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Abstract

We evaluate the stabilization properties of several rules and instruments to reduce government debt in a Currency Union, like the EMU. In a two-country New-Keynesian DSGE model, with a debt-elastic government bond spread and incomplete international financial markets, we study the effects of government debt deleveraging, under different scenarios for fiscal policy coordination. We find that greater stabilization is achieved when the two countries coordinate by stabilizing net exports. Moreover, we find that taxes are a better instrument for deleveraging compared to government transfers. Our policy prescriptions for the Euro Area are to reduce government debt less during recessions and liquidity traps, and to do so using distortionary taxes, while concentrating on reducing international demand imbalances.

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1 Introduction

In the wake of the Covid-19 pandemic, which pushed many countries to stimulate the economy through large fiscal expansions, the increase in government debt poses an important challenge in understanding how to deal with it in the future, in particular how to reduce government debt ~~to make it more sustainable~~, while at the same time stabilizing the economy, especially in the Euro Area. Our work can then give some insight on the stabilization properties of different deleveraging schemes with varying levels of coordination among countries sharing the same currency, as in the Euro Area. At the same time, the Next Generation EU initiative can be seen as a temporary and initial version of an EU fiscal capacity that could develop into a version similar to our Full Fiscal Union, which can itself give some insight on the stabilization properties of a common fiscal capacity.

The situation of high sovereign debt in most EMU (European Economic and Monetary Union) countries requires to consider which is the best way and timing to reduce public debt. As Ferrero (2009) shows, there are important welfare gains in adopting a flexible fiscal policy, because by constraining fiscal policy, deleveraging creates a trade-off between discipline and stabilization. This paper studies government debt deleveraging in a two-country Currency Union where financial markets are incomplete and different government debt levels determine a government bond spread[1]. Governments choose the level of government consumption and transfers, which are financed only by distortionary taxes on labour income and firm sales and by short-term government bonds. We calibrate the model to Germany (country H) and the Rest of the Euro Area (country F, namely France, Italy and Spain). We assume that the Rest of the Euro Area (country F) needs to reduce its public debt, using alternatively distortionary taxation or government transfers. This also allows us to study the stabilization properties of different instruments used to deleverage, as tax-based consolidations affect more prices and create more distortions on labour supply, compared to expenditure-based consolidations, which have mainly an income effect. Being part of a Currency Union, government debt deleveraging creates spillover effects on Germany (country H) through international financial and goods markets, mainly by moving relative prices. Although Germany was the country that in the years from 2011 to 2019 actually reduced its debt-to-GDP ratio, our simulations of debt reduction in other Euro Area countries give some insight on the effects of future fiscal consolidations, mostly needed in countries that, unlike Germany, have a very high public debt.

Given the discussion on the reduction of international trade imbalances and on the need for a common fiscal capacity in the EMU, we analyze the stabilization properties (in terms of deviation of output and inflation from their symmetric flexible-price steady-state equilibrium) of deleveraging under three different degrees for fiscal policy coordination: in a Pure Currency Union (PCU), which describes the current scenario without coordination among countries, each country follows a

[1] This assumption implies that the international trade of foreign bonds is subject to intermediation costs, following mainly De Paoli (2009), which in turn follows Turnovsky (1985). As explained in Ghironi (2006) and Schmitt-Grohé and Uribe (2003) this assumption ensures stationarity of net foreign assets and is an assumption also used in Eggertsson, Ferrero and Raffo (2014).

countercyclical fiscal policy to reduce its domestic output gap[2], while in a Coordinated Currency Union (CCU), which describes a scenario where countries coordinate their fiscal policies, the two countries aim to reduce their net exports gap independently, and in a Full Fiscal Union (FFU), which describes a scenario where countries coordinate and also pool their resources in a central fiscal capacity, in addition to reducing their net exports gap, the two countries consolidate their budget constraints. Therefore, only in a Full Fiscal Union the two countries coordinate in terms of both cyclical and deleveraging policies. This reflects, to some extent, the discussion in the EU on fiscal policy coordination among countries, in this case by reducing current account imbalances through the reduction of trade imbalances, and on a central fiscal capacity, as in the case of our Full Fiscal Union. We also study deleveraging during recessions driven by preference shocks in the presence of the Zero Lower Bound.

Our analysis shows that deleveraging amplifies the overall volatility of the economy. To that extent, the role of deflationary pressures induced by the deleveraging shock is crucial in its international transmission, with and without other shocks. After a deleveraging shock to country F, it is country H which falls into a recession, because goods imported from country F are now more convenient, lowering substantially net exports in country H due to its greater openness to trade. At the same time, GDP in country F increases, as also found in Cogan et al. (2013), mainly induced by the deflationary effect on net exports through the terms of trade. This mechanism is also underlined by Coenen, Mohr and Straub (2008) and Forni, Gerali and Pisani (2010). Thus, reducing the volatility of net exports is key for the purpose of stabilization and can be done in two different ways: when transfers are used to deleverage, a Full Fiscal Union stabilizes more, while when taxes are used to deleverage a Coordinated Currency Union creates more stabilization. In particular, the expansion in GDP in country F increases the tax base, pushing the fiscal authority to reduce the tax rates to balance the government budget, so as to follow the debt path imposed by the deleveraging rule. As argued by Vogel (2012) and Ferrara and Tirelli (2017), if fiscal policy is able to anticipate the reduction in taxes while simultaneously cutting government spending, this highly reduces the cost of deleveraging. Because of their ability to reduce the deflation induced by the debt reduction in country F and, in turn, its effects on country H, distortionary taxes are always a better instrument for deleveraging because they stabilize the economy more than other instruments. Additionally, in line with in't Veld (2013), we show that deleveraging amplifies the length and the depth of liquidity traps and recessions induced by demand shocks. However, while in in't Veld (2013) a temporary fiscal stimulus in countries running a current account surplus induces only a small improvement in current account deficits in the periphery, we find a similar result when country H partially bears the cost of deleveraging in country F by consolidating budget constraints.

Our research is related to two different strands of literature: one that has focused on Currency Union DSGE models and another one on government debt deleveraging models. We follow the open economy approach of Galí (2009), but in a two-country Currency Union with only distortionary

[2]The output gap and net exports gap are defined as the deviation of output and net exports from their symmetric flexible–price equilibrium.

taxes as sources of government revenue like in Ferrero (2009). To this framework, in our previous work, Cole, Guerello and Traficante (2020), we added home bias in consumption, to allow for deviations from Purchasing Power Parity, and targeting rules for fiscal policy, to allow governments to coordinate following three scenarios describing different degrees of coordination. In this paper we extend the latter model with a debt-elastic government bond spread and incomplete international financial markets, to account for international financial frictions, following Hjortsø (2016), and to study the effects of the international transmission of a deleveraging shock. We also extend our analysis to the case of a Zero Lower Bound occasionally binding constraint on interest rates.

This paper is also related to the literature on government debt deleveraging, which has been investigated thoroughly after the Great Recession and the sovereign debt crisis in the Euro Area. Some recent papers, such as Ferrara and Tirelli (2017), analyze the fiscal policy mix for public debt deleveraging and its consequences in a closed economy, while Almeida et al. (2013) considers a small open economy setup. However, the slump driven by a fiscal consolidation, as observed in these papers, is only partially confirmed by the literature on deleveraging in large open economies, such as Coenen, Mohr and Straub (2008), Forni, Gerali and Pisani (2010), Vogel (2012) and Cogan et al. (2013), which underline that fluctuations in net exports reduce recessions.

Like Coenen, Mohr and Straub (2008) and Erceg and Lindé (2013), our paper deals with government debt deleveraging in a Currency Union, and the debt reduction we treat reproduces what member countries in the Euro Area are requested to do in order to reach the target of 60% of GDP, as stated in the Maastricht Treaty. We analyze the stabilization properties of different paths for dynamic government debt deleveraging and study the best taxation and spending combinations. To this extent, Kirsanova et al. (2007) shows substantial welfare gains if government expenditure, contrasting asymmetric shocks, responds to differences in inflation and the terms of trade, especially when there is slow debt consolidation. We contribute to this literature by taking into account the possibility of a Redemption Fund, as outlined in Van Rompuy et al. (2012), or the more recent Next Generation EU, in the form of a Full Fiscal Union. The possibility for countries in dire circumstances, as recessions or tight fiscal consolidations, to obtain a partial debt relief through transfers by a central EMU government may be welfare improving for the union as a whole, especially in the face of asymmetric shocks.

Our model is structured to allow for spillovers from monetary to fiscal policy and viceversa, and from one country to another through country-specific fiscal policies. Nominal rigidities, in the form of staggered price adjustments, generate real effects of monetary policy, while distortionary taxation generates non-Ricardian effects of fiscal policy. This framework allows to study the interaction between country-specific fiscal policies where, in the absence of the nominal exchange rate as an automatic stabilizer, fiscal policies influence each other through their effects on output and the terms of trade. Incomplete international financial markets with a government bond spread and government debt deleveraging dynamics provide additional frictions in the economy which can be addressed by monetary or fiscal authorities[3].

The remainder of the paper is structured as follows. Section 2 describes the general model and different degrees of fiscal policy coordination (PCU, CCU and FFU) with different deleveraging instruments (Government Transfers and Taxes). Section 3 shows the calibration of the parameters and steady state stances of the model. Section 4 provides numerical simulations under different coordination schemes and different fiscal policy instruments for deleveraging, also during liquidity traps. Section 5 collects the main conclusions and provides possible extensions, while some appendices present some details about the model and a sensitivity analysis[4].

2 A Two-Country Currency Union Model

The EMU is represented by two countries (or groups of countries) of different size forming a Currency Union. Both economies are assumed to share identical preferences, technology and market structure, but may be subject to different shocks, price rigidities, initial conditions and fiscal stances. The two countries are indexed by H and F for Home and Foreign. We can think of country H as Germany, the Core, and country F as the Rest of the Euro Area (France, Italy and Spain). The EMU is inhabited by a continuum of infinitely-lived households of measure one, indexed by $i \in [0, 1]$. Each household owns a monopolistically competitive firm producing a differentiated good, indexed by $j \in [0, 1]$. The population on the segment $[0, h)$ belongs to country H while the population on the segment $[h, 1]$ belongs to country F, so that the relative size of country H is $h \in [0, 1]$, while the relative size of country F is $1 - h$. This is true for both households and firms. Each country has an independent Fiscal Authority, while the Currency Union shares a common Monetary Authority. Monetary policy is designed to set the interest rate for the whole Currency Union following an Inflation Targeting Regime.

The main novelty of our theoretical model is given by the characterization of the fiscal policy scenarios and coordination stratiegies in the monetary union. Fiscal policy is designed following the Stability and Growth Pact and Fiscal Compact rules, by imposing that the target Government Debt-to-GDP ratio is about 60% and that countries must either adopt a balanced budget or adopt the Debt Brake rule, which implies that if government debt-to-GDP is more than 60% it should decrease by 5% of the excess every year. In our model there is a stabilization rule for fiscal policy, in a setting where one country has to deleverage, while the other country balances the budget. As mentioned in the introduction, we consider three different scenarios in terms of fiscal policy coordination. Our aim is to provide an analysis that includes the current EMU architecture (PCU) without coordination on a common variable and also two other scenarios with higher degrees of coordination. The maximum level of coordination is described in the Full Fiscal Union scenario, where countries target a common variable and consolidate their budget constraints.

[3] This topic is of interest also for macro-prudential authorities. However, in this paper we do not consider macro-prudential policies.

[4] Appendix A.1 collects all equilibrium conditions of the model used for the simulations, while Appendix A.2 describes the symmetric flexible–price steady-state equilibrium on which the model is calibrated. Appendix B describes a sensitivity analysis to changes in some key parameters of the model.

In describing the model and monetary and fiscal policies in detail in what follows, we denote variables referred to the *Foreign* country with a star (*) and, given symmetry between the two countries, we show the main equations only for country F.

2.1 Households

In each country there is a continuum of households, which gain utility from private consumption and disutility from labour, consume goods produced in both countries with home bias, supply labour to domestic firms, and collect profits from those firms. Households can trade a complete set of one-period state-contingent claims only within their own country. Households in country H can purchase one-period bonds issued by both countries' governments, while households in country F can only purchase one-period bonds issued by their own country's government, subject to their budget constraints. This assumption introduces a friction in international financial markets so that they are not complete and spreads between government bond yields can occur. This reflects the fact that Germany (country H) is a net creditor on international financial markets, while the Rest of the Euro Area (country F) is a net debtor on international financial markets.

In our model we include financial intermediaries for the sole purpose of creating international financial frictions and a government bond spread. However, we do not have a fully detailed banking sector: All the financial intermediaries in the model are owned by the households in country H, earn profits on all the internationally traded bonds, by collecting savings from households in country H at the interest rate set by the Central Bank, and lending to the government in country F at the interest rate paid on its government bonds. The government in country H collects savings directly from households in country H at no extra cost. This distinction between country H and country F allows to create a spread on the interest rate of country F.

The only financial assets traded internationally are government bonds issued by country F, $B^i_{F,t}$, which are purchased by households in country H in period t. These bonds yield a net return, i^*_t, which depends on the risk-free rate, i_t (defined below), and on a transaction cost for households in country H on purchases of government bonds issued by country F, δ_t. This transaction cost can be interpreted as a risk premium paid by the more indebted country and is given by the following expression:

$$\delta_t = \delta^B \left(\frac{B^{*G}_{t-1}}{P^*_{H,t-1} Y^*_{t-1}} - \frac{B^{*G}}{P^*_H Y^*} \right) \tag{2.1.1}$$

where $\frac{B^{*G}_{t-1}}{P^*_{H,t-1} Y^*_{t-1}}$ is the overall government debt-to-GDP for country F in period $t-1$ and variables without subscript t are their respective steady state values. The previous equation shows how this government bond spread is increasing in the deviation of government debt-to-GDP from steady state by δ^B, which represents the sensitivity of the government bond spread to deviations of government debt-to-GDP from its target long–run value. This reflects a positive interest rate spread as a risk premium paid on bonds issued by a country which has a government debt-to-GDP ratio higher than the 60% target.

Each household in country F, indexed by $i \in [h, 1]$ seeks to maximize the present-value utility[5]:

$$E_0 \left\{ \sum_{t=0}^{\infty} \beta^t \xi_t^* \left[\frac{(C_t^{*i})^{1-\sigma} - 1}{1-\sigma} - \frac{(N_t^{*i})^{1+\varphi}}{1+\varphi} \right] \right\} \tag{2.1.2}$$

where $\beta \in [0, 1]$ is the common discount factor, which households use to discount future utility, σ is the inverse of the elasticity of intertemporal substitution (it is also the Coefficient of Relative Risk Aversion (CRRA)), φ is the inverse of the Frisch elasticity of labour supply and ξ_t^* is a preference shock to *Foreign* households. This preference shock is assumed to follow the AR(1) process in logs:

$$\xi_t^* = (\xi_{t-1}^*)^{\rho_\xi^*} e^{\varepsilon_{\xi,t}} \tag{2.1.3}$$

where $\rho_\xi^* \in [0, 1)$ is a measure of persistence of the shock and $\varepsilon_{\xi,t}$ is a zero mean white noise process. N_t^{*i} denotes hours of labour supplied by households in country F, while C_t^{*i} is a composite index for private consumption defined by:

$$C_t^{*i} \equiv \left[(1-\alpha^*)^{\frac{1}{\eta}} (C_{H,t}^{*i})^{\frac{\eta-1}{\eta}} + \alpha^{*\frac{1}{\eta}} (C_{F,t}^{*i})^{\frac{\eta-1}{\eta}} \right]^{\frac{\eta}{\eta-1}} \tag{2.1.4}$$

where the parameter $\eta > 0$ measures the substitutability between domestic and foreign goods (international trade elasticity), and $\alpha^* \in [0, 1]$ is a measure of openness of the *Foreign* economy to international trade. Equivalently $(1 - \alpha^*)$ is a measure of the degree of home bias in consumption in country F. When α^* tends to zero the share of goods from country H in consumption baskets of country F vanishes and the country ends up in autarky, consuming only domestic goods. If $1-\alpha^* > 1-h$ there is home bias in consumption in country F, because the share of consumption of domestic goods is greater than the share of production of domestic goods. The same applies to the *Home* parameter of openness to international trade $\alpha \in [0, 1]$ for country H, except for the fact that if $1 - \alpha > h$ there is home bias in consumption in country H. $C_{H,t}^{*i}$ is an index of consumption of domestic goods for households in country F, given by the constant elasticity of substitution (CES) function (also known as Dixit and Stiglitz (1977) aggregator function):

$$C_{H,t}^{*i} \equiv \left(\left(\frac{1}{1-h} \right)^{\frac{1}{\varepsilon}} \int_h^1 C_{H,t}^{*i}(j)^{\frac{\varepsilon-1}{\varepsilon}} \, dj \right)^{\frac{\varepsilon}{\varepsilon-1}} \tag{2.1.5}$$

where $j \in [0, 1]$ denotes a single good variety of the continuum of differentiated goods produced in the world economy and the parameter $\varepsilon > 1$ measures the elasticity of substitution between varieties produced within a given country. $C_{F,t}^{*i}$ is an index of imported goods for households in

[5] We choose to specify additively separable period utility of the type with Constant Relative Risk Aversion (CRRA), so with constant elasticity of intertemporal substitution and with constant elasticity of labour supply.

country F, given by the analogous CES function:

$$C_{F,t}^{*i} \equiv \left(\left(\frac{1}{h}\right)^{\frac{1}{\varepsilon}} \int_0^h C_{F,t}^{*i}(j)^{\frac{\varepsilon-1}{\varepsilon}} \, dj \right)^{\frac{\varepsilon}{\varepsilon-1}} \tag{2.1.6}$$

Households in country F maximize their present-value utility, equation 2.1.2, subject to the following sequence of budget constraints:

$$\int_h^1 P_{H,t}^*(j) C_{H,t}^{*i}(j) \, dj + \int_0^h P_{F,t}^*(j) C_{F,t}^{*i}(j) \, dj + D_t^{*i} + B_{F,t}^{*i} \\ \leq \frac{D_{t-1}^{*i}}{\mathcal{Q}_{t-1,t}^*} + B_{F,t-1}^{*i}(1 + i_{t-1}^*) + (1 - \tau_t^{*w}) W_t^* N_t^{*i} + T_t^{*i} + \Gamma_t^{*i} \tag{2.1.7}$$

for $t = 0, 1, 2, \dots,$ where $P_{H,t}^*(j)$ is the price of variety j in country F, $P_{F,t}^*(j)$ is the price of variety j imported from country H, D_{t-1}^{*i} is the portfolio of state-contingent claims purchased by the household in period $t-1$, $\mathcal{Q}_{t-1,t}^*$ is the stochastic discount factor for households in country F, which is different for households in the two countries and represents the price of state-contingent claims or equivalently, at the optimum, the inverse of the gross return on state-contingent claims, $B_{F,t}^{*i}$ are government bonds issued by country F and purchased by the household in country F in period t, i_{t-1}^* is the net return on government bonds issued by country F, W_t^* is the nominal wage for households in country F, T_t^{*i} denotes lump-sum transfers from the government to households, Γ_t^{*i} denotes the share of profits net of taxes to households from ownership of firms and $\tau_t^{*w} \in [0, 1]$ is a marginal tax rate on labour income paid by households to the government. As shown by the budget constraint, households in country F can only purchase government bonds issued by their own country and, differently from households in country H, they do not pay any transaction costs on the purchase of these government bonds.

Households in country H maximize their present-value utility subject to the following different sequence of budget constraints:

$$\int_0^h P_{H,t}(j) C_{H,t}^i(j) \, dj + \int_h^1 P_{F,t}(j) C_{F,t}^i(j) \, dj + D_t^i + B_{H,t}^i + B_{F,t}^i \\ \leq \frac{D_{t-1}^i}{\mathcal{Q}_{t-1,t}} + B_{H,t-1}^i(1 + i_{t-1}) + B_{F,t-1}^i(1 + i_{t-1}^*)(1 - \delta_{t-1}) + (1 - \tau_t^w) W_t N_t^i + T_t^i + \Gamma_t^i + \mathcal{I}_t^{*i} \tag{2.1.8}$$

for $t = 0, 1, 2, \dots,$ where $P_{H,t}(j)$ is the price of variety j in country H, $P_{F,t}(j)$ is the price of variety j imported from country F, D_{t-1}^i is the portfolio of state-contingent claims purchased by the household in period $t-1$, $\mathcal{Q}_{t-1,t}$ is the stochastic discount factor for households in country H, $B_{H,t}^i$ are government bonds issued by country H and purchased by the household in period t, i_{t-1} is the nominal interest rate set by the Central Bank in period $t-1$, which is also the net return on government bonds issued by country H, W_t is the nominal wage for households in country H,

T_t^i denotes lump-sum transfers from the government to households, Γ_t^i denotes the share of profits net of taxes to households from ownership of firms, $\mathcal{I}_t^i$ denotes the share of profits to households in country H from ownership of the financial intermediaries and $\tau_t^w \in [0,1]$ is a marginal tax rate on labour income paid by households to the government.

All variables are expressed in units of the union's currency. Last but not least, households in country F are subject to the following solvency constraint, for all t, that prevents them from engaging in Ponzi-schemes:

$$\lim_{T\to\infty} E_t \left\{ \mathcal{Q}^*_{t,T} D_T^{*i} \right\} \geq 0 \tag{2.1.9}$$

Aggregating the intratemporal optimality condition yields the aggregate labour supply equation for households in country F:

$$N_t^* = (1-h)^{1+\frac{\sigma}{\varphi}} (C_t^*)^{-\frac{\sigma}{\varphi}} \left[(1-\tau_t^{*w}) \frac{W_t^*}{P_t^*} \right]^{\frac{1}{\varphi}} \tag{2.1.10}$$

where N_t^* is aggregate labour supply and C_t^* is aggregate consumption for households in country F. Aggregating the intertemporal optimality condition for households in country F, taking conditional expectations on both sides of the equation and using the no-arbitrage condition between government bonds and state-contingent claims yields:

$$\frac{1}{1+i_t^*} = E_t\{\mathcal{Q}^*_{t,t+1}\} = \beta E_t \left\{ \frac{\xi^*_{t+1}}{\xi^*_t} \left(\frac{C^*_{t+1}}{C^*_t} \right)^{-\sigma} \frac{1}{\Pi^*_{t+1}} \right\} \tag{2.1.11}$$

where $\frac{1}{1+i_t^*} = E_t\{\mathcal{Q}^*_{t,t+1}\}$ is the price of a one-period government bond issued by country F paying off one unit of the union's currency in $t+1$ and $\Pi^*_{t+1} \equiv \frac{P^*_{t+1}}{P^*_t}$ is gross CPI inflation in country F. In country H the same equation is given by:

$$\frac{1}{(1+i_t^*)(1-\delta_t)} = \frac{1}{1+i_t} = E_t\{\mathcal{Q}_{t,t+1}\} = \beta E_t \left\{ \frac{\xi_{t+1}}{\xi_t} \left(\frac{C_{t+1}}{C_t} \right)^{-\sigma} \frac{1}{\Pi_{t+1}} \right\} \tag{2.1.12}$$

where $\frac{1}{1+i_t} = E_t\{\mathcal{Q}_{t,t+1}\}$ is the price of a one-period government bond issued by country H paying off one unit of the union's currency in $t+1$ and $\Pi_{t+1} \equiv \frac{P_{t+1}}{P_t}$ is gross CPI inflation in country H. The previous equation implies that the interest rate paid on government bonds issued by country F is increasing in the transaction cost δ_t or in the government bond spread $(1+i_t^*)\delta_t$.

The financial intermediaries, owned by the households in country H, earn profits on all the internationally traded bonds, $B^i_{F,t-1}$, by collecting savings from households in country H at the interest rate set by the Central Bank, i_{t-1}, and lending to the government in country F at the interest rate paid on its government bonds, i^*_{t-1}. The aggregate profits of these financial intermediaries are then given by:

$$\mathcal{I}_t \equiv \int_0^h \mathcal{I}_t^i \, di \equiv h\mathcal{I}_t^i \equiv B_{F,t-1} \left[(1+i^*_{t-1}) - (1+i_{t-1}) \right] = B_{F,t-1}(1+i^*_{t-1})\delta_{t-1} \tag{2.1.13}$$

where $B_{F,t-1} \equiv \int_0^h B^i_{F,t-1}\, di \equiv hB^i_{F,t-1}$ are aggregate bonds issued by the government in country F and purchased by households in country H and where the government bond spread for country F, on which financial intermediaries make profits, is given by $(1+i^*_{t-1})\delta_{t-1}$.

2.2 Firms

In country F there is a continuum of firms indexed by $j \in [h, 1]$, each producing a differentiated good with the same technology represented by the following production function:

$$Y^*_t(j) = A^*_t N^*_t(j) \tag{2.2.1}$$

where A^*_t represents the level of technology in country F, which evolves exogenously over time following the AR(1) process in logs:

$$A^*_t = (A^*_{t-1})^{\rho^*_a} e^{\varepsilon_{a,t}} \tag{2.2.2}$$

where $\rho^*_a \in [0, 1]$ is a measure of persistence of the shock and $\varepsilon_{a,t}$ is a zero mean white noise process.

From the production function we can derive labour demand for individual firms in country F and the nominal and real marginal costs of production, which are equal across firms in each country and are given by:

$$N^*_t(j) = \frac{Y^*_t(j)}{A^*_t} \implies MC^{*n}_t = \frac{W^*_t}{A^*_t} \implies MC^*_t = \frac{W^*_t}{A^*_t P^*_{H,t}} \tag{2.2.3}$$

Aggregating individual labour demand across firms in each country yields the aggregate labour demand for country F:

$$N^*_t \equiv \int_h^1 N^*_t(j)\, dj = \int_h^1 \frac{Y^*_t(j)}{A^*_t}\, dj = \frac{Y^*_t}{A^*_t} \int_h^1 \frac{1}{1-h} \left(\frac{P^*_{H,t}(j)}{P^*_{H,t}} \right)^{-\varepsilon} dj = \frac{Y^*_t}{A^*_t} d^*_t \tag{2.2.4}$$

where Y^*_t is aggregate output in country F, given by:

$$Y^*_t \equiv \left(\left(\frac{1}{1-h} \right)^{\frac{1}{\varepsilon}} \int_h^1 Y^*_t(j)^{\frac{\varepsilon-1}{\varepsilon}}\, dj \right)^{\frac{\varepsilon}{\varepsilon-1}} \tag{2.2.5}$$

and where the term:

$$d^*_t \equiv \int_h^1 \frac{1}{1-h} \left(\frac{P^*_{H,t}(j)}{P^*_{H,t}} \right)^{-\varepsilon} dj \tag{2.2.6}$$

represents relative price dispersion across firms in country F. In a symmetric flexible–price equilibrium this relative price dispersion is equal to one.

Aggregating over all $j \in [h, 1]$ firm j's period t profits net of taxes in country F, substituting in labour demand, marginal costs, the demand function for output, using the definition of $P^*_{H,t}$ and substituting in price dispersion yields aggregate profits net of taxes in country F:

$$\Gamma_t^* = (1-\tau_t^{*s})P_{H,t}^*Y_t^* - P_{H,t}^*MC_t^*Y_t^*d_t^* = P_{H,t}^*Y_t^*(1-\tau_t^{*s} - MC_t^*d_t^*) \tag{2.2.7}$$

where τ_t^{*s} is the marginal tax rate on firm sales in country F.

Following Calvo (1983), each firm in country F may reset its price with probability $1-\theta^*$ in any given period. Thus, each period a fraction $1-\theta^*$ of randomly selected firms reset their price, while a fraction θ^* keep their prices unchanged. As a result, the average duration of a price in country F is given by $(1-\theta^*)^{-1}$, and θ^* can be seen as a natural index of price stickiness for country F. In country H each firm may reset its price with probability $1-\theta$ in any given period. This allows for the two countries to have different degrees of price rigidity, consistently with the empirical evidence (see calibration in Section 3).

A firm in country F re-optimizing in period t will choose the price $\bar{P}_{H,t}^*$ that maximizes the current market value of the profits net of taxes generated while that price remains effective. Formally, it solves the problem:

$$\max_{\bar{P}_{H,t}^*} \sum_{k=0}^{\infty} \theta^{*k} E_t \left\{ \mathcal{Q}_{t,t+k}^* Y_{t+k|t}^*(j) \left[(1-\tau_{t+k}^{*s})\bar{P}_{H,t}^* - MC_{t+k}^{*n} \right] \right\} \tag{2.2.8}$$

subject to the sequence of demand constraints:

$$Y_{t+k|t}^*(j) = \left(\frac{\bar{P}_{H,t}^*}{P_{H,t+k}^*} \right)^{-\varepsilon} \frac{Y_{t+k}^*}{h} \tag{2.2.9}$$

for $k = 0,1,2,\ldots$, where $\mathcal{Q}_{t,t+k}^*$ is the households' stochastic discount factor in country F for discounting k-period ahead nominal payoffs from ownership of firms, defined by:

$$\mathcal{Q}_{t,t+k}^* = \beta^k \frac{\xi_{t+k}^*}{\xi_t^*} \left(\frac{C_{t+k}^*}{C_t^*} \right)^{-\sigma} \frac{P_t^*}{P_{t+k}^*} \tag{2.2.10}$$

for $k = 0,1,2,\ldots$, and where $Y_{t+k|t}^*(j)$ is the output in period $t+k$ for firm j which last reset its price in period t.

The optimal price chosen by firms in country F can then be expressed as a function of only aggregate variables:

$$\bar{P}_{H,t}^* = \frac{\varepsilon}{\varepsilon - 1} \frac{\sum_{k=0}^{\infty} (\beta\theta^*)^k E_t \left\{ \frac{\xi_{t+k}^* (C_{t+k}^*)^{-\sigma}}{P_{t+k}^*} \frac{Y_{t+k}^*}{(P_{H,t+k}^*)^{-\varepsilon}} MC_{t+k}^{*n} \right\}}{\sum_{k=0}^{\infty} (\beta\theta^*)^k E_t \left\{ \frac{\xi_{t+k}^* (C_{t+k}^*)^{-\sigma}}{P_{t+k}^*} \frac{Y_{t+k}^*}{(P_{H,t+k}^*)^{-\varepsilon}} (1-\tau_{t+k}^{*s}) \right\}} \tag{2.2.11}$$

2.3 Prices and International Assumptions

Several international identities and assumptions need to be spelled out in order to link the *Home* economy to the *Foreign* one and to be able to close the model. The Consumer Price Index (CPI) for country F is given by:

$$P_t^* \equiv \left[(1-\alpha^*)(P_{H,t}^*)^{1-\eta} + \alpha^*(P_{F,t}^*)^{1-\eta}\right]^{\frac{1}{1-\eta}} \tag{2.3.1}$$

while the Consumer Price Index (CPI) for country H is given by:

$$P_t \equiv \left[(1-\alpha)(P_{H,t})^{1-\eta} + \alpha(P_{F,t})^{1-\eta}\right]^{\frac{1}{1-\eta}} \tag{2.3.2}$$

where $P_{H,t}^*$ is the domestic price index or Producer Price Index (PPI) in country F and $P_{F,t}^*$ is a price index for goods imported from country H, respectively defined by:

$$P_{H,t}^* \equiv \left(\frac{1}{1-h}\int_h^1 P_{H,t}^*(j)^{1-\varepsilon}\,dj\right)^{\frac{1}{1-\varepsilon}} \tag{2.3.3}$$

$$P_{F,t}^* \equiv \left(\frac{1}{h}\int_0^h P_{F,t}^*(j)^{1-\varepsilon}\,dj\right)^{\frac{1}{1-\varepsilon}} \tag{2.3.4}$$

while $P_{H,t}$ is the domestic price index or Producer Price Index (PPI) in country H and $P_{F,t}$ is a price index for goods imported from country F, respectively defined by:

$$P_{H,t} \equiv \left(\frac{1}{h}\int_0^h P_{H,t}(j)^{1-\varepsilon}\,dj\right)^{\frac{1}{1-\varepsilon}} \tag{2.3.5}$$

$$P_{F,t} \equiv \left(\frac{1}{1-h}\int_h^1 P_{F,t}(j)^{1-\varepsilon}\,dj\right)^{\frac{1}{1-\varepsilon}} \tag{2.3.6}$$

Although deviations from *Purchasing Power Parity* (PPP) may arise because of home bias in consumption, we assume that the *Law of One Price* (LOP) holds for every single good j, which implies:

$$P_{H,t}(j) = P_{F,t}^*(j) \qquad \text{and} \qquad P_{F,t}(j) = P_{H,t}^*(j) \tag{2.3.7}$$

for all $j \in [0,1]$, where $P_{H,t}(j)$ (or $P_{F,t}^*(j)$ for goods imported from country F) is the price of good j in country H and $P_{F,t}(j)$ (or $P_{H,t}^*(j)$ for goods produced in country F) is the price of good j in country F in terms of the union's currency. Plugging the previous expressions into the definitions of $P_{H,t}$ and $P_{F,t}$, equations 2.3.5 and 2.3.6, respectively yields:

$$P_{H,t} = P_{F,t}^* \qquad \text{and} \qquad P_{F,t} = P_{H,t}^* \tag{2.3.8}$$

The *terms of trade* are defined as the price of foreign goods in terms of home goods, for households in country H and in country F, and are given respectively by:

$$\mathcal{S}_t \equiv \frac{P_{F,t}}{P_{H,t}} \qquad \text{and} \qquad \mathcal{S}_t^* \equiv \frac{P_{F,t}^*}{P_{H,t}^*} \tag{2.3.9}$$

Combining the previous result with the definition of the terms of trade for countries H and F yields:

$$\mathcal{S}_t = \frac{P_{F,t}}{P_{H,t}} = \frac{P^*_{H,t}}{P^*_{F,t}} = \frac{1}{\mathcal{S}^*_t} \tag{2.3.10}$$

The relationship between PPI inflation and CPI inflation in country F is given by:

$$\Pi^*_t = \Pi^*_{H,t} \left[\frac{1-\alpha^* + \alpha^*(\mathcal{S}^*_t)^{1-\eta}}{1-\alpha^* + \alpha^*(\mathcal{S}^*_{t-1})^{1-\eta}}\right]^{\frac{1}{1-\eta}} \tag{2.3.11}$$

while dividing the terms of trade in period t by the terms of trade in period $t-1$ yields a relationship showing the evolution of the terms of trade over time:

$$\frac{\mathcal{S}^*_t}{\mathcal{S}^*_{t-1}} = \frac{\Pi^*_{F,t}}{\Pi^*_{H,t}} = \frac{\Pi_{H,t}}{\Pi^*_{H,t}} \implies \mathcal{S}^*_t = \frac{\Pi_{H,t}}{\Pi^*_{H,t}}\mathcal{S}^*_{t-1} \tag{2.3.12}$$

as a function of PPI inflation in both countries H and F.

The *Real Exchange Rate* between the *Home* country and country F is the ratio of the two countries' CPIs, expressed both in terms of the union's currency, and is defined by:

$$Q_t \equiv \frac{P^*_t}{P_t} = \mathcal{S}_t \left[\frac{1-\alpha^* + \alpha^*(\mathcal{S}^*_t)^{1-\eta}}{1-\alpha + \alpha(\mathcal{S}_t)^{1-\eta}}\right]^{\frac{1}{1-\eta}} = \frac{1}{Q^*_t} \tag{2.3.13}$$

where the difference between the real exchange rate and the terms of trade is given by the degree of openness of the two countries and the international trade elasticity. If the countries both have complete home bias ($\alpha = \alpha^* = 0$), then they are in autarky and the real exchange rate is exactly equal to the terms of trade, because the CPI and PPI are the same in each country. Based on our calibration ($\alpha = 0.52 < 1 - h = 0.6$), the real exchange rate increases when the terms of trade increase, meaning they both move in the same direction, so we can just track the movements in the terms of trade, without having to keep track of the movements in the real exchange rate.

Net Exports are defined as domestic production minus domestic consumption, which is equal to exports minus imports, and are given in real terms (divided by $P^*_{H,t}$ for country F and by $P_{H,t}$ for country H, denoted with a tilde) for country F by:

$$\widetilde{NX}^*_t = Y^*_t - \frac{P^*_t}{P^*_{H,t}}C^*_t - G^*_t = Y^*_t - \left[1-\alpha^* + \alpha^*(\mathcal{S}^*_t)^{1-\eta}\right]^{\frac{1}{1-\eta}} C^*_t - G^*_t \tag{2.3.14}$$

where net exports are shown to be a function of the country's degree of openness and the terms of trade, other than domestic production and public and private domestic consumption.

Net Foreign Assets are given by the sum of private and public assets held abroad and are given in real terms by:

$$\widetilde{NFA}_t \equiv \tilde{B}_{F,t} \qquad \text{and} \qquad \widetilde{NFA}^*_t \equiv \tilde{B}^*_{F,t} - \tilde{B}^{*G}_t \tag{2.3.15}$$

Since government bonds issued by country H are not traded internationally, while government bonds issued by country F are traded internationally, the market clearing conditions for these bonds are given by:

$$B_{H,t} - B_t^G = 0 \qquad B_{F,t} + B_{F,t}^* - B_t^{*G} = 0 \tag{2.3.16}$$

From the households' budget constraint, substituting in firm profits and labour income, the expression for transfers backed out from the government budget constraint, the market clearing condition for government bonds issued by country H, the definitions of net exports and net foreign assets, yields a relationship between net foreign assets, net exports and the balance of payments for country F, which in real terms can be rewritten as:

$$\widetilde{NFA}_t^* = (1 + i_{t-1}^*) \frac{\widetilde{NFA}_{t-1}^*}{\Pi_{H,t}^*} + \widetilde{NX}_t^* \tag{2.3.17}$$

2.4 Central Bank and Monetary Policy

The unique Central Bank in the Currency Union sets monetary policy by choosing the nominal interest rate to target *union-wide* CPI inflation through a Taylor rule. We assume that the Central Bank cares only about inflation, reflecting the fact that price stability is the primary objective of the ECB.

Monetary policy follows an Inflation Targeting Regime of the kind:

$$\beta(1 + i_t) = \left(\frac{\Pi_t^U}{\Pi^U}\right)^{\phi_\pi(1-\rho_i)} [\beta(1 + i_{t-1})]^{\rho_i} \tag{2.4.1}$$

where union-wide inflation is defined as the population-weighted geometric average of the CPI inflation in the two countries:

$$\Pi_t^U \equiv (\Pi_t)^h (\Pi_t^*)^{1-h} \tag{2.4.2}$$

while variables without subscripts t denote their respective steady state levels, ϕ_π represents the responsiveness of the interest rate to inflation and ρ_i is a measure of the persistence of the interest rate over time (interest rate smoothing).

In the alternative scenario, the Central Bank has a Zero Lower Bound (ZLB) constraint on its policy rate, the nominal interest rate: $i_t \geq 0$, which makes the rule become:

$$i_t = \max\{\tilde{i}_t, 0\} \tag{2.4.3}$$

$$\beta(1 + \tilde{i}_t) = \left(\frac{\Pi_t^U}{\Pi^U}\right)^{\phi_\pi(1-\rho_i)} \left[\beta(1 + \tilde{i}_{t-1})\right]^{\rho_i}$$

2.5 Government and Fiscal Policy in a Pure Currency Union

National governments choose the level of government consumption and transfers, which are financed by distortionary taxes on labour income and firm sales and by short-term government bonds. In a Pure Currency Union (uncoordinated fiscal policy) each government chooses the amount of government consumption for domestic stabilization purposes, by setting it to target the output gap, while using government transfers to deleverage government debt (in the transfer scenario in country F) and using a mix of distortionary tax rates on labour income and firm sales to finance remaining government expenditure or to finance also deleveraging (in the distortionary tax scenario in country F). In a Pure Currency Union country H balances its budget and country F deleverages its government debt, managing fiscal policy independently without cooperating, because each country only cares about stabilizing its own domestic demand. The output gap is defined as the log deviation of output from its symmetric flexible–price equilibrium, which is a distorted steady state because the tax on firm sales does not offset distortions from monopolistic competition. Governments aim to stabilize the economy by reducing the output gap, using government consumption to absorb excess domestic supply, while using transfers or a mix of taxes to reduce government debt (in country F) or to balance the budget (in country H).

In country F the government finances a stream of public consumption G_t^* and transfers T_t^* subject to the following sequence of budget constraints:

$$\int_h^1 P_{H,t}^*(j) G_t^*(j)\, dj + \int_h^1 T_t^{*i}\, di + B_{t-1}^{*G}(1 + i_{t-1}^*) = B_t^{*G} + \tau_t^{*s} P_{H,t}^* Y_t^* + \int_h^1 \tau_t^{*w} W_t^* N_t^{*i}\, di \quad (2.5.1)$$

where the right hand side represents government income from taxation and newly issued government bonds, while the left hand side represents total government spending on consumption and transfers, and on government bonds due at the end of period t, including interest. B_t^{*G} are government bonds issued by country F in period t, while all other variables are as explained above. Government consumption, G_t^*, is given by the following CES function, where we assume that the government purchases only goods produced domestically (complete home bias):

$$G_t^* \equiv \left(\left(\frac{1}{1-h} \right)^{\frac{1}{\varepsilon}} \int_h^1 G_t^*(j)^{\frac{\varepsilon-1}{\varepsilon}}\, dj \right)^{\frac{\varepsilon}{\varepsilon-1}} \quad (2.5.2)$$

In the transfer scenario, fiscal policy in country F chooses government consumption to stabilize the output gap countercyclically, through the fiscal rule:

$$\frac{G_t^*}{G^*} = \left(\frac{Y_t^*}{Y^*} \right)^{-\psi_y^*(1-\rho_g^*)} \left(\frac{G_{t-1}^*}{G^*} \right)^{\rho_g^*} \quad (2.5.3)$$

while using real transfers $\tilde{T}_t^*$ to deleverage its government debt, through the debt rule:

$$\frac{\tilde{B}_{t-1}^{*G}}{\Pi_{H,t}^*} - \tilde{B}_t^{*G} = \gamma^* \left(\frac{\tilde{B}_{t-1}^{*G}}{\Pi_{H,t}^*} - \tilde{B}^{*G} \right) \quad (2.5.4)$$

and varying equally the tax rates on labour income and firm sales to finance the remaining government expenditure, through the tax rules[6]:

$$\tau_t^{*w} - \tau^{*w} = \tau_t^{*s} - \tau^{*s} \qquad (\tau_t^{*s} + \tau_t^{*w} MC_t^* d_t^*) Y_t^* - (\tau^{*s} + \tau^{*w} MC^*) Y^* = G_t^* - G^* \tag{2.5.5}$$

where $\rho_g^* \in [0,1]$ is a measure of persistence of the government consumption process. Variables without subscripts t represent their respective steady state level, while $\psi_y^* \geq 0$ represents the responsiveness of government consumption to variations of the output gap and $\gamma^* \in [0,1]$ is the desired share of reduction per period of the excess real government debt with respect to steady state.

In the distortionary tax scenario, fiscal policy in country F chooses government consumption to stabilize the output gap countercyclically, as in the transfer scenario, but this time keeping real transfers constant and varying equally the tax rates on labour income and firm sales to deleverage its government debt and to finance all government expenditure, through the tax rule:

$$\tau_t^{*w} - \tau^{*w} = \tau_t^{*s} - \tau^{*s} \tag{2.5.6}$$

In country H the government similarly finances a stream of public consumption G_t and transfers T_t subject to the following sequence of budget constraints:

$$\int_0^h P_{H,t}(j) G_t(j)\, dj + \int_0^h T_t^i\, di + B_{t-1}^G (1 + i_{t-1}) = B_t^G + \tau_t^s P_{H,t} Y_t + \int_0^h \tau_t^w W_t N_t^i\, di \tag{2.5.7}$$

In the transfer scenario, fiscal policy in country H chooses government consumption to stabilize the output gap countercyclically, through the fiscal rule:

$$\frac{G_t}{G} = \left(\frac{Y_t}{Y}\right)^{-\psi_y(1-\rho_g)} \left(\frac{G_{t-1}}{G}\right)^{\rho_g} \tag{2.5.8}$$

while keeping real transfers constant, as the government does not need to deleverage its government debt, and maintaining a balanced budget, through the debt rules:

$$\tilde{B}_t^G = \frac{\tilde{B}_{t-1}^G}{\Pi_{H,t}} \qquad \tilde{T}_t = \tilde{T} \tag{2.5.9}$$

while varying equally the tax rates on labour income and firm sales to finance all government expenditure, through the tax rule:

$$\tau_t^w - \tau^w = \tau_t^s - \tau^s \tag{2.5.10}$$

In the distortionary tax scenario, fiscal policy in country H chooses government consumption to stabilize the output gap countercyclically, while keeping real transfers constant, maintaining a

[6] Cole, Guerello and Traficante (2020) shows that the amplification of shocks is increased by financing government expenditure with a higher share of taxes on labour income compared to taxes on firm sales, because taxes on labor income are much more distortionary than taxes on firm sales, but in our simulations here government expenditure is financed equally by both taxes, as the tax mix does not affect qualitatively the dynamics.

balanced budget and varying equally the tax rates on labour income and firm sales to finance all government expenditure, just as in the transfer scenario, since the government does not need to deleverage.

2.6 Government and Fiscal Policy in a Coordinated Currency Union

If the Governments of the two countries choose to coordinate, they will use their fiscal instruments to target a common objective, while maintaining independent budget constraints. In a Coordinated Currency Union, instead of using government consumption to stabilize the domestic output gap countercyiclically, they will use the same fiscal instrument to stabilize the net exports gap procyclically, while using government transfers to deleverage government debt (in the trasfer scenario in country F) and using a mix of distortionary tax rates on labour income and firm sales to finance the remaining government expenditure or to finance also deleveraging (in the distortionary tax scenario in country F). This represents the act of coordinating policies on a common objective, which depends on the interactions between the two economies, while the budget constraints of the two fiscal authorities remain unmodified. In a Coordinated Currency Union country H balances its budget and country F deleverages its government debt, still managing fiscal policy independently, but coordinating by stabilizing their trade flows. The net exports gap is defined as the log deviation of net exports from their symmetric flexible–price equilibrium, which is a distorted steady state. Governments aim to stabilize the economy by reducing the net exports gap instead of the output gap, using government consumption to absorb excess international supply, while using a mix of taxes to reduce government debt (in country F) or to balance the budget (in country H).

We choose the net exports gap as a common objective to coordinate on because one of the main concerns emerging in the Euro Area in the past years is the deep asymmetry between Core countries, such as Germany, running trade surpluses and the Rest of the Euro Area running trade deficits. In particular, these imbalances in the Euro Area have grown considerably, creating concerns about economic growth and stability with a common currency[7].

In the transfer scenario, fiscal policy in country F chooses government consumption to stabilize its real net exports gap procyclically, through the fiscal rule:

$$\frac{G_t^*}{G^*} = \left(\frac{\widetilde{NX}_t^*}{\widetilde{NX}^*} \right)^{\psi_{nx}^*(1-\rho_g^*)} \left(\frac{G_{t-1}^*}{G^*} \right)^{\rho_g^*} \tag{2.6.1}$$

while using real transfers $\tilde{T}_t^*$ to deleverage its government debt and varying equally the tax rates on labour income and firm sales to finance the remaining government expenditure, just as in a Pure Curreny Union. In the same transfer scenario, fiscal policy in country H chooses government

[7]For references on current account imbalances in the Euro Area see Kollmann et al. (2014) and Schmitz and Von Hagen (2011), while we follow Hjortsø (2016) in our idea to coordinate fiscal policies by reducing international demand imbalances.

consumption to stabilize its real net exports gap procyclically, through the fiscal rule:

$$\frac{G_t}{G} = \left(\frac{\widetilde{NX}_t}{\widetilde{NX}}\right)^{\psi_{nx}(1-\rho_g)} \left(\frac{G_{t-1}}{G}\right)^{\rho_g} \tag{2.6.2}$$

while keeping real transfers $\tilde{T}_t$ constant and maintaining a balanced budget by varying equally the tax rates on labour income and firm sales to finance the remaining government expenditure, just as in a Pure Curreny Union, where $\psi_{nx} \geq 0$ for country H and $\psi^*_{nx} \geq 0$ for country F represent the responsiveness of government consumption to variations of the real net exports gap.

In the distortionary tax scenario, fiscal policy in country F chooses government consumption to stabilize its real net exports gap procyclically, as in the transfer scenario (see equation 2.6.1), but this time keeping real transfers constant and varying equally the tax rates on labour income and firm sales to deleverage its government debt and finance all government expenditure, just as in a Pure Currency Union. In the same distortionary tax scenario, fiscal policy in country H chooses government consumption to stabilize its real net exports gap procyclically, as in the transfer scenario (see equation 2.6.2), while keeping real transfers constant, maintaining a balanced budget and varying equally the tax rates on labour income and firm sales to finance all government expenditure, just as in a Pure Currency Union.

2.7 Government and Fiscal Policy in a Full Fiscal Union

If instead of considering two fiscal authorities managing fiscal policy independently, one for each country, or coordinating their policies, but with two separate budget constraints, we consider only one fiscal authority managing fiscal policy for both countries at the same time in a coordinated manner and with a consolidated budget constraint, then we can think of it as an extreme case of fiscal policy coordination. A Full Fiscal Union uses a consolidated budget constraint to finance local government consumption for international stabilization purposes, by setting it to target the net exports gap, as in a Coordinated Currency Union, while using government transfers in both countries to deleverage the government debt of country F (in the transfer scenario) and using a mix of distortionary tax rates on labour income and firm sales to finance the remaining government expenditure (in the transfer scenario) or to finance also deleveraging (in the distortionary tax scenario), but varying symmetrically the fiscal instruments across countries, so as to use union-wide resources to finance the government expenditure in both countries and the deleveraging in country F. In this case government debt will be aggregated across countries and both governments will contribute to deleveraging. Nonetheless, given that financial markets are still incomplete, there continue to be two separate government bonds for the two countries, which pay different interest rates and so have different bond yields[8]. In a Full Fiscal Union country H and country F do not manage fiscal policy independently anymore and, while coordinating by stabilizing their trade

[8] Cole, Guerello and Traficante (2020) shows the effects of different degrees of fiscal policy coordination with complete financial markets.

flows, they also harmonize the movements of their fiscal instruments to finance both countries' expenditures and to reduce the government debt of country F, as if there were only one country. In this case country H pays partially for the deleveraging of country F, as in a transfer union, where transfers from one government to the other are conditional on government debt and the business cycle.

A Full Fiscal Union uses local government spending to manage fiscal policy at the union level with a consolidated budget constraint, by financing streams of local public consumption, G_t and G_t^*, subject to the consolidated budget constraint of the two national fiscal authorities that can be expressed in real terms as:

$$G_t + \tilde{T}_t + S_t(G_t^* + \tilde{T}_t^*) + i_{t-1}\frac{\tilde{B}_{t-1}^G}{\Pi_{H,t}} + \frac{i_{t-1}+\delta_{t-1}}{1-\delta_{t-1}}\frac{S_{t-1}\tilde{B}_{t-1}^{*G}}{\Pi_{H,t}^*} =$$
$$(\tau_t^s + \tau_t^w MC_t d_t)Y_t + (\tau_t^{*s} + \tau_t^{*w} MC_t^* d_t^*)S_t Y_t^* + \tilde{B}_t^G - \frac{\tilde{B}_{t-1}^G}{\Pi_{H,t}} + S_t\tilde{B}_t^{*G} - \frac{S_{t-1}\tilde{B}_{t-1}^{*G}}{\Pi_{H,t}^*} \quad (2.7.1)$$

where variables with a tilde are in real terms (divided by $P_{H,t}$ for country H and by $P_{H,t}^*$ for country F), and where the left hand side represents current government expenditure and interest payments on outstanding debt, while the right hand side represents government financing of that expenditure through taxes and the possible variation of overall government debt.

We assume that union-wide fiscal policy chooses government consumption in each country to stabilize its real net exports gap procyclically, as in a Coordinated Currency Union, through the fiscal rules 2.6.1 and 2.6.2. In the transfer scenario, real transfers are used symmetrically in both countries to deleverage the government debt of country F, while maintaining the government debt of country H constant, through the debt rules:

$$\frac{\tilde{B}_{t-1}^{*G}}{\Pi_{H,t}^*} - \tilde{B}_t^{*G} = \gamma^*\left(\frac{\tilde{B}_{t-1}^{*G}}{\Pi_{H,t}^*} - \tilde{B}^{*G}\right) \qquad \tilde{B}_t^G = \frac{\tilde{B}_{t-1}^G}{\Pi_{H,t}} \quad (2.7.2)$$

$$\tilde{T}_t - \tilde{T} = \tilde{T}_t^* - \tilde{T}^* \quad (2.7.3)$$

while varying symmetrically also across countries the tax rates on labour income and firm sales to finance the remaining government expenditure, through the tax rules:

$$\tau_t^w - \tau^w = \tau_t^s - \tau^s \qquad \tau_t^{*w} - \tau^{*w} = \tau_t^w - \tau^w \qquad \tau_t^{*s} - \tau^{*s} = \tau_t^s - \tau^s \quad (2.7.4)$$

$$(\tau_t^s + \tau_t^w MC_t d_t)Y_t + (\tau_t^{*s} + \tau_t^{*w} MC_t^* d_t^*)S_t Y_t^* - (\tau^s + \tau^w MC)Y - (\tau^{*s} + \tau^{*w} MC^*)Y^* = G_t + G_t^* - G - G^* \quad (2.7.5)$$

In the distortionary tax scenario, union-wide fiscal policy chooses government consumption in each country to stabilize its real net exports gap procyclically, as in the transfer scenario, through the fiscal rules 2.6.1 and 2.6.2, but this time keeping real transfers constant, while maintaining the government debt of country H constant and reducing the government debt of country F through the debt rules 2.7.2. In this way, the tax rates on labour income and firm sales vary simmetrically also across countries, to finance all government expenditure including deleveraging, through the tax rules 2.7.4.

3 Calibration

The model is calibrated[9] mainly following Ferrero (2009). We consider the top 4 Euro Area countries, which account for about 80% of Euro Area GDP and we divide them into the Rest of the Euro Area (namely, France, Italy and Spain), country F, and the Core (namely Germany), country H. The size of country H is set according to the relative GDP size to $h = 0.4$, as Germany accounts for about 30% of Euro Area GDP. Most of the parameters governing the economies of the two countries are set symmetrically, with the exception of the degree of price rigidity, which has been set such that in country H the average duration of a price is 4 quarters while in country F it is 5 quarters. The gross markup $\frac{\varepsilon}{\varepsilon-1}$ has been set to 1.1, which implies a net markup of 10%, and the discount factor has been chosen to match a compounded annual interest rate of 2%.

The parameters for monetary policy follow the optimal operational rule criteria[10], as shown in Schmitt-Grohé and Uribe (2004)[11], so we set the response of the interest rate to inflation to $\phi_\pi = 1.5$, and the interest rate smoothing parameter to $\rho_i = 0.8$. These theoretical values are supported by empirical evidence for the Euro Area by, among others, Adjemian, Darracq Pariés and Moyen (2007), who estimated the coefficients of several optimal operational rules between 0.6 and 0.9 for the interest rate smoothing parameter and between 1.5 and 2 for the inflation response parameter.

We estimate the sensitivity of the transaction cost δ_t, or of the government bond spread, to deviations of government debt-to-GDP from steady state and find that for every ten percentage points increase in government debt-to-GDP the government bond spread increases by 9 basis points, according to which we set $\delta^B = 0.009$[12]. The transaction cost δ_t responds to the debt-to-GDP ratio in deviation from Maastricht Treaty's objective of 60%. Specifically, we assume an AR(1) process for the transaction cost, measured by the spread between the long run interest rates on government bonds for France, Italy and Spain and the same yield for Germany[13]. We regress the time series of the spreads of the 4 countries in a Panel VAR(1), including also the government debt-to-GDP ratio, as a difference from the Maastricht Treaty's objective of 60%. In order to account for the effects of the European sovereign debt crisis, the data on the spread has been demeaned and country fixed

[9]The calibration is done on the steady state values described in Appendix A.2.

[10]An interest-rate rule is operational when it satisfies three requirements. First, the nominal interest rate is set as a function of a few readily observable macroeconomic variables. Second, the operational rule must induce an equilibrium satisfying the zero lower bound on nominal interest rates. Third, an operational interest-rate rule must induce a unique rational expectations equilibrium.

[11]While the original version of the Taylor principle includes also a response to the output gap, Schmitt-Grohé and Uribe (2004) find that a standard Taylor rule with an inflation coefficient of 1.5 and an output coefficient of 0.5 fails to be operational.

[12]This result is in line with Hjortsø (2016), which finds a similar sensitivity to be 0.01 instead of 0.009.

[13]The data is collected by Eurostat and reported in the ECB statistical data warehouse. It is harmonized to assess the convergence of Member Countries. The sample spans from 2002 to 2015 at quarterly frequency.

effects have been introduced in the model to account for the initial conditions (i.e. few countries show historically high levels of government debt). The estimated pooled elasticity of the spread to the debt-GDP ratio in deviation from the 60% target is used to calibrate the sensitivity of the transaction cost described above.

Table 1 collects all calibrated parameters and steady state stances.

Table 1: Calibrated Parameters and Steady State Stances.

Parameters	Description	Country H	Country F
h	Relative size of domestic economy	0.4	0.6
β	Discount factor	0.995	0.995
ε	Elasticity of substitution of domestic goods	11	11
$\frac{\varepsilon}{\varepsilon-1}$	Gross Price Mark-Up	1.1	1.1
η	Elasticity of international substitution	4.5	4.5
σ	Inverse Elasticity of intertemporal substitution	3	3
φ	Inverse Frisch Elasticity of labour supply	2	2
θ	Degree of price rigidity	3/4	4/5
δ^B	Sensitivity of bond spread to debt-to-GDP	0.009	0.009
γ	Desired reduction of excess gov. debt-to-GDP	0	0.05
α	Openness of domestic economy	0.52	0.361
$\frac{\alpha}{h}$	Relative openness of domestic economy	1.3	0.6017
$\frac{1-\alpha}{h}$	Home bias	1.2	1.065
ψ_y	Responsiveness of fiscal pol. to output gap	0.067	0.061
ψ_{nx}	Responsiveness of fiscal pol. to net exports gap	0.043	0.014
ϕ_π	Responsiveness of monetary pol. to inflation	1.5	1.5
ρ_i	Interest Rate smoothing parameter	0.8	0.8
ρ_ξ	Persistence of preference shock	0.94	0.8
ρ_a	Persistence of technology shock	0.58	0.70
σ_ξ	Standard deviation preference shock	0.0024	0.0086
σ_a	Standard deviation technology shock	0.0087	0.0033
$corr_\xi$	Correlation preference shock	0.625	0.625
$corr_a$	Correlation technology shock	0.418	0.418
Steady State Ratios	**Description**	**Country H**	**Country F**
$(1+i)^4-1$	Annualized Interest Rate	2%	2%
τ^w	Tax Rate on wage income	40.6%	27.9%
τ^s	Tax Rate on firm sales	2.5%	19.5%
$\tau^w MC + \tau^s$	Tax Revenues-to-GDP	38.49%	39.92%
$\frac{G}{Y}$	Government consumption-to-GDP	18.7%	21.9%
$\frac{\tilde{T}}{Y}$	Real Transfers-to-GDP	18.58%	16.81%
$\frac{\tilde{NX}}{Y}$	Net Exports-to-GDP	1.73%	-1.14%
$\frac{C}{Y}$	Consumption-to-GDP	79.58%	79.24%
$\alpha^* \frac{C^*}{Y}$	Exports-to-GDP	43.1%	27.47%

The calibration of the two countries mainly differs in the fiscal policy parameters, calibrated to values observed before the Covid pandemic. In particular, the government consumption-to-GDP ratios have been set respectively to 18.7% for Germany and 21.9% for the Rest of the Euro Area, according to the pre-pandemic average from 2008 to 2016 (source ECB-SDW). The marginal tax rates on labour income have been set respectively to 40.61% for Germany and 27.94% for the Rest of the Euro Area in accordance to the average from 2008 to 2016 of the labour income tax wedges, excluding social security contributions made by the employer, for the median individual, as reported in OECD (2015). The marginal tax rate on firm sales has been set to 19.5% for the Rest of the Euro Area according to the average from 2008 to 2016 of the VAT rate for France, Italy and Spain as reported in Eurostat, European-Commission et al. (2015), while it has been calibrated for Germany to match the average ratio of net exports-to-GDP of 1.73% observed from 2008 to 2016[14]. Since we do not assume a positive long–run growth in productivity for Germany, we replicate the production slack in the Rest of the Euro Area (i.e. a substantially lower competitiveness) by setting the marginal tax rate on firm sales to 2.5%, as if there were a production incentive, although the observed VAT rate for Germany is 19%[15]. This calibration implies a steady state tax revenue-to-GDP ratio of respectively 38.49% for Germany and 39.92% for the Rest of the Euro Area, clearly in line with the data observed over the past decades for Germany (38.72%) and for France, Italy and Spain (39.15%).

Finally, the annualized steady state value of government debt-to-GDP in both countries is set to roughly 60% as stated in the Maastricht Treaty. In the simulations, the Rest of the Euro Area (country F) starts with a higher level of government debt-to-GDP, equal to roughly 80%, in line with the average pre-pandemic level of government debt-to-GDP for France, Italy and Spain. The desired fraction of reduction of excess government debt for the Rest of the Euro Area is set to $\gamma^* = 0.05$, corresponding to a 5% yearly reduction of excess government debt[16], to comply with the Debt Brake Rule in the Stability and Growth Pact and the Fiscal Compact.

The transfers-to-GDP ratios have been set such that the government deficit is zero in steady state. Henceforth, the overall calibration of the fiscal sector implies a steady state ratio of transfers-to-GDP of respectively 18.58% for country H and 16.81% for country F, and a steady state ratio of current expenditure-to-GDP of respectively 37.28% for country H and 38.71% for country F. This calibration is broadly in line with the observed data from 2008 to 2016 for the subsidies-to-GDP ratio (26.85% for Germany and 24.69% for the Rest of the Euro Area) and the current expenditure (less interest)-to-GDP ratio (35.54% for Germany and 36.85% for the Rest of the Euro Area).

The calibration of the fiscal policy parameters, ψ_y and ψ_{nx}, comes from the welfare optimization carried out in Cole, Guerello and Traficante (2020) under complete financial markets in a model with complete markets and no deleveraging shocks[17]. We can interpret these fiscal parameters as the best choice for the government in the absence of financial frictions.

The parameters of openness have been set to match an export-to-GDP ratio $\left(\frac{\alpha^* C^*}{Y}\right)$ of roughly

[14] The average current account to GDP ratio observed from 2008 to 2016 for Germany is roughly 6.36%. However, we adjust the data for the overall trade weight with France, Italy and Spain (26%).

[15] The large difference in tax rates across countries and across types of tax revenues is not relevant for the design of fiscal rules since we consider all tax rates as percentage point differences from steady state.

[16] This corresponds to a similar reduction in government debt-to-steady state GDP.

[17] The fiscal policy parameters have been selected by maximizing the unconditional expectation of lifetime utility of the households of both countries, as explained in section 5 of Cole, Guerello and Traficante (2020).

43% for Germany[18] taken from the aggregate demand equation, while for the Rest of the Euro Area the parameter of openness is recovered by equating per-capita consumption across countries, which yields the following equation:

$$\alpha^* = \frac{h}{1-h}\left[\alpha + \frac{\left(\frac{1-\frac{G}{Y}}{1-\frac{G^*}{Y^*}}\right)\left(\frac{(1-\tau^w)(1-\tau^s)}{(1-\tau^{*w})(1-\tau^{*s})}\right)^{\frac{1}{\varphi}} - 1}{1+\frac{h}{1-h}\left(\frac{1-\frac{G}{Y}}{1-\frac{G^*}{Y^*}}\right)\left(\frac{(1-\tau^w)(1-\tau^s)}{(1-\tau^{*w})(1-\tau^{*s})}\right)^{\frac{1}{\varphi}}}\right] \tag{3.0.1}$$

Consequently, home biases are given by $\frac{1-\alpha}{h} = 1.2$ and $\frac{1-\alpha^*}{1-h} = 1.065$. Since both home biases are larger than one it means that the share of consumption of domestic goods is higher than the share of production of domestic goods. Based on our calibration, the real exchange rate increases as the terms of trade increase because the degree of openness of country H ($\alpha = 0.52$) is less than the size of country F ($1-h = 0.6$)[19].

The international trade elasticity or the elasticity of substitution between foreign and domestic goods is set to 4.5 for both countries, but we run a sensitivity analysis in Appendix B, where we show that if the international trade elasticity is lower, consumption moves more, but affects less movements in net exports, because they respond less to movements in the terms of trade. Nonetheless the differences are quite small so all our results still hold.

Regarding the dynamic parametrization of the shocks, both technology and preference shocks are assumed to follow a VAR(1) process that generally allows for both direct spillovers and second order correlation of the innovations. However, the structure has been restricted for all shocks to exclude direct spillovers. With the exception of the preference shocks, whose dynamics have been calibrated following Kollmann et al. (2014), the parameters characterizing the dynamics of the technology shocks have been estimated. For the estimation we employed the time series for Germany, France, Italy and Spain of labour productivity per hours worked for the technology shocks. All the series are chain-linked volumes re-based in 2010, seasonally adjusted and filtered by means of a Hodrick-Prescott filter. The sample considered spans at quarterly frequency from 2002 Q1 to 2015 Q3. The correlation between preference shocks is set in order to maximize the simulated correlation between output in the two countries, as in Cole, Guerello and Traficante (2020).

4 Numerical Simulations

We simulate the model numerically using Dynare[20] (Adjemian et al., 2011), which takes a first-order approximation of the model around its non–stochastic symmetric flexible–price steady state equilibrium with zero inflation and constant government debt at 60% of GDP for country H and at 80% of GDP for country F. We show the impulse response functions of the main variables after a deleveraging shock which forces country F to reduce persistently its debt–to–GDP ratio from 80%

[18] The value recovered from the data as the average from 2008 to 2016 is 43.5%.

[19] See Cole, Guerello and Traficante (2020) for details.

[20] All the equilibrium conditions of the model used for the simulations are shown in Appendix A.1.

to the Maastricht Treaty requirement of 60%[21]. Specifically, we study the stabilization properties of a range of fiscal policy specifications: different fiscal policy instruments for deleveraging and different scenarios for fiscal policy coordination. Moreover, to align our simulations to the current reality, we assess the effects of deleveraging while the Central Bank is pushed into a liquidity trap (due to the policy rate hitting the Zero Lower Bound) by a negative international demand shock. We also conduct some sensitivity analyses to key parameters which are reported in Appendix B. These exercises show that our results are robust to different calibrations, even if they entail a different magnitude for international spillovers.

4.1 The Transmission of a Deleveraging Shock

In this Section we analyze the transmission of a stochastic shock to the government debt target in country F, which brings it persistently from 80% to 60% of GDP. We assume that the deleveraging scheme works as follows: Country F has to reduce its government debt by 5% of the excess every year until it reaches 60%, as stated in the Maastricht Treaty. This implies that the fiscal authority after the shock must adjust taxes and, eventually, government transfers to balance the budget and to reduce the debt-to-GDP ratio.

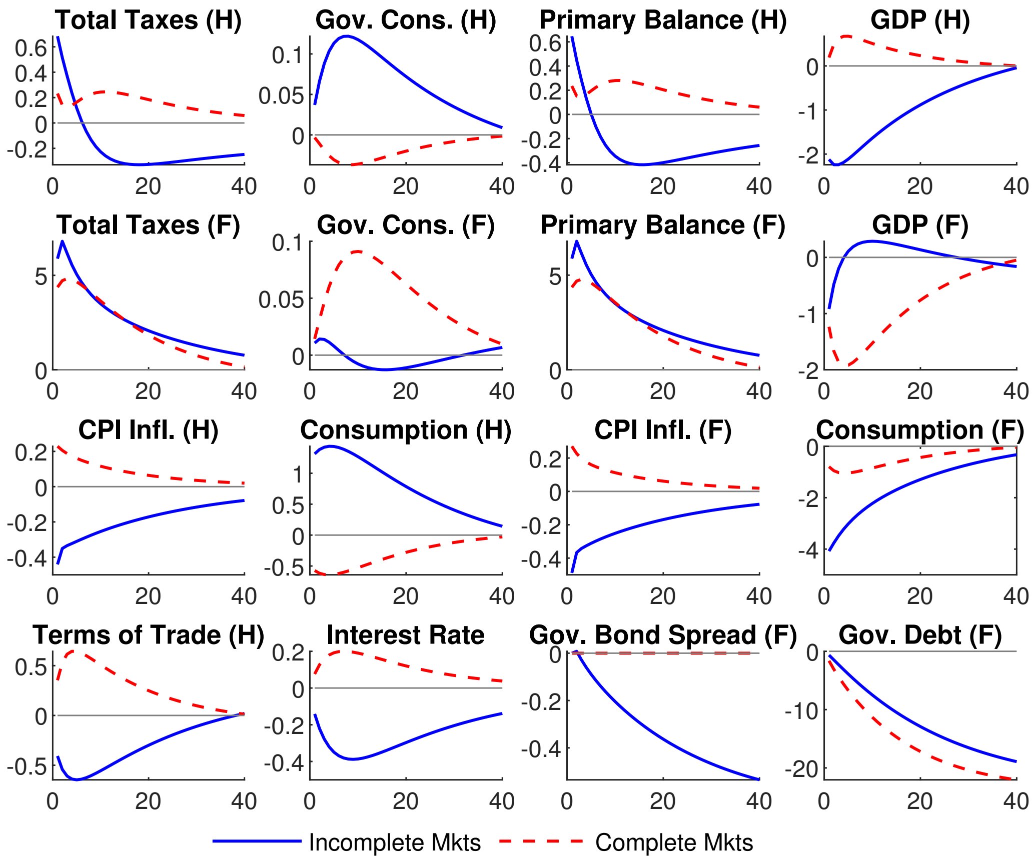
Figure 1 reports the dynamics of the model in a Pure Currency Union, when taxes are used to deleverage the government debt of country F. In the figure we also compare the dynamics under our baseline incomplete markets specification (solid blue line) with the case of complete financial markets (dashed red line)[22], highlighting some key features in the transmission mechanism of the deleveraging shock.

In our baseline incomplete markets specification (solid blue line), the shock to the government debt target in country F implies that from the first quarter onwards taxes (the deleveraging instrument) strongly increase to reduce government debt by the desired amount. Therefore, the cost of deleveraging affects the economy through a negative wealth effect on households' consumption in country F. Furthermore, since the government bond spread depends on the distance between the actual debt-to-GDP ratio and its long-run target, it falls following the deleveraging shock. Notice that on impact private consumption in country F decreases, but since the wealth effect on consumption is partially balanced by the response of the bond spread to a decreasing debt-to-GDP ratio, consumption gradually reverts back to its long run level. This mechanism is also amplified by the strong response of monetary policy to the deflationary pressure induced by the deleveraging process. The decrease in the interest rate set by the Central Bank further decreases the interest

[21] More in detail, the variable that is shocked is $\frac{B^{*G}}{P_H^* Y^*}$. We assume an AR(1) process for this shock with a very high persistence and a response to the unit innovation equal to the gap between the 80% steady state value and the 60% target.

[22] We also simulated the model in the case of a high debt-elastic government bond spread, corresponding to a calibration of $\delta^B = 0.0125$. This corresponds to a spread of 100 instead of 72 basis points with an 80% debt-to-GDP level and a 60% target. In this case the results differ only quantitatively with respect to the benchmark framework under incomplete markets. More in detail, we find that if the spread is more sensitive to the divergence of the debt–to–GDP ratio with respect to the 60% target, output in country F actually increases despite the increase in taxation. Strangely enough, after a deleveraging shock to country F, it is country H which falls into a recession, through a large terms of trade effect, while country F has a boom.

Figure 1: Deleveraging with Taxes in a Pure Currency Union.
Quarterly values in % deviation from s.s. except Taxes, Interest Rate, Inflation Rates and Gov. Bond Spread in p.p. difference from s.s.



rate in country F ($i_t^* \approx i_t + \delta_t$), because both its components decrease. While the government bond spread decreases because country F reduces its debt target, the deflationary pressures implied by the shock push the Central Bank to lower the policy rate. The transmission of the deleveraging shock to country H is mainly affected by monetary policy and relative prices. Due to the reduction in the policy rate, private consumption in country H increases. However, the reduction in labour supply induces a contraction in output and consequently an increase in taxation to balance the budget. Relative prices are the other variable which determines the recession in country H. Due to greater price flexibility in country H and a deeper deflation in country F, the terms of trade (prices in country F relative to country H) fall, mitigating the initial recession in country F and contributing significantly to the recession in country H. Consequently, the drop in net exports in country H drives the persistent recession in the Core and boosts labour supply and thus GDP in the Rest of the Euro Area. These dynamics are the result of a huge deleveraging shock (admittedly not very realistic) and of the great openness to trade in the Euro Area which amplifies movements in relative prices. Even if such a huge and sudden deleveraging process is unlikely to occur, this excercise highlights the importance of relative prices in driving the international transmission of shocks[23].

When markets are complete there is no spread (dashed red line), and, as a consequence, the negative wealth effect on consumption in country H is not balanced by a decrease in the bond spread. Contrarily to what occurs with incomplete markets, there is co-movement in private consumption across countries so that the negative wealth effect makes consumption fall in both countries. The Central Bank now increases the common interest rate to counteract the increase in prices. Inflationary pressures are larger in country F determining an improvement in the terms of trade, and inducing a recession in country F and an expansion in country H. Overall, we observe lower volatility and mostly opposite dynamics in the economy.

The primary balance shows the fiscal policy stance of each country, indicating that country F chooses a restrictive fiscal policy, which is necessary to reduce its government debt. Country H, on the other hand, chooses a restrictive fiscal policy with complete markets and an expansionary one with incomplete markets. This is mainly given by the comovement of consumption when markets are complete, bringing taxes to increase to counteract the fall in consumption (the tax base), while consumption increases, bringing taxes to fall, when markets are incomplete. In country H government consumption also influences the primary balance, as it follows the opposite direction of GDP.

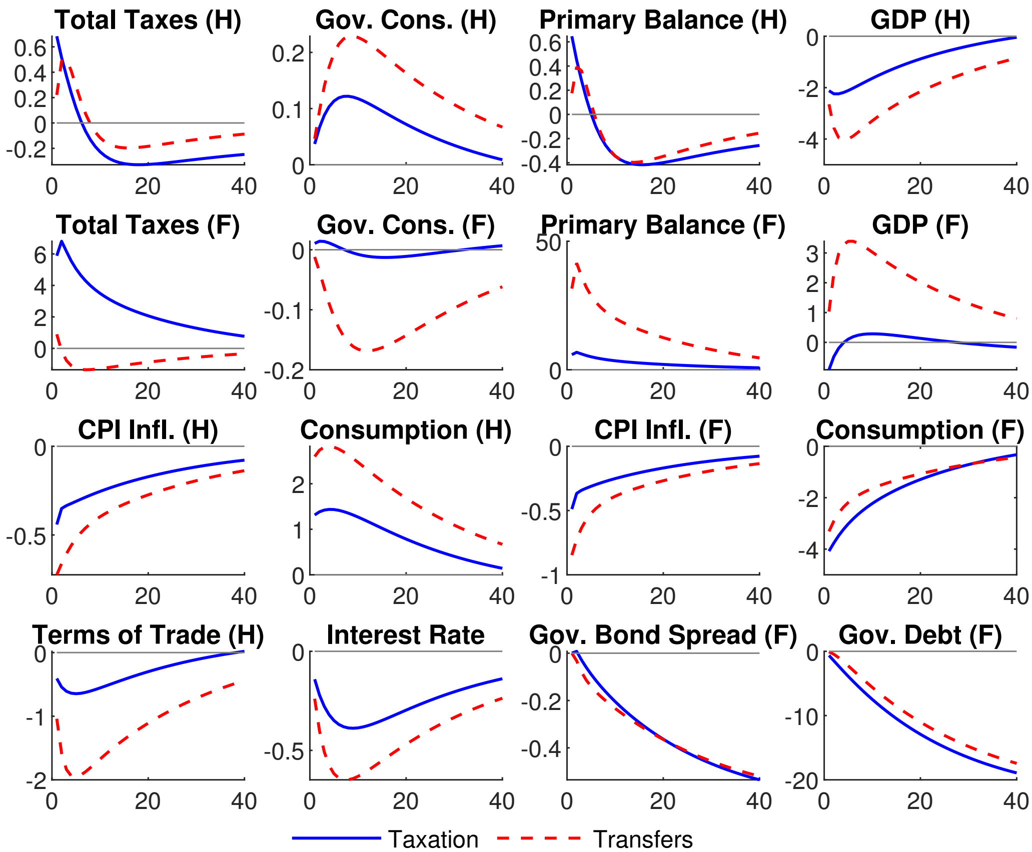
4.2 Instruments for Deleveraging

Figure 2 shows the comparison between different fiscal instruments for deleveraging, after the shock to the government debt target described above, in a Pure Currency Union. The increase in distortionary taxes (solid blue line), rather than the fall in government transfers (dashed red line), produces very different dynamics, changing spillovers across countries, which is why we study the stabilization properties of the different instruments.

In Figure 2 we can see that, independently of the instrument used to deleverage, most of the dynamics are driven by the strong deflationary pressure produced by the deleveraging shock, which makes prices in country F decrease more than in country H, as shown by the movements in the terms of trade and in CPI inflation in both countries. Using distortionary taxes (solid blue line), rather than government transfers (dashed red line), to deleverage government debt produces more stable dynamics for most variables, except for consumption in country F. At the same time distortionary taxes affect the dynamics of the economy more than other instruments, but depending on their effect on consumption and especially prices, they can reduce the deflationary pressure, bringing in the end to a gain in stabilization. Figure 2 shows that most variables follow qualitatively very similar paths, except for the fiscal policy instruments in country F, which behave differently by construction. Taxes move even when government transfers are used to deleverage, because they need to counteract the effect of the change in GDP and government consumption on the government budget constraint. Instead, when taxes are used to deleverage, their responses change sign in

[23] Figure 7 in Appendix B shows that having the same price rigidity does not affect our results in a significant way.

Figure 2: Instruments for Deleveraging in a Pure Currency Union.
Quarterly values in % deviation from s.s. except Taxes, Interest Rate, Inflation Rates and Gov. Bond Spread in p.p. difference from s.s.



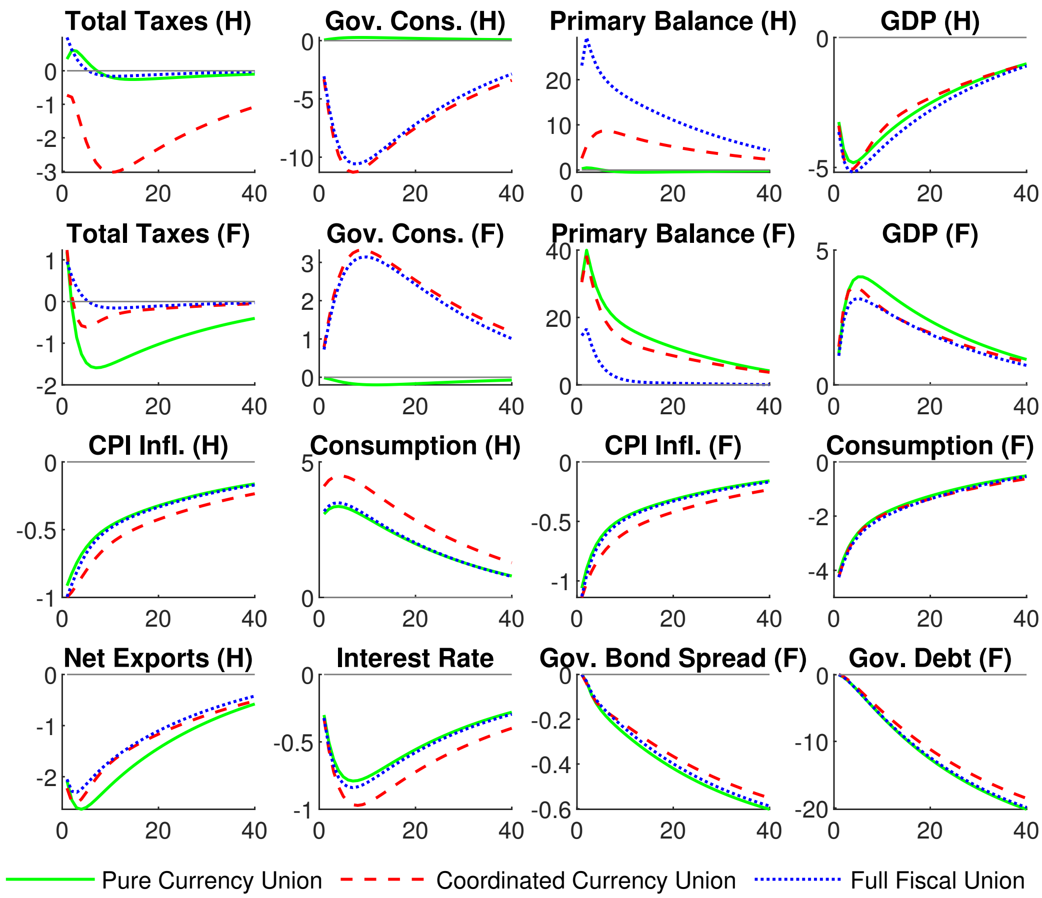
country F and increase instead of decrease, reducing consumption in country F and reducing the volatility in the terms of trade, by partially counteracting deflationary pressures, which stabilizes more net exports and GDP in both countries[24].

To conclude, using distortionary taxes to deleverage, rather than government transfers, stabilizes the economy the most in all scenarios, because deflationary pressures in country F are reduced and so are international spillovers, through the stabilization of the terms of trade.

The primary balance in country F is more stable, with a slightly restrictive fiscal stance, when taxes are used to deleverage, as their increase counteracts the fall in consumption (the tax base). On the other hand, when transfers are used to deleverage the primary balance turns very restrictive, as GDP increases more and government consumption falls less. In country H the primary balance

[24]Also in a Full Fiscal Union (not shown here) distortionary taxes generally provide more stabilization for the same reason described above, but there is also less divergence for most variables in the paths given by the different instruments for deleveraging, compared to a Pure Currency Union. This is mainly because the tax rates and the fiscal instruments for deleveraging move symmetrically across countries, which stabilizes more relative prices and thus international spillovers, reducing the volatility in most variables.

Figure 3: Coordination of Deleveraging with Government Transfers.
Quarterly values in % deviation from s.s. except Taxes, Interest Rate, Inflation Rates and Gov. Bond Spread in p.p. difference from s.s.

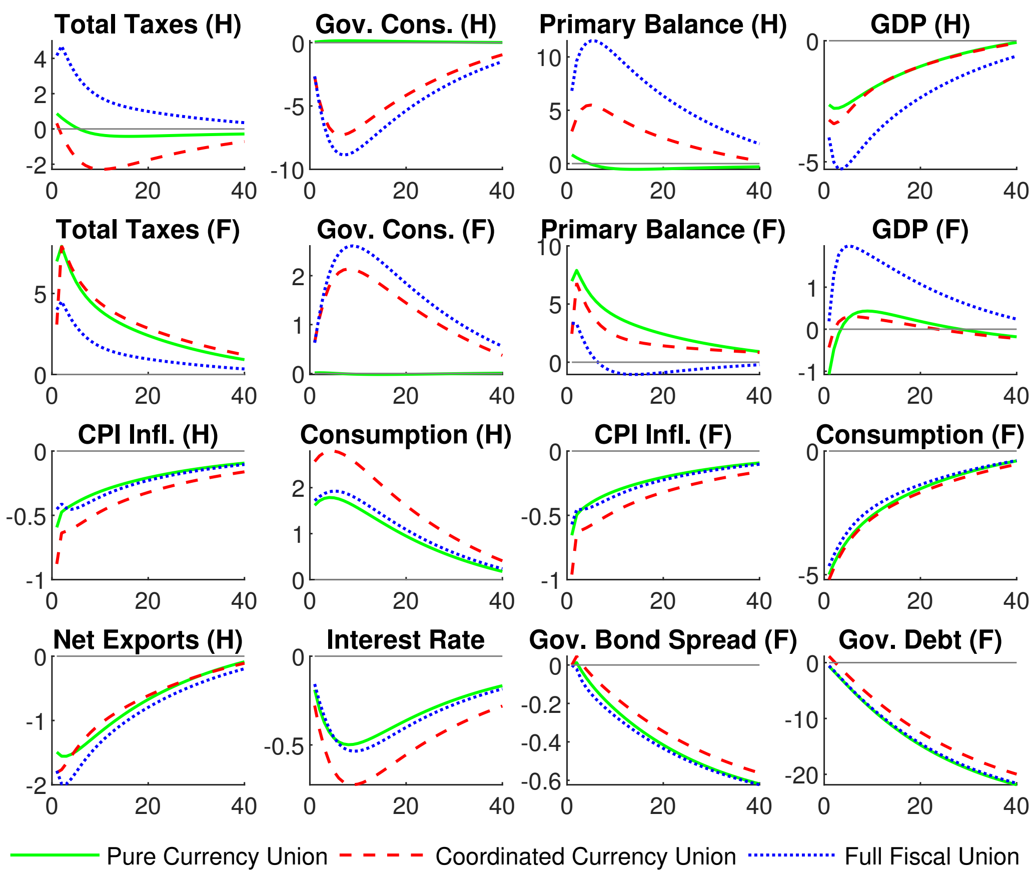


starts restrictive and then turns expansionary, following the increase and fall in taxes over time.

4.3 Coordination of Deleveraging

Figures 3 and 4 compare the three different degrees of fiscal policy coordination, while using government transfers or taxes, respectively, to deleverage the government debt of country F. We can see in both cases how there is very little difference in the dynamics of most variables, in particular output and net exports, between a Full Fiscal Union (dotted blue line) and a Coordinated Currency Union (dashed red line). This tells us that coordinating fiscal policy, by targeting the net exports gap or by additionally consolidating budget constraints and moving tax rates jointly, makes little difference. Only when using government transfers to deleverage, we can see that a Pure Currency Union (solid green line) is the worst in terms of stabilization of the deleveraging shock, at least for net exports and thus GDP. This is because stabilizing net exports reduces the volatility of distortionary taxes in country F and consequently stabilizes relative prices and international

Figure 4: Coordination of Deleveraging with Taxes.
Quarterly values in % deviation from s.s. except Taxes, Interest Rate, Inflation Rates and Gov. Bond Spread in p.p. difference from s.s.



spillovers.

Looking at Figure 3 we can see the stabilization properties of the different degrees of fiscal policy coordination when deleveraging with government transfers. It is clear that a Full Fiscal Union stabilizes more the deleveraging shock, at least for GDP in country F. The fact that both countries coordinate or also share the cost of deleveraging reduces the amplification of the shocks, mainly driven by the opposite movements in government consumption in both countries, which, becoming highly procyclical instead of countercyclical, reduce the movements in net exports and consequently the negative spillovers driven by the terms of trade. In a Coordinated Currency Union, nonetheless, since taxes fall more in country H, there is higher deflation.

Figure 4 confirms that most of the reasoning done when using real transfers to deleverage still holds when using taxes to finance all expenditure and deleveraging. In this case, a Coordinated Currency Union (dashed red line) seems to produce more stabilization than a Full Fiscal Union (dotted blue line), except for inflation. This is mainly given by the greater movements of taxes in country H in a Full Fiscal Union, which amplifies movements in relative prices and thus international trade and GDP, bringing to more amplified dynamics compared to a Coordinated Currency Union. This brings us to think that it might not be convenient to consolidate budget constraints across countries in this case, as it reduces the stabilization gains from coordinating on the net exports gap. Furthermore, there is little difference between stabilizing output and net exports, except for there is greater deflation in the latter case.

Even though the stabilization properties of the different degrees of coordination depend on the instruments used to deleverage, we can clearly see that a Coordinated Currency Union stabilizes GDP more than other levels of coordination in both cases of deleveraging with transfers and taxes.

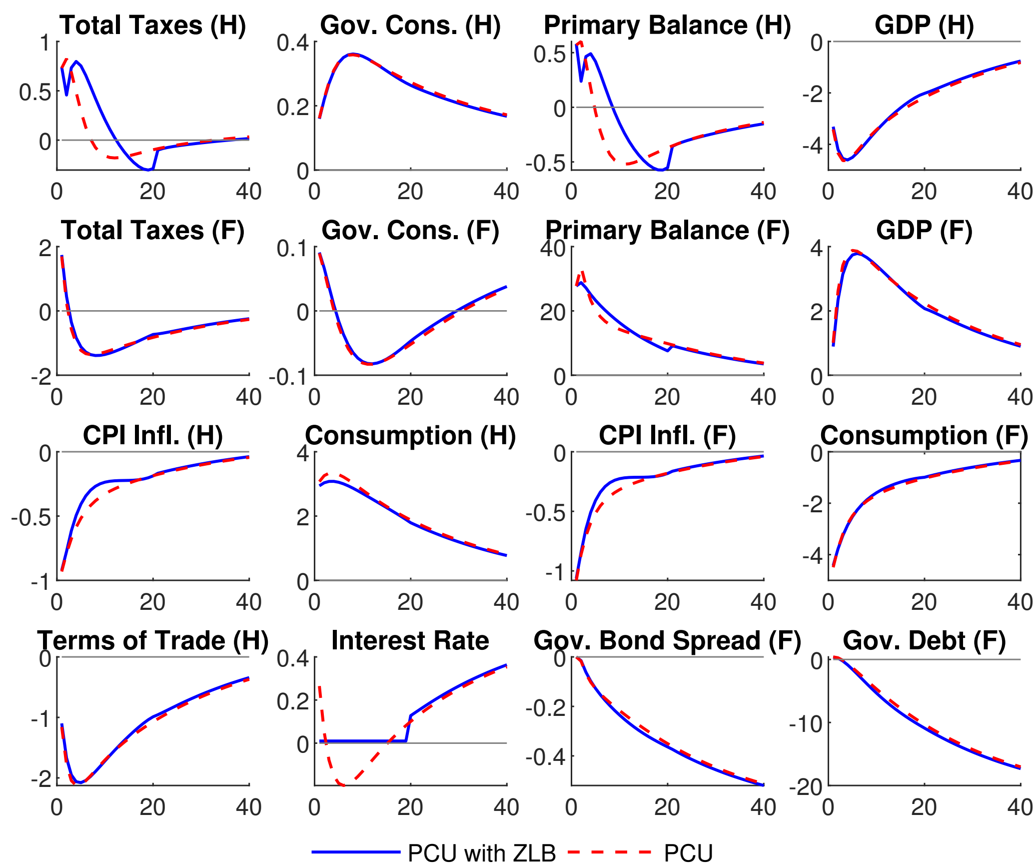
The primary balance in both countries and in both cases is relatively more stable in a Coordinated Currency Union, although always restrictive, as most of the stabilization is gained by taxes moving in the same direction as GDP, other than by government consumption targeting the net exports gap. It seems that greater stabilization of GDP is reached by moving the primary balance quite symmetrically in both countries.

4.4 Deleveraging at the Zero Lower Bound

It is well known that a monetary policy which aims at price stability typically offsets the stimulative effect of an expansionary fiscal policy. An expansionary fiscal action leads to an increase in inflationary pressure, which in turn leads to movements in the real interest rate. If the Central Bank is active, the inflationary pressures lead to upward movements in the real interest rate, while if the Central Bank is passive the real interest rate decreases, amplifying the effect of the fiscal stimulus. As argued by Coenen et al. (2012), although in Europe this amplification mechanism is less pronounced than in the US, due to greater nominal rigidities, it is quite relevant for the transmission of fiscal shocks. Furthermore, Erceg and Lindé (2014) points out to the fact that an economy emerges faster from a liquidity trap if government spending rises and in such a trap sizeable government spending can stimulate a much larger response in tax revenues than in normal times, making fiscal expansions less costly. The flip-side of these arguments, which are relevant when deleveraging government debt, is that spending cuts have larger effects if the nominal interest rate is up against the Zero Lower Bound (ZLB), while they may also prolong the recession and boost real government debt, creating a vicious debt-deflation loop. Given that in the last decade the European Central Bank's monetary policy has been constrained by the ZLB, we decide to compare the dynamics of the economy in a Pure Currency Union and a Full Fiscal Union during a liquidity trap. In this scenario, the nominal interest rate hits the ZLB for some quarters, while the monetary policy rule moves from equation 2.4.1 to equation 2.4.3.

To account for this new feature of the model, we exploit the algorithm proposed by Guerrieri and Iacoviello (2015) to deal with occasionally binding constraints. Their tool is built on the insight that occasionally binding constraints, like the ZLB, can be handled as different regimes of the same model: under one regime the occasionally binding constraint is slack and a rational expectations solution exists, while under the other regime the same constraint is binding and the model might yield multiple solutions. However, if a shock moves the model from the slack regime to the binding

Figure 5: Deleveraging with Transfers at the ZLB in a Pure Currency Union - Preference shock in country F.
Quarterly values in % deviation from s.s. except Taxes, Interest Rate, Inflation Rates and Gov. Bond Spread in p.p. difference from s.s.

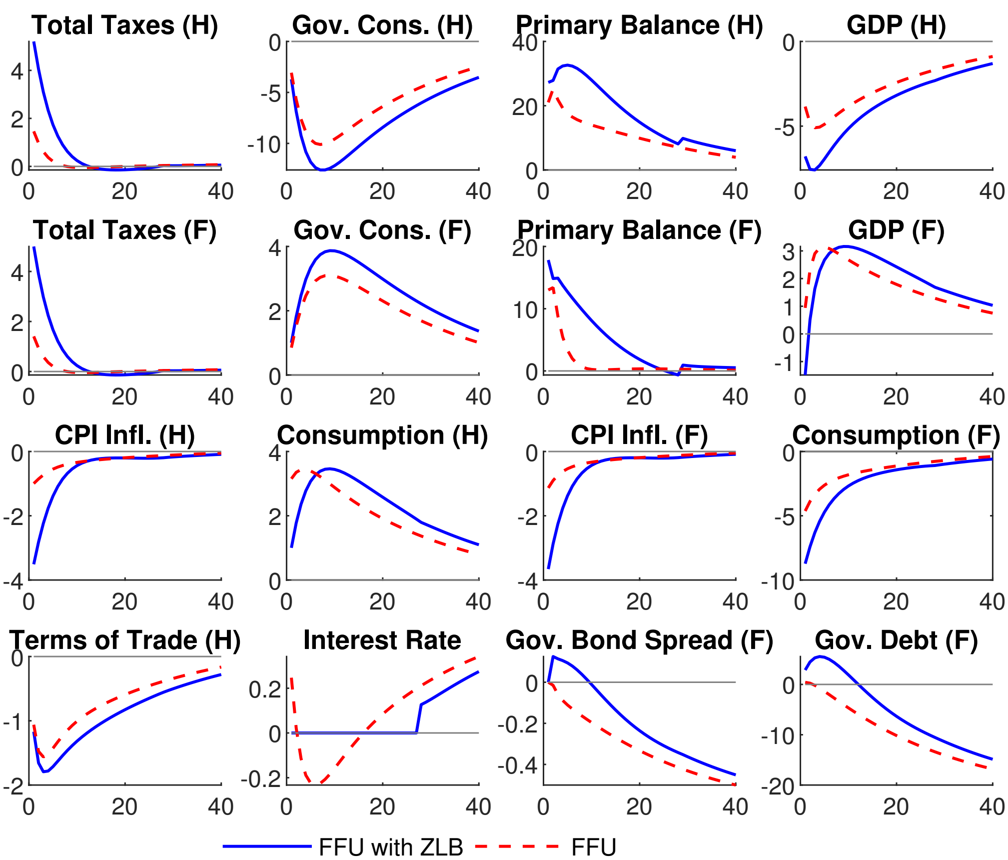


one, it must return to the original regime in a finite time horizon. The model is linearized around the non-stochastic steady state of the slack regime and the piecewise linear solution method proposed involves a first order approximation of the model around the same point for each regime. Since the dynamics in a regime may crucially depend on its expected duration, which in turn depends on the state vector, the solution is highly non-linear. This solution method allows to compute endogenously the length of the liquidity trap, which might differ across fiscal policy scenarios.

The liquidity trap is designed by assuming that a negative preference shock hits the economy in country F while the Government is starting to deleverage[25]. The deflationary pressure pushes the Central Bank to lower the nominal interest rate until reaching the ZLB from the first period of simulation. The duration of the liquidity trap, roughly 20-30 quarters, strongly depends on

[25]We consider a unit innovation entering both the deleveraging shock and the preference shock equations. The unit innovation is multiplied by the standard deviation for the preference shock equation and by the gap between the steady state value and the debt target for the deleveraging shock equation. This implies that there is a correlation between the deleveraging shock and the preference shock and allows to simulate the two shocks simultaneously.

Figure 6: Deleveraging with Transfers at the ZLB in a Full Fiscal Union - Preference shock in country F.
Quarterly values in % deviation from s.s. except Taxes, Interest Rate, Inflation Rates and Gov. Bond Spread in p.p. difference from s.s.



the deleveraging shock and on the degree of fiscal policy coordination between the two countries. Figures 5 and 6 respectively compare the impulse responses after a shock to the government debt target in country F (other than the preference shock) with and without the ZLB constraint, in both a Pure Currency Union and a Full Fiscal Union. In particular, due to international spillovers, deflationary pressures arise also in country H, even if the preference shock hits only country F directly. Although monetary policy, constrained by the ZLB, is not able to offset the additional deflationary pressures in country H, the movements of taxes in country H are able to offset them. In this case, the main driver of deflation is still the deleveraging shock.

We can observe that in a Pure Currency Union, the ZLB constraint marginally affects the dynamics of the economy. In a Full Fiscal Union, instead the ZLB constraint produces different dynamics by increasing the volatility of the terms of trade and thus GDP, while increasing deflation in both countries. This is because in a Full Fiscal Union the deflationary pressures are stronger and the shadow interest rate (interest rate without ZLB) falls more, because, as fiscal instruments move in the same direction across countries, country H is not able to offset international spillovers.

With the ZLB, despite the tight deleveraging rule, government debt in country F grows in real terms for some quarters, because the deflationary pressures are not offset by the Central Bank, which is constrained. As argued in Erceg and Lindé (2014), when the ZLB constraint is binding the government deficit decreases by less in response to spending cuts, if the tax base responds more to swings in output. Looking at Figures 5 and 6, it is possible to observe that the temporary increase in government debt is much more pronounced in a Full Fiscal Union because government consumption in both countries becomes highly pro-cyclical, creating further deflation compared to a Pure Currency Union. Therefore, the liquidity trap lasts longer in a Full Fiscal Union (almost 30 quarters) than in a Pure Currency Union (about 20 quarters). Furthermore, since the deflationary pressures are highly exacerbated in the former case, the interest rate is stuck at zero in the presence of the constraint for more than it is in negative territory in the absence of the constraint, making the overall process self-reinforcing. This is much more pronounced in a Full Fiscal Union, while in a Pure Currency Union the timing of the exit from zero or negative rates is almost the same. Moreover, the fact that in a Full Fiscal Union the interest rate becomes positive later in the presence of the ZLB constraint than without it, increases the persistence of most variables away from the steady state, as the Central Bank is not able to stabilize enough the economy, because it cannot respond to the deflationary pressures for a longer time than expected. We can conclude that, although coordination is generally desirable, it seems to be costly when the economy is constrained by the presence of the ZLB.

Although in a Pure Currency Union the ZLB does not affect the dynamics much, it does so more in a Full Fiscal Union. In the latter, in fact, less amplified movements of the primary balance in both countries create relatively more stable dynamics of GDP, which in this case are associated to less amplified movements in taxes and government consumption, reducing spillovers across countries, as shown by the movements in the terms of trade.

5 Conclusions and Possible Extensions

This research was conducted to assess the effects of government debt deleveraging in the Euro Area, as often requested by the European Commission, and the stabilization properties of different fiscal instruments for deleveraging and different degrees of fiscal policy coordination, to bring to the proper government debt management in a Currency Union. We build an open economy (Germany vs. the Rest of the Euro Area) DSGE model of a Currency Union with incomplete international financial markets, to study the effects of government debt deleveraging in the Rest of the Euro Area and its spillovers towards Germany. This analysis highlights the central role played by international demand imbalances for the transmission of a deleveraging shock. In fact, it is actually Germany that, given its large openness to trade, falls in a recession due to the drop in net exports, while the Rest of the Euro Area does not.

We show that the best instrument for deleveraging are distortionary taxes in all degrees of fiscal policy coordination, because they counteract the deflationary effect of the deleveraging shock more than other instruments. This result partially contrasts with the findings in the open economy literature, which point out to the fact that expenditure-based consolidations are preferable, but, as stated in Coenen, Mohr and Straub (2008) and Erceg and Lindé (2013), tax based consolidations in a Currency Union make the reactions of GDP, inflation and the terms of trade smoother.

The effects of international spillovers are amplified when the two countries do not coordinate their fiscal policies (as in a Pure Currency Union), because deflation is reduced more in country H, but relative prices move more, amplifying movements in net exports. As a consequence, we find that coordinating on the net exports gap when deleveraging generally provides more stabilization at the cost of higher deflation.

We also consider the implications of the Zero Lower Bound constraint in the deleveraging dynamics and consequent spillovers. We observe that deflationary pressures are much stronger in a Full Fiscal Union in the presence of the ZLB because the Central Bank cannot counteract deflation in the presence of the constraint, and, as fiscal instruments like taxes move symmetrically across countries, the Government of the country which is not deleveranging would but can't offset the additional international spillovers, although it would bring to higher volatility in the terms of trade and in net exports. Furthermore, the liquidity trap lasts longer in a Full Fiscal Union because government consumption becomes highly procyclical, creating further deflation.

Our policy prescriptions for the Euro Area are to reduce government debt less during recessions and liquidity traps, to do so using distortionary taxes, while concentrating on reducing international demand imbalances.

Our model focuses on a very specific design of fiscal policy, and one can consider coordination strategies which are different from ours, also by studying their optimal design. Moreover, introducing physical capital in the production function might revert the prediction on spillovers because investment strongly responds to falling interest rates. We also do not consider household heterogeneity and the distributional consequences of fiscal consolidations, which might enhance the stabilization properties of expenditure-based fiscal consolidations, if the transfers are finalized at reducing inequalities, and might smooth international spillovers because of the reduced wealth effect. Finally, the model includes financial intermediaries for the sole purpose of creating international financial frictions, while a fully specified banking sector is not considered, although it would amplify the shocks rather than change their direction, hence reinforcing our results. A more extended model, however, might provide insights on how breaking the doom-loop, along with fiscal coordination, would help to address the negative consequences of a deleveraging shock (i.e. the debt-deflation spiral).

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A Mathematical Appendix

As a tool for students and researchers studying models of this kind for the first time, we provide as many mathematical derivations of the model as possible here in the Appendix. This is meant for learning purposes, especially concerning complex models with cumbersome mathematical derivations.

A.1 Equilibrium Conditions

Here we collect all the equilibrium conditions of the full model, differentiating between a Pure Currency Union, a Coordinated Currency Union and a Full Fiscal Union and between different policy rules.
The equilibrium conditions of the model are grouped into the following blocks:

Aggregate Demand Block
The aggregate demand block is composed of aggregate demand in both countries H:

$$Y_t = \left[1-\alpha+\alpha(\mathcal{S}_t)^{1-\eta}\right]^{\frac{\eta}{1-\eta}} \left[(1-\alpha)C_t + \alpha^*(\mathcal{S}_t)^{\eta}\left(\frac{1-\alpha^*+\alpha^*(\mathcal{S}_t)^{\eta-1}}{1-\alpha+\alpha(\mathcal{S}_t)^{1-\eta}}\right)^{\frac{\eta}{1-\eta}} C_t^*\right] + G_t \quad \text{(A.1.1)}$$

and F:

$$Y_t^* = \left[1-\alpha^*+\alpha^*(\mathcal{S}_t)^{\eta-1}\right]^{\frac{\eta}{1-\eta}} \left[(1-\alpha^*)C_t^* + \alpha(\mathcal{S}_t)^{-\eta}\left(\frac{1-\alpha^*+\alpha^*(\mathcal{S}_t)^{\eta-1}}{1-\alpha+\alpha(\mathcal{S}_t)^{1-\eta}}\right)^{\frac{\eta}{\eta-1}} C_t\right] + G_t^* \quad \text{(A.1.2)}$$

while the evolution of private consumption is given by the households' Euler Equation in countries H:

$$\frac{1}{1+i_t} = \beta E_t\left\{\frac{\xi_{t+1}}{\xi_t}\left(\frac{C_{t+1}}{C_t}\right)^{-\sigma}\frac{1}{\Pi_{t+1}}\right\} \quad \text{(A.1.3)}$$

and F:

$$\frac{1-\delta_t}{1+i_t} = \beta E_t\left\{\frac{\xi_{t+1}^*}{\xi_t^*}\left(\frac{C_{t+1}^*}{C_t^*}\right)^{-\sigma}\frac{1}{\Pi_{t+1}^*}\right\} \quad \text{(A.1.4)}$$

while the relationship between CPI inflation and PPI inflation is given by:

$$\Pi_t = \Pi_{H,t}\left[\frac{1-\alpha+\alpha(\mathcal{S}_t)^{1-\eta}}{1-\alpha+\alpha(\mathcal{S}_{t-1})^{1-\eta}}\right]^{\frac{1}{1-\eta}} \quad \text{(A.1.5)}$$

in country H and:

$$\Pi_t^* = \Pi_{H,t}^*\left[\frac{1-\alpha^*+\alpha^*\mathcal{S}_t^{\eta-1}}{1-\alpha^*+\alpha^*(\mathcal{S}_{t-1})^{\eta-1}}\right]^{\frac{1}{1-\eta}} \quad \text{(A.1.6)}$$

in country F, and the evolution of the terms of trade is given by:

$$\mathcal{S}_t = \frac{\Pi^*_{H,t}}{\Pi_{H,t}} \mathcal{S}_{t-1} \tag{A.1.7}$$

while the exogenous demand shocks evolve according to:

$$\xi_t = (\xi_{t-1})^{\rho_\xi} e^{\varepsilon_{\xi,t}} \tag{A.1.8}$$

$$\xi^*_t = (\xi^*_{t-1})^{\rho^*_\xi} e^{\varepsilon_{\xi,t}} \tag{A.1.9}$$

Aggregate Supply Block

The aggregate supply block is composed of the aggregate supply equation for country H:

$$\left(\frac{1 - \theta(\Pi_{H,t})^{\varepsilon - 1}}{1 - \theta} \right)^{\frac{1}{1-\varepsilon}} = \frac{\varepsilon}{\varepsilon - 1} \frac{K_t}{F_t} \tag{A.1.10}$$

where:

$$K_t = \xi_t (C_t)^{-\sigma} Y_t MC_t + \beta\theta E_t \left\{ \frac{(\Pi_{H,t+1})^{\varepsilon+1}}{\Pi_{t+1}} K_{t+1} \right\} \tag{A.1.11}$$

$$F_t = \xi_t (C_t)^{-\sigma} Y_t (1 - \tau^s_t) + \beta\theta E_t \left\{ \frac{(\Pi_{H,t+1})^{\varepsilon}}{\Pi_{t+1}} F_{t+1} \right\} \tag{A.1.12}$$

and marginal cost in country H is given by:

$$MC_t = \frac{(Y_t)^\varphi (d_t)^\varphi (C_t)^\sigma}{(1 - \tau^w_t)(A_t)^{1+\varphi}(h)^{\varphi+\sigma}} \left[1 - \alpha + \alpha(\mathcal{S}_t)^{1-\eta} \right]^{\frac{1}{1-\eta}} \tag{A.1.13}$$

and the aggregate supply equation for country F:

$$\left(\frac{1 - \theta^*(\Pi^*_{H,t})^{\varepsilon - 1}}{1 - \theta^*} \right)^{\frac{1}{1-\varepsilon}} = \frac{\varepsilon}{\varepsilon - 1} \frac{K^*_t}{F^*_t} \tag{A.1.14}$$

where:

$$K^*_t = \xi^*_t (C^*_t)^{-\sigma} Y^*_t MC^*_t + \beta\theta^* E_t \left\{ \frac{(\Pi^*_{H,t+1})^{\varepsilon+1}}{\Pi^*_{t+1}} K^*_{t+1} \right\} \tag{A.1.15}$$

$$F^*_t = \xi^*_t (C^*_t)^{-\sigma} Y^*_t (1 - \tau^{*s}_t) + \beta\theta^* E_t \left\{ \frac{(\Pi^*_{H,t+1})^{\varepsilon}}{\Pi^*_{t+1}} F^*_{t+1} \right\} \tag{A.1.16}$$

and marginal cost in country F is given by:

$$MC^*_t = \frac{(Y^*_t)^\varphi (d^*_t)^\varphi (C^*_t)^\sigma}{(1 - \tau^{*w}_t)(A^*_t)^{1+\varphi}(1-h)^{\varphi+\sigma}} \left[1 - \alpha^* + \alpha^*(\mathcal{S}_t)^{\eta-1} \right]^{\frac{1}{1-\eta}} \tag{A.1.17}$$

while the evolution of price dispersion is given by:

$$d_t = \theta d_{t-1}(\Pi_{H,t})^{\varepsilon} + (1-\theta)\left[\frac{1-\theta(\Pi_{H,t})^{\varepsilon-1}}{1-\theta}\right]^{\frac{\varepsilon}{\varepsilon-1}} \tag{A.1.18}$$

for country H, and:

$$d_t^* = \theta^* d_{t-1}^*(\Pi_{H,t}^*)^{\varepsilon} + (1-\theta^*)\left[\frac{1-\theta^*(\Pi_{H,t}^*)^{\varepsilon-1}}{1-\theta^*}\right]^{\frac{\varepsilon}{\varepsilon-1}} \tag{A.1.19}$$

for country F, while the levels of technology evolve exogenously according to:

$$A_t = (A_{t-1})^{\rho_a} e^{\varepsilon_{a,t}} \tag{A.1.20}$$

$$A_t^* = (A_{t-1}^*)^{\rho_a^*} e^{\varepsilon_{a,t}} \tag{A.1.21}$$

Net Exports, Net Foreign Assets and the Balance of Payments

Real Net Exports for country H are given by:

$$\widetilde{NX}_t = Y_t - \left[1-\alpha+\alpha(\mathcal{S}_t)^{1-\eta}\right]^{\frac{1}{1-\eta}} C_t - G_t \tag{A.1.22}$$

Real Net Foreign Assets for country H are given by:

$$\widetilde{NFA}_t = \tilde{B}_{F,t} \tag{A.1.23}$$

The real Balance of Payments for country H is given by:

$$\widetilde{BP}_t = \widetilde{NX}_t + \frac{i_{t-1}+\delta_{t-1}}{1-\delta_{t-1}}\frac{\widetilde{NFA}_{t-1}}{\Pi_{H,t}} \tag{A.1.24}$$

so that real Net Foreign Assets for country H evolve according to:

$$\widetilde{NFA}_t = \frac{\widetilde{NFA}_{t-1}}{\Pi_{H,t}} + \widetilde{BP}_t = \frac{1+i_{t-1}}{1-\delta_{t-1}}\frac{\widetilde{NFA}_{t-1}}{\Pi_{H,t}} + \widetilde{NX}_t \tag{A.1.25}$$

while the transaction cost is given by:

$$\delta_t \equiv \delta^B\left(\frac{B_{t-1}^{*G}}{P_{H,t-1}^* Y_{t-1}^*} - \frac{B^{*G}}{P_H^* Y^*}\right) \tag{A.1.26}$$

Monetary Policy

Monetary policy sets the nominal interest rate following the rule:

$$\beta(1+i_t) = \left(\frac{\Pi_t^U}{\Pi^U}\right)^{\phi_\pi(1-\rho_i)} [\beta(1+i_{t-1})]^{\rho_i} \tag{A.1.27}$$

where union-wide CPI inflation is defined by:

$$\Pi_t^U \equiv (\Pi_t)^h (\Pi_t^*)^{1-h} \tag{A.1.28}$$

while in the alternative Zero Lower Bound scenario it follows the rule:

$$i_t = \max\{\tilde{i}_t, 0\} \tag{A.1.29}$$

$$\beta(1+\tilde{i}_t) = \left(\frac{\Pi_t^U}{\Pi^U}\right)^{\phi_\pi(1-\rho_i)} \left[\beta(1+\tilde{i}_{t-1})\right]^{\rho_i}$$

Fiscal Policy in a Pure Currency Union - Transfer Scenario
Fiscal policy sets government consumption following the rules:

$$\frac{G_t}{G} = \left(\frac{Y_t}{Y}\right)^{-\psi_y(1-\rho_g)} \left(\frac{G_{t-1}}{G}\right)^{\rho_g} \tag{A.1.30}$$

$$\frac{G_t^*}{G^*} = \left(\frac{Y_t^*}{Y^*}\right)^{-\psi_y^*(1-\rho_g^*)} \left(\frac{G_{t-1}^*}{G^*}\right)^{\rho_g^*} \tag{A.1.31}$$

while using real transfers to deleverage government debt in country F and balance the budget in country H, through the debt rules:

$$\tilde{B}_t^G = \frac{\tilde{B}_{t-1}^G}{\Pi_{H,t}} \qquad \frac{\tilde{B}_{t-1}^{*G}}{\Pi_{H,t}^*} - \tilde{B}_t^{*G} = \gamma^* \left(\frac{\tilde{B}_{t-1}^{*G}}{\Pi_{H,t}^*} - \tilde{B}^{*G}\right) \tag{A.1.32}$$

and varying the tax rates on labour income and firm sales to finance the remaining government expenditure, through the tax rules:

$$\tau_t^w - \tau^w = \tau_t^s - \tau^s \qquad \tau_t^{*w} - \tau^{*w} = \tau_t^{*s} - \tau^{*s} \qquad (\tau_t^{*s} + \tau_t^{*w} MC_t^* d_t^*) Y_t^* - (\tau^{*s} + \tau^{*w} MC^*) Y^* = G_t^* - G^* \tag{A.1.33}$$

with the following budget constraints:

$$G_t + \tilde{T}_t + i_{t-1}\frac{\tilde{B}_{t-1}^G}{\Pi_{H,t}} = \tau_t^s Y_t + \tau_t^w MC_t d_t Y_t + \tilde{B}_t^G - \frac{\tilde{B}_{t-1}^G}{\Pi_{H,t}} \tag{A.1.34}$$

$$G_t^* + \tilde{T}_t^* + \frac{i_{t-1} + \delta_{t-1}}{1-\delta_{t-1}} \frac{\tilde{B}_{t-1}^{*G}}{\Pi_{H,t}^*} = \tau_t^{*s} Y_t^* + \tau_t^{*w} MC_t^* d_t^* Y_t^* + \tilde{B}_t^{*G} - \frac{\tilde{B}_{t-1}^{*G}}{\Pi_{H,t}^*} \tag{A.1.35}$$

Fiscal Policy in a Pure Currency Union - Distortionary Tax Scenario

Fiscal policy sets government consumption following the rules:

$$\frac{G_t}{G} = \left(\frac{Y_t}{Y}\right)^{-\psi_y(1-\rho_g)} \left(\frac{G_{t-1}}{G}\right)^{\rho_g} \tag{A.1.36}$$

$$\frac{G_t^*}{G^*} = \left(\frac{Y_t^*}{Y^*}\right)^{-\psi_y^*(1-\rho_g^*)} \left(\frac{G_{t-1}^*}{G^*}\right)^{\rho_g^*} \tag{A.1.37}$$

while using equally taxes on labour income and on firm sales to deleverage government debt in country F and balance the budget in country H, through the debt rules:

$$\tilde{B}_t^G = \frac{\tilde{B}_{t-1}^G}{\Pi_{H,t}} \qquad \frac{\tilde{B}_{t-1}^{*G}}{\Pi_{H,t}^*} - \tilde{B}_t^{*G} = \gamma^* \left(\frac{\tilde{B}_{t-1}^{*G}}{\Pi_{H,t}^*} - \tilde{B}^{*G}\right) \tag{A.1.38}$$

and keeping real transfers constant, through the tax rules:

$$\tau_t^w - \tau^w = \tau_t^s - \tau^s \qquad \tilde{T}_t = \tilde{T} \qquad \tau_t^{*w} - \tau^{*w} = \tau_t^{*s} - \tau^{*s} \qquad \tilde{T}_t^* = \tilde{T}^* \tag{A.1.39}$$

Fiscal Policy in a Coordinated Currency Union - Transfer Scenario

Fiscal policy sets government consumption following the rules:

$$\frac{G_t}{G} = \left(\frac{\widetilde{NX}_t}{\widetilde{NX}}\right)^{\psi_{nx}(1-\rho_g)} \left(\frac{G_{t-1}}{G}\right)^{\rho_g} \tag{A.1.40}$$

$$\frac{G_t^*}{G^*} = \left(\frac{S\,\widetilde{NX}_t}{S_t\,\widetilde{NX}}\right)^{-\psi_{nx}^*(1-\rho_g^*)} \left(\frac{G_{t-1}^*}{G^*}\right)^{\rho_g^*} \tag{A.1.41}$$

while using real transfers to deleverage government debt in country F and balance the budget in country H, through the debt rules:

$$\tilde{B}_t^G = \frac{\tilde{B}_{t-1}^G}{\Pi_{H,t}} \qquad \frac{\tilde{B}_{t-1}^{*G}}{\Pi_{H,t}^*} - \tilde{B}_t^{*G} = \gamma^* \left(\frac{\tilde{B}_{t-1}^{*G}}{\Pi_{H,t}^*} - \tilde{B}^{*G}\right) \tag{A.1.42}$$

and varying the tax rates on labour income and firm sales to finance the remaining government expenditure, through the tax rules:

$$\tau_t^w - \tau^w = \tau_t^s - \tau^s \qquad \tau_t^{*w} - \tau^{*w} = \tau_t^{*s} - \tau^{*s} \qquad (\tau_t^{*s} + \tau_t^{*w} MC_t^* d_t^*) Y_t^* - (\tau^{*s} + \tau^{*w} MC^*) Y^* = G_t^* - G^* \tag{A.1.43}$$

with the following budget constraints:

$$G_t + \tilde{T}_t + i_{t-1} \frac{\tilde{B}_{t-1}^G}{\Pi_{H,t}} = \tau_t^s Y_t + \tau_t^w MC_t d_t Y_t + \tilde{B}_t^G - \frac{\tilde{B}_{t-1}^G}{\Pi_{H,t}} \tag{A.1.44}$$

$$G_t^* + \tilde{T}_t^* + \frac{i_{t-1}+\delta_{t-1}}{1-\delta_{t-1}} \frac{\tilde{B}_{t-1}^{*G}}{\Pi_{H,t}^*} = \tau_t^{*s} Y_t^* + \tau_t^{*w} MC_t^* d_t^* Y_t^* + \tilde{B}_t^{*G} - \frac{\tilde{B}_{t-1}^{*G}}{\Pi_{H,t}^*} \tag{A.1.45}$$

Fiscal Policy in a Coordinated Currency Union - Distortionary Tax Scenario

Fiscal policy sets government consumption following the rules:

$$\frac{G_t}{G} = \left(\frac{\widetilde{NX}_t}{\widetilde{NX}} \right)^{\psi_{nx}(1-\rho_g)} \left(\frac{G_{t-1}}{G} \right)^{\rho_g} \tag{A.1.46}$$

$$\frac{G_t^*}{G^*} = \left(\frac{S\,\widetilde{NX}_t}{S_t\,\widetilde{NX}} \right)^{-\psi_{nx}^*(1-\rho_g^*)} \left(\frac{G_{t-1}^*}{G^*} \right)^{\rho_g^*} \tag{A.1.47}$$

while using equally taxes on labour income and on firm sales to deleverage government debt in country F and balance the budget in country H, through the debt rules:

$$\tilde{B}_t^G = \frac{\tilde{B}_{t-1}^G}{\Pi_{H,t}} \qquad \frac{\tilde{B}_{t-1}^{*G}}{\Pi_{H,t}^*} - \tilde{B}_t^{*G} = \gamma^* \left(\frac{\tilde{B}_{t-1}^{*G}}{\Pi_{H,t}^*} - \tilde{B}^{*G} \right) \tag{A.1.48}$$

and keeping real transfers constant, through the tax rules:

$$\tau_t^w - \tau^w = \tau_t^s - \tau^s \qquad \tilde{T}_t = \tilde{T} \qquad \tau_t^{*w} - \tau^{*w} = \tau_t^{*s} - \tau^{*s} \qquad \tilde{T}_t^* = \tilde{T}^* \tag{A.1.49}$$

Fiscal Policy in a Full Fiscal Union - Transfer Scenario

Fiscal policy sets government consumption following the rules:

$$\frac{G_t}{G} = \left(\frac{\widetilde{NX}_t}{\widetilde{NX}} \right)^{\psi_{nx}(1-\rho_g)} \left(\frac{G_{t-1}}{G} \right)^{\rho_g} \tag{A.1.50}$$

$$\frac{G_t^*}{G^*} = \left(\frac{S\,\widetilde{NX}_t}{S_t\,\widetilde{NX}} \right)^{-\psi_{nx}^*(1-\rho_g^*)} \left(\frac{G_{t-1}^*}{G^*} \right)^{\rho_g^*} \tag{A.1.51}$$

while using real transfers equally in both countries to deleverage the government debt of country F, while country H maintains its government debt constant, through the debt rules:

$$\frac{\tilde{B}_{t-1}^{*G}}{\Pi_{H,t}^*} - \tilde{B}_t^{*G} = \gamma^* \left(\frac{\tilde{B}_{t-1}^{*G}}{\Pi_{H,t}^*} - \tilde{B}^{*G} \right) \qquad \tilde{B}_t^G = \frac{\tilde{B}_{t-1}^G}{\Pi_{H,t}} \qquad \tilde{T}_t - \tilde{T} = \tilde{T}_t^* - \tilde{T}^* \tag{A.1.52}$$

and varying equally across countries the tax rates on labour income and firm sales to finance the remaining government expenditure, through the tax rules:

$$\tau_t^w - \tau^w = \tau_t^s - \tau^s \qquad \tau_t^{*w} - \tau^{*w} = \tau_t^w - \tau^w \qquad \tau_t^{*s} - \tau^{*s} = \tau_t^s - \tau^s \tag{A.1.53}$$

$$(\tau_t^s+\tau_t^w MC_t d_t)Y_t+(\tau_t^{*s}+\tau_t^{*w} MC_t^* d_t^*)S_t Y_t^*-(\tau^s+\tau^w MC)Y-(\tau^{*s}+\tau^{*w} MC^*)Y^* = G_t+G_t^*-G-G^* \tag{A.1.54}$$

with the following consolidated budget constraint:

$$G_t+\tilde{T}_t+S_t(G_t^*+\tilde{T}_t^*)+i_{t-1}\frac{\tilde{B}_{t-1}^G}{\Pi_{H,t}}+\frac{i_{t-1}+\delta_{t-1}}{1-\delta_{t-1}}\frac{S_{t-1}\tilde{B}_{t-1}^{*G}}{\Pi_{H,t}^*}=$$
$$(\tau_t^s+\tau_t^w MC_t d_t)Y_t+(\tau_t^{*s}+\tau_t^{*w} MC_t^* d_t^*)S_t Y_t^*+\tilde{B}_t^G-\frac{\tilde{B}_{t-1}^G}{\Pi_{H,t}}+S_t\tilde{B}_t^{*G}-\frac{S_{t-1}\tilde{B}_{t-1}^{*G}}{\Pi_{H,t}^*} \tag{A.1.55}$$

Fiscal Policy in a Full Fiscal Union - Distortionary Tax Scenario

Fiscal policy sets government consumption following the rules:

$$\frac{G_t}{G}=\left(\frac{\widetilde{NX}_t}{\widetilde{NX}}\right)^{\psi_{nx}(1-\rho_g)}\left(\frac{G_{t-1}}{G}\right)^{\rho_g} \tag{A.1.56}$$

$$\frac{G_t^*}{G^*}=\left(\frac{S\,\widetilde{NX}_t}{S_t\,\widetilde{NX}}\right)^{-\psi_{nx}^*(1-\rho_g^*)}\left(\frac{G_{t-1}^*}{G^*}\right)^{\rho_g^*} \tag{A.1.57}$$

while keeping real transfers constant and varying equally across countries the tax rates on labour income and firm sales to deleverage the government debt of country F, while country H maintains its government debt constant, through the debt rules:

$$\frac{\tilde{B}_{t-1}^{*G}}{\Pi_{H,t}^*}-\tilde{B}_t^{*G}=\gamma^*\left(\frac{\tilde{B}_{t-1}^{*G}}{\Pi_{H,t}^*}-\tilde{B}^{*G}\right)\qquad \tilde{B}_t^G=\frac{\tilde{B}_{t-1}^G}{\Pi_{H,t}}\qquad \tilde{T}_t=\tilde{T}\qquad \tilde{T}_t^*=\tilde{T}^* \tag{A.1.58}$$

and also varying equally across countries the tax rates on labour income and firm sales to finance the remaining government expenditure, through the tax rules:

$$\tau_t^w-\tau^w=\tau_t^s-\tau^s\qquad \tau_t^{*w}-\tau^{*w}=\tau_t^w-\tau^w\qquad \tau_t^{*s}-\tau^{*s}=\tau_t^s-\tau^s \tag{A.1.59}$$

We can now define an equilibrium for the Currency Union.

Definition 1 (Equilibrium)**.** *An Imperfectly competitive equilibrium is a sequence of stochastic processes:*
$\mathcal{X}_t\equiv\{Y_t,Y_t^*,C_t,C_t^*,\Pi_{H,t},\Pi_{H,t}^*,\Pi_t,\Pi_t^*,\Pi_t^U,S_t,K_t,K_t^*,F_t,F_t^*,MC_t,MC_t^*,d_t,d_t^*,\widetilde{NX}_t,\widetilde{NFA}_t,\widetilde{CA}_t\}$ *and exogenous disturbances:*
$\mathcal{Z}_t\equiv\{\xi_t,\xi_t^*,A_t,A_t^*\}$ *satisfying equations A.1.1 through A.1.25 and the definition of union-wide inflation A.1.28, given initial conditions:*
$\mathcal{I}_{-1}\equiv\{C_{-1},C_{-1}^*,\Pi_{H,-1},\Pi_{H,-1}^*,S_{-1},d_{-1},d_{-1}^*,\widetilde{NFA}_{-1}\}$*, plus monetary and fiscal policies:*
$\mathcal{P}_t\equiv\{i_t,G_t,G_t^*,\tilde{T}_t,\tilde{T}_t^*,\tau_t^s,\tau_t^{*s},\tau_t^w,\tau_t^{*w},\tilde{B}_t^G,\tilde{B}_t^{*G}\}$ *specified in equation A.1.27 or A.1.29 for monetary policy and in equations A.1.30 through A.1.59 for the various specifications of fiscal policy, for*

$t \geq 0$.

A.2 The Steady State

We describe the symmetric (in terms of per capita consumption and prices) non-stochastic flexible–price steady state with constant government debt and zero inflation. We focus on the perfect foresight steady state and equilibrium deviations from it, given by different shocks. *Perfect Foresight* is a viable assumption because, despite the uncertainty to which price-setters are subject, it disappears in the aggregate due to the further assumption that there is a large number (more accurately, a continuum) of firms, as explained in Calvo (1983).

The symmetric non-stochastic flexible–price steady state with constant government debt and zero inflation is defined by the following assumptions and equations[26].

All shocks are constant at their long-run levels of 1:

$$\xi = \xi^* = A = A^* = 1 \tag{A.2.1}$$

There is no inflation and no price dispersion:

$$\Pi_H = \Pi_H^* = \Pi = \Pi^* = \Pi^U = 1 \implies d = d^* = 1 \tag{A.2.2}$$

The terms of trade and the real exchange rate are equal to 1:

$$S = 1 \implies Q = 1 \tag{A.2.3}$$

Per-capita consumption is equal across countries:

$$\frac{C}{h} = \frac{C^*}{1-h} \tag{A.2.4}$$

Aggregate demand in each country is given by:

$$Y = (1-\alpha)\, C + \alpha^* C^* + G \tag{A.2.5}$$

$$Y^* = (1-\alpha^*)\, C^* + \alpha C + G^* \tag{A.2.6}$$

Combining the previous equations we can derive per-capita consumption in each country as a function of output and government spending and equate the two to derive an equation linking output and government spending in the two countries:

$$Y = \frac{(1-\alpha)h + \alpha^*(1-h)}{(1-\alpha^*)(1-h) + \alpha h}\left[Y^* - G^*\right] + G \tag{A.2.7}$$

[26] In the list of equations below, D and D^* are not present since they are state-contingent claims and, as a consequence, after aggregating they are equal to zero in each country.

From the Euler Equations:

$$\frac{1}{1+i} = \beta \implies i = \frac{1}{\beta} - 1 \tag{A.2.8}$$

Recalling marginal costs in steady state from price-setting:

$$MC = \frac{\varepsilon - 1}{\varepsilon}(1 - \tau^s) \tag{A.2.9}$$

$$MC^* = \frac{\varepsilon - 1}{\varepsilon}(1 - \tau^{*s}) \tag{A.2.10}$$

Marginal costs are also given by labour market equilibrium:

$$MC = \frac{(Y)^\varphi (C)^\sigma}{(1 - \tau^w)(h)^{\varphi+\sigma}} \tag{A.2.11}$$

$$MC^* = \frac{(Y^*)^\varphi (C^*)^\sigma}{(1 - \tau^{*w})(1 - h)^{\varphi+\sigma}} \tag{A.2.12}$$

Equating the two marginal cost expressions for each country yields consumption in terms of output:

$$C = \left[\frac{\varepsilon - 1}{\varepsilon} \frac{(1 - \tau^s)(1 - \tau^w)(h)^{\varphi+\sigma}}{(Y)^\varphi}\right]^{\frac{1}{\sigma}} \tag{A.2.13}$$

$$C^* = \left[\frac{\varepsilon - 1}{\varepsilon} \frac{(1 - \tau^{*s})(1 - \tau^{*w})(1 - h)^{\varphi+\sigma}}{(Y^*)^\varphi}\right]^{\frac{1}{\sigma}} \tag{A.2.14}$$

Deriving per-capita consumption in the two countries and equating the two yields an equation linking output in the two countries:

$$Y^* = \frac{1 - h}{h} \left[\frac{(1 - \tau^{*s})(1 - \tau^{*w})}{(1 - \tau^s)(1 - \tau^w)}\right]^{\frac{1}{\varphi}} Y \tag{A.2.15}$$

In steady state real net exports are given by:

$$\widetilde{NX} = Y - C - G \tag{A.2.16}$$

while real net foreign assets are:

$$\widetilde{NFA} = \tilde{B}_F \tag{A.2.17}$$

The real balance of payments is given by:

$$\widetilde{BP} = \widetilde{NX} + \left(\frac{1}{\beta} - 1\right) \widetilde{NFA} \tag{A.2.18}$$

while from the budget constraints of households and governments, or equivalently from the evolution of net foreign assets:

$$\widetilde{NFA} = \widetilde{NFA} + \widetilde{BP} \tag{A.2.19}$$

which implies that in steady state the balance of payments must be zero and so net exports pin down net foreign assets:

$$\widetilde{BP} = 0 \implies \widetilde{NX} = -\left(\frac{1}{\beta} - 1\right)\widetilde{NFA} \tag{A.2.20}$$

while the transaction cost in steady state is zero ($\delta = 0$), because debt is constant at steady state level and there are no shocks.

The household budget constraints in steady state for countries H and F are given by:

$$C = \left(\frac{1}{\beta} - 1\right)(\tilde{B}_H + \tilde{B}_F) + \tilde{T} + Y(1 - \tau^s - \tau^w MC) \tag{A.2.21}$$

$$C^* = \left(\frac{1}{\beta} - 1\right)\tilde{B}^*_F + \tilde{T}^* + Y^*(1 - \tau^{*s} - \tau^{*w} MC^*) \tag{A.2.22}$$

Instead the government budget constraints of the two countries in steady state read:

$$G + \tilde{T} + \left(\frac{1}{\beta} - 1\right)\tilde{B}^G = Y(\tau^s + \tau^w MC) \tag{A.2.23}$$

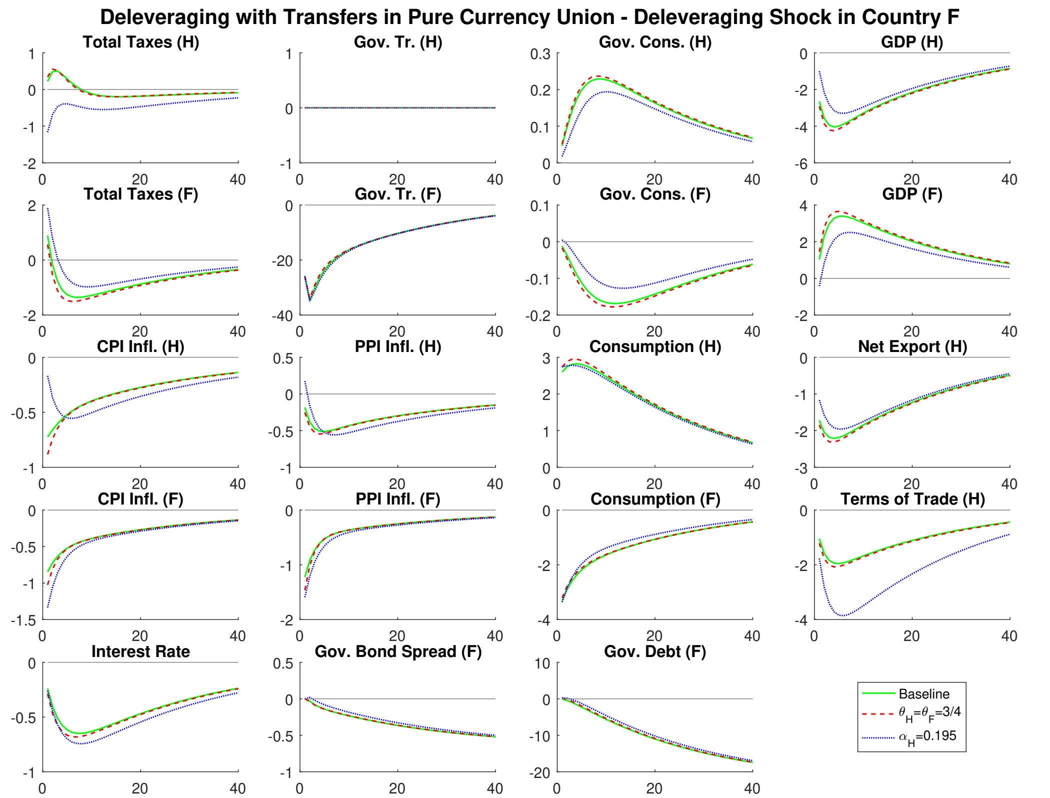
$$G^* + \tilde{T}^* + \left(\frac{1}{\beta} - 1\right)\tilde{B}^{*G} = Y^*(\tau^{*s} + \tau^{*w} MC^*) \tag{A.2.24}$$

B Sensitivity Analysis

Looking at the previous literature, such as Coenen et al. (2012), it turns out that the strength of international spillovers and the effect of having a supranational policy rate is affected by a few key parameters. For this reason, we decide to perform a robustness check to prove that our results hold even if some features of the model are different.

Since international spillovers mainly depend on the difference in price rigidity and on the difference in the openness to trade of the two countries, Figure 7 reports the impulse responses after a shock to the government debt target in a Pure Currency Union in which country F's deleveraging is financed with government transfers. The Figure compares the impulse responses in the baseline calibration (solid green line) to the case in which price rigidity is the same in both countries, ($\theta_H = \theta_F = 3/4$, as in the calibration for country H, dashed red line) and to the case in which both economies are almost closed (dotted blue line). This almost closed economy calibration is based on a lower value for the export–to–GDP ratio in country H (17.2%), which corresponds exclusively to intra-Euro Area exports. Such a calibration implies higher home biases for both countries (2 for country H and 1.42 for country F). We observe that openness drives our results more than the difference in price rigidity because the differences between the solid green lines and the dashed red lines are negligible, while several variables fluctuate less if the economies are almost closed ($\alpha = \alpha_H = 0.195$ and $\alpha* = \alpha_F = 0.148$). However, spillovers are sizeable even with a high home bias in consumption. In this case, the terms of trade and the interest rate are more volatile, while GDP in both countries is more stable. The higher volatility in the terms of trade does not translate

Figure 7: Sensitivity of Deleveraging with Transfers to θ and α in Pure Currency Union



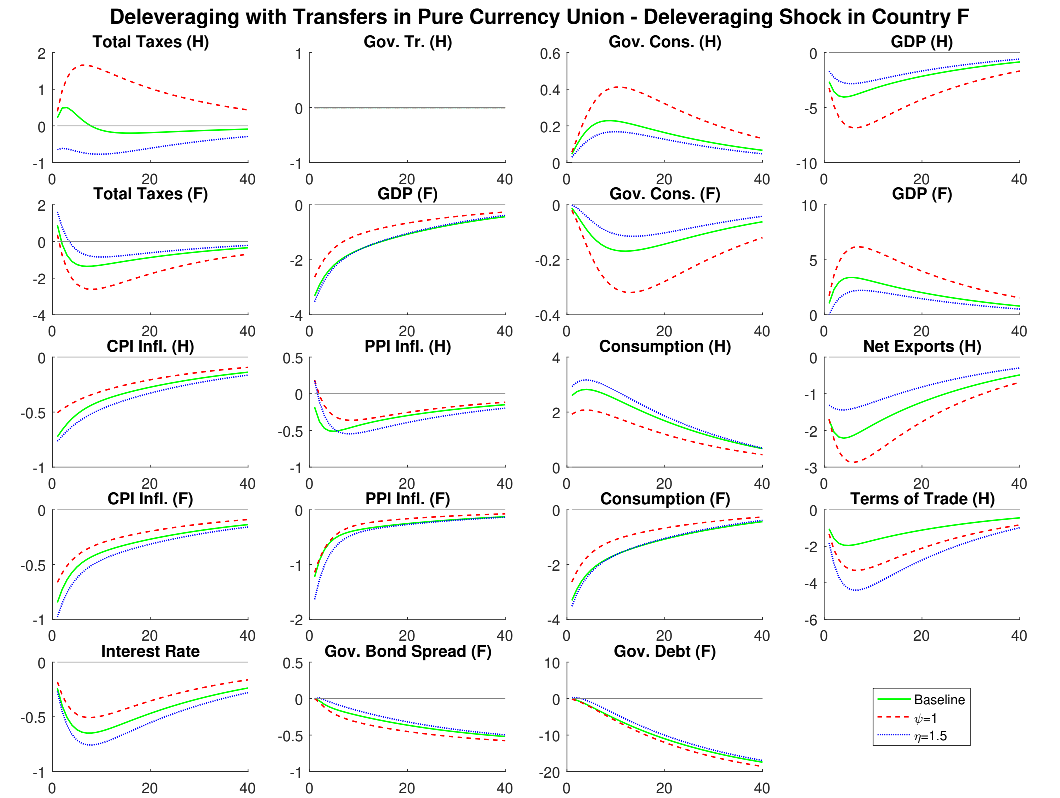
Quarterly values in % deviation from s.s. except Taxes, Interest Rate, Inflation Rates and Gov. Bond Spread in p.p. difference from s.s.

into a higher volatility in GDP because net exports are much less sensitive to movements in price differentials when the economies are almost closed.

The literature (see Hjortsø (2016) and Galí and Gertler (2010)) shows that the elasticity of substitution between domestic and imported goods is a key parameter for any open economy DSGE model, since it affects how demand for different goods responds to relative prices. This parameter, together with the Frisch elasticity of labour supply and the elasticity of intertemporal substitution, determines whether a higher trade openness has large or small effects on output and inflation, because all these parameters influence the slope of the labour supply curve in the same direction as openness does. Indeed, the more substitutable the goods (or the larger the share of imported goods) the flatter the curve and, hence, consumers change more their domestic and imported quantities in order to smooth aggregate consumption.

Figure 8 shows the impulse responses after a shock to the government debt target in a Pure Currency Union when deleveraging is achieved with government transfers. We can notice that if the international trade elasticity is low (dotted blue line), consumption moves more, but affects less movements in net exports, because they respond less to movements in the terms of trade, given a lower substitutability between international goods. As a consequence the response of GDP is

Figure 8: Sensitivity of deleveraging with Transfers to φ and η in a Pure Currency Union



Quarterly values in % deviation from s.s. except Taxes, Interest Rate, Inflation Rates and Gov. Bond Spread in p.p. difference from s.s.

smoother in both cases, while inflation and the interest rate move more. The opposite is true when the labour supply elasticity is high (dashed red line) because the movements in the terms of trade translate into more amplified movements in net exports, allowing consumption to be smoothed and inflation volatility to be reduced, but at the cost of higher volatility in output.