

Accounting for Spanish economic development 1850-2019

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17 January 2023

Online at https://mpra.ub.uni-muenchen.de/116025/ MPRA Paper No. 116025, posted 18 Jan 2023 05:18 UTC

APPENDIX FOR "ACCOUNTING FOR SPANISH ECONOMIC DEVELOPMENT 1850-2019"

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I. Data

Data are provided by Prados de la Escosura (2017, 2020) and Prados de la Escosura and Rosés (2021).

Population, L_t , is population between 16 – 65 years at a time t. Hours Worked at a time t, H_t , are total hours worked at a time t. Therefore, the fraction of time devoted to work, $H_{L,t}$, equals the hours worked per capita divided by the available hours per person (2/3 of the total, 8760 hours per year, $H_A = 5840$), $H_{L,t} = H_t/(L_t H_A)$.

Real output, real consumption and real investment are computed deflating the corresponding nominal magnitudes by the implicit deflator of GDP: $Y_t = Y_{N,t}/P_t$, $C_t = C_{N,t}/P_t$ and $X_t = X_{N,t}/P_t$. We detrend real output, real investment and real consumption per capita by $(1 + \gamma_y)^t$ and the fraction of time per capita devoted to work by $(1 + \gamma_h)^t$. Values of γ_y and γ_h are calibrated in section 4.1.

The depreciation rate is obtained as the percentage that the consumption of fixed capital represents of the net capital stock. The labour share as well as the capital share and the land share are provided by Prados de la Escosura and Rosés (2021), and we compute $s_{k,t} = 1 - s_{h,t}$. Capital stock is computed using the perpetual inventory method according to the move law in section 3 of the main text. We do not use data for the stock of available land. According our production function, we can compute the ratio of effective land to effective capital from the ratio of land share to capital share: $\frac{q_T T}{q_K K} = \frac{s_T}{s_K}$.

We normalize so that both detrended output per capita and detrended time devoted to work per capita have a mean value of 1 over the sample period. The data are available in Fundación Rafael del Pino.

II. ESTIMATING THE PRODUCTION FUNCTION

If the production function is VES, then the first order condition for capital (4) can be log-linearized in the following way

$$\log s_{K_A,t} = \beta_0 + \beta_1 t + \rho \log \frac{K_t}{Y_t} + \frac{\omega \rho}{\psi} \log \left(1 + \frac{s_{T,t}}{s_{K,t}}\right) + \rho(\omega - 1) \log \frac{K_t}{H_t} + \epsilon_t,$$

where $\beta_0 = \log \alpha \omega$, $\beta_1 = (1 - \omega) \log(1 + \gamma)$ and $\epsilon_t = \rho (\omega \log q_{K,t} + (1 - \omega) \log z_t)$. Here, $s_{K,t}$ is the capital share, $s_{T,t}$ is the land share and $s_{K_A,t}$ is the aggregate share of capital and land, $s_{K_A,t} = s_{T,t} + s_{K,t} = 1 - s_{h,t}$, where $s_{h,t}$ is the labour share. Capital stock is computed using the perpetual inventory method according to the move law specified in section 3 of the main text.

We introduce two dummy variables to account for the mean changes in periods 1950– 1974 and 1975–2019. The previous equation is estimated by both Ordinary Least Squares (OLS) and three-stage generalized instrumental variables (GIV) approach developed by Fair (1970) and applied by Antras (2004) and Young (2013) in the present context. The stages are as follows. First, run a two-stage least squares regression (2SLS). We use two groups of instrumental variables. On the one hand, the lagged dependent and independent variables and, on the other hand, a group of variables taken from Prados de la Escosura (2017). In particular, we use the taxes on production and imports, gross value added in industry and consumption of fixed capital. The three variables are expressed as ratios to GDP and in logs. Second, estimate an AR(1)regression of the 2SLS residuals: $\epsilon_t = \varkappa \epsilon_{t-1} + u_t$. Third, use the estimated coefficient from the second stage to estimate

$$\log s_{K_A,t} - \hat{\varkappa} \log s_{K_A,t} = \beta_0 + \beta_1 t + \rho \left(\log \frac{K_t}{Y_t} - \hat{\varkappa} \log \frac{K_{t-1}}{Y_{t-1}} \right) + \frac{\omega \rho}{\psi} \left(\log \left(1 + \frac{s_{T,t}}{s_{K,t}} \right) - \hat{\varkappa} \log \left(1 + \frac{s_{T,t-1}}{s_{K,t-1}} \right) \right) + \rho(\omega - 1) \left(\log \frac{K_t}{H_t} - \hat{\varkappa} \log \frac{K_{t-1}}{H_{t-1}} \right) + \epsilon_t.$$

The third stage is, therefore, a feasible generalized least squares (FGLS) treatment assuming an AR(1) process for the errors. The 2SLS and FGLS components are to account, respectively, for endogeneity and serial correlation in the OLS residuals. The results of the estimations are displayed in Table A.1.

| Equation | OLS (VES) | OLS (CES) | GIV ^b (VES) | GIV ^b (CES) | GIV ^C (VES) | GIV ^C (CES) |
|--|---------------|---------------|------------------------|------------------------|------------------------|------------------------|
| <u> </u> | 2.437** | 2.372** | 0.404** | 0.362** | 0.524** | 0.621** |
| Constant | (0.892) | (0.893) | (0.088) | (0.077) | (0.111) | (0.068) |
| D | -0.283^{**} | -0.255^{**} | -0.031 | -0.021 | -0.062 | 0.005 |
| $Dummy_{1950-1974}$ | (0.052) | (0.048) | (0.024) | (0.020) | (0.023) | (0.017) |
| Dummer | -0.19^{*} | -0.056 | -0.011 | 0.010 | -0.051 | 0.002 |
| $Dummy_{1975-2019}$ | (0.11) | (0.055) | (0.046) | (0.025) | (0.047) | (0.02) |
| 4 | 0.006** | 0.007^{**} | 0.001^{**} | 0.001^{**} | 0.001 | 0.000 |
| t | (0.002) | (0.001) | (0.000) | (0.000) | (0.001) | (0.000) |
| lm(1 + c / c) | 0.602** | 0.564^{**} | 0.508^{**} | 0.510^{**} | 0.521^{**} | 0.383** |
| $\ln(1+s_T/s_K)$ | (0.154) | (0.152) | (0.085) | (0.084) | (0.083) | (0.06) |
| $\ln K/V$ | -0.399^{**} | -0.324^{**} | 0.410^{**} | 0.388^{**} | -0.401^{**} | -0.319^{**} |
| $\ln K/Y$ | (0.137) | (0.127) | (0.129) | (0.122) | (0.123) | (0.1) |
| $\ln K/H$ | 0.096 | | 0.086^{**} | | 0.179 | |
| $\ln K/H$ | (0.06) | | (0.164) | | (0.134) | |
| R^2 | 0.43 | 0.424 | 0.33 | 0.328 | 0.338 | 0.34 |
| | | | Model P | arameters | | |
| ρ | -0.399 | -0.324 | -0.410 | -0.388 | -0.401 | -0.319 |
| ψ | -0.503 | -0.574 | -0.638 | -0.761 | -0.426 | -0.832 |
| ω | 0.759 | | 0.790 | | 0.553 | |
| Significant at 10% (*) and at 5% (**). | | | | | | |

TABLE A.1ESTIMATION RESULTS^a. Anual data 1850-2019.

^a IBM Corp. Released 2016. IBM SPSS Statistics for Windows, Version 24.0. Armonk, NY: IBM Corp.

^b Instruments are lagged dependent and independent variables.

^c Instruments are logs of: ratio of taxes on production and imports to GDP, ratio of gross added value in the industrial sector to GDP and ratio of consumption of fixed capital to GDP.

III. CONTRIBUTION OF THE WEDGES

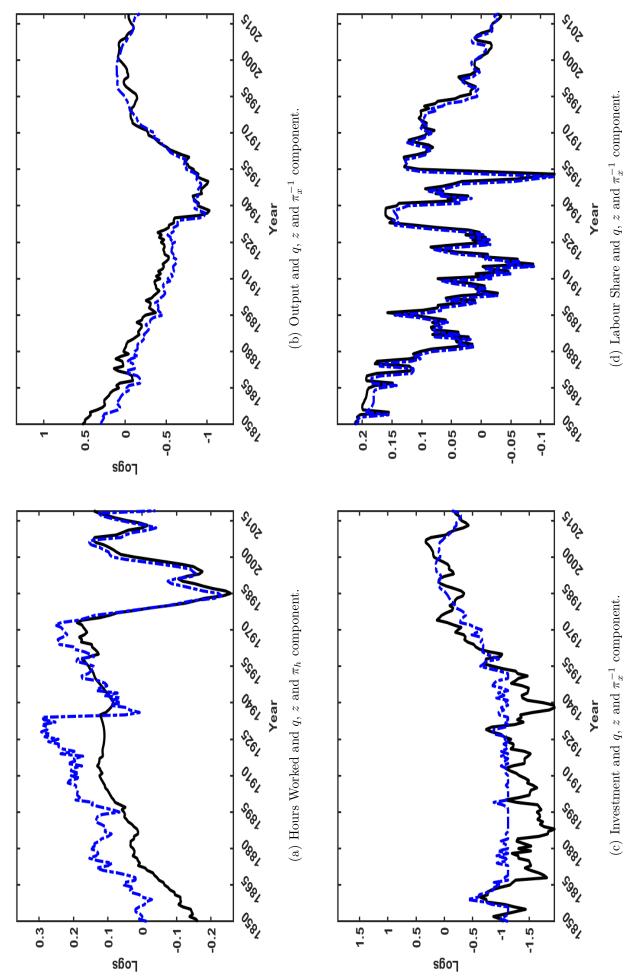
Labour. The investment wedge was the main force driving evolution of detrended hours worked per capita above its potential path from 1850 to the Civil War, although the efficiency wedges also contributed it (see Fig. A.3, panels (a), (b) and (e)). From the end of the First World War, the contribution of the labour-efficiency wedge would have driven to a strong increase in hours worked per capita, but the contribution of the labour wedge worked in the opposite direction and the evolution of hours worked per capita remained around its potential trend (see Fig. A.3, panels (b), (d) and (e)). In the crisis of the 1930s (Great Depression and Civil War), worsening of the labour would have caused a big fall of detrended hours worked per capita, but they did not fall a lot mainly due to the counterweight of the capitalefficiency wedge (see Fig. A.3, panels (a) and (d)). However, after the Civil War and until the mid-1970s, the labour wedge underwent a big improvement leading to the evolution of detrended hours worked per capita to exceed its potential evolution, but mostly before the sixties, inasmuch as the wedge-alone components due to the investment and labour-efficiency wedges reduced the expansive effect of the labour wedge after the end of the fifties (see Fig. A.3, panels (b), (d) and (e)). From the mid-1970s, movements of the labour wedge accurately accounted for the oscillations of detrended hours worked per capita (see Fig. A.3, Panel (d)). In particular, movements of the labour wedge accounted for (a) the strong fall of the detrended hours worked per capita from the mid-1970s to the mid-1980s; (b) the significant recovery from the mid-1980s to the Great Recession and (c) the fall in the Great Recession and its subsequent recovery.

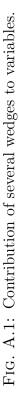
Output. The decline of the investment wedge was the main force slowing output growth from 1850 to the First World War (see Fig. A.4, Panel (e)). From 1850 to 1868, the fall of the labour-efficiency wedge strengthened the slowdown and, from the mid-1870s to the First World War, the capital-efficiency wedge did it (see Fig. A.4, panels (a) and (b)). From the late 19th century to the Great Depression, the contribution of the labour-efficiency wedge would have driven growth of detrended output per capita above its potential growth, but the opposite contributions of the capital-efficiency and investment wedges prevented it until the end of the First World War (see Fig. A.4, panels (a), (b) and (c)). The main force driving growth slowdown of output during the Great Recession and the Civil War was the strong fall of the labour-efficiency wedge (see Fig. A.4, Panel (b)) and the labour wedge played a significant, but secondary role (see Fig. A.4, Panel (d)). Growth acceleration of output between the end of the 1950s and the mid-1970s was primarily due to the substantial increase in the labour-efficiency wedge (see Fig. A.4, Panel (b)). The labour and capital-efficiency wedges played significant, but secondary roles (see Fig. A.4, panels (a) and (d)); earlier, the recovery of output growth from the beginning of the 1950s was mainly driven by the investment wedge (see Fig. A.4, Panel (e)). The leading cause of the growth slowdown of output from the mid-1970s (and, in particular, during the Great Recession) was the decline of the capital-efficiency wedge (see Fig. A.4, Panel (a)). From the mid-1970s, the contributions of the capital- and labour-efficiency wedges to the evolution of output

growth go in opposite directions and, while the contribution of the former slows down output growth, the contribution of the latter accelerates it (see Fig. A.4, Panel (a) and (b)). The latter predominates until the beginning of the 21st century, but the former predominates thereafter. Consequently, the joint contribution of these two wedges (or the contribution of the TFP) slows down output growth from the beginning of the 21st century (see Fig. A.4, Panel (c)). From the mid-1970s, oscillations in the labour wedge played a significant role in output movements (see Fig. A.4, Panel (d)). In particular, they played a significant role in accounting for (a) output growth slowdown in the 1970s until the mid-1980s; (b) recovery of output growth from the mid-1980s to the Great Recession; (c) growth slowdown of output in the Great Recession and its subsequent recovery.

Investment. Detrended investment per capita remained low until the second half of the 20th century due to the low level of the investment wedge, but it increased from the middle of the 20th century because the investment wedge underwent a significant increase in the 1940s and 1950s (see Fig. A.5, Panel (e)). Detrended investment per capita underwent a slight increase from the end of the 19th century to the Great Depression mainly driven by the increase of the labourefficiency wedge, but, during the Great Depression and the Civil War, it fell due to the decrease of this same wedge (see Fig. A.5, Panel (b)). Its recovery after the Civil War was mostly driven by the increase in the investment wedge until the mid-1950s (see Fig. A.5, Panel (e)) and its boom from the end of 1950s to the beginning of the 1970s was primarily driven by the capital-efficiency wedge and secondarily by the labour wedge (see Fig. A.5, panels (a) and (d)). After the end of the 1970s, the contributions of the capital- and labour-efficiency wedges to the evolution of investment go in opposite directions and, while the contribution of the former boosts investment, the contribution of the latter depresses it. Consequently, the joint contribution of these two wedges (or the contribution of the TFP) becomes negative from the beginning of the 21^{st} century (see Fig. A.5, Panels (a), (b) and (c)). Oscillations in the labour wedge largely drove detrended investment per capita movements from the mid-70s to the Great Recession (see Fig. A.5, Panel (d)). In particular, it accounted for (a) investment growth slowdown in the 70s until the mid-80s; (b) investment growth recovery from the mid-80s to the Great Recession; (c) investment growth slowdown of investment during the Great Recession.

Labour share. Primarily, the capital-efficiency wedge and, secondarily, the capital investment wedge contributed to depressing the labour share during the second half of the 19^{th} century and up to the First World War (see Fig. A.6, panels (a) and (e)). After the mid-1970s, the decline of the capital-efficiency wedge driven the fall of the labour share (see Fig. A.6, panels (e)). However, the increase in the labour share from the First World War to the Civil War was mainly driven by the increase of the labour-efficiency wedge, although the capital-efficiency wedge played a significant, but secondary role (see Fig. A.6, panels (a) and (b)). The labour wedge has not played any significant role in accounting for the evolution of the labour share from 1850 to 2019 (see Fig. A.6, panel (d)).





IV. Other production functions: Ces and CD

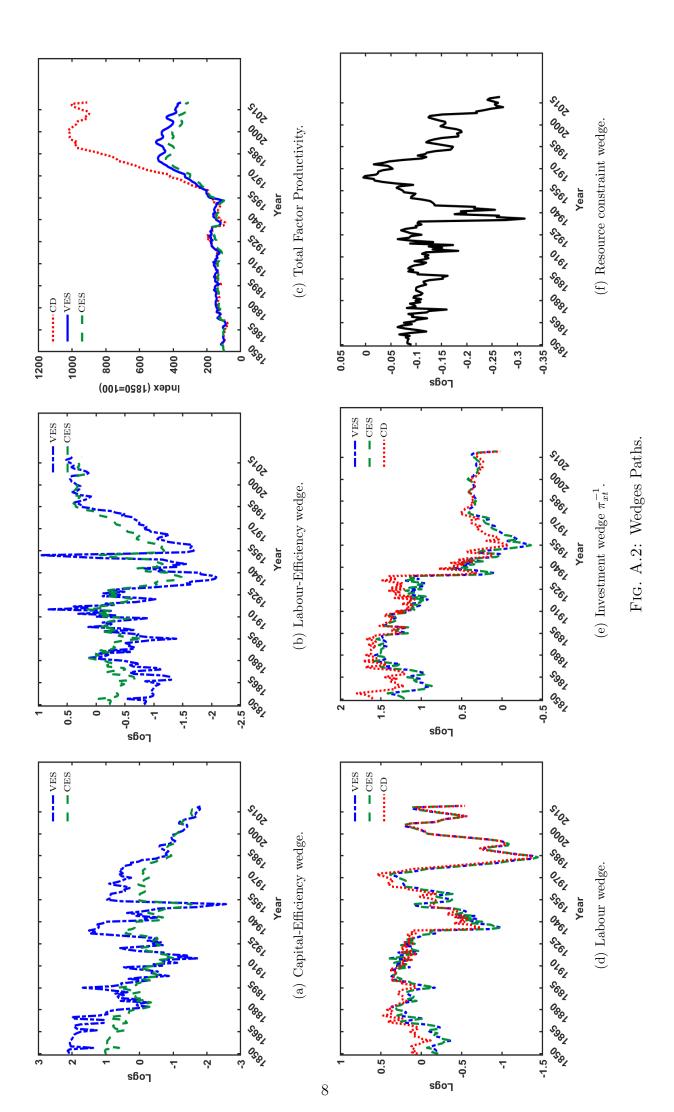
We have simulated the model for three alternative specifications of the production function. In particular, we consider VES, CES, and CD production functions. The main results of our simulations are quite robust to the specification of the production function. We display the evolution of the wedges in Fig. A.2 and the results in Fig. A.3-A.6. We do not divide the variables by its neoclassical transitional components, neither the wedge-alone components by its steady value to can display all simulations in the same figures.

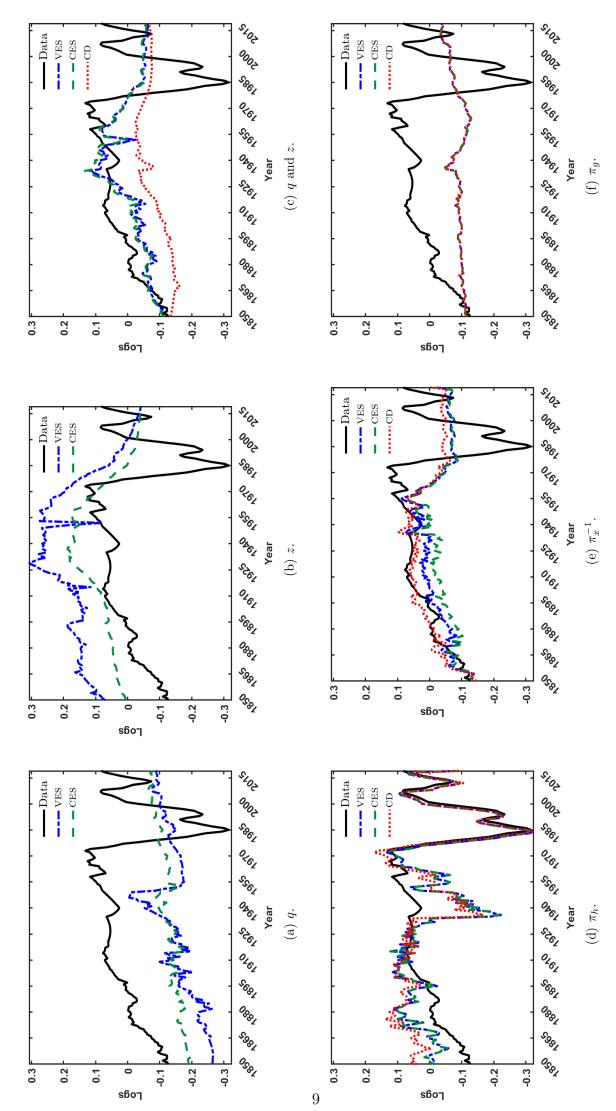
The results of the simulations with VES and CES production functions are very similar and, in our opinion, they do not deserve further comments. The results are more different in the CD case. The reasons of these differences are pointed out by del Río and Lores (2021).

First, the CD case does not allow breaking down TFP) into its two components: the labour-and capital-efficiency wedges. The TFP evolution can hide opposite behaviours of its components, which is especially notorious from the end of the 1970s. Thereafter, the growth rate of detrended TFP decreased markedly and detrended TFP even declined from the end of the 1990s. This TFP growth slowdown hides the fact that the labour efficiency wedge grew at a good pace until the mid-1980s and remained roughly stable, but oscillatory, thereafter, while the capital efficiency wedge began a continued and persistent decline from the mid-1970s. A similar fact is observed from the end of the 19th century to the First World War. However, regardless of the chosen production function, the evolution of detrended TFP contributed to depress output growth since the beginning of the 21st century.

Second, in the CD case, output elasticities for factors are constant and, then, changes in factor shares are reflected in movements of the labour and investment wedges. Along the period considered, Spanish factor shares did not remain constant. Therefore, it might have large differences in the wedges and its contribution if they are computed assuming a CD production function or other production functions, like the VES or the CES. However, even if the differences are notorious, they are not very significant.

We can point out some little differences between the contribution of the wedges in the CD case and the VES and CES cases. In the CD case: (i) the contribution of TFP to the increase in hours worked per capita from the end of the First World War to the Great Depression was lower; (ii) the contribution of TFP to low output growth between the beginning of the last quarter of the 19th century to the First World War was higher; (iii) TFP contributed to depress detrended investment per capita from the end of the 19th to the Great Depression.







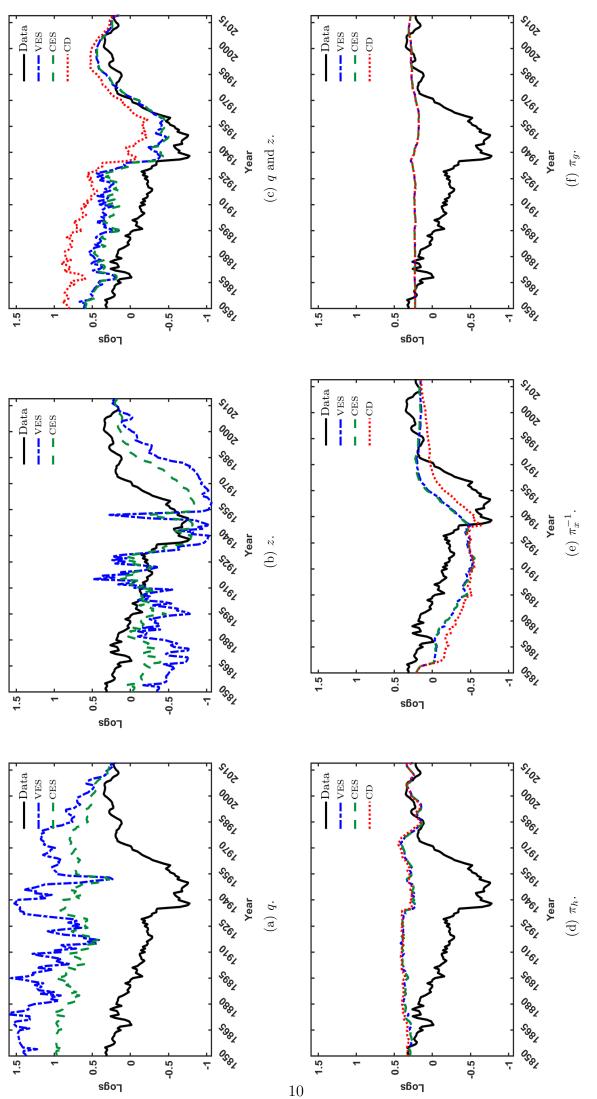
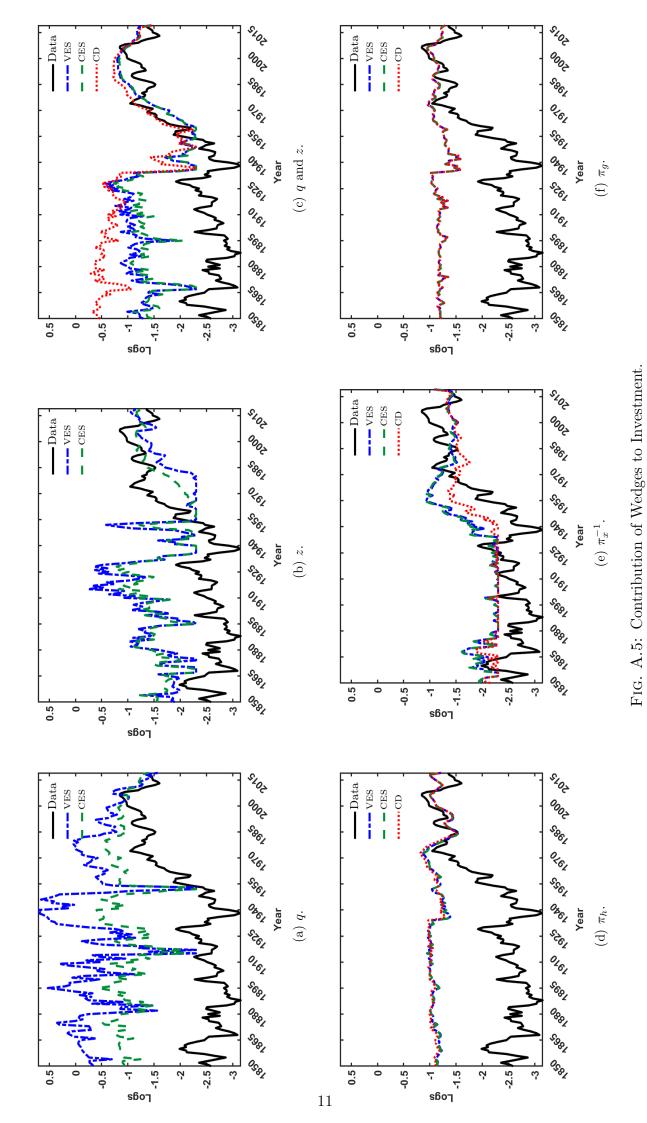


FIG. A.4: Contribution of Wedges to Output (Detrended).



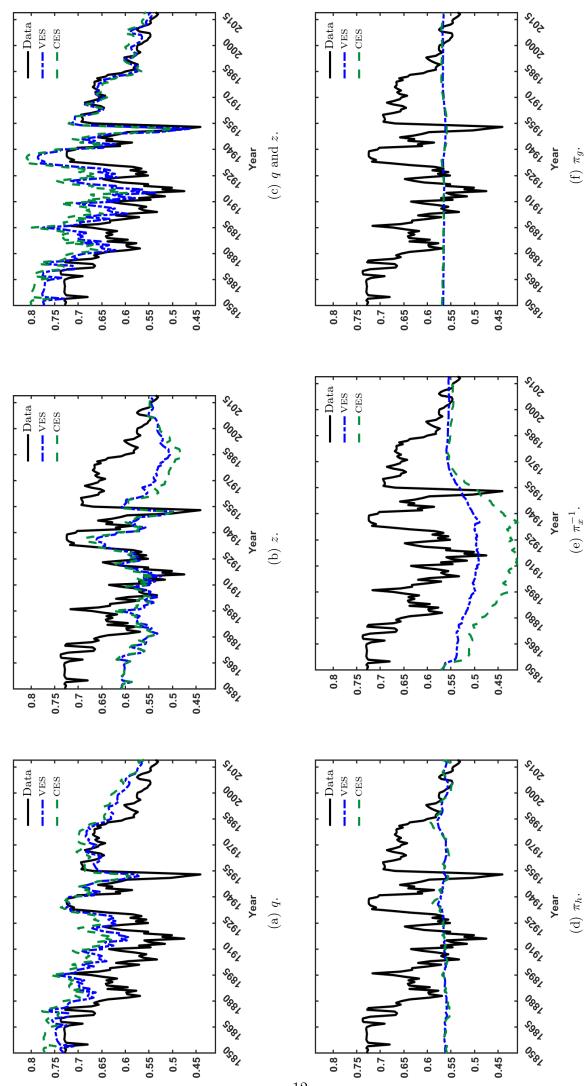


FIG. A.6: Contribution of Wedges to Labour Share.

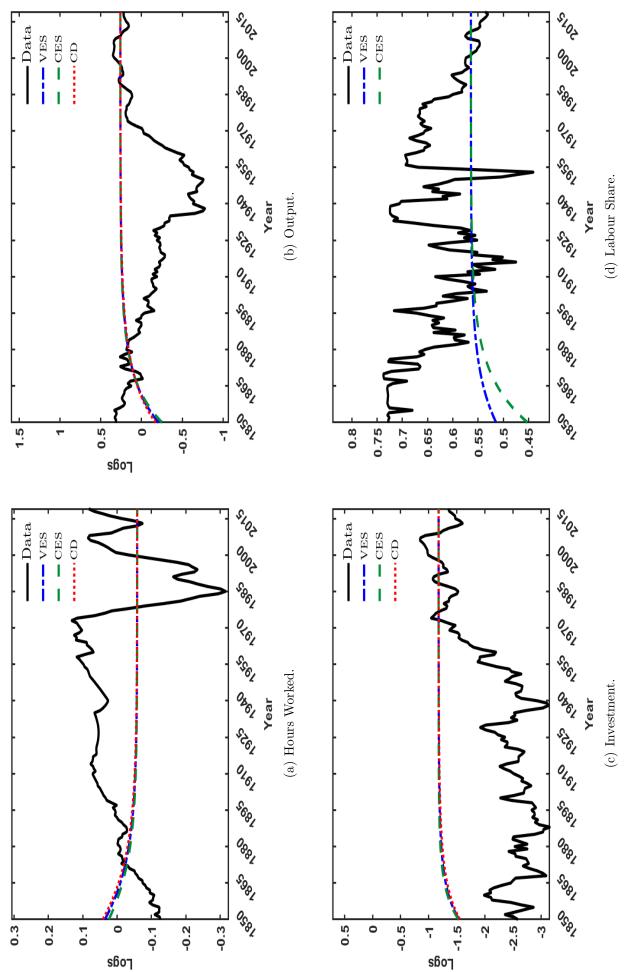
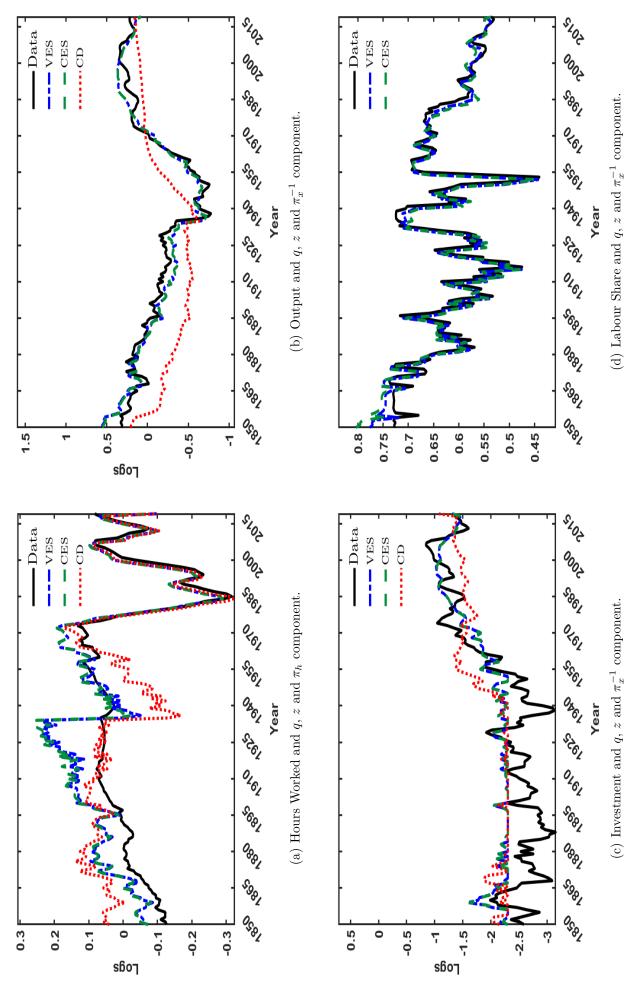
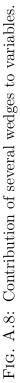


FIG. A.7: Contribution of dynamic transition to variables.





V. KPR UTILITY FUNCTION

We assume a KPR utility function:

$$U_{t} = \sum_{t=0}^{\infty} L_{t} \beta^{t} \left(\log C_{L,t} - \frac{H_{L,t}^{1-\nu}}{1-\nu} \right),$$

where $0 < \beta < 1$ is the discount factor, $-1/\nu$ is the Frisch elasticity of the labour supply and $C_{L,t} = C_t/L_t$ is consumption per capita. This utility function is the limiting case of the one we use as the base case when $\sigma \longrightarrow 1$. Table A.2 summarizes parameters and all BGP values of this case. We do not detrend hours worked per capita because, according to the model, if a KPR utility function is assumed, then hours worked per capita do not display any long-run trend. As can be seen in Fig. A.10 hours worked per capita displayed a decreasing trend from 1850 to mid-1970s.

The results of the simulation with the KPR utility function are displayed in Fig. A.10-A.13.

We only get a significantly different result in our simulations. In particular, the labour wedge almost exclusively drives the evolution of hours worked per capita along the whole period 1850-2019. In particular, it accounts for the continued decline of the hours worked per capita until de mid-1970s. If a BP utility function is assumed, then the contribution of the labour wedge remains roughly stable until the Great Recession, and the investment wedge primarily accounts for the upward deviation of hours worked per capita from its potential evolution. Therefore, we conclude that the labour wedge mostly drives the potential evolution of the hours worked per capita until the 1970s, but not their deviations from its potential path.

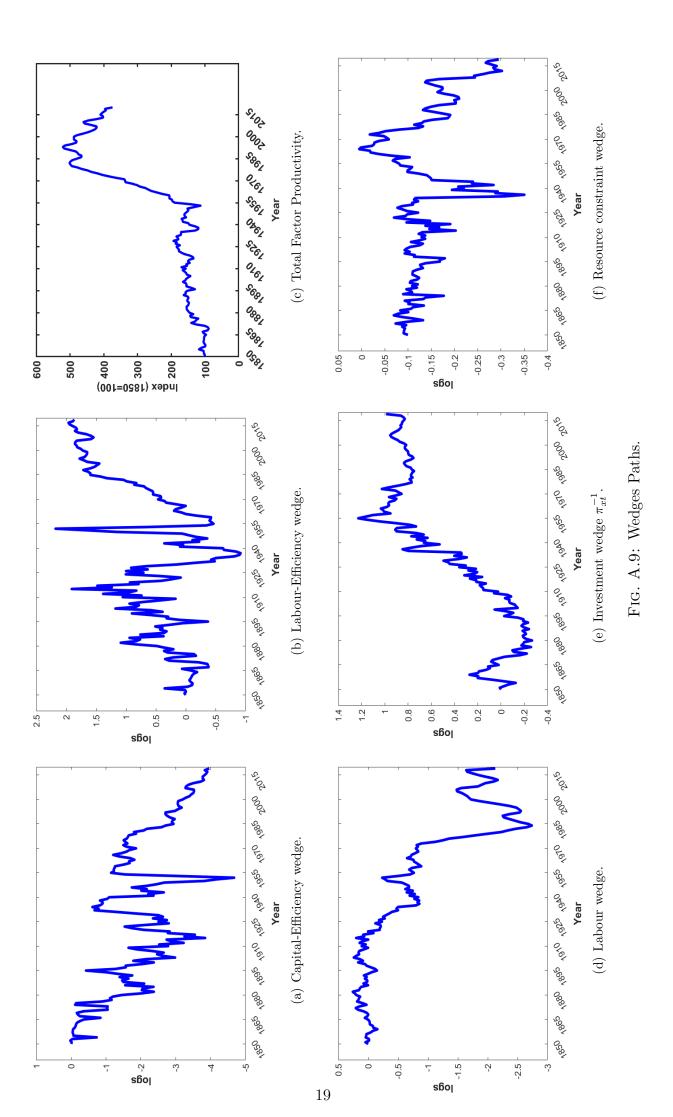
| Parameter | Description | | | |
|---------------------------------|--|---------|--|--|
| γ | Growth Rate of Labour-Aumenting Technical Progress | 0.0172 | | |
| γ_y | Growth Rate of Output per Capita | | | |
| γ_h | Growth Rate of Hours per Capita | | | |
| η | Population Growth Rate | 0.0073 | | |
| δ | Depreciation Rate of Capital | 0.0329 | | |
| λ | Adjustment Cost Paramenter | 4.3448 | | |
| κ | Adjustment Cost Paramenter | 0.0575 | | |
| ho | Production Function Parameter | -0.3735 | | |
| lpha | Production Function Parameter | 0.6225 | | |
| ω | Production Function Parameter | 0.7000 | | |
| ψ | Land Production Function Parameter | -0.6223 | | |
| u | Frisch Elasticity Parameter | | | |
| σ | Intertemporal Elasticity of Substitution Parameter | | | |
| eta | Discount Factor | 0.9781 | | |
| BGP variables | | | | |
| \overline{q} | Capital Efficiency Wedge | 0.2420 | | |
| z | Labour Efficiency Wedge | 1.7762 | | |
| π_h | Labour Wedge | 0.2988 | | |
| π_x | Investment Wedge | 1.4442 | | |
| π_g | Resource Constraint Wedge | | | |
| k/y | Capital-Output Ratio | | | |
| y | Output per Capita | | | |
| h | Hours Worked per Capita | | | |
| x/y | Investment Rate | 0.2379 | | |
| c/y Consumption to Output Ratio | | 0.5943 | | |
| ε | Capital-Output Elasticity | | | |
| <i>i</i> | Interest Rate | 0.0400 | | |

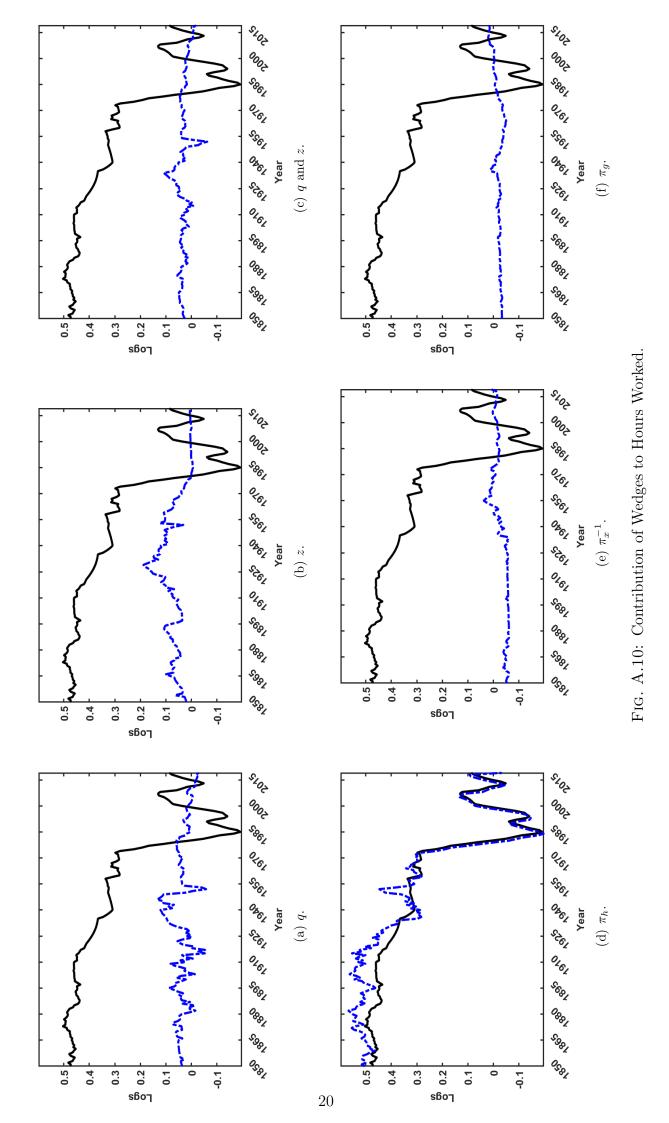
| TABLE A | .3 |
|---------------------------|---------------|
| $\hat{\phi}$ -STATISTICS. | $\sigma = 1.$ |

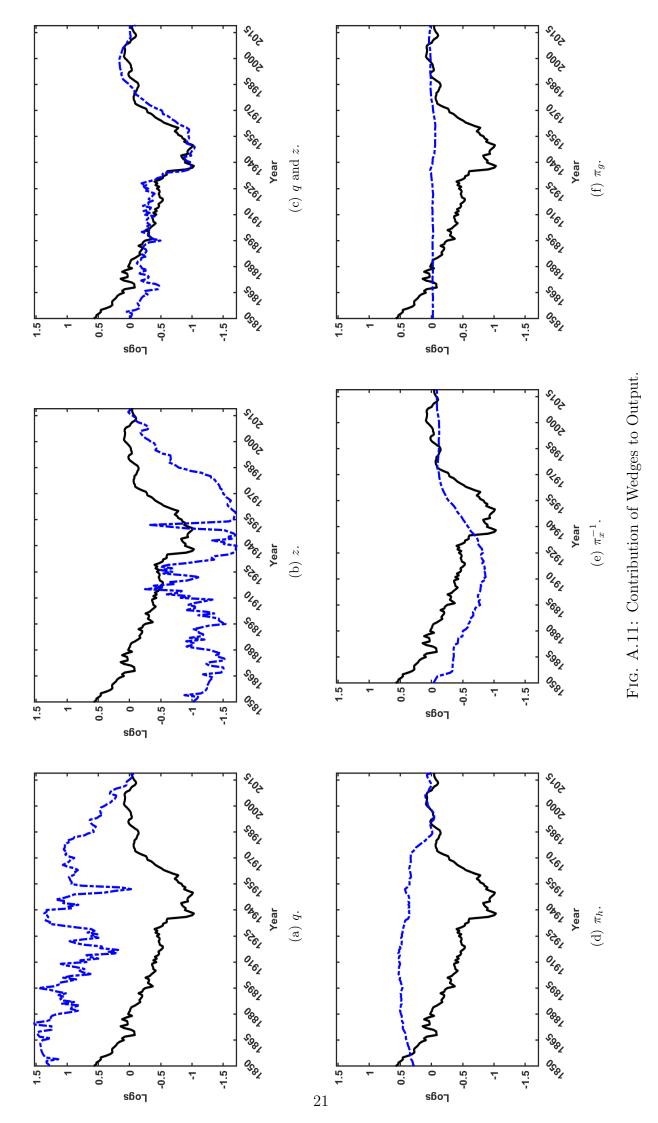
| Variable | $\hat{\phi}_q^{\mathbf{y}}$ | $\hat{\phi}_z^{\mathbf{y}}$ | $\hat{\phi}_{\pi_h}^{\mathbf{y}}$ | $\hat{\phi}_{\pi_x}^{\mathbf{y}}$ | $\hat{\phi}_{\pi_g}^{\mathbf{y}}$ |
|-------------|-----------------------------|-----------------------------|-----------------------------------|-----------------------------------|-----------------------------------|
| | ψq | 1 | $\frac{\varphi_{\pi_h}}{850-201}$ | | $\varphi \pi_g$ |
| ĥ | 0.022 | 0.027 | 0.920 | 0.015 | 0.016 |
| \hat{y} | 0.022 | 0.021 0.071 | 0.320 0.117 | 0.421 | 0.348 |
| \hat{x} | 0.036 | 0.249 | 0.062 | 0.421 0.537 | 0.116 |
| \hat{s}_h | 0.517 | 0.243 0.163 | 0.118 | 0.078 | $0.110 \\ 0.124$ |
| <u> </u> | 0.011 | | 850-187 | | 0.121 |
| \hat{h} | 0.008 | 0.008 | 0.974 | 0.005 | 0.005 |
| \hat{y} | 0.027 | 0.000 0.017 | 0.431 | 0.136 | 0.389 |
| \hat{x} | 0.021 | 0.481 | 0.431 0.031 | 0.412 | 0.059 |
| \hat{s}_h | 0.880 | 0.049 | 0.023 | 0.412 0.021 | 0.003 0.027 |
| <u> </u> | 0.000 | | 875-189 | | 0.021 |
| \hat{h} | 0.019 | 0.022 | $\frac{0.932}{0.932}$ | 0.013 | 0.014 |
| \hat{y} | 0.010 | 0.011 | 0.002 0.043 | 0.010 0.054 | 0.883 |
| \hat{x} | 0.028 | 0.196 | 0.040 | 0.666 | 0.070 |
| \hat{s}_h | 0.415 | 0.178 | $0.010 \\ 0.157$ | 0.000 0.078 | 0.070 0.172 |
| <u> </u> | 0.110 | | 895-19 | | 0.112 |
| ĥ | 0.026 | 0.028 | 0.909 | 0.017 | 0.020 |
| \hat{y} | 0.036 | 0.088 | 0.078 | 0.309 | 0.489 |
| \hat{x} | 0.019 | 0.169 | 0.027 | 0.305 0.735 | 0.050 |
| \hat{s}_h | 0.170 | 0.239 | 0.253 | 0.080 | 0.259 |
| <u> </u> | 0.110 | | 914-192 | | 0.200 |
| ĥ | 0.024 | 0.048 | 0.888 | 0.018 | 0.021 |
| \hat{y} | 0.035 | 0.410 | 0.043 | 0.322 | 0.021 0.190 |
| \hat{x} | 0.019 | 0.031 | 0.016 | 0.903 | 0.031 |
| \hat{s}_h | 0.096 | 0.333 | 0.253 | 0.072 | 0.247 |
| <u> </u> | 0.000 | | 929-194 | | 0.211 |
| \hat{h} | 0.020 | 0.031 | 0.929 | 0.009 | 0.011 |
| \hat{y} | 0.015 | 0.110 | 0.044 | 0.731 | 0.100 |
| \hat{x} | 0.024 | 0.453 | 0.065 | 0.350 | 0.108 |
| \hat{s}_h | 0.691 | 0.190 | 0.050 | 0.023 | 0.046 |
| -11 | | | 940-195 | | |
| ĥ | 0.028 | 0.041 | 0.895 | 0.019 | 0.017 |
| \hat{y} | 0.039 | 0.239 | 0.074 | 0.481 | 0.167 |
| \hat{x} | 0.039 | 0.488 | 0.079 | 0.231 | 0.163 |
| \hat{s}_h | 0.369 | 0.238 | 0.157 | 0.091 | 0.145 |
| | | 19 | 959-197 | | |
| ĥ | 0.010 | 0.012 | 0.965 | 0.008 | 0.006 |
| \hat{y} | 0.030 | 0.043 | 0.092 | 0.524 | 0.311 |
| \hat{x} | 0.048 | 0.158 | 0.130 | 0.370 | 0.294 |
| \hat{s}_h | 0.896 | 0.023 | 0.030 | 0.022 | 0.028 |
| | | 19 | 974-200 |)7 | |
| ĥ | 0.067 | 0.071 | 0.732 | 0.070 | 0.060 |
| \hat{y} | 0.007 | 0.006 | 0.163 | 0.271 | 0.553 |
| \hat{x} | 0.016 | 0.022 | 0.518 | 0.133 | 0.311 |
| \hat{s}_h | 0.407 | 0.064 | 0.227 | 0.130 | 0.171 |
| | 2007-2019 | | | | |
| ĥ | 0.159 | 0.172 | 0.321 | 0.155 | 0.193 |
| \hat{y} | 0.073 | 0.061 | 0.253 | 0.358 | 0.255 |
| \hat{x} | | 0.070 | 0.126 | 0.283 | 0.203 |
| | 0.318 | 0.070 | 0.120 | 0.200 | 0.203 |

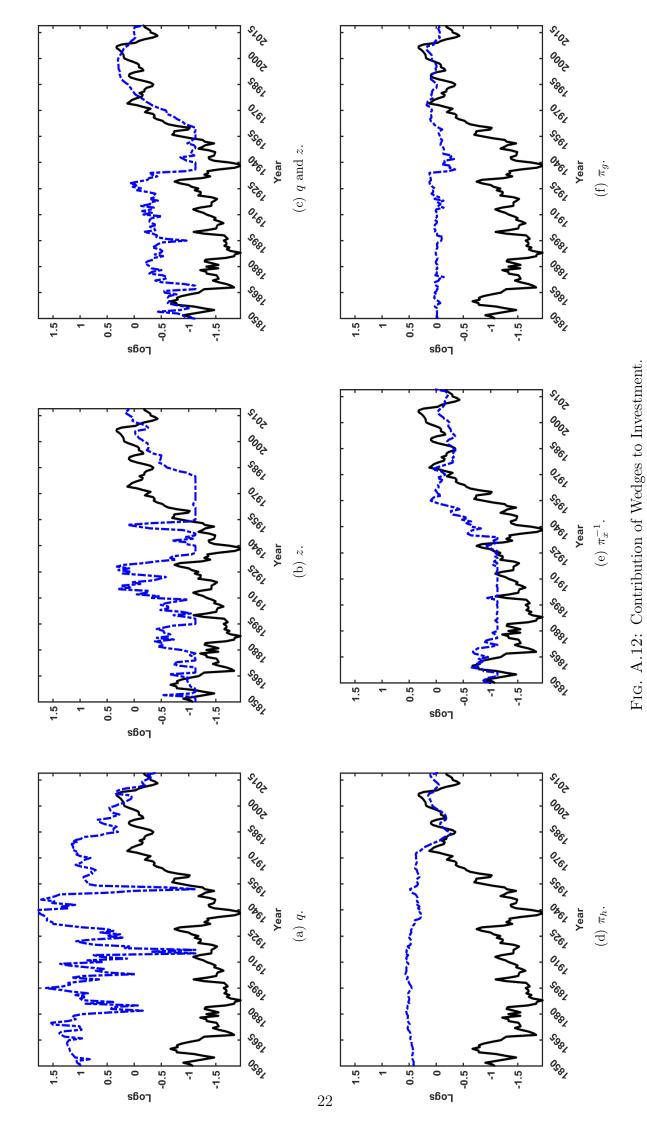
TABLE A.4 Correlation Coeficients between Non-Transitional Component and Normalized Wedge-Alone Component of Variables (logs). $\sigma = 1$.

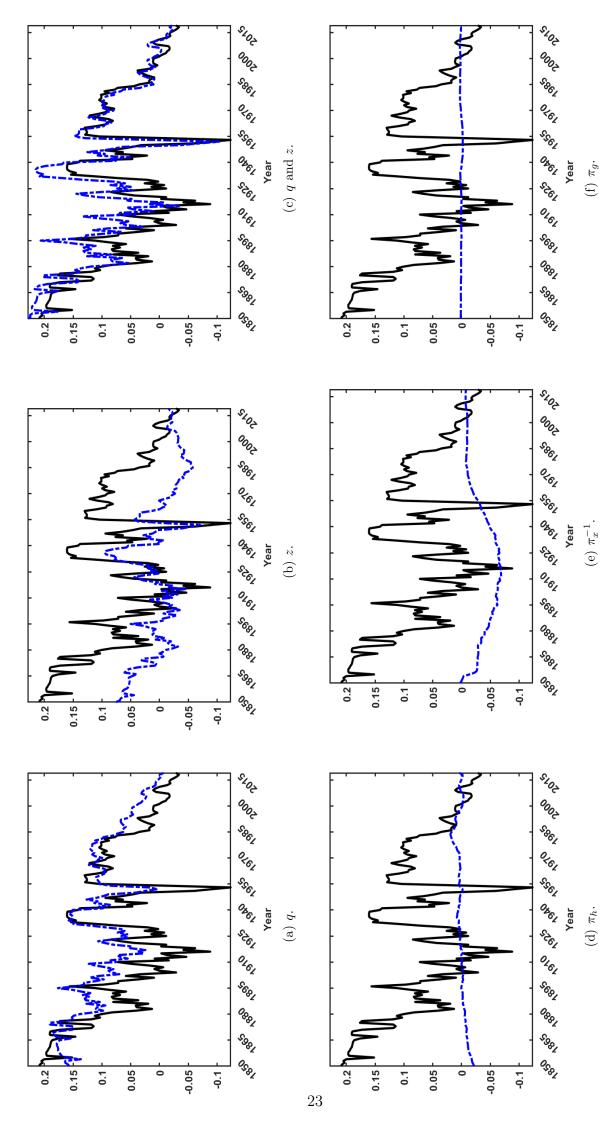
| Variable | $r\left(\hat{\mathbf{y}}, \hat{\mathbf{y}}_q\right)$ | $r\left(\hat{\mathbf{y}}, \hat{\mathbf{y}}_z\right)$ | $r\left(\hat{\mathbf{y}}, \hat{\mathbf{y}}_{\pi_h}\right)$ | $r\left(\hat{\mathbf{y}},\hat{\mathbf{y}}_{\pi_x} ight)$ | $r\left(\hat{\mathbf{y}}, \hat{\mathbf{y}}_{\pi_g}\right)$ | |
|---------------------|--|--|--|--|--|--|
| | 1850-2019 | | | | | |
| ĥ | 0.290 | 0.612 | 0.987 | -0.593 | -0.627 | |
| \hat{y} | 0.083 | 0.344 | -0.330 | 0.423 | 0.455 | |
| \hat{x} | -0.352 | 0.220 | -0.769 | 0.763 | 0.535 | |
| \hat{s}_h | 0.874 | 0.657 | -0.378 | 0.118 | -0.008 | |
| | | | 1850-187 | | | |
| ĥ | 0.206 | 0.258 | 0.666 | -0.254 | 0.520 | |
| \hat{y} | -0.469 | 0.867 | -0.913 | 0.841 | -0.433 | |
| \hat{x} | -0.222 | -0.053 | -0.685 | 0.177 | 0.170 | |
| \hat{s}_h | 0.026 | 0.678 | -0.567 | 0.500 | 0.643 | |
| | | | 1875-189 | 5 | | |
| ĥ | -0.032 | -0.819 | 0.756 | -0.120 | -0.666 | |
| \hat{y} | 0.063 | -0.231 | -0.060 | 0.900 | -0.865 | |
| \hat{x} | -0.060 | -0.401 | 0.574 | 0.111 | 0.137 | |
| \hat{s}_h | 0.784 | 0.324 | -0.402 | 0.507 | -0.203 | |
| | | | 1895-191 | | | |
| ĥ | -0.248 | -0.155 | 0.606 | -0.010 | -0.476 | |
| \hat{y} | 0.728 | -0.774 | -0.524 | 0.676 | -0.635 | |
| $\hat{y} \ \hat{x}$ | -0.418 | 0.610 | 0.416 | 0.134 | 0.273 | |
| \hat{s}_h | 0.729 | 0.417 | 0.233 | 0.484 | -0.597 | |
| | | | 1914-192 | | | |
| \hat{h} | -0.408 | -0.917 | 0.865 | -0.465 | 0.614 | |
| \hat{y} | 0.274 | 0.010 | -0.558 | 0.379 | 0.648 | |
| \hat{x} | 0.418 | 0.248 | -0.662 | 0.651 | 0.721 | |
| \hat{s}_h | 0.634 | 0.532 | 0.463 | 0.666 | 0.350 | |
| | | | 1929-194 | 0 | | |
| ĥ | -0.779 | 0.703 | 0.805 | -0.559 | -0.429 | |
| \hat{y} | -0.677 | 0.862 | 0.919 | -0.924 | 0.060 | |
| $\hat{y} \ \hat{x}$ | -0.892 | 0.917 | 0.857 | -0.587 | 0.690 | |
| \hat{s}_h | 0.958 | 0.776 | 0.779 | 0.621 | -0.416 | |
| | | | 1940-195 | 59 | | |
| \hat{h} | -0.483 | -0.142 | 0.348 | 0.690 | -0.841 | |
| \hat{y} | 0.211 | -0.342 | -0.544 | 0.514 | -0.072 | |
| \hat{x} | -0.195 | -0.270 | -0.014 | 0.871 | 0.567 | |
| \hat{s}_h | 0.842 | 0.705 | 0.690 | -0.002 | 0.504 | |
| | 1959-1974 | | | | | |
| \hat{h} | 0.334 | 0.359 | 0.280 | -0.020 | -0.184 | |
| \hat{y} | 0.554 | 0.938 | -0.436 | 0.990 | 0.951 | |
| \hat{x} | 0.561 | -0.126 | 0.310 | -0.163 | 0.921 | |
| \hat{s}_h | 0.122 | 0.673 | 0.339 | -0.557 | -0.484 | |
| | 1974-2007 | | | | | |
| \hat{h} | 0.373 | 0.842 | 0.958 | 0.660 | -0.506 | |
| \hat{y} | -0.728 | 0.769 | -0.075 | 0.025 | 0.533 | |
| \hat{x} | -0.338 | 0.500 | 0.733 | 0.806 | 0.780 | |
| \hat{s}_h | 0.974 | -0.107 | 0.673 | 0.810 | 0.312 | |
| | | | 2007-201 | .9 | | |
| ĥ | 0.497 | -0.480 | 0.598 | 0.636 | -0.773 | |
| \hat{y} | 0.753 | -0.723 | 0.613 | -0.332 | 0.211 | |
| \hat{x} | 0.807 | -0.792 | 0.470 | 0.608 | 0.852 | |
| \hat{s}_h | 0.803 | 0.447 | 0.400 | -0.245 | 0.836 | |

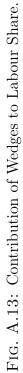


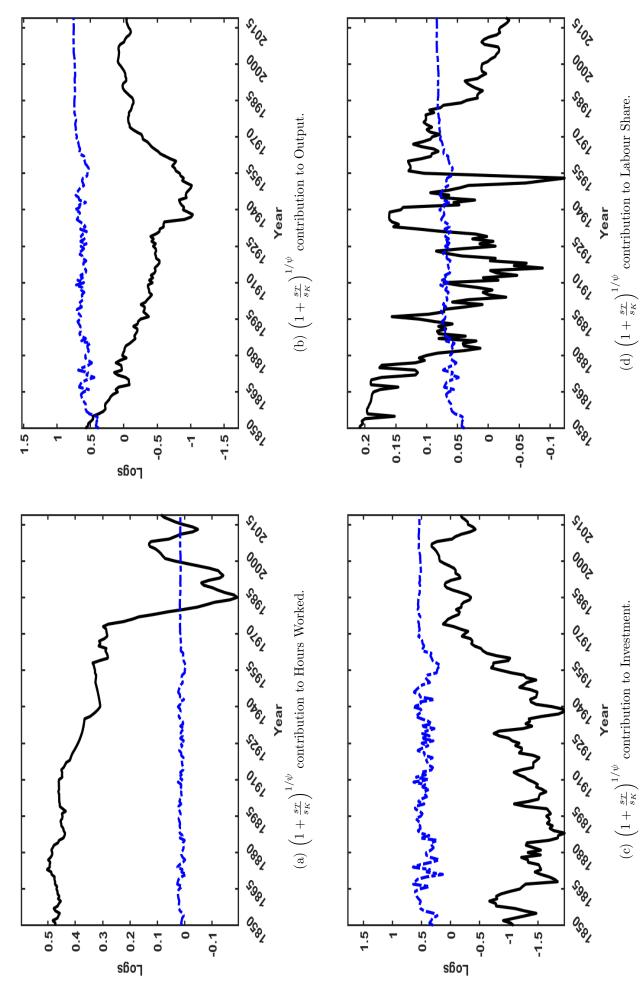


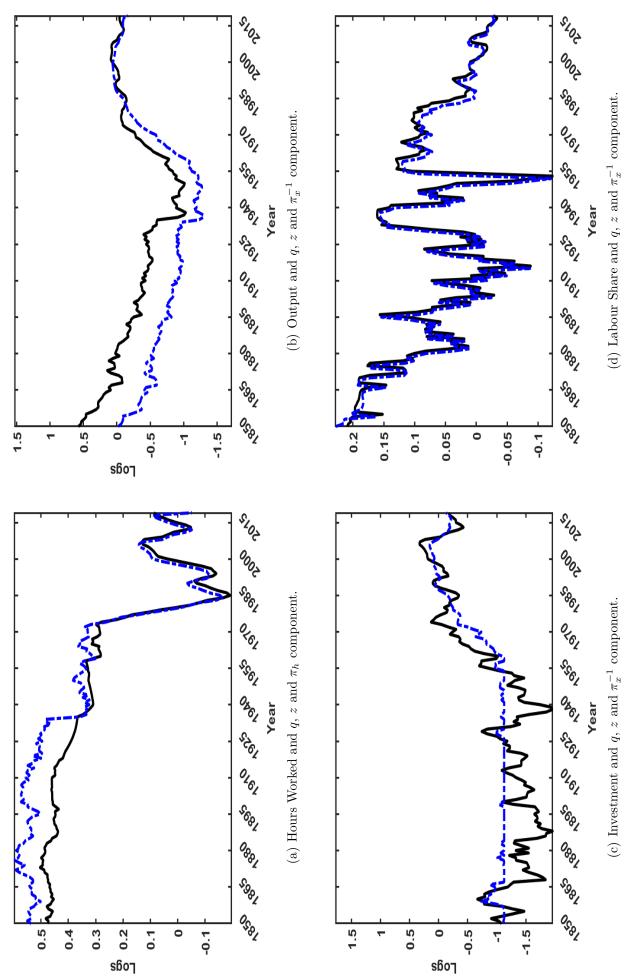


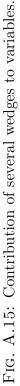












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