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# International mixed triopoly, privatization and subsidization

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#### Abstract

This paper examines privatization in an international mixed triopoly model with a state-owned firm, a domestic firm and a foreign private firm to reassess the welfare effect of production subsidies. The main result of the paper is that if optimal domestic subsidies are used before and after privatization, then privatization improves domestic social welfare. The paper finds that this result is quite different from that of the existing domestic mixed oligopoly model.

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# 1. Introduction

The seminal work by White (1996) investigates how production subsidies influence the privatization decision in a quantity-setting mixed oligopoly market and presents the following three main results. First, if production subsidies are utilized before and after privatization, privatization does not change economic welfare. Second, if production subsidies are utilized only before privatization, there is a reduction in social welfare. Third, the production subsidy contributes to overall efficiency in a mixed oligopoly market because of cost distribution effects. Poyago-Theotoky (2001) and Myles (2002) demonstrate that the optimal production subsidy is identical irrespective of whether (i) a public firm moves simultaneously with n private firms, (ii) it acts as a Stackelberg leader, or (iii) all firms behave as profit-maximizers. These studies do not consider foreign firms.

In general, many firms compete not only with domestic firms but also with foreign firms. Therefore, by building upon White (1996), we consider four games: the first two games are international mixed triopoly in which a state-owned public firm, a domestic private firm and a foreign private firm compete with each other, and the second two games are international private triopoly in which the state-owned public firm is privatized. We examine the effects of domestic production subsidies in an international mixed triopoly model with a foreign private firm regarding privatization.

The remainder of this paper is organized as follows. In Section 2, we describe the basic setting considered in this study. Section 3 solves and compares the four games. Finally, Section 4 concludes the paper.

## 2. Basic setting

There are three firms: a domestic private firm (firm D), a foreign private firm (firm F) and a state-owned public firm (firm S). The firms produce perfectly substitutable goods. Throughout this paper, subscripts D, F and S represent firm D, firm F and firm S, respectively. We do not consider the possibility of entry or exit. The basic setting is taken from White (1996). The market demand function is given by P = a - Q, where Q is total output:  $Q = q_D + q_F + q_S$ . The firms have identical technologies represented by a quadratic cost function  $C(q_i) = F + \frac{1}{2}kq_i^2$ , i = D, F, S. We assume that F = 0 and k = 1 with no loss of generality.

Firm D's profit is given by

$$\pi_{\rm D} = q_{\rm D} \left( a - Q \right) - \frac{1}{2} q_{\rm D}^2 + s q_{\rm D} \,, \tag{1}$$

where s is the subsidy for each unit of output. Firm D aims to maximize (1).

Firm F's profit is given by

$$\pi_{\rm F} = q_{\rm F} \left( a - Q \right) - \frac{1}{2} q_{\rm F}^2. \tag{2}$$

Firm F aims to maximize (2).

Firm S's profit is given by

$$\pi_{\rm s} = q_{\rm s} \left( a - Q \right) - \frac{1}{2} q_{\rm s}^2 + s q_{\rm s} \,, \tag{3}$$

and domestic social welfare is given by

$$W = CS + \pi_{\rm S} + \pi_{\rm D} - s(q_{\rm S} + q_{\rm D}), \qquad (4)$$

where  $CS = \frac{1}{2}Q^2$  denotes domestic consumer surplus. Domestic social welfare is the sum of domestic consumer surplus and two domestic firms' profits.

# **3. Results**

In this section, we consider the following four regimes: unsubsidized international mixed triopoly, subsidized international mixed triopoly, unsubsidized international private triopoly, and subsidized international private triopoly.

#### 3.1. Unsubsidized international mixed triopoly

We present the Cournot equilibrium values of outputs, profits, consumer surplus and welfare when there is no subsidy (s = 0):

$$q_{\rm D}^{\rm M}(0) = q_{\rm F}^{\rm M}(0) = \frac{1}{6}a, \quad q_{\rm S}^{\rm M}(0) = \frac{1}{3}a,$$
 (5)

$$Q^{\rm M}(0) = \frac{2}{3}a\,,\tag{6}$$

$$\pi_{\rm D}^{\rm M}(0) = \pi_{\rm F}^{\rm M}(0) = \frac{1}{24}a^2, \quad \pi_{\rm S}^{\rm M}(0) = \frac{1}{18}a^2,$$
(7)

$$CS^{\rm M}(0) = \frac{2}{9}a^2, \qquad (8)$$

$$W^{\rm M}(0) = \frac{23}{72}a^2.$$
<sup>(9)</sup>

Notice that  $q_{\rm S}^{\rm M}(0)$  is higher than  $q_{\rm D}^{\rm M}(0)$  and  $q_{\rm F}^{\rm M}(0)$ . Also notice that firm S makes a strictly positive profit.

#### 3.2. Subsidized international mixed triopoly

We consider the international mixed triopoly game when the government considers setting a production subsidy. The game has two stages. In the first stage, the government sets the production subsidy to maximize domestic social welfare. In the second stage, firms simultaneously and independently choose their output levels. We solve for the subgame perfect equilibrium through backward induction. Starting from the second stage, we obtain the second-stage Cournot equilibrium outputs in terms of the subsidy *s*:

$$q_{\rm D}^{\rm M}(s) = \frac{a+3s}{7}, \quad q_{\rm F}^{\rm M}(s) = \frac{2a-s}{14}, \quad q_{\rm S}^{\rm M}(s) = \frac{6a-3s}{14},$$
 (10)

We consider the first stage of the game. In the first stage, taking into account how firms will react to the subsidy, the government sets the subsidy to maximize (4). We obtain the welfare-maximizing subsidy as follows:

$$s^{M^*} = \frac{22}{53}a . (11)$$

Notice that  $s^{M^*}$  is strictly positive. Therefore, we obtain the following subgame perfect equilibrium values:

$$q_{\rm D}^{\rm M}(s^{\rm M^*}) = \frac{17}{53}a, \quad q_{\rm F}^{\rm M}(s^{\rm M^*}) = \frac{6}{53}a, \quad q_{\rm S}^{\rm M}(s^{\rm M^*}) = \frac{18}{53}a, \quad (12)$$

$$Q^{\rm M}(s^{\rm M^*}) = \frac{41}{53}a\,,\tag{13}$$

$$\pi_{\rm D}^{\rm M}(s^{\rm M^*}) = \frac{867}{5618}a^2, \quad \pi_{\rm F}^{\rm M}(s^{\rm M^*}) = \frac{54}{2809}a^2, \quad \pi_{\rm S}^{\rm M}(s^{\rm M^*}) = \frac{450}{2809}a^2, \tag{14}$$

$$CS^{\rm M}(s^{\rm M^*}) = \frac{1681}{5618}a^2,\tag{15}$$

$$W^{\rm M}(s^{\rm M^*}) = \frac{954}{2809}a^2.$$
(16)

Note that  $W^{M}(s^{M^*})$  is higher than  $W^{M}(0)$ . We state the following proposition.

**Proposition 1:** If optimal domestic subsidies are used in an international mixed triopoly, then all firms have different equilibrium outputs,  $q_{\rm D}^{\rm M}(s^{\rm M^*}) - q_{\rm D}^{\rm M}(0) > 0$ ,  $q_{\rm F}^{\rm M}(s^{\rm M^*}) - q_{\rm F}^{\rm M}(0) < 0$ , and  $q_{\rm S}^{\rm M}(s^{\rm M^*}) - q_{\rm S}^{\rm M}(0) > 0$ .

Notice that this result differs from Proposition 1 in White (1996).

#### 3.3. Unsubsidized international private triopoly

In the subsection, we solve for the Cournot equilibrium outcomes of the unsubsidized private triopoly game. All triopolists maximize their own profits. We obtain the following values:

$$q_{\rm D}^{\rm P}(0) = q_{\rm F}^{\rm P}(0) = q_{\rm S}^{\rm P}(0) = \frac{1}{5}a, \qquad (17)$$

$$Q^{\rm P}(0) = \frac{3}{5}a\,,\tag{18}$$

$$\pi_{\rm D}^{\rm P}(0) = \pi_{\rm F}^{\rm P}(0) = \pi_{\rm S}^{\rm P}(0) = \frac{3}{50}a^2, \qquad (19)$$

$$CS^{P}(0) = \frac{9}{50}a^{2}, \qquad (20)$$

$$W^{\rm P}(0) = \frac{3}{10}a^2.$$
(21)

We compare the subsidized international mixed triopoly outcomes with those of the unsubsidized international private triopoly.

**Proposition 2:** If optimal domestic subsidies are used before but not after privatization of firm S in an international mixed triopoly, then  $W^{P}(0) - W^{M}(s^{M^*}) < 0$ .

Note that the use of optimally chosen subsidies makes the international mixed triopoly game unambiguously better than the unsubsidized international private triopoly.

#### 3.4. Subsidized international private triopoly

In this subsection, we follow the methodology of Subsection 3.2. Starting from the second stage, we obtain the second-stage Cournot equilibrium outputs in terms of the subsidy s:

$$q_{\rm D}^{\rm P}(s) = q_{\rm S}^{\rm P}(s) = \frac{2a+3s}{10}, \quad q_{\rm F}^{\rm P}(s) = \frac{a-s}{5},$$
 (22)

We consider the first stage of the game. In the first stage, taking into account how firms will react to the subsidy, the government sets the subsidy to maximize (4). We obtain the welfare-maximizing subsidy as follows:

$$s^{P^*} = \frac{2}{5}a \,. \tag{23}$$

Note that  $s^{M^*}$  is strictly positive. We now obtain the following subgame perfect

equilibrium values:

$$q_{\rm D}^{\rm P}(s^{\rm P^*}) = q_{\rm S}^{\rm P}(s^{\rm P^*}) = \frac{8}{25}a, \quad q_{\rm F}^{\rm P}(s^{\rm P^*}) = \frac{3}{25}a,$$
 (24)

$$Q^{\rm P}(s^{\rm P*}) = \frac{19}{25}a\,,\tag{25}$$

$$\pi_{\rm D}^{\rm P}(s^{\rm P*}) = \pi_{\rm S}^{\rm P}(s^{\rm P*}) = \frac{96}{625}a^2, \quad \pi_{\rm F}^{\rm P}(s^{\rm P*}) = \frac{27}{1250}a^2, \tag{26}$$

$$CS^{P}(s^{P^*}) = \frac{361}{1250}a^2, \qquad (27)$$

$$W^{\rm P}(s^{\rm P*}) = \frac{17}{50}a^2.$$
<sup>(28)</sup>

By comparing (17) - (21) with (24) - (28), the effects of the subsidy on the private triopoly results are stated by the following proposition.

**Proposition 3:** If optimal domestic subsidies ate used in an international private triopoly, then  $Q^{P}(s^{P^*}) - Q^{P}(0) > 0$  and  $W^{P}(s^{P^*}) - W^{P}(0) > 0$ 

We find that this proposition is essentially the same as Proposition 2 in White (1996).

Finally, we compare the two subsidized games, which will indicate the welfare effects of privatization when subsidies are used before and after the action.

**Proposition 4:** In the two subsidized games,  $s^{M^*} - s^{P^*} > 0$ ,  $Q^M(s^{M^*}) - Q^P(s^{P^*}) > 0$ ,  $CS^M(s^{M^*}) - CS^P(s^{P^*}) > 0$ , and  $W^M(s^{M^*}) - W^P(s^{P^*}) < 0$ .

Note that privatization improves domestic social welfare. Therefore, we find that this result is quite different from Proposition 4 in White (1996), which demonstrates that the optimal subsidy, output and welfare are unchanged.

# 4. Conclusion

We have examined the effects of domestic production subsidies in an international mixed triopoly model with a state-owned firm, a domestic private firm and a foreign private firm. We have shown that if optimal domestic subsidies are used before and after privatization, then privatization increases domestic social welfare. We have found that our result is quite different from that of the existing domestic mixed oligopoly model.

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