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ACCOUNTING FOR SPANISH ECONOMIC DEVELOPMENT 1850-2019

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Abstract

By conducting wedge-growth accounting, we assess the contribution of the economic forces (expressed as wedges in the equilibrium conditions of the neoclassical growth model) driving Spanish economic growth from 1850 to 2019. We find that declining investment and capital-efficiency wedges slowed down Spanish economic growth and downsized the labour share from 1850 to the First World War. The crisis of the 1930s (Great Depression and Civil War) was primarily driven by the decrease of the labour-efficiency wedge. The simultaneous increase of both efficiency wedges drove the Spanish economic miracle of the 1960s, which was preceded by a large increase in the investment wedge, resulting in a significant rise of the investment rate. From the mid-1970s, the declining capital-efficiency wedge was the primary force driving the fall of the labour share and the output growth slowdown. However, the labour wedge drove the medium-term fluctuations of output, labour, and investment.

Keywords: Growth Accounting, Capital-Efficiency Wedge, Labour-Efficiency Wedge, Labour Wedge, Investment Wedge, Resource Constraint Wedge, Output, Labour Share, Hours Worked, Investment, Spanish economic growth, Wedge-growth accounting.

JEL classification: E13, E17, E25, O41, O47.

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1 Introduction

Traditional growth accounting (TGA) (see Solow, 1957) and wedge-growth accounting (WGA) possess several similarities, yet differ in important ways. TGA examines the impact of changes in productive inputs and their quality improvements on output growth, whereas WGA takes a first step in analysing the primary forces driving economic growth by expressing them as wedges in the equilibrium conditions of the neoclassical growth model and evaluating their effect on the evolution of labour, output, and investment. WGA provides a rigorous approach to data analysis, but results may be highly dependent on the assumptions made regarding functional forms of the production and utility functions. To counteract this issue, our analysis uses multiple functional forms. Additionally, WGA does not identify the structural causes of events, so we review both macroeconomic literature and literature on Spanish economic history to interpret our findings.

The objective of this work is to analyse the primary forces driving the Spanish economic development. We utilize data from Prados de la Escosura (2017, 2020) and Prados de la Escosura and Rosés (2021) to conduct a WGA analysis of the Spanish economy from 1850 to 2019.

The WGA method developed by del Río and Lores (2021) consists of two stages. In the first stage, the complete neoclassical growth model is used to identify wedges in the equilibrium conditions which reconcile the model with data on production, labour, resource allocation and the income distribution. Specifically, the five wedges considered are capital-efficiency, labour-efficiency, investment, labour and resource constraint. The capital-efficiency (resp. labour-efficiency) wedge is the ratio of capital (resp. hours) actually employed in production and the optimal capital (resp. hours) needed to achieve a certain level of output with a given factor income distribution. By assigning the output elasticities for factors as weights, the generalized geometric mean of the capital- and labour-efficiency wedges equals total factor productivity (TFP). The labour wedge is the gap between the consumption-leisure marginal substitution rate and the marginal product of labour. The investment wedge represents the gap between the inter-temporal marginal substitution rate and the return on capital, while the resource wedge captures the difference between output and its allocation to consumption and investment. In
the second stage, the model is solved by simulation to establish the contribution of the wedges to the variation of the variables around their potential evolution paths. The wedge-alone components of the variables are computed by simulating the model for the measured values of each wedge, keeping the values of the other wedges constant. The wedge-alone components show the contributions of the wedges to the movements of the variables around their potential evolution paths.

The WGA method is closely inspired by the business cycle accounting method developed by Chari et al. (2007). The WGA method differs from Chari et al. (2007) in two ways. First, the deterministic version of the neoclassical growth model, which is more suitable for economies in transition that are moving away from balanced growth path (BGP) than the standard stochastic version used by Chari et al. (2007), is used. Second, unlike Chari et al. (2007)'s method, which produces a single efficiency wedge reflecting TFP, the WGA method requires the model to be consistent with the evolution of income factor shares, enabling the decomposition of TFP into the capital-efficiency and labour-efficiency wedges. Identifying both efficiency wedges is important because the TFP evolution can conceal very different, even divergent, behaviours of both efficiency wedges. As our results attest, this empirically relevant possibility is key to understand the stagnation of Spanish TFP from the early 1980s.

Due to the long period of time analysed, our model considers a utility function which allows it to match the decreasing trend of hours worked per capita in the Spanish economy from 1850 to 2019. Boppart and Krusell (2020) (BP) discuss the properties of this utility function. Generally, wedge-growth and business cycle accounting exercises are implemented using the kind of utility functions proposed by King et al. (1988) (KPR) which implies that hours worked per capita remain constant along a BGP. To the best of our knowledge, we are the first to conduct WGA with a BP utility function. Using either utility function has a significant effect on the calculation of the wedges and their contributions, particularly on the labour wedge. In the Appendix, we display the results of the WGA analysis conducted using a KPR utility function. Considering both utility functions has led us to the conclusion that the labour wedge has been the main driver of the potential evolution of hours worked per capita from 1850 to 2019, but not of their

\footnote{See del Río and Lores (2021, 2023) for a more exhaustive explanation.}
deviations from its potential path until the mid-1970s.

Over the past 150 years, the Spanish economy has gone through a transformation from a pre-industrial, underdeveloped economy to a developed, industrial economy. This is why, unlike most previous studies in this field, we include land as a productive factor in the production function. We also establish an equivalence between our aggregate production function with capital and land and a two-sector economy with an agricultural sector using land and an industrial sector using capital. We then analyse the impact of the modernization of the agricultural sector on economic growth during the Spanish economic miracle of the 1960s.\(^2\) Our findings show that it had a significant, but secondary, contribution to output growth during the Spanish economic miracle. More importantly, it helped to reduce output volatility since the 1960s.

We estimate a Variable Elasticity of Substitution (VES) production function for the Spanish economy from 1850 to 2019. This production function includes land, capital, and labour as productive factors. Our results indicate that land, capital, and labour are gross complements. To check the robustness of our results and compare them to other studies in the literature, we also implement WGA exercises using a Cobb-Douglas (CD) and a Constant Elasticity of Substitution (CES) production functions. The results are displayed in the Appendix.

Although there is some debate, empirical evidence increasingly indicates that capital and labour are gross complements (see Chirinko, 2008). Del Río and Lores (2019) also estimate a VES production function for the United States and they also find that capital and labour are gross complements. The literature on housing production has mainly analysed the elasticity of substitution between land and other productive factors. Most papers find an elasticity of substitution less than one; however, as pointed out by Epple et al. (2010), these estimates face serious problems of measurement errors.\(^3\)

To the best of our knowledge, this is the first time that a WGA exercise has been conducted for such a long period of time, providing an initial insight into the forces that have shaped the Spanish economy during 150 years. Other authors have studied the

\(^2\)Spanish economic miracle refers to the period of exceptionally rapid growth and development across all major areas of economic activity in Spain from the mid-1950s to the mid-1970s, during the latter part of the Francoist regime. The economic boom was brought to an end by the 1970s international oil and stagflation crises.

\(^3\)McDonald (1981) reviews the early empirical literature on housing production.
Spanish or other economies, but with a focus on specific events or shorter time frames. Our results are generally consistent with those of these previous studies (see the next section), but our longer temporal focus allows us to put them into perspective.

Our analysis suggests that the Spanish economy has gone through four distinct phases since 1850. (i) From 1850 to the 1920s, Spanish economic growth was hindered by the poor performance of the investment and capital-efficiency wedges, which we believe is related to the underdeveloped Spanish financial sector. (ii) The second quarter of the 20th century was a time of crisis and stagnation (Great Depression, Civil War and Autarky) driven by a sharp decline in the efficiency wedges. (iii) The third quarter of the 20th century was the time of the Spanish economic miracle, which was based in the improvement in the investment wedge and driven by the notable increase in the labour-efficiency wedge. We believe that the financial and political stability of the Franco regime from the late 1940s likely reduced firms’ investment costs and provided the Spanish economy with the necessary foundations to start the growth and development process. We also believe that the impressive growth of the labour-efficiency wedge from the late 1950s to the mid-1970s reflects the deep modernization and industrialization of the Spanish economy, which was characterized by an intense process of technological adoption and a substantial increase in the human capital of the Spanish labour force. (iv) From the mid-1970s to 2019, growth of TFP and output slowed down. Furthermore, economic growth was very fluctuating, especially in hours worked. We find that the main driving force of the growth slowdown of TFP and output was a declining capital-efficiency wedge. However, the medium-term fluctuations in output, investment, and labour were primarily driven by the labour wedge, which may point to the structural deficiencies of the Spanish labour market.

The remainder of this study is organized as follows. We discuss our main findings in section 2. Section 3 describes the model. We calibrate the model and compute the wedges in section 4. Section 5 simulates the model to assess the contribution of each wedge to the evolution of output, hours worked, investment and labour share in Spain from 1850 to 2019. In section 6, we develop an equivalence result between the aggregate production function and a two-sector economy and evaluate the impact of the modernization of the agricultural sector on economic growth. In section 7, we look at the Spanish economic growth between 1850 and 2019 in light of the results of our analysis, the macroeconomic literature and the literature on Spanish economic history. Section 8 concludes.
2 Findings in context

The forces driving Spanish modernization and industrialization may be similar to those of other economies that experienced economic miracles. We find that the Spanish economic miracle from the end of 1950 to mid-1970s was primarily driven by the labour-efficiency wedge and secondarily by the capital-efficiency wedge. However, the investment wedge improved significantly in the 1940s, driving the recovery after the Civil War and preceding the miracle. Lu (2012) found that the capital wedge was the primary driver of rapid growth in the early stages of East Asian countries’ development (Hong Kong, Singapore, South Korea, and Taiwan), but TFP growth became the primary driver in the later stages. Konya (2013) found that reducing distortions in the capital and labour wedges would lead to significant productivity gains in six European countries. Cheremukhin et al. (2015, 2017) used a two-sectors neoclassical growth model to account for the economic growth of China and Russia. They found that structural change from agricultural to industry was key, and entry barriers and monopoly power in the non-agricultural sector were the main reason for Tsarist Russia’s failure to industrialize before the First World War.

Some works explore the role of wedges in certain episodes of Spanish economic history. Giménez and Montero (2015) find that the Spanish Great Depression was mainly driven by the fall of $\text{tfp}$. This is consistent with our finding that the output downturn during the Spanish Great Depression was primarily driven by the fall of the labour-efficiency wedge and secondarily by the labour wedge. Rodríguez-López and Solís-García (2016) apply the business cycle accounting methodology to the Spanish economy after 1976 and find that the recession of the 1970s and the Great Recession were mainly driven by the labour wedge. We find that, from the mid-1970s, oscillations of output, labour, and investment of the Spanish economy were driven by the movements of the labour wedge. Although we find that the decrease in the capital-efficiency wedge played a primary role in accounting for output growth slowdown from the mid-1970s and, particularly, during the Spanish Great Recession, we also find that the labour wedge played a significant, but secondary role in accounting for the output growth slowdown and almost exclusively drove the fall in hours worked per capita. Unlike Brinca et al. (2016), we do not find

4 Chari et al. (2002) find that movements of labour, output, and investment in the U.S. Great Depression were mostly driven by the fall of TFP and the labour wedge.
that the investment wedge played any significant role in accounting for the evolution of output during the Spanish Great Recession. However, as argued by del Río and Lores (2023), distortions on investment can be be manifested in the capital-efficiency wedge and/or the investment wedge.

Even if the aims of TGA and WGA differ, our results are consistent with the findings of Prados de la Escosura and Rosés (2021), who conduct a TGA exercise for the Spanish economy from 1850 to 2019. For the whole period, they find that half of Spanish labour productivity growth between 1850 and 2019 resulted from capital deepening, one-third from TFP and labour quality contributed the rest. They also find that (i) the productivity acceleration in the 1920s and (ii) from the mid-1950s to the mid-1980s (period including the Spanish economic miracle) was mainly due to TFP growth as well as (iii) TFP stagnated after the mid-1980s. They also find that the Spanish output downturn during the Great Depression and the Great Recession resulted from the sharp decline in TFP. The stagnation of Spanish TFP has been highlighted by many authors, raising an intriguing question.\footnote{See, for example, Boldrin et al. (2010) and Rodríguez-López and Solís-García (2016).} We find that, (i) from the end of the 19th century to the Great Depression, the contribution of the labour-efficiency wedge drove output growth above its potential growth (although it was curbed by the contributions of the capital-efficiency and investment wedges until the end of the First World War). Our results show that (ii) the Spanish economic miracle was primarily driven by the labour-efficiency wedge and secondarily by the capital-efficiency wedge, which translated into a large increase of TFP. We also find that, (iii) since the mid-1970s, the paths of capital-efficiency and labour-efficiency wedges diverged. The capital-efficiency wedge decreased, slowing down TFP and output growth, while the labour-efficiency wedge increased until the early 1990s and remained roughly constant since then. This has resulted in the stagnation or even decline of detrended TFP, which is the primary cause of the output growth slowdown after the mid-1970s. Our work raises the question of this divergence.

There are some similarities between our results and the findings of WGA conducted by del Río and Lores (2021) for the United States. In particular, they also find that lowering the capital-efficiency wedge mostly drove the output growth slowdown and the labour share in the United States from de mid-1970s. Moreover, they find that the
capital-efficiency wedge was the main force driving the U.S. labour share between 1950 and 2017, like us for Spain between 1850 and 2019.

3 The Model

We develop a one-sector neoclassical growth model with five exogenous wedges. Output, $Y_t$, can be allocated to consumption, $C_t$, investment, $X_t$, and other ends, $G_t$. The resource constraint is $Y_t = C_t + X_t + G_t$. We rewrite it as $\pi_{g,t} Y_t = C_t + X_t$, where $\pi_{g,t} = 1 - \frac{G_t}{Y_t}$ is called the resource wedge. Capital accumulation involves quadratic investment adjustment costs, $K_{t+1} = X_t + (1 - \delta_t) K_t - \frac{\lambda}{2} \left( \frac{X_t}{K_t} - \kappa \right)^2 K_t$, where $\lambda > 0$, $\kappa > 0$ and $0 < \delta_t < 1$ is the depreciation rate of capital at a time $t$.

The representative household at a time $t$ is composed of $L_t$ members and $L_{t+1} = (1 + \eta_{t+1}) L_t$ where $\eta_{t+1}$ is the population growth rate between $t$ and $t+1$. Each member of the representative household is endowed with one unit of time that can be shared between labour, $1 < H_{L,t} < 0$, in return for a wage, $W_t$, and leisure, $1 - H_{L,t}$. Therefore, $H_t = L_t H_{L,t}$ is time offered in the labour market by the representative household.

The intertemporal utility function of the representative household is

$$U_t = \sum_{t=0}^{\infty} L_t \beta^t \left( \frac{C_{L,t}^{1-\sigma} - 1}{1 - \sigma} - \frac{H_{L,t}^{1-\nu}}{1 - \nu} \right),$$

where $0 < \beta < 1$ is the discount factor, $1/\sigma$ is the intertemporal elasticity of substitution, $-1/\nu$ is the Frisch elasticity of the labour supply, $L_t$ are the members of household and $C_{L,t} = C_t / L_t$ is consumption per capita. If $\sigma$ goes to 1, then the standard KPR utility function emerges.

The household budget constraint is $L_t C_{L,t} + \pi_{x,t} X_t = \pi_{h,t} W_t H_{L,t} L_t + r_t K_t + B_t$, where $\pi_{h,t}$ is the labour wedge, $\pi_{x,t}^{-1}$ is the investment wedge and $B_t$ are lump-sum transfers, $B_t = (1 - \pi_{h,t}) W_t H_{L,t} L_t - (1 - \pi_{x,t}) X_t - (1 - \pi_{g,t}) Y_t$. This transfer system guarantees the consistency of the household budget constraint and the resource constraint.

A perfectly competitive representative firm produces output, $Y_t$, according to a neoclassical production function. It uses capital, land, and labour as production factors, $Y_t = F \left( K_{A,t}, (1 + \gamma)^t z_t H_t \right)$ where $\gamma \geq 0$ is the rate of labour-augmenting technical
change, $z_t$ is the labour-efficiency wedge and $K_{A,t}$ is a neoclassical aggregator of capital, $K_t$, and land, $T_t$, displaying constant returns to scale, $K_{A,t} = D(q_{K,t}K_t, q_{T,t}T_t)$, with $q_{K,t}$ and $q_{T,t}$ being the efficiency indices of capital and land at a time $t$.

Under perfect competition, the rental prices of capital and land equal their marginal productivities, $r_{T,t} = r_K D(q_{K,t}K_t, q_{T,t}T_t)$, where $Y = K, T, i = 1, 2$ and $r_{K,A,t}$ is the rental price of $K_A$, which equals its marginal productivity, $r_{K,A,t} = F_1(K_{A,t}, (1 + \gamma t)^t z_t H_t)$. Considering that $D_1$ and $D_2$ are homogeneous of degree 0, then it follows from the previous optimality conditions that the ratio of effective land to effective capital is a function of the ratio of the land share, $s_{T,t} = r_{T,t}T_t/Y_t$, to the capital share, $s_{K,t} = r_{K,t}K_t/T_t$:

$$\frac{q_{T,t}}{q_{K,t}K_t} = S\left(\frac{s_{T,t}}{s_{K,t}}\right).$$

Therefore, the production function can be rewritten as a function of capital and labour, $Y_t = F(q_t K_t, (1 + \gamma t)^t z_t H_t)$, where $q_t = q_{K,t}D\left(1, S\left(\frac{s_{T,t}}{s_{K,t}}\right)\right)$ is the capital-efficiency wedge which has two components: the former is the efficiency index of capital, $q_{K,t}$, and the latter reflects the ratio of effective land to effective capital, $D\left(1, S\left(\frac{s_{T,t}}{s_{K,t}}\right)\right)$. Moreover, TFP is the generalized geometric mean of the capital- and labour-efficiency wedges: $A_t = q_t^\varepsilon z_t^{1-\varepsilon_t}$, where $\varepsilon_t$ is output elasticity for capital and $1 - \varepsilon_t$ is output elasticity for labour.

Along a BGP, output per capita, consumption per capita, investment per capita and capital per capita change at the gross rate $1 + \gamma_y = (1 + \gamma)^{\frac{1-\sigma}{\gamma - \sigma}}$ and hours worked per capita change at the gross rate $1 + \gamma_h = (1 + \gamma)^{\frac{1-\sigma}{\gamma - \sigma}} = (1 + \gamma_y)/(1 + \gamma)$. Hereafter, a lower-case variable ($y, c, x, k$ and $h$) denotes the corresponding upper-case variable ($Y, C, X, K$ and $H$) detrending and per capita.

We can rewrite the resource constraint, the production function and the move law of detrended capital per capita as

$$c_t + x_t = \pi_{g,t}y_t,$$  \hspace{1cm} (1)

$$y_t = f(q_t k_t, z_t h_t).$$ \hspace{1cm} (2)

and

$$(1 + \eta_{t+1}) (1 + \gamma_y) k_{t+1} = x_t + (1 - \delta_t) k_t - \frac{\lambda}{2} \left(\frac{x_t}{k_t} - \kappa\right)^2 k_t.$$ \hspace{1cm} (3)

If $\sigma$ goes to 1, then hours worked per capita remain constant along a BGP, which is the case of the KPR utility function.
The representative firm equals marginal productivities of factors to their rental prices \((r_t \text{ and } W_t)\) to maximize its profits,

\[
\varepsilon_t = r_t \frac{k_t}{y_t} = 1 - s_t \tag{4}
\]

and

\[
1 - \varepsilon_t = w_t \frac{h_t}{y_t} \equiv s_t, \tag{5}
\]

where \(w_t = W_t / (1 + \gamma)^t\) is detrended wage per worked hour, \(s_t\) is labour share,

\[
\varepsilon_t = \frac{q_t k_t f_1 (q_t k_t, z_t h_t)}{f (q_t k_t, z_t h_t)} \tag{6}
\]

is output elasticity for capital and \(1 - \varepsilon_t\) is output elasticity for labour. According to the first order conditions (4) and (5), the factor shares equal output elasticities for factors and add to 1.

The first-order conditions of the household problem are

\[
\frac{1}{\beta} (1 + \gamma_y) \left( \frac{c_{t+1}}{c_t} \right)^\sigma = 1 + i_{t+1}, \tag{7}
\]

\[
r_{t+1} = \frac{\pi_x,t (1 + i_{t+1})}{1 - \lambda \left( \frac{x_{t+1}}{k_{t+1}} - \kappa \right)} + \frac{\pi_{x,t+1}}{1 - \lambda \left( \frac{x_{t+1}}{k_{t+1}} - \kappa \right)},
\]

\[
\cdot \left[ \frac{\lambda}{2} \left( \frac{x_{t+1}}{k_{t+1}} - \kappa \right)^2 - \lambda \left( \frac{x_{t+1}}{k_{t+1}} - \kappa \right) \frac{x_{t+1}}{k_{t+1}} - (1 - \delta_{t+1}) \right], \tag{8}
\]

and

\[
c_t^\sigma h_t^{-\nu} = \pi_{h,t} w_t. \tag{9}
\]

Equation (7) is the Euler equation and states that the marginal rate of intertemporal substitution equals the gross interest rate, \(1 + i_{t+1} = \frac{p_t}{p_{t+1}},\) where \(p\) is the Arrow-Debreu price of the composite commodity. According to equation (8) the rental price of capital equals its user cost, which includes the interest rate, the depreciation rate of capital, the investment wedge and the investment adjustment costs. Finally, equation (9) establishes
that the wage adjusted by the labour wedge equals the marginal rate of substitution between consumption and labour.

Given the seven exogenous variables \( \{ q_t, z_t, \pi_{g,t}, \pi_{x,t}, \pi_{h,t}, \delta_t, \eta_{t+1} \}_{t=0}^{\infty} \), equation system (1)-(9) with an initial condition for the detrended capital per capita, \( k_0 \), and the transversality condition characterize the dynamic equilibrium of the economy.

Along a BGP, both the capital depreciation rate and population growth rate remain constant, \( \eta_t = \eta \) and \( \delta_t = \delta \) along with the five wedges \( (\pi_{g,t}, \pi_{x,t}^{-1}, \pi_{h,t}, q_t, \text{ and } z_t) \) and the variables \( c_t, x_t, k_t, h_t, w_t, r_t, y_t \) and \( i_t \). We assume that \( \kappa = (1 + \eta) (1 + \gamma_y) - (1 - \delta) \) which implies that \( \kappa \) equals the ratio of investment to capital along a BGP and the investment adjustment costs cease.

Given \( q, z, \pi_x, \pi_h, \) and \( \pi_g \), the following equations characterize a BGP:

\[
\frac{(1 + \gamma_y)^\sigma}{\beta} = 1 + i \tag{10}
\]

\[
(1 + \eta) (1 + \gamma_y) - (1 - \delta) = \frac{x}{k} \tag{11}
\]

\[
\varepsilon y = \pi_x (i + \delta) k \tag{12}
\]

\[
(1 - \varepsilon) y = wh \tag{13}
\]

\[
\sigma h^{-\nu} = \pi_h w \tag{14}
\]

\[
\varepsilon = \frac{qk f_1(qk, zh)}{f(qk, zh)} \tag{15}
\]

\[
y = f(qk, zh) \tag{16}
\]

\[
c + x = \pi_g y \tag{17}
\]

Equation (10) is the Euler equation. According to equation (11) the accumulation of capital is such that the ratio of investment to capital is constant. Equations (12) and (13) are the profit-maximizing conditions of the representative firm. Equation (14) represents the leisure-income choice. The equation (15) offers the output elasticity for capital. Equation (16) is the production function and, finally, equation (17) is the resource constraint.

\(^7\)If \( q_t \) is constant, then technical change is purely labour-augmenting, which is a necessary condition for the existence of a BGP (see Uzawa, 1961 and Jones and Scrimgeour, 2008).
4 Quantitative implementation

Our quantitative exercise consists of three steps. First, we calibrate the model. Second, we calculate the wedges that allow the model to reproduce the observed paths of output, consumption, investment, hours (all per capita), and labour share. Finally, we simulate the evolution of the economy with one or several wedges. We use a data set elaborated from Prados de la Escosura (2017, 2020) and Prados de la Escosura and Rosés (2021) which is described in section I of the Appendix.

To carry out our quantitative exercise, we assume a VES production function which, written in terms of the detrended variables per capita, is

\[ y_t = \left[ \alpha (q_t k_t)^{\omega \rho} (z_t h_t)^{(1-\omega)\rho} + (1 - \alpha) (z_t h_t)^{\rho} \right]^{\frac{1}{\rho}} \]

where \( \alpha \in (0, 1) \), \( \omega \in (0, 1) \), and \( \rho \in (-\infty, 1] \). Moreover, if the previous production function is a reduced form of a production function aggregating capital and land as \( K_{A,t} = \left[ (q_{K,t} K_t)^{\psi} + (q_{T,t} T_t)^{\psi} \right]^{\frac{1}{\psi}} \), where \( \psi \in (-\infty, 1] \), then the capital-efficiency wedge is \( q_t = \left( 1 + \frac{s_{T,t}}{s_{K,t}} \right)^{\frac{1}{\psi}} q_{K,t} \). The output elasticity for capital of this VES production function is \( \varepsilon_t = \alpha \omega \left( q_t \frac{k_t}{y_t} \right)^{\rho} \left( \frac{q_t k_t}{z_t h_t} \right)^{(\omega-1)\rho} \). If \( \omega = 1 \), then the production function is CES and when \( \rho \to 0 \), the production function converges to a CD production function with output elasticity for capital \( \alpha \omega \), i.e. \( y_t = A_t k_t^{\alpha \omega} h_t^{1-\alpha \omega} \), where \( A_t \) is the only efficiency wedge and reflects TFP.

To check the robustness of our results, we calibrate and simulate the model for the three cases: VES, CES and CD. However, the discussion of the results in the CES and CD cases is left for the section IV of the Appendix. We also leave for the section V of the Appendix the case of the KPR utility function.

4.1 Calibration

We restrict the model’s parameter values to be compatible with observations of the Spanish economy from 1850 to 2019. In line with Rodríguez-López and García (2016), we set \( \nu = -3 \) to have a Frisch elasticity of 1/3. We set \( \sigma = 1.8658 \) and \( \gamma = 0.0208 \). These values, considering that \( (1 + \gamma)^{\frac{1}{1-\nu}} = 1 + \gamma_h \) and \( 1 + \gamma_g = (1 + \gamma)(1 + \gamma_h) \), imply that
\(\gamma_h = -0.003525\) and \(\gamma_y = 0.0172\) which are the annual average growth rates of the hours worked per capita and output per capita in the period 1850 – 2019, respectively. The annual interest rate is set \(i = 0.04\). The population growth rate and the depreciation rate of capital are set \(\eta = 0.0063\) and \(\delta = 0.0365\), which are the annual averages in the period. We set \(\lambda = 0.25/\kappa\) (where \(\kappa = (1 + \eta) (1 + \gamma_y) - (1 - \delta)\)) to obtain the elasticity of the price of capital with respect to the investment-capital ratio to be 0.25 (see Brinca et al., 2016).

We log-linearize the first order condition for capital to estimate the parameters of the production function by Ordinary Least Squares and Generalized Instrumental Variables. Furthermore, we use the last method to treat with endogeneity (which is almost inevitable in a production function framework) and serial correlation. The estimation is described in section II of the Appendix. For the base case, we use a VES production function with \(\rho = -0.3735\) and \(\omega = 0.7\), which are the averages of the estimated values in Table A.1 of the Appendix. The VES production function nests the CES \((\omega = 1)\) and CD \((\rho \rightarrow 0\) with output elasticity for capital \(\alpha\omega)\) functions. Then we set \(\alpha = 0.6225\) to have \(\alpha\omega\) equal to the average labour share, 0.564, in the period 1987-2019.

The BGP values for the wedges \(q, z, \pi_h, \pi_x^{-1}\) and \(\pi_g\) are calculated by solving the equation system (10)-(17) to reproduce the 1987 – 2019 sample averages for detrended hours worked, detrended output per capita, investment rate, consumption to output rate and labour share. In the CD case, it is only calibrated a BGP value for one efficiency wedge, \(A\). Table 1 summarizes parameters and all BGP values.

### 4.2 Computing the wedges

In this section, we compute the paths of the wedges and detrended capital per capita consistent with the Spanish data for the period 1850-2019 on hours worked per capita, labour share, consumption per capita, investment per capita, output per capita, the population growth rate and the capital depreciation rate assuming that the economy converges to the previously calibrated BGP. The results are not very sensitive to the choice of the final BGP due to the high speed of convergence.
To compute the wedges, we solve the equilibrium equation system

\[ c_t + x_t = \pi_{g,t} y_t \]  

(18)

\[
(1 + \eta_{t+1}) (1 + \gamma_y) k_{t+1} = x_t + (1 - \delta_t) k_t - \frac{\lambda}{2} \left( \frac{x_t}{k_t - \kappa} \right)^2 k_t
\]  

(19)

\[
y_t = \left[ \alpha \left( q_t k_t h_t^{(1 - \omega)} \right)^\rho + (1 - \alpha) (z_t h_t)^\rho \right]^\frac{1}{\rho}\]  

(20)

\[
1 - s_t = \alpha \omega \left( \frac{z_t}{y_t} \right)^\rho \left( \frac{q_t k_t}{z_t h_t} \right)^{(\omega - 1)\rho}
\]  

(21)

\[
\frac{\pi_{x,t}}{1 - \lambda \left( \frac{z_t}{y_t} - \kappa \right)} \frac{(1 + \gamma_y)^\sigma c_{t+1}}{c_t} = (1 - s_{t+1}) \frac{y_{t+1}}{k_{t+1}} - \frac{\pi_{x,t+1}}{1 - \lambda \left( \frac{z_{t+1}}{y_{t+1}} - \kappa \right)}.
\]  

(22)

\[
ce_t h_t^{1 - \nu} = \pi_{h,t} s_t y_t
\]  

(23)

for \( k_{t+1}, \pi_{x,t}, \pi_{h,t}, \pi_{g,t}, q_t, \) and \( z_t, \) given an initial condition for capital, \( k_0, \) obtained from the data set such that \( k_0/y_0 = 1.26 \) (see Section I in the Appendix), and the observed paths of \( \eta_{t+1}, y_t, c_t, x_t, s_t, \delta_t \) and \( h_t \) in the 1850 – 2019 period as well as their assumed paths for \( t > 2019, \)

\[
j_t = j_T e^{-\left( t - T \right)} + j \left( 1 - e^{-\left( t - T \right)} \right)
\]

where \( j_t \) is \( \eta_{t+1}, c_t, x_t, y_t, s_t, \delta_t \) or \( h_t \) at period \( t \geq T, \) \( T = 2019 \) and \( j \) is the constant calibrated value above. We set \( \iota = 0.03, \) which is around the speed of convergence estimated in most works (see Barro and Sala-i-Martin, 1995). Our method allows us to compute converging paths of wedges from the initial period until infinity. In practice, we have computed 1000 periods.

Equation (18) is the resource constraint. Equation (19) is the capital accumulation law. Equation (20) is the production function. Equation (21) is the first-order condition for capital. Equation (22) is the Euler condition. Finally, equation (23) is the household condition for the optimal time allocation.
To compute the wedges in the CES case, the only changes required in the equation system (18)-(23) are related to the production function. Equations (20) and (21) are
\[ y_t = \left[ \alpha (q_t k_t)^\rho + (1 - \alpha) (z_t h_t)\right]^\frac{1}{\rho} \] and
\[ 1 - s_t = \alpha \left( q_t \frac{k_t}{z_t} \right)^\rho. \] Two efficiency wedges are computed, as in the VES case. Differentiating any neoclassical production function with constant returns to scale, it follows that TFP, \( A_t \), equals the weighted geometric mean of both efficiency wedges, being the weights the output elasticities for factors:
\[ A_t = \frac{\hat{q}_t^{\hat{q}_t} \hat{z}_t^{1-\hat{q}_t}}{\hat{q}_t^{\hat{q}_t} \hat{z}_t^{1-\hat{q}_t}}. \] The efficiency wedges depend on the assumed production function, but the labour and investment wedges do not. Indeed, any neoclassical production function with constant returns to scale and variable output elasticities for factors produces the same labour and investment wedges.\(^8\)

In the CD case, the model is not required to be consistent with income factor distribution and, consequently, a single efficiency wedge can be computed. To compute the wedges, the equation system (18)-(23) requires the following changes: equation (20) for the production function is
\[ y_t = A_t^{\alpha \omega} h_t^{1-\alpha \omega}, \] where \( A_t \) is the efficiency wedge, equation (21) is \( 1 - s = \alpha \omega \), and in equation (23), \( s = 1 - \alpha \omega. \)

In the CD case, the investment and labour wedges reflect changes in income factor shares. From equation (23), it follows that \( \pi^{cd}_{h,t} = \frac{\omega}{\alpha \omega} \pi_{h,t} \), where \( \pi^{cd}_{h,t} \) is the CD labour wedge and \( \pi_{h,t} \) is the labour wedge computed with any other production function with variable elasticities for factors and assuming that the marginal productivities of factors equal the rental prices (consequently, the output elasticities for factors equal the income factor shares). Therefore, when the labour share undergoes significant changes, the evolution of both labour wedges significantly differs. In particular, if the labour share decreases (resp. increases), then \( \pi^{cd}_{h,t} \) decreases (resp. increases) regarding \( \pi_{h,t}. \) Although it is not so evident, changes in factor shares also affect the calculation of the investment wedge in the opposite sense how they affect the labour wedge. In particular, if the labour share decreases (resp. increases), then the CD investment wedge increases (resp. decreases) regarding the investment wedge computed assuming that the output elasticities for factors equal the factor income shares (the VES and CES cases). To show this point, we assume that the depreciation rate of capital is 1 and there not exist adjustment costs of investment. Under these assumptions, it follows from equation (23)

\(^8\)Note that (22) and (23) do not involve the production function.
that \((\pi_{x,t}^{\text{cd}})^{-1} = \pi_{x,t}^{-1} \frac{1-x_t+1}{1-\alpha\omega}\), where \((\pi_{x,t}^{\text{cd}})^{-1}\) is the CD investment wedge and \(\pi_{x,t}^{-1}\) is the investment wedge computed with any other production function with variable elasticities for factors and assuming that the marginal productivities of factors equal the rental prices. If the depreciation rate of capital is lower than 1 or there exist adjustment costs of investment, then the previous relationship is only approximated.

As argued by del Río and Lores (2023), in the CD case, it is implicitly assumed that movements in the factor income shares are led by market-frictions or non-competitive forces reflected in the factor wedges and moving away the factor rental prices from the marginal productivities of factors. Therefore, the labour wedge reflects the gap between the marginal productivity of labour and the real wage in addition to the gap between the marginal rate of substitution between consumption and leisure and the real wage. However, Karabarbounis (2014) argues that explanations of the labour wedge based on departures of the representative firm’s marginal productivity of labour from the real wage are rejected by data because the labour share of income is not strongly procyclical.

The paths of the wedges for the VES case are displayed in Fig. 1. Our analysis reveals that the behaviour of the Spanish TFP has been the result of two opposing forces. On the one hand, the capital-efficiency wedge underwent a continuous and prolonged decline since the mid-1970s. On the other hand, the labour-efficiency wedge increased until the early 1990s and then remained roughly stable, albeit oscillating. The net result was that detrended TFP initially stagnated, then, from the late 1990s, declined. If we focus our attention on the Great Recession, both wedges also exhibited opposite behaviours: the capital-efficiency wedge decreased, and the labour-efficiency wedge increased (see Fig. 1, panels (a), (b) and (c)).

In Fig. 2, we display the computed labour-efficiency wedge and the labour quality indices of Prados de la Escosura and Rosés (2010). We think that the comparison supports our breakdown of the TFP into two components (the labour- and capital-efficiency wedges) because the evolution of the income-based labour quality index roughly approximates the evolution of the labour-efficiency wedge computed in our work: it grows very slowly until the 1950s and then underwent a strong increase until the mid-1980s, thereafter it remains roughly stable. However, the index does not track the strong fall of the labour-efficiency wedge from the Great Depression to the 1940s. The income-based labour quality index is a better proxy of the labour-efficiency wedge because it reflects all factors influencing
labour efficiency, not only formal education.

5 Accounting for movements of labour, output, investment and labour share

We simulate the model to assess how the wedges’ evolution can account for the evolution of output, hours worked, investment and labour share in Spain from 1850 to 2019. In particular, we compute the so-called wedge-alone components and the neoclassical transitional components.

The neoclassical transitional component of a variable is the result of simulating the equilibrium equation system (18)-(23), given $k_0$ in its sample initial value (which has been used to compute the wedges in previous section) and assuming that all wedges, as well as both the capital depreciation and population growth rates, remain in their steady values. For example, the neoclassical transitional component of detrended output per capita is the result of simulating, given $k_0$ in its initial value, the equilibrium equation system (18)-(23), assuming that $q_t = q$, $\pi_{h,t} = \pi$, $z_t = z$, $\pi_{g,t} = \pi_g$, $\pi_{x,t} = \pi_x$, $\eta_t = \eta$ and $\delta_t = \delta$.

We call it neoclassical transitional component because it is the convergence path of the variable to its steady value along the BGP due to that the initial stock of detrended capital per capita differs from its steady value. The neoclassical transitional component represents the potential evolution path of the variable without any change in the fundamentals of the economy. For example, if the considered variable is output per capita, then its neoclassical transitional component represents the so-called potential output per capita.

The wedge-alone component of a variable due to a wedge $i$ is the result of simulating, given $k_0 = k$ in the steady state, the equilibrium equation system (18)-(23), assuming that the wedge $i$ follows the computed path of the wedge in the previous section while the remaining wedges, as well as both the depreciation and population growth rates, remain in their steady values. For example, in panel (a) of Fig. 4, the simulated path of detrended output per capita is the result of simulating, given $k_0 = k$ in the steady state, the equilibrium equation system (18)-(23), assuming that $q$ follows the computed path.
above (displayed in Fig. 1, panel (a)) and $\pi_{h,t} = \pi$, $z_t = z$, $\pi_{g,t} = \pi_g$, $\pi_{x,t} = \pi_x$, $\eta_t = \eta$ and $\delta_t = \delta$.

The wedge-alone components do not include the neoclassical transitional components because they are computed assuming that the initial stock of detrended capital per capita equals its steady value. Therefore, we must subtract the neoclassical transitional components from the evolution of the variables to assess the contribution of the wedge-alone components. We do it by computing the non-transitional components of the variables. In particular, we divide the variable by its neoclassical transitional component. For example, if $y$ is detrended output per capita and $y_T$ its neoclassical transitional component, then its non-transitional component is $\hat{y} = y/y_T$ or, in logs, $\log \hat{y} = \log y - \log y_T$.

As we have divided the variables by their transitional components, we have to divide the wedge-alone components by the BGP steady values to carry out the comparison. Therefore, we compute the normalized wedge-alone components of the variables. In particular, we divide the wedge-alone components of the variables by the steady values of the variables along the BGP. For example, if $y_i$ is the wedge-alone component of detrended output per capita due to wedge $i$ and $y^*$ is the steady value of detrended output per capita, then the normalized wedge-alone component of detrended output per capita due to wedge $i$ is $\hat{y}_i = y/y^*$ or, in logs, $\log \hat{y}_i = \log y_i - \log y^*$.

On one hand, the non-transitional component of a variable reflects movements of the variable around its neoclassical transitional component. For example, for output per capita, it reflects its movements around its potential evolution path. On the other hand, the normalized wedge-alone components of the variables reflect the contributions of changes in the wedges to movements of the variables around their neoclassical transitional components. For example, the contribution of changes in the investment wedge to movements of output per capita around its potential evolution path. Therefore, we must compare the normalized wedge-alone components with the non-transitional components to assess the contributions of the wedges to movements of the variables around their potential paths.
5.1 Contribution of the wedges

The normalized wedge-alone components and the non-transitional components of the variables (output, labour, investment, and labour share) for the VES case are displayed in each panel of Fig. 3-6. The non-transitional component of a variable is displayed with a solid line, and its normalized wedge-alone components are displayed with dashed-solid lines. We display the results of our experiments for the CES and CD cases in Section IV of the Appendix.

To summarize the contribution of the normalized wedge-alone components to movements of the variables around their neoclassical transitional components, we define and compute the $\hat{\phi}$-statistic. It captures how closely a normalized wedge-alone component tracks the non-transitional component of a variable. The $\hat{\phi}$-statistic for $\hat{y}_{i,t}$ of wedge $i$ is as follows,

$$\hat{\phi} = \frac{1}{\sum_i (\log \hat{y}_{i,t} - \log \hat{y}_{i,t})}{\sum_i (1/\sum_t (\log \hat{y}_{i,t} - \log \hat{y}_{i,t}))},$$

where $\hat{y}_{i,t} \in \{\hat{h}_{i,t}, \hat{y}_{i,t}, \hat{x}_{i,t}, \hat{s}_{i,t}\}$ is the normalized wedge-alone component of a variable due to wedge $i$ and $\hat{y}_t \in \{\hat{h}_t, \hat{y}_t, \hat{x}_t, \hat{s}_t\}$ is the non-transitional component of the same variable. The statistic lies in $[0,1]$, sums to one across the five wedges, and reaches its maximum value of 1 when a particular normalized wedge-alone component tracks the non-transitional component perfectly. We compute this statistic for hours worked, output, investment, and labour share.\(^9\)

The $\hat{\phi}$-statistic is similar to the $\phi$-statistic proposed by Brinca et al. (2016), but both the observed variables and wedge-alone components are normalized, respectively, by their neoclassical transitional components and their steady valued along a BGP. Variables are in logs. Therefore, the $\hat{\phi}$-statistic subtracts the difference between the steady value of the variable and its neoclassical transitional component to the difference between the wedge-alone component and the variable.

As pointed out by Fehrle and Huber (2022), the $\hat{\phi}$-statistic is inadequate to distinguish between procyclical and countercyclical contributions of the wedges. For this reason, we also compute the Pearson correlation coefficients between the non-transitional component

\(^9\)The values of the logarithms of the variables are normalized so that their value is zero in the initial year of each period for which the $\phi$-statistic is computed.
of a variable and their normalized wedge-alone components. The values of the \( \hat{\phi} \)-statistic and the Pearson correlation coefficients are displayed in Tables 2 and 3. We display both statistics for the whole period 1850-2019 and for several sub-periods. Both statistics are imperfect measures of the contributions of the wedges. It is convenient to consider both statistics together with a close look at the figures to get a better idea of these contributions.

Here, we set forth a summary of our results. A more comprehensive discussion of the results is developed in section III of the Appendix. The main findings of our analysis can be summarized as follows:

i. The decline of the labour-efficiency wedge slowed down output growth from 1850 to the mid-1870s. From 1850 to 1875, the Pearson correlation coefficient between output and its component due to the labour-efficiency wedge is 0.834 (see Table 3) and the \( \hat{\phi} \)-statistic for output of the labour-efficiency wedge is 0.381 (see Table 2).

ii. The decline in the investment wedge weighed down Spanish economic growth in the second half of the 19th century until the First World War, especially from the mid-1870s. From this time, the decline of the capital-efficiency wedge strengthened the slowdown (see Fig. 4, panels (a) and (e)). From 1850 to 1875, the Pearson correlation coefficient between output and its component due to the investment wedge is 0.801, from 1875 to 1895, 0.898, and, from 1895 to 1914, 0.692 (see Table 3). From 1850 to 1875, the \( \hat{\phi} \)-statistic for output of the investment wedge is 0.407, from 1875 to 1895, 0.819, and, from 1895 to 1914, 0.801 (see Table 2).

iii. From the end of the 19th century to the First World War, the labour-efficiency wedge drove output growth above its potential growth. However, the contributions of the capital-efficiency and investment wedges were negative, preventing output growth from exceeding its potential growth until after the First World War (see Fig. 4, panels (a), (b) and (e)). From 1895 to 1914, the Pearson correlation coefficients between output and its components due to the labour-efficiency wedge, capital-efficiency wedge and investment wedge are \(-0.740\), 0.704 and 0.692 respectively (see Table 3). After the end of the First World War, the contributions of the capital-efficiency and investment wedges to output growth became positive, leading
to a slight recovery in output growth until the Great Depression (see Fig. 4, panels (a) and (e)).

iv. The output growth downturn in the crisis of the 1930s (Great Depression and Civil War) was primarily driven by the falling labour-efficiency wedge, while in the Great Recession were primarily driven by the falling capital-efficiency wedge (see Fig. 4, panels (a) and (b)). The labour wedge played a significant, but secondary role in both crises, more intensely depressing employment (see Fig. 4, panel (d), and Fig. 3, panel (d)). In both crises, the efficiency wedges moved in opposite directions. The Pearson correlation coefficients indicate that the labour-efficiency wedge had a positive correlation with output in the Great Depression, 0.865, while the capital-efficiency wedge had a negative correlation with output, −0.720. Conversely, the capital-efficiency wedge had a positive correlation with output in the Great Recession, 0.751, while the labour-efficiency wedge had a negative correlation, −0.722 (see Table 3).

v. The Spanish economic miracle of the 1960s was primarily driven by the increase in the labour-efficiency wedge and secondarily by the increase in the capital-efficiency wedge (see Fig. 4, panels (b) and (d)). The increase of both efficiency wedges translated in a significant increase of $\text{tfp}$ (see Fig. 4, panel (c)). From 1959 to 1974, the $\hat{\phi}$-statistic for output of the labour-efficiency (resp. capital-efficiency) wedge is 0.505 (resp. 0.319) (see Table 2). During this period, the improvement of the labour wedge had a significant positive impact on hours worked per capita (see Fig. 3, panel (d)).

vi. In the 1940s, a significant increase in the investment wedge preceded the Spanish economic miracle, leading to a substantial rise in the investment rate by the end of the decade (see Fig. 5, Panel (e)). The $\hat{\phi}$-statistic for output of the investment wedge is 0.172 from 1940 to 1959 (see Table 2), and the Pearson correlation coefficient between output and its component due to the investment wedge is 0.491 (see Table 3).

vii. From the early 1970s, the capital-efficiency wedge’s decline contributed to $\text{tfp}$ stagnation and output growth slowdown, while the labour-efficiency wedge worked
in the opposite direction (see Fig. 1, panels (a), (b) and (c), and Fig. 4, panels (a), (b) and (c)). The Pearson correlation coefficients between output and its components due to the capital-efficiency and labour-efficiency wedges from 1994 to 2007 are $-0.710$ and $0.763$, respectively, and from 2007 to 2019, they are $-0.751$ and $0.722$ (see Table 3). This same pattern of behaviour for both wedges and their contributions was also observed from the end of the 19th century to the First World War. From 1895 to 1914, the Pearson correlation coefficients between output and its components due to the capital-efficiency and labour-efficiency wedges were $-0.704$ and $0.740$, respectively (see Table 3).

viii. From the mid-1970s onwards, the labour wedge has been the primary factor driving the wide medium-term fluctuations in labour, output, and investment (see panels (d) of Figs. 3-6). The $\hat{\phi}$-statistics for labour, output, and investment of the labour wedge for the period 1974-2007 are $0.574$, $0.420$ and $0.521$ respectively, and for the period 2007-2019, they are $0.430$, $0.399$ and $0.273$ (see Table 2). The Pearson correlation coefficients between labour, output, and investment and their components due to the labour wedge for the period 1974-2007 were $0.954$, $0.351$ and $0.781$ respectively, and for the period 2007-2019, they were $0.550$, $0.387$ and $0.344$ (see Table 2).

ix. The capital-efficiency wedge was the primary factor driving the evolution of the Spanish labour share from 1850 to 2019, particularly its decline after the end of the 1970s. Together with the investment wedge, it also drove the decline of the labour share from the last quarter of the 19th century to the First World War (see Fig. 6, panels (a) and (e)). The $\hat{\phi}$-statistic for the labour share of the capital-efficiency wedge over the entire period 1850-2019 was $0.185$ (see Table 2), and the Pearson correlation coefficient was $0.871$ (see Table 3).

These main findings are robust to the specification of the production function as well as to the assumption of a KPR utility function, which is usual in the literature. Additionally, the analysis of the KPR shows that the labour wedge mostly drives the potential evolution of the hours worked per capita until the 1970s, but not their deviations from its potential path. We display the results of our simulations for three production functions (VES,
CES and CD) in Figs. A.2-A.8 in the section IV of the Appendix. The results of the simulations with a KPR utility function are displayed in Figs. A.9-A.13 in the section V of the Appendix.

6 Structural change and the great moderation

We demonstrate that a two-sector model produces our aggregate production function. This equivalence result is useful for connecting wedges to the economic, institutional, and technological processes that create them. In this instance, the equivalence result links wedges to structural change, i.e. the transition from an agricultural sector that primarily uses land and labour as production factors to an industrial sector that uses capital and labour.

The equivalence result reveals that structural change is reflected in both capital- and labour-efficiency wedges. Consider an economy with two intermediate sectors, 1 and 2. Sector 1 uses land, $T$, and labour, $\mu_1 H$, as productive factors and sector 2 uses capital, $K$, and labour, $\mu_2 H$, where $\mu_1 > 0$ and $\mu_2 > 0$ are the fractions of labour employed in sector 1 and sector 2, respectively, being $\mu_1 + \mu_2 = 1$. The production functions of both intermediate sectors are $Y_i = \left[ (\Upsilon_\omega (\mu_i H)^{1-\omega} + (\mu_i H)^{\rho}) \right]^{\frac{1}{\rho}}$, where $i = 1, 2$, $\Upsilon = T$ if $i = 1$ and $\Upsilon = K$ if $i = 2$. Production of final output requires both intermediate goods: $Y = [Y_1^\rho + Y_2^\rho]^{\frac{1}{\rho}}$. If the intermediate production functions are substituted into the production function of the final good, after a little of algebra, we have the production function $Y = \left[ \alpha K_A^{\psi} (zH)^{(1-\omega)p} + (1-\alpha) (zH)^{\rho} \right]^{\frac{1}{\rho}}$ where $K_A = [(q_K K)^{\psi} + (q_T T)^{\psi}]^{\frac{1}{\psi}}$, $\psi = \omega \rho$, $z = ((1-\mu_2)^{\rho} + \mu_2^\rho)^{\frac{1}{\rho}}$, $q_T = \mu_2^{\frac{1}{\rho}} z^{\frac{1-\omega}{\rho}}$ and $q_K = (1-\mu_2)^{\frac{1}{\rho}} z^{\frac{1-\omega}{\rho}}$. Finally, as argued in Section 3, we can rewrite previous production function as a function of capital and labour $Y = \left[ \alpha (q K)^{\omega p} (zH)^{(1-\omega)p} + (1-\alpha) (zH)^{\rho} \right]^{\frac{1}{\rho}}$, being $q = (1 + s_T / s_K)^{\frac{1}{\rho}} q_K$.

Previous analysis identify two sources of structural change: a change in the ratio of effective land to effective capital, $\frac{q_T}{q_K} = (\frac{z_T}{z_K})^{\frac{1}{\rho}}$, which is reflected in the first component of the capital-efficiency wedge, and a change in the fraction of labour employed in both sectors, $\mu_i$, which is reflected in both the second component of the capital-efficiency and the labour-efficiency wedge.

We focus on the first source of structural change, which reflects the modernization
of the agricultural sector. Using data on income factors shares, we compute the ratio of effective land to effective capital. Fig. 7 shows that this ratio remained trendily constant, although very volatile, until the end of the 1950s when the Spanish economic miracle started. During the 1960s and until the mid-1970s, however, it underwent a large increase. This suggests that the Spanish economic miracle involved a strong process of increasing efficiency and modernization of the agricultural sector. After the 1960s, volatility of this ratio significantly decreases. Therefore, structural change may have contributed to the Great Moderation in output volatility.  

Fig. 8 displays the output per capita and the \( (1 + s_T/s_K)^\frac{1}{\psi} \) series filtered with the Hodrick-Prescott filter, which shows the apparent reduction in volatility of both series after the 1970s. A test on the equality of the variances of each series between the periods 1850-1959 and 1974-2019 allows us to reject the null hypothesis of equality for both series (Table 4).

To assess the impact of the modernization of the agricultural sector during the Spanish economic miracle, we simulate the response of the model to changes in the ratio of effective land to effective capital following the same method described above. In particular, we simulate for the observed path of \( (1 + s_T/s_K)^\frac{1}{\psi} \) holding constant \( q_k \) and the other wedges. The results of the simulations are displayed in Fig. 9. The first noteworthy feature is the lower volatility of the component of output due to the modernization of the agricultural sector after the 1960s. Before 1959, its variance is 0.20, and after 1974, 0.012. Therefore, we can conclude that the modernization of the Spanish agricultural sector significantly contributed to reducing output volatility. We also find that the modernization of the Spanish agricultural sector had a significant, but secondary, contribution to the increase of detrended output per capita and detrended investment per capita (see Fig. 9, panels (b) and (c)). Between 1959 and 1974, the former grew 62% and the latter 94%, whereas the components of output and investment due to the modernization of the agricultural sector grew 12.76% and 27.68%, respectively. Moreover, the modernization of the agricultural sector also slightly contributed to increasing the labour share (see Fig. 9, panel (d)).

10The term “Great Moderation” refers to a significant reduction in output volatility that occurred during the mid-1980s in u.s. and other countries (see, for example, Bernanke, 2004 or Stock and Watson, 2005).

11The series has been filtered with the Hodrick-Prescott filter to compute the variances before 1959 and after 1974.
7 The Spanish economic growth 1850-2019

In this section, we interpret Spanish economic growth between 1850 and 2019 in light of the results of our analysis and drawing on the macroeconomic theories and economic history.

7.1 19th century: the investment burden

From 1850 to the mid-1870s, slow Spanish output growth was largely driven by a decline in the labour-efficiency wedges. Some authors point to Spain’s difficulty in implementing the agricultural advances achieved in Northern Europe during the 17th and 18th centuries, due to the prevalence of summer droughts in around 90% of Spanish territory (see Tortella, 1994). Other authors suggest inefficient property rights hindered investment and mechanization in the agricultural sector (see García Sanz, 1985, and Simpson, 1997). Historians generally agree that land reforms (the disposal of ecclesiastical and municipal land and the abolition of seigneurial jurisdiction, the sheep grazing association of the Mesta (1836), the entail of estates of the nobility (1836-41), and the tithes (1841)) failed to spark an agricultural revolution (see Nadal, 1973). Additionally, the mismanagement of the country’s mineral resources and the lack of forward and backward linkages may have had a negative impact on efficiency (see Harvey and Taylor, 1987).

The prolonged decline in the investment wedge hindered Spanish economic growth in the second half of the 19th century until the First World War, particularly from the last quarter of the 19th century. From this time, this was compounded by a simultaneous decrease in the capital-efficiency wedge, further exacerbating the negative effects on growth. Furthermore, during the second half of the nineteenth century and the first third of the twentieth century, the investment wedge remained relatively low, weighing on investment during this period.

As argued by Chari et al. (2002), an economy with the type of credit market frictions and investment costs considered by Carlstrom and Fuerst (1997) is equivalent to a growth model with investment wedges. Del Río and Lores (2023), argue that higher costs of investment can lower either the capital-efficiency wedge, the investment wedge or both. Banerjee and Duflo (2005) exhaustively review the extensive theoretical literature that
considers financial frictions a central issue in economic development, and Levine (2005) outlines five ways through which financial development influences economic outcomes.

The second half of the 19th century saw a series of major financial and economic crises, including the famines of 1857 and 1868 (see Fuentes, 2007), the financial crisis of 1866 (see Flandreau and Ugolini, 2014, and Fuentes, 2007), the Long Depression of 1873–1896 (see Capie and Wood, 1997), the colonial wars and the disaster of 1898. Comín and Cuevas (2017) report two debt crises (1861–1881 and 1885–1902) and five bank crises (1865, 1881–1882, 1890, 1913 and 1914) in Spain from 1850 to the First World War. Betrán et al. (2012) found that from 1850 to 2000, Spanish financial crises were longer and more frequent than those in the rest of the world. After the unstable bimetallic standard implemented in 1868, Spain suspended gold convertibility in 1883, which, according to Martín-Aceña (1993), resulted in a sharp decline in foreign capital inflows. Cuevas (2018) analyses the strong negative impact of the 1866 financial crisis on the expansion of the Spanish financial system and its backwardness relative to other European countries between 1780 and 1874. Pérez (1997) points out the inflationary bias of the Spanish financial system, centered around the Bank of Spain from 1874 and at the service of public finances. Martín-Aceña (1987) analyses the development of the Spanish financial system after 1830 and highlights its weaknesses.

Therefore, financial frictions raising firm’s investment costs might have been a major impediment to investment during the latter half of the 19th century. Other factors that could have hindered investment include high energy costs (see Pollard, 1981), an inefficient and costly transport system (see Simpson, 1997), and a low level of public investment in areas such as transport, health, and education (see Comín, 1994, 1995).

From the late 19th century to the First World War, our analysis reveals that the labour-efficiency wedge drove detrended output per capita growth above its potential. However, the opposing contributions of the capital-efficiency and investment wedges prevented output growth from exceeding its potential until the end of the First World War, likely reflecting the persistence of 19th century distortions. From the First World War to the Great Depression, both capital-efficiency and investment wedges improved, resulting in a recovery of output growth.

According to Tortella (1994) the south-western European countries (Spain, Portugal, and Italy) began a halting recovery relative to the north-western European countries
from 1900 to 1930. He argues that from the First World War the Spanish economy experienced significant structural change, expressed in the decline of the active population in agriculture and the increase in literacy. Prados de la Esclosura and Rosés (2021) report an acceleration of Spanish growth between 1920 and 1929, with an average annual growth rate of 4.1 percent compared to 1.2 percent between 1893 and 1913. Rosés et al. (2010) argue that the initial industrialization of Spain was concentrated in some few regions, but since 1900 manufacturing production spread to others many locations. Martínez-Galarraga et al. (2015), using parametric and non-parametric techniques, argue that the construction of new transport networks coupled with changes in trade policies drove economic integration of Spanish regions, which boosted economic growth and stabilized per capita income disparities between 1900 and 1930.

7.2 The labour-efficiency wedge and the Spanish economic miracle

The late 1950s to mid-1970s saw a period of rapid economic growth, driven primarily by an increase in the labour-efficiency wedge and secondarily by an increase in the capital-efficiency wedge. This might reflect the modernization and industrialization of the Spanish economy, which saw a rapid adoption of new technologies and a substantial increase in human capital (reported by Prados de la Escosura and Rosés, 2021), allowing it to converge towards the world technological frontier.

Sanchís-Llopis (2006) found that, although sharper in some manufacturing industries, transport and communications, the increase in TFP spread to most production sectors during the Spanish economic miracle, and concluded that productivity was intensely affected by spillover effects linked to new technologies, such as changes in firm organization, improvements in human capital, economies of scale, mass production processes and different efficiency improvements, which can be identified as disembodied technological change. The autarkic policies of the Franco regime in the 1940s led to an international isolation of the Spanish economy, but this was reversed in the early 1950s, particularly with the 1959 Stabilization and Liberalization Plan. The 1950s saw a strong increase in imports and exports, as well as foreign investment (see Prados de la Escosura et al., 2011).
Nelson and Phelps (1966) were pioneers in analyzing the interaction between human capital and technology adoption and its relevance to technological catch-up and economic growth. Bils and Klenow (2000) and Vandenbussche et al. (2006) have both recently addressed the same question. McGrattan and Prescott (2009) provided a model in which foreign direct investment is a channel for technology absorption.

7.3 The investment wedge and the foundations of the Spanish economic miracle

From the late 1940s to the early 1950s, the investment wedge experienced a significant rise, which was followed by a substantial increase in the investment rate in the second half of the 20th century. The improvement in the investment wedge could be attributed to various factors that WGA cannot identify due to its intrinsic limitations. However, according to the theory, the reduction of firms’ investment costs could be reflected in the increase in the investment wedge (see Chari et al., 2002). In light of the findings of some articles, we believe that economic and political reforms, such as easing financial frictions, promoting political stability, and securing property rights, may have reduced firms’ investment costs, thus encouraging investment and laying the foundations for the Spanish economic miracle of the 1960s.

In 1946, Franco’s government passed a banking regulation law that replaced the regulations of 1921 and 1931. According to Martín-Aceña et al. (2014), this law provided the Spanish financial system with stability and encouraged the channelling of savings to industrial investments, despite its interventionist nature. Calvo-González (2007) argues that the intensification of the Cold War at the end of the 1940s led to the United States taking an interest in Spain as a military ally, thereby helping to consolidate Franco’s regime and increasing economic confidence (particularly with regards to security of property rights under the dictatorial regime). This helps to explain why economic growth resumed in Spain before any significant changes to the autarkic economic policies of the Francoist regime were implemented. Calvo-González (2007) shows that the volatilities of both the peseta-dollar exchange and the spread between the unofficial price of gold in Madrid and the market price in Zurich significantly reduced at the end of 1952, when it was widely expected that the negotiations underway would lead to the signature of the
**Pacto de Madrid** between the United States and Spain in 1953, as it did. He also finds a positive and statistically significant increase in Madrid stock returns following the signature of the **Pacto de Madrid** and that the lower volatility of the exchange rate fostered investment.\(^{12}\) Prados de la Escosura et al. (2011) argue that the gradual reduction of economic distortions by the Francoist regime before the major reform of the 1959 Stabilization and Liberalization Plan fostered economic growth from the early 1950s. Fraile (1999) reports the strong anti-market attitude of the Francoist regime and its economic policies that threatened property rights from the Civil War up to the early 1950s. Buera and Shin (2013) analyses the process of economic development following large-scale reforms that reduce economic distortions in the presence of financial frictions. Their model accounts for the dynamics of TFP and the increase in investment rates in countries that experienced economic miracles. Similarly, Giné and Townsend (2004)’s model analyses the effects of financial liberalization in Thailand from 1976 to 1996. Jeong and Townsend (2007) find that financial deepening was a key factor in the rapid output and TFP growth in Thailand between 1976 and 1996.\(^{13}\) Besley and Ghatak (2010) argue that strengthening property rights boosts investment by limiting expropriation, facilitating the trade of assets, and improving their collateralizability, which reduces financial frictions.

### 7.4 The labour wedge and the strong oscillations from the mid-1976

Our analysis revealed that the strong fluctuations in Spanish employment, output, and investment from the mid-1970s were mainly driven by the labour wedge. Specifically, the deterioration of the labour wedge was responsible for the crisis of the 1970s, while its improvement drove the recovery of the 1980s and 1990s. Although the labour wedge was a major factor in the output growth slowdown during the Great Recession, the primary cause was the decline in capital-efficiency wedge, but the labour wedge was almost exclusively responsible for the decrease in hours worked per capita.

\(^{12}\) In November 1950, the United States supported a vote in the U.N. General Assembly invalidating the 1946 resolution, which excluded Spain from this organization, while the **Pacto de Madrid** (September 1953) committed the United States to provide an unspecified amount of aid in return for the right to establish four military bases in Spain.

\(^{13}\) See also Jeong and Townsend (2008).
Our findings are in line with those of Rodríguez-López and Solís-García (2016), who find that the Spanish recession of the 1970s and the Spanish Great Recession were largely driven by the labour wedge. They argue that taxes and labour market institutions are likely responsible for the labour wedge movements. Boldrin et al. (2010) attribute the strong fluctuations in Spanish productivity and employment from 1978 to 2008 to the Spanish labour market’s features, particularly the dual system that protects permanent workers at the expense of temporary ones and the inefficient collective wage bargaining system.

7.5 The wedges, the Great Depression and the Great Recession

We find that the poor performance of output and investment in Spain during the Great Depression and the Civil War is mainly driven by worsening labour-efficiency wedges, while the fall of hours worked is mainly attributed to the labour wedge.

The Civil War caused immense losses of lives and physical capital, but, in addition, the exile and the post-war political repression resulted in a significant destruction of human capital (see Prados de la Escosura and Rosés, 2010). Giménez and Montero (2015) argue that frictions and a monopsonistic structure of the labour market contribute to explain the behaviour of output, employment, and investment during the Spanish Great Depression. Jorge-Sotelo (2019) further argues that, during the Spanish Great Depression, private investment experienced a sharp decline because bank loans contracted severely due to the limitations on how much emergency liquidity could be provided by the Banco de España.

Our results are consistent with the theory proposed by Chari et al. (2002) that the Spanish Great Depression was caused by monetary limitations leading to bank loan constraints, resulting in a misallocation of investments across firms (input-friction theory) in a context of wage stickiness (sticky-wage theory) and increasing cartelization and unionization (cartelization theory). However, the investment-friction theory, which suggests that monetary contractions increase frictions in capital markets and the firm investment costs, which produce investment-driven downturns in output (see Bernanke and Gertler, 1989, and Carlstrom and Fuerst, 1997), does not appear to be a promising explanation for the Spanish Great Depression because we do not find that the investment or capital-
efficiency wedges played a significant role in this crisis.

We find that the Spanish Great Recession was primarily driven by the capital-efficiency wedge and secondarily by the labour wedge. Del Río and Lores (2023) argue that financial frictions, which increase investment costs, can be reflected in the capital-efficiency wedge. This suggests that the investment-friction theory may help to account for the Spanish Great Recession. Additionally, the decline in the capital-efficiency wedge during the Great Recession may be attributed to the devaluation of real estate capital. During the Spanish Great Recession, we find a rebound in the growth of the labour-efficiency wedge. Hospido and Moreno-Galbis (2015) argue that this might be due to less-productive massive temporary job destruction and an increased weight of large firms with higher TFP.

7.6 The capital-efficiency wedge, the growth slowdown and the labour share

Our results indicate that the growth slowdown after the mid-1970s was mainly driven by the decline in the capital-efficiency wedge, which, although partially offset by an increase in labour-efficiency, caused stagnation in Spanish TFP.

Díaz and Franjo (2016) argue that the stagnation of Spanish TFP between 1996 and 2007 was caused by a low rate of investment-specific technological change and an inefficiently high investment in residential structures, which was fostered by subsidies to the housing purchase. Pérez and Benages (2017) find that Spanish capital productivity experienced a persistent decline between 1981 and 2014, attributing it to overinvestment in residential structures, the low investment in intangible assets due to the prevalence of traditional sectors, the low weight of large firms, and the importance of micro enterprises. García-Santana et al. (2020) argue that TFP stagnation during the 1995-2007 period was caused by cronyism, which drove misallocation of production factors across firms. Moral-Benito (2018) argues that evidence based on firm-level data suggests that resource misallocation across firms is the main cause of TFP deterioration between 1995 and 2007, and that the allocation of credit to low-productivity but high-collateral firms can partly explain the resource misallocation across firms. Koch et al. (2021) provide structural estimates of TFP and document that most of the productivity decline in Spanish manufacturing in their sample could be attributed to firms that never used robots.
We find that the output growth slowdown and the decline in the Spanish labour share from the mid-1970s onwards was largely driven by the falling capital-efficiency wedge. At a microeconomic level, Koch et al. (2021) estimate that the labour cost share decreases by almost 7% following robot adoption by Spanish firms. Koh et al. (2020) suggest that the decline of the labour share in the United States is due to the transition to an economy more heavily reliant on intellectual property assets. From the late 19th century to the First World War, the capital-efficiency wedge, in conjunction with the investment wedge, also drove the fall of the Spanish labour share.

8 Conclusion

By conducting a wedge-growth accounting analyses, we have identified the forces driving Spanish economic growth between 1850 and 2019. Our analysis provides a guide for possible explanatory theories of the various vicissitudes experienced by the Spanish economy throughout this long period of economic development. Nevertheless, wedge-growth accounting does not provide the causes of events; rather, it indicates where we should focus our attention to find a precise explanation.

Many economists and historians have asked whether the 19th century was a period of economic failure for Spain. To answer this question, we can compare the Spanish economy to other neighbouring economies or evaluate it in light of the neoclassical growth model. According to neoclassical theory, the potential growth rate of output per capita of an economy is determined by the growth rate of output per capita along the transitional path to the Balanced Growth Path of the economy. Our calculations show that the output per capita growth rate in the second half of the 19th century until the First World War was lower than the potential economic growth rate. Thus, we can conclude that this period was a relative failure. Prados de la Escosura and Sánchez-Alonso (2020) also find this relative failure when comparing the GDPs of Spain and the U.S.

Our results indicate that the decline in the labour-efficiency wedge hindered Spanish economic growth from 1850 to the mid-1870s. This weak behaviour of the labour-efficiency wedge may be attributed to the failure of the “liberal” land reforms of the first half of the 19th century, as suggested by the existing literature. Additionally, we find that the decline of the investment and capital-efficiency wedges impeded output growth.
from the second half of the 19th century until the First World War. According to the theory, higher investment costs can lower both wedges. In light of existing research on the economic vicissitudes of the 19th century, we believe that the financial frictions resulting from the underdeveloped Spanish financial system may have been the primary cause of the high investment costs.

From the late 1950s to the mid-1970s, the Spanish economy experienced an impressive period of economic growth. Our results indicate that this was driven by both labour-efficiency and capital-efficiency wedges. Our analysis shows that this economic miracle was preceded by a significant increase in the investment wedge, which translated into a sharp rise in the investment rate from the mid-20th century. In light of existing research, it was likely due to greater political and financial stability, which strengthened property rights and reduced firms’ investment costs. We believe that this stability and security enabled the deep process of modernization, industrialization, and technological adoption of the 1960s, which was reflected in the strong increase of both efficiency wedges and the rapid convergence of the Spanish economy to the world technological frontier.

Since the mid-1970s, Spanish economic growth slowed down and TFP stagnated. Our analysis finds that both the stagnation in TFP and the output growth slowdown were driven by the prolonged and persistent decline in the capital-efficiency wedge. In contrast, the labour-efficiency wedge contributed positively to TFP and output growth. Thus, the Spanish productivity puzzle is not only composed of the causes of the stagnation of TFP and the slowdown in growth, but also of the reasons why the capital- and labour-efficiency wedges are exhibiting such different behaviour. Interestingly, both wedges also showed an opposite behaviour from the end of the 19th century to the First World War. However, the growth of output per capita from the mid-1970s slightly exceeded its potential growth, unlike in the second half of the 19th century. This was due to the investment wedge not being as negative as in the second half of the 19th century.

Our analysis shows that the Spanish economy in the second half of the 19th century and after the early 1980s experienced similar growth patterns in some respects. This brings up the issue of what is causing these similarities. In both time periods, the capital-efficiency and labour-efficiency wedges moved in opposite directions, and the decrease in the capital-efficiency wedge was mainly responsible for the decrease in the labour share. However, from the mid-1970s, the output per capita growth slightly exceeded its potential
growth, unlike in the second half of the 19th century. This was because the behaviour of the investment wedge was not as negative as it was in the second half of the 19th century.

We find that the labour wedge have played a major role in accounting for the pronounced medium-run fluctuations in production, labour, and investment in the Spanish economy since the mid-1970s. This suggests that the causes for these wide oscillations might be found in the shortcomings of the Spanish labour market, as pointed out by many economists.

From the mid-1970s onwards, we find that the capital-efficiency wedge of the Spanish economy experienced a continued decline (see del Río and Lores, 2021, find a similar fact for the United States). Some economists have investigated the misallocation of investments in the Spanish economy during this period. We hypothesize that this misallocation of capital could account for the decline in the capital-efficiency wedge. Further research into its causes and implications for capital efficiency might help to better understanding the Spanish economic growth slowdown since the mid-1970s. We believe that this research should take into account the opposite behaviour of the capital-efficiency and labour-efficiency wedges.

We have established an equivalence result between our aggregate production function with capital and land and a two-sector economy with an agricultural sector using land and an industrial sector using capital. In light of this equivalence result, we have determined that modernization of the agricultural sector contributed significantly, though secondarily, to the increase in output growth during the Spanish economic miracle. Moreover, it substantially moderated output volatility after the 1960s. While our research has provided valuable insight into the role of structural change in Spanish economic development, we did not have analysed all changes involved by the structural change and further research is needed to quantify the consequences of the structural change for Spanish economic development.
References


Table 1

**MODEL PARAMETERS AND bgp VARIABLES.**

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**bgp variables**

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Table 3
**Correlation Coefficients between Non-Transitional Component and Normalized Wedge-Alone Component of Variables (logs).**

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<td>1875-1895</td>
<td>0.541</td>
<td>0.521</td>
<td>-0.317</td>
<td>0.609</td>
<td>0.726</td>
<td></td>
</tr>
<tr>
<td>1895-1914</td>
<td>0.517</td>
<td>0.554</td>
<td>0.623</td>
<td>0.545</td>
<td>0.189</td>
<td></td>
</tr>
<tr>
<td>1914-1929</td>
<td>-0.174</td>
<td>-0.829</td>
<td>0.352</td>
<td>0.484</td>
<td>0.624</td>
<td></td>
</tr>
<tr>
<td>1929-1940</td>
<td>-0.844</td>
<td>0.181</td>
<td>0.527</td>
<td>-0.250</td>
<td>-0.207</td>
<td></td>
</tr>
<tr>
<td>1940-1959</td>
<td>-0.756</td>
<td>-0.081</td>
<td>0.679</td>
<td>0.498</td>
<td>-0.959</td>
<td></td>
</tr>
<tr>
<td>1959-1974</td>
<td>0.585</td>
<td>-0.629</td>
<td>0.725</td>
<td>-0.778</td>
<td>0.440</td>
<td></td>
</tr>
<tr>
<td>1974-2007</td>
<td>0.382</td>
<td>0.050</td>
<td>0.954</td>
<td>0.246</td>
<td>-0.187</td>
<td></td>
</tr>
<tr>
<td>2007-2019</td>
<td>0.535</td>
<td>0.047</td>
<td>0.550</td>
<td>0.327</td>
<td>-0.657</td>
<td></td>
</tr>
</tbody>
</table>
Table 4
Volatility break test.\textsuperscript{a}

<table>
<thead>
<tr>
<th></th>
<th>1850-1959</th>
<th>1959-2019</th>
<th>$\frac{\sigma_1^2}{\sigma_2^2}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\sigma_{y}^2$</td>
<td>0.278</td>
<td>0.112</td>
<td>2.477</td>
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<tr>
<td>$\sigma_{q/q}^2$</td>
<td>1.007</td>
<td>0.045</td>
<td>22.245</td>
</tr>
</tbody>
</table>

\textsuperscript{a} Series in logs filtered using the HP filter.
The critical $F_{108,60}$ value at $\alpha = 0.025$ is 1.591.
(a) Capital-Efficiency wedge.

(b) Labour-Efficiency wedge.

(c) Total Factor Productivity.

(d) Labour wedge.

(e) Investment wedge $\pi_{xt}^{-1}$.

(f) Resource constraint wedge.

Fig. 1: Wedges Paths.
Fig. 2: Labour-efficiency wedge and quality indices for labour.
Fig. 3: Contribution of Wedges to Hours Worked.
Fig. 4: Contribution of Wedges to Output (Detrended).
Fig. 5: Contribution of Wedges to Investment.
Fig. 6: Contribution of Wedges to Labour Share.
Fig. 7: Structural change, \((1 + \frac{s_{T,t}}{s_{K,t}})^{\frac{1}{\psi}}\).

Fig. 8: Output per capita and structural change filtered with H-P.
Fig. 9: Contribution of structural change to variables.

(a) $\left(1 + \frac{\sigma_T}{\sigma_K}\right)^{1/\psi}$ contribution to Hours Worked.

(b) $\left(1 + \frac{\sigma_T}{\sigma_K}\right)^{1/\psi}$ contribution to Output.

(c) $\left(1 + \frac{\sigma_T}{\sigma_K}\right)^{1/\psi}$ contribution to Investment.

(d) $\left(1 + \frac{\sigma_T}{\sigma_K}\right)^{1/\psi}$ contribution to Labour Share.