Can the stock market boost economic growth? Evidence from the Mexican real estate investment trust (REIT)

Razo-De-Anda, Jorge Omar and Cruz-Aké, Salvador and Venegas-Martínez, Francisco

Instituto Politécnico Nacional, Instituto Politécnico Nacional, Instituto Politécnico Nacional

1 June 2022

¿Can the stock market boost economic growth? evidence from the Mexican real estate investment trust (REIT)

Jorge Omar Razo-De-Anda[*]   *Salvador Cruz-Aké*[**]
Francisco Venegas-Martínez[***]

Abstract

This paper develops a stochastic dynamic general equilibrium model to assess the impact of Real Estate Investment Trust (REIT) in the growth rate of the real estate sector through direct investment in infrastructure. Based on the theoretical relationships that the model provides we show empirical evidence, through a quantile econometric analysis of time series, of the positive impact of the REITs in the construction sector. The growth in the construction sector comes from the demand for real estate by those trusts, which would lead to a price increase, promoting gross fixed capital formation, and increasing the value of output in the construction industry.

Keywords: real estate investment trust, real estate markets, financial markets, general equilibrium.
JEL classification: R33, D53, C02.

* Profesor-investigador en la Escuela Superior de Economía del Instituto Politécnico Nacional. <jorgerazodeanda@gmail.com>.

** Profesor-investigador en la Escuela Superior de Economía del Instituto Politécnico Nacional. <salvador.ake22@gmail.com>.

*** Profesor-investigador en la Escuela Superior de Economía del Instituto Politécnico Nacional. fvenegas1111@yahoo.com.mx

¿Puede el mercado de valores impulsar el crecimiento económico? evidencia del fideicomiso de inversión inmobiliaria de México (FIBRA)

Resumen

Este trabajo desarrolla un modelo estocástico y dinámico de equilibrio general útil para evaluar el impacto que tienen los Fideicomisos de Inversión en Bienes Raíces (FIBRASs) en el crecimiento del sector inmobiliario a través de la inversión directa en infraestructura. Con base en las relaciones teóricas que provee el modelo propuesto se muestra evidencia empírica, a través de un análisis econométrico quantilico de series de tiempo, del impacto positivo de las FIBRAs en el sector de la construcción. Esto se debe al crecimiento de la demanda de bienes inmuebles por parte de los fideicomisos, lo que conduce a un aumento de precios promoviendo la formación bruta de capital fijo e incrementando el valor de la producción en la industria de la construcción.

Palabras clave: fideicomiso de inversión en bienes raíces, mercados de inmuebles, mercados financieros, equilibrio general.
Clasificación JEL: R33, D53, C02.

1. Introduction

There is a robust and studied link between the construction and the growth of the Gross Domestic Product (GDP), works as those published by (Chiang, Tao, & Wong, 2015), (Ngowi, Pienaar, Talukhaba, & Mbachu, 2005) or (Donaubauer, Meyer, & Nunnenkamp, 2016) give empirical evidence of their importance and its linked externalities such as the rise of the demand on related industries or places where the construction is placed. Mexico is not the exception for this link, notwithstanding the severe affectation of the construction sector (public or private) during the economic world crisis in 2008-2009, and the slow recovery, the ties between the construction sector and the GDP remained. The empirical evidence provided by (Lozano Montero, Godínez López, & Albor Guzmán, 2018) or (Connolly, 2018) shows that the relationship remains present even at the worst part of an economic recession and fructifies on the expansive part of the business cycle.

In the aftermath of the subprime crises (2011), the Mexican Stock Exchange launched its first publicly traded Real Estate Trust Certificates (REIT UNO, 2018), the Mexican equivalent of the Real Estate Investment Trust (REIT) with

the intention of revitalizing the Mexican construction sector. The main idea of this legal figure is to provide liquidity to the real estate market and associated to that liquidity, promote the investment and growth of that economic activity, for more details see (Cruz García, 2018), (Washburn, 2013) or (Poo Rubio & Rocha Chiu, 2018). There is some empirical evidence like those presented by (Virani & Kaur, 2015), (Reddy & Wong, 2016) or (Bai & Zhu, 2017) that shows the effect of this instrument on the liquidity of the market and its dependence of common market factors such as the interest rates and the risk premium. On the other hand, authors as (Dick, Rafferty, Toner, & Wright, 2017), (Nasieku & Wanyonyi, 2016) or (Wang, Cohen, & Glascock, 2018) show that the REITs influence the real estate industry, and through them, they affect other actual economic activities and variables. As the reader may see, the REITs are tax transparent and supposedly liquid financial instruments that may affect the real part of the economy through the size of its investments and its externalities. All the previous mentioned characteristics imply that an integral study of these instruments may include the government influence on it, the portfolio decision of a rational consumer that has access to a broad and complete financial market that incorporates nonrisky assets and risky assets related to the real economy. Papers as (Le Blanc & Lagarenne, 2004), (Edelstein & Magin, 2013) or (Booth, 2002) made some analysis on theoretical models that cover some of this issues but do not explicitly take account for all of them at the same time.

The inclusion of REITs in an investment portfolio is not as simple as it may appear at first sight. The REITs are conceptualized in several ways; the most common is as a structured instrument that includes a risky bond (the rents have default risk and a recovery rate) and small capital stock (the buildings can have a capital gain) or a real option. The value of the capital gain and the expected present value of the rents is supposedly reflected by the Net Assets Value (NAV) as on a small-cap stock but without its liquidity problems. A more in-depth insight into the several forms to assess a REIT is in (Dubreuille, Cherif, & Bellalah, 2016), (Krewson-Kelly & Thomas, 2016) or (Cruz-Aké, García-Ruiz, & Venegas-Martínez, 2016).

The purpose of this paper is to study if the inclusion of this tax transparent investment vehicle has influenced the Mexican construction sector using a theoretical and econometric analysis. The theoretical analysis (Dynamic General Stochastic Equilibrium Model, DGSEM) seeks to obtain empirical evidence on the influence of macroeconomic variables such as the interest rate, the risk premium, the cost of capital in the real economy and the role of the government. This paper is organized as follows, in section two we develop a Dynamic General Stochastic Equilibrium Model (DGSEM) to explain the effect of the REITs on the construction sector. In section three, we present an

econometric analysis to provide empirical evidence to back our theoretical model that considers the effect of the possible extreme market values of REITS. Finally, in section four, we offer conclusions, limitations and some future research.

2. Dynamic general stochastic equilibrium model

In this section, we develop a Dynamic General Stochastic Equilibrium Model (DGSEM) to assess the impact of the inclusion of a REIT in a rational consumer environment where the consumer – investor have access to a REIT certificate, a capital risky asset and a government credit risk-free bond. In our model, there are two kinds of rational producers: one produces goods and the other real estate. The model allows determining, in a general stochastic equilibrium model, the optimal proportion that the rational consumer – investor will invest in REITs certificates. This paper assumes that the economy produces a general consumption good and the representative firm participates in the building sector. On the consumer - investor budget constraint, we include the yields of the construction sector. Our paper considers random innovations (diffusion process) to model the daily fluctuations of the REITs certificates.

2.1 Assumptions and economy features

We suppose that the economy is populated by identical endless life agents, this can is equivalent to saying that the family heads are interested not only in their satisfaction but the wellness of all his future generations, so his planning horizon is infinite. Theoretically, this assumption is an extreme application of the intergenerational altruism. For more details on this concept, the interested reader may see (Bernheim, 1989), (Galperti & Strulovici, 2017) or (Kotlikoff, 2016). All our economic agents have the same preferences and endowments, for that reason, it is possible to take a representative agent. We also assume that the agents are economically rational and they have a von Neumann–Morgenstern type utility function with the consumption as a single argument. The individual has a relative risk aversion given by the constant q. We also assume that the consumer discounts his consumption along the time using a rate of δ, this number is a representation of the consumer's anxiety for present consumption. The functional form of the agent's total discounted utility is:

$$U_0 = E_0 \left[\int_0^\infty \frac{c_t^{1-\theta} - 1}{1 - \theta} \, e^{-\delta t} \, \mathrm{d}t \right] \tag{1}$$

Individ-ual wealth comes through four sources: a government bond (free of credit risk), B_{lt}, the value of the capital used to produce a generic product different from the real estate, K_t, a risky bond perpetual bond representing the rents related to the real estate, B_t, and an option, P_t, whose underlying asset is the risky bond.

The idea of separating the real estate value into a risky perpetual bond, B_t, and an option, P_t comes from the concepts of usufruct[1] and naked ownership[2] respectively. The tenants may use the real estate as long as they pay rent and their contract remains in force, this means that a default on the lease or an empty property will result in a loss of value of the risky bond. In the same way, the owner may sell the whole real estate and its associated contracts, obtaining a capital gain, if its market price is higher than the expected present value of the property's rents represented by the bond, B_t. As the reader may see, the valuation of the bond is the usufruct's market price, and the option is the contingent capital gain if the owner decides to sell at the market's price. By summing those values (a structured note), the valuer will get the Net Asset Value, NAV,[3] of the property. The structured note approach for the NAV value, the sum of the risky bond value and its option, is used to capture the real value of that part of the consumer's wealth, this is:

$$a_t = Bl_t + K_t + B_t + P_t \tag{2}$$

Taking the total derivative of the last equation to get a motion expression for the consumer's wealth we get:[4]

$$da_t = a_t\left[N_{Bl}\,dR_{Bl} + N_k\,dR_k + N_B\,dR_B + N_p\,dR_p\right] - c_t(1 + \tau_c)dt - d\tau_t \tag{3}$$

where $N_{jt} = \frac{j_t}{a_t}$ represents the portion of the wealth invested in the j-th asset at the time t. In the same way dR_{jt} represents the instant return of each of the portfolios assets. Trying to make a more realistic modelling, we include the motion equation of an *ad valorem* tax over the wealth, $d\tau_t = r_\tau\,a_t\,d_t + \sigma_\tau\,a_t\,dW_{\tau,t}$, and a similar motion equation for the *ad valorem* tax on consumption, τ_c, this is an effort similar to those in (Pereira & Shoven, 1988), (Banerjee & Basu, 2015) or (Papageorgiou, 2014). In both cases, the random variable $dW_{\tau,t}$ is a standard brownian motion; for more details see (Mikosch, 1988) or (Karatzas & Shreve, 2012).

---

[1] The right of a person to use the goods of another and enjoy their benefits, with the obligation of conserving them and taking care of them as if they were their own.
[2] Naked owners are the owners of the property subject to the usufructuary's rights.
[3] The Net Asset Value is the actual market value of the real estate portfolio in a REIT, it captures the capital gain of the property and the present value of the expected rents.
[4] See, for instance, Venegas-Martínez (2008), (2006) and (2001).

2.2 Necessary condition to the consumer's problem

Consider the functional that represents the expected present value of the "representative rational consumer's utility", this funcional is the objective function of the stochastic dynamic problem faced by the consumers when their income is randomly determined by the yields of their investments, this problem is the classical example of a stochastic optimization problem in continuous time, for more details see (Chang, 2004) or (Yong & Zhou, 1999). The fact of using an isoelastic[5] utility function is not significative to the maximization problem as long as it covers the economic and formal requirements of being a growing at decreasing rates function.[6] Provided with the fact that the candidate for the solution of the optimization problem takes the form of the objective function, our problem is:

$$J(a_t, t) = E_t \left[\int_t^\infty \frac{c_s^{1-\theta} - 1}{1 - \theta} e^{-\delta s} ds \right] \qquad (4)$$

where, as before, c_t, is the consumption at the instant t, θ is the consumer's risk aversion parameter and δ is the subjective consumer's discount rate. By applying the recursivity principle, the mean value theorem for an integral and the Frechét differential to the second term, we got:

$$J(a_t, t) = \max_{c_t} E_t \left[\frac{c_t^{1-\theta} - 1}{1 - \theta} e^{-\delta t} dt + o(dt) + J(a_t, t) + dJ(a_t, t) + o(dt) \right] \qquad (5)$$

Simplifying, applying the Itô's lemma, taking the expectations and dividing by dt we obtain the Hamilton –Jacobi–Bellman equation. This last expression is the necessary condition to achieve the optimum of the stochastic dynamic problem

$$0 = \frac{c_t^{1-\theta} - 1}{1 - \theta} e^{-\delta s} + \frac{\partial J}{\partial t} + \frac{\partial J}{\partial a_t} \mu_a a_t + \frac{1}{2} \frac{\partial^2 J}{\partial a^2} \sigma_{a_t}^2 a_t^2. \qquad (6)$$

The candidate to solve the last equation, $J(a_t, t) = V(a_t)e^{-\delta t}$, so that:

$$0 = \frac{c_t^{1-\theta} - 1}{1 - \theta} e^{-\delta s} - \delta V(a_t)e^{-\delta t} + V'(a_t)e^{-\delta t} \mu_a a_t + \frac{1}{2} V''(a_t)e^{-\delta t} \sigma_{a_t}^2 a_t^2, \qquad (7)$$

---

[5] The isoelastic function is a generalization of the hyperbolic absolute risk aversion.
[6] If this requirement is not accomplished, the integral is not defined.

We will use this condition to attain the general equilibrium. To continue setting the model, we need to develop dynamic stochastic equations to all the richness source in the economy.

2.3 Interest rate dynamics

To set the general equilibrium model is essential to determine a term structure that allows the rational agent to allocate resources a long time to stabilize his consumption. With the intention of providing the model of a simple but theoretically sound term structure, we propose that, in this economy, that the real interest rate (obtained by discounting the inflation rate) follows a diffusion process with a constant variance of the form:

$$\mathrm{d}r_t = \mu(r_t, t)\mathrm{d}t + \sigma(r_t, t)\mathrm{d}W_t, \tag{8}$$

where μ (r_t, t) is the drift associated to the short interest rate and $\sigma(r_t, t)$ is a function that drives the short run volatility in the interest rate. This framework is compatible with any mean reversion interest rate model as long as it keeps a constant variance, v.g. (Merton, 1973), (Vasicek, 1977) or (Kalotay, Williams, & Fabozzi, 1993). To keep the exercise as simple as possible, consider a zero coupon bond that pays, at maturity, a single monetary unit. By using this assumption, we can get the value of the bond, this is:

$$B(t, T) = e^{R(t,T)(T-t)} \tag{9}$$

To make the calculations to obtain the forward rate, we consider a single monetary unit of investment, continuously capitalizable interest rate, on a period of (t, T). The last assumption takes into an expression of the interest rate as an average of the continuous forward rates, this is:

$$I(t, T) = e^{R(t,T)(T-t)} \tag{10}$$

where

$$R(t, T) = \frac{1}{T - t} \int_t^T f(t, s)\mathrm{d}s$$

To make explicit the form of the term structure, we propose the use of a (Vasicek, 1977) model.

$$R(t, T) = \frac{1}{T - t} \int_t^T \left[(r_s - b)e^{-a(s-t)} + b \right]\mathrm{d}s \tag{11}$$

After solving the last integral, we obtain $R(t,T) = \dfrac{D(t,T)r_t}{T-t} - \dfrac{A(t,T)}{T-t}$, where $D(t,T) = \left(\dfrac{1-e^{-a(T-t)}}{a}\right)$ and $A(t,T) = -b(T-t) + \dfrac{b}{a}(1-e^{-a(T-t)})$. This sets the interest rate path of a (Vasicek, 1977) mean reversion model. We emphasize that the small chance of getting a negative real interest rate to be consistent with the reality of the financial markets when the central banks decide to rise the short-run interest rate and the old bonds were issued with fixed coupon rates or when there is a sudden inflation upsurge.

2.4 The yield of the different assets on the financial market
2.4.1 The real yield of a credit risk-free bond

At this point in the paper, we will make explicit the yield of each of the assets that may be part of the "rational consumer's" portfolios. In the first place, we will stipulate the real return of a Credit risk-free bond, this is.

$$dR_{Bl} = \frac{dBl_t}{Bl_t} = r_l dt \tag{12}$$

2.4.2 The real yield of a credit risky bond

With the idea of being congruent with the rest of the model, we propose that the yield of the risky credit bond should be determined with a (Vasicek, 1977) short rate model, this is

$$dr_t = a_r(b_r - r_t)dt + \sigma_{rt}\,dW_{r_t,t}, \tag{13}$$

where a is the adjustment speed to the long run rate, b. This assumption yield to a bond's price of the form

$$B_t = f(r_t, t) = e^{A_r(t,T)-r_t D_r(t,T)} \tag{14}$$

Applying the Itô's lemma to calculate the correspondent stochastic integrodifferential equation, we get

$$dB_t = \left\{\left[\frac{\partial A_r}{\partial t} - r_t\frac{\partial D_r}{\partial t}\right]B_t - D_r B_t\,[a_r(b_r - r_t)] + \frac{1}{2}D_r{}^2 B_t \sigma_{rt}{}^2\right\}dt - D_r B_t \sigma_{rt}\,dW_{r_t,t} \tag{15}$$

By solving this differential equation, we get a similar set of components for the dynamic of the risky bond rate. In this case, it is important to stress that although the risky and the riskless term structures have the same form

(they may not be parallel curves), the risky term structure is always above the government one because of the non-zero default risk. The default risk part of this interest rate usually creates different parameters for the Vasicek model. The a (speed of convergence to the long run rate) and b (long-run rate) parameters are different and usually create divergent curves. As before, we get

$$D_r = \frac{1 - e^{-a_r(T-t)}}{a_r} \tag{16}$$

$$A_r = \frac{1}{a_r^2}[D_r(T - t) - T + t]\left(a_r^2 b_r - \frac{1}{2}\sigma_{rt}^2\right) - \frac{\sigma_{rt}^2 D_r^2(t, T)}{4a_r} \tag{17}$$

After calculating the corresponding partial derivatives on (19), we obtain the yield of the risky bond, this is

$$dR_B = \frac{dB_t}{B_t} = r_t dt - \sigma_{Bt} dW_{r_t, t} \tag{18}$$

where $\sigma_{Bt} = \sigma_{rt} D_r$

2.4.3 Return on capital

The capital yield follows a stochastic process of the form

$$dR_k = r_k dt + \sigma_k dW_{k, t} \tag{19}$$

where $dW_{k,t}$ is a standardized brownian motion.

2.4.4 Return on the NAV's option

At last, we propose a functional form for the NAV's yield. In this case, we stress the unique nature of this asset under the paper's assumptions context. In this model, the NAV's option models the capital gain on the REIT's asset (the building), this means that we have a sell option on the building capital (k) that is attached to the risky bond that represents the rent's cash flow.

This put option (the REIT has the right, but it is not bound to sell) will be exercised if the building's market value is higher than the net present value of all the expected rents. By now, we postulate that the diffusion process of the NAV is given by:

$$dR_P = r_P dt - \sigma_P dW_{P, t} \tag{20}$$

2.5 *The consumption optimization problem*

With all the previous work in hand, we can follow the standard procedure to solve the stochastic optimization problem. To do so, we substitute all de asset's yields into the richness equation, this is

$$\frac{\mathrm{d}a_t}{a_t} = \mu_{a_t}\mathrm{d}t + \sigma_{a_t}\mathrm{d}W_{a_t,t} \tag{21}$$

where

$$\mu_a = N_{Bl}r_l + N_k r_k + N_B r_t + N_p r_P - \frac{c_t}{a_t}(1 + \tau_c) - r_\tau \tag{22}$$

$$\sigma_{a_t} = N_k \sigma_k\, \mathrm{d}W_{k,t} - N_B \sigma_{Bt}\mathrm{d}W_{r,t} - N_p \sigma_P\, \mathrm{d}W_{P,t} - \sigma_\tau \mathrm{d}W_{\tau,t} \tag{23}$$

Consider the Hamilton-Jacobi-Bellman as the optimum of the previously stated consumer's problem

$$0 = \max_{c_t} E\left[\frac{c_t^{1-\theta} - 1}{1 - \theta} e^{-\delta s}\mathrm{d}t + o(\mathrm{d}t) + \left(\frac{\partial J}{\partial t} + \frac{\partial J}{\partial a_t}\mu_a a_t + \frac{1}{2}\frac{\partial^2 J}{\partial a^2}\sigma_{a_t}^2 a_t^2 \right)\mathrm{d}t + \frac{\partial J}{\partial a_t}\sigma_{a_t} a_t \mathrm{d}W_{a_t,t} \right] \tag{24}$$

By using the typical candidate $J(a_t,t) = V(a_t)e^{-\delta t} = \left[\frac{a_t^{1-\theta} - 1}{1 - \theta}\right]e^{-\delta t}$, and substituting it and its derivatives into the HJB, we get:

$$
\begin{aligned}
= &\frac{c_t^{1-\theta} - 1}{1 - \theta} - \delta\left[\frac{a_t^{1-\theta} - 1}{1 - \theta}\right] + a_t^{1-\theta}\left[N_{Bl}r_l + N_k r_k + N_B r_t + N_p r_P - \frac{c_t}{a_t}(1 + \tau_c) - r_\tau\right] \\
&- \frac{\theta}{2}a_t^{1-\theta}\left[N_k{}^2\sigma_k{}^2 - 2N_k N_B \sigma_{K,Bt} - 2N_k N_p \sigma_{K,P} - 2N_k \sigma_{K,\tau} + N_B{}^2\sigma_B{}^2\right. \\
&\left.+ 2N_B N_P \sigma_{B,P} + 2N_B \sigma_{B,\tau} + N_P{}^2\sigma_P{}^2 + 2N_P \sigma_{P,\tau} + \sigma_\tau{}^2\right] \\
&+ \Phi\left[1 - N_{Bl} + N_k + N_B + N_p\right]
\end{aligned}
\tag{25}
$$

We proceed to obtain the gradient vector, this is

$$0 = \frac{\partial H}{\partial c_t} = \frac{1}{c_t^\theta} - \frac{(1 + \tau_c)}{a_t^\theta} \Longrightarrow c_t = \left[\frac{1}{(1 + \tau_c)}\right]^{\frac{1}{\theta}} a_t \tag{26}$$

$$0 = \frac{\partial H}{\partial N_K} = r_k - \theta\left[N_k \sigma_k{}^2 - N_B \sigma_{K,B} - N_P \sigma_{K,P} - \sigma_{K,\tau}\right] - \Phi \tag{27}$$

$$0 = \frac{\partial H}{\partial N_{Bl}} = a_t^{1-\theta}r_l - \Phi \tag{28}$$

$$0 = \frac{\partial H}{\partial N_B} = r_t - \theta\left[-N_k \sigma_{K,B} + N_B \sigma_B{}^2 + N_p \sigma_{B,P} + \sigma_{B,\tau}\right] - \Phi \tag{29}$$

$$0 = \frac{\partial H}{\partial N_p} = r_P - \theta\left[-N_k \sigma_{K,P} + N_B \sigma_{B,P} + N_p \sigma_P{}^2 + \sigma_{P,\tau}\right] - \Phi \tag{30}$$

To get the optimal investment paths, we solved the linear problem and obtained multiple solutions. Those various solutions mean that, at the optimum, all the risk premiums regarding risk must be the same and therefore the consumer is indifferent between them. The optimization conclude that the inclusion of a REIT in the consumer's portfolio does not drive him away from the optimal path. So we can infer that there may be an asset combination that will generate optimal paths for wealth and consumption because the consumer is taking into account all the assets considering their risk. The planning horizon is not an issue in this case because of the intergenerational altruism. The inclusion of a different planning horizon may induce different results; those will be analyzed in other paper.

2.6. Goods and real estate producers

For the production side of the model, we suppose that a single consumption good is produced and that the yield of each share (capital stocks, k_t) is determined by the production function and the dividend policy of the industry. On (Turnovsky, 1993) the author considers that the production of the consumption goods follows a stochastic process

$$dy_t = \gamma k_{t,c} dt + \gamma k_{t,c} \sigma_y \, dW_{y,t} \tag{31}$$

where γ is the expected average of the marginal return on capital. In a similar manner, the yield that a firm pays for the capital is defined by

$$dR_{k,c} = (1 - \tau_y) \frac{dv_t}{k_{t,c}} + \frac{du_t}{u_t} \tag{32}$$

in this case, dv_t is the average change in dividends and du_t is the average marginal change in share's price. Under this assumptions, the firm pays capital gains depending on the net dividends (after taxes) and the changes on share's prices.

In the sake of simplicity, let us say that at time t, the number of floating shares is constant and denoted by N, this is $k_{t,c} = Nu_t$, so we have:

$$dk_{t,c} = N du_t + u_t dN \tag{33}$$

The previous equations explain that the firm's equity is represented by the stock's price multiplied by the number of outstanding shares. Assuming the number of outstanding shares as constant, a change in the value of the equity comes from a shift in the share's price. On the other hand, the net product of

the firm (taking away the taxes) is used to pay dividends to shareholders or reinvesting on equity

$$(1 - \tau_{ic})dy_{t,c} = dv_t + dk_{t,c} \tag{34}$$

Provided with $N = \dfrac{k_{t,c}}{u_t}$, we can plug this definition on the change of the equity's value (32) and then get:

$$dk_{t,c} = \frac{k_{t,c}}{u_t} du_t \tag{35}$$

By substituting the change of the equity's value (34) on the yield of the stock price, u_t, we get the c hange of the stock price as a function of the dividend paid by the firm, v_t, the equity invested, k_t, and the tax structure, τ_{ic}, over the production, $y_{t,c}$, this is:

$$\frac{du_t}{u_t} = \frac{(1 - \tau_{ic})dy_{t,c} - dv_t}{k_{t,c}} \tag{36}$$

Now, we add the share's price movement into the equity yield, $R_{k,c}$, equation

$$dR_{k,c} = -\tau_y \frac{dv_t}{k_{t,c}} + \frac{(1 - \tau_{ic})dy_{t,c}}{k_{t,c}}, \tag{37}$$

Under the last assumption, we suppose that the dividends, v_t, are a constant proportion of the net income, y_t, thus $dv_t = \alpha(1-\tau_{ic}) dy_{t,c}$. By substituting the dividends, v_t, equation into the yield equation (36) we get.

$$dR_{k,c} = \left(1 - \alpha\tau_y\right)(1 - \tau_{ic})\frac{dy_{t,c}}{k_{t,c}} \tag{38}$$

If we take the product's dynamic equation and plug it into the last equation (marginal return of capital), we get a stochastic dynamic expression for the capital return, this expression includes the income tax, the dividends and capital gains.

$$dR_{k,c} = \left(1 - \alpha\tau_y\right)(1 - \tau_{ic})\gamma k_{t,c}, dt + \left(1 - \alpha\tau_y\right)(1 - \tau_{ic})\gamma k_t \sigma_y \, dW_{y,t} \tag{39}$$

Taking into account the capital's yield original form we can observe that it has the same structure as any stochastic differential equation, this is a deterministic trend and a stochastic part

$$dR_{k,c} = r_{k,c}dt + \sigma_{k,c} \, dW_{k,c,t} \tag{40}$$

where $r_{k,c} = \left(1 - \tau_y\right)(1 - \tau_{ic})\gamma k_{t,c}$ and $\sigma_{k,c}dW_{k,t} = \left(1 - \tau_y\right)(1 - \tau_{ic})\gamma k_{t,c}\sigma_y \, dW_{y,c,t}$.

An identical analysis was practiced to the real estate sector; the resultant stochastic differential equation has the same structure and components that the one for the capital yields (equation 42), this is $dR_{k,i} = r_{k,i}dt + \sigma_{k,i} dW_{k,i,t}$.

2.7. Government's budget restriction

In this part of the paper, we introduce the government's role and behavior in the economy. In the sake of simplicity, we assume that the government gets its money through the solely means: tax revenue and debt emission as in (Venegas-Martínez & Polanco-Gaytán, 2011), those assumptions are resumed in

$$dg - d\tau_1 - d\tau_2 + Bl_t\, dR_{Bl} = dBl_t \tag{41}$$

where dg is the marginal change in the public expense, $d\tau_1$ is the marginal change in the tax revenue on consumption and $d\tau_2$ is the marginal change on tax revenue on firm's activity. We model the marginal change on the public expense as a diffusion process. However, this public expense should represent a fixed part of the gross domestic product, because of that, we can express it as

$$dg = \bar{g}\gamma k_t dt + \gamma k_t \sigma_g\, dW_{g,t} \tag{42}$$

Equation 41 postulate that $\bar{g}$ is a constant fraction of the expected average marginal product that is dedicated to the public expense. In the other hand, we define the tax side of the problem. The consumers faces taxes on earned interest, on consumption, on wealth and on capital gains

$$d\tau_1 = i\tau_y\widehat{N_{Bl}}a_t dt + \alpha\tau_y(1 - \tau_{ic})\gamma k_t\big[dt + \sigma_y\, dW_{y,t}\big] + r_\tau a_t dt + \sigma_\tau a_t dW_{\tau,t} + c_t\tau_c dt \tag{43}$$

It is essential to specify that there are two kinds of firms; thus two types of corporate taxpayers; some of them pay taxes on goods production, τ_{ic}, and the rest of the firms pay taxes on the real estate production, τ_{ii}, this can be written as:

$$d\tau_2 = \tau_{ic}\gamma k_t dt + \tau_{ic}\gamma k_t\sigma_y\, dW_{yc,t} + \tau_{ii}\gamma k_t dt + \tau_{ii}\gamma k_t\sigma_y\, dW_{yi,t} \tag{44}$$

or equivalently,

$$d\tau_2 = \tau_{ic}\gamma k_t\big[dt + \sigma_y\, dW_{y,t}\big] + \tau_{ii}\gamma k_t\big[dt + \sigma_y\, dW_{yi,t}\big] \tag{45}$$

2.8. Macroeconomic equilibrium

In this section, we will determine the optimal capital formation path, for more details see (Venegas - Martínez & Polanco - Gaytán, 2011), (Smets & Wouters, 2003) or (Wickens, 2012). To determine the capital formation we will use the income identity, this is:

$$dy_t = c_t dt + dk_t + dg_t \tag{46}$$

we substitute the consumption, production and government equations into the capital formation dynamic stochastic equation, and taking expectations.

$$E\left[\frac{dk_t}{k_t}\right] = \gamma(1 - \bar{g}) - \left(\left[\frac{1}{(1 + \tau_c)}\right]^{\frac{1}{\theta}} \frac{1}{\widehat{N_k}}\right) \tag{47}$$

With this result, we can observe that there is a relation between the capital growth rate and the REIT´s. Our model shows that the REIT´s yield depends on the stock´s share of the individual wealth and the consumption, so it is theoretically proved that the REIT´s affect the capital formation. On the next section, we will measure this dependence by using quantile regression.

3. Empirical analysis

The goal of this section is to find empirical evidence of the relevance of the investment on REIT´s on the real estate capital formation. In the econometric analysis we use monthly observations from the Construction Gross Domestic Product, CGDP, and the Global Index of Economic Activities, IGAE, against the Total Net Asset Value, NAV.

3.1. Causality and cointegration

The NAVS series was calculated based on the certificates of participation and the market prices of the REITs[7] registered as of December 30, 2018. The estimate consisted of multiplying the volume of certificates of participation (obtained from Economatica) in circulation at the monthly closing of each of the REIT times their corresponding market price (obtained from Investing). Finally, the monthly NAVs of each of the REITs were added to obtain the current market values of the REITs assets in Mexico. On the other hand, the GDP of the construction sector corresponds to the private construction of industrial, commercial and services real

---

[7] Denomination of the Mexican REIT by its acronyms. Fideicomisos de Inversión en Bienes Raíces

estate. The above considering that the complexes that acquire or build by REITs represent the role they are engaged in (leasing of commercial real estate, services and industrial). Additionally, the IGAE is introduced as an additional independent variable that represents, to some extent, the preliminary results of the national monthly GDP, since the investment depends directly on the level of income. We describe the data and their sources on table 1 and show their general behavior on graph 1 and 2. The data were taken from the investing portal and the National Institute of Statistics and Geography. The observations are monthly from January 2014 to December 2018.

Table 1
Variables and sources for econometric analysis

| Acronym	Name	Source
NAV	REIT´s Net Asset Value	Investing
IGAE	Global Index of Economic Activities	INEGI*
CGDO	Construction Gross Domestic Product	INEGI*

Source: own elaboration. *National Institute of Statistics and Geography.

a)

b)

c)
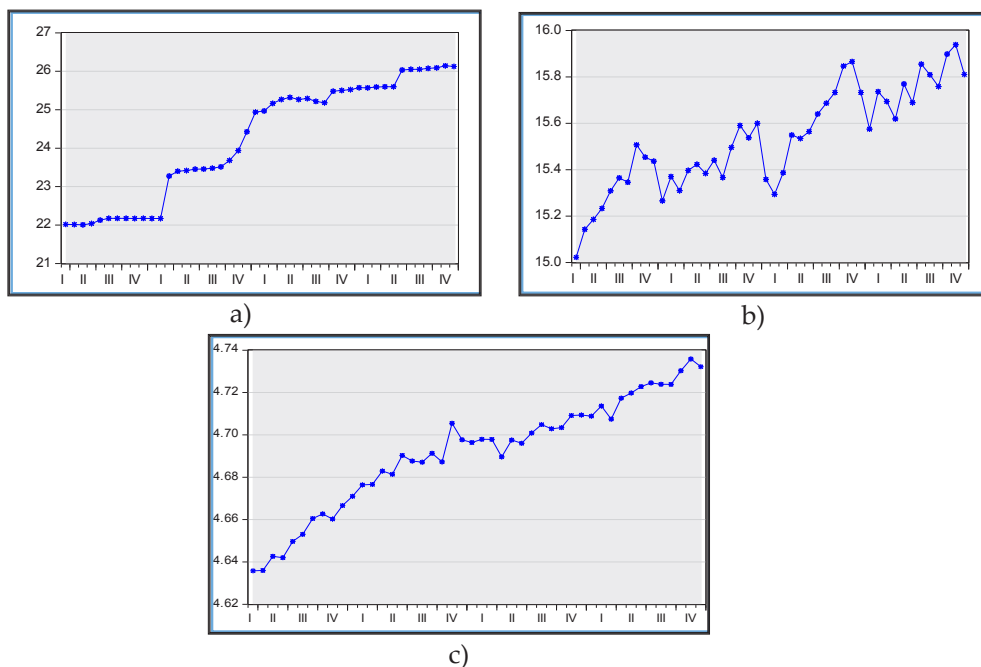
Source: own elaboration with data from INEGI and Investing. Panel a) Average Net Asset Value of REIT Market. Panel b) GDP construction sector. Panel c) Mexican GDP. Series set in log terms.

Figure 1
General behavior of the analyzed variables

We started the analysis using a linear regression between the Construction GDP, CGDP, and the rest of the variables. The results showed that the residues under the tests of Lagrange and the Durbin Watson statistic present first and second order serial autocorrelation. and under the ARCH tests for one and two lags indicate that there is heteroscedasticity. We do not present[8] the linear regression results because of its problems (heteroscedasticity and non-normality). The main result on this analysis is the presence of extreme values, those extreme observations may explain the non-normality but the heteroscedasticity problem suggest the presence of differentiated effects, related to the dimension of one or several variables.
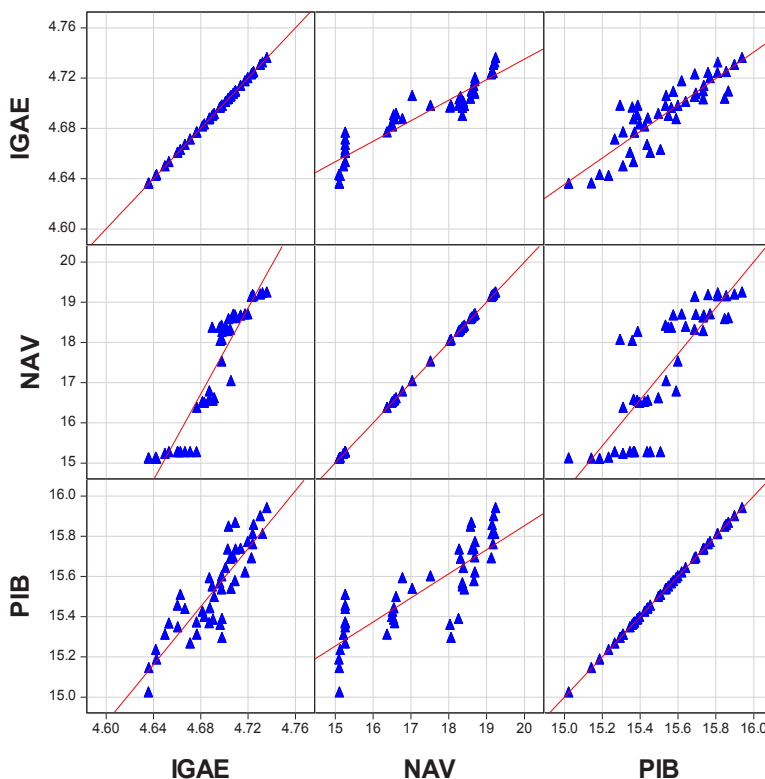
Source: own elaboration.

Figure 2
Dispersion matrix between the GDP of the construction series, net asset value, global economic activity index

---

[8] The results are available upon request by email.

Those ideas led us to examine our data through the quantile regression analysis. In this sense, a Granger Causality test was performed in order to justify the temporal relationship of the series; the tests can be checked in the table 2. Further, a cointegration test is performed under the Johansen approach to eliminate the possibility of spurious regression (Greene, 2012), whereby it is necessary that all series have the same order of integration. In this sense, to determine the stationarity of a series, the Augmented Dickey-Fuller test is used (Greene, 2012).

Table 2
Granger causality test over PIB, NAV, IGAE

Granger Causality Test		
Null hypothesis	F statistic	Prob.
NAV does not cause IGAE	1.59843	0.2131
IGAE does not cause NAV	4.12451	0.0486
PIB does not cause IGAE	2.09364	0.1553
IGAE does not cause PIB	5.48308	0.024
PIB does not cause NAV	2.04767	0.1598
NAV does not cause PIB	10.2659	0.0026

Source: own elaboration. Specification with the first lag determined by the Akaike and Schwartz criteria of optimal lag. The shaded cells denote rejection of the null hypothesis at 5%.

Table 3
Stationarity test over PIB, NAV, IGAE

Augmented dickey-fuller test (levels)				
Serie	Critical values for significance level			T-statistic
	10%	5%	1%	
PIB	-3.186854	-3.513075	-4.17564	-3.952899
NAV	-3.186854	-3.513075	-4.17564	-2.037037
IGAE	-3.186854	-3.513075	-4.17564	-3.062975

Augmented dickey-fuller test (First difference)				
Serie	Critical values for significance level			T-statistic
	10%	5%	1%	
PIB	-3.188259	-3.515523	-4.180911	-7.756435
NAV	-3.188259	-3.515523	-4.180911	-6.769168
IGAE	-3.188259	-3.515523	-4.180911	-10.92288

Source: own elaboration. The test was performed considering linear trend and intercept.

As it can be seen in the table 3, the series are integrated in order one. Once it has been determined that the series are I (1), we proceed to determine if there is cointegration by the Johansen method. With respect to the trace statistic, the Johansen cointegration test, in table 4, indicates that there are at least two cointegration relationships or long-term relationship between the variables, thereby ruling out that a regression between these variables is considered spurious.

Table 4

Johansen cointegration test over PIB, NAV, IGAE

Contegration rank test (trace)				
Null hypothesis	Eigenvalue	Trace statistic	Critical value	Prob.**
None *	0.350274	34.84575	29.79707	0.012
At least 1 *	0.242557	15.87276	15.49471	0.0439
At least 2	0.079591	3.649237	3.841466	0.0561

Trace test show that there exist almost two cointegration equations at 0.05 level
* Denotes rejection at 0.05
**MacKinnon-Haug-Michelis (1999) p-values

Unrestricted cointegration rank test (maximum eigenvalue)				
Null Hypothesis	Eigenvalue	Max eigen statistic	Critical value	Prob.**
None	0.350274	18.97299	21.13162	0.0976
At least 1	0.242557	12.22353	14.2646	0.1025
At least 2	0.079591	3.649237	3.841466	0.0561

Max eigen test show that there no exist cointegration equations at 0.05 level
* Denotes rejection at 0.05
**MacKinnon-Haug-Michelis (1999) p-values.

Source: own elaboration. It is considered deterministic linear trend and lags in the first difference.

With the previous tests it is determined that there is a causality in Granger's sense of the Net Asset Values and the Global Index of Economic Activity towards construction GDP. On the other hand, the Johansen Cointegration test indicates a long-term balance, thereby ruling out the possibility of spurious regressions.

In the following section we proceed to perform a robust regression, because the series and residues do not follow a normal distribution, as well as the linear model present problems of heteroscedasticity. For this, a regression by quantiles is proposed, which considers the empirical distributions of the series allowing to estimate the different parameters and also evaluate the different impacts for extreme values of the distribution.

3.2. *Quantile regression model*

Publicly traded assets such as REITs participation certificates hardly show signs of normality, so the use of econometric tools based on Gaussian disturbances can lead to a biased analysis. On the other hand, the estimated effects of the linear models only consider the average effects, however, there may be different behaviors in the tails of the distribution. One of the questions that arise in this work is whether the long-term effect between the construction sector and the net value of the fibers is stable considering the possible extreme values of the latter for being subject to speculation. If this relation varies in the different points of the distribution, in periods of stress in the financial markets, it could mean a contraction of the product of the construction sector and therefore of the economic activity. This is why regression by quantiles is used, which, considering the different parts of the empirical distributions of the series, allows evaluating the impacts of the regressors on the values of the dependent variable (Konker and Basset, 1978). It is worth mentioning that the regression by quantiles, unlike regressions by least squares, seeks to minimize the absolute error to the different quantiles of the distribution, in contrast to the minimization of the square of errors or residues. The results are presented in table 5.

Table 5
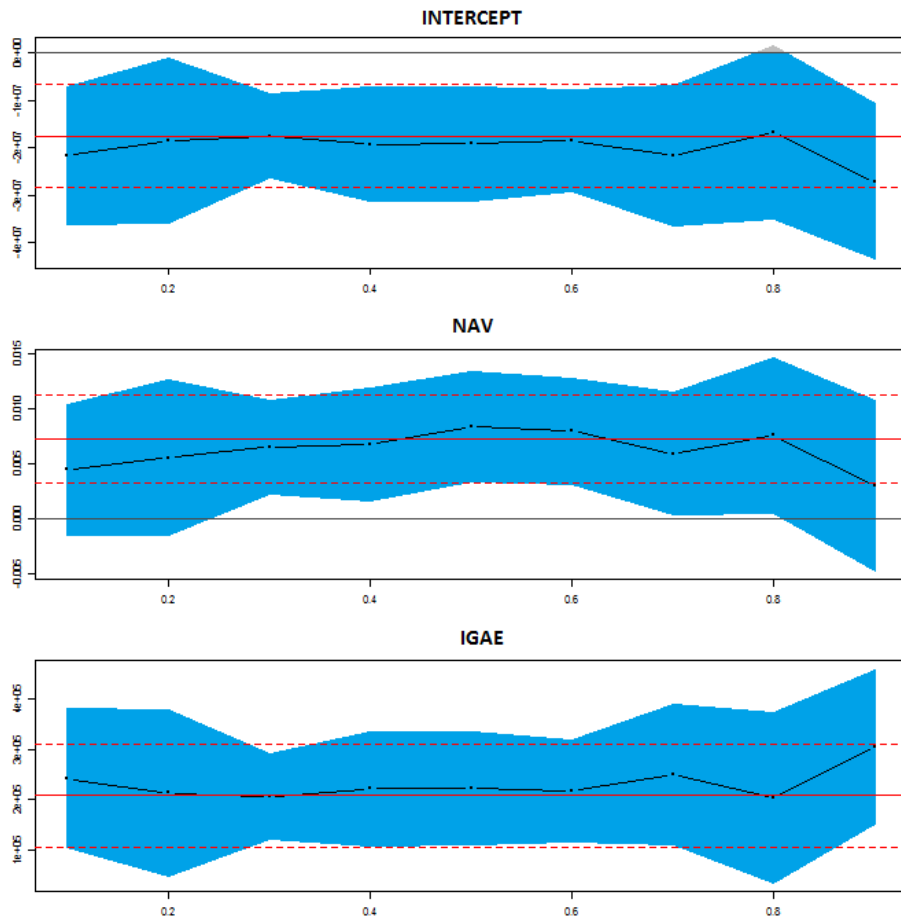Quantile regression estimates

Quantile	Intercept			NAV			IGAE		
	value	t-value	Prob.	value	t-value	Prob.	value	t-value	Prob.
0.10	-21677870.00	-2.47	0.02	0.00	1.25	0.22	242497.70	2.87	0.01
0.2	-18569470.00	-1.76	0.09	0.01	1.31	0.20	213700.70	2.12	0.04
0.30	-17464650.00	-3.20	0.00	0.01	2.56	0.01	205671.60	4.00	0.00
0.4	-19209450.00	-2.61	0.01	0.01	2.18	0.03	222702.60	3.22	0.00
0.50	-19184950.00	-2.64	0.01	0.01	2.78	0.01	222410.50	3.24	0.00
0.6	-18436320.00	-2.84	0.01	0.01	2.75	0.01	217193.90	3.52	0.00
0.70	-21650900.00	-2.40	0.02	0.01	1.76	0.09	249440.10	2.93	0.01
0.8	-16780610.00	-1.51	0.14	0.01	1.77	0.08	204519.70	1.97	0.05
0.90	-27176700.00	-2.72	0.01	0.00	0.65	0.52	305764.50	3.25	0.00

Source: own elaboration with R-Project. Shaded cells denote statistical significance in the estimated parameters at 5%.

Based on the previous table, it is possible to observe that the estimates are significant for the central quantiles, that is, the extreme values of the NAVs do not have a significant impact on the GDP of the construction. Additionally, it is possible to observe that the estimates made through the regression by

quantiles are kept constant through the different quantiles, that is, the impact that an increase of 1000 pesos has on the Net Asset Value, the GDP of the construction will increase by approximately .01, regardless of the distribution quantile. Graph 3 also shows that quantile estimates yield similar estimates to those obtained by the least squares method.

Figure 3

Parameter estimation of independent variables and their 95% confidence bands. Quantile regression

Source: own elaboration. Estimate made with the R-Project. The red line indicates the Regression by the method of Ordinary least squares and the red dotted line indicates its 95% confidence band respectively.

From the above it is concluded under the quantile regression approach that the extreme values of the NAVs do not affect the GDP of the construction, and that the values in the central quantiles do not present a significant variation, so it is possible to use the method of least squares to be able to establish predictions of the behavior of this variable.

4. Conclusions

The main result of this research is that REITS have a direct positive effect on the Gross Domestic Product of the construction sector, and that there is a long-term balance between the aforementioned variables. The foregoing proves that the hypothesis raised in this paper is accepted.

Likewise, under the development of a Stochastic Dynamic General Equilibrium Model it was theoretically determined that there is a relationship between the Gross Formation of Fixed Capital of the economy and the investment in the REITS Participation Certificates. The inclusion of CBFIs in an investment portfolio of a rational agent represents a type of hedge against possible variations of the remaining assets in the portfolio. The foregoing as a consequence of the fact that there are infinite possible combinations between the available assets and the REITS that can generate the same returns by having the same risk premium. As an example it is possible to observe the diversification of the SIEFORES 'portfolios in Mexico, with a participation of the CBFIs of the REITS in ever greater proportion and with relatively stable yields.

Finally, under the quantile regression approach, empirical evidence was found that the extreme values, whether positive or negative, that present the Net Values of the Assets that belong to the Real Estate Investment Trusts do not have a significant relationship, or In other words, they do not impact the Gross Domestic Product of the Construction sector in any way. This demonstrates that a sharp rise or fall in the net values of the assets at market prices would not have repercussions on the real variables such as the product.

References

Bai, Q., & L. Zhu (2017). The Effects of Industry Specific and Local Economic Factors on Credit Default Swap Spreads: Evidence from REITs. *Journal of Financial Services Research*, vol. 54, No. 5, pp. 1-29.

Banerjee, S., & P. Basu (30 de May de 2015). *National Council of Applied Economic Research*. Obtenido de A dynamic stochastic general equilibrium model for India: saber.eaber.org/sites/default/files/documents/1432552897Working%20Paper%20Sheshdari%20Banerjee.pdf.

Bernheim, B. D. (1989). Intergenerational altruism, dynastic equilibria and social welfare. *The review of economic studies*, vol. 56, No. 1, pp. 119-128.

Booth, P. (2002). Real estate investment in an asset/liability modeling context. *Journal of Real Estate Portfolio Management*, Vol. 8, No.3 , 183-198.

Chang, F. R. (2004). *Stochastic Optimization in Continuous Time.* Cambridge, UK: Cambridge University Press.

Chiang, Y.-H.; L. Tao, & F. K. Wong (2015). Causal relationship between construction activities, employment and GDP: The case of Hong Kong. *Habitat international*, vol. 46, No.1, pp. 1-12.

Connolly, P. (18 de July de 2018). *Recent trends in the Mexican construction industry and outlook for the 21st century: Its image, employment prospects and skill requirements.* Obtenido de Unpublished report for the ILO (2001): https://www.researchgate.net/profile/Priscilla_Connolly/publication/313702987_Recent_trends_in_the_Mexican_construction_industry_and_outlook_for_the_21st_Century_its_image_employment_prospects_and_skill_requirements/links/58a35fef458515d15fd9c73c/Recent-t.

Cruz García, A. F. (18 de July de 2018). *Private equity for real estate in Mexico: overview, challenges and opportunities*. Obtenido de Diss. Massachusetts Institute of Technology: https://dspace.mit.edu/handle/1721.1/101326.

Cruz-Aké, S.; R. S. García-Ruiz, & F. Venegas-Martínez (2016). Mexican REITs (REITS) in retirement funds (AFORES): different pricing approaches and market risk measurement implications. *International Journal of Bonds and Derivatives*, vol. 2, No.3, pp. 211-232.

Dick, B.; M. Rafferty; P. Toner, & S. Wright (2017). Financialisation and labour in the Australian commercial construction industry. *The Economic and Labour Relations Review*, vol. 28, No. 4, pp. 500-518.

Donaubauer, J.; B. E. Meyer, & P. Nunnenkamp (2016). A new global index of infrastructure: Construction, rankings and applications. *The World Economy*, Vol. 39, No. 2, pp. 236-259.

Dubreuille, S.; M. Cherif, & M. Bellalah (2016). Real Options: An Alternative Valuation Model for the US REIT Market. *International Journal of Business.* vol. 21, No. 1, pp. 42-54.

Edelstein, R., & K. Magin (2013). The equity risk premium for securitized real estate: The case for us real estate investment trusts. *Journal of Real Estate Research*, vol. 35, No. 4, pp. 393-406.

Galperti, S., & B. Strulovici (2017). A theory of intergenerational altruism. *Econometrica*, vol. 85, No. 4, pp. 1175-1218.

Greene, W. H. (2012). *Econometric Analysis*. USA: Prentice Hall.

Kalotay, A. J.; G. O. Williams, & F. J. Fabozzi (1993). A Model for Valuing Bonds and Embedded Options. *Financial Analysts Journal. CFA Institute Publications*, vol. 49, No. 3, pp. 35-46.

Karatzas, I., & S. Shreve (2012). *Brownian motion and stochastic calculus,* vol. 113. New York: Springer Science & Business Media.

Kotlikoff, L. J. (2016). *Essays on saving, bequests, altruism, and life-cycle planning*. Massachusetts: MIT Press.

Krewson-Kelly, S., & R. B. Thomas (2016). *The Intelligent REIT Investor: How to Build Wealth with Real Estate Investment Trusts*. Hboken, New Jersey: John Wiley & Sons, Incorporated, 2016.

Le Blanc, D., & C. Lagarenne (2004). Owner-occupied housing and the composition of the household portfolio: the case of France. *The Journal of Real Estate Finance and Economics,* vol. 29, No. 3, pp. 259-275.

Lozano Montero, E.; R. Godínez López, & S. M. Albor Guzmán (15 de July de 2018). Public-Private Associations in Mexico. *Financing and Social Benefits in Road Infrastructure Projects*. Obtenido de https://papers.ssrn.com/sol3/papers.cfm?abstract_id=3100568.

Merton, R. C. (1973). Theory of Rational Option Pricing. *Bell Journal of Economics and Management Science*. The RAND Corporation, vol. 4, No. 1, pp. 141-183.

Mikosch, T. (1988). *Elementary stochastic calculus, with finance in view*, vol. 6. Singapore: World Scientific Publishing Company.

Nasieku, T., & R. M. Wanyonyi (2016). The relationship between mortgage finance and real estate development in Kenya. *Journal of Social Science Research,* vol. 10, No. 1, pp. 1950-1955.

Ngowi, A. B.; E. Pienaar; A. Talukhaba, & J. Mbachu (2005). The globalisation of the construction industry, a review. *Building and Environment,* No. 40, vol. 1, pp. 135-141.

Papageorgiou, D. (30 de May de 2014). *BoGGEM*: No. 182. Obtenido de A dynamic stochastic general equilibrium model for policy simulations: saber.eaber.org/sites/default/files/documents/1432552897Working%20Paper%20Sheshdari%20Banerjee.pdf.

Pereira, A. M., & J. B. Shoven (1988). Survey of dynamic computational general equilibrium models for tax policy evaluation. *Journal of Policy modeling*, vol. 10, No 3, pp. 401-436.

Poo Rubio, A. M., & L. Rocha Chiu (18 de July de 2018). *Evaluación de los Fideicomisos de Inversión en Bienes Raíces (REITS) como mecanismos de financiamiento en edificación*. Obtenido de http://zaloamati.azc.uam.mx/handle/11191/5769.

Reddy, W., & W. Wong (13 de July de 2016). *Australian interest rate movements and A-REITs performance: A sectoral analysis.* Obtenido de AsRES 21st International Conference. Asian Real Estate Society.

Turnovsky, S. J. (1993). Macroeconomic policies, growth, and welfare in a stochastic economy. *International Economic Review*, vol. 31, No. 4, pp. 953-981.

Vasicek, O. (1977). An equilibrium characterization of the term structure. *Journal of financial economics*, vol. 5, No. 2, pp. 177-188.

Virani, S., & P. Kaur (2015). Real Estate Investment Trusts (REITs)–An Innovative Approach to Real Estate Sector. *Annual Research Journal of Symbiosis Centre for Management Studies*, vol. 3, pp. 155-165.

Wang, C.; J. Cohen, & J. L. Glascock (23 de May de 2018). *Geographically Overlapping Real Estate Assets, Liquidity Spillovers, and Liquidity Multiplier Effects.* Obtenido de SSRN: https://papers.ssrn.com/sol3/papers.cfm?abstract_id=3154088.

Washburn, M. (2013). How REIT Uno's offering transforms Mexico's securities market. *International Financial Law Review*, April 2013, NA.

Yong, J., & X. Y. Zhou (1999). *Stochastic controls: Hamiltonian systems and HJB equations*, vol. 43. New York, USA: Springer Science & Business Media.

Venegas-Martínez, F. (2008). Riesgos financieros y económicos: Productos derivados y decisiones económicas bajo incertidumbre. Cengage Learning. México.

Venegas-Martínez, F. (2006). Stochastic temporary stabilization: Undiversifiable devaluation and income risks. *Economic Modelling,* vol. 23, No. 1, pp. 157-173.

Venegas-Martínez, F. (2001). Temporary stabilization: A stochastic analysis, *Journal of Economic Dynamics and Control*, vol. 25, No. 9, pp. 1429-1449.