Demand theory for poverty and affluence

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DEMAND THEORY FOR POVERTY AND AFFLUENCE

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TWO-PAGE SUMMARY

PROPOSITION 1. THE SHAPE OF THE UTILITY FUNCTION.

An individual's experience of consumption, Xi, of a commodity i, (good, service or event), Xi ≥ 0, for i = 1, 2, …, m, can be represented by a continuous, smooth, single-valued, utility function, Ui = U(Xi), $0 \leq Ui \leq 1$, shaped like a leaning 'S' curve, bounded below and above, but marginal utility, Ui', is always less than infinity.

U_i"	> 0	> 0	= 0	< 0	< 0	< 0
U_i'	= 0	> 0	> 0	> 0	= 0	< 0
U_i	= 0	> 0	> 0	> 0	= 1	$1 > U_i > 0$
X_i	$X_i = 0$	$0 < X_i < \mu_i$	$X_i = \mu_i$	$\mu_i < X_i < sat_i$	$X_i = sat_i$	$X_i >$ finite sat_i
Individual experience:	Minimum:	deprived	Point of inflection: subsistence	sufficiency	Maximum: satiated	surfeit

A point of inflection occurs at Xi = μi, (where μi is a subsistence parameter comparable to the survival parameters in some econometric demand models). Utility reaches a maximum of 1 at Xi = sati, and the consumer is satiated in Xi. If sati < ∞, as Xi increases, utility decreases and is diminishing.

This theory is based on bounded cardinal utility. The steepness of the slope around the point of inflection, represented by σ_1, represents intensity-of-need for the ith commodity. The smaller is σ_1, the more intense is the need.

PROPOSITION 2. THE SEPARABILITY OF COMMODITIES.

The utilities of a group of commodities that satisfy the same need are multiplicatively related (with or without dependence), and the utilities of groups of satisfiers, each group satisfying a different need, are additively related. It is assumed that there is a finite number of fundamental human needs, which are universal and a-historic.
Needs are satisfied by an infinite diversity of culturally-determined satisfiers.

The following may be noted from the **INDIFFERENCE CURVE MAPS** created from an additive utility function, (see figures 2 and 5):

* The subsistence parameters, **X1 = μ1 and X2 = μ2, form borders** on the left and lower part of the indifference map, representing deprivations in X1 and X2.

* **A straight-line indifference curve, AB**, has slope $-\sigma_2/\sigma_1$, and separates convex-to-the-origin indifference curves from the concave-to-the-origin ones in the non-solution 'dysfunctional poverty' space, OAB. AB could be regarded as an absolute poverty line, APL. Convex and concave curves together lead to discontinuities.

* In the convex-to-the-origin part of the indifference curve map, each commodity can provide **ultra-superior, superior-normal, inferior-normal and inferior Giffen** experiences. These are marked for X1 on figures 2 and 5. The experience of a good as 'inferior' can be regarded as 'functional poverty'.

For the **LEISURE-CONSUMPTION** choice in figure 5, (with consumption, X2 ≥ 0, on the vertical axis), leisure, (0 ≤ X1 ≤ 168 hours pw, on the horizontal axis), is experienced as both inferior-normal and inferior Giffen when X1 ≥ μ1 for X2 ≥ μ2, (ie. the consumer is not deprived of X1, but is deprived of X2).

* In figure 5, the point (μ1, μ2) is labelled E, and EF is the locus of points on the indifference curves where dX2/dX1 = $-\sigma_2/\sigma_1$. The convex-to-the-origin indifference curves part of the map can now be divided into **4 areas, labelled as L, M, N and R**.
Let us assume that the utility function is maximised subject to a linear budget.

The **LINEAR BUDGET** is expressed by X2 = (Z1 – X1).p1/p2 + Z2, where Z1 is an endowment of time in a given period, (eg. 168 hours pw), priced at p1, and Z2 is an endowment of material goods, priced at p2. (Z1 – X1) measures hours of paid work; (Z1 – X1).p1/p2 is real earnings; Z2.p2 is unearned income.

Backward-bending LABOUR SUPPLY (Ls) CURVES are derived, see Figure 6; with real wage rate, p1/p2 on the vertical axis; parameter σ_2/σ_1 can be interpreted as a 'natural wage-rate'; and labour hours, (Z1 – X1), with parameter (Z1 – μ1), on the horizontal axis.

* **Note**: The **four areas, L, M, N and R** from the indifference map can be identified on the Ls curves diagram. Area R leads to elastic *backward-sloping curves* for high-wages, deprived of leisure; Areas N and M yield inelastic, high- and low-waged Ls curves respectively, and neither is deprived of either leisure or consumption; Area L leads to elastic, low-waged, Ls curves, deprived of income but not of leisure.

* **An envelope curve** bounds the lower limit of the Ls curves, associated with the change from inferior to superior characteristics, representing X2 = μ2, for X1 > μ1.

* **The intercepts on the p1/p2 axis represent the 'reservation wage'**, the consumer's minimum acceptable wage-rate. It can be shown that the reservation wage is a U-shaped function of Z2, being highest when Z2 = 0, reaching a minimum when Z2 = μ2, and increasing again for Z2 > μ2.

* **POLICY IMPLICATIONS**. A low endowment of material goods**,** such that 0 < Z2 < A, (via a state benefit, for instance), leads to a polarised outcome in terms of consumption. Faced with a high wage, (p1/p2 > σ_2/σ_1), the consumer's solution could be in area R of the indifference map, deprived of leisure. Faced with a low wage, the consumer remains unemployed, and deprived of consumption, as a non-tangential corner solution. An individual 'chooses' his/her cheaper deprivation. This may explain how governments can spend a lot of money, while still keeping people well below subsistence.

* For A < Z2 < μ2, the low-waged individual, working part-time and responding to a change in wage rates, would still be deprived of consumption. A National Minimum Wage (NMW) could provide an incentive for an individual to work longer hours.

* For Z2 = μ2, the consumer, who could be either low- or high-waged, would be deprived of neither consumption nor leisure.

DEMAND THEORY FOR POVERTY AND AFFLUENCE[1]

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ABSTRACT:

This paper extends demand theory into the neglected area of increasing marginal utility, representing 'deprivation'. Proposition 1 suggests that an individual's consumption of a commodity could be represented by a bounded cardinal, S-shaped utility function. Proposition 2 indicates when to add and when to multiply utilities. Added S-shaped utilities lead to both convex- and concave-to-the-origin indifference curves, the latter space defining dysfunctional poverty. An absolute poverty line can be identified. A given commodity could potentially provide all of superior, inferior or Giffen *experiences* within the convex-to-the-indifference curve space. The derived structural forms, including labour supply, reveal discontinuities, envelope curves and high elasticities associated with deprivation. The two propositions together yield the familiar results of traditional neoclassical theory and provide an integrating framework for analysing utility and demand. A functional form for additive S-shaped utilities with meaningful, estimable parameters is derived in the appendix, and is used to create the diagrams.

(150 words)

KEYWORDS: Increasing marginal utility, additive utilities, absolute poverty line, Giffen good.

JEL classification: D11, J22.

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DEMAND THEORY FOR POVERTY AND AFFLUENCE

> 'How can we convince a sceptic that this "law of demand' is really true of all consumers, all times, all commodities? Not by a few (4 or 4,000) selected examples, surely. Not by a rigorous theoretical proof, for none exists – it is an empirical rule. Not by stating, what is true, that economists believe it, for we could be wrong. Perhaps as persuasive proof as is readily summarised is this: if an economist were to demonstrate its failure in a particular market at a particular time, he would be assured of immortality, professionally speaking, and a rapid promotion. Since most economists would not dislike either reward, we may assume that the total absence of exceptions is not from lack of trying to find them.'
>
> (Stigler, 1966: p.24)

I. INTRODUCTION

When neoclassical economists rejected the concept of cardinal utility because it cannot be measured, in favour of formal axiomatic demand theory, they also rejected a host of information contained in the *shape* of the utility function. Neoclassical economists have also assumed that marginal utility (MU) always diminishes as consumption increases. This may be true for economists, most of whom have had comfortable life-styles. But, is this also true of very poor people, experiencing deprivation? Suppose that, for them, MU increases as consumption increases. The change from one state to the other must also represent a significant experience, that is, a subsistence threshold.

Let us also suppose that a minimum and a maximum (satiation) level of utility can be assumed, then bounded cardinal utility and interpersonal comparisons become feasible. These two thoughts led to the first proposition, giving rise to the possibility of an S-shaped utility function representing an individual's experience of consuming a commodity (good, service or event).

The second proposition arose from the question of whether the S-shaped utility functions should be added or multiplied, and the realisation that it is not a question of either/or, but when should bounded cardinal S-shaped utility functions be added, and when multiplied? The two propositions about utility, one about the shape of a bounded cardinal utility function and the other specifying their separability conditions, offer an integrating framework for analysing utility and demand, and would seem to present an additional perspective on demand theory.

The two propositions seem to bring the whole range of possibilities of concavity, convexity, independence, complementarity and substitutability, out of the shadows and into centre stage of demand theory assumptions. They will be seen to integrate and provide a context for many of the statements in demand theory about the relationship between MU, separability, price-effects and unearned income-effects. The analysis of price effects into income- and substitution-effects, for both additive and multiplicative utilities, is essentially as exercise in the topology of the assumed underlying utility function.

Majumdar (1958) had compared different approaches to utility, summarised here in Table 1 with the addition of the row 'Bounded Cardinalist'.

TABLE 1. APPROACHES TO UTILITY

	Column A Introspective	Column B Behaviourist
Ordinalist	1915 Slutsky 1934 Hicks-Allen Indifference curves Occam's razor	1947 Samuelson: Revealed Preference Analysis
Bounded Cardinalist (S-shaped utility)	1968 Van Praag 1988 Miller	
Cardinalist	1871 Menger 1871 Walras 1874 Jevons 1890 Marshall	von Neumann-Morgenstern Neocardinalists Uncertainty

Columns A and B in Table 1 represent the different methodological approaches to utility theory, and the uses that have been made of it. In column B, economists merely attempt to observe and describe outcomes. The theories in column A are introspective in that they attempt to provide explanations and greater understanding, for which a level of abstraction is necessary. The theory presented here starts with plausible psychological assumptions and attempts to predict their consequences. It is found to encompass the traditional neoclassical demand theory as a special case. It integrates many current piecemeal results, explains some of the anomalies that arise with the traditional theory, and offers some further insights and novel predictions of its own.

The traditional utility debate has been conducted in terms of cardinal or ordinal utilities. The approach adopted here, following the early seminal work of Van Praag (1968), recognises the possibility of an intermediate state in the form of bounded cardinal utility, which arises as soon as satiation is admitted, either at infinite consumption (as in Van Praag's case), or for finite consumption. This present work closely relates to Van Praag's ground-breaking work, in which he explored the outcomes from a bounded cardinal utility function based on *multiplicative* versions of the lognormal distribution function, representing S-shaped utility, satiated at infinity.

Thus, there are three ways in which this present work differs from that of Van Praag.

- He states 'There may be finite satiation points; if there is no satiation point we call the improper point $\infty = (\infty, \ldots, \infty)$ a satiation point' (Van Praag, 1968, p.6) because he assumes that if over-consumption (i.e. a surfeit) occurs then it is as a result of factors other than price[2]. In contrast, the theory presented here assumes that satiation could

[2] Satiation can occur in three ways:
* at finite consumption, with over-consumption accompanied by both a reduction in utility, $U_i < 0$, and a negative price, such as a wager, where the consumer is effectively being paid to consume more;
* at finite consumption, but over-consumption is not accompanied by a reduction in utility, $U_i = 0$;
* at infinite consumption.

If consumers were rational, then over-consumption would not take place, and it could be difficult to estimate the satiation parameter. But, consumers are not rational, and there is plenty of evidence revealing that many over-indulge in both food and alcohol, for example, posing risks to health, evidence that finite satiation is a reality in some dimensions of utility. Also, over-consumption will occur as a result of other factors, apart from price.

occur at finite consumption, even if negative prices are not observed, (although negative prices could occur, as the result of a wager, for instance). This theory assumes that satiation can occur at either finite consumption (in at least one dimension of need), or at infinity (in at least one dimension of need).

- The theory presented here distinguishes between utilities being additive and being multiplicative, although only the case of additive utilities is explored here. Van Praag assumes that the relationship between the utilities from commodities is multiplicative, both without and with dependence (substitutes and complements).
- He makes a case for using the log-normal distribution function as his functional form (pp.81, 86, 119), whereas for additive utilities, the normal distribution function is much more tractable.

The theory presented here is seen to encompass Van Praag's theory, which is applied extensively on continental Europe (Van Herwaarden *et al*, 1981; Hagenaars, 1986).

The plan of the rest of the paper is as follows. Section II sets out the two propositions about utility described above, providing the foundation for this extension of utility theory. In section III, the indifference curves resulting from the propositions are examined, which are found to exhibit all of ultra-superior, superior-normal, inferior-normal and inferior-Giffen behaviour, depending on the combination of the good with that of another satisfier. A potential absolute poverty line (or plane) is identified. In section IV, the introduction of a linear budget yields maps of Engels and demand curves, each displaying discontinuities and an envelope curve, followed by a note about satiation and interpersonal comparisons. The leisure-consumption choice is explored in section V finding similar results for supply curves as for demand curves. The conclusion in section VI summarises the results, indicating some areas of further theoretical exploration, suggesting how to test the theory empirically and finishing with a range of policy areas to which the extended theory could be applied.

In the appendix, a functional form for a utility function with meaningful, estimable parameters is derived, that meets most of the conditions of proposition 1, and fulfils the additive part of the separability proposition. A demand equation and a labour supply equation is derived from the utility function, and the properties of both utility function and the derived structural equations are explored. This functional form is used to create figures 2-7.

II. THE TWO PROPOSITIONS ABOUT UTILITY

Proposition 1 emphasises that diminishing MU is only part of the consumption experience, as Hicks (1939) pointed out, and though it may be the most frequently occurring in a prosperous society, (especially for most economists), increasing MU also yields some interesting and important phenomena for economists to examine.

Proposition 1. The Leaning S-shaped Utility Function.

The first proposition is as follows:

> 'An individual's experience of consumption, X_i, of a commodity i, (good, service or event), $X_i \geq 0$, for i = 1, 2, …, m, can be represented by a continuous, smooth, single-valued, utility function, $U_i = U(X_i)$, $0 \leq U_i \leq 1$, that has the shape of a

leaning ‘S’ curve, bounded below and above, but marginal utility, U_i’, is always less than infinity.’

This is summarised in Table 2 and illustrated in Figure 1 below.

The fact that MU has a minimum and a maximum implies that utility is cardinal, but bounded above and below, as in the seminal work of Van Praag (1968). It is assumed that MU is never infinite.

TABLE 2. SIGNS ASSOCIATED WITH THE LEANING S-SHAPED UTILITY FUNCTION, $U_i(X_i)$.

U_i”	> 0	> 0	$= 0$	< 0	< 0	< 0
U_i’	$= 0$	$0 < U_i < \infty$	$0 < U_i < \infty$	$0 < U_i < \infty$	$= 0$	< 0
U_i	$= 0$	> 0	> 0	> 0	$= 1$	$1 > U_i > 0$
X_i	$X_i = 0$	$0 < X_i < \mu_i$	$X_i = \mu_i$	$\mu_i < X_i < sat_i$	$X_i = sat_i$	$X_i >$ finite sat_i
Experience of individual:	zero consumption	deprivation	subsistence	sufficiency	satiation	surfeit

The shape of the utility function is associated with different experiences. A point of inflection occurs at $X_i = \mu_i$, representing a ‘subsistence’ threshold comparable to the committed consumption level, or survival parameter, in the Stone-Geary utility function from which the Linear Expenditure System (LES) is derived. Consuming less than this, where MU is increasing, implies ‘deprivation’. Consumption greater than the subsistence threshold, where MU is positive but diminishing, may be labelled ‘sufficiency’. The point of maximum utility yields ‘satiation’ in that particular commodity, while consumption greater than a finite satiation point is obviously in ‘surfeit’.

The steeper the slope around the subsistence threshold, the more intense is the desire for, and the satisfaction gained from consuming, that commodity. It is important that the desire for the commodity is fully satisfied before finite satiation is experienced, and this places some restrictions on the relationship between the subsistence threshold, μ_i, the level of consumption associated with satiation, sat_i, and the measure of intensity of need, σ_i.

This latter condition suggests that a distribution function (DF) provides a better representation of the main part of the S-shaped function than would part of a frequency function. Van Praag (1968) gave a persuasive argument for choosing the DF of the lognormal distribution, (LN-DF), confirmed empirically by Van Herwaarden and Kapteyn (1981).

The functional form developed in the appendix, and used to produce figures 2-7, is based on the DF of the normal distribution, (N-DF), (but it does not have any statistical connotations in this context). It does not match up with proposition 1 exactly, for two reasons:

i) $-\infty \leq X_i \leq +\infty$, implying that consumption could take negative values; and,

ii) the use of any DF does not allow for the exploration of the case where satiation occurs for finite values of X_i.

That X_i may take negative values could be explained by ‘free satisfiers’. That is, some fulfilment of a need may be provided by natural circumstances, such as where a warm climate can heat a home before consumption of fuel is required.

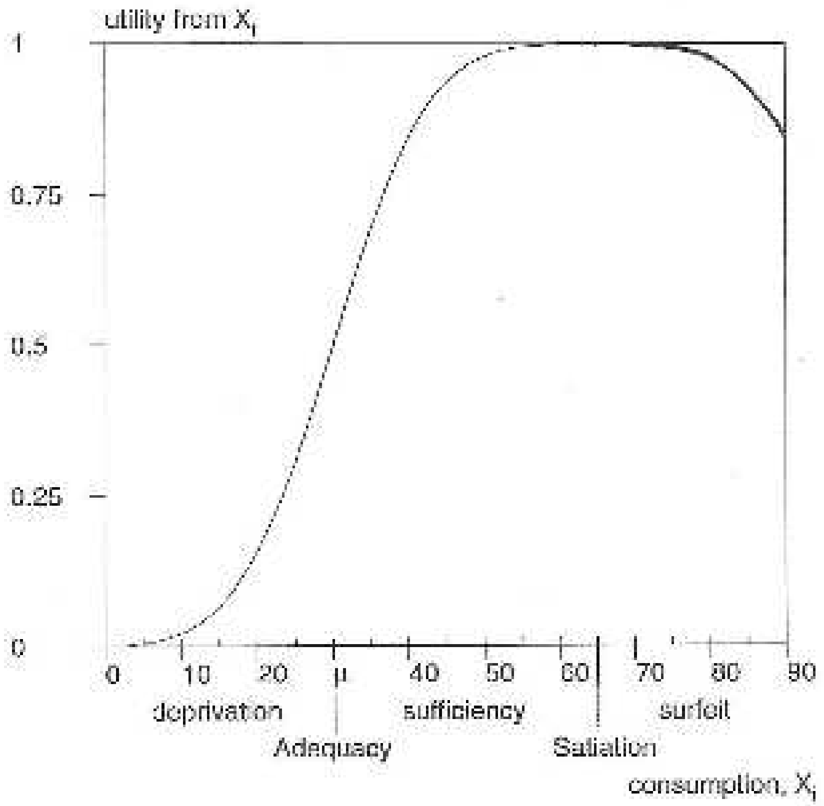

Fig. 1.- The leaning S shaped utility function, μ = 30, σ = 10.

The N-DF was chosen for pragmatic reasons, because it is fairly tractable, and is useful for illustrating many aspects of the theory, providing a reasonable approximation for the part of the leaning S-shape around the subsistence threshold. Further, it has the added advantage that its two parameters, μ_i and σ_i, have important economic interpretations, and are potentially estimable. The parameter μ_I is the survival level or subsistence threshold.

Parameter σ_i is a measure of the intensity-of-need for commodity i. The range of X_i covered by $\mu_i \pm 1.96.\sigma_i$ indicates where MU is experienced most intensely. The smaller is σ_i, the steeper is the slope of the $U_i(X_i)$ function around the parameter μ_I, and the more intense is the need. "Commodities with a large variance are commodities for which satisfaction comes rather slowly … Commodities with a small variance are commodities … of which one is quickly satisfied. For instance, life necessities have presumably a small variance." (Van Praag, 1968, p.34). This raises an interesting question, which could be explored using this functional form, 'Would a commodity, or group of commodities, to which many people are addicted have an even smaller variance than life necessities?'

Each parameter can vary over time for each individual, and between different groups of people, according to demographic variables and other experiences.

Proposition 2. Separability.

Proposition 2 states that

> 'the utilities of a group of commodities that satisfy the same need are multiplicatively related (with or without dependence), and the utilities of groups of satisfiers, each group satisfying a different need, are additively related'.

The discussion of 'separability' and 'the grouping of commodities' in the economics literature (Green, 1976; Deaton and Muellbauer, 1980) often comes across as though they are secondary after-thoughts, unconnected with the axioms of demand theory. Discussion centres on whether the utilities gained from the consumption of different commodities are additive or multiplicative. However, it is not a question of either/or, but rather 'when are utilities additive, and when are they multiplicative?'

It is assumed here, following Mallman and Nudlar (1986), and to a lesser extent Maslow (Lutz and Lux, 1979), that there is a finite number of fundamental human needs and that these are universal and ahistoric. Needs are satisfied by an infinite diversity of culturally-determined satisfiers. Needs cannot be observed directly, but only through the effects of their satisfiers, or lack thereof. The nine, finite, fundamental human needs are for permanence (or subsistence); for protection; for affection; for understanding (one's environment); for participation (in one's community); for leisure; for creation; for identity (or meaning); and for freedom, (Max-Neef, 1986, p.49).

If needs can be defined and identified in terms of their additivity, then two needs could be confirmed as such by comparing the results of additive and multiplicative versions of the same utility function. By extension, it would also be possible to test the hypothesis that the number of needs is finite.

The functional form in the appendix is based on a 2-variable, additive, normal DF, utility function, (2.Add.N-DF), whereas Van Praag and the Leyden school concentrated on an n-variable, multiplicative, lognormal distribution function, utility function, (n.Mult.LN-DF).

III. THE PREDICTIONS OF THE TWO PROPOSITIONS

Indifference Curves

These two propositions together define the utility function for a vector of commodities, and the indifference curves associated with it. Some of the consequents of these propositions are the starting point for traditional neoclassical theory. The continuity, smoothness, and single-valued nature of proposition 1, (including the restriction that the MU from consuming a commodity is less than infinity), preserve the continuity, smoothness and transitivity properties of the indifference curves, as assumed in the axioms of the traditional axiomatic ordinal theory (Green, 1976, pp. 22-42).

The separability proposition gives rise to two very different types of indifference curve maps. The multiplicative one is very like the familiar representative convex-to-the-origin indifference curves found in textbooks, and will not be discussed here. Additive utilities provide more interest. This is illustrated in Figure 2[3], for two commodities, X_1 and X_2, satisfying needs 1 and 2 respectively.

The first feature to note is that the indifference curves close to the origin are concave-to-the-origin or (quasi-concave). However, their shapes are changing so that those further from the origin become less concave, until there is a straight-line indifference curve, which divides the concave from the convex-to-the-origin indifference curves further away from the origin. Thus, the straight-line indifference curve, labelled AB in figure 2, divides the indifference curve map into a triangular area OAB (of which it forms the hypotenuse), and a more familiar type of indifference curve map. The triangle OAB is comparable with the border found in the Stone-Geary utility function from which the Linear Expenditure System (LES) model is derived. It is as though the inner axes of the Stone-Geary indifference curve map have been prised open in order to form the straight-line AB.

The triangle OAB represents a non-solution space, except for corner solutions on the axes, which are non-tangential ‘choices’, representing extreme deprivation in one or both dimensions of need, and can be interpreted as ‘dysfunctional poverty’. Poverty is a multi-dimensional function of all needs. For complex functional forms, a natural **absolute poverty line**, **APL**, could be defined as a locus of points dividing concave from convex indifference curves, that would be neither a straight line, nor co-incidental with any one indifference curve. For this special 2-variable case, derived from the distribution function for a *symmetric* frequency function, the APL is the straight-line indifference curve and it can be shown that $APL = \mu_1/\sigma_1 + \mu_2/\sigma_2$. Might this be a reasonable approximation for the APL for all functional forms? By extension, for a finite, k-need system, $APL = \sum \mu_i / \sigma_i$, for all i = 1, 2, … m. The intercept of this *plane* on each axis, is σ_i.APL. The combination of concave and convex indifference curves, represented here by the straight-line indifference curve, is the source of discontinuity in the structural forms derived from the utility function.

The slope of the straight-line indifference curve is $-\sigma_2/\sigma_1$, and it provides a measure of the consumer’s relative intensities-of-need or relative preferences between commodities. The smaller the value of σ_1, the greater the slope of the straight-line indifference curve (measured at corner A), the greater the intensity-of-need for commodity X_1 compared with X_2. Estimates of these ratios might enable one to rank needs, and to test Maslow’s hypothesis (Lutz and Lux, 1979) that there is a hierarchy of needs, without resorting to assumptions about lexicographic orderings of preferences.

[3] Figures 2 - 6 were created using Seppo Mustonen’s SURVO software.

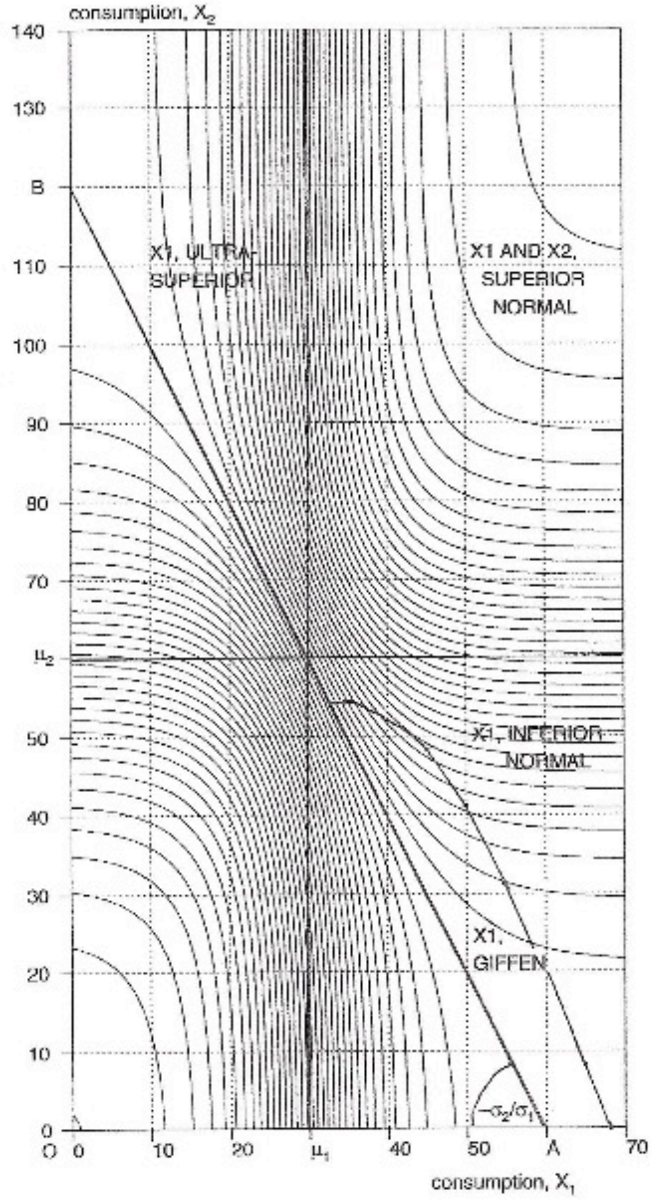

Fig. 2. Indifference curves. $\mu_1 = 30$, $\sigma_1 = 10$.
$\mu_2 = 60$, $\sigma_2 = 20$.
U takes values from 0.005 to 0.995 in steps of 0.015.

The second feature to note is that the two subsistence thresholds, where $X_1 = \mu_1$ and $X_2 = \mu_2$, further divide the indifference map into a left-hand border and a lower border, separating deprivation from sufficiency in each need.

The third feature refers to the characteristics of X_1 and X_2 in the convex-to-the-origin part of the indifference curve map. It can be shown that in the top right-hand quadrant of Figure 2, both commodities are experienced as superior normal goods, (additivity and positive diminishing marginal utilities always yield superior normal characteristics).

In the lower border, the consumer is deprived of X_2, and is experiencing increasing MU. In all of that part of the lower border where the indifference curves are convex-to-the-origin (to the right of line AB), commodity X_1 is experienced as an inferior good. This experience might be thought of as functional poverty. In the small area adjacent to the straight-line indifference curve, indicated in Figure 2, X_1 is experienced as a Giffen good (Dougan, 1982; Silberberg *et al*, 1984). This confirms that the Giffen experience is one in which the consumer is able to fulfil that need sufficiently (need 1 in this example), as anticipated by Berg (1987), but the satisfier X_1 has low unit value, and the consumer is deprived in another dimension of need (need 2). That the Giffen experience is associated with a straight-line indifference curve, adjacent to a triangular non-solution space, was anticipated by Davies (1994).

In that part of the left-hand border where the indifference curves are convex-to-the-origin, the consumer experiences commodity X_1 as a deprivation, (with increasing MU), and, following Hirschleifer's terminology (1976, chap.4), X_1 is here termed an ultra-superior good. Kohli (1985) calls this experience an 'anti-Giffen good', but 'anti-inferior' would be more accurate.

This outcome emphasises that a *commodity* should not be categorised as one of superior, inferior normal or a Giffen good, because it could be all of these, depending on its combination with another good. Rather than categorising the commodity, it is the consumer's *experience of fulfilment of that need* that should be categorised as ultra-superior, superior, inferior normal or Giffen, according to his/her relative prices and income circumstances. This would appear to confirm Spiegel's belief 'that Giffen goods are far more pervasive than is generally believed' (1994, p.137). That the challenge of formulating a utility function for the elusive Giffen *good* (as opposed to the pervasive Giffen *experience*) continues to engage economists is evidenced by Sørensen (2007), Jensen *et al* (2008), Moffatt (2012), Haagsma (2012) and Biederman (2015).

The theory based on these two propositions predicts the already well-established facts, encompassing the traditional static, axiomatic, ordinal theory of demand as a special case, while challenging some of its more restrictive implicit (and explicit) assumptions. For instance, it challenges the assumption that MU is everywhere diminishing and thus the convexity axioms. The convexity assumption of neoclassical demand theory seems to be based on the statement that 'maximising utility will usually yield a solution for an indifference curve that is convex to the origin (for positive prices), and thus indifference curves must be everywhere convex to the origin'. This is true for all multiplicative utility functions, and, in fact, the indifference curves for *both* additive or multiplicative utility functions that exclude the possibility of increasing MU are, indeed, everywhere convex to the origin.

Satiation

Indifference curves for satiation at finite levels of consumption and surfeits with $U_i' < 0$ are likely to look much the same for additive as for multiplicative utilities. Surfeits in two dimensions would yield elliptical indifference curves. A surfeit in one dimension and sufficiency in another would yield partial ellipses. Choices in this part of the indifference curve map could be observed if an individual were persuaded to consume a surfeit of a commodity by the inducement of a negative price. A trivial example is provided by someone accepting a bet to prove that s/he could drink a certain amount of alcoholic drink in a given time without being sick.

Satiation is often rejected in the traditional ordinal theory on the grounds that, for positive prices, it would never be chosen. This argument fails to distinguish between the possibility of satiation existing though rarely manifested, and of it not even existing. If the possibility of the satiation of a need exists (and patently it does), then it could delimit some areas of the indifference curve map as being outwith the solution space for positive prices. The null hypothesis that either satiation does not occur, or it occurs only at an infinite level of consumption can be tested empirically against the alternative that it occurs with finite consumption. If the null hypothesis is rejected, it could have profound implications for the consumer's saving behaviour, for instance. Also, it is possible that, in some circumstances, an individual's endowment of leisure could be in surfeit. Many interns pay for the privilege of working in a company for a limited period, which could be regarded as a negative wage rate.

Interpersonal Comparisons of Welfare

The assumptions that the utility function $U_i = U(X_i)$ reaches a minimum, $U_i = 0$, at $X_i \leq 0$ and a maximum, $U_i = 1$, at $X_i = sat_i$ where sat_i is finite, or $sat_i = \infty$, are necessary conditions for utility to be bounded below and above. Despite the difficulty in observing either satiation or zero consumption (because we receive free satisfiers from our environment - for instance, warmth from the sun reduces the amount of fuel we might need to consume before reaching our subsistence threshold), these assumptions allow for the possibility of standardising utility over a range of $0 \leq U_i \leq 1$, say, for comparing the utility attained in fulfilling one fundamental need, or over the range $0 \leq \sum Ui \leq 1$, permitting interpersonal comparisons of welfare (utility). Evaluations of individual welfare functions of income, based on the shape of the distribution function of a lognormal distribution, have already been carried out very successfully by members of the Leyden School, (see Van Praag and Kapteyn, 1994, for instance), based on the seminal work of Van Praag (1968).

Each individual might be able to experience satiation, but it is not assumed that the level of utility experienced as satiation is the same for each person. However, it is assumed that it is possible to compare each individual's utility with his/her maximum attainable.

IV. PREDICTIONS ABOUT THE DERIVED FUNCTIONAL FORMS

Linear Budget and Engels Curves

Given prices, p_1 and p_2, a linear budget constraint can be expressed in terms of its allocation of expenditure, $Y = X_1.p_1 + X_2.p_2$.

A key parameter associated with the consumption of X_i is μ_i, and similarly a key concept associated with income is the 'subsistence income' or 'survival income', defined as $\mu_1.p_1 + \mu_2.p_2$, where the consumer is just able to fulfil his/her subsistence levels of consumption.

Any budget constraint that passes through the co-ordinate (μ_1, μ_2) is a subsistence or survival budget, including the budget that is co-incidental with the straight-line indifference curve, AB.

The surplus of the consumer's budget, over his/her survival budget is called supernumerary income, as in the LES, denoted here as H, where

$$H = (X_1 - \mu_1).p_1 + (X_2 - \mu_2).p_2 = Y - \mu_1.p_1 - \mu_2.p_2.$$

Novel phenomena

Each of the structural forms derived for X_1 from the utility function will display three features:
1) a set of discontinuities occur, caused by the combination of concave and convex indifference curves, when consumers in poverty have to resort to corner solutions, or self-provisioning;
2) deprivation leads to high elasticities; and
3) an envelope curve is identified.

The Engels curves, plotting the consumption of commodity X_1 on the vertical axis against income on the horizontal one, for given levels of price, p_1, are illustrated in Figure 3. The downward-sloping Engels curves, where commodity X_1 is being experienced as an inferior good, are obvious. Here the consumer has a low income, less than survival income, but s/he is able to consume sufficient of X_1, when it has a relatively low price, ($p_1/p_2 < \sigma_2/\sigma_1$).

Surprisingly, perhaps, the Engels curves for commodity X_1 when it is being experienced as an ultra superior good, that is, when the consumer is deprived in X_1 but has an income greater than survival level, the income elasticity of demand for X_1 is greater than 1. Perversely, the commodity is labelled a 'luxury good' in this circumstance, (although to someone who is deprived of a commodity, the experience of increased consumption of that commodity, due to an increase in income, must feel luxurious).

When income is greater than survival income, and the consumer is sufficiently satisfied by consumption of the commodity, $X_1 > \mu_1$, the Engels curves have positive income elasticities of demand that are less than 1, (X_1 becomes a 'necessity').

On examination of Figure 3, a set of discontinuities can be identified, associated with the straight-line indifference curve. If tangential solutions cannot be found in the functional poverty space, consumers with less than survival income can gain their highest utility from corner solutions in the dysfunctional poverty space. If relative price, $p_1/p_2 \geq \sigma_2/\sigma_1$, the consumer can gain more utility from spending the total available income on (inadequate amounts of) X_2, with $X_1 = 0$. If relative price, $p_1/p_2 \leq \sigma_2/\sigma_1$, then all of the income is spent on (equally inadequate amounts of) X_1, along the X_1 axis, with $X_2 = 0$.

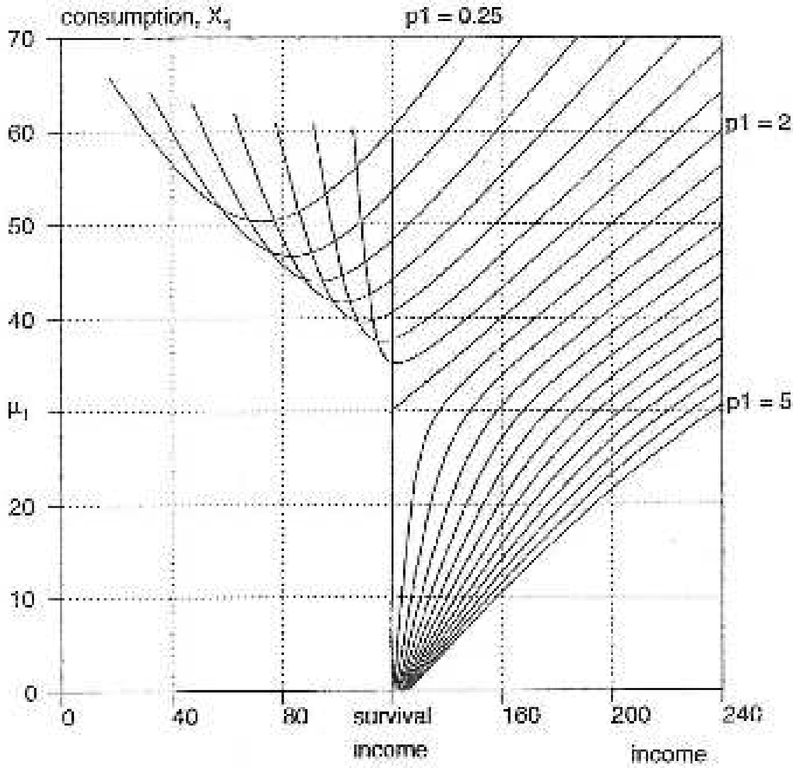

Fig. 3.- Engels curves, generated by equation (4). $\mu_1 = 30$, $\sigma_1 = 10$, $\mu_2 = 60$, $\sigma_2 = 20$, $p_2 = 1$.
p1 takes values from 0.25 to 5.00 in steps of 0.25.

Demand curves

The demand curves for commodity X_1 are illustrated in Figure 4. Consumption of X_1 on the horizontal axis is plotted against relative price, p_1/p_2, on the vertical axis, for different levels of income. A few economists at various times in the past (Stonier and Hague, 1980, p.77;

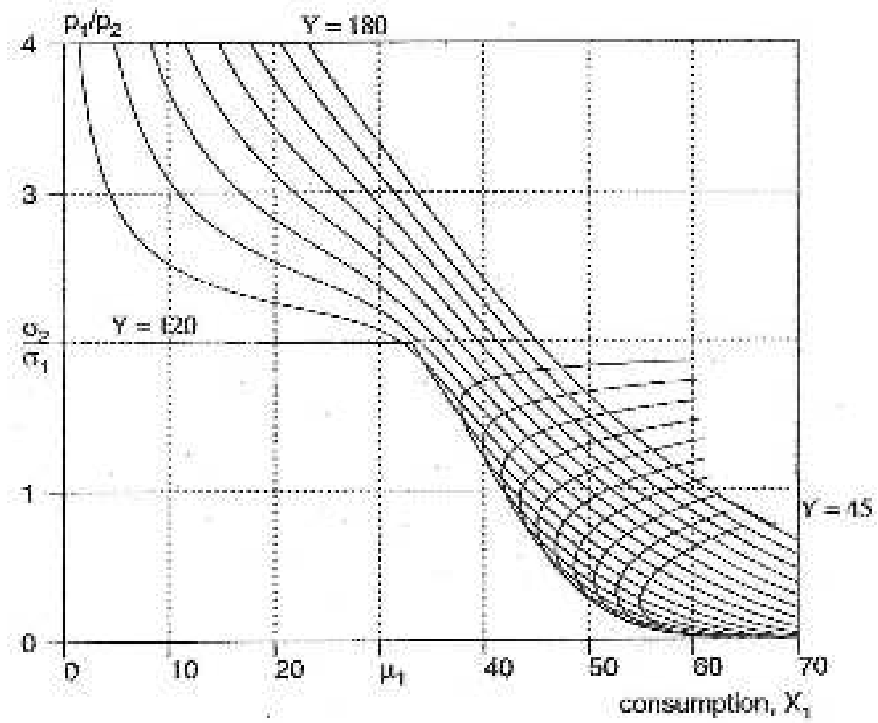

Fig. 4. Demand curves, generated by equation (4): $\mu_1 = 30$, $\sigma_1 = 10$, $\mu_2 = 60$, $\sigma_2 = 20$, $p_2 = 1$.
Y takes values from 45 to 180 in steps of 7.5 units.

Hirschleifer, 1976, pp. 98 and 114) have tried to draw a series of demand curves for a commodity as it transforms from superior to inferior normal to a Giffen good. The two key parameters are again $X_1 = \mu_1$, and the relative intensity-of-need parameters, σ_2/σ_1. The demand curves seem familiar, apart from some in the lower part of the graph where $p_1/p_2 < \sigma_2/\sigma_1$, where X_1 is being experienced as an inferior good, having a normal slope at relatively low prices, but suddenly bends forwards as relative price, p_1/p_2, rises, when X_1 is experienced as a Giffen good. Figure 4 also indicates that a part of some of the demand curves can be slightly concave to the origin.

The division between the concave and convex-to-the-origin indifference curves, represented in this example by the straight-line indifference curve, leads to discontinuities in the demand function, assuming a linear budget. For commodity X_1, the conditions for the corner solutions in the dysfunctional poverty space are the same as for the Engels curves, when income is less than survival income, and the actual outcomes again depend on relative prices compared with σ_2/σ_1.

This instability can lead to an apparent instability in behaviour; that is, large reactions can occur in response to small changes in relative prices. If prices were to waver slightly around σ_2/σ_1, then behaviour could appear to oscillate markedly. In some sense, σ_2/σ_1 can be regarded as a 'natural price'.

With additive utilities, the two goods are net substitutes for each other.

Envelope Curves

Another unexpected novel phenomena predicted by the two propositions is the presence of two envelope curves, each providing minimum bounds (for tangential solutions), one to the demand curves for $p_1/p_2 \leq \sigma_2/\sigma_1$, and the other to the Engels curves for $H < 0$. With hindsight, it should have been intuitively obvious that there might be an envelope curve on demand curves, if one assumes that the demand curves slope down from left to right. If they shift to the right as income increases for superior goods, and they also shift to the right as income decreases for inferior goods, then the envelope must occur on the boundary between a good being inferior and its being superior.

A second method of demonstrating this is through the process of deriving the envelope curve. The demand equation for X_1 is differentiated with respect to Y, and the derivative is set equal to 0, (and then expressed in terms of Y). The expression for Y is then substituted into the X_1 demand equation to obtain the envelope curve. However, this is the condition for the threshold between X_1 being superior and its being inferior, ie coincidental with the boundary $X_2 = \mu_2$, for $X_1 > \mu_1$.

Using similar reasoning, it is easy to prove that the envelope curve for the Engels curve is coincidental with the threshold between X_1 being inferior normal and its being inferior Giffen.

V. APPLICATION TO LEISURE-CONSUMPTION CHOICE.

Empirical evidence has rejected the assumption of additive utilities for consumption and leisure within the Stone-Geary (LES) model (Blundell, 1988). This is to be expected, since it is based only on diminishing marginal utilities and is therefore almost impossible to distinguish between multiplicative and additive separabilities. However, the *ex ante* predictions for labour supply based on the two propositions presented in section II above suggest otherwise.

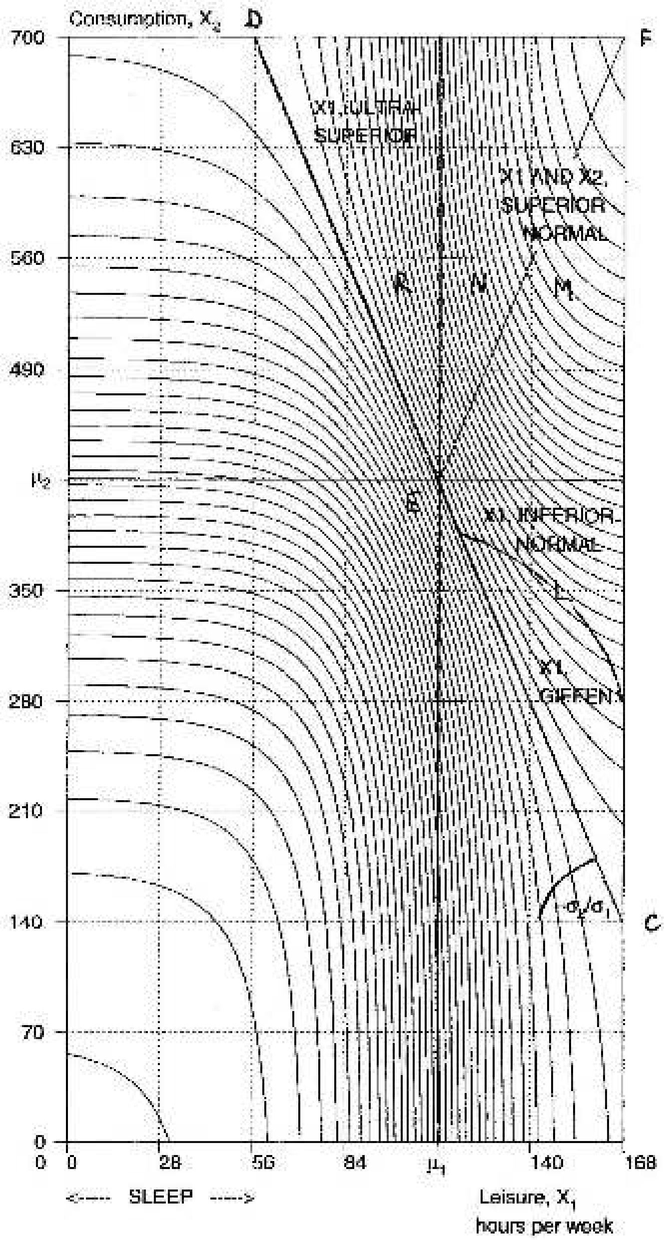

Fig. 5. Indifference curves for Consumption Leisure choice.
$\mu_1 = 112$, $\sigma_1 = 28$.
$\mu_2 = 420$, $\sigma_2 = 140$,
U takes values from 0.005 to 0.995 in steps of 0.015.

The indifference curve map for the consumption-leisure choice is almost the same as for any other pair of additive utilities. X_1 now becomes leisure and X_2 becomes consumption.

However, leisure is different from other commodities because it is constrained to a time limit in a given time period, such as 168 hours per week. The other difference is that, normally, the linear budget is given by the expenditure from money income, $Y = X_1.p_1 + X_2.p_2$. Now, Y represents a set of endowments, (Z_1, Z_2), where Z_1 is the maximum time available for leisure in a given time period, and Z_2 is a flow of material endowments. These are valued at p_1 and p_2 respectively, such that $Z_2.p_2$ represents unearned income. Now the linear budget becomes:

$$Z_1.p_1 + Z_2.p_2 = X_1.p_1 + X_2.p_2$$

or

$$X_2 = (Z_1 - X_1).p_1/p_2 + Z_2$$

The time not spent on leisure, $(Z_1 - X_1)$, is assumed to be spent working for pay, p_1. $(Z_1 - X_1).p_1$ is earnings.

The parameters, μ_1 and μ_2 are subsistence parameters, committed leisure and subsistence consumption respectively. The ratio of parameters, σ_2/σ_1, represents 'relative want intensity', in this case of leisure over consumption. The lower the value of σ, the more highly valued is that need. If $\sigma_2/\sigma_1 > 1$, then leisure is valued more highly than consumption, and this leads to a higher wage economy to compensate for the loss of the leisure.

The characteristics of the commodities noted on the indifference curve map in Figure 2 above apply equally to consumption and leisure. Each can display ultra-superior, superior normal, inferior normal and inferior Giffen experience. The subsistence parameters, $X_1 = \mu_1$ and $X_2 = \mu_2$, divide the map into four parts, and the left hand and lower borders represent deprivations of leisure and consumption respectively. The main difference now is that $X_1 = Z_1$ provides a constraint, which, in this example, cuts the straight-line indifference curve at point A.

The indifference curve map may be further divided by an income-consumption locus of points where the budget line is parallel to the straight-line indifference curve, $p_1/p_2 = \sigma_2/\sigma_1$. This locus is a straight-line labelled EF, where E is point (μ_1, μ_2). For this functional form, EF and EB are symmetric about $X_1 = \mu_1$, and EF and EA are symmetric around $X_2 = \mu_2$. EF divides that part of the indifference curve map where leisure acts as a superior good, into two areas. The map containing convex-to-the origin indifference curves is now divided into four parts, which are labelled, L, M, N and R in Figure 5.

In area L, leisure acts as an inferior good and represents low-wage solutions, $p_1/p_2 < \sigma_2/\sigma_1$.
In area M, leisure acts as a superior good but also represents low-wage solutions.
In area N, leisure acts as a superior good and represents high-wage solutions, $p_1/p_2 > \sigma_2/\sigma_1$.
In area R, leisure acts as an ultra-superior good, and also represents high-wage solutions.

Figure 6 illustrates the set of labour supply curves that are derived from this type of additive utility function (with a linear budget). The horizontal axis denotes hours-worked-for-pay, labelled $(Z_1 - X_1)$, and has a parameter $(Z_1 - \mu_1)$. The vertical axis measures real wage–rates, p_1/p_2, and the parameter on the vertical axis is σ_2/σ_1. This latter may be interpreted as a 'natural wage-rate'. Each curve is associated with a different level of unearned income, $Z_2.p_2$.

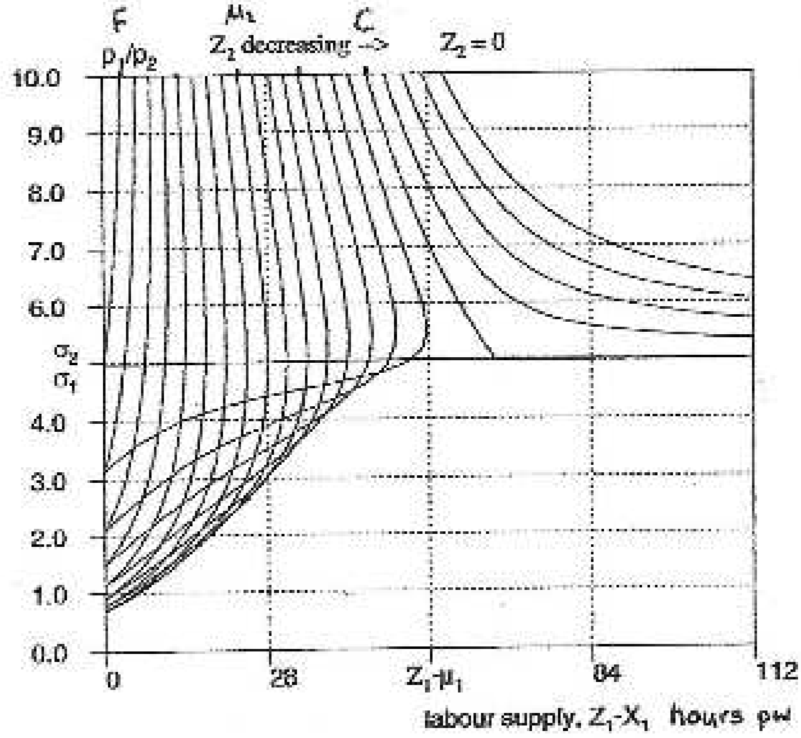

Fig. 6.- Labour supply curves, generated by equation (4a):
$\mu_1 = 112$, $Z_1-\mu_1 = 56$, $\sigma_1 = 28$.
$\mu_2 = 420$, $\sigma_2 = 140$, $p_2 = 1$.
Unearned income, Z2, takes values from 0 to 700 in steps of 35 units.

The labour supply curves for unearned income A < $Z_2.p_2 < \mu_2$, are *backward-bending curves*, but different sections of them can be identified as solutions associated with areas L, M and N on the indifference curve map.

Area L on the indifference curve map leads to the *highly-elastic* labour supply curves in response to wage-rate changes facing a worker, who is low-paid ($p_1/p_2 < \sigma_2/\sigma_1$), and 'part-time', $(Z_1 - X_1) < (Z_1 - \mu_1)$.

Area M leads to the mainly *inelastic* curves in response to wage-rate changes for a low-waged ($p_1/p_2 < \sigma_2/\sigma_1$), part-time worker, $(Z_1 - X_1) < (Z_1 - \mu_1)$. Areas L and M on the indifference curve map lead to solutions in the same bottom-left-hand quadrant of the labour supply curve figure.

Area N leads to the *inelastic* curves for higher real wage-rates, $p_1/p_2 > \sigma_2/\sigma_1$, where hours are also $(Z_1 - X_1) < (Z_1 - \mu_1)$.

Lastly, area R leads to the *backward-sloping* curves of a worker facing high real wage-rates, $p_1/p_2 > \sigma_2/\sigma_1$, and excessive hours, mostly $(Z_1 - X_1) > (Z_1 - \mu_1)$, worked by self-employed people, for instance, and others, that are initially inelastic but become *very elastic* as wage rates decrease and hours-worked-for-pay increase.

The discontinuity associated with the straight-line indifference curve, at $p_1/p_2 = \sigma_2/\sigma_1$, is also apparent for labour supply curves, but there are some differences. Firstly, the poverty space has been truncated at $X_1 = 168$ hours pw. The right-hand axis, at $X_1 = 168$, measures endowments of goods, Z_2, or unearned income[4], $Z_2.p_2$. Secondly, the intercept on $X_1 = 168$ hours, (marked at A), where paid work hours are zero, is now the most important part of the boundary of the poverty space.

There is an envelope curve below the labour supply curves, for $(Z_1 - X_1) < (Z_1 - \mu_1)$, that differentiates between leisure-exhibiting inferior and superior characteristics. It can be shown that this occurs where $X_2 = \mu_2$, (for $X_1 > \mu_1$). Once a worker has attained his/her subsistence level of consumption, his/her behaviour changes, from elastic responses to wage-rate changes to inelastic responses. The envelope curve illustrates why a government has to exert (sometimes brutal) conditionality if it wishes to coerce people to work for longer hours than their natural limits.

A second novel prediction is that the minimum acceptable wage (reservation wage), below which it is not worth someone working for pay, and which is indicated by the intercept of a labour supply curve on the real-wage axis, is a U-shaped function of unearned income, $Z_2.p_2$. The reservation wage = σ_2/σ_1 when $Z_2.p_2$ is at the point labelled A on the indifference curve map. As shown on Figure 7, the reservation wage decreases as $Z_2.p_2$ increases, until $Z_2.p_2 = \mu_2$, where the reservation wage is at its minimum. It then increases as $Z_2.p_2$ increases further until it is at the point labelled F on the indifference curve map, when it takes the value of σ_2/σ_1 again. In fact, the reservation wage extends steeply above σ_2/σ_1 for both $Z_2.p_2 \leq$ A and $Z_2.p_2 \geq$ F.

The reservation wage, voluntary and involuntary employment, and participation decisions all arise naturally from the context of the theory. The functional form is capable of illustrating a wide variety of the observed facts associated with labour supply economics. Similarly, the roles of the parameters (committed leisure, subsistence consumption, and the consumer's consumption-leisure preference parameters) are unambiguous and estimable for a group of similar individuals. The theory has interesting implications for tax and benefit policy.

[4] It is conceivable that $Z_2.p_2$ could even take negative values, if an individual were paying interest on a loan taken out in order to study to attain a higher-paid wage-rate.

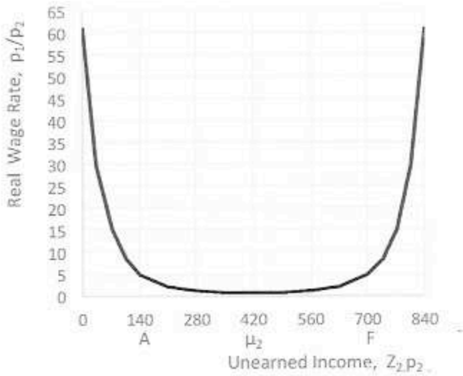

Fig.7. The Reservation Wage is a U-shaped function of unearned income, $Z_2.p_2$

$\mu_1 = 112$, $sigma_1 = 28$
$\mu_2 = 420$, $sigma_2 = 140$
$sigma_2/sigma_1 = 5$, $p_2 = 1$

VI. CONCLUSION

The two propositions taken together, covering all the cases encompassed by the shape and separability of utilities, would appear to present another perspective on utility and demand theory. Together they extend the range of consumption in traditional demand theory to predict the reactions of individuals when experiencing *deprivation with respect to a need*. Many former anomalies, including 'Giffen goods', and very elastic labour supply curves, are now revealed as aspects of behaviour when consumers suffer from deprivation in at least one dimension of need. The theory provides a multi-dimensional definition for an absolute poverty line, reveals discontinuities in the derived Engels, demand and labour supply curves, and identifies envelope curves associated with them.

The theory predicts many already well-established facts and does not refute most of the traditional static ordinal theory of demand, but encompasses it as a special case, while challenging some of its more restrictive implicit (and explicit) assumptions. Specifically, combining increasing MU and additive utilities challenges the convexity axiom. It also encompasses the approach of the Leyden School. In Table 3 below, the traditional neoclassical theory occurs within the diminishing MU proposition row, while the Leyden School is associated with the multiplicative column.

Table 3. PROPOSITIONS FOR UTILITY THEORY

PROPOSITIONS:	**Additive functions**	**Multiplicative functions**
Diminishing MU	Traditional neoclassical and ordinal theories	
Diminishing and increasing MU	Miller propositions: deprivation of needs	Leyden School

Demand theory, having provided the basis for many important related topics, including studies of consumer behaviour, poverty, inequality, labour economics, tax and benefit systems, health, wellbeing and happiness, individual welfare functions, social choice, public economics and economic development, over many decades, seems to have exhausted its possibilities and become less fashionable in recent decades. Maybe the theory presented here could provide the impetus for a surge of new interest?

Clearly, there is scope for theoretical explorations of various aspects of this theory. A list could include the following:
* What results of the axiomatic theory are not encompassed by this theory?
* What are the implications of the two propositions, if any, for general equilibrium analysis or optimal taxation theory?
* What are the properties of the contract curves derived from Edgeworth boxes, when one or both parties are deprived of one or other of the needs for which commodities are being traded? Could it be used to define exploitation?

My own future plan is to test the two fundamental propositions empirically, in two stages:

* To test how well the very-non-linear functional form[5] derived in the Appendix fits the data for any two appropriate variables[6], (the obvious examples are leisure and consumption)[7], compared with other direct explicit functional forms[8] based on diminishing MU and/or multiplicativity, and also compared with flexible functional forms (Deaton *et al*, 1980).
* If the new functional form performs well, then it could be used to test proposition 1, with the null hypothesis, H_o, representing current demand theory, based on diminishing MU (which implies that μ_i would not take a positive value):

$$H_o: \quad \mu_i = 0 \qquad vs \qquad H_a: \; \mu_i > 0 \qquad \text{for } i = 1, 2.$$

The functional form derived here provides estimable parameters with realistic psychological interpretations. Both the survival levels (subsistence parameters), μ_1, μ_2, and the intensity-of-need parameters, σ_2/σ_1 and σ_2, could be estimated for different groups of people in the dataset.

In addition to providing new opportunities for exploring labour supply empirically, and examining the individual's experience of deprivations and poverty, this theory also has important policy implications, for instance, predicting the differential effects of tax and benefit proposals for a population with a wide variety of wage rates and unearned incomes.

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[5] Ideally, the separability assumption would be tested for pairs, or two groups, of commodities by using both additive and multiplicative versions of the same functional forms, and testing between them, when suitable functional forms have been developed.
[6] With additive utilities, the shapes of the indifference curves for any two needs are independent of other needs. Thus, a pair of commodities satisfying two different needs could be studied independently of commodities satisfying other needs.
[7] It could also test whether housing and/or insurance, (representing satisfiers of the need for security and protection), is additive with other forms of consumption. Or, are different types of addictions additively or multiplicatively separable? Test Maslow's 'Hierarchy of Needs'. How many 'needs' can be identified?
[8] It is possible that estimates from the Linear Expenditure System could severely underestimate μ_i.

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APPENDIX: Specification of a functional form for additive utilities.

This functional form, derived by adding two utilities, each representing the consumption of a commodity, each satisfying a different need, was developed in Miller (1985). It is based on the normal distribution function and will be referred to as the '2-variable, additive, normal-distribution function, utility function', ('2 Add N-DF', for short). It does not fit the first proposition completely, because utility from the consumption of the commodities is not zero when consumption is zero. This could represent the fact that sometimes free satisfiers may fulfil a need before measured ones are used, for instance, the warmth of the sun heating one's home during daytime, but needing to consume fuel in the evening. Nor does it feature satiation at finite consumption levels. However, it is useful because it is tractable, and because it contains meaningful, estimable parameters for subsistence or adequacy, μ_i, and intensity-of-need, σ_i, for both commodities. It also fulfils the other conditions for proposition 1 and for additive utilities. It was used to create figures 2, 3 and 4 above. Clearly, it would be desirable to develop further tractable functional forms, which will embody both propositions fully.

The additive N-DF utility function is defined as the sum of the distribution functions for the normal distribution (which have no statistical connotations in the present context), of consumption levels X_i, $-\infty < X_i < +\infty$, where the i'th commodity fulfils the i'th need. The sum is scaled such that utility, U, lies between 0 and 1.

The '2 Add N-DF' utility function fits the 'two needs, two commodities' case.

$$U(X_1, X_2) = \tfrac{1}{2} F_1(X_1) + \tfrac{1}{2} F_2(X_2)$$

(1) $$U(X_1, X_2) = \tfrac{1}{2}\int_{-\infty}^{X_1} \frac{e^{-(R_1-\mu_1)^2/2\sigma_1^2}}{\sigma_1.\sqrt{2\pi}}.dR_1 + \tfrac{1}{2}\int_{-\infty}^{X_2} \frac{e^{-(R_2-\mu_2)^2/2\sigma_2^2}}{\sigma_2.\sqrt{2\pi}}.dR_2$$

where U, $0 \le U \le 1$, is utility,

$\mu_1, \mu_2 \ge 0$ are subsistence parameters representing 'adequacy' thresholds, and

$\sigma_1, \sigma_2 > 0$ are parameters representing intensity of need for commodities 1 and 2.

The utility function, together with the budget constraint, represents the structural form of the model.

A *linear budget constraint* is expressed in the form of allocation of expenditure, Y, on the amounts, X_1 and X_2, of two commodities, at prices p_1 and p_2 respectively:

$$Y = X_1.p_1 + X_2.p_2,$$

where Y, p_1 and $p_2 \ge 0$.

Maximising $U = F(X_1, X_2)$ subject to the budget constraint Y, and using the Lagrangian multiplier method gives the optimality condition as

(2) $$\left(\frac{X_2-\mu_2}{\sigma_2}\right)^2 = \left(\frac{X_1-\mu_1}{\sigma_1}\right)^2 + 2.\ln\left(\frac{\sigma_1.p_1}{\sigma_2.p_2}\right)$$

Equation (2) also gives the locus of points describing the *income-consumption locus* for a given price ratio, p_1/p_2.

Using the following short-hand notation for elements that appear frequently

$ONE = \sigma_1.p_1$
$TWO = \sigma_2.p_2$
$H = Y - \mu_1.p_1 - \mu_2.p_2$ (supernumerary income)

and substituting for $X_2 = Y/p_2 - (p_1/p_2).X_1$ into equation (2) yields

(3) $$\left[\frac{\left(H-(X_1-\mu_1).p_1\right)}{TWO}\right]^2 = \left[\frac{(X_1-\mu_1).p_1}{ONE}\right]^2 + 2.\ln\left(\frac{ONE}{TWO}\right)$$

This 'implicit demand equation' is a quadratic equation in $(X_1 - \mu_1).p_1$, which is solved using the negative square root, yielding the *expenditure equation* for the first commodity:

(4) $$X_1.p_1 = \mu_1.p_1 + \frac{H.ONE^2 - ONE.TWO\sqrt{\left[H^2 + \left(ONE^2 - TWO^2\right).2.\ln\left(\frac{ONE}{TWO}\right)\right]}}{\left(ONE^2 - TWO^2\right)}$$

In order to accommodate the effect of constraining $X_2 \geq 0$, equation (4) must be qualified such that $0 \leq X_1 \leq Y/p_1$. Thus, if $X_1 < 0$, put $X_1 = 0$, and if $X_1 > Y/p_1$, put $X_1 = Y/p_1$. Similarly, $0 \leq X_2 \leq Y/p_2$. These give corner solutions on the axes outwith the non-solution space.

Equation (4) is the solution to a quadratic equation in $(X_1 - \mu_1).p_1$, and gives two solutions. The negative root maximises utility. The expenditure equations for X_1 and X_2 are symmetric and homogeneous of degree zero in p_1, p_2 and Y. The two demand equations represent the reduced form of the model.

There are several different ways of arranging equation (4) to simplify it, or to facilitate an intuitive understanding of it, or to work out the best way to estimate it. For instance, X_1 can be expressed in terms of Y, p_1/p_2, p_1 and p_2. It is very non-linear in every version.

When both $Y \geq H$, and the budget line is parallel to the straight line indifference curve, and thus $(\sigma_1.p_1)/(\sigma_2.p_2) = 1$, and using the negative root, the expenditure equation simplifies to

(5) $$X_1.p_1 = \mu_1.p_1 + \frac{H\ .\ ONE}{(ONE + TWO)}$$

By expressing equation (2) in terms of p_1, and differentiating with respect to X_2 and X_1, dX_1/dX_2 can be found by implicit differentiation. By setting $dX_1/dX_2 = 0$, the locus for the threshold between *X_1 being superior and its being inferior*, on the indifference curve map, is found to be coincidental with $X_2 = \mu_2$, for $X_1 > \mu_1$.

By differentiating X_1 in equation (4) with respect to Y, and setting the partial derivative equal to zero, one obtains the condition

(6) $$H = ONE \times \sqrt{\left[-2 \times \ln\left(\tfrac{ONE}{TWO}\right)\right]}, \qquad if\left(\tfrac{ONE}{TWO}\right) < 1.$$

Substituting for H from equation (6) into equation (4) gives the *envelope curve on the demand equations*, for $p_1/p_2 \;\leq\; \sigma_2/\sigma_1$.

(7) $$X_1 = \mu_1 + \frac{ONE \times \sqrt{\left[-2 \times \ln\left(\tfrac{ONE}{TWO}\right)\right]}}{p_1}.$$

In order to obtain the locus for the *boundary between the inferior normal and inferior Giffen experience in X_1*, the following procedure is adopted.

The equation for the budget line is rearranged as $p_1 = (Y - X_2.p_2)/X_1$, and p_1 is substituted into the optimality equation (2) above, resulting in equation (8), which, when solved numerically, gives *the price-consumption loci* for given income levels on the indifference curve map.

(8) $$e^{(X_2-\mu_2)^2/2\sigma_2^2-(X_1-\mu_1)^2/2\sigma_1^2} = \left(\frac{(Y - X_2.p_2)}{X_1.p_2}\right)\left(\frac{\sigma_1}{\sigma_2}\right)$$

Equation (8) is rearranged in terms of Y, which is then differentiated with respect to X_1 and X_2. Using implicit differentiation, dX_1/dX_2 is obtained and set equal to zero, eliminating Y. This gives the locus of points for the boundary:

(9) $$\left(\frac{X_1 - \mu_1}{\sigma_1}\right)^2 = \left(\frac{X_2 - \mu_2}{\sigma_2}\right)^2 + 2 \times \ln\left(\frac{X_1}{\sigma_1}\right) + 2 \times \ln\left(\frac{\mu_2 - X_2}{\sigma_2}\right), \qquad \text{for } X_2 < \mu_2.$$

Equation (9) has to be solved numerically to find the solutions for X_1. It gives two solutions, one each side of the straight-line indifference curve. The locus cuts the straight-line indifference curve at

$$½\left(\mu_1 + \sqrt{\left[\mu_1^2 + 4 \times \sigma_1^2\right]}\right), \qquad \mu_2 - \left(\frac{\sigma_2}{2\sigma_1}\right) \times \left(\sqrt{\left[\mu_1 + 4\sigma_1^2\right]} - \mu_1\right)$$

* * * * * * * * *

The Consumption-Leisure choice

The 2-V.A.N-DF.U-Fn can be applied to the choice between Consumption, ($X_2 \geq 0$), and Leisure, X_1, and to the Labour Supply equations, as follows:

The usual budget constraint is $Y = X_1.p_1 + X_2.p_2$, where Y is income, but now it becomes $Y = Z_1.p_1 + Z_2.p_2$, where Z_1 and Z_2 comprise a set of endowments: Z_1 of time, (in this case 168 hours per week), priced at p_1; Z_2, of material resources priced at p_2, ($Z_2.p_2$ could be unearned income)). Thus, the appropriate linear budget constraint is

$$X_2 = (Z_1 - X_1)p_1/p_2 + Z_2.$$

$(Z_1 - X_1)$ represents hours worked for pay, and $(Z_1 - X_1).p_1$ is earnings, where $0 \leq X_1 \leq Z_1$.

Supernumerary income, H, in this case becomes

$$H = (Z_1 - \mu_1).p_1 + (Z_2 - \mu_2).p_2$$

The **labour supply equation** can be obtained by subtracting $X_1.p_1$ from $Z_1.p_1$ in equation (4) and substituting for $H = (Z_1 - \mu_1)\, p_1 + (Z_2 - \mu_2)\, p_2$.

$$\text{(4a)} \quad (Z_1 - X_1).p_1 = \frac{-(Z_1 - \mu_1).p_1.\text{TWO}^2 - (Z_2 - \mu_2).p_2.\text{ONE}^2 - \text{ONE x TWO}.\sqrt{[\text{Bracket}]}}{(\text{ONE}^2 - \text{TWO}^2)}$$

where [Bracket] = $[H^2 + (\text{ONE}^2 - \text{TWO}^2).2.\ln(\text{ONE/TWO})]$. The negative root is used.

The envelope curve on the Labour Supply curves is given by

$$\text{(7a)} \quad (Z_1 - X_1).p_1 = (Z_1 - \mu_1).p_1 - \text{ONE x } \sqrt{[-2 \text{ x } \ln(\text{ONE/TWO})]}.$$

It can be shown that this locus of points representing the difference between superior and inferior characteristics is co-incidental with the line dividing the two areas on the indifference curve map, ie $X_2 = \mu_2$, for $X_1 \geq \mu_1$.

The **reservation wage** is a function of unearned endowments, Z_2, and the real wage rate, p_1/p_2. It can be obtained by setting $(Z_1 - X_1) = 0$ in equation (7a) and rearranging it in terms of p_1, leading eventually to

$$\text{(10a)} \quad \left(\frac{z_2 - \mu_2}{\sigma_2}\right)^2 - \left(\frac{z_1 - \mu_1}{\sigma_1}\right)^2 = 2 \text{ x } \ln\left(\frac{\sigma_1.p_1}{\sigma_2.p_2}\right)$$

Rearranging this in terms of p_1 gives

$$\text{(11a)} \quad p_1 = \left(\frac{\sigma_2.p_2}{\sigma_1}\right) + \sqrt{e^{(z_2-\mu_2)^2/\sigma_2^2-(z_1-\mu_1)^2/\sigma_2^2}}\,.$$

This quadratic in Z_2 can be solved using the negative root, yielding an expression for the U-shaped reservation wage, p_1, which is symmetric about $Z_2 = \mu_2$ for the 2-V.A.N-DF.U-Fn model.

All the other results in equations (5) – (9) for the 2-V.A.N-DF.U-Fn case hold for consumption and leisure, with the above qualifications.

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The utility function given in equation (1) could be adjusted as follows so that X_i represents a vector (group) of k_i commodities, X_{i1}, X_{i2}, … X_{iki}, all satisfying the same need in different ways. The multiplicative utility function in the 'two needs, two commodities' case is the '2-G.M.N-DF.U-Fn'.

(1c)
$$\mathrm{U} = ½\int_{-\infty}^{X_{11}} \ldots \int_{-\infty}^{1k1} . \frac{\left|V_1\right|^{1/2} e^{-1/2(R_1-\mu_1)'V_1^{-1}(R_1-\mu_1)}}{(2\pi)^{k1\div 2}} .dR_{11}.dR_{12}\ldots dR_{1k1}$$

$$+ ½\int_{-\infty}^{X_{21}} \ldots \int_{-\infty}^{X_{2k2}} . \frac{\left|V_2\right|^{1/2} e^{-1/2(R_2-\mu_2)'V_2^{-1}(R_2-\mu_2)}}{(2\pi)^{k2/2}} .dR_{21}.dR_{22}\ldots dR_{2k2}$$

where μ_1 and μ_2 are column vectors of survival levels, and V_1 and V_2 are the variance-covariance matrices for the first and second groups of commodities respectively. This formulation specifies additive utilities between groups, and multiplicative utilities within a group of commodities. The demand equation for this functional form has not yet been obtained.

* * * *

In the 'three needs, three commodities' case, the '3-V.A.N-DF.U-Fn' yields the following 'implicit demand equation for X_1':

(3c)
$$\left(H-(X_1-\mu_1)p_1\right)\times ONE = TWO\times\sqrt{\left[\left((X_1-\mu_1)p_1\right)^2 + 2\times ONE^2\times\ln\left(\tfrac{ONE}{TWO}\right)\right]}$$

$$+THREE\times\sqrt{\left[\left((X_1-\mu_1)p_1\right)^2 + 2\times ONE^2\times\ln\left(\tfrac{ONE}{THREE}\right)\right]},$$

where $H = Y - \mu_1.p_1 - \mu_2.p_2 - \mu_3.p_3$,

and THREE $= \sigma_3.p_3$.

In order to obtain an expression for the 'explicit demand equation' for X_1, the quartic expression in $(X_1 - \mu_1).p_1$ in equation (3b), must be solved.

(1,470 words - appendix)
(9,060 words in total).