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ACCOUNTING FOR THE ROLE OF INVESTMENT FRICTIONS IN RECESSIONS

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Abstract

We conduct Business Cycle Accounting analyses for both the Euro Area and the United States. If the observed changes in the factor income shares reflect the frictionless competitive adjustment of productive factors, then we find that the capital-efficiency wedge was the main force driving the output growth slowdown during the u.s. Great Recession, with the labour and investment wedges being significant, but secondary forces. The countercyclical evolution of the labour-efficiency wedge helped to mitigate the output growth slowdown. Our results suggest that the investment frictions, which raise the firm's costs of investment, may be the primary cause of the u.s. Great Recession. However, in the u.s. 1982 Recession and the Euro Area Great Recession, the labour-efficiency wedge was the main driving force of the output growth slowdown, with the labour wedge being a significant, but secondary force and the investment wedge being negligible.

Keywords: Business Cycle Accounting, Capital-Efficiency Wedge, Labour-Efficiency Wedge, Labour Wedge, Investment Wedge, Resource Constraint Wedge, Productivity, Labour Share, Hours Worked, Great Recession.

JEL classification: E1, E3, O4.

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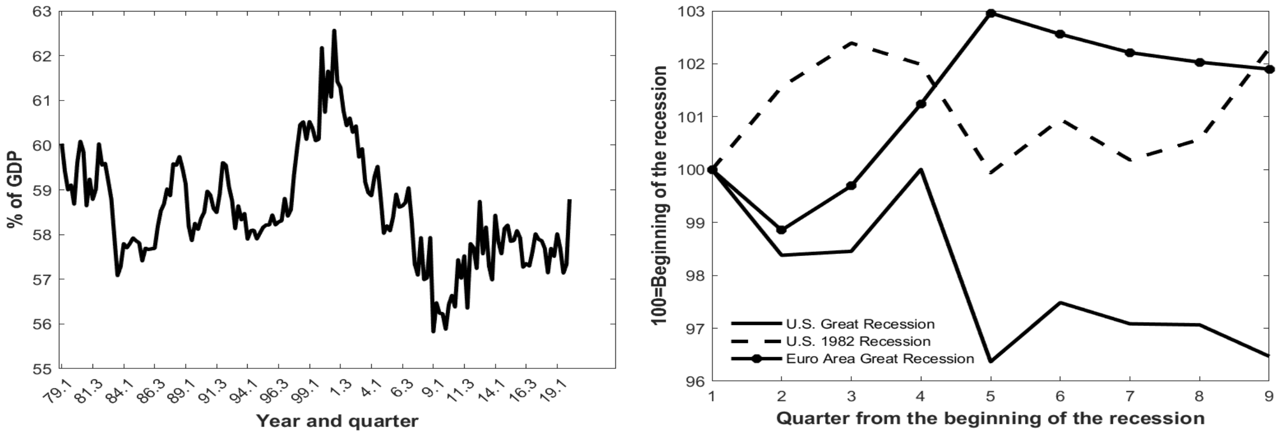
1 Introduction

The Business Cycle Accounting (BCA) method developed by Chari et al. (2002) and (2007) uses the equilibrium conditions of the dynamic stochastic general equilibrium (DSGE) model to measure the wedges representing the overall distortions to the relevant equilibrium conditions of the model. Usually, four wedges are computed (labour wedge, investment wedge, efficiency wedge and resource constraint wedge) and then fed back into the model one at a time to assess how much of the observed movements of macroeconomic variables can be attributed to each wedge. The BCA method allows revealing the mechanisms through which fundamental processes drive economic fluctuations. Therefore, it is a methodology for determining the most promising kind of theories regarding the primary characteristics of economic fluctuations and has been used in many countries and periods (see, for example, Brinca (2013) (2014)), Cavalcanti (2007), Chakraborty and Otsu (2013), Chakraborty (2006), Cho and Doblas-Madrid (2013), Kersting (2008), Kobayashi and Inaba (2006), Otsu (2010), Rodríguez-López and Solís-García (2016), and Sustek (2011)). Brinca et al. (2020) provide a summary of this literature.

The objective of this paper is to carry out a BCA exercise for the two main recent American recessions (the 1982 Recession and the Great Recession) and the Great Recession in the Euro Area (formed by 12 countries) with the aim of accounting for the role played by the efficiency wedges of labour and capital as well as discussing the relative importance of the investment and labour wedges. We perform our exercise under two alternative assumptions: (i) considering that either the marginal productivities of factors equal the factor rental prices and thus the labour wedge only reflects the gap between the real wage and the marginal rate of substitution between consumption and leisure, and (as argued by Karabarbounis (2014)) (ii) the marginal productivities of factors differ from the factor rental prices due to market-frictions or non-competitive forces (as implicitly is assumed by Chari et al. (2002) and (2007), and Brinca et al. (2016)) and thus, in addition, the labour wedge also reflects the gap between the marginal productivity of labour and the real wage.

We must underscore that we modify the Chari et al. (2007)'s BCA method, including in the analysis the factor income distribution. Therefore, unlike previous BCA exercises, we can compute two efficiency wedges, one for labour and another for capital. To compute both wedges, we make the model consistent with the trend and cyclical behaviour of the factor income shares. Computing two efficiency wedges can empirically be very relevant. If a single efficiency wedge is computed, its movements might conceal very different movements in the capital-efficiency and labour-efficiency wedges. Therefore, the same behaviour of the efficiency wedge might be explained by different mechanisms and theories. In particular, movements in the efficiency wedges of capital and labour might go in opposite directions and cancel each other out. In this case, we might conclude that changes in the efficiency wedge are not relevant in accounting for economic fluctuations, even if the changes in both the labour-efficiency and capital-efficiency wedges were playing a very active role. Indeed, we find that it might have been the case during the U.S Great Recession.

We assume that the marginal productivities of factors equal their rental prices to compute both efficiency wedges. Under this assumption, movements in the factor income shares are led by the frictionless competitive adjustment of the productive factors. However, Chari et al. (2007) implicitly assumes that the factor rental prices differ from the marginal productivities of factors. Moreover, they assume a Cobb-Douglas (CD) production function. Therefore, movements in the factor income shares are led by market-frictions or non-competitive forces moving away the factor rental prices from the marginal productivities of factors. This means that the labour and investment wedges computed by Chari et al. (2007) reflect movements in the factor income shares. This is not the case with the investment and labour wedges computed using our method (we clarify this point in the next section). That is not a mere theoretical curiosity because, from the fifties, the U.S. labour share underwent a significant decline as well as important oscillations (see Fig. 1, panel (a)). Moreover, the labour share underwent a significant decrease during the U.S Great Recession, but it increased during the Euro Area Great Recession and the U.S. 1982 Recession (see Fig. 1, panel (b)). Indeed, we find that assuming that the movements in the factor income shares are due either to market-frictions and non-competitive forces, or to the frictionless competitive adjustment of the productive factors have significant consequences for the computation of the labour and investment wedges and their effects for the economic fluctuations. Although many authors find evidence of a decline in the labour share in the United States and other countries, there is some



(a) The US labour share.

(b) The labour share in three recessions.

Fig. 1: The labour share.

discussion on the subject. Elsby et al. (2013) show that the labour share decreased in most U.S. industries during 1987–2011. In the same vein, del Río and Lores (2019) show that the U.S. labour share fell by 4.99% during 1998-2015. Moreover, the labour share in the U.S. private sector (i.e. excluding the government sector) fell by 6.25%. During this period, del Río and Lores (2019) find that the labour share fell in 40 of 60 industries of the U.S. private sector. Oberfield and Raval (2014) find that the decline in labour share for the U.S manufacturing sector was higher than the fall of the labour share in the whole economy. However, Gomme and Rupert (2004) find that the labour share of the U.S nonfinancial corporate business sector and the labour share of the U.S economy excluding government and housing sectors remained roughly stable from the end of the seventies to the early 2000s. Rognlie (2015) argues that the increase in the U.S net capital share from the mid-1980s turns out to come entirely from the housing sector, and Koh et al. (2020) argue that intellectual property capital entirely accounts for the decline in U.S. labour share, which is secularly constant for structures and equipment capital. Lawrence (2015)) argues that the decline in the ratio of effective capital to effective labour in a context of biased technical progress and gross complementarity between capital and labour can account for much of the recent fall in U.S. labour share at both the aggregate and industry levels. However, Karabarbounis and Neiman (2014) argue that the decline in the relative price of investment, which induces capital deepening in a context in which capital and labour are gross substitutes, explains roughly half of the observed decline in labour share. Some authors find that the decline of the labour share is not confined to the U.S.. For the OECD countries, Jones (2003) reports empirical evidence of an increase in the capital shares, and Cho et al. (2017) find that the labour share significantly decreased after the mid-1970s. Karabarbounis and Neiman (2014), using a dataset of 59 countries, find that the labour share in the corporate sector declined significantly in the majority of countries since the early 1980s. Rodriguez and Jayadev (2010) estimate a declining average trend in labour shares using an equally weighted set of 129 countries.

We specify a DSGE model with both the CD and Variable Elasticity of Substitution (VES) production functions to perform our BCA exercises. A VES production function as the one used in this work arises from a Constant Elasticity of Substitution (CES) production function with variable utilization of capital (see Section 3). Both VES and CES production functions were estimated by del Río and Lores (2019) for the U.S. economy. In both cases, they estimate that capital and labour are gross complements. Here, we use their estimates. Although there is some debate, empirical evidence increasingly indicates that the elasticity of substitution between capital and labour is lower than 1. Based on a literature survey, Chirinko (2008) concludes that the weight of the evidence suggests an elasticity of substitution in the range $0.4-0.6$. Our parametrization of the VES production function implies that the elasticity of substitution between capital and labour is around 0.7.

In the VES case, we assume that the marginal productivities of factors equal the factor rental prices. Therefore, the factor income shares are led by the frictionless competitive adjustment of factors. This assumption has two implications. First, it allows computing two efficiency wedges: one for capital and another for labour. Second, the labour wedge only reflects the gap between the marginal rate of substitution between consumption and leisure and the real wage. To this regard, we follow del Río and Lores (2021) who use a VES production function to compute two efficiency wedges in addition to the resource constraint, labour, and investment wedges in a neoclassical growth model to analyse the U.S. economic growth after the Second World War.

In the standard CD case, a single efficiency wedge can be computed. Moreover, since output elasticities for factors are constant, then changes in the factor shares must necessarily be driven by market-frictions or non-competitive forces moving away the factor rental prices from their marginal productivities. Therefore, in the CD case, the labour wedge reflects the gap between the marginal productivity of labour and the real wage in addition to the gap between the marginal rate of substitution between consumption and leisure and the real wage. However, Karabarbounis (2014) argues that explanations of the labour wedge based on departures of the representative firm's marginal productivity of labour from the real wage are rejected by data because the labour share of income is not strongly procyclical. In particular, Karabarbounis (2014) concludes: "As a result, models that generate volatile and countercyclical labour wedges by modifying the firm side of the neoclassical growth model are rejected by the data. The most promising explanations of the labour wedge should be able to generate large deviations between the real wage and the household's measured marginal rate of substitution".

We compare the results obtained with both specifications. The CD specification is the standard case in the literature. In particular, it is the specification used by Chari et al. (2007). Therefore, the wedges computed in the CD case corresponds to the wedges computed with the Chari et al. (2007)'s method.

If the observed changes in the factor income shares reflect the frictionless competitive adjustment of productive factors, then we find that during the U.S. Great Recession (i) the capital-efficiency wedge was the main force driving the output growth slowdown, (ii) the labour and investment wedges were significant, but secondary forces, and (iii) the countercyclical evolution of the labour-efficiency wedge helped to reduce output growth slowdown. [1] However, we find that, in the U.S. 1982 Recession and the Euro Area Great Recession, (i) the labour-efficiency wedge was the main driving force of the output growth slowdown, (ii) the labour was a significant, but secondary force, and (iii) the investment wedge was a negligible force.

Brinca et al. (2016) find that the efficiency wedge played a negligible role in the U.S. Great Recession. Arellano et al. (2019) build a model consistent accounting for the output growth slowdown during the U.S. Great Recession, consistently with the Brinca et al. (2016)'s findings. They developed a model in which a deterioration of the financial frictions is related to a worsening of the labour wedge and downturns in aggregate labour and output, with small movements in TFP. However, our results reveal

[1] del Río and Lores (2021) find that the productivity slowdowns of the U.S. economy in the seventies and after the end of the nineties were driven by the drop in the capital-efficiency wedge.

that the Brinca et al. (2016)'s result might be due to that the evolution of the efficiency wedge conceals movements of both the capital-efficiency and labour-efficiency wedges going in opposite directions and cancelling each other out. Moreover, as argued in the next section, financial frictions raising the firm's cost of investment can be manifested in either the capital-efficiency wedge or the investment wedge or both. Therefore, in contrast to the results obtained by Brinca et al. (2016), our results suggest that the frictions in capital markets of the kind proposed by the *investment-friction theory* might have been a prominent force driving the U.S. Great Recession.[2] Chari et al. (2002) called the models with financial frictions raising the firm's investment costs and causing investment-driven downturns in output the *investment-friction theory*. Some eminent examples of these models are Gertler (1989), Carlstrom and Fuerst (1997), Kiyotaki and Moore (1997), Bernanke et al. (1999) and Gertler and Kiyotaki (2009).[3]

The remainder of this paper is organized as follows. Section 2 discusses the implications of the specification of the production function in measuring the wedges, the relationship between the wedges and the prices of capital assets, and the relationship between the investment costs and both the capital-efficiency and investment wedges. Section 3 describes the model. In section 4, we describe the BCA procedure and the building of data. In section 5, we assess the role of the wedges in the three considered recessions. Finally, section 6 concludes.

2 Technology, prices and wedges

The BCA method developed by Chari et al. (2007) allows computing a single efficiency wedge reflecting Total Factor Productivity (TFP) as well as estimating an autoregressive stochastic process of order one for it. There are infinite combinations of the labour-efficiency and capital-efficiency wedges consistent with the efficiency wedge. Chari et al. (2007)'s method ignores the factor income distribution. However, introducing in the analysis an assumption on the factor income distribution (in particular, on the relationship

[2] As discussed in the next section, the drop of the capital-efficiency wedge is consistent with the observed decline of the prices of houses and capital assets during the U.S. Great Recession.

[3] Christiano et al. (2015) assert that the vast bulk of movements in economic activity during the Great Recession were due to financial frictions.

between the output elasticities for factors and the factor shares) allows identifying the particular combination of the labour-efficiency and capital-efficiency wedges consistent with the total efficiency wedge and the factor income distribution.

Consider the following neoclassical production function with constant returns to scale,

$$Y_t = A_t F(q_t K_t, z_t(1+\gamma)^t H_t) = A_t z_t F(\frac{q_t}{z_t} K_t, (1+\gamma)^t H_t), \tag{1}$$

Diamond and McFadden (1965)'s impossibility theorem states the impossibility of simultaneously identifying A, q and z. Chari et al. (2007) evade the impossibility assuming that the efficiency of capital relative to labour is constant for all t (i.e. $q_t = z_t = 1$ for all t). They compute A_t from (1) using data for Y_t, K_t and H_t. The single efficiency wedge computed by Chari et al. (2007) reflects Total Factor Productivity (TFP). Here, we drop the technological assumption on the constancy of the efficiency of capital relative to labour. We assume that q_t/z_t is such that the output elasticity for capital equals the capital share,

$$\varepsilon\left(\frac{q_t}{z_t}\frac{K_t}{H_t}\right) = \varepsilon_{k,t}. \tag{2}$$

Then, normalizing $A_t = 1$, we compute q_t and z_t from (1) and (2) using in addition data for factor income distribution. Computing q_t and z_t requires departing from the CD specification of the production function. The reason is that output elasticities for factors are constant. They do not depend on the ratio of effective capital to effective labour, $q_t K_t / z_t H_t$.

The single efficiency wedge computed with the Chari et al. (2007)'s BCA method is proportional to the weighted geometric average of both capital-efficiency and labour-efficiency wedges, $A_t = A_0 q_t^{\varepsilon_t} z_t^{1-\varepsilon_t}$ where A_0 is a constant. Consider the production function (1) with $A_t = 1$ and, alternatively, with $q_t = z_t = 1$. Differentiating, it follows that the growth rate of TFP ($g_{A,t}$) is

$$g_{Y,t} - \varepsilon_t g_{K,t} - (1-\varepsilon_t) g_{H,t} = g_{A,t} = \varepsilon_t g_{q,t} + (1-\varepsilon_t)\, g_{z,t}.$$

Therefore, integrating, it follows that A_t is proportional to the weighted geometric average of the capital-efficiency and labour-efficiency wedges.

The single efficiency wedge might conceal very different behaviours of the capital-efficiency and labour-efficiency wedges. For example, if g_q and g_z are moving in opposite directions, it might compute a low value for g_A, even if g_q, and g_z, are experiencing large variations. Therefore, to calculate $g_{A,t}$ ignoring $g_{q,t}$ and $g_{z,t}$ might lead to the wrong conclusion that changes in the efficiency wedges of the factors are not significant in accounting for economic fluctuations.

Computing $g_{A,t}$ requires the output elasticities for factors. Chari et al. (2007) assume a CD production function. Then, the output elasticities for labour and capital are constants ($\varepsilon_t = \varepsilon$) and

$$\widetilde{g}_{A,t} = g_{Y,t} - \varepsilon g_{K,t} - (1-\varepsilon)\, g_{H,t}.$$

Another alternative is to assume that marginal productivities of factors equal the factor rental prices. Therefore, the output elasticities for factors equal the factor shares (i.e., $\varepsilon_t = \varepsilon_{k,t}$ and $1-\varepsilon_t = \varepsilon_{h,t}$). In this case,

$$\overline{g}_{A,t} = g_{Y,t} - (1-\varepsilon_{h,t})\, g_{K,t} - \varepsilon_{h,t} g_{H,t}.$$

If the labour share, ε_h, goes down and the ratio of capital to worked hours is increasing, $g_{K,t} - g_{H,t} > 0$, then $\overline{g}_{A,t}$ decreases relative to $\widetilde{g}_{A,t}$. Therefore, assuming a constant output elasticity for labour might lead to underestimating the drop in the efficiency wedge, if the decline in the labour share is reflecting that output elasticity for labour is falling.

Moreover, as pointed out by del Río and Lores (2021), assuming constant output elasticities for factors might lead to overstating the fall of the labour wedge and to understating the fall of the investment wedge whether the labour share decreases. We focus on the labour wedge to clarify this point.

The first-order condition determining the time allocation of a household with additive log utilities of consumption, c_t, and leisure, $1-h_t$, is

$$\mu \frac{c_t}{y_t} \frac{h_t}{1-h_t} = \pi_{h,t}^s \varepsilon_{h,t}, \tag{3}$$

where $\mu > 0$ is the value of leisure relative to consumption, y_t is output, $\pi_{h,t}^s$ is the labour supply wedge reflecting distortions in the labour supply, and $\varepsilon_{h,t} = w_t h_t / y_t$ is the labour share. The first-order condition for labour of a firm is

$$\pi_{h,t}^d(1-\varepsilon_t) = \varepsilon_{h,t}, \tag{4}$$

where $\pi_{h,t}^d$ is the labour demand wedge, which reflects distortions of the labour demand, and $1-\varepsilon_t$ is the elasticity of output for labour. Combining equations (3) and (4), we have the total labour wedge

$$\pi_{h,t} \equiv \pi_{h,t}^s \pi_{h,t}^d = \frac{\mu}{1-\varepsilon_t} \frac{c_t}{y_t} \frac{h_t}{1-h_t}, \tag{5}$$

which reflects distortions in both the labour supply and demand. It follows from (3), (4), and (5) that the labour supply wedge, $\pi_{h,t}^s$, does not depend on the shape of the production function, but the demand labour wedge, $\pi_{h,t}^d$, and thus the total labour wedge, $\pi_{h,t}$, they do.

To compute the labour wedge, most of the authors (excepting del Río and Lores (2021), as far as we know) specify a CD production function and thus assume that the elasticity of output for labour, $1-\varepsilon_t$, is constant. In this case, movements in the labour share are driven by changes in the labour demand wedge π_h^d (see equation (4) and reflect frictions or non-competitive forces moving away the rental prices from their marginal productivities.

However, another alternative assumption is that output elasticity for labour equals the labour share (i.e. $1-\varepsilon_t = \varepsilon_{h,t}$). In this case, rental prices and marginal productivities move together and, thus, changes in the labour share are entirely driven by competitive forces modifying output elasticities for factors, see equation (4). Consequently, the labour wedge,

$$\pi_{h,t} = \pi_{h,t}^s = \frac{\mu}{\varepsilon_{h,t}} \frac{c_t}{y_t} \frac{h_t}{1-h_t}, \tag{6}$$

exclusively reflects distortions of the labour supply (i.e., $\pi_{h,t}^d = 1$). Nevertheless, to implement this identifying assumption, the CD production function must be laid aside and assume any other neoclassical production function with variable output elasticities for factors. Although the efficiency wedges do, the labour and investment wedges computed in this second way do not depend on the production function chosen.

In this article, we compute the labour and investment wedges under both assumptions.

It follows from (5) and (6) that the relationship between both labour wedges is

$$\pi_{h,t} = \pi^s_{h,t}\varepsilon_{h,t}/(1-\varepsilon),$$

where $\pi_{h,t}$ is the labour wedge computed under the CD assumption and thus by Chari et al. (2002) and (2007) and Brinca et al. (2016), whereas $\pi^s_{h,t}$ is the labour wedge computed under the VES assumption assuming that the marginal productivities of factors equal the factor rental prices (using a VES production function or any other with variable output elasticities for factors).

Therefore, when the labour share undergoes significant changes, the evolution of both labour wedges will significantly differ. In particular, if the labour share decreases (resp. increases), then $\pi_{h,t}$ decreases (resp. increases) regarding $\pi^s_{h,t}$.

Although it is not so evident, changes in the labour share (and then in the capital share) also affect the calculation of the investment wedge (see del Río and Lores (2021)) just in the opposite sense how they affect the labour wedge. In particular, if the labour share decreases (resp. increases), then the investment wedge computed assuming constant output elasticities for factors increases (resp. decreases) regarding the investment wedge computed assuming that the output elasticities for factors equal the factor income shares. To show this point, we assume that the depreciation rate of capital is 1 and there not exist adjustment costs of investment. The Euler equation of a perfect foresight model with additive log utilities of consumption and leisure implies that the investment wedge is given by

$$\pi^{-1}_{x,t} = \varepsilon_{t+1}\frac{y_{t+1}}{k_{t+1}}\frac{c_t}{c_{t+1}}\frac{\beta}{1+\gamma}$$

Brinca et al. (2016) assume a CD production function and then $\varepsilon_{t+1} = \varepsilon$ for all t. However, we assume that $\varepsilon_{t+1} = \varepsilon_{k,t+1}$. Therefore, the relationship between the investment wedge computed under the former assumption (and by Brinca et al. (2016)), $\widetilde{\pi}^{-1}_{x,t}$, and the investment wedge computed under the latter assumption, $\overline{\pi}^{-1}_{x,t}$, is

$$\frac{\widetilde{\pi}^{-1}_{x,t}}{\overline{\pi}^{-1}_{x,t}} = \frac{\varepsilon}{\varepsilon_{k,t}}.$$

If the depreciation rate of capital is lower than 1 or there exist adjustment costs of investment, then the previous relationship is only approximated.

We display the relationship between the wedges computed for the U.S. economy under the CD assumption and the VES assumption in Fig. 2. Consistently with the discussion above, the ratio is above (resp. below) the bisector during the Great Recession and below (resp. above) the bisector during the 1982 Recession in the case of the investment (resp. labour) wedge (see Fig. 2, panels (b) and (c)). During the Great Recession (resp. 1982 Recession), TFP computed in the CD case is higher (resp. lower) than the TFP computed in the VES (see Fig. 2, panel (a)). More interestingly, the relationship is horizontal during the U.S. Great Recession, which means that, in the CD case, TFP changed very little during the U.S. Great Recession, much less than in the VES case.

Now, we discuss the relationship between the capital-efficiency wedge, the investment wedge and the price of the capital assets. Consider that the capital services at time t are

$$Y_t = q_t K_t,$$

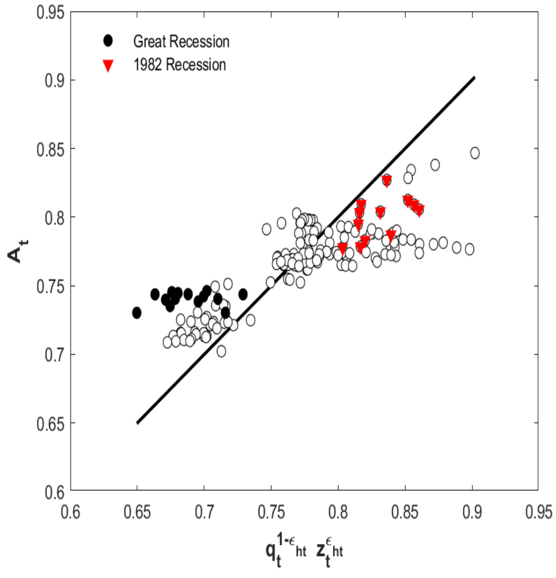
where q_t is the capital-efficiency wedge and K_t is the stock of capital which evolves according to

$$K_{t+1} = b_t I_t + (1 - \delta) K_t,$$

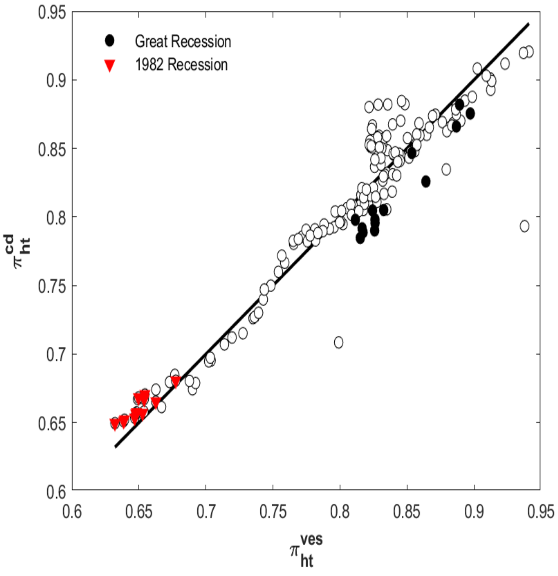
where b_t is the efficiency of investment and $0 < \delta < 1$ is the physical depreciation rate of capital. Under perfect competition, the price of capital assets at time t is

$$p_t = \frac{q_{t+1} b_t}{r_{t+1} + \delta_{e,t+1}},$$

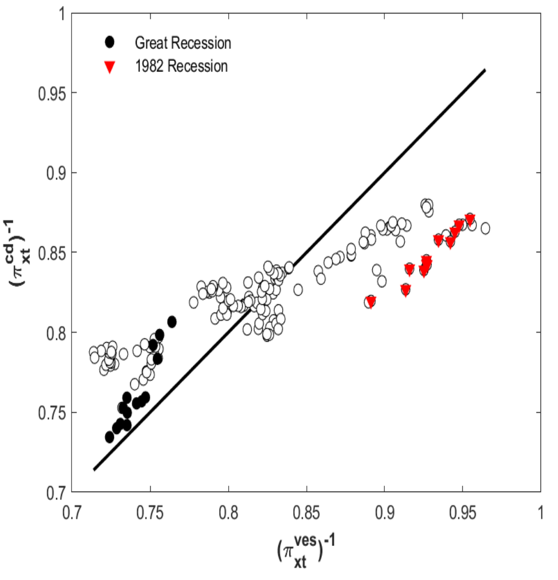
where $\delta_{e,t} = 1 - (1 - \delta) \frac{p_{t+1}}{p_t} \frac{b_t}{b_{t+1}}$ is the economic depreciation rate of capital. If $Y_t = C_t + I_t$ is the resource constraint, then both the evolution law of the stock of capital and the resource constraint can be rewritten in terms of the efficiency-adjusted investment, $X_t = b_t I_t$, as



(a) Total Factor Productivity.



(b) Labour Wedge.



(c) Investment Wedge.

Fig. 2: CD wedges vs. VES wedges.

follows:

$$K_{t+1} = X_t + (1-\delta)K_t$$

and

$$Y_t = C_t + \frac{1}{b_t}X_t,$$

which means that the investment wedge (which along this work is denoted by π_x^{-1}) at time t is $\pi_{x,t}^{-1} = b_t$. Therefore, according to the theory, the relative price of capital assets, p, is an increasing function of the capital-efficiency wedge and the investment wedge.

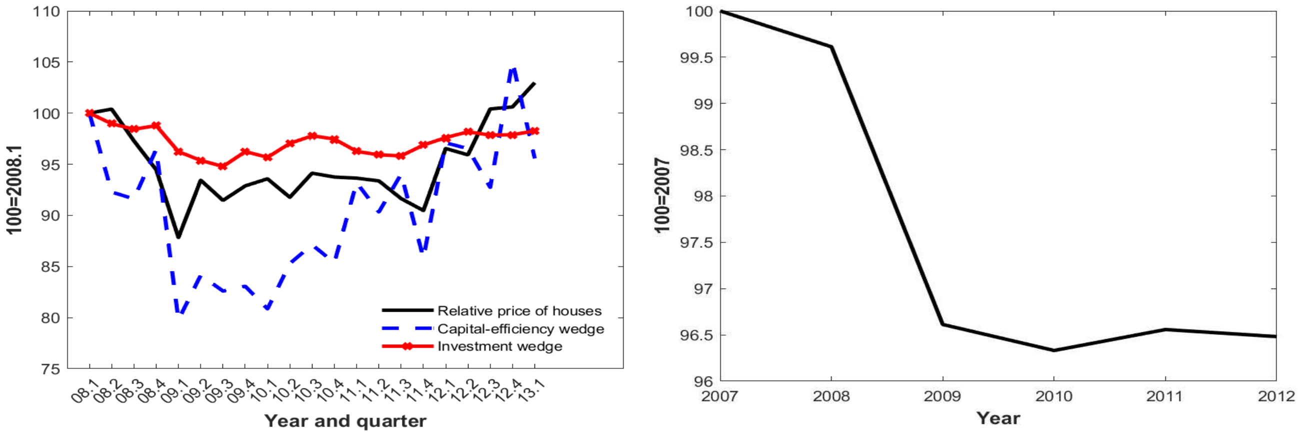
In Fig. 3, panel (a), we display the evolution of the U.S. relative housing price during the U.S. Great Recession, as well as the evolutions of the investment and capital-efficiency wedges computed below under the VES assumption. The evolutions of both wedges and the relative housing price are very related and consistent with theory. In Fig. 3, panel (b), we display the evolution of the relative price of U.S capital assets during the years of the Great Recession. Its drop is also consistent with theory and our computation of the wedges.[4]

As pointed out by Chari et al. (2002), most of financial frictions discussed by the *investment friction theory* end up affecting the economy by raising the firm's cost of investment, from 1 to $1+\tau_x$. Chari et al. (2002) claim that these costs of investment show up as an investment wedge in the BCA exercise. However, this statement must be qualified. We argue below that if the costs linked to the financial frictions are not included in the measured investment, the costs of financial frictions show up as an investment wedge. Conversely, if the cost of financial frictions are included in the measured investment, then these costs show up as a capital-efficiency wedge.

Consider the resource constraint

$$C_t + (1+\tau_{xt})X_{1,t} = Y_t, \tag{7}$$

[4]We have built the U.S. relative housing price deflating the Median Sales Price of Houses Sold for the United States (provided by the Federal Reserve Bank of St. Louis) by the implicit deflator of GDP. We use annual data from the National Income and Product Accounts (NIPA) to build the relative price of U.S. capital assets. First, we have built a price index of the capital assets dividing the NIPA current-cost net stock of fixed assets and consumer durable goods by the NIPA chain-Type Quantity Indexes for Net Stock of Fixed Assets and Consumer Durable Goods. Second, we have divided this index by the implicit deflator of GDP.



(a) The relative price of capital, q_t and $\pi_{xt}^{-}1$. (b) The relative price of capital assets.

Fig. 3: The relative price of capital, capital-efficiency wedge and investment wedge.

where C_t is consumption, $X_{1,t}$ is investment, $1 + \tau_{xt}$ are the investment costs,

$$Y_t = F(K_{1,t}, H_t), \tag{8}$$

is output, which is a function of labour H_t and capital $K_{1,t}$ that evolves according to

$$K_{1,t+1} = X_{1,t} + (1 - \delta)K_{1,t}. \tag{9}$$

If the measured investment is $X_{1,t}$ and the path of the stock of capital is built using $X_{1,t}$, then the investment wedge is $\pi_{x,t}^{-1} = (1+\tau_x t)^{-1}$. However, if the measured investment is $X_t = (1 + \tau_x)X_{1,t}$ and the path of the stock of capital is built using X_t, then the three previous equations can be rewritten as

$$C_t + X_t = Y_t, \tag{10}$$

$$Y_t = F(q_t K_t, H_t), \tag{11}$$

and

$$K_{t+1} = X_t + (1 - \delta_t)K_t, \tag{12}$$

where $q_t = (1+\tau_{x,t-1})^{-1}$ is the capital-efficiency wedge and reflects the costs of investment,

$K_t = (1 + \tau_{x,t-1})K_{1,t}$ is the stock of capital adjusted by the costs of investment and $1 - \delta_t = (1 - \delta)(1 + \tau_{x,t})(1 + \tau_{x,t-1})^{-1}$ is the economic depreciation rate of capital.

3 The Model

Our model is a neoclassical dynamic growth model with stochastic variables, henceforth called 'wedges'. These wedges are 'distortions' and represent policies and institutions which affect productivity, hours worked, capital accumulation, and resource constraint.

Output, Y_t, is allocated to consumption, C_t, investment, I_t, and other purposes, G_t. A perfectly competitive representative firm produces output according to a neoclassical production function with constant returns to scale. It uses capital, K_t, and labour, H_t, as productive factors. Labour equals time worked per worker, h_t, times the number of workers (which equals population), L_t: $H_t = h_t L_t$. The number of workers (or population) grows at the constant rate η, $\frac{L_{t+1}}{L_t} = 1+\eta > 0$. Detrended output per worker, $y_t = \frac{Y_t}{(1+\gamma)^t L_t}$, is given by

$$y_t = A_t f(q_t k_t, z_t h_t), \tag{13}$$

where $\gamma \geq 0$ is the rate of labour-augmenting technological progress, q_t is the *capital-efficiency wedge*, z_t is the *labour-efficiency wedge*, A_t is the (total) *efficiency wedge* and $k_t = \frac{K_t}{(1+\gamma)^t L_t}$ is detrended capital per capita.

The representative firm hires capital and labour to equalize its marginal productivities to their rental prices (r_t and W_t),

$$\varepsilon_t = r_t \frac{k_t}{y_t} \equiv \varepsilon_{k,t} \tag{14}$$

and

$$1 - \varepsilon_t = w_t \frac{h_t}{y_t} \equiv \varepsilon_{h,t}, \tag{15}$$

where $w_t = \frac{W_t}{(1+\gamma)^t}$ is detrended wage per worked hour, ε_t is output elasticity for capital

$$\varepsilon_t = \frac{f_1\left(\frac{q_t k_t}{z_t h_t}, 1\right)}{f\left(1, \frac{z_t h_t}{q_t k_t}\right)} \equiv \varepsilon\left(\frac{q_t k_t}{z_t h_t}\right) \tag{16}$$

and $1-\varepsilon_t$ is output elasticity for labour. According to the first-order conditions (14) and (15), the capital share, $\varepsilon_{k,t}$, equals the output elasticity for capital and the labour share, $\varepsilon_{h,t}$, equals the output elasticity for labour. Moreover, both factor shares sum to 1.

We consider two alternative specifications of the production function. The representative firm produces output according to either a VES or a CD production function. If the production function is VES, then output per worker is given by

$$y_t = \left[\alpha \left(q_t k_t\right)^{\omega\psi} \left(z_t h_t\right)^{(1-\omega)\psi} + (1-\alpha)\left(z_t h_t\right)^{\psi}\right]^{\frac{1}{\psi}},$$

where $\psi \leq 1$, $0 < \alpha < 1$, and $0 < \omega < 1$. The VES production function might result from a CES production function $y_t = \left[\alpha \left(q_t u_t k_t\right)^{\psi} + (1-\alpha)\left(z_t h_t\right)^{\psi}\right]^{\frac{1}{\psi}}$ where the utilization rate of capital is an increasing function of the ratio of efficient labour to efficient capital, $u_t = \left(q_t k_t / z_t h_t\right)^{\omega - 1}$. If the production function is CD, then detrended output per worker is given by

$$y_t = A_t k_t^{\varepsilon} h_t^{1-\varepsilon}$$

where $0 < \varepsilon < 1$ and A_t is the *efficiency wedge*. If the production function is VES, then output elasticity for capital is variable and given by

$$\varepsilon_t = \alpha\omega \left(\alpha + (1-\alpha)\frac{z_t h_t}{q_t k_t}\right)^{-1}$$

and if the production function is CD, then ouput elasticity for capital is constant and given by

$$\varepsilon_t = \varepsilon.$$

Remark. In general, the output elasticities for factors are functions of the capital-labour ratio (both factors adjusted by their efficiency). Therefore, assuming that the factor income shares equal the output elasticities for factors, it is generally possible to calibrate q_t/z_t. However, in the CD case, q_t and z_t cannot be identified because output elasticities for factors do not depend on the ratio q_t/z_t.

The resource constraint in terms of the detrended variables per capita is:

$$c_t + x_t + g_t = y_t, \tag{17}$$

where $c_t = \frac{C_t}{(1+\gamma)^t L_t}$ is detrended consumption per capita, $x_t = \frac{X_t}{(1+\gamma)^t L_t}$ is detrended investment per capita, and $g_t = \frac{G_t}{(1+\gamma)^t L_t}$ is the *resource constraint wedge*.

The move law of detrended capital per capita is

$$(1+\eta)(1+\gamma) k_{t+1} = x_t + (1-\delta) k_t - \frac{\phi}{2}\left(\frac{x_t}{k_t} - \kappa\right)^2 k_t \tag{18}$$

where $0 < \delta < 1$ is the economic depreciation rate of capital and capital accumulation includes quadratic adjustment costs: $\phi > 0$ and $\kappa > 0$.

The representative household at time t is composed of L_t members. Each member of the representative household is endowed with one unit of time, which can be shared between leisure, $1-h_t$, and labour, $0 < h_t < 1$, in return for a wage W_t. The intertemporal utility function of the representative household is

$$U_t = E_t \left\{ \sum_{t=0}^{\infty} L_t \beta^t \left[\log C_{L,t} + \mu \log(1-h_t)\right] \right\}$$

where $C_{L,t} = \frac{C_t}{L_t}$ is consumption per capita, $0 < \beta < 1$ is the discount factor, and $\mu > 0$ is the value of leisure relative to consumption. The household budget constraint is

$$L_t C_{L,t} + \pi_{x,t} X_t = \pi_{h,t} W_t h_t L_t + r_t K_t + B_t$$

where B_t are lump-sum transfers, $\pi_{h,t}$ is the *labour wedge*, and $\pi_{x,t}^{-1}$ is the *investment wedge*.

The first-order conditions characterizing a maximum of the household problem are

$$\frac{(1+\gamma)}{\beta}\frac{1}{c_t} = E_t \left\{ \frac{1+i_{t+1}}{c_{t+1}} \right\} \tag{19}$$

$$r_{t+1} = \frac{\pi_{x,t}(1+i_{t+1})}{1-\phi\left(\frac{x_t}{k_t} - \kappa\right)} + \frac{\pi_{x,t+1}}{1-\phi\left(\frac{x_{t+1}}{k_{t+1}} - \kappa\right)} \cdot$$

$$\cdot \left[\frac{\phi}{2}\left(\frac{x_{t+1}}{k_{t+1}} - \kappa\right)^2 - \phi\left(\frac{x_{t+1}}{k_{t+1}} - \kappa\right)\frac{x_{t+1}}{k_{t+1}} - (1-\delta_{t+1})\right] \tag{20}$$

and

$$\mu \frac{c_t}{1-h_t} = \pi_{h,t} w_t \tag{21}$$

Here i_{t+1} is the interest rate at time $t+1$. Equation (19) is the Euler equation, according to which expected (discounted) marginal utilities are equal over time. Equation (20) establishes that the rental price of capital equals its user cost which, in addition to the interest rate and the economic depreciation rate, also includes the investment wedge and the investment adjustment costs. Equation (21) states that the marginal rate of substitution between consumption and leisure equals the wage adjusted by the labour wedge.

The exogenous states (the wedges) follow a five or four dimensional vector autoregressive of order one where the error process is assumed to be multivariate normal with mean zero and variance and covariance matrix $V = QQ'$, as described below:

$$s_{t+1} = P_0 + P s_t + Q v_{t+1} \tag{22}$$

in which $s_t = (\log q_t, \log z_t, \log \pi_{ht}, \log \pi_{xt}, \log g_t)$ in the VES case and $s_t = (\log A_t, \log \pi_{ht}$, $\log \pi_{xt}$, $\log g_t)$ in the CD case.

The system of equations (13)-(22) characterizes the equilibrium of the economy.

4 The business cycle accounting procedure

Given the values for the parameters in Table 1, the model is solved for the steady-state quantities and the equilibrium is found. Equilibrium decision rules are derived assuming that the exogenous states follow the previous five or four dimensional vector autoregressive of order one where the matrices of autoregressive coefficients P_0 and P can be estimated.

The data are used as observables and the Kalman filter is used to back out the wedges. The procedure involves: (i) solving the model for steady state quantities; (ii) computing decision rules by log-linearization around the steady state; and (iii) building a state space representation of the model, with a matrix for the laws of motion for the state variables, which are subject to Gaussian innovations and a matrix with the optimal choices for output, hours, investment, government consumption and the output elasticity for labour as a function of the states. The likelihood of the innovations being jointly normal is computed and the optimization program concerns the choice of the parameters of the VAR, i.e., the matrices P and Q, such that the likelihood is maximized. The vector P_0 is set such that $P_0 = (I - P)^{-1}E[s_t]$. We explain the choice of $E[s_t]$ in the next section. For a detailed list of different MLE estimation methods see Adjemian et al. (2022) and the references contained therein.

We extend the procedure proposed by Chari et al. (2007) in the wedge measurement step when the VES technology is assumed. Like them, we measure the government consumption wedge directly from the data, as explained in section 4.2. To obtain the values of the other wedges, we use the data and the model's decision rules. With y_t^d, h_t^d, x_t^d, g_t^d and ε_t^d denoting the data for the model variables and the observed labour share. $y(k_t, s_t)$, $h(k_t, s_t)$, $x(k_t, s_t)$ and $\varepsilon(k_t, s_t)$ denoting the decision rules of the model and being k_0^d an initial condition for capital, the realized wedge series s_t^d solve the system given by

$$y_t^d = y(k_t^d, s_t^d), \quad h_t^d = h(k_t^d, s_t^d), \quad x_t^d = x(k_t^d, s_t^d), \quad \varepsilon_t^d = \varepsilon_t \left(k_t^d, s_t^d\right) \tag{23}$$

with $g_t = g_t^d$ and

$$(1+\eta)\,(1+\gamma)\,k_{t+1}^d = x_t^d + (1-\delta)\,k_t^d - \frac{\phi}{2}\left(\frac{x_t^d}{k_t^d} - \kappa\right)^2 k_t^d, \quad \text{given } k_0^d$$

We use these values for the wedges in our experiments. Finally, we perform simulations to see to what extent models with just one wedge or a combination of wedges can replicate observed data. Hence, new decision rules are computed, setting the wedges, that are excluded in a specific simulation exercise, to their unconditional mean values throughout the simulation procedure. Since they no longer are random variables in the simulations, the computation of new equilibrium decision rules and allocations in the simulated economies ensures that the agent's expectations are consistent with the model.

4.1 Parameterization and Calibration

The parameters held fixed across U.S. and the Euro Area are as follows: the annualized discount factor $\beta = 0.975$, the annualized depreciation rate $\delta = 0.05$, and the relative value of leisure $\mu = 2.5$. We set $\psi = -2$ and $\omega = 0.5$, which are values near del Río and

Lores (2019)'s estimates. In particular, these authors estimate $\psi = -1.9$ and $\omega = 0.36$. We consider a value of ω higher than the estimated value above of del Río and Lores (2019) because to have well-defined series for z_t and q_t, it is necessary that $\omega > \varepsilon_t$ for all t. Other parameters are specific to the U.S. or the Euro Area: η is the average growth rate of population, γ the average growth rate of output per capita, and ϕ and κ the adjustment cost coefficients. We normalize so that detrended log output has a mean value of zero over the sample period.

If κ equals the investment-capital ratio along a BGP, then it follows from (18) that $\kappa = (1+\eta)(1+\gamma) - (1-\delta)$. The adjustment costs function is quadratic, which is typical in macroeconomic literature and is also used in Brinca et al. (2016). To perform the quantitative analyses, we follow Brinca et al. (2016) and set $\phi = 0.25/\kappa$ to obtain the elasticity of the price of capital regarding the investment-capital ratio of 0.25. Parameter α is set so that $\alpha\omega$ equals the sample average for capital share. Therefore, when $\psi \longrightarrow 0$, the VES production function converges to a CD production function with output elasticity for capital $\alpha\omega$.

The mean values of exogeneous states $E[s_t]$ are calculated by solving the equation system (13)-(22) when $v_t = E[v_t] = 0$ to reproduce the sample averages for detrended hours worked, detrended output per capita, investment rate, consumption to output rate and capital share. In the CD case, it is only calibrated a mean value for one efficiency wedge, A. To compare the VES and CD economies, we choose the CD production function parameter ε equal to the output elasticity for capital in the BGP of the VES economy. Table 1 summarizes the parameters held constant across countries and those that are specific to U.S. and the Euro Area.

4.2 Data

Here, we describe the data used in our quantitative analysis. The Euro Area is made up of 12 countries (Austria, Belgium, Finland, France, Germany, Greece, Ireland, Italy, Luxemburg, Netherlands, Portugal, and Spain). The value of a variable in the Euro Area is the sum of the values of that variable in the 12 countries. Data for the United Stats are provided by the National Income and Products Accounts (NIPA) and for Europe by Eurostat (EU).

Taxes

United States and *Euro Area*. We follow Prescott (2004) to compute taxes. We compute net direct taxes on personal expenditures in consumption and services, T_N^p, as a fraction of taxes on production and imports plus subsidies $T_N^p = \mu T_N$, where $\mu = \left(\frac{2}{3} + \frac{1}{3}\left(G_N^d + G^{nds}\right) / \left(G_N^d + G^{nds} + G_N^x\right)\right)$, G_N^{nds} are nominal expenditures in nondurable consumption and services, G_N^d are nominal expenditures in durable consumer goods, and G_N^x is nominal EU gross capital formation (resp. nominal NIPA gross private domestic investment). Durable consumer goods are excluded from consumption and included in investment, and taxes on nondurable consumption and services, T_N^c, are then $T_N^c = T_N^p G_N^{nds} / \left(G_N^{nds} + G_N^d\right)$, while taxes on durable consumption are $T_N^d = T_N^p G_N^d / \left(G_N^{nds} + G_N^d\right)$. Net direct taxes on investment equal the taxes on production and imports plus subsidies less taxes on nondurable consumer goods and services: $T_N^x = T_N - T_N^c$.

Services of durables consumer goods and nominal output

United States and *Euro Area*. The nominal stock of durable goods is computed using the perpetual method of inventory, $K_{t+1}^d = G_{N,t}^d - T_{N,t}^d + (1 - \delta_d) K_t^d$ where $\delta_d = 0.0694$ and $K_0^d = \left(G_{N,0}^d - T_{N,0}^d\right) / (g_d + \delta_d)$, where g_d is the average quarterly growth rate of $G_N^d - T_N^d$ during the sample period for the United States (resp. the Euro Area). The flow of nominal services of durable consumer goods is $S_N^d = (i + \delta_d) K^d$, where $i = 0.00985$ is the interest rate. Nominal output, Y_N, is nominal NIPA (resp. EU) GDP, GDP_N, less NIPA (resp. EU) taxes on production and imports plus subsidies, T_N, plus the flow of services of durable consumer goods, S_N^d: $Y_N = GDP_N - T_N + S_N^d$.

Consumption and investment

United States and *Euro Area*. From the expenditures on nondurables and services we subtract taxes and, following Brinca et al. (2016), add the flow of services of durable consumer goods, whereupon nominal consumption is $C_N = G_N^{nds} - T_N^c + S_N^d$. Following Brinca et al. (2016), we add personal expenditures in durable consumer goods to investment and take away taxes. Nominal Investment in the Euro Area is $X_N = G_N^x - T_N^x + G_N^d$ and in the United States is $X_N = G_N^x + G_N^{gx} - T_N^x + G_N^d$, where G^{gx} is nominal NIPA government gross investment.[5]

Population and labour

United States. We follow Cociuba et al. (2018) in measuring population and worked hours.Population, L, is the civilian noninstitutional population aged $16-65$ years (data are taken from the Bureau of Labour Statistics) plus military personnel on active duty (data are from the Defense Manpower Data Center, Office of the Secretary of Defense, and U.S. Department of Defense). Worked hours, H, are total persons at work 16 years and over multiplied by the average hours worked per week by total persons at work and by the number of weeks of a quarter, $\frac{52}{4}$. Series of persons at work and average hours worked per week are provided by the Bureau of labour Statistics. Therefore, worked hours per capita are $\frac{H}{L}$ and the fraction of time devote to work, h, equals the worked hours per capita divided by the available hours per person ($\frac{2}{3}$ of the total hours, 2184 hours per quarter, $H_A = 1456$), $h = H/(LH_A)$.

Euro Area. Population, L, is the EU population aged $16-65$ years for the 12 countries of the Euro Area. Worked hours are $H = (v_s L_s + v_e L_e)\,\frac{52}{4}$, where L_s the EU self-employed workers, L_e are EU employees, v_s are EU average week hours worked by self-employed worker, v_e are EU average week hours worked by employees, and $\frac{52}{4}$ is the number of weeks of a quarter. v_s and v_e are the weighted averages of the average week hours in the 12 countries of the Euro Area, being the weights the self-employed workers in the country and the employees in the country relative to the total self-employed workers and the total employees in the Euro Area. Therefore, worked hours per capita are $\frac{H}{L}$ and the fraction of time devoted to work, h, equals the worked hours per capita divided by the available hours per person ($\frac{2}{3}$ of the total hours, 2184 hours per quarter, $H_A = 1456$), $h = H/(LH_A)$.

[5]Under our definitions of consumption and investment, the resource constraint wedge, g, equals government consumption plus net exports (both in terms per capita and detrended).

Factor shares

United States. We construct the factor shares using an economy-wide definition standard in the macroeconomics literature (see Koh et al. (2020)). Data are taken from NIPA. We split the proprietors' income into capital and labour incomes using the factor shares of the unambiguous income of the economy. In particular, with Unambiguous Capital Income $(UCI) = S_N^d$ + Rental Income + Corporate Profits + Net Interest + Current Surplus Government Enterprises + Business Current Transfers Payments + Statistical Discrepancy and Unambiguous Income $(UI) = UCI$ + Depreciation (DEP) + Compensation of Employees (CE), we define the factor share of the unambiguous capital income as $\theta = \frac{UCI+DEP}{UI}$ and the Ambiguous Capital Income as $ACI = \theta PI$, where PI is Proprietors' Income. Then, the capital share is $\varepsilon_K = \frac{UCI+DEP+ACI}{Y_N}$ and $\varepsilon_L = 1 - \varepsilon_K$ is the labour share.

Euro Area. The labour share is computed imputing to self-employed workers the same wage per hour as employees, $\varepsilon_L = W_N/Y_N\,(1 + v_s L_s/v_e L_e)$, where W_N is the EU compensation of employees.

Real variables

United States and *Euro Area.* Real output, real consumption, and real investment are computed deflating the corresponding nominal magnitudes by the implicit deflator of GDP (provided by NIPA or EU): $Y = Y_N/P$, $C = C_N/P$, and $X = X_N/P$.

5 The economic recessions and the role of the wedges

In this section, we focus on the role of the wedges in the two main recessions that the U.S. economy faced after the World War II, excepting the recent COVID recession. The first is the recession in the late seventies, which we call the 1982 Recession, and the second is the Great Recession, which began in 2008. Furthermore, we also analyse the Great Recession in the Euro Area. The wedge alone components together with the corresponding observed variables (output, labour, and investment) are displayed in each panel of Figs. 4 to 9. The values of the ϕ-statistics are displayed in Table 2. The ϕ-statistic was defined by

Brinca et al. (2016) and is intended to capture how closely a particular component tracks the underlying variable.[6] The ϕ-statistic for $\mathbf{y}_{i,t}$ of wedge i is as follows,

$$\phi_i^{\mathbf{y}} = \frac{1/\sum_t \left(\mathbf{y}_{i,t} - \mathbf{y}_t\right)^2}{\sum_j \left(1/\sum_t \left(\mathbf{y}_{i,t} - \mathbf{y}_t\right)^2\right)},$$

where $\mathbf{y}_{i,t} \in \{h_{i,t}, y_{i,t}, x_{i,t}\}$ is the wedge-alone component of a variable due to wedge i and $\mathbf{y}_t \in \{h_t, y_t, x_t\}$ is the corresponding observed same variable. The growth rates from the peak to the trough of worked hours of the variables and their wedge-alone components for the three recessions are displayed in Table 3.

5.1 The U.S. Great Recession

CD case. The labour wedge was the main force driving the evolution of output during the U.S. Great Recession (see Fig. 5, panel (c)). As shown in Table 3, between 2008.1 and 2009.4 the decline in the labour wedge accounted for around 79% of the fall in detrended output per capita. As shown in Table 2, the value of the ϕ-statistic of the labour wedge for output is 0.60. The investment wedge also played a significant but secondary role in accounting for the evolution of output. Its ϕ-statistic for output is 0.24 and, between 2008.1 and 2009.4, its decline accounted for 50% of the fall in detrended output per capita (see Table 3). During the U.S. Great Recession, the main force driving the evolution of hours worked per capita was the labour wedge (its ϕ-statistic for labour is 0.94) while the main force driving the evolution of detrended investment per capita was the investment wedge (its ϕ-statistic was 0.68). However, the efficiency wedge played a negligible role; as shown in Table 2, the values of the ϕ-statistics of the efficiency wedge for labour, output and investment are 0.01, 0.11, and 0.07.

VES case. The main force driving the output growth slowdown during the U.S. Great Recession between 2008.1 and 2009.4 was the decline of the capital-efficiency wedge (see Fig. 4, panel (c)). As shown in Table 3, the fall in the capital-efficiency wedge accounted for 155% of the decline in detrended output per capita between 2008.1 and 2009.4, while

[6]The ϕ-statistic has the desirable feature that it lies in $[0, 1]$, sums to one across the four wedges, and when a particular output component tracks a variable perfectly, then its value is 1.

the drops of the investment and labour wedges accounted for 32% and 55%, respectively. The ϕ-statistic for output of the capital-efficiency wedge is 0.34, higher than the values of the ϕ-statistics for output of the other wedges (see Table 2). The contributions of the investment and labour wedges to the recession in terms of the evolution of output and investment were similar (see Fig. 4, panel (c)), but the drop of the labour wedge had a higher impact on the fall of hours worked (see Fig. 4, panels (d) and (e)). The fall in the investment (resp. labour) wedge between 2008.1 and 2009.4 accounted for 16% (resp. 65%) of the decline in hours worked per capita, 32% (resp. 55%) of the decline in detrended output per capita, and 31% (resp. 34%) of the decline in detrended investment per capita (see Table 3). As shown in Table 2, the ϕ-statistics for labour, output, and investment of the labour (resp. investment) wedge are 0.61, 0.33 and 0.16 (resp. 0.12, 0.22 and 0.17). As shown in Fig. 4 and Table 3, the countercyclical evolution of the labour-efficiency wedge contributed to reducing the fall of the labour, output, and investment during the U.S. Great Recession.

Comparing the specifications. In the VES case regarding the CD case, the role of the labour wedge is reduced. Therefore, it does not play a prominent, but a secondary, role in accounting for the evolution of output and the importance of the labour and investment wedges becomes quite similar. Furthermore, in the CD case, the role played by the fall in the efficiency wedge is negligible, but the VES case reveals that this result might be due to that the labour and capital-efficiency wedges were moving in opposite directions and cancelling each other out.

5.2 The 1982 Recession

CD case. The main force driving the evolution of output and investment during the 1982 U.S. Recession was the efficiency wedge (see Fig. 7, panels (c) and (e)). As shown in Table 2, the ϕ-statistic of the efficiency wedge for output is 0.72 and for investment is 0.71, higher than the values of the ϕ-statistics for output and investment of the other wedges. The drop in the efficiency wedge accounted for 68% and 66% of the falls in detrended output per capita and detrended investment per capita betTable 3). The labour wedge played a prominent role in accounting for the evolution of hours worked per capita. As shown in Table 2, its ϕ-statistic for labour is 0.54. The investment wedge played a negligible role in accounting for the evolution of labour, output and investment during the U.S. 1982 Recession (see Fig. 7, panels (c), (d) and (e)). Its ϕ-statistics for labour, output and investment are 0.16, 0.07 and 0.12 (see Table 2).

VES case. The labour-efficiency wedge was the main force driving the evolution of output and investment during the U.S. 1982 Recession (see Fig. 6, panel (c)). In particular, the drop in the labour-efficiency wedge accounted for 40% and 38% of the falls in detrended output per capita and detrended investment per capita between 1980.1 and 1982.4 (see Table 3). The ϕ-statistic of the labour-efficiency wedge for output is 0.44 and for investment 0.55 (see Table 2). The labour wedge played a prominent role in accounting for the evolution of hours worked (its ϕ-statistic for labour is 0.76) and a significant but secondary role in accounting for the evolution of output (its ϕ-statistic for output is 0.33). The investment and capital-efficiency wedges played negligible roles in accounting for the evolution of the variables during the 1982 U.S. Recession. The ϕ-statistics of the investment (resp. capital-efficiency) wedge for labour, output, and investment are 0.08,0:07 and 0.11 (resp. 0.05, 0.08 and 0.07).

Comparing the specifications. The labour wedge gains prominence in the VES case regarding the CD case. The VES case reveals that the fall in the efficiency wedge is mainly driven by the labour-efficiency wedge.

5.3 The Great Recession in the Euro Area

CD case. The main force driving the evolution of output and investment during the Euro Area Great Recession was the efficiency wedge (see Fig. 9, panels (c) and (e)). As shown in Table 2, the ϕ-statistic of the efficiency wedge for output is 0.56 and for investment is 0.35, higher than the values of the ϕ-statistics for output and investment of the other wedges. The drop in the efficiency wedge accounted for 68.2% and 44.5% of the falls in detrended output per capita and detrended investment per capita between 2008.1 and 2009.4 (see Table 3). The investment and labour wedges played significant but secondary roles in accounting for the evolution of output. The ϕ-statistic of the investment (resp. labour) wedge for output is 0.21 (resp. 0.17), see Table 2. The labour wedge played a prominent role in accounting for the evolution of hours worked per capita. As shown in Table 2, its ϕ-statistic for labour is 0.82.

VES case. The labour-efficiency wedge was the main force driving the evolution of output and investment during the Euro Great Recession (see Fig. 8, panels (c) and (e)). As shown in Table 2, its ϕ-statistic for output is 0.73 and 0.43 for investment (see Table 2). The labour wedge played a significant but secondary role in accounting for the evolution of output and investment (its ϕ-statistic is 0.17 for output and 0.24 for investment) and a prominent role in accounting for the evolution of labour (its ϕ-statistic for labour is 0.86). The investment wedge played a negligible role in accounting for the evolution of output and labour, but it played a more significant role in accounting for the evolution of investment (see Fig. 8, panels (c), (d) and (e)). The capital-efficiency wedge displayed a countercyclical behaviour. It played a negligible role in accounting for the evolution of labour, output, and investment during the Euro Area Great Recession (see Fig. 8, panels (c), (d) and (e)). As shown in Table 2, the ϕ-statistics of the investment (resp. capital-efficiency) wedge for labour, output, and investment are 0.06, 0.05 and 0.16 (resp. 0.02, 0.02 and 0.05).

Comparing the specifications. In the VES case regarding the CD case, the role of the labour wedge is strengthened and the role of the investment wedge is reduced. Furthermore, the VES case reveals that the fall in the efficiency wedge is mainly driven by the labour-efficiency wedge.

6 Conclusion

We conduct BCA exercises for the Euro Area and the United States under two alternative hypotheses. On the one hand, we assume that movements of the factor shares are driven by market-frictions or non-competitive forces reflected in the labour and investment wedges. This is the standard assumption in the BCA literature. On the other hand, we assume that movements in the factor shares are driven by the frictionless competitive adjustment of the productive factors. We modify the Chari et al. (2007)'s BCA method to compute two efficiency wedges, one for capital and another for labour. In particular, to compute both efficiency wedges, we require the model to be consistent with the trend and cyclical behaviour of the factor income shares. We have computed the contribution of both efficiency wedges to the evolution of labour, output, and investment during the Great Recession (in both the Euro Area and the United States) and the U.S. 1982 Recession.

If movements in factor shares are driven by market-frictions or non-competitive forces, our results broadly confirm previous findings by Brinca et al. (2016). In particular, we find that the labour wedge played a prominent role in accounting for the evolution of output in the U.S. Great Recession, whereas the investment wedge played a significant but secondary role and the role played by the efficiency wedge was negligible. However, we find that the efficiency wedge was the main force driving the evolution of output in the Euro Area Great Recession and the 1982 U.S. Recession.

If movements in factor shares are driven by the frictionless competitive adjustment of factors, the overall picture sharply changes. In particular, we find that, during the U.S. Great Recession, the capital-efficiency wedge played a prominent role in accounting for the output growth slowdown, whereas the countercyclical evolution of the labour-efficiency wedge contributed to mitigating it and the labour and investment wedges played significant but secondary roles. However, we find that the prominent force driving the output growth slowdown in both the Euro Area Great Recession and the 1982 U.S. Recession was the labour-efficiency wedge, whereas the labour wedge was a significant but secondary force and the investment wedge a negligible one.

Our results have three main implications. First, they show that the same evolution of the efficiency wedge might conceal very different behaviours of the capital-efficiency and labour-efficiency wedges and hence just analysing the impact of the efficiency wedge on the macroeconomic variables might lead to a poor understanding of the economic recessions. In particular, the negligible role played by the efficiency wedge in accounting for the evolution of output during the U.S. Great Recession found by previous works might be explained by the movement in opposite directions of both the labour and capital-efficiency wedges. Second, we find that the capital-efficiency wedge played a prominent role in accounting for the output growth slowdown during the U.S. Great Recession, whereas the role played by the investment wedge was significant but secondary, which means that the frictions in the capital markets of the kind proposed by the so-called *investment-friction theory* that produce investment-driven downturns in output might have been a prominent force driving the U.S. Great Recession. Finally, our results suggest that the U.S. Great Recession may have involved some propagation mechanisms significantly different from those working during the Euro Area Great Recession and the U.S. 1982 Recession because in the U.S. Great Recession, in addition to the significant role played by the investment wedge, the efficiency wedges moved in the opposite directions to how they did in the other two recessions.

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Table 1
MODEL PARAMETERS AND BGP VARIABLES (ANNUALIZED)

Model parameters held constant across countries			
Parameter	**Description**		
δ	Depreciation Rate of Capital	0.0500	
ψ	Production Function Parameter	-2.0000	
ω	Production Function Parameter	0.5000	
β	Discount Factor	0.9750	
μ	Relative Value of Leisure	2.5000	
Model parameters specific to each country		**USA**	**EA 12**
η	Population Growth Rate	0.090	0.0002
γ	Growth Rate of Output per Worker	0.0198	0.0017
α	Production Function Parameter	0.8318	0.7526
ϕ	Adjustment Cost Parameter	12.5574	18.9162
BGP **variables**			
h	Hours Worked per Capita	0.2403	0.2503
k/y	Capital-Output Ratio	3.5261	4.7830
x/y	Investment Rate	0.2808	0.2529
c/y	Consumption-to-Output Ratio	0.5931	0.5361
ε	Output Elasticity for Labour	0.5841	0.6253

Table 2

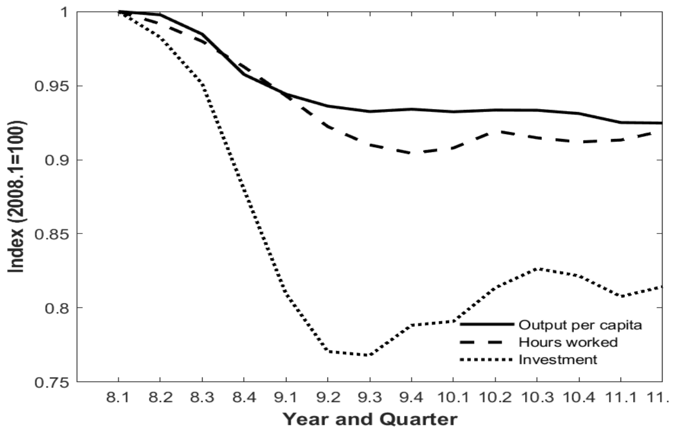
ϕ-STATISTICS

	VES					CD			
Variable	$\phi_q^{\mathbf{y}}$	$\phi_z^{\mathbf{y}}$	$\phi_{\pi_h}^{\mathbf{y}}$	$\phi_{\pi_x}^{\mathbf{y}}$	$\phi_g^{\mathbf{y}}$	$\phi_A^{\mathbf{y}}$	$\phi_{\pi_h}^{\mathbf{y}}$	$\phi_{\pi_x}^{\mathbf{y}}$	$\phi_g^{\mathbf{y}}$
	U.S. Great Recession								
h	0.14	0.07	0.61	0.12	0.06	0.01	0.94	0.04	0.01
y	0.34	0.02	0.33	0.22	0.08	0.11	0.60	0.24	0.06
x	0.56	0.02	0.16	0.17	0.09	0.07	0.17	0.68	0.08
	U.S. 1982 Recession								
h	0.05	0.06	0.76	0.08	0.04	0.23	0.54	0.16	0.07
y	0.08	0.44	0.33	0.07	0.08	0.72	0.15	0.07	0.06
x	0.07	0.55	0.18	0.11	0.09	0.71	0.11	0.12	0.07
	E.A12 Great Recession								
h	0.02	0.04	0.86	0.06	0.02	0.09	0.82	0.06	0.04
y	0.02	0.73	0.17	0.05	0.03	0.56	0.17	0.21	0.06
x	0.05	0.43	0.24	0.16	0.12	0.35	0.25	0.23	0.17

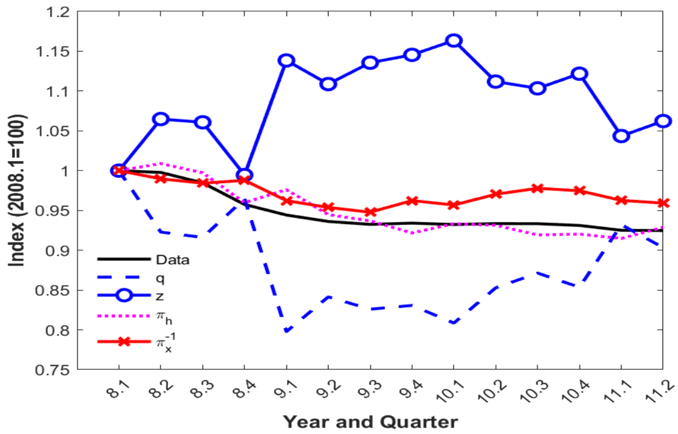
Table 3

WEDGE-ALONE COMPONENTS, DEVIATION FROM PEAK (HOURS TROUGH)

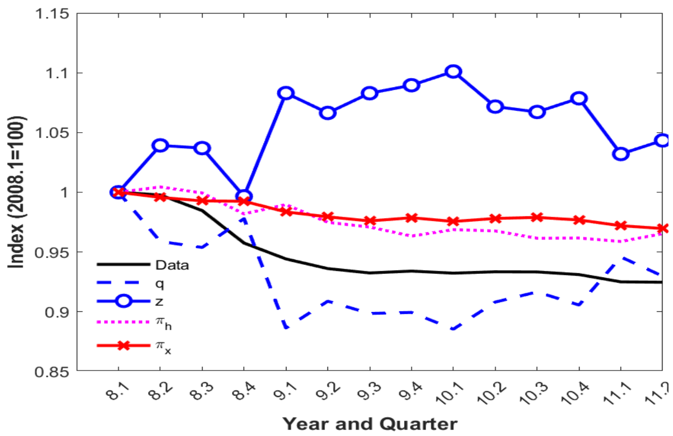
		VES					CD			
	Data	$\mathbf{y}_{qt}$	$\mathbf{y}_{zt}$	$\mathbf{y}_{\pi_h t}$	$\mathbf{y}_{\pi_x t}$	$\mathbf{y}_{gt}$	$\mathbf{y}_{At}$	$\mathbf{y}_{\pi_h t}$	$\mathbf{y}_{\pi_x t}$	$\mathbf{y}_{gt}$
		U.S. Great Recession								
h	-10.06	-2.72	0.75	-6.54	-1.63	1.94	0.38	-9.14	-4.36	2.22
y	-6.82	-10.59	8.58	-3.73	-2.15	0.83	-0.39	-5.40	-3.40	1.01
x	-23.79	-18.13	18.02	-8.08	-7.34	-2.17	0.49	-9.71	-17.05	-1.81
		U.S. 1982 Recession								
h	-5.99	-1.02	-0.47	-3.35	-2.96	0.24	-2.21	-2.64	-2.85	0.28
y	-9.61	-2.29	-3.86	-3.48	0.62	0.01	-6.52	-2.79	-0.91	0.06
x	-23.65	-5.37	-8.92	-4.43	-9.49	-0.23	-15.58	-3.39	-9.25	-0.21
		E.A12 Great Recession								
h	-3.40	1.06	-0.27	-4.11	-0.80	0.90	-0.70	-2.72	-0.05	1.09
y	-4.88	2.09	-4.64	-2.55	-0.48	0.54	-3.33	-1.63	-1.62	0.68
x	-17.83	5.82	-11.31	-6.36	-3.37	-2.11	-7.94	-4.19	-3.06	-1.64



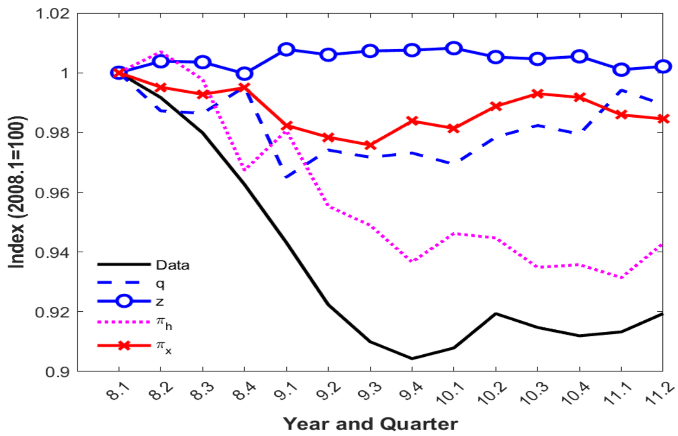
(a) Output per capita, hours worked, investment per capita.



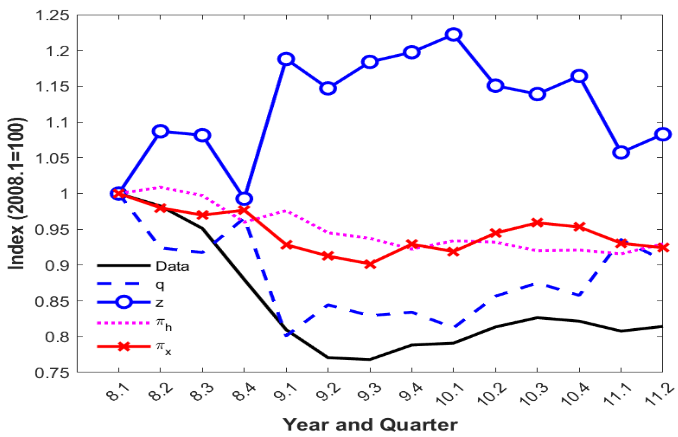
(b) Output per capita and four wedges.



(c) Output per capita and its components.

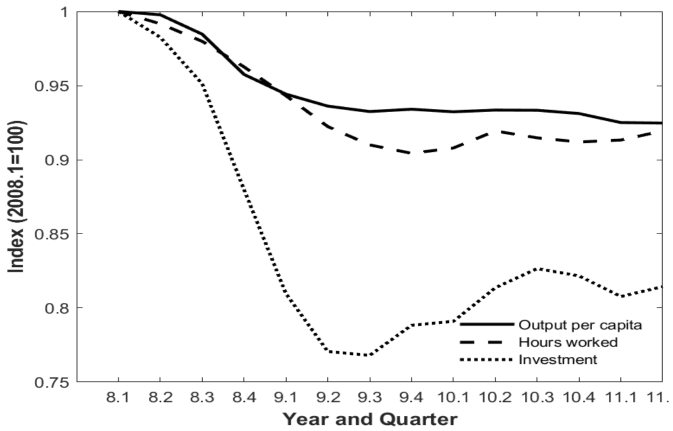


(d) Hours worked and its components.

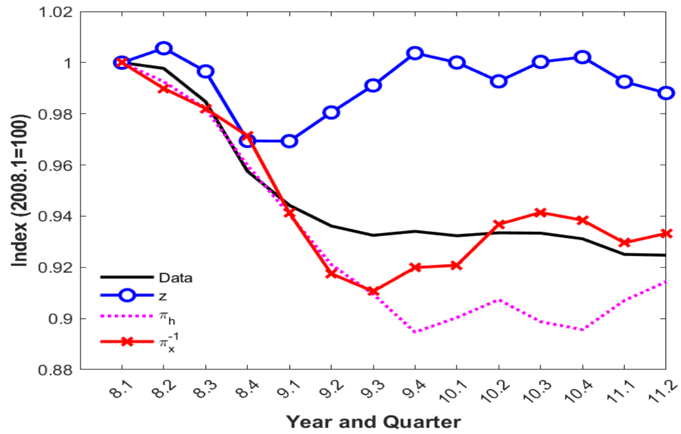


(e) Investment per capita and its components.

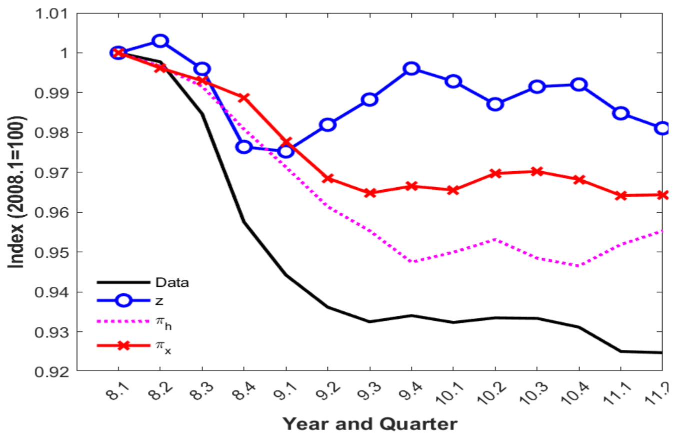
Fig. 4: The U.S. Great Recession with VES production function.



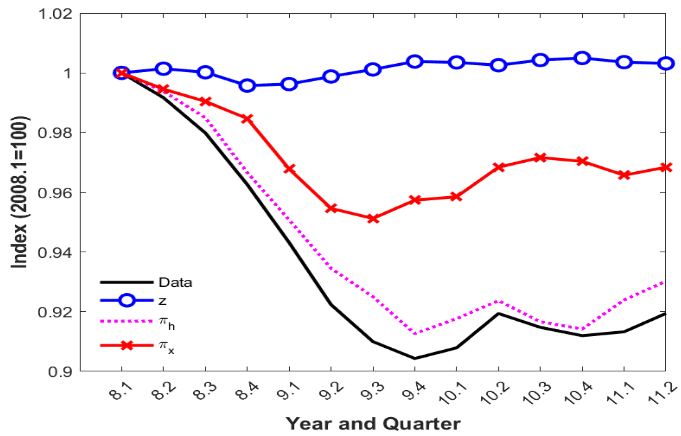
(a) Output per capita, hours worked, investment per capita.



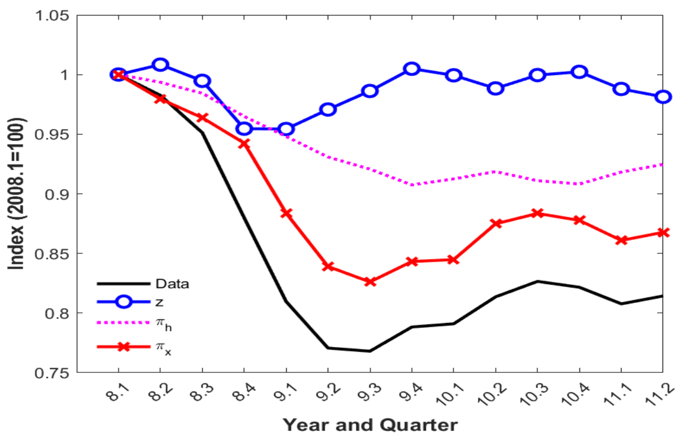
(b) Output per capita and four wedges.



(c) Output per capita and its components.

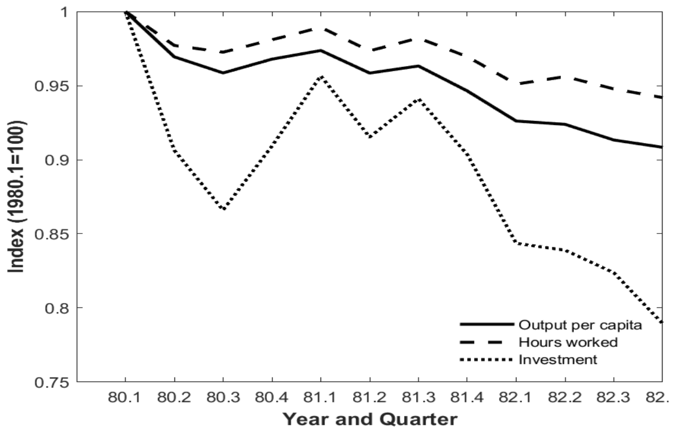


(d) Hours worked and its components.

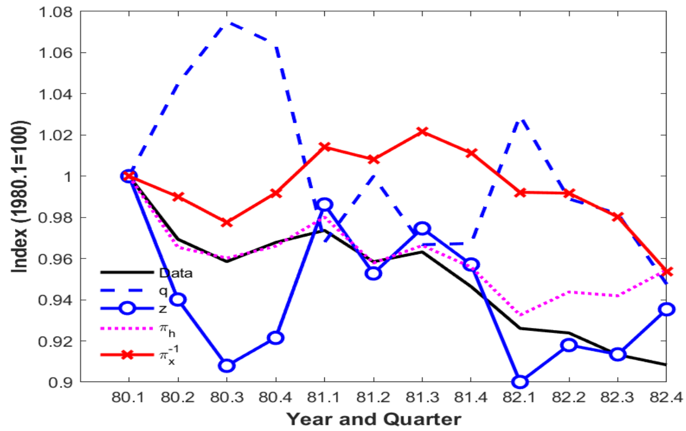


(e) Investment per capita and its components.

Fig. 5: The U.S. Great Recession with CD production function.



(a) Output per capita, hours worked, investment per capita.



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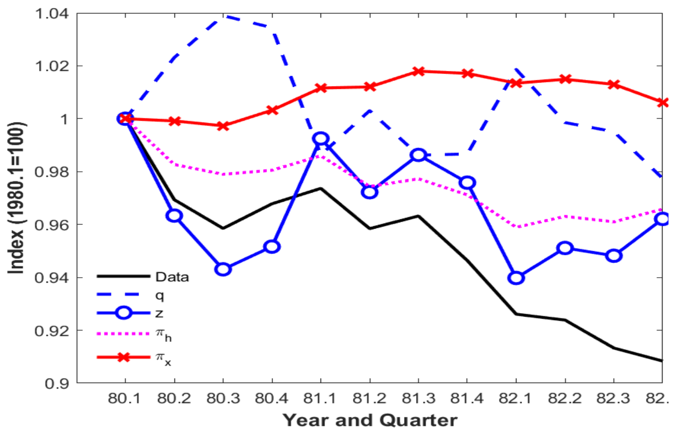
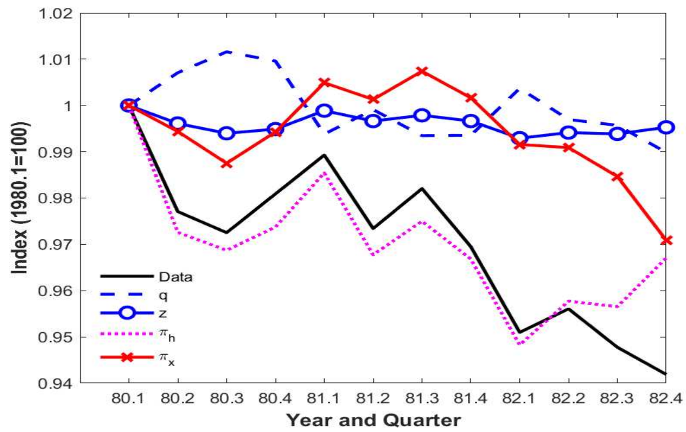
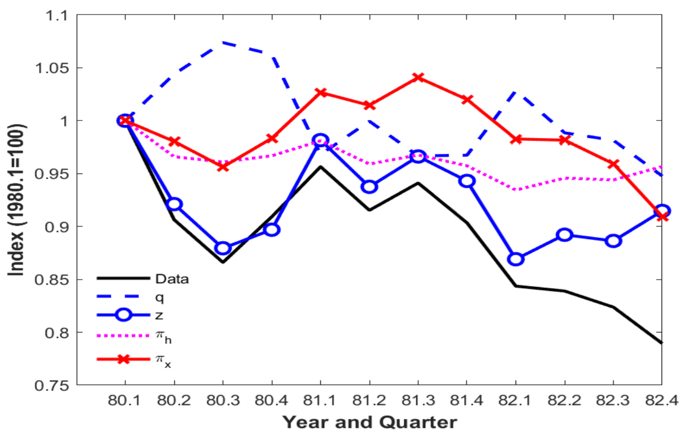


(c) Output per capita and its components.

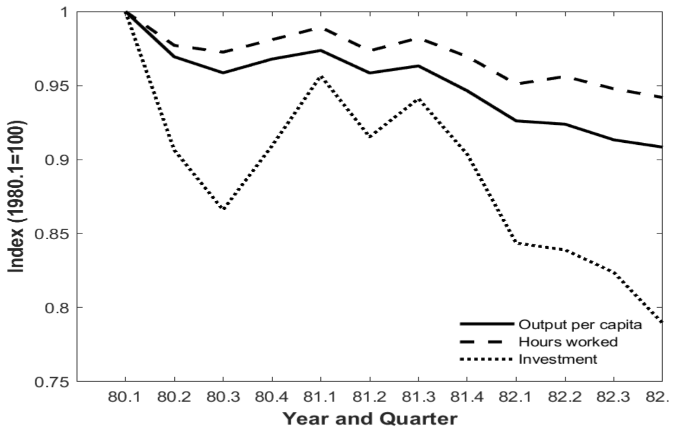


(d) Hours worked and its components.

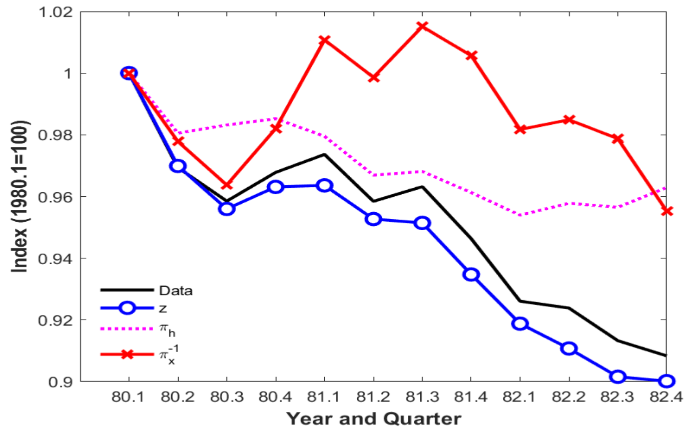


(e) Investment per capita and its components.

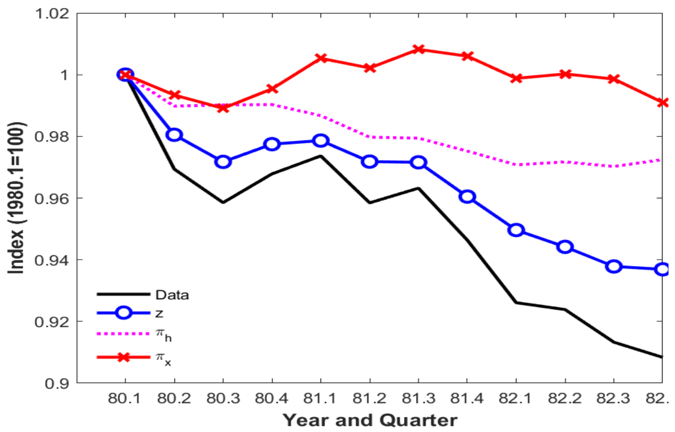
Fig. 6: The U.S. 1982 Recession with VES production function.



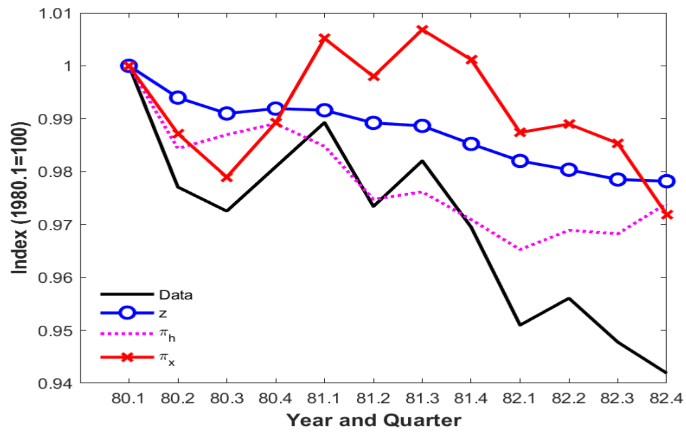
(a) Output per capita, hours worked, investment per capita.



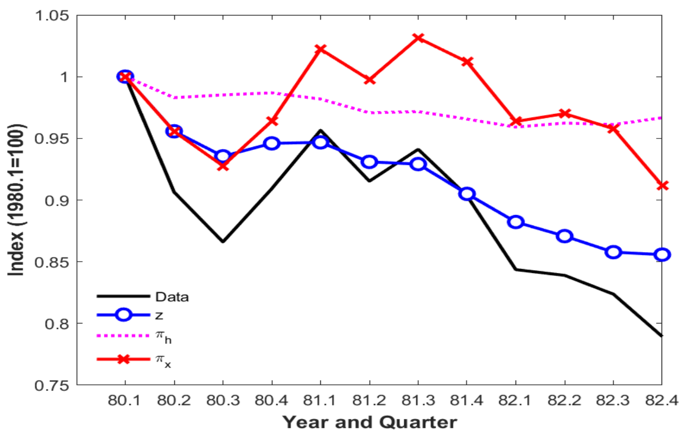
(b) Output per capita and four wedges.



(c) Output per capita and its components.

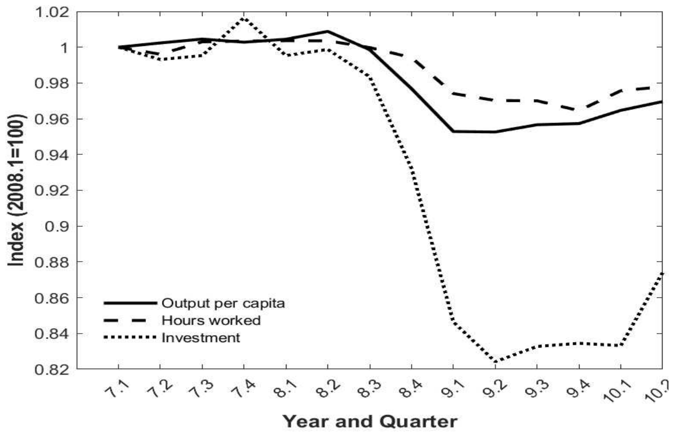


(d) Hours worked and its components.

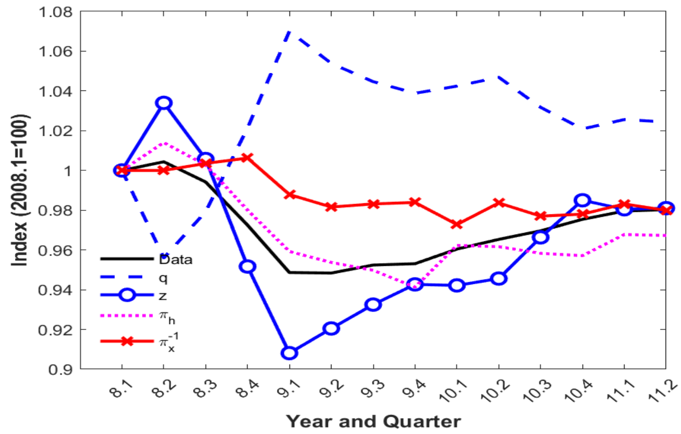


(e) Investment per capita and its components.

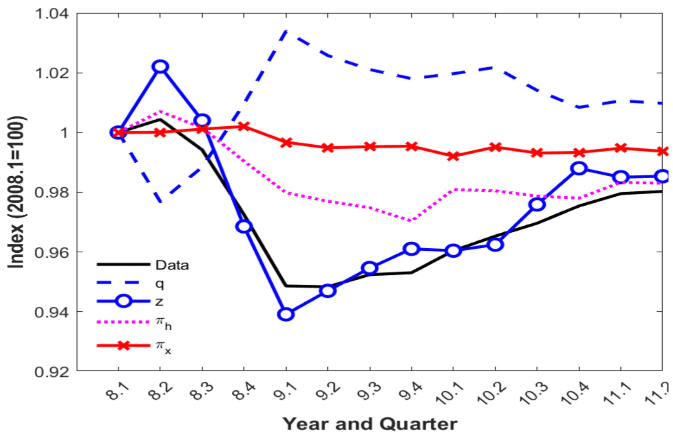
Fig. 7: The U.S. 1982 Recession with CD production function.



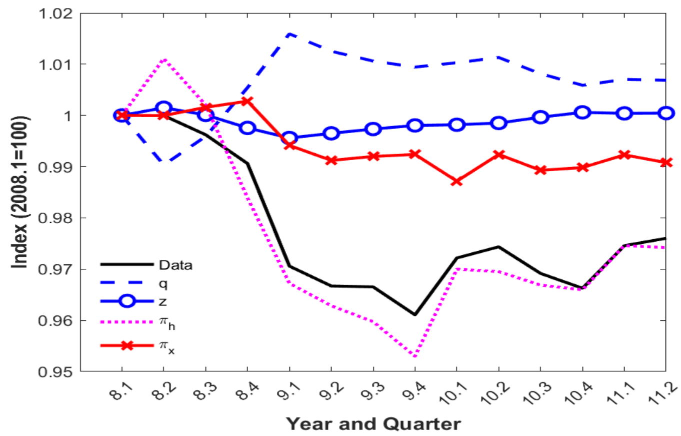
(a) Output per capita, hours worked, investment per capita.



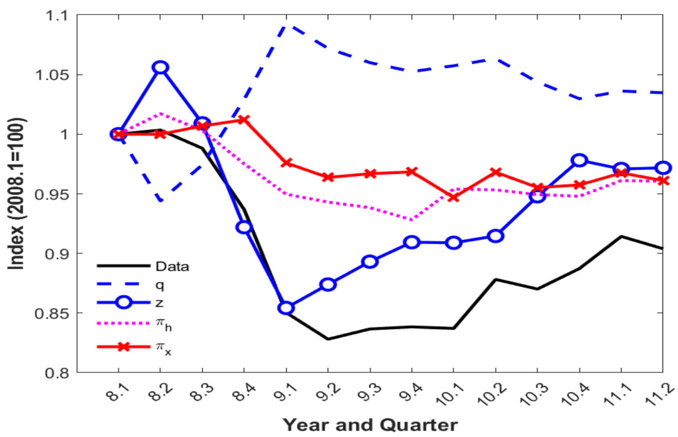
(b) Output per capita and four wedges.



(c) Output per capita and its components.

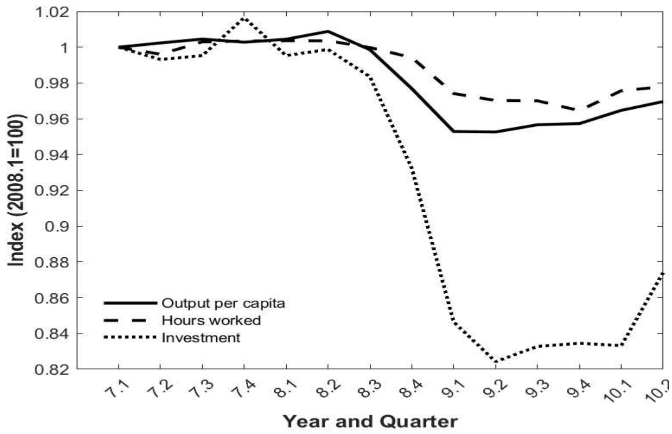


(d) Hours worked and its components.

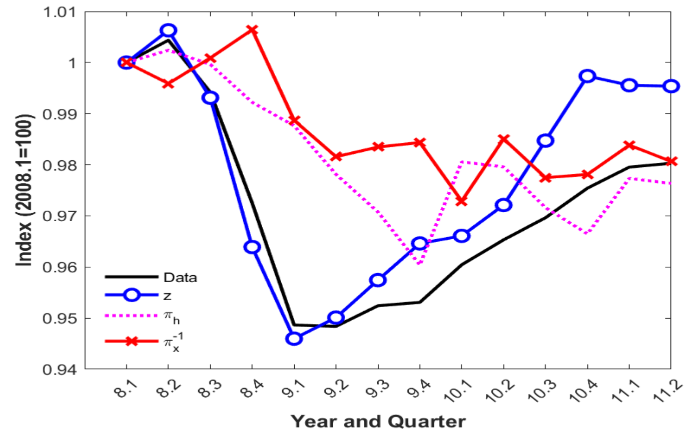


(e) Investment per capita and its components.

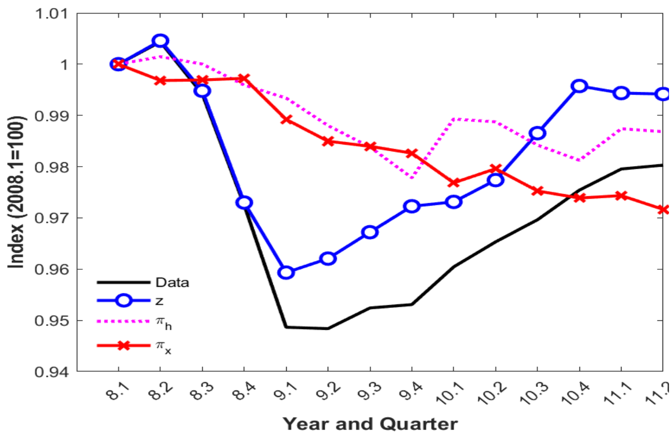
Fig. 8: The Euro Area 12 Great Recession with VES production function.



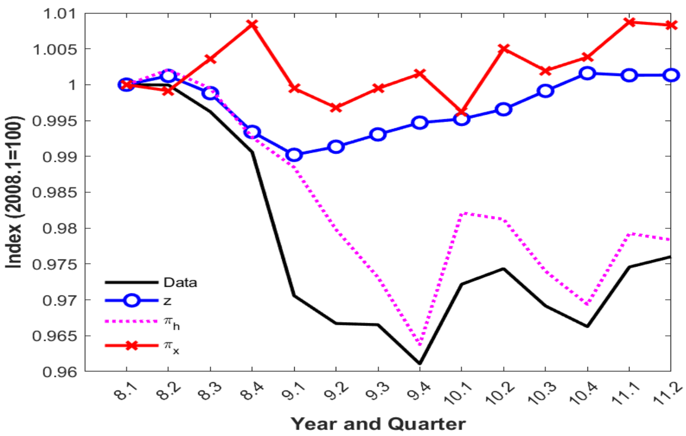
(a) Output per capita, hours worked, investment per capita.



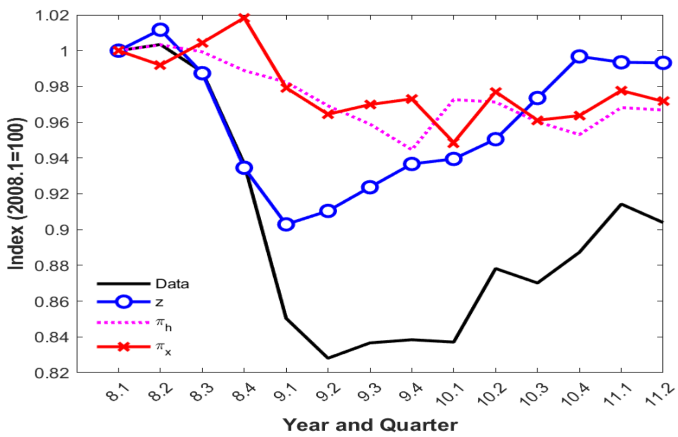
(b) Output per capita and four wedges.



(c) Output per capita and its components.



(d) Hours worked and its components.



(e) Investment per capita and its components.

Fig. 9: The Euro Area 12 Great Recession with CD production function.

Table 4

PARAMETERS OF THE VAR(1). U.S.†

Coefficients Matrix P	Coefficient Matrix Q, V = QQ'
Panel A: VES case	
$\begin{pmatrix} \underset{(.0355)}{.9300} & \underset{(.0953)}{.1102} & \underset{(.1345)}{.0425} & \underset{(.1718)}{.0692} & \underset{(.0858)}{.0699} \\ \underset{(.0302)}{.0472} & \underset{(.0830)}{.9117} & \underset{(.1138)}{-.0594} & \underset{(.1446)}{-.1033} & \underset{(.0729)}{-.0256} \\ \underset{(.0151)}{.0335} & \underset{(.0287)}{.0951} & \underset{(.0001)}{.9300} & \underset{(.0480)}{.0951} & \underset{(.0188)}{-.0254} \\ \underset{(.0067)}{.0070} & \underset{(.0114)}{.0018} & \underset{(.0144)}{-.0493} & \underset{(.0163)}{.9300} & \underset{(.0092)}{.0049} \\ \underset{(.0135)}{-.0132} & \underset{(.0291)}{-.1022} & \underset{(.0277)}{-.0640} & \underset{(.0479)}{.0548} & \underset{(.0001)}{.9300} \end{pmatrix}$	$\begin{pmatrix} \underset{(.0060)}{.0713} & 0 & 0 & 0 & 0 \\ \underset{(.0016)}{-.0575} & \underset{(.0052)}{.0093} & 0 & 0 & 0 \\ \underset{(.0723)}{-.0050} & \underset{(.0745)}{-.0060} & \underset{(.0015)}{.0191} & 0 & 0 \\ \underset{(.2311)}{.0012} & \underset{(.2388)}{-.0049} & \underset{(.1756)}{.0025} & \underset{(.0026)}{.0084} & 0 \\ \underset{(.0899)}{-.0003} & \underset{(.0954)}{-.0033} & \underset{(.0585)}{-.0070} & \underset{(.1630)}{.0021} & \underset{(.0020)}{.0243} \end{pmatrix}$
$E[s_t] = [-2.646,\ 1.434,\ -.220,\ .204,\ -2.063]$	
Panel B: CD case	
$\begin{pmatrix} \underset{(.0004)}{.8770} & \underset{(.0123)}{.0108} & \underset{(.0199)}{-.0953} & \underset{(.0070)}{.0297} \\ \underset{(.0245)}{.0164} & \underset{(.0003)}{.9820} & \underset{(.0219)}{.0164} & \underset{(.0052)}{-.0138} \\ \underset{(.0252)}{-.0454} & \underset{(.0117)}{-.0117} & \underset{(.0239)}{.9570} & \underset{(.0070)}{-.0050} \\ \underset{(.0881)}{-.1596} & \underset{(.0312)}{-.0839} & \underset{(.0827)}{-.1440} & \underset{(.0006)}{.9480} \end{pmatrix}$	$\begin{pmatrix} \underset{(.0007)}{.0095} & 0 & 0 & 0 \\ \underset{(.0633)}{-.0030} & \underset{(.0007)}{.0086} & 0 & 0 \\ \underset{(.1075)}{-.0043} & \underset{(.1147)}{-.0038} & \underset{(.0022)}{.0110} & 0 \\ \underset{(.0734)}{-.0017} & \underset{(.0714)}{-.0106} & \underset{(.1127)}{.0044} & \underset{(.0015)}{.0207} \end{pmatrix}$
$E[s_t] = [-.450,\ -.220,\ .204,\ -2.063]$	

†Quarterly data 1979:1-2020:4. Numbers in parentheses are standard deviations of parameters estimated by MLE.

Table 5

PARAMETERS OF THE VAR(1). EURO AREA 12.†

Coefficients Matrix P	Coefficient Matrix Q, V = QQ'
Panel A: VES case	
$\begin{pmatrix} \underset{(.3145)}{-.0708} & \underset{(.3164)}{-.6845} & \underset{(.3010)}{.4101} & \underset{(.6047)}{1.0007} & \underset{(.1888)}{.1755} \\ \underset{(.2484)}{-.2891} & \underset{(.2901)}{.6460} & \underset{(.2839)}{-.1479} & \underset{(.3260)}{.0664} & \underset{(.0766)}{-.0056} \\ \underset{(.1449)}{-.2539} & \underset{(.1565)}{-.0297} & \underset{(.1591)}{.7869} & \underset{(.2272)}{-.0297} & \underset{(.0639)}{-.0063} \\ \underset{(.0839)}{-.2385} & \underset{(.0885)}{-.1762} & \underset{(.1027)}{-.0951} & \underset{()}{.9800} & \underset{(.0332)}{-.0028} \\ \underset{(.4007)}{.8512} & \underset{(.3864)}{.5497} & \underset{(.3789)}{-.5305} & \underset{(.8775)}{-1.2546} & \underset{(.2732)}{.7933} \end{pmatrix}$	$\begin{pmatrix} \underset{(.0022)}{.0217} & 0 & 0 & 0 & 0 \\ \underset{(.0801)}{-.0121} & \underset{(.0023)}{.0173} & 0 & 0 & 0 \\ \underset{(.1474)}{-.0021} & \underset{(.0921)}{.0077} & \underset{(.0015)}{.0087} & 0 & 0 \\ \underset{(.1561)}{.0039} & \underset{(.1870)}{.0073} & \underset{(.1699)}{.0007} & \underset{(.0015)}{.0058} & 0 \\ \underset{(.1549)}{-.0002} & \underset{(.1418)}{.0080} & \underset{(.1635)}{-.0096} & \underset{(.1778)}{.0157} & \underset{(.0028)}{.0112} \end{pmatrix}$
$E[s_t] = [-2.957,\ 1.452,\ -.408,\ .002,\ -1.593]$	
Panel B: CD case	
$\begin{pmatrix} \underset{(.2695)}{.6445} & \underset{(.3566)}{-.2391} & \underset{(.3481)}{.0116} & \underset{(.0852)}{-.0295} \\ \underset{(.1238)}{.1029} & \underset{(.1751)}{.7182} & \underset{(.1962)}{.1029} & \underset{(.0434)}{-.0511} \\ \underset{(.1056)}{-.1000} & \underset{(.1403)}{-.1945} & \underset{(.1604)}{.8314} & \underset{(.0480)}{-.0421} \\ \underset{(.2601)}{-.0492} & \underset{(.2989)}{-.2968} & \underset{(.6105)}{-.4019} & \underset{(.1432)}{.9033} \end{pmatrix}$	$\begin{pmatrix} \underset{(.0017)}{.0202} & 0 & 0 & 0 \\ \underset{(.1041)}{.0069} & \underset{(.0016)}{.0088} & 0 & 0 \\ \underset{(.1490)}{.0065} & \underset{(.2363)}{.0007} & \underset{(.0018)}{.0068} & 0 \\ \underset{(.1834)}{-.0001} & \underset{(.1527)}{-.0131} & \underset{(.3136)}{.0189} & \underset{(.0030)}{.0121} \end{pmatrix}$
$E[s_t] = [-.332,\ -.408,\ .002,\ -1.593]$	

†Quarterly data 2005:1-2020:3. Numbers in parentheses are standard deviations of parameters estimated by MLE.