(1-\tau_t)w_{ut} + T_t + (1+r)b_t\right]. \tag{30}$$

Similarly, the corresponding equation when she received $b_t \geq \overline{e}$ and thus is a skilled worker is

$$b_{t+1} = \lambda\left[(1-\tau_t)w_{st} + T_t + (1+r)(b_t - \overline{e})\right]. \tag{31}$$

$F_{t+1} \geq F_t$, i.e., $H_{t+1} \geq H_t$, holds when all the children of skilled workers can afford education, i.e., for any lineage satisfying $b_t \geq \overline{e}$, $b_{t+1} \geq \overline{e}$. From (31), this is the case if

$$\lambda\left[(1-\tau_t)w_{st} + T_t\right] \geq \overline{e}. \tag{32}$$

[27] Asset income is assumed to be non-taxed to make these results unchanged and to be consistent with the fact that it is less heavily taxed than labor income.

When this condition holds, $H_{t+1} = F_{t+1} > H_t = F_t$ is true when there exist lineages satisfying $b_t < \overline{e}$ and $b_{t+1} \geq \overline{e}$. From (30), such lineages exist only if $\lambda\{(1-\tau_t)w_{ut} + T_t + (1+r)b_t\} \geq \overline{e}$ for some $b_t < \overline{e}$, which is the case when ($\lambda(1+r) < 1$ is assumed)

$$\frac{\lambda}{1-\lambda(1+r)}\,[(1-\tau_t)w_{ut} + T_t] > \overline{e}. \tag{33}$$

By contrast, $F_{t+1} = F_t \Leftrightarrow H_{t+1} = H_t$ if $b_{t+1} = \lambda\{(1-\tau_t)w_{ut} + T_t + (1+r)b_t\} < \overline{e}$ is true for any $b_t < \overline{e}$, which is the case when

$$\frac{\lambda}{1-\lambda(1+r)}\,[(1-\tau_t)w_{ut} + T_t] \leq \overline{e}. \tag{34}$$

Thus, when $F < H^*$ and the disposable income (net of asset income) of skilled workers is sufficiently high, if the net disposable income of unskilled workers is high enough, $H(=F)$ increases over time; otherwise, it is constant. Because the disposable incomes depend on $H = F$, whether H increases or not is determined by the level of F. The next lemma shows that, for given values of p and q and under some conditions, if the initial level of F is sufficiently high, H increases over time and reaches H^* eventually; otherwise, it stays low.

Lemma 2 *Suppose that Assumption 1 holds and fix values of p and q.*

(i) If $\overline{e}$ (λ) is sufficiently but not extremely small (large), there exists $\underline{F} \in (0, H^*)$ such that when $F_0 \in (\underline{F}, H^*)$, H increases over time and converges to H^* and when $F_0 \leq \underline{F}$, $H_t \leq \underline{F}$ for any t.[28] *When $F_0 \in (\underline{F}, H^*)$, the utility of everyone converges to the same level in the long run.*

(ii) When $\beta > (\leq)\gamma$, $\underline{F}$ is lowest when $p = q = 1$ and is highest when $p = q = 0$ (when $p = q = 0$ and $p = 0$, $q = 1$).[29]

Proof. See Appendix C. ■

When the initial proportion of those who have enough wealth for education is sufficiently high, i.e., $F_0 \in (\underline{F}, H^*)$, the share of skilled workers and thus the level of the unskilled wage are relatively high in the initial period. Thus, unskilled workers with relatively large wealth can make large enough transfer for their children to take education and become skilled workers, as long as the transfer motive λ is sufficiently strong or the cost of education $\overline{e}$ is sufficiently low. Hence, $H = F$ and thus w_u increase, which further induces the upward mobility of children of unskilled workers. In this way, H increases over time and eventually reaches H^* at which people are indifferent between taking education and not.[30,31] Further, because within-class disparities in transfers diminish over time from (30) and (31), the welfare level of everyone becomes equal in the long run.

[28] When $p = q = 0$ or when $\beta \leq \gamma$ and $p = 0$, $q = 1$, in which $\tau = 0$, $H_t = F_0$ for any t when $F_0 \leq \underline{F}$. In other cases, in which $\tau > 0$, one cannot rule out the possibility that $H \leq \underline{F}$ increases or decreases *temporarily*. See the proof of the lemma on this point. Further, the possibility that multiple values of H^* exist cannot be ruled out in these cases. If multiple values of H^* exist, H converges to the highest H^* when $F_0 \in (\underline{F}, H^*)$. If $\overline{e}$ (λ) is extremely small (large), from any $F_0 < H^*$, H increases over time and converges to H^*.

[29] Magnitude relations among H^* for different values of p and q are clear when $\frac{A_s}{A_u}$ is small enough that $\overline{H} \leq \frac{\beta+\gamma}{2\beta}$ (the case of Figure 3 below) or when $\overline{H} > \frac{\beta+\gamma}{2\beta}$ and H^* for $p = q = 1$ is smaller than $\frac{\beta+\gamma}{2\beta}$: H^* for $p = q = 1$ is smallest, H^* for $p = q = 0$ is largest, and H^* for $p = 0$, $q = 1$, which decreases with $\Delta\widetilde{S}_N$, is in the middle. When $\overline{H} > \frac{\beta+\gamma}{2\beta}$ and H^* for $p = q = 1$ is greater than $\frac{\beta+\gamma}{2\beta}$ (the case of Figure 4 below), relations are generally ambiguous.

[30] After H reaches H^*, F continues to increase and converges to 1.

[31] When $p = q = 0$ holds at $H = H^*$, the net *economic* return to education is negative, in other words, earnings net of educational expenditure of skilled workers is *lower* than earnings of unskilled workers, i.e., $w_s - (1+r)\overline{e} < w_u$. This happens because education has an non-economic benefit, i.e., higher status, as well. For other values of p and q, whether the long-run net economic return is positive or not is not clear.

By contrast, when $F_0 < \underline{F}$, generally, the unskilled wage is low enough that children of unskilled workers do not receive enough wealth for education. Thus, H does not increase continuously and $H < \underline{F} < H^*$ always holds,[32] which implies that the utility enjoyed by skilled workers is higher than that of unskilled workers even in the long run.

That is, the initial proportion of those who can afford education, F_0, determines the dynamics and long-run level of the skilled workers' share and inter-class welfare disparity. Further, the threshold level of F, $\underline{F}$, differs depending on values of p and q. It is lowest when $p=q=1$ and when $\beta > (\leq) \gamma$, is highest when $p=q=0$ (when $p=q=0$ and $p=0$, $q=1$). In other words, under universal national (class) identity, the minimum level of F_0 for the upward mobility and the long-run welfare equalization is lowest (highest). This is because the tax rate is highest (zero) and thus income transfers to the unskilled are largest (not conducted) under the identity.

5 Main results

Based on Lemma 2 and the results in Section 3, many of which are illustrated in Figures 1 and 2, this section examines how the endogenous evolution of the skilled workers' share and exogenous skill-biased technical change affect identity, redistribution, and development. Henceforth, the following assumption on identity dynamics is imposed.

Assumption 3 *When the society is in an equilibrium with particular values of p and q, it stays in the same equilibrium in subsequent periods, as long as the equilibrium continues to exist.*

It states that values of p and q do not change over time, as long as they are equilibrium values.

5.1 Effect of H on identity, redistribution, and development

For ease of analysis, first, the effect of H is examined for given levels of productitivies A_s and A_u. As in Section 3.2, the analysis focuses on the case $\beta > \gamma$; however, the following results are mostly unchanged when $\beta \leq \gamma$.[33]

The next proposition analyzes how the dynamics and long-run levels of H, earnings, and earnings inequality and the long-run interclass disparity in welfare depend on F_0 and $\Delta\widetilde{S_N} \equiv \widetilde{S_N} - \widetilde{S_C}$. Unlike Lemma 2, p and q are endogernized by taking into account the results in Section 3.

Proposition 2 *Suppose that $\beta > \gamma$, Assumptions 1–3, and the existence conditions for $\underline{F}$ in Lemma 2 hold and the society starts with $F_0 < H^*$.*[34]

(i) When $F_0 \leq \underline{F}$, $H_t \leq \underline{F}$ for any t. Because H is low, Y is low, w_s is high, w_u is low, and inter-class disparities in earnings and welfare are large.

(ii) When $F_0 > \underline{F}$, H increases over time and converges to H^* almost always.[35] *Accordingly, Y increases, w_s decreases, w_u increases, and the earnings disparity falls over time, and the welfare level of everyone becomes equal in the long run.*

[32] As mentioned in footnote 28, $H < \underline{F}$ possibly increases or decreases temporarily.

[33] When $\beta \leq \gamma$, two results in (ii) of Proposition 3 below do not hold: since $\tau = 0$ when $p = 0$, $q = 1$, τ and thus the speed of convergence to H^* do not change when the society shifts from $p = 0$, $q = 1$ to $p = q = 0$.

[34] H^* is H^* for values of p and q in the initial period. The same is true for $\underline{F}$ of the equations for F_0 and H_t below.

[35] As explained in footnote 39 attached to the next proposition, when the society starts with $p = 0$, $q = 1$, H could stop increasing after it shifts to $p = q = 0$.

(*iii*) *As* $\Delta\widetilde{S_N}$ *is higher,* $\underline{F}$ *is lower and thus* $F_0 \geq \underline{F}$ *is more likely to hold. In particular, when* $\Delta\widetilde{S_N}$ *is very high (low),* $p = q = 1$ ($p = q = 0$) *and thus* $\underline{F}$ *is lowest (highest).*

Proof. See Appendix C. ■

When the initial proportion of those who can afford education is sufficiently low, i.e., $F_0 \leq \underline{F}$, the skilled workers' share stays low, i.e., $H_t \leq \underline{F}$ for any t. Hence, output (thus per capita income) is low, the skilled (unskilled) wage is high (low), and inter-class disparities in earnings and welfare are large. The society is in "poverty trap". By contrast, when the initial proportion of such individuals is sufficiently high, i.e., $F_0 > \underline{F}$, the skilled workers' share increases over time and reaches H^* almost always. Accordingly, output (thus per capita income) increases, the skilled (unskilled) wage decreases (increases), and the earnings disparity falls over time, and the welfare level of everyone becomes equal in the long run. The result is standard in Galor-Zeira type models, in which indivisibilities in human capital investment and credit constraints are present, and consistent with empirical findings such as Litschig and Lombardi (2019) and Aiyar and Ebeke (2020).[36]

The latter positive scenario is more likely to be realized as $\Delta\widetilde{S_N} \equiv \widetilde{S_N} - \widetilde{S_C}$, the difference between the exogenous component of the national status and that of the class status, is higher: $\underline{F}$ is lower and thus $F_0 > \underline{F}$ holds with smaller F_0, i.e., with the worse initial condition, when $\Delta\widetilde{S_N}$ is higher. In particular, when the exogenous status difference is very high (low), everyone identifies with the nation (their class) and the tax rate is highest (zero); thus, $\underline{F}$ is lowest (highest) and the good outcomes are most (least) likely to be realized (see Figure 3 below).

The previous proposition shows that the dynamics and long-run levels of the economic outcomes depend on F_0 and $\Delta\widetilde{S_N}$ because these variables affect whether $F_0 > \underline{F}$ holds or not. The next proposition that focuses on the case $F_0 > \underline{F}$ shows that the dynamics and long-run outcomes are also influenced by the level of $\Delta\widetilde{S_N}$ through its effect on the *evolutions* of social identity and redistributive taxation. Further, it shows the possibility of multiple equilibria.

Proposition 3 *Suppose that the assumptions and conditions of Proposition 2 hold and the society starts with an* $F_0 \in (\underline{F}, H^*)$.[37]

(*i*) *If* $\Delta\widetilde{S_N}$ *is very high (low),* $p = q = 1$ ($p = q = 0$) *and* τ *is high* ($\tau = 0$) *all the time; thus, the disposable income of unskilled workers is high (low) for given* H *and* H *converges to* H^* *quickly (slowly).*

(*ii*) *Otherwise, when* $\Delta\widetilde{S_N}$ *is relatively high (low), the society generally shifts from* $p = 0$, $q = 1$ *to* $p = q = 1$ ($p = q = 0$) *eventually.*[38] *The shift increases* τ (*decreases* τ *to* 0) *and speeds up (slows*

[36] Litschig and Lombardi (2019), based on Brazilian sub-national data for the period 1970−2000, show that sub-national units with a higher share of income going to the middle quintile at the expense of the bottom quintile in 1970 grew faster subsequently, while places with a higher initial share of income going to the top at the expense of the middle did not grow faster. Further, they find that the positive effect of a higher share of the middle quintile is observed only for places in which those in the middle quintile are poor. The model can yield a similar result if the initial distribution of wealth is such that many individuals in the middle quintile have b slightly below $\overline{e}$ and those in the top quintile have $b > \overline{e}$. Aiyar and Ebeke (2020), using cross-county data, find that the negative effect of income inequality on growth is stronger when the degree of the inequality of opportunity, which is measured by father-son correlations of income and education, is higher. In the model, when $F < H^*$, the intergenerational correlations are high and the effect of increased inequality through lowered τ on H is negative, whereas when $F \geq H^*$, the correlations are low, particularly for those with $b \geq \overline{e}$, and the effect of increased inequality on H is zero or small.

[37] $\underline{F}$ and H^* are the ones for values of p and q in the initial period.

[38] As illustrated in Figure 3 (Figure 4), when $\frac{A_s}{A_u}$ is small (large) enough that $\overline{H} \leq (>) \frac{\beta+\gamma}{2\beta}$ and $\Delta\widetilde{S_N}$ is intermediate for the range of $\Delta\widetilde{S_N}$ considered in (ii), the society could be in $p = 0$, $q = 1$ ($p = 1$, $q = 0$) in the long run.

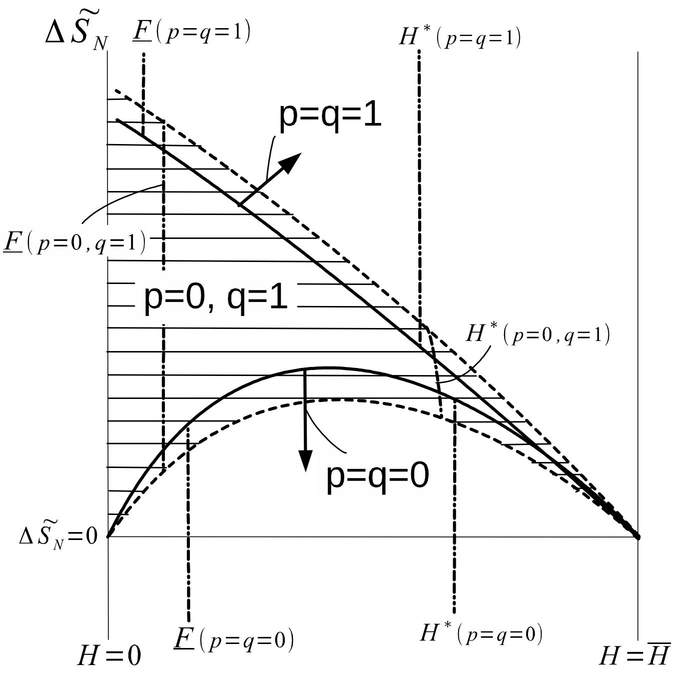

Figure 3: Dynamics when $\beta > \gamma$ and $\frac{A_s}{A_u}$ is relatively low

or stops) the convergence to H^*.[39]

(iii) When $\Delta\widetilde{S_N}$ *is not very high or low, multiple equilibria could exist for given* $\Delta\widetilde{S_N}$ *and* $H = F$. *The dynamics and long-run outcomes differ depending on which equilibrium is realized initially.*

Proof. See Appendix C. ■

Figure 3 illustrates the dynamics when $\frac{A_s}{A_u}$ is small enough that $\overline{H} \leq \frac{\beta+\gamma}{2\beta}$, based on Figure 1 in Section 3.2. The proposition examines the situation in which F_0 is located at the right side of the particular $\underline{F}$ for p and q realized in the initial period. Thus, $H = F$ moves rightward in the figure, as long as values of p and q are unchanged and $H < H^*$. When the initial values of p and q cease to be an equilibrium, $H = F$ continues to move rightward if H is greater than the $\underline{F}$ for the new values of p and q.

If $\Delta\widetilde{S_N}$ is very high (low), everyone always identifies with the nation (their class), i.e., $p=q=1$ ($p = q = 0$), and thus redistributive taxation is large in scale (not implemented). Hence, the disposable income of unskilled workers is high (low) for given H and thus the convergence of H to H^* and the equalization of welfare occur quickly (slowly).

Otherwise, when $\Delta\widetilde{S_N}$ is relatively high (low), the society generally shifts from the equilibrium in which the skilled identify with their class and the unskilled identify with the nation, i.e., $p=0$, $q=1$, to the one in which everyone identifies with the nation (their class) eventually. Before the shift, the tax rate and thus the dynamics do not depend on $\Delta\widetilde{S_N}$. When $\Delta\widetilde{S_N}$ is relatively high, the shift increases the scale of redistributive taxation and *speeds up* the convergence to H^*, whereas when $\Delta\widetilde{S_N}$ is relatively low, the shift leads to no taxation and *slows or stops* the convergence.[40]

[39] The increase of H could stop after the shift to $p = q = 0$ when $\Delta\widetilde{S_N}$ is in the range in which $p_0 = 0$, $q_0 = 1$ and F_0 is greater than $\underline{F}$ for $p = 0, q = 1$ but is smaller than $\underline{F}$ for $p = q = 0$. See Figures 3 and 4.

[40] The result that $\tau = 0$ after the shift to $p = q = 0$ is due to several assumptions that make the model analytically

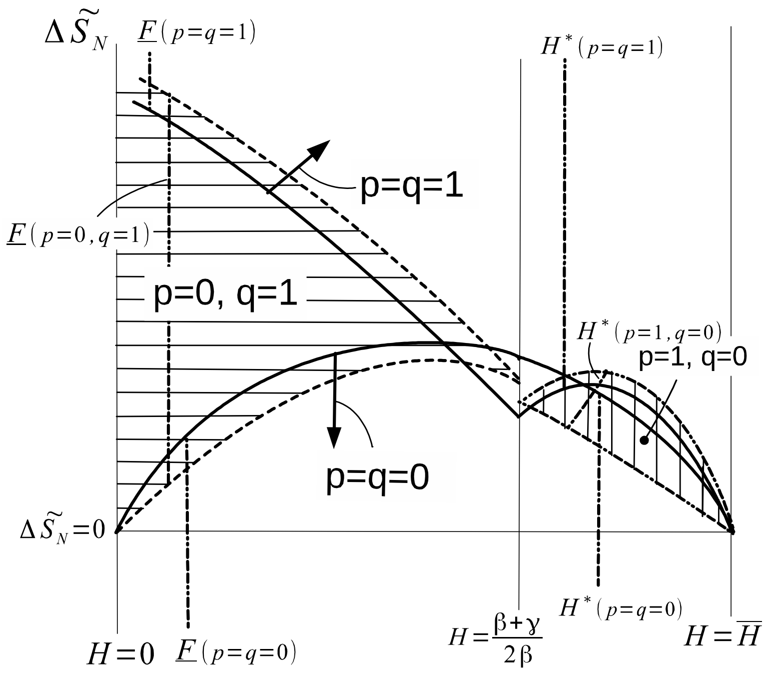

Figure 4: Dynamics when $\beta > \gamma$, $\frac{A_s}{A_u}$ is relatively high, and γ is small

In the latter case, the increase of H could stop after the shift when $\Delta\widetilde{S_N}$ is in the range in which $p_0=0$, $q_0=1$ and F_0 is greater than $\underline{F}$ for $p=q=0$ but is smaller than $\underline{F}$ for $p=0, q=1$.

Multiple equilibria could exist for given $\Delta\widetilde{S_N}$ and $H=F$ when $\Delta\widetilde{S_N}$ is not very high or low. The figure shows that when $\Delta\widetilde{S_N}$ is particularly high (low) for the range of $\Delta\widetilde{S_N}$ in which $p=0$, $q=1$ is an equilibrium, i.e., in the region close to the upper (lower) dividing line for $p=0$, $q=1$, both $p=q=1$ ($p=q=0$) and $p=0$, $q=1$ are equilibria. The dynamics and long-run outcomes differ depending on which equilibrium *happens to* be realized initially. When the society is in the upper (lower) region with multiple equilibria and $p_0=0$, $q_0=1$ happens to be the initial equilibrium, the convergence to H^* is more difficult (easier), i.e., $\underline{F}$ is greater (smaller), and occurs more slowly (faster) than when $p_0=q_0=1$ ($p_0=q_0=0$).

Figure 4 illustrates the dynamics when $\frac{A_s}{A_u}$ is large enough that $\overline{H} > \frac{\beta+\gamma}{2\beta}$ *and* the degree of status concern γ is small enough that $H^* > \frac{\beta+\gamma}{2\beta}$, based on Figure 2.[41] (When γ is not small, the dynamics are very similar to the previous case.) Although the figure looks more complicated, the above results remain unchanged. The main difference is that $p=1$, $q=0$, not $p=0$, $q=1$, could hold in the long run when $\Delta\widetilde{S_N}$ is in the intermediate range.

Common to both figures, the society shifts from $p=0$, $q=1$ to $p=q=1$ ($p=q=0$) when $\Delta\widetilde{S_N}$ is relatively but not extremely large (small). This can be explained as follows. An increase in H shakes existing social identities of *both classes*: the class (national) identity of skilled (unskilled) workers becomes weaker in the sense that the utility gain from identifying with their class (the nation)

tractable. In particular, if ϕ for the unskilled is greater than for the skilled, i.e., the distribution of σ_i (the individual-specific parameter that measures the voter's "ideological" bias toward party 2) is narrower for the unskilled, the weight on the unskilled is greater in the social welfare function and $\tau > 0$ holds. However, qualitative results do not change.

[41] For the case illustrated in Figure 4, relations among H^* for different values of p and q are ambiguous except that H^* for $p=1$, $q=0$, which increases with $\Delta\widetilde{S_N}$, on the upper (lower) dividing line for $p=1$, $q=0$ is greater than H^* for $p=q=1$ (smaller than H^* for $p=q=0$).

rather than with the nation (their class) decreases. As was explained more in detail in Section 3.2, the class identity of skilled workers weakens because their increasing share in the population and the falling inter-class wage disparity decreases the perceived distance of these workers to the "average national" and raises the status of the nation relative to that of their class. The national identity of unskilled workers weakens because their decreasing share in the population increases the distance to the "average national", which dominates other effects operating in the opposite direction. If the exogenous component of the national status is high relative to that of the class status, the utility gain from identifying with their class is small for the skilled; thus, they change identities and universal national identity is realized. Otherwise, the unskilled change identities and everyone identifies with their class.

5.1.1 Dependence of perceived distance on non-economic attributes

So far, for ease of presentation, the perceived distance term of the utility function depends only on the difference in disposable income between oneself and the identity group. It would be more plausible, however, to assume that, similarly to Shayo (2009), the perceived distance depends also on differences in non-economic attributes that represent class-specific culture, norms of behavior, values, etc., acquired through education and work. Suppose, for simplicity, that individuals belonging to a particular class share the same non-economic attribute. Then, whether an individual in class C ($C = S, U$; S [U] is for skilled [unskilled]) possesses the non-economic characteristics specific to each class or not is expressed by the following indicators:

$$q_C^S = 1 (= 0) \text{ and } q_C^U = 0 (= 1) \text{ for } C = S (= U). \tag{35}$$

And, the perceived distance between an individual in class C ($C = S, U$) and group G ($G = C, N$; N is for the nation) is given by

$$d_{CG} = \omega_q \left(\left| q_C^S - q_G^S \right| + \left| q_C^U - q_G^U \right| \right) + \left| y_C - y_G \right|, \tag{36}$$

where ω_q is the weight on differences in the indicators (the weight on the difference in disposable income is normalized to 1), and q_G^S (q_G^U) are the group G's average value of the non-economic attribute for class S (class U). In particular, $q_S^S = 1$, $q_S^U = 0$ ($q_U^S = 0$, $q_U^U = 1$), and $q_N^S = H$, $q_N^U = 1 - H$.

A decrease in ω_q implies that people care less about inter-class differences in culture, norms of behavior, values, etc. It may also be interpreted as declines in class-specific culture, norms, and values, and the homogenization of these attributes because the indicators do not capture the quantitative importance of the attributes. The next proposition shows that a decrease in ω_q has similar effects to an increase in $\Delta\widetilde{S_N}$ on the dynamics and long-run levels of identity, redistribution, and development.

Proposition 4 *Suppose that the assumptions and conditions of Proposition 2 hold.*

(*i*) *As ω_q is smaller, $\underline{F}$ is lower and thus $F_0 \geq \underline{F}$ is more likely to hold. In particular, when ω_q is very small (large), $p = q = 1$ ($p = q = 0$) and thus $\underline{F}$ is lowest (highest).*

(*ii*) *Suppose that the society starts with an $F_0 \in (\underline{F}, H^*)$.*

  (*a*) *If ω_q is very small (large), $p = q = 1$ ($p = q = 0$) and τ is high ($\tau = 0$) all the time; thus, the disposable income of unskilled workers is high (low) for given H and H converges to H^* quickly (slowly).*

  (*b*) *Otherwise, when ω_q is relatively small (large), the society generally shifts from $p = 0$, $q = 1$ to $p = q = 1$ ($p = q = 0$) eventually. The shift increases τ (decreases τ to 0) and speeds up (slows or stops) the convergence to H^*.*

(*c*) *When ω_q is not very small or large, multiple equilibria could exist for given ω_q and $H = F$. The dynamics and long-run outcomes differ depending on which equilibrium is realized initially.*

Proof. See Appendix C. ■

Graphically, this result holds because all the dividing lines of Figures 3 and 4 shift downward when there is a decline in ω_q.

5.1.2 Implications

The result suggests that large cross-country differences in the level and speed of economic development might be due to differences in the exogenous component of the national status (relative to that of the class status) and in inter-class distances in culture, norms, and values or the salience of the distances in people's minds, as well as differences in the initial distributions of wealth and access to advanced technology. In many developing countries, the belief that people share a glorious history, rich culture, or a "right" sense of values is weak and inter-class differences in culture, norms, and values are large or thought to be serious. According to the model, these make the national status low or the perceived distance to the other class large, the formation of common national identity difficult, and as a result, the scale of redistribution limited, the upward mobility of the poor through education and the pace of development slow.

This implication of the model is consistent with empirical findings. First, empirical works indicate that national identity promotes growth and development by increasing income redistribution and stimulating educational investment. Various works (Chen and Li, 2009; Transue, 2007; Qari, Konrad and Geys, 2012; Singh, 2015) suggest that national identity has a positive effect on redistribution (see footnote 17 in Section 3.1 for details). The analysis by Berg et al. (2018), using cross-country data covering a large number of countries, indicates that income redistribution, unless very large-scale, makes economic growth faster and more sustainable by lowering income inequality. They also find that lower inequality is associated with higher years of education. Further, Hanushek and Woessmann (2012a), based on data of 64 countries, find that educational achievement, measured by cognitive skills, has a large effect on growth. Second, there is suggestive evidence for Latin American countries that income redistribution is small in scale and difficult to expand due to weak sense of common identity resulting from severe social divisions. Goni, Lopez, and Serven (2011) find that, while market inequality is not very different between Latin American and Western European countries, after-tax after-transfer inequality is much higher in the former group of countries because transfers are small in scale and not well targeted to the poor. This is also the case when public expenditures on education and health as well as cash transfers are considered.[42] Blofield and Juan Pablo (2011), based on World Values Survey data, find that a large share of the population in Latin American countries prefer even *higher* income inequalities than very high levels of current inequalities (at the same time, a sizable portion of people support lower inequalities), whereas in European countries, people tend to accept the status quo. And, experts on Latin America (O'Donnell, 1998; Vilas, 1997) argue that policies that seriously deal with poverty and inequality are difficult to form because the people do not have the feeling of common belonging or broad solidarity due to sharp social divisions or polarization.

The result also indicates the critical importance of *nation-building policies*, such as school education and government propaganda that emphasize common history, culture, and values and

[42] Hanushek and Woessmann (2012b) find that poor growth and development performance of Latin American nations can be explained mostly by poor educational achievement (cognitive skills). This finding, together with the limited scale of redistribution, suggests that increased redistribution might improve the economic outcomes by enabling the poor to access quality education.

policies promoting between-group contact that might make inter-class distances in culture, norms, and values less salient in people's minds, for countries with low national status or large (or salient) inter-class distances in these attributes. According to the model, the policies would lift the national status or diminish (or deemphasize) the inter-class differences, thereby contributing to the formation of common identity and the better economic outcomes. Londoño-Vélez (2022) finds that a Colombian financial aid reform increasing the share of low-income students at an elite university caused high-income students to interact more with low-income peers and raise support for progressive redistribution. Her finding can be explained by the present model. Various studies, though not class-related, indicate that nation-building policies can effectively fortify national identity. Chen, Lin, and Yang (2020) examine a curriculum reform that introduced a large amount of Taiwan-related contents in the history subject for junior high school students and find that students under the new curriculum are much more likely to hold exclusive Taiwanese identity rather than dual identities of Taiwanese and Chinese. Blouin and Mukand (2019), based on field and lab experiments in post-genocide Rwanda, show that an exposure to government radio propaganda weakened the salience of ethnic identity and increased interethnic trust and cooperation. By examining data on a lottery that allocates conscripts to different regions in Spain, Cáeres-Delpiano et al. (2021) find that, for men from regions with a weak Spanish identity, being assigned to military service in a different region increases national identity greatly and persistently.

Finally, according to the model, when the skilled workers' share and thus the level of development are high, everyone is identified with the nation (their class), if $\Delta\widetilde{S_N}$ and ω_q are high (low). Classic modernization theories in political science (Deutsch, 1953; Gellner, 1983; Weber, 1979), based on the past experience of Europe, argues that modernization (including industrialization and universal education) leads to widespread national identity at the expense of subnational identities (Robinson, 2014). The result suggests that these theories hold only when the national status is relatively high or inter-class distances in culture, norms, and values are relatively small or of little concern.

5.1.3 Time-varying exogenous variables

So far, productitivies A_s, A_u, and the cost of education $\overline{e}$ have been time-invariant. The results do not change qualitatively, as long as these variables grow over time at the same constant rate and the exogenous components of status, $\widetilde{S_N}$ and $\widetilde{S_C}$ (thus $\Delta\widetilde{S_N}$), are multiplied by a variable that also grows at the same rate.[43,44] It would be reasonable to suppose that $\overline{e}$ grows at the same rate as the productivities and thus earnings, considering that the main cost of education is the cost of hiring teachers and other staff. The assumption on the status variables would also be plausible because the importance of exogenous factors affecting status, such as culture, history, and values, in one's welfare seems not to diminish with economic growth in the real society.

[43] Suppose that these variables grow at g and let $A_{st} = g^t A_s$, $A_{ut} = g^t A_u$, and $\overline{e_t} = g^{t-1}\overline{e}$ (note that $\overline{e_t}$ is the cost in period $t-1$). Then, given H_t, w_{st}, w_{ut}, and T_t also grow at g. Denote detrended endogenous variables with tilde, e.g., $\widetilde{w_{st}} \equiv \frac{w_{st}}{g^t}$ and $\widetilde{b_t} \equiv \frac{b_t}{g^{t-1}}$. By dividing both sides of the equations by g^t, (28) and (29) respectively are expressed as $\widetilde{b_{t+1}} = \lambda\{(1-\tau_t)\widetilde{w_{ut}} + \widetilde{T_t} + (1+r)\frac{\widetilde{b_t}}{g}\}$ and $\widetilde{b_{t+1}} = \lambda\{(1-\tau_t)\widetilde{w_{st}} + \widetilde{T_t} + (1+r)\frac{1}{g}(\widetilde{b_t} - \overline{e})\}$. Then, the conditions for $H_{t+1} = F_{t+1} > H_t = F_t$ become $\lambda[(1-\tau_t)\widetilde{w_{st}} + \widetilde{T_t}] \geq \overline{e}$ and $\frac{\lambda}{1-\frac{\lambda}{g}(1+r)}\left[(1-\tau_t)\widetilde{w_{ut}} + \widetilde{T_t}\right] > \overline{e}$, which are very similar to (32) and (33) respectively. Results on identity choice and the tax rate do not change because given H, all the terms in the utility function grow at g. Results on earnings and welfare disparities do not change when relative measures are used. Results on output, earnings, disposable incomes, and the long-run welfare level hold when they are detrended.

[44] In the model in which the perceived distance depends also on differences in non-economic indicators (Section 5.1.1), the weight on differences in the indicators ω_q also needs to be multiplied by such a variable.

5.2 Effect of SBTC on identity, redistribution, and development

Skill-biased technical change (SBTC) is another main driving force of economic growth and development besides human capital accumulation. Hence, this section examines the effect of increasing $\frac{A_s}{A_u}$ for given H. For simplicity, exogenous variables such as A_u and $\Delta\widetilde{S_N}$ are assumed to be constant: as mentioned in Section 5.1.3, qualitative results do not change when A_u grows over time at a constant rate and the exogenous components of status, $\widetilde{S_N}$ and $\widetilde{S_C}$ (thus $\Delta\widetilde{S_N}$), are multiplied by a variable that also grows at the same rate. Unlike the original setting, $\bar{e}$ is assumed to be proportional to w_s because the main cost of education in the real world is the cost of hiring teachers, skilled workers, and a change in $\frac{A_s}{A_u}$ affects w_s. The next proposition summarizes the result.

Proposition 5 *Suppose that the assumptions and conditions of Proposition 2 hold. Then, an increase in* $\frac{A_s}{A_u}$ *has the following effects.*

(*i*) *The inter-class disparity in earnings increases.*

(*ii*) *All the dividing lines for identity choice shift upward on the* $(H, \Delta\widetilde{S_N})$ *plane. Hence, when the rise of* $\frac{A_s}{A_u}$ *continues, the society shifts to an equilibrium with weaker national identity eventually, unless* $\Delta\widetilde{S_N}$ *is small or very large:*

(*a*) *If* $H < \frac{\beta+\gamma}{2\beta}$ *or* $\frac{A_s}{A_u}$ *is small enough that* $\overline{H} \leq \frac{\beta+\gamma}{2\beta}$, *the society shifts from* $p=q=1$ *to* $p=0$, $q=1$ (*from* $p=0, q=1$ *to* $p=q=0$) *when* $\Delta\widetilde{S_N}$ *is relatively large* (*small*).[45]

(*b*) *Otherwise, it shifts from* $p=q=1$ *to* $p=q=0$ *when* $\Delta\widetilde{S_N}$ *is relatively small.*[46]

(*iii*) *Given* p *and* q, τ *rises but the inter-class disparity in welfare increases. When the identity shift occurs,* τ *falls and the welfare disparity increases.*

(*iv*) *When the cost of education is sufficiently high,*[47] $\underline{F}_t$ *increases and for* $F_0 > \underline{F}_0$, *the speed of the convergence to* H^*, *which increases with* $\frac{A_s}{A_u}$, *slows down. The latter is particularly so when the identity shift occurs.*

Proof. See Appendix C. ■

SBTC increases the wage differential between the skilled and the unskilled. In response to the increased disparity, given p and q, the tax rate is raised (unless $p = q = 0$), but the expansion of redistribution is not large enough to offset the increased wage inequality. Hence, the perceived distances of each class to the other class and thus to the "average national" increase. SBTC also raises (lowers) the status of the skilled (unskilled) class relative to that of the nation due to the increased inter-class differential in disposable income. Hence, the national identity of skilled workers becomes weaker in the sense that the utility gain from identifying with the nation rather than with their class decreases. As for the unskilled, the lowered relative status of their class partially offsets the increased perceived distance to the "average national", but the latter effect dominates (due to $\beta > \gamma$) and their national identity also becomes weaker. As a result, all the dividing lines for identity choice in Figures 3 and 4 shift upward. Therefore, when increased $\frac{A_s}{A_u}$ continues, the society shifts to an equilibrium with weaker national identity and the *lower* rate of redistributive tax at some point, unless $\Delta\widetilde{S_N}$ is small or very large.

[45] To be precise, when $\overline{H} > \frac{\beta+\gamma}{2\beta}$ and $H(< \frac{\beta+\gamma}{2\beta})$ is close to $\frac{\beta+\gamma}{2\beta}$, the shift from $p = q = 1$ to $p = q = 0$ and the shift from $p = 0$, $q = 1$ to $p = q = 1$, then to $p = q = 0$ are also possible (see Figure 4).

[46] To be precise, the shift from $p = q = 1$ to $p = 1$, $q = 0$, then to $p = q = 0$ can also occur (see Figure 4).

[47] To be accurate, this is true when the constant s of $\bar{e}_t = sw_{st-1}$ is sufficiently large.

The increased differential in disposable income enlarges the disparity in welfare. The effect is particularly large when the identity shift occurs because it reduces the scale of redistribution.

SBTC also affects the dynamics and long-run outcomes. Because SBTC raises the cost of education, which is proportional to the skilled wage, at a higher rate than the unskilled wage, it increases $\underline{F}_t$, which implies that the escape from "poverty trap" becomes more difficult for the society starting with a relatively small share of those who can afford education. Further, the pace of convergence to H^* becomes slower when $F_0 > \underline{F}_0$. This is particularly so when the identity shift occurs and thus the scale of redistribution shrinks.

The result shows that as technology is more skill biased, national identity is harder to be realized and redistribution is less effective, thus the upward mobility and equalization of welfare are harder to be realized or occur more slowly. The higher skill biasedness of current technology, as well as weaker sense of shared history, culture, and values and greater inter-class differences in culture, norms, and values, might contribute to slower paces of the upward mobility and development of many developing countries than the paces experienced by developed countries during the period of modernization.

As for advanced economies, income equality has increased greatly during the last several decades, but demand for and the scale of income redistribution have not increased or even decreased by some measures (Ashok et al., 2016; Piketty, Saez, and Stantcheva, 2014). To explain this fact, Windsteiger (2022) develops a model in which individuals are segregated according to income and interact mainly with those with similar income levels, thus they have biased information on income inequality. As a result, under some conditions, increased inequality leads to a fall in perceived inequality, thus decreases in support for and the level of redistribution. The present model, after a slight modification, can also explain the fact. Suppose that, unlike the original setting, individuals are heterogeneous in the weight on the exogenous component of group status, δ, whose distribution is continuous. Then, unless $\Delta\widetilde{S_N}$ is small or very large, as $\frac{A_s}{A_u}$ increases, p and q and thus τ could decrease continuously.

6 Conclusion

This paper developed a dynamic model of income redistribution and educational investment augmented with social identification and explored the interaction among identity, redistribution, and development theoretically. Analysis showed that, given the skilled workers' share, the rate of redistributive taxation is higher as the proportion of people identifying with the nation is higher. When the exogenous component of the national status is higher (which would be the case when, for example, people have stronger pride in the nation), or when inter-class differences in culture, norms, and values are smaller or less salient in people's minds, the society is less likely to be in "poverty trap", and under a favorable initial condition that avoids the trap, the speed of an increase in the skilled workers' share and thus the pace of development are faster because of stronger national identity and greater redistribution. As skill-biased technical change (SBTC) proceeds, the society becomes more likely to be in "poverty trap", and when the initial condition is favorable, the upward mobility of the poor becomes slower. Further, when SBTC continues, the society generally shifts to an equilibrium with a smaller portion of people identifying with the nation, which decreases the tax rate and the scale of redistribution and makes the negative effects of SBTC stronger.

The result suggests that large cross-country differences in the level and speed of development might be partly due to differences in the exogenous component of the national status (relative to that of the class status) and in inter-class distances in culture, norms, and values or people's concerns with the distances. In many developing countries, the belief that people share a glorious history, rich culture, or a “right” sense of values is weak and inter-class differences in culture, norms, and values are large or thought to be serious. According to the model, these make the formation of common national identity difficult, and as a result, the scale of redistribution limited, the upward mobility of the poor and the pace of development slow. In these countries, nation-building policies, such as school education and government propaganda that emphasize common history, culture, and values and policies promoting between-group contact that might make the inter-class distances less salient in people’s minds would be critical for good outcomes. Technology available for present developing countries is much more skill-biased than the one developed countries used when they underwent the modernization and industrialization of the economies. The result on SBTC suggests that this might be another reason for the slower pace of development, particularly of the poor, in many developing countries.

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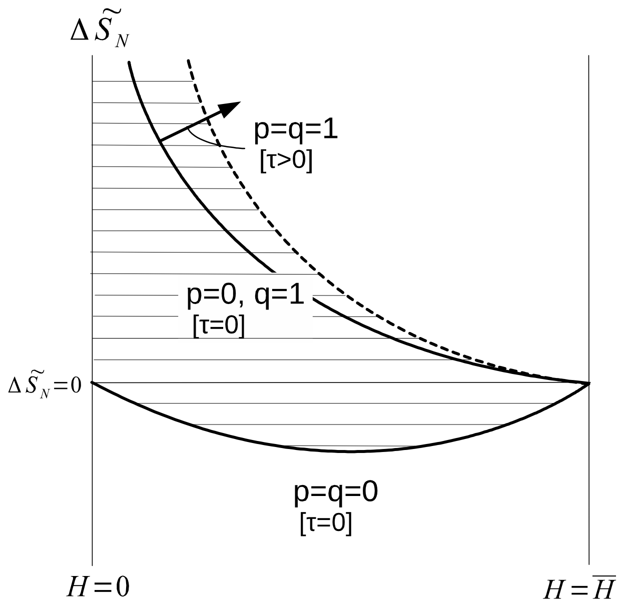

Figure 5: Equilibria when $\beta \leq \gamma$

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Appendix A Propositions A1 and A2

This Appendix presents results on shapes of dividing lines for quilibria, which are the basis for Figures 1 and 2 in Section 3.2. The next proposition presents the result when $\beta \leq \gamma$. This case is not discussed in the main text because its result is less interesting and would be less realistic, but it is presented first due to simplicity.

Proposition A1 *Suppose that $\beta \leq \gamma$ and Assumption 2 holds.*

(i) $p = q = 1$, $p = 0$, $q = 1$, and $p = q = 0$ can be stable equilibria.

(ii) The dividing line for $p = q = 1$ and the upper dividing line for $p = 0$ and $q = 1$ decrease with H, go to $+\infty$ as $H \to 0$, and go to 0 as $H \to \overline{H}$, where $\overline{H}$ is H satisfying $H = a(H) \Leftrightarrow w_s = w_u$, on the $(H, \Delta\widetilde{S_N})$ plane.

(iii) There exists $H^\sharp \in (0, \overline{H})$ such that the dividing line for $p = q = 0$ and the lower dividing line for $p = 0$ and $q = 1$ decrease (increase) with H for $H < (>) H^\sharp$. They go to 0 as $H \to 0$ and $H \to \overline{H}$.

(*iv*) *On the* $(H, \Delta\widetilde{S_N})$ *plane, the dividing line for* $p = q = 0$ *and the lower dividing line for* $p = 0$ *and* $q = 1$ *are the same; they are located below the dividing line for* $p = q = 1$ *and the upper dividing line for* $p = 0$ *and* $q = 1$*; the dividing line for* $p = q = 1$ *is located below the upper dividing line for* $p = 0$ *and* $q = 1$*.*

Proof. See Appendix C. ■

Based on the proposition, Figure 5 illustrates combinations of H and $\Delta\widetilde{S_N}$ such that equilibria exist when $\beta \leq \gamma$. There are two differences from the figures when $\beta > \gamma$ (Figures 1 and 2 in Section 3.2). First, the dividing line for $p = q = 0$ decreases (increases) with H for relatively small (large) H. This implies that everyone identifies with their class only when $\Delta\widetilde{S_N} < 0$, i.e., the exogenous component of the class identity $\widetilde{S_C}$ is greater than that of the national identity $\widetilde{S_N}$, which would be unlikely for unskilled workers. The relation with H is opposite to the case $\beta > \gamma$, because the effect of H on status dominates the effect on perceived distance when $\beta \leq \gamma$, where the former effect implies that when H is relatively small (large), an increase in H increases (decreases) the national status relative to the class status and thus the utility from identifying with the nation. Second, the lower dividing line for $p = 0$, $q = 1$ coincides with the dividing line for $p = q = 0$ because τ when $p = 0$, $q = 1$ is 0 when $\beta \leq \gamma$. Except these points, the figure is similar to the one when $\beta > \gamma$ and A_s is relatively low (Figure 1).

The next proposition presents the result when $\beta > \gamma$. Based on (i) of the proposition, Figure 1 in Section 3.2 illustrates combinations of H and $\Delta\widetilde{S_N}$ such that equilibria exist when $\frac{A_s}{A_u}$ is relatively low, and based on (ii) and (iii) of the proposition, Figure 2 illustrates the combinations when $\frac{A_s}{A_u}$ is relatively high.

Proposition A2 *Suppose that* $\beta > \gamma$ *and Assumption 2 holds.*

(*i*) *When* $\frac{A_s}{A_u}$ *is small enough that* $\overline{H} \leq \frac{\beta+\gamma}{2\beta}$*, Proposition A1 applies except the following.*

- (*a*) *The dividing line for* $p = q = 0$ *and the lower dividing line for* $p = 0$ *and* $q = 1$ *are different. The former increases (decreases) with* H *for* $H < (>) H^{\sharp}$*. The latter increases (decreases) with* H *for small (large) enough* H*.*[48]
- (*b*) *On the* $(H, \Delta\widetilde{S_N})$ *plane, the dividing line for* $p = q = 0$ *is located above the lower dividing line for* $p = 0$ *and* $q = 1$*; when* H *is relatively high, the dividing line for* $p = q = 0$ *could be located above the dividing line for* $p = q = 1$ *and the upper dividing line for* $p = 0$ *and* $q = 1$*.*

(*ii*) *When* $\frac{A_s}{A_u}$ *is large enough that* $\overline{H} > \frac{\beta+\gamma}{2\beta}$ *and* $H \leq \frac{\beta+\gamma}{2\beta}$*, the results are same as (i) except the following.*

- (*a*) *When* A_s *is large enough that* $H^{\sharp} \geq \frac{\beta+\gamma}{2\beta}$*, the dividing line for* $p = q = 0$ *and the lower dividing line for* $p = 0$ *and* $q = 1$ *increase with* H*.*
- (*b*) *When* H *is relatively high, the dividing line for* $p = q = 0$ *is located above the dividing line for* $p = q = 1$ *and the upper dividing line for* $p = 0$ *and* $q = 1$*. The lower and upper dividing lines for* $p = 0$ *and* $q = 1$ *intersect at* $H = \frac{\beta+\gamma}{2\beta}$*.*

(*iii*) *When* $\overline{H} > \frac{\beta+\gamma}{2\beta}$ *and* $H > \frac{\beta+\gamma}{2\beta}$*,*

- (*a*) $p = q = 1$*,* $p = 1$ *and* $q = 0$*, and* $p = q = 0$ *can be stable equilibria.*
- (*b*) *The dividing line for* $p = q = 0$ *and the lower dividing line for* $p = 1$ *and* $q = 0$ *decrease with* H *and go to* 0 *as* $H \to \overline{H}$ *on the* $(H, \Delta\widetilde{S_N})$ *plane.*

[48] When H is intermediate, the relationship with H is not analytically clear.

(c) *The dividing line for $p = q = 1$ and the upper dividing line for $p = 1$ and $q = 0$ decrease with H for sufficiently large H and go to 0 as $H \to \overline{H}$. They could increase with H for sufficiently small H.*[49]

(d) *The dividing line for $p = q = 0$ is located above the lower dividing line for $p = 1$ and $q = 0$; the dividing line for $p = q = 1$ is located below the upper dividing line for $p = 1$ and $q = 0$; when H is relatively low (high), the dividing line for $p = q = 0$ is located above (below) the dividing line for $p = q = 1$ and the upper dividing line for $p = 1$ and $q = 0$; the lower and upper dividing lines for $p = 1$ and $q = 0$ intersect at $H = \frac{\beta+\gamma}{2\beta}$.*[50]

Proof. See Appendix C. ■

Appendix B Determination of H^* and Proof on the value of H

This Appendix formally explains how H^* is determined and proves that $H = F$ ($H = H^*$) holds when F is small (large). By substituting (26) and (27) into (24), the indirect utility function equals

$$v_{CG} = (1-\tau)w_C + T + (1+r)a - \beta d_{CG} + \gamma S_G. \tag{37}$$

From this equation, (9)–(12), (28), and (29),

$$\begin{aligned}
v_{SN} &= (1+\gamma)\left(\tau - \frac{1}{2}\tau^2\right)\overline{w} + (1-\tau)w_s - \beta(1-\tau)(w_s - \overline{w}) + \gamma\left[\delta\widetilde{S_N} + (1-\tau)\overline{w}\right] + (1+r)a, \\
&= (1+\gamma)\left(\tau - \frac{1}{2}\tau^2\right)\overline{w} + (1-\tau)w_s - \beta(1-\tau)(w_s - \overline{w}) + \gamma\left[\delta\widetilde{S_N} + (1-\tau)\overline{w}\right] + (1+r)(b - \overline{e}), \quad (38) \\
v_{SS} &= (1+\gamma)\left(\tau - \frac{1}{2}\tau^2\right)\overline{w} + (1-\tau)w_s + \gamma\left[\delta\widetilde{S_C} + (1-\tau)w_s\right] + (1+r)(b - \overline{e}), \quad (39) \\
v_{UN} &= (1+\gamma)\left(\tau - \frac{1}{2}\tau^2\right)\overline{w} + (1-\tau)w_u - \beta(1-\tau)(\overline{w} - w_u) + \gamma\left[\delta\widetilde{S_N} + (1-\tau)\overline{w}\right] + (1+r)b, \quad (40) \\
v_{UU} &= (1+\gamma)\left(\tau - \frac{1}{2}\tau^2\right)\overline{w} + (1-\tau)w_u + \gamma\left[\delta\widetilde{S_C} + (1-\tau)w_u\right] + (1+r)b. \quad (41)
\end{aligned}$$

H^* when $p = q = 1$ is H satisfying $v_{SN} = v_{UN}$, thus from (38) and (40), H satisfying

$$\begin{aligned}
&(1-\tau)w_s - \beta(1-\tau)(w_s - \overline{w}) - (1+r)\overline{e} = (1-\tau)w_u - \beta(1-\tau)(\overline{w} - w_u) \\
&\Leftrightarrow [1 - \beta(1-2H)](1-\tau)(w_s - w_u) - (1+r)\overline{e} = 0, \quad (42) \\
&\text{where } \tau = \tfrac{2\beta}{1+\gamma}(a(H) - H).
\end{aligned}$$

H^* when $p = q = 0$ is H satisfying $v_{SS} = v_{UU}$, thus from (39) and (41), H satisfying

$$(1+\gamma)(w_s - w_u) - (1+r)\overline{e} = 0. \tag{43}$$

H^* when $p = 0$, $q = 1$ is H satisfying $v_{SS} = v_{UN}$, thus from (39) and (40), H satisfying

$$\begin{aligned}
&(1-\tau)w_s + \gamma\left[\delta\widetilde{S_C} + (1-\tau)w_s\right] - (1+r)\overline{e} = (1-\tau)w_u - \beta(1-\tau)(\overline{w} - w_u) + \gamma\left[\delta\widetilde{S_N} + (1-\tau)\overline{w}\right] \\
&\Leftrightarrow [1 + \beta H + \gamma(1-H)](1-\tau)(w_s - w_u) - (1+r)\overline{e} - \gamma\delta\Delta\widetilde{S_N} = 0, \quad (44) \\
&\text{where } \tau = \tfrac{\beta-\gamma}{1+\gamma}(a(H) - H) \text{ when } \beta > \gamma \text{ and } \tau = 0 \text{ when } \beta \leq \gamma.
\end{aligned}$$

[49] When H is intermediate, the relationship with H is not analytically clear.

[50] When H is intermediate, the relative positions of these dividing lines are not analytically clear.

H^* when $p=1$, $q=0$ is H satisfying $v_{SN}=v_{UU}$, thus from (38) and (41), H satisfying

$$(1-\tau)w_s-\beta(1-\tau)(w_s-\overline{w})+\gamma\left[\delta\widetilde{S_N}+(1-\tau)\overline{w}\right]-(1+r)\overline{e}=(1-\tau)w_u+\gamma\left[\delta\widetilde{S_C}+(1-\tau)w_u\right]$$

$$\Leftrightarrow [1-\beta(1-H)+\gamma H](1-\tau)(w_s-w_u)-(1+r)\overline{e}+\gamma\delta\Delta\widetilde{S_N}=0, \tag{45}$$

$$\text{where } \tau=\frac{\beta+\gamma}{1+\gamma}(a(H)-H).$$

H^* exists because the LHSs of (42)−(45) go to $+\infty$ as $H\to 0$ from $\lim_{H\to 0}(w_s-w_u)=+\infty$, and the LHSs become negative as $H\to\overline{H}$ from $\lim_{H\to\overline{H}}(w_s-w_u)=0$. To be more precise, the latter is true when $p=0$, $q=1$, which is an equlibrium only when $\beta>\gamma$ from Propositions A1 and A2, because $\Delta\widetilde{S_N}$ in (44) is positive from (22). It is true when $p=1$, $q=0$ because the LHS of (45) is smaller than $[1-\beta(1-H)+\gamma H](1-\tau)(w_s-w_u)-(1+r)\overline{e}+(\beta-\gamma)(1-\tau)H(w_s-w_u)=[1-\beta(1-2H)](1-\tau)(w_s-w_u)-(1+r)\overline{e}$ from (23).

Proof that $H=F$ ($H=H^*$) when F is small (large)

When $p=q=0$, $H=F$ ($H=H^*$) holds when $v_{SS}\geq(<)v_{UU}$ is satisfied with $H=F$, i.e., $(1+\gamma)(w_s-w_u)-(1+r)\overline{e}\geq(<)0$ with $H=F$ from (43). Because w_s-w_u decreases with H, this is the case when $F\leq(>)H^*$. Similarly, when $p=0$, $q=1$ and $\beta\leq\gamma$ (thus $\tau=0$), $H=F$ ($H=H^*$) holds when $F\leq(>)H^*$ from (44).

For other cases, the relation between H and the LHSs of the above equations determining H^* is generally unclear. Whether $H=F$ or $H=H^*$ is clear only for sufficiently small or large F. As for the range of F satisfying $H=F$, the following is true. When $p=0$, $q=1$ and $\beta>\gamma$, from (22), the LHS of (44) is greater than $[1-\beta(1-2H)](1-\tau)(w_s-w_u)-(1+r)\overline{e}>(1-\beta)(1-\tau)(w_s-w_u)-(1+r)\overline{e}$. Hence, $H=F$ holds at least for F weakly smaller than the smallest H satisfying $(1-\beta)(1-\tau)(w_s-w_u)-(1+r)\overline{e}=0$, which is smaller than H^* for $p=q=0$. It is easy to see that a similar result holds when $p=q=1$ from (42). When $p=1$, $q=0$, from (23), the LHS of (45) is greater than $(1+\gamma)(1-\tau)(w_s-w_u)-(1+r)\overline{e}$. Hence, $H=F$ holds at least for F weakly smaller than the smallest H satisfying $(1+\gamma)(1-\tau)(w_s-w_u)-(1+r)\overline{e}=0$, which is smaller than H^* for $p=q=0$.

As for the range of F satisfying $H=H^*$, the following is true. When $p=0$, $q=1$ and $\beta>\gamma$, $H=H^*$ at least for F weakly greater than H^* for $p=q=0$. This is because, from (22), the LHS of (44) is smaller than $(1+\gamma)(1-\tau)(w_s-w_u)-(1+r)\overline{e}$, which is smaller than the LHS of (43). Similarly, when $p=q=1$ and $\beta\leq\gamma$, $H=H^*$ at least for F weakly greater than H^* for $p=q=0$ because the LHS of (42) is smaller than that of (43) from $[1-\beta(1-2H)](1-\tau)(w_s-w_u)<(1+\gamma)(w_s-w_u)$. When $p=q=1$ and $\beta>\gamma$, the LHS of (42) is smaller than $(1+\beta)(w_s-w_u)-(1+r)$, thus $H=H^*$ at least for F weakly greater than H satisfying $(1+\beta)(w_s-w_u)-(1+r)=0$, which is greater than H^* for $p=q=0$. A similar result holds when $p=1$, $q=0$ also because the LHS of (45) is smaller than $[1-\beta(1-2H)](1-\tau)(w_s-w_u)-(1+r)\overline{e}$, which is smaller than $(1+\beta)(w_s-w_u)(w_s-w_u)-(1+r)\overline{e}$.

Appendix C Proofs of lemmas, propositions, and a corollary

Proof of Lemma 1. Suppose that $q\in(0,1)$ is an equilibrium, which implies that unskilled workers are indifferent between the two identities. Then, from (11) and (12),

$$-\beta(1-\tau)(\overline{w}-w_u)+\gamma\left[\delta\widetilde{S_N}+(1-\tau)\overline{w}+T\right]=\gamma\left[\delta\widetilde{S_C}+(1-\tau)w_u+T\right]$$

$$\Leftrightarrow \gamma\delta\Delta\widetilde{S_N}=(\beta-\gamma)(1-\tau)(\overline{w}-w_u), \tag{46}$$

where from (17),

$$\tau = 1 - \frac{1}{(1+\gamma)\overline{w}}\begin{pmatrix}[Hw_s+(1-H)w_u]+\gamma\{H[p\overline{w}+(1-p)w_s]+(1-H)[q\overline{w}+(1-q)w_u]\} \\ -\beta\{Hp(w_s-\overline{w})+(1-H)q(\overline{w}-w_u)\}\end{pmatrix}.$$

From the above equation, τ increases (decreases) with q when $\beta > (<)\gamma$. Thus, when q increases, the RHS of (46) decreases and thus identifying with the nation becomes more attractive than identifying with their class. This means that the equilibrium is unstable. (Further, when $\beta = \gamma$, (46) holds only when $\Delta\widetilde{S_N} = 0$.) $p \in (0,1)$ is not a stable equilibrium can be proved similarly. ■

Proof of Proposition 1. (i) Values of τ are obtained from (17). $a(H) \equiv \frac{\alpha(A_sH)^{\frac{\sigma-1}{\sigma}}}{\alpha(A_sH)^{\frac{\sigma-1}{\sigma}}+(1-\alpha)[A_u(1-H)]^{\frac{\sigma-1}{\sigma}}} \geq H \Leftrightarrow w_s \geq w_u$ from (4) and (5). (ii) Straightforward from (i) except that $p = 1, q = 0$ is not an equilibrium when $\beta \leq \gamma$, which is shown in the proof of Proposition A1 (i). (iii) From (i)(b), $a(H) - H = 0$ at $H = 0, a(H)$, and

$$\begin{aligned} a'(H) &= \frac{\sigma-1}{\sigma}\alpha(A_sH)^{\frac{\sigma-1}{\sigma}}\frac{\left\{\alpha(A_sH)^{\frac{\sigma-1}{\sigma}}+(1-\alpha)[A_u(1-H)]^{\frac{\sigma-1}{\sigma}}\right\}\frac{1}{H}-\left\{\alpha(A_sH)^{\frac{\sigma-1}{\sigma}}\frac{1}{H}-(1-\alpha)[A_u(1-H)]^{\frac{\sigma-1}{\sigma}}\frac{1}{1-H}\right\}}{\left\{\alpha(A_sH)^{\frac{\sigma-1}{\sigma}}+(1-\alpha)[A_u(1-H)]^{\frac{\sigma-1}{\sigma}}\right\}^2} \\ &= \frac{\sigma-1}{\sigma}\frac{\alpha(A_sH)^{\frac{\sigma-1}{\sigma}}}{\alpha(A_sH)^{\frac{\sigma-1}{\sigma}}+(1-\alpha)[A_u(1-H)]^{\frac{\sigma-1}{\sigma}}}\frac{(1-\alpha)[A_u(1-H)]^{\frac{\sigma-1}{\sigma}}}{\alpha(A_sH)^{\frac{\sigma-1}{\sigma}}+(1-\alpha)[A_u(1-H)]^{\frac{\sigma-1}{\sigma}}}\frac{1}{H(1-H)} \\ &= \frac{\sigma-1}{\sigma}\frac{a(H)[1-a(H)]}{H(1-H)} > 0, \end{aligned} \tag{47}$$

which implies that

$$\lim_{H\to 0}[a'(H)-1] = \frac{\sigma-1}{\sigma}\lim_{H\to 0}\left[\frac{a(H)}{H}\right]-1$$

$$= \frac{\sigma-1}{\sigma}\lim_{H\to 0}\left\{\frac{1}{H}\frac{1}{1+\frac{1-\alpha}{\alpha}\left[\frac{A_u(1-H)}{A_sH}\right]^{\frac{\sigma-1}{\sigma}}}\right\}-1 = +\infty, \tag{48}$$

$$\lim_{H\to\overline{H}}[a'(H)-1] = -\frac{1}{\sigma} < 0. \tag{49}$$

Further,

$$\begin{aligned} a''(H) &= \frac{\sigma-1}{\sigma}\frac{H(1-H)a'(H)[1-2a(H)]-a(H)[1-a(H)](1-2H)}{[H(1-H)]^2} \\ &= \frac{\sigma-1}{\sigma}a(H)[1-a(H)]\frac{\frac{\sigma-1}{\sigma}[1-2a(H)]-(1-2H)}{[H(1-H)]^2} \\ &= \frac{\sigma-1}{\sigma}a(H)[1-a(H)]\frac{-\frac{1}{\sigma}[1-2a(H)]-2(a(H)-H)}{[H(1-H)]^2} < 0. \end{aligned} \tag{50}$$

Hence, there exists unique $H^+ \in (0, \overline{H})$ satisfying $a'(H) - 1 = 0$, and $a(H) - H$ increases (decreases) with H for $H < (>)H^+$. This implies that $\frac{d\tau}{dH} > (<)0$ for $H < (>)H^+$. ■

Proof of Corollary 1 . (i) The result on inequality is from $(1-\tau)w_s + T - [(1-\tau)w_u + T] = (1-\tau)(w_s-w_u)$ and Proposition 1 (ii). The result on the disposable income of skilled workers holds because $(1-\tau)w_s + T = (1-\tau)w_s + \left(\tau-\frac{1}{2}\tau^2\right)\overline{w}$ decreases with τ from $-w_s + (1-\tau)\overline{w} < 0$. (ii) The derivative of $(1-\tau)w_u + T = (1-\tau)w_u + \left(\tau-\frac{1}{2}\tau^2\right)\overline{w}$ with respect to τ equals $-w_u + (1-\tau)\overline{w}$, which is positive under Assumption 1 because

$$
\begin{aligned}
-w_u+(1-\tau)\,\overline{w}>0 &\Leftrightarrow \tau<\frac{\overline{w}-w_u}{\overline{w}}\\
&\Leftrightarrow \tau<\frac{H\Big\{\alpha(A_sH)^{\frac{\sigma-1}{\sigma}}\frac{1}{H}-(1-\alpha)[A_u(1-H)]^{\frac{\sigma-1}{\sigma}}\frac{1}{1-H}\Big\}}{\alpha(A_sH)^{\frac{\sigma-1}{\sigma}}+(1-\alpha)[A_u(1-H)]^{\frac{\sigma-1}{\sigma}}}\ \text{(from (3) and (5))}\\
&\Leftarrow \frac{2\beta}{1+\gamma}(a(H)-H)<\frac{1}{1-H}(a(H)-H)\ \text{(from Proposition 1)},
\end{aligned}
\tag{51}
$$

where the last inequality is true under Assumption 1. ■

Proof of Lemma 2. (i) When $p=q=0$ or when $\beta\leq\gamma$ and $p=0$, $q=1$, $\tau=0$ from Proposition 1 and thus $(1-\tau)w_s+T=w_s$, which equals $w_u+\frac{(1+r)\overline{e}}{1+\gamma}$ ($w_u+\frac{(1+r)\overline{e}+\gamma\delta\Delta\widetilde{S_N}}{1+\beta H+\gamma(1-H)}$) at $H=H^*$ when $p=q=0$ from (43) (when $\beta\leq\gamma$ and $p=0$, $q=1$ from (44)). Because w_s decreases with H from (4), (32) is satisfied for any $H\leq H^*$ if it holds at $H=H^*$, where the condtion can be expressed as $\lambda w_u\geq\left[1-\frac{\lambda(1+r)}{1+\gamma}\right]\overline{e}$ when $p=q=0$ and as $\lambda\left[w_u+\frac{\gamma\delta\Delta\widetilde{S_N}}{1+\beta H+\gamma(1-H)}\right]\geq\left[1-\frac{\lambda(1+r)}{1+\beta H+\gamma(1-H)}\right]\overline{e}$ when $p=0$, $q=1$. Because w_s-w_u decreases with H and w_u increases with H from (4) and (5), H^* and w_u at $H=H^*$ decrease with $\overline{e}$. Hence, (32) holds when $\overline{e}$ is sufficiently small or λ is sufficiently large. Because $w_s<w_u+(1+r)\overline{e}$ at $H=H^*$, (33) too holds at $H=H^*$. For $H<H^*$, the LHS of (33) increases with H. Thus, if (34) holds at $H=0$, there exists $\underline{F}\in(0,H^*)$ such that $\frac{\lambda}{1-\lambda(1+r)}w_u=\overline{e}$ at $H=\underline{F}$; $\frac{\lambda}{1-\lambda(1+r)}w_u>(\leq)\overline{e}$ and thus $H=F<H^*$ increases over time (is time-invariant) for $H>(\leq)\underline{F}$. Because $w_u=(1-\alpha)^{\frac{\sigma}{\sigma-1}}A_u$ at $H=0$, the condition holds if $\overline{e}$ is not too small or λ is not too large so that $\frac{\lambda}{1-\lambda(1+r)}(1-\alpha)^{\frac{\sigma}{\sigma-1}}A_u<\overline{e}$ is true.

When values of p and q are such that $\tau>0$, whether $(1-\tau)w_s\geq(1-\tau)w_u+(1+r)\overline{e}$ at $H=H^*$ or not is unclear from (42), (44), and (45). Consider the case in which $(1-\tau)w_s\geq(1-\tau)w_u+(1+r)\overline{e}$ for any $H\leq H^*$ first. In this case, $\underline{F}$ smaller than the one for $\tau=0$ exists if the above conditions hold. This is because (34) holds at $H=0$ from $(1-\tau)w_u+T=w_u$ at $H=0$, $\frac{\lambda}{1-\lambda(1+r)}[(1-\tau)w_u+T]>\frac{\lambda}{1-\lambda(1+r)}w_u\geq\overline{e}$, i.e., (33), holds for H weakly greater than $\underline{F}$ for $\tau=0$, where $(1-\tau)w_u+T>w_u$ under Assumption 1 from the proof of Corollary 1 (ii), and (32) holds from $(1-\tau)w_s\geq(1-\tau)w_u+(1+r)\overline{e}$. (Since the relation between $(1-\tau)w_u+T$ and H is unclear, the possibility that there exist multiple levels of H satisfying $\frac{\lambda}{1-\lambda(1+r)}[(1-\tau)w_u+T]=\overline{e}$ cannot be excluded; by definition, $\underline{F}$ is the highest H satisfying the equation. Because the relation between $(1-\tau)w_s+T$ and H too is ambiguous, the possible existence of $H<\underline{F}$ satisfying $\lambda[(1-\tau)w_s+T]=\overline{e}$ too cannot be ruled out, although this is unlikely given that w_s is large for small H. Thus, the possibility that $H<\underline{F}$ increases or decreases temporarily cannot be ruled out.)

Next, consider the case in which $(1-\tau)w_s<(1-\tau)w_u+(1+r)\overline{e}$ for some $H\leq H^*$. As in the previous case, if the conditions for values of p and q such that $\tau=0$ are satisfied, (34) holds at $H=0$ and (33) holds for H weakly greater than $\underline{F}$ for $\tau=0$. By contrast, these conditions do not assure that (32) is true. Because $(1-\tau)w_s+T=(1-\tau)w_s+\left(\tau-\frac{1}{2}\tau^2\right)\overline{w}>\frac{1}{2}\left(w_s+\frac{3}{4}\overline{w}\right)>\frac{7}{8}\overline{w}$ from Assumption 2 and $\overline{w}$ increases with H from (3), (4), and (5), (32) holds if $\lambda\frac{7}{8}\overline{w}\geq\overline{e}$ holds for the minimum H satisfying $(1-\tau)w_s=(1-\tau)w_u+(1+r)\overline{e}$, which is denoted by $H^\dagger$. Because $(1-\tau)(w_s-w_u)$ decreases with H at $H=H^\dagger$, $H^\dagger$ decreases with $\overline{e}$. Hence, (32) is true if $\overline{e}$ is sufficiently small or λ is sufficiently large.

The result on the utility is because within-class disparities in transfers diminish over time from (30) and (31).

(ii) The result for $\underline{F}$ is straightforward from the proof of (i) and Corollary 1 (ii), and the result for H^* is straightforward from the definition of H^* and Proposition 1. ■

Proof of Proposition 2. (i) The result on H is straightforward from Lemma 2 (i) when values of p and q are time-invariant, which is the case when $p=q=0$ in the initial period from footnote 28 attached to the lemma. For other values of p_0 and q_0, as mentioned in the footnote, the possibility that $H \leq \underline{F}$ increases or decreases temporarily cannot be ruled out. Then, from Figures 1 and 2 based on Proposition A2, values of p and q may change. When the shift from $p=0, q=1$ to $p=q=0$ occurs, H remains smaller than $\underline{F}$ for $p=0, q=1$, because H is time-invariant after the shift to $p=q=0$ from footnote 28. The same is true for the shift from $p=q=1$ to $p=q=0$. When the shift from $p=q=1$ to $p=0, q=1$ occurs, H remains smaller than $\underline{F}$ for $p=q=1$, because $\underline{F}$ for $p=0, q=1$ is greater than $\underline{F}$ for $p=q=1$ from Lemma 2 (ii). By contrast, when $p_0=0, q_0=1$ and F_0 is smaller than $\underline{F}$ for $p=0, q=1$ and greater than $\underline{F}$ for $p=q=1$, the possibility that the shift from $p=0, q=1$ to $p=q=1$ occurs and H converges to H^* cannot be ruled out. (The shift from or to $p=1, q=0$ is not considered, because $p=1, q=0$ is realized only for large H when $\beta > \gamma$ and $\frac{A_s}{A_u}$ is relatively high,)

The results on Y, w_s, w_u, and the earnings disparity are straightforward from (3)−(5). The welfare disparity is greater than the long-run level for $F_0 > \underline{F}$, which is 0 from (ii). Thus, the disparity is large in the sense that it is greater than the one for $F_0 > \underline{F}$ when H is sufficiently large.

(ii) The result on H is from Figures 1 and 2 based on Proposition A2, Lemma 2 (i), and Assumption 3. The result on the long-run welfare is from Lemma 2 (i), and the results on w_s, w_u, and the earnings disparity are from (4) and (5). The result on Y is from $\frac{dY}{dH} > 0$, which is proved as follows. $\frac{dY}{dH} > 0 \Leftrightarrow w_s > w_u$ because, from (3),

$$\begin{aligned}\frac{dY}{dH} &= \left\{\alpha(A_sH)^{\frac{\sigma-1}{\sigma}} + (1-\alpha)[A_u(1-H)]^{\frac{\sigma-1}{\sigma}}\right\}^{\frac{\sigma}{\sigma-1}-1}\left\{\alpha(A_sH)^{\frac{\sigma-1}{\sigma}}\frac{1}{H} - (1-\alpha)[A_u(1-H)]^{\frac{\sigma-1}{\sigma}}\frac{1}{1-H}\right\} \\ &= w_s - w_u \text{ (from (4) and (5))}, \qquad (52)\end{aligned}$$

where $w_s > w_u$ from $H < \overline{H}$.

(iii) The results are from Lemma 2 (ii) and Proposition A2 (Figures 1 and 2). ■

Proof of Proposition 3. (i) and (ii) The results on identities and τ are from Figures 1 and 2 that are based on Proposition A2, Proposition 1, and Assumption 3. Convergence to H^* is from Proposition 2 (ii), except the stop of the convergence when $\Delta\widetilde{S_N}$ is relatively low, which is explained in footnote 39 attached to the proposition. (iii) Existence of multiple equilibria are from Figures 1 and 2 (Proposition A2). Assumption 3 implies the persistent effect of the initial equilibrium on the subsequent dynamics and long-run outcomes. ■

Proof of Proposition 4. First, it is proved that shapes of the dividing lines for each combination of p and q are similar to those under the original setting. From (35) and (36), the utility of an individual in class C ($C = S, U$) identifying with social group G ($G = C, N$), u_{CG}, which is given by (9)−(12) under the original setting, is expressed as:

$$u_{SN} = (1+\gamma)\left(\tau - \frac{1}{2}\tau^2\right)\overline{w} + (1-\tau)w_s - \beta\left[2\omega_q(1-H) + (1-\tau)(w_s - \overline{w})\right] + \gamma\left[\delta\widetilde{S_N} + (1-\tau)\overline{w}\right], \quad (53)$$

$$u_{SS} = (1+\gamma)\left(\tau - \frac{1}{2}\tau^2\right)\overline{w} + (1-\tau)w_s + \gamma\left[\delta\widetilde{S_C} + (1-\tau)w_s\right], \quad (54)$$

$$u_{UN} = (1+\gamma)\left(\tau - \frac{1}{2}\tau^2\right)\overline{w} + (1-\tau)w_u - \beta\left[2\omega_q H + (1-\tau)(\overline{w} - w_u)\right] + \gamma\left[\delta\widetilde{S_N} + (1-\tau)\overline{w}\right], \quad (55)$$

$$u_{UU} = (1+\gamma)\left(\tau - \frac{1}{2}\tau^2\right)\overline{w} + (1-\tau)w_u + \gamma\left[\delta\widetilde{S_C} + (1-\tau)w_u\right]. \quad (56)$$

From these equations, the condition for $p = q = 0$ equals ($\tau = 0$ from Proposition 1 (i)(a))

$$p = q = 0 \text{ iff } \gamma\delta\Delta\widetilde{S_N} \le \min\{[(w_s-w_u)(\beta+\gamma)+2\beta\omega_q](1-H), [(w_s-w_u)(\beta-\gamma)+2\beta\omega_q]H\}$$

$$\Leftrightarrow \gamma\delta\Delta\widetilde{S_N} \le [(w_s-w_u)(\beta-\gamma)+2\beta\omega_q]H \quad \text{for } H \le \min\left\{H_{00}^{\flat}, \overline{H}\right\} \tag{57}$$

$$\text{and } \gamma\delta\Delta\widetilde{S_N} \le [(w_s-w_u)(\beta+\gamma)+2\beta\omega_q](1-H) \quad \text{for } H \in \left[H_{00}^{\flat}, \overline{H}\right] \text{when } \overline{H} > H_{00}^{\flat}, \tag{58}$$

$$\text{where } H_{00}^{\flat} \text{ is } H \text{ satisfying } H = \frac{1}{2}\left\{1+\frac{\gamma}{\beta}\frac{w_s-w_u}{(w_s-w_u)+2\omega_q}\right\}.$$

$H_{00}^{\flat}$ is unique because $\frac{1}{2}\left\{1+\frac{\gamma}{\beta}\frac{w_s-w_u}{(w_s-w_u)+2\omega_q}\right\}$ equals $\frac{\beta+\gamma}{2\beta}$ at $H=0$ and decreases with H. Note that $\overline{H} \le (>) H_{00}^{\flat} \Leftrightarrow \overline{H} \le (>) \frac{1}{2}$ and $\frac{\beta+\gamma}{2\beta}$ of the original model is replaced by $H_{00}^{\flat}$.

As with the corresponding equations of the original model, the RHS of (57) when $\beta > \gamma$ increases (decreases) with H for small (large) H and the RHS of (58) decreases with H. The former can be shown as follows. The proof of (iii) of Proposition A1 shows that there exists $H^{\sharp} \in (0, \overline{H})$ such that $\frac{d[(w_s-w_u)H]}{dH} > (<) 0$ for $H < (>) H^{\sharp}$. Further, $\frac{d^2[(w_s-w_u)H]}{dH^2} < 0$ for $H \ge H^{\sharp}$ can be easily proven from the equations in the proof. Hence, unless $\beta\omega_q$ is very large, there exists $H^{\sharp\sharp} \in (H^{\sharp}, \overline{H})$ such that the RHS of (57) when $\beta > \gamma$ increases (decreases) with H for $H < (>) H^{\sharp\sharp}$.

The condition for $p = q = 1$ equals

$$p = q = 1 \text{ iff } \gamma\delta\Delta\widetilde{S_N} \ge \max\{[(1-\tau)(w_s-w_u)(\beta+\gamma)+2\beta\omega_q](1-H), [(1-\tau)(w_s-w_u)(\beta-\gamma)+2\beta\omega_q]H\}$$

$$\Leftrightarrow \gamma\delta\Delta\widetilde{S_N} \ge [(1-\tau)(w_s-w_u)(\beta+\gamma)+2\beta\omega_q](1-H) \quad \text{for } H \le \min\left\{H_{11}^{\flat}, \overline{H}\right\} \tag{59}$$

$$\text{and } \gamma\delta\Delta\widetilde{S_N} \ge [(1-\tau)(w_s-w_u)(\beta-\gamma)+2\beta\omega_q]H \quad \text{for } H \in \left[H_{11}^{\flat}, \overline{H}\right] \text{when } \overline{H} > H_{11}^{\flat}, \tag{60}$$

$$\text{where } \tau = \frac{2\beta}{1+\gamma}(a(H)-H) \text{ and } H_{11}^{\flat} \text{ is } H \in (0, H_{00}^{\flat}) \text{ satisfying } H = \frac{1}{2}\left\{1+\frac{\gamma}{\beta}\frac{(1-\tau)(w_s-w_u)}{(1-\tau)(w_s-w_u)+2\omega_q}\right\}.$$

$H_{11}^{\flat} < H_{00}^{\flat}$ because $\frac{1}{2}\left\{1+\frac{\gamma}{\beta}\frac{(1-\tau)(w_s-w_u)}{(1-\tau)(w_s-w_u)+2\omega_q}\right\} < \frac{1}{2}\left\{1+\frac{\gamma}{\beta}\frac{w_s-w_u}{(w_s-w_u)+2\omega_q}\right\}$. $\overline{H} \le (>) H_{11}^{\flat} \Leftrightarrow \overline{H} \le (>) \frac{1}{2}$.

As with the corresponding equations of the original model, the RHS of (59) decreases with H because it equals the original equation plus $2\beta\omega_q(1-H)$ and the RHS of (60) increases with H for small H and decreases with H for H close to $\overline{H}$. To be precise, the RHS of (60) increases with H at least for $H \le H^{\sharp\sharp} \in (H^{\sharp}, \overline{H})$ from $\frac{d[(1-\tau)(w_s-w_u)H]}{dH} > 0$ at least for $H \le H^{\sharp}$ (the proof of Proposition A2 (i)(a)) and the above proof on the RHS of (57), and it decreases with H for H close to $\overline{H}$ from $\frac{d[(1-\tau)(w_s-w_u)H]}{dH} = \frac{d[(w_s-w_u)H]}{dH}$ at $H = \overline{H}$ and the proof on the RHS of (57).

The condition for $p = 0, q = 1$ equals

$$p=0, q=1 \text{ iff } \gamma\delta\Delta\widetilde{S_N} \le [(1-\tau)(w_s-w_u)(\beta+\gamma)+2\beta\omega_q](1-H) \tag{61}$$

$$\text{and } \gamma\delta\Delta\widetilde{S_N} \ge [(1-\tau)(w_s-w_u)(\beta-\gamma)+2\beta\omega_q]H, \text{ where } \tau = \frac{\beta-\gamma}{1+\gamma}(a(H)-H). \tag{62}$$

This occurs only for $H \le \min\left\{H_{01}^{\flat}, \overline{H}\right\}$, where $H_{01}^{\flat}$ is $H \in (H_{11}^{\flat}, H_{00}^{\flat})$ satisfying $H = \frac{1}{2}\left\{1+\frac{\gamma}{\beta}\frac{(1-\tau)(w_s-w_u)}{(1-\tau)(w_s-w_u)+2\omega_q}\right\}$ with $\tau = \frac{\beta-\gamma}{1+\gamma}(a(H)-H)$. From the above proofs on the equations for $p = q = 0$ and $p = q = 1$, the relations between the RHSs of (61) and (62) and H are similar to those under the original setting.

Finally, the condition for $p = 1, q = 0$ equals

$$p=1, q=0 \text{ iff } \gamma\delta\Delta\widetilde{S_N} \ge [(1-\tau)(w_s-w_u)(\beta+\gamma)+2\beta\omega_q](1-H) \tag{63}$$

$$\text{and } \gamma\delta\Delta\widetilde{S_N} \le [(1-\tau)(w_s-w_u)(\beta-\gamma)+2\beta\omega_q]H, \text{ where } \tau = \frac{\beta+\gamma}{1+\gamma}(a(H)-H). \tag{64}$$

This happens only for $H \in \left[H_{10}^{\flat}, \overline{H}\right]$, where $H_{10}^{\flat}$ is $H \in (H_{11}^{\flat}, H_{01}^{\flat})$ satisfying $H = \frac{1}{2}\left\{1 + \frac{\gamma}{\beta}\frac{(1-\tau)(w_s - w_u)}{(1-\tau)(w_s-w_u)+2\omega_q}\right\}$ with $\tau = \frac{\beta+\gamma}{1+\gamma}(a(H) - H)$, thus only when $\overline{H} > H_{10}^{\flat} \Leftrightarrow \overline{H} > \frac{1}{2}$. The relations between the RHSs of these equations and H are similar to those under the original setting from the above proofs on the equations for $p = q = 0$ and $p = q = 1$.

From these results, the figure that illustrates combinations of H and $\Delta\widetilde{S_N}$ such that each equilibrium exists when $\overline{H} \leq \frac{1}{2}$ is very similar to Figures 1 and 3. (The difference is that the highest value of H such that the dividing line for $p = q = 0$ is upward sloping and the corresponding H for the lower dividing line for $p = 0, q = 1$ are greater.) Shapes of the dividing lines of the figure when $\overline{H} > \frac{1}{2}$ too are similar to those of Figures 2 and 4, although, unlike these figures, the critical value of H at which the equation for the dividing line for $p = q = 1$ changes, the one above which $p = 1, q = 0$ holds, the one below which $p = 0, q = 1$ holds, and the one at which the equation for the dividing line for $p = q = 1$ changes are all different, i.e., $H_{11}^{\flat} < H_{10}^{\flat} < H_{01}^{\flat} < H_{00}^{\flat}$.

A decrease in ω_q decreases the RHSs of (57)–(64). Hence, given H, $\Delta\widetilde{S_N}$ satisfying the equations decrease, i.e., all the dividing lines shift downward on the $(H, \Delta\widetilde{S_N})$ plane. ■

Proof of Proposition 5. (i) From (4) and (5),

$$\frac{dw_s}{d\left(\frac{A_s}{A_u}\right)^{\frac{\sigma-1}{\sigma}}} - \frac{dw_u}{d\left(\frac{A_s}{A_u}\right)^{\frac{\sigma-1}{\sigma}}} = A_u \alpha (H)^{\frac{\sigma-1}{\sigma}} (\Gamma)^{\frac{\sigma}{\sigma-1}-2} \left\{ \begin{array}{c} \frac{1}{H}\left[\alpha\left(\frac{A_s}{A_u}\right)^{\frac{\sigma-1}{\sigma}}(H)^{\frac{\sigma-1}{\sigma}} + (1-\alpha)(1-H)^{\frac{\sigma-1}{\sigma}} + \frac{1}{\sigma-1}\left(\frac{A_s}{A_u}\right)^{\frac{\sigma-1}{\sigma}}\alpha(H)^{\frac{\sigma-1}{\sigma}}\right] \\ -\frac{1}{\sigma-1}(1-\alpha)(1-H)^{\frac{\sigma-1}{\sigma}}\frac{1}{1-H} \end{array} \right\}$$

$$= A_u \alpha (H)^{\frac{\sigma-1}{\sigma}} (\Gamma)^{\frac{\sigma}{\sigma-1}-2} \left\{ \begin{array}{c} \frac{1}{H}\left[\alpha\left(\frac{A_s}{A_u}\right)^{\frac{\sigma-1}{\sigma}}(H)^{\frac{\sigma-1}{\sigma}} + (1-\alpha)(1-H)^{\frac{\sigma-1}{\sigma}}\right] \\ + \frac{1}{\sigma-1}\left[\left(\frac{A_s}{A_u}\right)^{\frac{\sigma-1}{\sigma}}\alpha(H)^{\frac{\sigma-1}{\sigma}}\frac{1}{H} - (1-\alpha)(1-H)^{\frac{\sigma-1}{\sigma}}\frac{1}{1-H}\right] \end{array} \right\} > 0, \tag{65}$$

where the last inequality sign is from $w_s > w_u$.

(ii) The RHSs of the conditions for identity choice, (18)–(23), are expressed as $(1-\tau)(w_s - w_u)$ times an expression that does not depend on A_s and A_u. Consider the case $p = q = 1$, in which $\tau = \frac{2\beta}{1+\gamma}(a(H) - H)$ from Proposition 1 (i). (Other cases can be proved similarly.) From (65) and the proposition ($\Gamma \equiv \alpha\left(\frac{A_s}{A_u}H\right)^{\frac{\sigma-1}{\sigma}} + (1-\alpha)(1-H)^{\frac{\sigma-1}{\sigma}}$),

$$\frac{d[(1-\tau)(w_s - w_u)]}{d\left(\frac{A_s}{A_u}\right)^{\frac{\sigma-1}{\sigma}}} = -\frac{2\beta}{1+\gamma}\frac{\alpha(H)^{\frac{\sigma-1}{\sigma}}(1-\alpha)(1-H)^{\frac{\sigma-1}{\sigma}}}{\left[\alpha\left(\frac{A_s}{A_u}H\right)^{\frac{\sigma-1}{\sigma}} + (1-\alpha)(1-H)^{\frac{\sigma-1}{\sigma}}\right]^2} A_u(\Gamma)^{\frac{\sigma}{\sigma-1}-1}\left[\alpha\left(\frac{A_s}{A_u}H\right)^{\frac{\sigma-1}{\sigma}}\frac{1}{H} - (1-\alpha)(1-H)^{\frac{\sigma-1}{\sigma}}\frac{1}{1-H}\right]$$

$$+ \left[1 - \frac{2\beta}{1+\gamma}(a(H) - H)\right] A_u \alpha (H)^{\frac{\sigma-1}{\sigma}} (\Gamma)^{\frac{\sigma}{\sigma-1}-2} \left\{ \begin{array}{c} \frac{1}{H}\left[\alpha\left(\frac{A_s}{A_u}\right)^{\frac{\sigma-1}{\sigma}}(H)^{\frac{\sigma-1}{\sigma}} + (1-\alpha)(1-H)^{\frac{\sigma-1}{\sigma}}\right] \\ + \frac{1}{\sigma-1}\left[\left(\frac{A_s}{A_u}\right)^{\frac{\sigma-1}{\sigma}}\alpha(H)^{\frac{\sigma-1}{\sigma}}\frac{1}{H} - (1-\alpha)(1-H)^{\frac{\sigma-1}{\sigma}}\frac{1}{1-H}\right] \end{array} \right\}$$

$$= A_u(\Gamma)^{\frac{\sigma}{\sigma-1}-2}\alpha(H)^{\frac{\sigma-1}{\sigma}} \left\{ \begin{array}{c} -\frac{2\beta}{1+\gamma}(1 - a(H))\left[\alpha\left(\frac{A_s}{A_u}H\right)^{\frac{\sigma-1}{\sigma}}\frac{1}{H} - (1-\alpha)(1-H)^{\frac{\sigma-1}{\sigma}}\frac{1}{1-H}\right] \\ + \left[1 - \frac{2\beta}{1+\gamma}(a(H) - H)\right]\frac{1}{H}\left[\alpha\left(\frac{A_s}{A_u}\right)^{\frac{\sigma-1}{\sigma}}(H)^{\frac{\sigma-1}{\sigma}} + (1-\alpha)(1-H)^{\frac{\sigma-1}{\sigma}}\right] \\ + \frac{1}{\sigma-1}\left[1 - \frac{2\beta}{1+\gamma}(a(H) - H)\right]\left[\left(\frac{A_s}{A_u}\right)^{\frac{\sigma-1}{\sigma}}\alpha(H)^{\frac{\sigma-1}{\sigma}}\frac{1}{H} - (1-\alpha)(1-H)^{\frac{\sigma-1}{\sigma}}\frac{1}{1-H}\right] \end{array} \right\}$$

$$= A_u(\Gamma)^{\frac{\sigma}{\sigma-1}-2}\alpha(H)^{\frac{\sigma-1}{\sigma}}\left\{\begin{array}{c}\left[1-\frac{2\beta}{1+\gamma}(1-H)\right]\alpha\left(\frac{A_s}{A_u}H\right)^{\frac{\sigma-1}{\sigma}}\frac{1}{H}+\frac{2\beta}{1+\gamma}(1-a(H))(1-\alpha)(1-H)^{\frac{\sigma-1}{\sigma}}\frac{1}{1-H}\\ +\left[1-\frac{2\beta}{1+\gamma}(a(H)-H)\right]\frac{1}{H}(1-\alpha)(1-H)^{\frac{\sigma-1}{\sigma}}\\ +\frac{1}{\sigma-1}\left[1-\frac{2\beta}{1+\gamma}(a(H)-H)\right]\left[\left(\frac{A_s}{A_u}\right)^{\frac{\sigma-1}{\sigma}}\alpha(H)^{\frac{\sigma-1}{\sigma}}\frac{1}{H}-(1-\alpha)(1-H)^{\frac{\sigma-1}{\sigma}}\frac{1}{1-H}\right]\end{array}\right\}>0, \quad (66)$$

where $1-\frac{2\beta}{1+\gamma}(1-H) > 0$ from Assumption 1. Hence, the dividing lines for identity choice shift upward. The result on the identity shift is from Proposition A2 in Appendix A or Figures 3 and 4.

(iii) [Result on τ] From Proposition 1, when $p = q = 0$ is not true, τ equals a constant times $a(H)-H$. The derivatives of $a(H) - H$ with respect to $\left(\frac{A_s}{A_u}\right)^{\frac{\sigma-1}{\sigma}}$ equal

$$\frac{d(a(H)-H)}{d\left(\frac{A_s}{A_u}\right)^{\frac{\sigma-1}{\sigma}}}=\frac{1}{\left\{\alpha\left(\frac{A_s}{A_u}H\right)^{\frac{\sigma-1}{\sigma}}+(1-\alpha)(1-H)^{\frac{\sigma-1}{\sigma}}\right\}^2}$$

$$\times\left\{\left[\alpha\left(\frac{A_s}{A_u}H\right)^{\frac{\sigma-1}{\sigma}}+(1-\alpha)(1-H)^{\frac{\sigma-1}{\sigma}}\right]\alpha(H)^{\frac{\sigma-1}{\sigma}}(1-H)-\left[\alpha\left(\frac{A_s}{A_u}H\right)^{\frac{\sigma-1}{\sigma}}(1-H)-(1-\alpha)(1-H)^{\frac{\sigma-1}{\sigma}}H\right]\alpha(H)^{\frac{\sigma-1}{\sigma}}\right\}$$

$$=\frac{\alpha(H)^{\frac{\sigma-1}{\sigma}}(1-\alpha)(1-H)^{\frac{\sigma-1}{\sigma}}}{\left\{\alpha\left(\frac{A_s}{A_u}H\right)^{\frac{\sigma-1}{\sigma}}+(1-\alpha)(1-H)^{\frac{\sigma-1}{\sigma}}\right\}^2}>0. \quad (67)$$

When the identity shift occurs, the result is straightforward from (ii) and Proposition 1 (ii).

[Result on the welfare disparity] The difference in welfare between skilled and unskilled workers equal to the LHSs of (42)−(45) in Appendix B. Given values of p and q, an increase in $\frac{A_s}{A_u}$ raises $(1-\tau)(w_s-w_u)$ of the LHSs from (66) in the proof of (ii) and thus the inter-class welfare disparity. (Increased $\frac{A_s}{A_u}$ does not affect $\bar{e}$ because it is proportional to w_s in the previous period.) The proof of the result when the identity shift occurs is as follows.

Shift from $p = q = 1$ to $p = q = 0$ (case mentioned in footnote 45): When $H \leq \frac{\beta+\gamma}{2\beta}$, the LHS of (43) is greater than that of (42), thus, together with the fact that $w_s - w_u$ increases with $\frac{A_s}{A_u}$ from (i), the identity shift increases the inter-class welfare disparity. When $H > \frac{\beta+\gamma}{2\beta}$, because the condition for $p = q = 1$ is (19) and the one for $p = q = 0$ is (21),

$$(w_{s,00}-w_{u,00})(\beta+\gamma)(1-H) \geq (1-\tau_{11})(w_{s,11}-w_{u,11})(\beta-\gamma)H, \quad (68)$$

where subscript 00 is for $p = q = 0$ and subscript 11 is for $p = q = 1$. The difference in the LHS of (43) and that of (42) equals

$$\begin{aligned}
& (1+\gamma)(w_{s,00}-w_{u,00})-[1-\beta(1-2H)](1-\tau_{11})(w_{s,11}-w_{u,11})\\
\geq\ & (1+\gamma)(w_{s,00}-w_{u,00})-[1-\beta(1-2H)](w_{s,00}-w_{u,00})\frac{(\beta+\gamma)(1-H)}{(\beta-\gamma)H} \text{ (from (68))}\\
=\ & \frac{w_{s,00}-w_{u,00}}{(\beta-\gamma)H}\{(1+\gamma)(\beta-\gamma)H-[1-\beta(1-2H)](\beta+\gamma)(1-H)\}\\
>\ & \frac{w_{s,00}-w_{u,00}}{H}\frac{\beta+\gamma}{2\beta}\{(1+\gamma)-(1-\beta+\beta+\gamma)\} = 0, \text{ (from } H>\frac{\beta+\gamma}{2\beta}\text{)}
\end{aligned} \quad (69)$$

where $[1-\beta(1-2H)](1-H)$ decreases with H from $H>\frac{\beta+\gamma}{2\beta}$.

Shift from $p = q = 1$ to $p = 0$, $q = 1$: From Propositions A1 and A2, $p = 0$, $q = 1$ is realized only for $H \leq \frac{\beta+\gamma}{2\beta}$. Given H, the LHS of (44) is lowest when $\gamma\delta\Delta\widetilde{S_N}=(\beta+\gamma)(1-\tau)(1-H)(w_s-w_u)$

from (22), in which case the LHS equals

$$
\begin{aligned}
&[1+\beta H+\gamma(1-H)]\,(1-\tau)(w_s-w_u)-(1+r)\overline{e}-(\beta+\gamma)(1-\tau)(1-H)(w_s-w_u)\\
&=[1-\beta(1-2H)](1-\tau)(w_s-w_u)-(1+r)\overline{e},
\end{aligned}
\tag{70}
$$

which is greater than the LHS of (42) because τ is lower when $p=0$, $q=1$ from Proposition 1 (ii) and w_s-w_u being increasing in $\frac{A_s}{A_u}$.

Shift from $p=0$, $q=1$ to $p=q=0$: Given H, the LHS of (44) is highest when $\gamma\delta\Delta\widetilde{S_N}=(\beta-\gamma)(1-\tau)H(w_s-w_u)$ from (22), in which case the LHS equals

$$
\begin{aligned}
&[1+\beta H+\gamma(1-H)](1-\tau)(w_s-w_u)-(1+r)\overline{e}-(\beta-\gamma)(1-\tau)H(w_s-w_u)\\
&=(1+\gamma)(1-\tau)(w_s-w_u)-(1+r)\overline{e},
\end{aligned}
\tag{71}
$$

which is smaller than the LHS of (43) from $\tau>0$ and w_s-w_u being increasing in $\frac{A_s}{A_u}$.

Shift from $p=q=1$ to $p=1$, $q=0$ (case mentioned in footnote 46): From Propositions A1 and A2, $p=1$, $q=0$ is realized only for $H>\frac{\beta+\gamma}{2\beta}$. The difference in the LHS of (45) and that of (42) equals

$$
\begin{aligned}
&[1-\beta(1-H)+\gamma H](1-\tau_{10})(w_{s,10}-w_{u,10})+\gamma\delta\Delta\widetilde{S_N}-[1-\beta(1-2H)](1-\tau_{11})(w_{s,11}-w_{u,11})\\
\geq\ &[1-\beta(1-H)+\gamma H](1-\tau_{10})(w_{s,10}-w_{u,10})\\
&+(1-\tau_{11})(w_{s,11}-w_{u,11})(\beta-\gamma)H-[1-\beta(1-2H)](1-\tau_{11})(w_{s,11}-w_{u,11})\ \text{(from (19))}\\
=\ &[1-\beta(1-H)+\gamma H][(1-\tau_{10})(w_{s,10}-w_{u,10})-(1-\tau_{11})(w_{s,11}-w_{u,11})]>0,
\end{aligned}
\tag{72}
$$

where the last inequality holds because $\tau_{11}>\tau_{10}$ from Proposition 1 (ii) and $w_{s,10}-w_{u,10}>w_{s,11}-w_{u,11}$ from w_s-w_u being increasing in $\frac{A_s}{A_u}$.

Shift from $p=1$, $q=0$ to $p=q=0$ (case mentioned in footnote 46): The difference in the LHS of (43) and that of (45) equals

$$
\begin{aligned}
&(1+\gamma)(w_{s,00}-w_{u,00})-\left\{[1-\beta(1-H)+\gamma H](1-\tau_{10})(w_{s,10}-w_{u,10})+\gamma\delta\Delta\widetilde{S_N}\right\}\\
&\geq(1+\gamma)(w_{s,00}-w_{u,00})-(w_{s,00}-w_{u,00})(\beta+\gamma)(1-H)-[1-\beta(1-H)+\gamma H](1-\tau_{10})(w_{s,10}-w_{u,10})\ \text{(from (21))}\\
&=[1-\beta(1-H)+\gamma H][(w_{s,00}-w_{u,00})-(1-\tau_{10})(w_{s,10}-w_{u,10})]>0,
\end{aligned}
\tag{73}
$$

where the last inequality holds from w_s-w_u being increasing in $\frac{A_s}{A_u}$.

(iv) [Result on the speed of convergence] From (30) and $\overline{e}_{t+1}=sw_{st}$, where s is a constant, in order for the child of an unskilled worker to be financially accessible to education, the following must hold for b_t the worker receives.

$$
\begin{aligned}
&\lambda\{(1-\tau_t)w_{ut}+T_t+(1+r)b_t\}\geq sw_{st}\\
&\Leftrightarrow\lambda(1+r)b_t\geq sw_{st}-\lambda\left[(1-\tau_t)w_{ut}+T_t\right].
\end{aligned}
\tag{74}
$$

If the RHS of the above equation increases with $\frac{A_s}{A_u}$, increased $\frac{A_s}{A_u}$ slows down the upward mobility of children of unskilled workers. When $p_t=q_t=0$ and thus $\tau_t=0$, the condition for the slowed mobility is (henceforth, time subscripts are omitted unless necessary) $s\frac{dw_s}{d\left(\frac{A_s}{A_u}\right)}-\lambda\frac{dw_u}{d\left(\frac{A_s}{A_u}\right)}>0\Leftrightarrow\frac{dw_s}{d\left(\frac{A_s}{A_u}\right)^{\frac{\sigma-1}{\sigma}}}/\frac{dw_u}{d\left(\frac{A_s}{A_u}\right)^{\frac{\sigma-1}{\sigma}}}>\frac{\lambda}{s}$, where the LHS of the last equation equals, from (4), (5), and (65),

$$\frac{\frac{dw_s}{d\left(\frac{A_s}{A_u}\right)^{\frac{\sigma-1}{\sigma}}}}{\frac{dw_u}{d\left(\frac{A_s}{A_u}\right)^{\frac{\sigma-1}{\sigma}}}} = \frac{\frac{1}{H}\left[\alpha\left(\frac{A_s}{A_u}\right)^{\frac{\sigma-1}{\sigma}}(H)^{\frac{\sigma-1}{\sigma}}+(1-\alpha)(1-H)^{\frac{\sigma-1}{\sigma}}+\frac{1}{\sigma-1}\left(\frac{A_s}{A_u}\right)^{\frac{\sigma-1}{\sigma}}\alpha(H)^{\frac{\sigma-1}{\sigma}}\right]}{\frac{1}{\sigma-1}(1-\alpha)(1-H)^{\frac{\sigma-1}{\sigma}}\frac{1}{1-H}}$$

$$= \frac{1-H}{H}\left[\sigma-1+\sigma\frac{\alpha\left(\frac{A_s}{A_u}\right)^{\frac{\sigma-1}{\sigma}}(H)^{\frac{\sigma-1}{\sigma}}}{(1-\alpha)(1-H)^{\frac{\sigma-1}{\sigma}}}\right]. \quad (75)$$

From (43) in Appendix B,

$$\begin{aligned} H_t &< H^* \Leftrightarrow (1+\gamma)(w_{st}-w_{ut}) > (1+r)sw_{st-1} \\ &\Rightarrow (1+\gamma)(w_{st}-w_{ut}) > (1+r)sw_{st} \text{ (since } H_t \geq H_{t-1}) \\ &\Leftrightarrow \frac{1-H}{H}\frac{\alpha\left(\frac{A_s}{A_u}\right)^{\frac{\sigma-1}{\sigma}}(H)^{\frac{\sigma-1}{\sigma}}}{(1-\alpha)(1-H)^{\frac{\sigma-1}{\sigma}}} > \frac{1+\gamma}{(1+\gamma)-(1+r)s} \text{ (from (4) and (5))}. \quad (76)\end{aligned}$$

Thus,

$$\begin{aligned} \frac{\frac{dw_s}{d\left(\frac{A_s}{A_u}\right)^{\frac{\sigma-1}{\sigma}}}}{\frac{dw_u}{d\left(\frac{A_s}{A_u}\right)^{\frac{\sigma-1}{\sigma}}}} &> \frac{1-H}{H}(\sigma-1)+\sigma\frac{1+\gamma}{(1+\gamma)-(1+r)s} \\ &> \frac{1+\gamma}{(1+\gamma)-(1+r)s} \text{ (from } \sigma \in (1,3]). \quad (77)\end{aligned}$$

Hence, the condition for the slowed upward mobility holds if

$$\frac{1+\gamma}{(1+\gamma)-(1+r)s} \geq \frac{\lambda}{s} \Leftrightarrow (1+\gamma)s - [(1+\gamma)-(1+r)s]\lambda \geq 0, \quad (78)$$

where $s \leq \lambda$ must be true because from (31) and $\tau_t = 0$, $b_{t+1} = \lambda\{w_{st}+(1+r)(b_t-\bar{e}_t)\} \geq \bar{e}_{t+1} = sw_{st}$ must hold for children of skilled workers to be accessible to education, which is necessary for H_t to non-decrease over time. The above inequality holds (does not hold) at $s = \lambda$ ($s = 0$) and the LHS of the second inequality increases with s. Hence, if s is sufficiently high, increased $\frac{A_s}{A_u}$ slows down the upward mobility of children of unskilled workers when $p = q = 0$.

When $p = q = 0$ does not hold and thus $\tau > 0$, the condition for the decreased mobility is $\frac{dw_s}{d\left(\frac{A_s}{A_u}\right)^{\frac{\sigma-1}{\sigma}}} / \frac{d[(1-\tau)w_u+T]}{d\left(\frac{A_s}{A_u}\right)^{\frac{\sigma-1}{\sigma}}} > \frac{\lambda}{s}$ from (74). This condition is less likely to hold than the condition when $p = q = 0$ because $\frac{d[(1-\tau)w_u+T]}{d\left(\frac{A_s}{A_u}\right)^{\frac{\sigma-1}{\sigma}}} > \frac{dw_u}{d\left(\frac{A_s}{A_u}\right)^{\frac{\sigma-1}{\sigma}}}$ from $\frac{d[(1-\tau)w_u+T]}{d\left(\frac{A_s}{A_u}\right)^{\frac{\sigma-1}{\sigma}}} = \frac{dw_u}{d\left(\frac{A_s}{A_u}\right)^{\frac{\sigma-1}{\sigma}}} + \frac{\partial(T-\tau w_u)}{\partial\tau}\frac{d\tau}{d\left(\frac{A_s}{A_u}\right)^{\frac{\sigma-1}{\sigma}}} + \frac{\partial(T-\tau w_u)}{\partial\left(\frac{A_s}{A_u}\right)^{\frac{\sigma-1}{\sigma}}}$, where $\frac{d(T-\tau w_u)}{d\tau} > 0$ from the proof of Corollary 1 (ii), $\frac{d\tau}{d\left(\frac{A_s}{A_u}\right)^{\frac{\sigma-1}{\sigma}}} > 0$ from (iii), and $\frac{\partial(T-\tau w_u)}{\partial\left(\frac{A_s}{A_u}\right)^{\frac{\sigma-1}{\sigma}}} = A_u\alpha(H)^{\frac{\sigma-1}{\sigma}}(\Gamma)^{\frac{\sigma}{\sigma-1}-2}\left\{\left(1-\frac{1}{2}\tau\frac{\sigma}{\sigma-1}\right)\Gamma+\frac{1}{\sigma-1}\left[\left(\frac{A_s}{A_u}\right)^{\frac{\sigma-1}{\sigma}}\alpha(H)^{\frac{\sigma-1}{\sigma}}-(1-\alpha)(1-H)^{\frac{\sigma-1}{\sigma}}\frac{H}{1-H}\right]\right\} > 0$ from $\sigma \leq 3$ and $\tau < \frac{1}{2}$ (Assumption 2). But the condition does hold when s is sufficiently high because $\frac{dw_s}{d\left(\frac{A_s}{A_u}\right)^{\frac{\sigma-1}{\sigma}}} > \frac{d[(1-\tau)w_s+T]}{d\left(\frac{A_s}{A_u}\right)^{\frac{\sigma-1}{\sigma}}} > \frac{d[(1-\tau)w_u+T]}{d\left(\frac{A_s}{A_u}\right)^{\frac{\sigma-1}{\sigma}}}$, where $\frac{dw_s}{d\left(\frac{A_s}{A_u}\right)^{\frac{\sigma-1}{\sigma}}} > \frac{d[(1-\tau)w_s+T]}{d\left(\frac{A_s}{A_u}\right)^{\frac{\sigma-1}{\sigma}}}$ from $\frac{d(-\tau w_s+T)}{d\left(\frac{A_s}{A_u}\right)^{\frac{\sigma-1}{\sigma}}} =$

$-\frac{d\left[\tau(w_s-w_u)(1-H)+\frac{1}{2}\tau^2(w_sH+w_u(1-H))\right]}{d\left(\frac{A_s}{A_u}\right)^{\frac{\sigma-1}{\sigma}}}<0$ ($\frac{d(w_s-w_s)}{d\left(\frac{A_s}{A_u}\right)^{\frac{\sigma-1}{\sigma}}}>0$ from (i)) and $\frac{d[(1-\tau)w_s+T]}{d\left(\frac{A_s}{A_u}\right)^{\frac{\sigma-1}{\sigma}}}>\frac{d[(1-\tau)w_u+T]}{d\left(\frac{A_s}{A_u}\right)^{\frac{\sigma-1}{\sigma}}}$ from $\frac{d[(1-\tau)(w_s-w_s)]}{d\left(\frac{A_s}{A_u}\right)^{\frac{\sigma-1}{\sigma}}}>0$, (66) in the proof of (ii).

When the identity shift occurs, the slowed upward mobility is more likely, i.e., it happens with smaller s, because τ falls and thus the change in $(1-\tau)w_u+T$, which could be positive or negative, is smaller than the change when τ is constant from $\frac{d(T-\tau w_u)}{d\tau}>0$.

[Result on H^*] H^* for different values of p and q are solutions to (42)$-$(45) in Appendix B. An increase in $\frac{A_s}{A_u}$ raises $(1-\tau)(w_s-w_u)$ in the LHSs of the equations from (66) in the proof of (ii). (Increased $\frac{A_s}{A_u}$ does not affect $\bar{e}$ because it is proportional to w_s in the previous period.) Although the relation between $(1-\tau)(w_s-w_u)$ and H is generally not clear, the fact that the LHSs of these equations equal $+\infty$ at $H=0$ and $-(1+r)\bar{e}<0$ at $H=\overline{H}$ implies that the LHSs decrease with H at $H=H^*$ (or when multiple levels of H^* exist, at the highest H^*, to which H converges). Hence Increased $\frac{A_s}{A_u}$ raises H^*.

[Result on $\underline{F}$] From (30), there exist lineages satisfying $b_t<\bar{e}_t=sw_{st-1}$ and $b_{t+1}\geq\bar{e}_{t+1}=sw_{st}$ only if $\lambda\{(1-\tau_t)w_{ut}+T_t+(1+r)b_t\}\geq sw_{st}$ for some $b_t<sw_{st-1}$, which is the case when

$$\begin{aligned}&\lambda\{(1-\tau_t)w_{ut}+T_t+(1+r)sw_{st-1}\}-sw_{st}>0\\ \Leftrightarrow\ &\lambda(1+r)sw_{st-1}>sw_{st}-\lambda[(1-\tau_t)w_{ut}+T_t].\end{aligned}\tag{79}$$

The equation corresponds to (33) when $\bar{e}$ is time-invariant, thus H_t satisfying it with equality is $\underline{F}_t$, though unlike before, it depends on H_{t-1}. Because the relation between the RHS of (79) and H_t is unclear, multiple values of H_t satisfying the equation with equality could exist; by definition, $\underline{F}_t$ is the highest value of such H_t, whose existence can be proved in a similar way as the proof of Lemma 2 (i) for the constant $\bar{e}$ case. Since the RHS equals $(s-\lambda)w_{st}\leq 0$ at $H_t=\overline{H}$ from $\tau_t=0$ (Proposition 1) and $\lambda\geq s$ (see the proof on the speed of convergence), the RHS decreases with H_t at $H_t=\underline{F}_t$. From the proof on the speed of convergence, the RHS of (79) increases with $\frac{A_{st}}{A_{ut}}$ when s is sufficiently high. Hence, $\underline{F}_t$ increases with $\frac{A_{st}}{A_{ut}}$. ■

Proof of Proposition A1. (i) $p=1$, $q=0$ cannot hold because the two conditions of (23) do not hold simultaneously when $\beta\leq\gamma$. (ii) Since $\beta\leq\gamma$, $\frac{\beta+\gamma}{2\beta}\geq 1$ and thus $H<\frac{\beta+\gamma}{2\beta}$ always holds. Hence, the RHS of the condition for $p=q=1$, (18), and that of the first condition for $p=0$, $q=1$, (22), equal $(\beta+\gamma)(1-\tau)(1-H)(w_s-w_u)$, where τ equals a constant times $a(H)-H$ from Proposition 1. In the following, the proof for the condition for $p=q=1$, where $\tau=\frac{2\beta}{1+\gamma}(a(H)-H)$, is provided.

From (47) in the proof of Proposition 1,

$$a'(H)=\frac{\sigma-1}{\sigma}\frac{a(H)[1-a(H)]}{H(1-H)}>0.\tag{80}$$

From (4) and (5) $(\Omega\equiv\alpha(A_sH)^{\frac{\sigma-1}{\sigma}}+(1-\alpha)[A_u(1-H)]^{\frac{\sigma-1}{\sigma}})$,

$$\begin{aligned}\frac{dw_s}{dH}&=\frac{1}{\sigma}\Omega^{\frac{\sigma}{\sigma-1}-2}\alpha(A_s)^{\frac{\sigma-1}{\sigma}}(H)^{-\frac{1}{\sigma}}\begin{pmatrix}\alpha(A_sH)^{\frac{\sigma-1}{\sigma}}\frac{1}{H}-(1-\alpha)[A_u(1-H)]^{\frac{\sigma-1}{\sigma}}\frac{1}{1-H}\\-\left\{\alpha(A_sH)^{\frac{\sigma-1}{\sigma}}+(1-\alpha)[A_u(1-H)]^{\frac{\sigma-1}{\sigma}}\right\}\frac{1}{H}\end{pmatrix}\\&=-\frac{1}{\sigma}\Omega^{\frac{\sigma}{\sigma-1}-2}(1-\alpha)[A_u(1-H)]^{\frac{\sigma-1}{\sigma}}\alpha(A_s)^{\frac{\sigma-1}{\sigma}}(H)^{-\frac{1}{\sigma}}\frac{1}{H(1-H)}<0.\end{aligned}\tag{81}$$

$$\frac{dw_u}{dH} = \frac{1}{\sigma}\Omega^{\frac{\sigma}{\sigma-1}-2}(1-\alpha)(A_u)^{\frac{\sigma-1}{\sigma}}(1-H)^{-\frac{1}{\sigma}}\begin{pmatrix} \alpha(A_sH)^{\frac{\sigma-1}{\sigma}}\frac{1}{H}-(1-\alpha)[A_u(1-H)]^{\frac{\sigma-1}{\sigma}}\frac{1}{1-H} \\ +\left\{\alpha(A_sH)^{\frac{\sigma-1}{\sigma}}+(1-\alpha)[A_u(1-H)]^{\frac{\sigma-1}{\sigma}}\right\}\frac{1}{1-H}\end{pmatrix}$$

$$= \frac{1}{\sigma}\Omega^{\frac{\sigma}{\sigma-1}-2}(1-\alpha)(A_u)^{\frac{\sigma-1}{\sigma}}(1-H)^{-\frac{1}{\sigma}}\alpha(A_sH)^{\frac{\sigma-1}{\sigma}}\frac{1}{H(1-H)} > 0. \tag{82}$$

From the above two equations,

$$\frac{d(w_s-w_u)}{dH} = -\frac{1}{\sigma}\Omega^{\frac{\sigma}{\sigma-1}-2}\alpha(A_s)^{\frac{\sigma-1}{\sigma}}(H)^{-\frac{1}{\sigma}}(1-\alpha)(A_u)^{\frac{\sigma-1}{\sigma}}(1-H)^{-\frac{1}{\sigma}}\frac{1}{H(1-H)}$$

$$= -\frac{1}{\sigma}\Omega^{\frac{\sigma}{\sigma-1}}\frac{a(H)(1-a(H))}{H^2(1-H)^2}. \tag{83}$$

Hence, from the above equation and (80),

$$\frac{d[(1-\tau)(1-H)(w_s-w_u)]}{dH} = -\frac{2\beta}{1+\gamma}\left\{\frac{\sigma-1}{\sigma}\frac{a(H)[1-a(H)]}{H(1-H)}-1\right\}$$

$$\times(1-H)\left\{\alpha(A_sH)^{\frac{\sigma-1}{\sigma}}+(1-\alpha)[A_u(1-H)]^{\frac{\sigma-1}{\sigma}}\right\}^{\frac{\sigma}{\sigma-1}-1}\left\{\alpha(A_sH)^{\frac{\sigma-1}{\sigma}}\frac{1}{H}-(1-\alpha)[A_u(1-H)]^{\frac{\sigma-1}{\sigma}}\frac{1}{1-H}\right\}$$

$$-\left[1-\frac{2\beta}{1+\gamma}(a(H)-H)\right]\left\{\alpha(A_sH)^{\frac{\sigma-1}{\sigma}}+(1-\alpha)[A_u(1-H)]^{\frac{\sigma-1}{\sigma}}\right\}^{\frac{\sigma}{\sigma-1}}\frac{1}{H^2(1-H)}\left[\frac{1}{\sigma}a(H)(1-a(H))+H(a(H)-H)\right]$$

$$= -\frac{\Omega^{\frac{\sigma}{\sigma-1}}}{H^2(1-H)}\begin{pmatrix} \frac{2\beta}{1+\gamma}(a(H)-H)\left\{\frac{\sigma-1}{\sigma}a(H)[1-a(H)]-H(1-H)\right\} \\ +\left[1-\frac{2\beta}{1+\gamma}(a(H)-H)\right]\left[\frac{1}{\sigma}a(H)(1-a(H))+H(a(H)-H)\right]\end{pmatrix}$$

$$= -\frac{\Omega^{\frac{\sigma}{\sigma-1}}}{H^2(1-H)}\begin{pmatrix} \frac{2\beta}{1+\gamma}(a(H)-H)\left\{\begin{matrix} a(H)[1-a(H)]-H(1-H)+H(a(H)-H) \\ -\left[\frac{1}{\sigma}a(H)(1-a(H))+H(a(H)-H)\right]\end{matrix}\right\} \\ +\left[1-\frac{2\beta}{1+\gamma}(a(H)-H)\right]\left[\frac{1}{\sigma}a(H)(1-a(H))+H(a(H)-H)\right]\end{pmatrix}$$

$$= -\frac{\Omega^{\frac{\sigma}{\sigma-1}}}{H^2(1-H)}\left\{\begin{matrix} \frac{2\beta}{1+\gamma}(a(H)-H)^2[1-a(H)] \\ +\left[1-\frac{4\beta}{1+\gamma}(a(H)-H)\right]\left[\frac{1}{\sigma}a(H)(1-a(H))+H(a(H)-H)\right]\end{matrix}\right\}, \tag{84}$$

which is negative since $1-\frac{4\beta}{1+\gamma}(a(H)-H)\geq 0$ from Assumption 2.

Further, from the first and second lines of (84) ($\overline{H}$ is H satisfying $H = a(H)$),

$$\lim_{H\to 0}[(1-\tau)(1-H)(w_s-w_u)] = +\infty, \quad \lim_{H\to\overline{H}}[(1-\tau)(1-H)H(w_s-w_u)] = 0. \tag{85}$$

(iii) The RHS of the condition for $p = q = 0$, (20), and that of the second condition for $p = 0$, $q = 1$, (22), equal $(\beta-\gamma)H(w_s-w_u) < 0$ since $\tau = 0$ in both cases when $\beta \leq \gamma$. From (83) in the proof of (ii) ($\Omega \equiv \alpha(A_sH)^{\frac{\sigma-1}{\sigma}}+(1-\alpha)[A_u(1-H)]^{\frac{\sigma-1}{\sigma}}$),

$$\frac{d[H(w_s-w_u)]}{dH} = \Omega^{\frac{\sigma}{\sigma-1}-1}\left\{\alpha(A_sH)^{\frac{\sigma-1}{\sigma}}\frac{1}{H}-(1-\alpha)[A_u(1-H)]^{\frac{\sigma-1}{\sigma}}\frac{1}{1-H}\right\}$$

$$-\frac{1}{\sigma}\Omega^{\frac{\sigma}{\sigma-1}-2}\alpha(A_s)^{\frac{\sigma-1}{\sigma}}(H)^{-\frac{1}{\sigma}}(1-\alpha)(A_u)^{\frac{\sigma-1}{\sigma}}(1-H)^{-\frac{1}{\sigma}}\frac{1}{1-H}$$

$$= \Omega^{\frac{\sigma}{\sigma-1}-1}\left\{\begin{matrix} \alpha(A_sH)^{\frac{\sigma-1}{\sigma}}\frac{1}{H}-(1-\alpha)[A_u(1-H)]^{\frac{\sigma-1}{\sigma}}\frac{1}{1-H} \\ -\frac{1}{\sigma}\frac{\alpha(A_sH)^{\frac{\sigma-1}{\sigma}}(1-\alpha)[A_u(1-H)]^{\frac{\sigma-1}{\sigma}}}{\alpha(A_sH)^{\frac{\sigma-1}{\sigma}}+(1-\alpha)[A_u(1-H)]^{\frac{\sigma-1}{\sigma}}}\frac{1}{H(1-H)^2}\end{matrix}\right\}, \tag{86}$$

where ($\overline{H}$ is H satisfying $H = a(H)$)

$$\lim_{H\to 0} H(w_s-w_u)=0,\ \lim_{H\to 0}\frac{d[H(w_s-w_u)]}{dH}=+\infty, \tag{87}$$

$$\lim_{H\to \overline{H}} H(w_s-w_u)=0,\ \lim_{H\to \overline{H}}\frac{d[H(w_s-w_u)]}{dH}=-\frac{1}{\sigma}\Omega^{\frac{\sigma}{\sigma-1}}\frac{1}{1-H}<0. \tag{88}$$

In the following, it is proved that there exists $H^\sharp \in (0,\overline{H})$ such that the first term of inside the big parenthesis of (86) is greater (smaller) than the second term for $H < (>) H^\sharp$. This implies that $\frac{d[H(w_s-w_u)]}{dH} > (<) 0$ for $H < (>) H^\sharp$ and thus, when $\beta \le \gamma$, the RHS of the condition for $p = q = 0$, (20), and that of the second condition for $p = 0$, $q = 1$ decrease (increase) with H for $H < (>) H^\sharp$.

The derivative of the first term with respect to H equals

$$-\frac{1}{\sigma}\frac{1}{H^2(1-H)^2}\left\{\alpha(A_sH)^{\frac{\sigma-1}{\sigma}}(1-H)^2+(1-\alpha)[A_u(1-H)]^{\frac{\sigma-1}{\sigma}}H^2\right\}<0. \tag{89}$$

The derivative of the second term with respect to H equals

$$\begin{aligned}
&\frac{1}{\sigma}\frac{\alpha(A_sH)^{\frac{\sigma-1}{\sigma}}(1-\alpha)[A_u(1-H)]^{\frac{\sigma-1}{\sigma}}}{\alpha(A_sH)^{\frac{\sigma-1}{\sigma}}+(1-\alpha)[A_u(1-H)]^{\frac{\sigma-1}{\sigma}}}\frac{3H-1}{H^2(1-H)^3}\\
&+\frac{1}{\sigma}\frac{\alpha(A_sH)^{\frac{\sigma-1}{\sigma}}(1-\alpha)[A_u(1-H)]^{\frac{\sigma-1}{\sigma}}}{H(1-H)^2}\frac{\sigma-1}{\sigma}\frac{\left\{\alpha(A_sH)^{\frac{\sigma-1}{\sigma}}+(1-\alpha)[A_u(1-H)]^{\frac{\sigma-1}{\sigma}}\right\}\left(\frac{1}{H}-\frac{1}{1-H}\right)-\left\{\alpha(A_sH)^{\frac{\sigma-1}{\sigma}}\frac{1}{H}-(1-\alpha)[A_u(1-H)]^{\frac{\sigma-1}{\sigma}}\frac{1}{1-H}\right\}}{\left\{\alpha(A_sH)^{\frac{\sigma-1}{\sigma}}+(1-\alpha)[A_u(1-H)]^{\frac{\sigma-1}{\sigma}}\right\}^2}\\
&=\frac{1}{\sigma}\frac{1}{H(1-H)^2}\frac{\alpha(A_sH)^{\frac{\sigma-1}{\sigma}}(1-\alpha)[A_u(1-H)]^{\frac{\sigma-1}{\sigma}}}{\alpha(A_sH)^{\frac{\sigma-1}{\sigma}}+(1-\alpha)[A_u(1-H)]^{\frac{\sigma-1}{\sigma}}}\left\{\frac{\sigma-1}{\sigma}\frac{-\alpha(A_sH)^{\frac{\sigma-1}{\sigma}}\frac{1}{1-H}+(1-\alpha)[A_u(1-H)]^{\frac{\sigma-1}{\sigma}}\frac{1}{H}}{\alpha(A_sH)^{\frac{\sigma-1}{\sigma}}+(1-\alpha)[A_u(1-H)]^{\frac{\sigma-1}{\sigma}}}+\frac{3H-1}{H(1-H)}\right\},
\end{aligned} \tag{90}$$

which is negative (positive) for small (large) H.

The difference between the derivative of the first term and that of the second terms is proportional to

$$\begin{aligned}
&-(1-H)\left\{\alpha(A_sH)^{\frac{\sigma-1}{\sigma}}(1-H)^2+(1-\alpha)[A_u(1-H)]^{\frac{\sigma-1}{\sigma}}H^2\right\}\\
&-\frac{\alpha(1-\alpha)(A_sH)^{\frac{\sigma-1}{\sigma}}[A_u(1-H)]^{\frac{\sigma-1}{\sigma}}}{\alpha(A_sH)^{\frac{\sigma-1}{\sigma}}+(1-\alpha)[A_u(1-H)]^{\frac{\sigma-1}{\sigma}}}\left\{\frac{\sigma-1}{\sigma}\frac{-\alpha(A_sH)^{\frac{\sigma-1}{\sigma}}H+(1-\alpha)[A_u(1-H)]^{\frac{\sigma-1}{\sigma}}(1-H)}{\alpha(A_sH)^{\frac{\sigma-1}{\sigma}}+(1-\alpha)[A_u(1-H)]^{\frac{\sigma-1}{\sigma}}}-(1-3H)\right\}.
\end{aligned} \tag{91}$$

In the following, it is proved that the difference is negative. This fact, together with the fact that the first term inside the big parenthesis of (86) is greater than the second term when $H \to 0$,

$$\begin{aligned}
&\lim_{H\to 0}\left\{\alpha(A_sH)^{\frac{\sigma-1}{\sigma}}\frac{1}{H}-(1-\alpha)[A_u(1-H)]^{\frac{\sigma-1}{\sigma}}\frac{1}{1-H}\right\}=\alpha(A_s)^{\frac{\sigma-1}{\sigma}}\lim_{H\to 0}\left(\frac{1}{H}\right)^{\frac{1}{\sigma}}-(1-\alpha)(A_u)^{\frac{\sigma-1}{\sigma}}\\
&>\lim_{H\to 0}\left\{\frac{1}{\sigma}\frac{\alpha(A_sH)^{\frac{\sigma-1}{\sigma}}(1-\alpha)[A_u(1-H)]^{\frac{\sigma-1}{\sigma}}}{\alpha(A_sH)^{\frac{\sigma-1}{\sigma}}+(1-\alpha)[A_u(1-H)]^{\frac{\sigma-1}{\sigma}}}\frac{1}{H(1-H)^2}\right\}=\frac{1}{\sigma}\alpha(A_s)^{\frac{\sigma-1}{\sigma}}\lim_{H\to 0}\left(\frac{1}{H}\right)^{\frac{1}{\sigma}},
\end{aligned} \tag{92}$$

implies that the first term is greater than the second term for $H < (>) H^\sharp$.

Let $J \equiv (A_sH)^{\frac{\sigma-1}{\sigma}}$ and $K \equiv [A_u(1-H)]^{\frac{\sigma-1}{\sigma}}$. If $-\alpha JH+(1-\alpha)K(1-H)\ge 0$, (91) is smaller than

$$\frac{1}{\alpha J+(1-\alpha)K}\left\{-(1-H)\left[\alpha J(1-H)^2+(1-\alpha)KH^2\right][\alpha J+(1-\alpha)K]+\alpha(1-\alpha)JK(1-3H)\right\}, \tag{93}$$

which is negative when $1-3H\leq 0$. When $1-3H>0$, if $J\geq K$, (93) is weakly smaller than

$$\frac{1}{\alpha J+(1-\alpha)K}\Big\{-(1-H)\Big[\alpha J(1-H)^2+(1-\alpha)KH^2\Big]K+\alpha(1-\alpha)JK(1-3H)\Big\}$$
$$=\frac{1}{\alpha J+(1-\alpha)K}\Big\{\alpha JK\Big[-(1-H)^3+(1-\alpha)(1-3H)\Big]-(1-H)(1-\alpha)K^2H^2\Big\}$$
$$=\frac{1}{\alpha J+(1-\alpha)K}\big\{\alpha JK\big[-\alpha(1-3H)-2H^2-(1-H)H^2\big]-(1-H)(1-\alpha)K^2H^2\big\}<0. \quad (94)$$

If $J<K$, (93) equals

$$\frac{1}{\alpha J+(1-\alpha)K}\Big(-(1-H)\Big\{\Big[\alpha J(1-H)^2+(1-\alpha)KH^2\Big]\alpha J+[(1-\alpha)KH]^2\Big\}-\alpha(1-\alpha)JK\big[(1-H)^3-(1-3H)\big]\Big)$$
$$=\frac{1}{\alpha J+(1-\alpha)K}\Big(-\alpha(1-\alpha)JK\big[2H^2+(1-H)H^2\big]-(1-H)\Big\{\Big[\alpha J(1-H)^2+(1-\alpha)KH^2\Big]\alpha J+[(1-\alpha)KH]^2\Big\}\Big)<0. \quad (95)$$

If $-\alpha JH+(1-\alpha)K(1-H)<0$, (91) is smaller than

$$\frac{1}{\alpha J+(1-\alpha)K}\Big\{-(1-H)\Big[\alpha J(1-H)^2+(1-\alpha)KH^2\Big][\alpha J+(1-\alpha)K]-\alpha(1-\alpha)JK\Big[\frac{-\alpha JH+(1-\alpha)K(1-H)}{\alpha J+(1-\alpha)K}-(1-3H)\Big]\Big\}$$
$$<\frac{1}{\alpha J+(1-\alpha)K}\Big\{-(1-H)\Big[\alpha J(1-H)^2+(1-\alpha)KH^2\Big]\frac{(1-\alpha)K}{H}+\alpha(1-\alpha)JK\,[H+(1-3H)]\Big\}$$
$$=\frac{1}{\alpha J+(1-\alpha)K}\frac{1}{H}\Big\{\alpha(1-\alpha)JK\Big[H(1-2H)-(1-H)^3\Big]-(1-H)[(1-\alpha)HK]^2\Big\}$$
$$=\frac{1}{\alpha J+(1-\alpha)K}\frac{1}{H}\Big(\alpha(1-\alpha)JK\big\{H(1-2H)-\big[1-3H+2H^2+(1-H)H^2\big]\big\}-(1-H)[(1-\alpha)HK]^2\Big)$$
$$=-\frac{1}{\alpha J+(1-\alpha)K}\frac{1}{H}\big\{\alpha(1-\alpha)JKH\big[(2H-1)^2+(1-H)H^2\big]+(1-\alpha)^2H^2(1-H)K^2\big\}<0. \quad (96)$$

(iv) The RHS of the condition for $p=q=0$ and that of the second condition for $p=0$, $q=1$ are the same from (20) and (22) because $\tau=0$ in both cases when $\beta\leq\gamma$. Hence, the dividing line for $p=q=0$ and the lower dividing line for $p=0$, $q=1$ are the same. Because the RHS of the condition for $p=q=0$ (and of the second condition for $p=0$ and $q=1$) is non-positive from $\beta\leq\gamma$, it is always smaller than the RHS of the condition for $p=q=1$ and that of the first condition for $p=0$, $q=1$. Thus, the dividing line for $p=q=0$ is located below the dividing line for $p=q=1$ and the upper dividing line for $p=0$, $q=1$ on the $(H,\Delta\widetilde{S_N})$ plane. From (18) and (22), the RHS of the condition for $p=q=1$ and that of the first condition for $p=0$, $q=1$ are the same except the value of τ, which is higher when $p=q=1$ from Proposition 1. Hence, the RHS of the former condition is smaller than that of the latter condition, that is, the dividing line for $p=q=1$ is located below the upper dividing line for $p=0$, $q=1$. ■

Proof of Proposition A2. (i) If H satisfying $H=a(H)$ is smaller than $\frac{\beta+\gamma}{2\beta}$, the equations for the dividing lines are the same as when $\beta\leq\gamma$. Hence, Proposition A1 applies except the following. (a) Because the RHS of the condition for $p=q=0$, (20), is positive from $\beta>\gamma$, the dividing line for $p=q=0$ increases (decreases) with H for $H<(>)H^\sharp$. Since $\beta>\gamma$, the RHS of the second condition for $p=0$, $q=1$, (22), equals $(\beta-\gamma)(1-\tau)H(w_s-w_u)$, where $\tau=\frac{\beta-\gamma}{1+\gamma}(a(H)-H)$ from Proposition 1.

From (80) and (83) in the proof of Proposition A1 (ii),

$$\frac{d[(1-\tau)H(w_s-w_u)]}{dH}=\left\{\alpha(A_sH)^{\frac{\sigma-1}{\sigma}}+(1-\alpha)[A_u(1-H)]^{\frac{\sigma-1}{\sigma}}\right\}^{\frac{\sigma}{\sigma-1}-1}$$

$$\times\left(\begin{array}{c} -\frac{\beta-\gamma}{1+\gamma}\left\{\frac{\sigma-1}{\sigma}\frac{\alpha(A_sH)^{\frac{\sigma-1}{\sigma}}}{\alpha(A_sH)^{\frac{\sigma-1}{\sigma}}+(1-\alpha)[A_u(1-H)]^{\frac{\sigma-1}{\sigma}}}\frac{(1-\alpha)[A_u(1-H)]^{\frac{\sigma-1}{\sigma}}}{\alpha(A_sH)^{\frac{\sigma-1}{\sigma}}+(1-\alpha)[A_u(1-H)]^{\frac{\sigma-1}{\sigma}}}\frac{1}{H(1-H)}-1\right\} \\ \times H\left\{\alpha(A_sH)^{\frac{\sigma-1}{\sigma}}\frac{1}{H}-(1-\alpha)[A_u(1-H)]^{\frac{\sigma-1}{\sigma}}\frac{1}{1-H}\right\} \\ +\left[1-\frac{\beta-\gamma}{1+\gamma}(a(H)-H)\right]\left\{\alpha(A_sH)^{\frac{\sigma-1}{\sigma}}\frac{1}{H}-(1-\alpha)[A_u(1-H)]^{\frac{\sigma-1}{\sigma}}\frac{1}{1-H}-\frac{1}{\sigma}\frac{\alpha(A_sH)^{\frac{\sigma-1}{\sigma}}(1-\alpha)[A_u(1-H)]^{\frac{\sigma-1}{\sigma}}}{\alpha(A_sH)^{\frac{\sigma-1}{\sigma}}+(1-\alpha)[A_u(1-H)]^{\frac{\sigma-1}{\sigma}}}\frac{1}{H(1-H)^2}\right\}\end{array}\right). \tag{97}$$

where the expression inside the big parenthesis equals

$$\begin{aligned}
&\left[1-\frac{2(\beta-\gamma)}{1+\gamma}(a(H)-H)\right]\left\{\alpha(A_sH)^{\frac{\sigma-1}{\sigma}}\frac{1}{H}-(1-\alpha)[A_u(1-H)]^{\frac{\sigma-1}{\sigma}}\frac{1}{1-H}-\frac{1}{\sigma}\frac{\alpha(A_sH)^{\frac{\sigma-1}{\sigma}}(1-\alpha)[A_u(1-H)]^{\frac{\sigma-1}{\sigma}}}{\alpha(A_sH)^{\frac{\sigma-1}{\sigma}}+(1-\alpha)[A_u(1-H)]^{\frac{\sigma-1}{\sigma}}}\frac{1}{H(1-H)^2}\right\}\\
&-\frac{\beta-\gamma}{1+\gamma}a(H)\left\{\frac{\sigma-1}{\sigma}\frac{(1-\alpha)[A_u(1-H)]^{\frac{\sigma-1}{\sigma}}}{\alpha(A_sH)^{\frac{\sigma-1}{\sigma}}+(1-\alpha)[A_u(1-H)]^{\frac{\sigma-1}{\sigma}}}\frac{1}{1-H}-1\right\}\left\{\alpha(A_sH)^{\frac{\sigma-1}{\sigma}}\frac{1}{H}-(1-\alpha)[A_u(1-H)]^{\frac{\sigma-1}{\sigma}}\frac{1}{1-H}\right\}\\
&\qquad-\frac{\beta-\gamma}{1+\gamma}(a(H)-H)\frac{1}{\sigma}\frac{\alpha(A_sH)^{\frac{\sigma-1}{\sigma}}(1-\alpha)[A_u(1-H)]^{\frac{\sigma-1}{\sigma}}}{\alpha(A_sH)^{\frac{\sigma-1}{\sigma}}+(1-\alpha)[A_u(1-H)]^{\frac{\sigma-1}{\sigma}}}\frac{1}{H(1-H)^2}\\
=&\left[1-\frac{2(\beta-\gamma)}{1+\gamma}(a(H)-H)\right]\left\{\alpha(A_sH)^{\frac{\sigma-1}{\sigma}}\frac{1}{H}-(1-\alpha)[A_u(1-H)]^{\frac{\sigma-1}{\sigma}}\frac{1}{1-H}-\frac{1}{\sigma}\frac{\alpha(A_sH)^{\frac{\sigma-1}{\sigma}}(1-\alpha)[A_u(1-H)]^{\frac{\sigma-1}{\sigma}}}{\alpha(A_sH)^{\frac{\sigma-1}{\sigma}}+(1-\alpha)[A_u(1-H)]^{\frac{\sigma-1}{\sigma}}}\frac{1}{H(1-H)^2}\right\}\\
&\qquad-\frac{\beta-\gamma}{1+\gamma}a(H)\left[\frac{1-a(H)}{1-H}-1\right]\left\{\alpha(A_sH)^{\frac{\sigma-1}{\sigma}}\frac{1}{H}-(1-\alpha)[A_u(1-H)]^{\frac{\sigma-1}{\sigma}}\frac{1}{1-H}\right\}\\
&+\frac{1}{\sigma}\frac{\beta-\gamma}{1+\gamma}\frac{\alpha(A_sH)^{\frac{\sigma-1}{\sigma}}(1-\alpha)[A_u(1-H)]^{\frac{\sigma-1}{\sigma}}}{\alpha(A_sH)^{\frac{\sigma-1}{\sigma}}+(1-\alpha)[A_u(1-H)]^{\frac{\sigma-1}{\sigma}}}\frac{1}{1-H}\left\{-\frac{a(H)-H}{H(1-H)}+\frac{\alpha(A_sH)^{\frac{\sigma-1}{\sigma}}\frac{1}{H}-(1-\alpha)[A_u(1-H)]^{\frac{\sigma-1}{\sigma}}\frac{1}{1-H}}{\alpha(A_sH)^{\frac{\sigma-1}{\sigma}}+(1-\alpha)[A_u(1-H)]^{\frac{\sigma-1}{\sigma}}}\right\}\\
=&\left[1-\frac{2(\beta-\gamma)}{1+\gamma}(a(H)-H)\right]\left\{\alpha(A_sH)^{\frac{\sigma-1}{\sigma}}\frac{1}{H}-(1-\alpha)[A_u(1-H)]^{\frac{\sigma-1}{\sigma}}\frac{1}{1-H}-\frac{1}{\sigma}\frac{\alpha(A_sH)^{\frac{\sigma-1}{\sigma}}(1-\alpha)[A_u(1-H)]^{\frac{\sigma-1}{\sigma}}}{\alpha(A_sH)^{\frac{\sigma-1}{\sigma}}+(1-\alpha)[A_u(1-H)]^{\frac{\sigma-1}{\sigma}}}\frac{1}{H(1-H)^2}\right\}\\
&\qquad+\frac{\beta-\gamma}{1+\gamma}a(H)\frac{a(H)-H}{1-H}\left\{\alpha(A_sH)^{\frac{\sigma-1}{\sigma}}\frac{1}{H}-(1-\alpha)[A_u(1-H)]^{\frac{\sigma-1}{\sigma}}\frac{1}{1-H}\right\},
\end{aligned}\tag{98}$$

where $1-\frac{2(\beta-\gamma)}{1+\gamma}(a(H)-H)\geq 0$ from Assumption 2.

Hence, the above expression and thus $\frac{d[(1-\tau)H(w_s-w_u)]}{dH}$ are positive at least when $H\leq H^{\sharp}\in(0,\overline{H})$ in which the first term of (98) is non-negative from the proof of Proposition A1 (iii), and they are negative when H is close to $\overline{H}$.

(b) Because $\tau>0$ when $p=0$, $q=1$ from $\beta>\gamma$, the RHS of the second condition for $p=0$, $q=1$, (22), is smaller than that of the condition for $p=q=0$, (20). The RHS of the condition for $p=q=0$ equals $(\beta-\gamma)(1-\tau)H(w_s-w_u)$, while the RHS of the condition for $p=q=1$, (18), and that of the first condition for $p=0$, $q=1$, (22), equal $(\beta+\gamma)(1-\tau)(1-H)(w_s-w_u)$. Because $(\beta-\gamma)H=(\beta+\gamma)(1-H)$ at $H=\frac{\beta+\gamma}{2\beta}$ and $\tau=0$ when $p=q=0$, for relatively high H, the RHS of the condition for $p=q=0$ could be greater than the RHSs of the other two conditions. By contrast, as Proposition A1, when H is relatively low, the RHS of the condition for $p=q=0$ is smaller than the RHSs of the other two conditions because from (ii) and (iii) of the proposition, the RHS of the former goes to 0 as $H\to 0$, while the RHSs of the other conditions go to $+\infty$ as $H\to 0$.

(ii) When $\overline{H}>\frac{\beta+\gamma}{2\beta}$ and $H\leq\frac{\beta+\gamma}{2\beta}$, similar to (i), the equations for the dividing lines are the same as when $\beta\leq\gamma$ and thus the results of (i) hold except the two points. (a) From (86) in the proof of Proposition A1 (iii),

$$H = H^{\sharp} \Leftrightarrow \alpha\left(\frac{A_s}{A_u}H\right)^{\frac{\sigma-1}{\sigma}}\frac{1}{H} - (1-\alpha)(1-H)^{\frac{\sigma-1}{\sigma}}\frac{1}{1-H} - \frac{1}{\sigma}\frac{\alpha\left(\frac{A_s}{A_u}H\right)^{\frac{\sigma-1}{\sigma}}(1-\alpha)(1-H)^{\frac{\sigma-1}{\sigma}}}{\alpha\left(\frac{A_s}{A_u}H\right)^{\frac{\sigma-1}{\sigma}} + (1-\alpha)(1-H)^{\frac{\sigma-1}{\sigma}}}\frac{1}{H(1-H)^2} = 0, \quad (99)$$

where the LHS of the equation decreases with H from the proof. Further, the derivative of the LHS with respect to $\alpha\left(\frac{A_s}{A_u}H\right)^{\frac{\sigma-1}{\sigma}}$ equals

$$\frac{1}{H} - \frac{1}{\sigma}\frac{\left\{\alpha\left(\frac{A_s}{A_u}H\right)^{\frac{\sigma-1}{\sigma}} + (1-\alpha)(1-H)^{\frac{\sigma-1}{\sigma}}\right\} - \alpha\left(\frac{A_s}{A_u}H\right)^{\frac{\sigma-1}{\sigma}}}{\left\{\alpha\left(\frac{A_s}{A_u}H\right)^{\frac{\sigma-1}{\sigma}} + (1-\alpha)(1-H)^{\frac{\sigma-1}{\sigma}}\right\}^2}\frac{(1-\alpha)(1-H)^{\frac{\sigma-1}{\sigma}}}{H(1-H)^2}$$

$$= \frac{1}{H}\left[1 - \frac{1}{\sigma}\frac{(1-a(H))^2}{(1-H)^2}\right] > 0. \quad (100)$$

Thus, $H^{\sharp}$ increases with $\frac{A_s}{A_u}$. From (98) in the proof of (i)(a) and (99), $\frac{d[(1-\tau)H(w_s-w_u)]}{dH} > 0$ when $H \leq H^{\sharp}$. Hence, when $\frac{A_s}{A_u}$ is large enough that $H^{\sharp} \geq \frac{\beta+\gamma}{2\beta}$, the dividing line for $p = q = 0$ and the lower dividing line for $p = 0$, $q = 1$ increase with H for $H \leq \frac{\beta+\gamma}{2\beta}$.

(b) When H is relatively high, the dividing line for $p = q = 0$ is definitely located above the other two dividing lines because given τ, the RHS of the condition for $p = q = 0$ is the same as the RHS of the condition for $p = q = 1$ and that of the first condition for $p = 0$, $q = 1$ at $H = \frac{\beta+\gamma}{2\beta}$ from (20) and (22) and $\tau = 0$ when $p = q = 0$. The last result is straightfowrard from (22).

(iii) (a) $p=0$, $q=1$ cannot hold because the two conditions of (22) do not hold simultaneously when $\beta>\gamma$. (b) When $H>\frac{\beta+\gamma}{2\beta}$, the dividing line for $p=q=0$ and the lower dividing line for $p=1$, $q=0$ equal $(\beta+\gamma)(1-\tau)(1-H)(w_s-w_u)$ from (21) and (23). Hence, the proof of Proposition A2 (ii) applies for them. (c) When $H>\frac{\beta+\gamma}{2\beta}$, the dividing line for $p=q=1$ and the upper dividing line for $p=1$, $q=0$ equal $(\beta-\gamma)(1-\tau)H(w_s-w_u)$ from (19) and (23). Hence, the proof of (i)(b) and (ii)(b) applies for them. When $\frac{A_s}{A_u}$ is small enough that $\overline{H}$ is close to $\frac{\beta+\gamma}{2\beta}$, the dividing lines decrease with H from the proof of (i)(a); hence, the term "could" is used in the last sentence of (c).

(d) From (21) and (23), the RHS of the condition for $p = q = 0$ and that of the second condition for $p = 1$, $q = 0$ are the same except the value of τ, which is 0 when $p = q = 0$. Hence, the RHS of the former condition is greater than that of the latter condition, that is, the dividing line for $p = q = 0$ is located above the lower dividinding line for $p = 1$, $q = 0$. From (19) and (23), the RHS of the condition for $p = q = 1$ and that of the first condition for $p = 1$, $q = 0$ are the same except the value of τ, which is higher when $p = q = 1$ from $\beta > \gamma$. Hence, the RHS of the former condition is smaller than that of the latter condition. From (21), (19), and (23), given τ, the RHS of the condition for $p = q = 0$ is smaller than the RHS of the condition for $p = q = 1$ and that of the first condition for $p = 1$, $q = 0$ when $H > \frac{\beta+\gamma}{2\beta}$ and they are equal at $H = \frac{\beta+\gamma}{2\beta}$, while $\tau = 0$ when $p = q = 0$. Hence, when H is relatively low, the dividing line for $p = q = 0$ is located above the dividling line for $p = q = 1$ and the upper dividinding line for $p = 1$, $q = 0$. From (4) and (5), the RHSs of all the conditions go to 0 as $H \to \overline{H}$. Further, because $\overline{H} > \frac{\beta+\gamma}{2\beta} > \frac{1}{2}$, $\left|\lim_{H\to\overline{H}} \frac{d[(1-H)(w_s-w_u)]}{dH}\right| < \left|\lim_{H\to\overline{H}} \frac{d[(1-\tau)H(w_s-w_u)]}{dH}\right|$ from (84) and (88). Hence, when H is relatively high, the dividing line for $p = q = 0$ is located below the other two dividling lines. The last result is straightfowrard from (23). ■