Impact of International Trade under Dual Labor Markets

Zhou, Haiwen

29 January 2023
Impact of International Trade under Dual Labor Markets

Haiwen Zhou

Abstract
A country’s unemployment rate can be affected by technology choice and the opening of international trade. This general equilibrium model examines the impact of international trade with the presence of dual labor markets in which manufacturing firms engage in oligopolistic competition and choose technologies with different marginal and fixed costs to maximize profits. In a closed economy, it is shown that an increase in labor market efficiency or a population increase induces manufacturing firms to adopt more advanced technologies and the wage rate in the manufacturing sector increases. With the existence of a continuum of technologies, technology choice is not a source of firm heterogeneity. The opening of international trade leads to an increase in the wage rate in the manufacturing sector and the price of the agricultural good. When countries are identical, international trade always increases national welfare.

Keywords: International trade, dual labor markets, oligopoly, technology choice, increasing returns

JEL Classification Numbers: F12, F16, F66, J64, E24

1. Introduction
There are two explanations for the wage gap between skilled and unskilled labor in some developed countries: technology choice and international trade. First, choice of advanced technologies may lead to a lower demand for unskilled workers. Second, the opening of international trade can reduce the demand for unskilled workers. The opening of trade interacts with technology choice. If different industries would pay the same wage rate, gaining jobs in some industries and losing jobs in some others will not be very controversial for countries engaging in international trade if the overall impact of international trade on employment is not large. With industries paying different wages, the impact of international trade on a country’s welfare can be controversial if a country gains jobs in low-wage sectors and loses jobs in high-wage sectors. Countries may also be concerned with how the opening of international trade affects the development and adoption of advanced technologies. With the presence of increasing returns in production, countries may have strong incentives to grab strategic industries with higher wages and better technological potentials under which firms engage in oligopolistic competition (Spencer and Brander, 1983).
In this paper, we study the impact of international trade on national welfare in a general equilibrium model with the presence of dual labor markets. While workers are identical in terms of qualifications, the equilibrium wage rate in the manufacturing sector will be higher than that in the agricultural sector because workers in the manufacturing sector could not be monitored perfectly and a higher wage rate is needed to prevent workers in the manufacturing sector from shirking (Shapiro and Stiglitz, 1984). Firms in the manufacturing sector engage in oligopolistic competition (Cournot competition) and choose outputs and technologies to maximize profits. To produce a manufactured good, there is a continuum of technologies, and a more advanced technology has a higher fixed cost but a lower marginal cost of production. With the existence of fixed costs, the manufacturing sector has increasing returns to scale in production.

In a closed economy, we study how a manufacturing firm’s equilibrium technology choice is affected by various parameters such as the level of labor market efficiency. First, we show that an increase in the level of labor market efficiency can induce manufacturing firms to adopt more advanced technologies. Second, we demonstrate that an increase in market size through an increase in population size can induce manufacturing firms to adopt more advanced technologies.

Interestingly, with the opening of international trade, we show that manufacturing firms in different countries choose the same technology even though countries differ in aspects such as their discount rates. When there are only two technologies, firms may choose different technologies with different marginal costs of production and thus firm heterogeneity results. In this model with a continuum of technologies, technology choice is not a source of firm heterogeneity. The opening of international trade leads to an increase in the price of the agricultural good and the wage rate in the manufacturing sector increases. The increase in the price of the agricultural good will reduce national welfare while the increase in the wage rate will increase national welfare. Overall, the impact of international trade on national welfare is ambiguous. However, international trade is always beneficial for each country when countries are identical.

This paper is related to the literature on dual labor markets, as studied in Bulow and Summers (1986). They show that the opening of international trade may not be beneficial to a country if a country loses high wage jobs and the wage difference between sectors is sufficiently large. There are two important differences between this paper and their model. First, production in

---

1 Oligopoly became an important type of market structure in developed countries such as the United States since the Second Industrial Revolution.
the manufacturing sector has constant returns and firms engage in perfect competition in their model. In this model, there are increasing returns in the manufacturing sector and firms engage in oligopolistic competition. The existence of increasing returns leads to the result that the ratio of the wage rate to the price of manufactured goods increases with the opening of international trade. Second, while technology choice is a channel for the opening of international trade to affect a country’s welfare in this model, technology choice is not addressed in their model.

In this model, the equilibrium wage rate in the manufacturing sector is higher than that in the agricultural sector. If employment in the agricultural sector is interpreted as unemployment as in Shapiro and Stiglitz (1984), this model is related to the literature on the impact of international trade on unemployment with the presence of efficiency wages (Copeland, 1989; Matusz, 1996; Hoon, 2001; Brecher and Chen, 2010; Davis and Harrigan, 2011). There are some significant differences between those papers and this one. First, while technology choice is not considered in those models, it is an essential aspect of this model. Second, in the above papers, firms engage in either perfect competition or monopolistic competition. In this paper, firms engage in oligopolistic competition. The motivation for adopting oligopoly as the type of market structure is as follows. With constant elasticity of demand, a firm’s output will be a constant if firms engage in monopolistic competition. In this model, a firm’s technology choice depends on its level of output. To ensure that a firm’s output changes with fundamentals such as population size even with constant elasticity of demand, oligopoly is chosen as the type of market structure. Zhou (2018) has addressed the impact of international trade in which firms engage in oligopolistic competition and unemployment is the result of the presence of efficiency wages. There are some essential differences between this model and Zhou (2018). First, technology choice is not addressed in that model. Second, the production of the agricultural good uses land only and does not use labor in that model. Thus, the impact of international trade on intersectoral labor mobility is not addressed in that model.

The plan of the paper is as follows. Section 2 establishes conditions for a representative consumer’s utility maximization, a representative manufacturing firm’s profit maximization, and market clearing conditions in a closed economy. Section 3 conducts comparative statics on the

---

2 There are alternative ways to model unemployment. First, impact of international trade on unemployment based on job search is studied in Davidson, Martin, and Matusz (1999). Second, Egger and Kreickemeier (2012) have explored the impact of international trade on unemployment in which unemployment results from the existence of fair wages.
impact of changes in key parameters such as an increase in population size to explore properties of the equilibrium. Section 4 studies the impact of international trade on technology choice and the wage rate in the manufacturing sector when countries differ in aspects such as the degree of patience of citizens. Section 5 discusses some possible generalizations and extensions of the model and concludes.

2. Equilibrium in a closed economy

There are two countries: home and foreign. In this section, we focus on the home country in autarky.

Time is continuous. Variables normally are not indexed by time if there is no confusion from doing this. Labor is the only factor of production. The size of the population is \( L \). There are two sectors: agriculture and manufacture. There is one agricultural good and its price is \( p_a \). The agricultural sector has constant returns to scale in production. Without loss of generality, we assume that each worker in the agricultural sector produces one unit of the agricultural good. Like He and Yu (2015) and Neary (2016), there is a continuum of manufactured goods indexed by \( \varpi \in [0,1] \). All manufactured goods have the same costs of production and enter a consumer’s utility function in a symmetric way.\(^3\) Production in the manufacturing sector requires monitoring to prevent shirking (Shapiro and Stiglitz, 1984) while there is perfect monitoring in the agricultural sector. In equilibrium, dual labor markets result. Employment in the manufacturing sector is more desirable because the equilibrium wage rate in the manufacturing sector will be higher than that in the agricultural sector even though workers are homogeneous (Bulow and Summers, 1986).

2.1. Utility maximization

A consumer’s consumption of the agricultural good in period \( t \) is \( c_a^t \) and her consumption of manufactured good \( \varpi \) is \( c_m^t(\varpi) \). The discount rate of an individual is \( \rho \). An individual’s cost of exerting effort is \( y \). If an individual shirks, the benefit from doing this is that the cost of exerting effort is saved. Let \( I \) denote an indicator function. If an individual does not shirk, \( I = 1 \); otherwise, \( I = 0 \). For the constants \( \alpha \in (0,1) \) and \( \sigma > 1 \), a representative consumer’s utility function is specified as

\[^3\] Like Neary (2016), the purpose of having a continuum of manufactured goods instead of only one manufactured good is to eliminate a manufacturing firm’s market power in the labor market.
\[
\int_0^\infty \left[ \left( c_t^l \right)^\alpha \left[ \int_0^1 c_m^t \frac{\sigma^{-1}}{\sigma} (\alpha) d\alpha \right]^{\sigma(1-\alpha)} - yI \right] e^{-\rho t} dt.
\] (1)

With the above homothetic utility function, utility maximization requires that a consumer spends \(\alpha\) percent of income on the agricultural good and \(1 - \alpha\) percent of income on manufactured goods. Also, the absolute value of a consumer’s elasticity of demand for a manufactured good is \(\sigma\).

The number of workers employed in the manufacturing sector is \(E\). A worker in the manufacturing sector caught shirking is immediately fired. This worker will find employment in the agricultural sector. If a worker does not shirk, the probability that this worker will be mistakenly viewed as shirking is \(z_1\). If a worker shirks, the probability that shirking is detected is \(z_2\). We assume that \(z_2 > z_1\). That is, if a worker shirks, the probability of being fired is higher. In addition to being fired from being viewed as shirking, the exogenous job separation rate for a worker in the manufacturing sector is \(q\). Let \(V_i\) denote a worker’s present value of working in the manufacturing sector, \(i\) for industry. Let \(V_a\) denote the present value of working in the agricultural sector, \(a\) for agriculture. For a worker, the benefit of shirking is \(y\) and the cost of shirking is the additional probability of being fired times the amount of capital loss \((z_2 - z_1)(V_i - V_a)\). A worker will not shirk if the lifetime utility from shirking is not higher than that from non-shirking:

\[
y \leq (z_2 - z_1)(V_i - V_a).\] (2)

The wage rate in the manufacturing sector is \(w\). For a worker employed in the manufacturing sector, the instant benefit is \(w\) and the probability of experiencing a shock in the change of asset value is \(q + z_1\). With discount rate times asset value equals flow benefits plus expected capital gain (Shapiro and Stiglitz, 1984), the asset equation for a worker employed in the manufacturing sector is

\[
\rho V_i = w + (q + z_1)(V_a - V_i).\] (3)

Since each agricultural worker produces one unit of output, the return for a worker employed in the agricultural sector is \(p_a\). For a worker in the agricultural sector, the instant benefit is \(p_a\). The instant probability for an individual in the agricultural sector to find a job in the manufacturing sector is \(\eta\), which is determined endogenously in equation (5) later. With discount rate times asset value equals flow benefit plus expected capital gain, the asset equation for a worker employed in the agricultural sector is

\[
\rho V_a = p_a + \eta(V_i - V_a).\] (4)

In equilibrium, no one shirks in this efficiency wage model. At each instant, the number of workers losing jobs in the manufacturing sector is $E(q + z_1)$. There are $L - E$ workers in the agricultural sector. With a job acquisition rate of $\eta$, the number of workers finding jobs in the manufacturing sector at each instant is $\eta(L - E)$. In a steady state, the number of workers employed in the manufacturing sector does not change. Thus, the flow out and flow in should be equal:

$$\eta(L - E) = E(q + z_1).$$  

(5)

In equilibrium, a manufacturing firm will adjust the wage rate in the manufacturing sector until (2) holds with equality. From equations (2), (3), (4), and (5), the non-shirking constraint for a worker in the manufacturing sector is

$$w - pa = \frac{yp}{2-x_1} + \frac{\beta(1+q)}{(2-x_1) L-E}.$$

(6)

Equation (6) in this model is the same as equation (6) in Bulow and Summers (1986, p. 383).  

2.2. Profit maximization

Like Spencer and Brander (1983) and Ishikawa, Sugita, and Zhao (2009), firms producing the same manufactured good are assumed to engage in Cournot competition. The number of identical firms producing manufactured good $\sigma$ is $m(\sigma)$, and $m \in \mathbb{R}^+$. To produce a manufactured good, a firm may choose from a continuum of technologies indexed by $n(\sigma) > 0$ (Zhou, 2004, 2009, 2011, 2013, 2019, 2021). A higher value of $n$ indicates a more advanced technology. For technology $n$, the fixed cost in terms of labor units is $f(n)$ and the marginal cost in terms of labor units is $\beta(n)$. We assume that $f$ and $\beta$ are twice continuously differentiable. To capture the substitution between fixed costs and marginal cost in production, we assume that a more advanced technology has a higher fixed cost but a lower marginal cost of production.  

---

4. Equation (6) shows that the role played by the price of the agricultural good in Shapiro and Stiglitz (1984) is different from that in Bulow and Summers (1986). In Shapiro and Stiglitz, the price of the agricultural good enters through the price index: a higher price of the agricultural good reduces the utility from working. In Bulow and Summers, the price of the agricultural good is the return from working in the agricultural sector. This difference in specification does not lead to significant difference in comparative static results between the two approaches.

5. This assumption on the tradeoff between fixed and marginal costs of production can be motivated by technology choice in the transportation sector. Compared with traditional technology relying on longshoremen, containerization is a technology with a much higher fixed cost (shown in specially designed container ships, cranes, and terminals) but a lower marginal cost of transporting goods.
is, \( f'(n) > 0 \) and \( \beta'(n) < 0 \). The price of manufactured good \( \sigma \) is \( p_1(\sigma) \). The level of output of a manufacturing firm is \( x(\sigma) \). For a manufacturing firm, its revenue is \( p_1(\sigma)x(\sigma) \) and its total cost is \( [f(n) + \beta(n)x]w \). Thus, its profit is \( p_1(\sigma)x(\sigma) - f(n)w - \beta(n)x(\sigma)w \). In a Cournot-Nash equilibrium, a manufacturing firm takes the wage rate, and outputs and technologies of other manufacturing firms as given and chooses its output and technology to maximize its profit.

A manufacturing firm’s optimal choice of output yields \( p_1 + x \frac{\partial p_1}{\partial x} - \beta(n)w = 0 \). Since the absolute value of the elasticity of demand for a manufactured good is \( \sigma \), a firm’s optimal choice of output yields

\[
p_i \left(1 - \frac{1}{m_\sigma}\right) = \beta w. \tag{7}
\]

A manufacturing firm’s optimal choice of technology requires

\[-f'(n) - \beta'(n)x = 0. \tag{8}\]

The corresponding second order condition for technology choice \(-f''(n) - \beta''(n)x < 0\) is valid.

### 2.3. Market-clearing conditions

To produce manufactured good \( \sigma \), each of the \( m(\sigma) \) manufacturing firms needs \( f(\sigma) + \beta(\sigma)x(\sigma) \) workers. Integrating over the range of manufactured goods, the number of workers employed in the manufacturing sector is

\[
E = \int_0^1 m(\sigma)[f(\sigma) + \beta(\sigma)x(\sigma)] \, d\sigma. \tag{9}
\]

For the market for the agricultural good, since each individual spends \( \alpha \) percent of income on the agricultural good and total income of this economy is \( wE + p_a(L - E) \), total demand for the agricultural good is \( \alpha[wE + p_a(L - E)] \). The number of workers employed in the agricultural sector is \( L - E \) and the value of total agricultural good is \( p_a(L - E) \). The clearance of the market for the agricultural good requires

\[
\alpha[wE + p_a(L - E)] = p_a(L - E). \tag{10}
\]

For the market for manufactured goods, since each individual spends \( 1 - \alpha \) percent of income on manufactured goods, total demand for manufactured goods is \( (1 - \alpha)[wE +
\( p_a(L - E) \). Total value of the supply of manufactured goods is

\[
\int_0^1 p_i(\omega)m(\omega)x(\omega)d\omega.
\]

The clearance of the market for manufactured goods requires

\[
(1 - \alpha)[wE + p_a(L - E)] = \int_0^1 p_i(\omega)m(\omega)x(\omega)d\omega. \tag{11}
\]

For convenience, the number of firms producing a manufactured good is a real number rather than restricted to be an integer. This number is determined by the zero-profit condition.\(^7\)

Zero profit for a manufacturing firm requires

\[
p_i x - fw - \beta x w = 0. \tag{12}
\]

We focus on a symmetric equilibrium in which all manufactured goods have the same price, output level, and number of producing firms. Since the measure of manufactured goods is one and all manufactured goods are symmetric, for simplicity, we drop the integration operator for the manufacturing sector. Equations (6)-(12) form a system of seven equations defining seven variables \( p_a, p_i, x, n, m, w, \) and \( E \) as functions of exogenous parameters. An equilibrium for a closed economy is a tuple \((p_a, p_i, x, n, m, w, E)\) satisfying equations (6)-(12). For the rest of the paper, a representative manufactured good is used as the numeraire. That is,

\[
p_i \equiv 1. \tag{13}
\]

3. Comparative statics

From equations (6)-(12), we can derive the following set of three equations defining three variables \( n, w, \) and \( p_a \) as functions of exogenous parameters:\(^8\)

\[
I_1 \equiv -f'(1 - \beta w) - \beta'fw = 0, \tag{14a}
\]

\[
I_2 \equiv 1 - \frac{y(z_1 + q)}{(w - p_a)(z_2 - z_1) - y\rho} - \frac{(1 - \alpha)p_a}{aw + (1 - \alpha)p_a} = 0, \tag{14b}
\]

\[
I_3 \equiv \frac{aw}{\sigma(1 - \beta w)^2} - (1 - \alpha)\left[ L - \frac{f}{\sigma(1 - \beta w)^2} \right]p_a = 0. \tag{14c}
\]

Partial differentiation of equations (14a)-(14c) with respect to \( n, w, p_a, L, \rho, \alpha, \) and \( q \) yields

---

\(^7\) See Zhang (2007) and Liu and Wang (2010) for examples of models in which firms engage in Cournot competition with free entry.

\(^8\) The derivation of (14a)-(14c) is as follows. First, equation (14a) comes from plugging the value of \( x \) from equation (12) into equation (8). Second, equation (14b) comes from plugging the value of \( m \) from equation (7), the value of \( x \) from equation (12) into equation (9), then plugging the resulting value of \( E \) into equation (6). Third, equation (14c) comes from dividing equation (10) by equation (11) and plugging the value of \( m \) from equation (7) and the value of \( x \) from equation (12) into the resulting equation.
\[
\begin{pmatrix}
\frac{\partial r_1}{\partial n} & \frac{\partial r_1}{\partial w} & 0 \\
0 & \frac{\partial r_2}{\partial w} & \frac{\partial r_2}{\partial p_a} \\
\frac{\partial r_3}{\partial n} & \frac{\partial r_3}{\partial w} & \frac{\partial r_3}{\partial p_a}
\end{pmatrix}
\begin{pmatrix}
dn \\
dw \\
dp_a
\end{pmatrix} = -\begin{pmatrix}0 \\
0 \\
0
\end{pmatrix}
dL - \begin{pmatrix}0 \\
0 \\
0
\end{pmatrix}
dp - \begin{pmatrix}0 \\
0 \\
0
\end{pmatrix}
d\alpha - \begin{pmatrix}0 \\
0 \\
0
\end{pmatrix}
dq.
\]

Let $\Delta$ denote the determinant of the matrix of endogenous variables of (15). According to the correspondence principle (Samuelson, 1983, chap. 9), stability requires that $\Delta < 0$.

Population size is related to market size. The following proposition studies the impact of an increase in population size.

Proposition 1: An increase in population size increases the wage rate in the manufacturing sector and the price of the agricultural good, and manufacturing firms choose more advanced technologies. An individual is better off with a larger population size. 

Proof: Plugging the value of $x$ from equation (12) into a firm's second order condition for technology choice $f''(n) - \beta'(n)x < 0$ yields $-f''(1 - \beta w) - \beta'' f w < 0$. Partial differentiating (14a) yields $\frac{\partial r_1}{\partial n} = -f''(1 - \beta w) - \beta'' f w < 0$. Also, $\frac{\partial r_2}{\partial w} = f'\beta - \beta' f > 0$ because $f' > 0$ and $\beta' < 0$.

Applying Cramer’s rule on (15) yields

\[
\frac{dn}{dL} = -\frac{\partial r_1 \partial r_2 \partial r_3}{\partial w \partial p_a \partial L} / \Delta > 0,
\]

\[
\frac{dp_a}{dL} = -\frac{\partial r_1 \partial r_2 \partial r_3}{\partial n \partial w \partial L} / \Delta > 0,
\]

\[
\frac{dw}{dL} = \frac{\partial r_1 \partial r_2 \partial r_3}{\partial n \partial p_a \partial L} / \Delta > 0.
\]

From the above two expressions, to demonstrate that $\frac{dw}{dL} > \frac{dp_a}{dL}$, it is needed that $-\frac{\partial r_2}{\partial p_a} > \frac{\partial r_2}{\partial w}$, or $\frac{y(z_1+q)(z_2-z_1)}{[(w-p_a)(z_2-z_1)-y\rho]^2} + \frac{(1-\alpha)aw}{[aw+(1-\alpha)p_a]^2} > \frac{y(z_1+q)(z_2-z_1)}{[(w-p_a)(z_2-z_1)-y\rho]^2} + \frac{(1-\alpha)\alpha p_a}{[aw+(1-\alpha)p_a]^2}$. This requires that $w > p_a$, which is always valid. From equation (6), $\frac{L}{L-E}$ increases if the gap between $w$ and $p_a$ is larger. ■

To understand Proposition 1, other things equal, an increase in population size increases the market size for manufactured goods. Thus, manufacturing firms choose more advanced technologies because the higher fixed costs can be spread over a higher level of output and the
average cost is lower. When the average cost decreases, the price of manufactured goods should decrease because firms earn zero profits. Since the price of a manufactured good is normalized to one, a lower price of manufactured goods shows up as a higher wage rate because the wage rate actually is the ratio between the wage rate and the price of a manufactured good. Since the agricultural sector has constant returns to scale, the average cost of production in the agricultural sector in terms of labor units is a constant. The manufacturing sector has increasing returns and the average cost decreases with population size. Thus, an increase in population size will reduce the relative price of manufactured goods. A lower relative price of manufactured goods is the same as a higher relative price of the agricultural good. That is, the price of the agricultural good increases with population size. Since the rate of wage increase is higher than the rate of increase in the price of the agricultural good, the real wage rate is higher. Also, the probability of finding a job in the manufacturing sector increases. Those factors lead to the result that an individual is better off with a higher population.

A higher discount rate means that an individual is less patient. Citizens in different countries may have different discount rates as shown in different saving rates among countries. The following proposition studies the impact of a change in the discount rate.

Proposition 2: An increase in the discount rate decreases the wage rate in the manufacturing sector and firms choose less advanced technologies. The impact on the price of the agricultural good is ambiguous.

Proof: Applying Cramer’s rule on (15) yields

\[
\frac{dn}{d\rho} = \frac{\partial r_1 \partial r_2 \partial r_3}{\partial w \partial \rho \partial p_a} / \Delta < 0, \\
\frac{dw}{d\rho} = -\frac{\partial r_1 \partial r_2 \partial r_3}{\partial n \partial \rho \partial p_a} / \Delta < 0, \\
\frac{dp_a}{d\rho} = \frac{\partial r_2 (\partial r_1 \partial r_3 - \partial r_1 \partial r_3 / \partial w \partial n)}{\partial \rho / \partial \rho / \partial \rho / \Delta}.
\]

Since the sign of \( \frac{\partial r_1 \partial r_2}{\partial n \partial w} - \frac{\partial r_1 \partial r_2}{\partial w \partial n} \) is ambiguous, the sign of \( \frac{dp_a}{d\rho} \) is ambiguous.

The intuition behind Proposition 2 is as follows. When the discount rate increases, an individual is less concerned with the future and the incentive to shirk is larger. Other things equal, the number of individuals employed in the manufacturing sector decreases. Through the non-
shirking constraint, the wage rate in the manufacturing sector decreases. A lower wage rate reduces a manufacturing firm’s incentive to adopt more advanced technologies because the marginal benefit from adopting more advanced technologies decreases. From equation (14b), when the discount rate increases, there are two effects on the price of the agricultural good. First, the direct effect (through \( \rho \) itself) is that it causes the price of the agricultural good to decrease. Second, the indirect effect (through \( w \), which is affected by \( \rho \) in equilibrium) is that a change in the discount rate causes the wage rate to decrease and a lower wage rate tends to cause the price of the agricultural good to increase. Without imposing additional structure on the model, it is unclear which effect dominates. Thus, the impact of an increase in the discount rate on the price of the agricultural good is ambiguous.

During the process of industrialization, the percentage of income spent on manufactured goods increases. The following proposition studies the impact of an increase in the percentage of income spent on the agricultural good.

Proposition 3: An increase in the percentage of income spent on the agricultural good decreases the wage rate in the manufacturing sector and firms choose less advanced technologies. Also, the price of the agricultural good decreases.

Proof: Applying Cramer’s rule on (15) yields

\[
\frac{dn}{\Delta} = -\frac{\partial r_1 \partial r_2 \partial r_3}{\partial \rho_a \partial \Delta / \partial \alpha} < 0,
\]

\[
\frac{dp_a}{\Delta} = -\frac{\partial r_1 \partial r_2 \partial r_3}{\partial n \partial \omega \partial \alpha} / \Delta < 0,
\]

\[
\frac{dw}{\Delta} = \frac{\partial r_1 \partial r_2 \partial r_3}{\partial n \partial \rho_a \partial \alpha} / \Delta < 0. \]

The intuition behind Proposition 3 is as follows. A decrease in the percentage of income spent on manufactured goods is like a decrease in population size. As market size for manufactured goods is smaller, manufacturing firms adopt less advanced technologies, and the real wage rate is lower. Since the relative cost of producing manufactured goods increases, the relative price of manufactured goods should increase. With the price of manufactured goods normalized to one and the price of the agricultural good thus measures the inverse of the relative price of manufactured goods to the agricultural good, a higher relative price of manufactured goods is the same as a lower price of the agricultural good.
With different cultures and institutions, countries differ in their labor market efficiencies. An increase in labor market efficiency can be captured by either an increase in $z_2$ or a decrease in $z_1$. The following proposition studies the impact of a change in labor market efficiency through an increase in $q$. A higher value of $q$ means that labor market efficiency decreases. Results will be the same if $z_1$ increases or $z_2$ decreases.

Proposition 4: A decrease in labor market efficiency leads manufacturing firms to adopt less advanced technologies and the wage rate in the manufacturing sector decreases. Impact on the price of the agricultural good is ambiguous.

Proof: Applying Cramer’s rule on (15) yields

$$\frac{dn}{dq} = \frac{\partial r_1 \partial r_2 \partial r_3}{\partial w \partial z_2 \partial p_a} / \Delta < 0,$$

$$\frac{dw}{dq} = \frac{-\partial r_1 \partial r_2 \partial r_3}{\partial n \partial z_2 \partial p_a} / \Delta < 0,$$

$$\frac{dp_a}{dq} = \frac{\partial r_2}{\partial q} \left( \frac{\partial r_1 \partial r_3}{\partial n \partial w} - \frac{\partial r_1 \partial r_3}{\partial w \partial n} \right) / \Delta.$$

Since the sign of $\frac{\partial r_1 \partial r_3}{\partial n \partial w} - \frac{\partial r_1 \partial r_3}{\partial w \partial n}$ is ambiguous, the sign of $\frac{dp_a}{dq}$ is ambiguous. ■

The intuition behind Proposition 4 is as follows. When the exogenous job separation rate increases, other things equal, the number of individuals employed in the manufacturing sector decreases. Through the non-shirking constraint, the wage rate in the manufacturing sector decreases. When the wage rate is lower, a firm’s incentive to adopt a more advanced technology will be lower because the marginal benefit from adopting a more advanced technology is smaller. When the exogenous job separation rate increases, the direct effect through $q$ is that it causes the price of the agricultural good to decrease while the indirect effect causes the price of the agricultural good to increase through a lower wage rate. Overall, an increase in the exogenous job separation rate has an ambiguous effect on the price of the agricultural good.

A country has a comparative advantage in producing manufactured goods if the relative price of manufactured goods to that of the agricultural good is lower. Since the price of a manufactured good is normalized to one in this model, a country has a comparative advantage in manufacturing if the price of the agricultural good is higher. From Proposition 1, a country with a larger population size will have a comparative advantage in producing manufactured goods. With
the opening of international trade, this country will export manufactured goods. From Proposition 4, an improvement in the level of labor market efficiency of a country does not necessarily increase a country’s comparative advantage in producing manufactured goods.

4. The impact of international trade

In this section, we study the impact when the home country opens trade with the foreign country. Foreign variables may carry an asterisk mark. For example, a foreign manufacturing firm’s technology is denoted by \( n^* \). The foreign country is assumed to have access to the same set of technologies as the home country. The two countries may differ in other aspects, such as population size, labor market efficiency, percentage of income spent on the agricultural good, and the degree of patience. With the opening of international trade, prices of the agricultural good and manufactured goods will be the same in the two countries because markets are integrated and there is no transportation cost.

4.1. Equilibrium conditions with international trade

Market for manufactured goods in the two countries are integrated. Like the derivation of equation (7), a domestic manufacturing firm’s optimal choice of output yields

\[
p_l \left( 1 - \frac{x}{\sigma (mx + m'x')} \right) = \beta(n)w.
\]

(16)

For equation (16), when the number of foreign firms is zero, this equation degenerates to equation (7) which is the equation for a domestic manufacturing firm’s optimal choice of output under autarky.

Similarly, a foreign manufacturing firm’s optimal choice of output yields

\[
p_l \left( 1 - \frac{\bar{x}}{\bar{\sigma} (\bar{m}x + \bar{m}'x')} \right) = \beta(n^*)\bar{w}.
\]

(16*)

For the world market for the agricultural good, domestic income is \( wE + p_a(L - E) \), foreign income is \( w^*E^* + p_a(L^* - E^*) \), and total world demand for the agricultural good is \( \alpha[wE + p_a(L - E)] + \alpha^*[w^*E^* + p_a(L^* - E^*)] \). In the home country, the number of workers employed in the agricultural sector is \( L - E \) and the counterpart in the foreign country is \( L^* - E^* \). Thus, the value of the total supply of the agricultural good in the world is \( p_a(L - E + L^* - E^*) \).

The clearance of the world market for the agricultural good requires

\[
\alpha[wE + p_a(L - E)] + \alpha^*[w^*E^* + p_a(L^* - E^*)] = p_a(L - E + L^* - E^*). \quad (17)
\]
For the world market for manufactured goods, total world demand for manufactured goods is 
\((1 - \alpha)[wE + p_a(L - E)] + (1 - \alpha^*)[w^*E^* + p_a(L^* - E^*)]\) and total value of world supply is 
\(\int_0^1 p_l(\omega)[mx + m^*x^*]d\omega\). The clearance of the world market for manufactured goods requires 
\((1 - \alpha)[wE + p_a(L - E)] + (1 - \alpha^*)[w^*E^* + p_a(L^* - E^*)]\)
\(= \int_0^1 p_l(\omega)[mx + m^*x^*]d\omega. \tag{18}\)

Like equation (6), the non-shirking constraint for a foreign worker is 
\(w^* - p_a = \frac{y(\sigma + 1) + L^*}{(z_2 - z_1)} \tag{6*}\)

Like equation (8), a foreign manufacturing firm’s optimal choice of technology yields 
\(f'(n^*) + \beta'(n^*)x^* = 0. \tag{8*}\)

Like equation (9), the level of employment in the manufacturing sector in the foreign country is the sum of demand from foreign manufacturing firms:
\(E^* = \int_0^1 m^*[f(n^*) + \beta(n^*)x^*]d\omega. \tag{9*}\)

Like equation (12), zero-profit for a foreign manufacturing firm requires 
\(p_lx^* - f(n^*)w^* - \beta(n^*)x^*w^* = 0. \tag{12*}\)

With the opening of international trade, equations (6), (8), (9), and (12) are still valid. Equations (6), (6*), (8), (8*), (9), (9*), (12), (12*), (16), (16*), (17), and (18) form a system of twelve equations defining twelve variables \(p_a, p_l, x, x^*, m, m^*, n, n^*, w, w^*, E, E^*\) as functions of exogenous parameters. An equilibrium with international trade is a tuple \((p_a, p_l, x, x^*, m, m^*, n, n^*, w, w^*, E, E^*)\) satisfying equations (6), (6*), (8), (8*), (9), (9*), (12), (12*), (16), (16*), (17), and (18).

4.2. Properties of the equilibrium with international trade

The following proposition shows that the opening of international trade induces manufacturing firms in different countries to choose the same technology.

Proposition 5: With the opening of international trade, \(n = n^*\).

Proof: From (16) and (16*), we get
\(mx + m^*x^* = \frac{f(n)w}{\sigma(1 - \beta(n)w)^2} \tag{19}\)
\(mx + m^*x^* = \frac{f(n^*)w^*}{\sigma[1 - \beta(n^*)w^*]^2}. \tag{19*}\)
From equation (8), \( x = \frac{-f'/\beta'}{f'/\beta f'}. \) Plugging this value of \( x \) into equation (12) yields \( w = \frac{f(n)w}{(1-\beta(n)w)^2} = \frac{(\beta f'-f\beta')f'}{\sigma f'\beta^t}. \) (20)

Similarly, by using (8*) and (12*), it can be shown that
\[
\Phi^* \equiv \frac{f(n^*)w^*}{\sigma[1-\beta(n^*)w^*)]^2} = \frac{[\beta(n^*)f'(n^*)-f(n^*)\beta'(n^*)]f'(n^*)}{\sigma f(n^*)\beta'(n^*)\beta(n^*)}. \quad (20^*)
\]

The right-hand side of (20) is solely a function of \( n \) and the right-hand of (20*) is solely a function of \( n^* \). If the right-hand side of (20) is a monotonic function of \( n \) and the right-hand of (20*) is a monotonic function of \( n^* \), then the two countries will choose the same level of technology in equilibrium. Differentiating (20) with respect to \( n \) yields \( \frac{\partial \Phi}{\partial n} = \frac{\partial \Phi}{\partial w} \frac{\partial w}{\partial n} + \frac{\partial \Phi}{\partial n}. \) Since \( \frac{\partial \Phi}{\partial w} > 0, \frac{\partial w}{\partial n} > 0, \) and \( \frac{\partial \Phi}{\partial n} = 0 \) from equation (8), it is clear that \( \frac{\partial \Phi}{\partial n} > 0. \) That is, \( \Phi \) is a monotonic function of \( n. \) Similarly, \( \Phi^* \) is a monotonic function of \( n^*. \) Thus, \( n = n^*. \)

Proposition 5 can be checked with special functional forms such as \( f(n) = n^\varphi \) and \( \beta(n) = n^{-\chi}, \) where \( \varphi \) and \( \chi \) are positive constants. With those functional forms, it can be shown that the right-hand side of (20) is an increasing function of \( n \) and the right-hand of (20*) is an increasing function of \( n^*. \) Since the two countries choose the same technology in the manufacturing sector, the wage rate in the manufacturing sector in the two countries will be equal after trade: \( w = w^*. \) With equal wage rate in the manufacturing sector, manufacturing firms in the two countries will produce the same level of output: \( x = x^*. \) This result that countries have the same wage rate after the opening of international trade is like the factor price equalization theorem in a Heckscher-Ohlin model. International trade leads to the equalization of prices of goods. Since countries are assumed to have the same production technologies in a Heckscher-Ohlin model, equalization of goods prices leads to the equalization of wage rate in different countries even though countries differ in factor endowments.

Technology choice could be a source of firm heterogeneity, as studied in Yeaple (2005) in which there is full employment. This model is different from Yeaple (2005) in which dual labor market is absent and all firms offer the same wage. More importantly, this model has a continuum of technologies while in Yeaple (2005) there are only two technologies. As illustrated in Gans (1998), the assumption of the existence of a continuum of technologies is less likely to lead to the
existence of multiple equilibria and thus firm heterogeneity compared with the assumption of the existence of only two technologies.

From the system of equations defining the equilibrium with international trade, we can derive the following system of equations of three equations defining three variables $n$, $w$, and $p$:\(^9\)

\[-f'(1 - \beta w) - \beta'f w = 0, \quad (21a)\]

\[\left[aw + (1 - \alpha)p_a\right]\left[1 - \frac{yL(z_1+q)}{(w-p_a)(z_2-z_1)-y}\right] - (1 - \alpha)p_aL \]

\[+[\alpha^*w + (1 - \alpha^*)p_a]\left[1 - \frac{yL'(z_1+q^*)}{(w-p_a)(z_2-z_1)-y\rho}\right] - (1 - \alpha^*)p_aL = 0, \quad (21b)\]

\[\frac{f}{\sigma(1-\beta w)^2} - \left[L - \frac{yL(z_1+q)}{(w-p_a)(z_2-z_1)-y}\right] - \left[L^* - \frac{yL'(z_1+q^*)}{(w-p_a)(z_2-z_1)-y\rho}\right] = 0. \quad (21c)\]

Now suppose the foreign country differs from the home country only in terms of the discount rate.\(^10\) When countries differ only in one aspect, such as population size, the analysis will be similar. Without loss of generality, assume that $\rho < \rho^*$. That is, residents in the home country are more patients than those in the foreign country. Let $\tau$ denote a positive number. When countries differ only in the discount rate, equations (21a)-(21c) simplify to the following equations (22a)-(22c) with $\tau = 1$:

\[\Omega_1 \equiv -f'(1 - \beta w) - \beta'f w = 0, \quad (22a)\]

\[\Omega_2 \equiv 1 - \frac{yL(z_1+q)}{(w-p_a)(z_2-z_1)-y\rho} + \tau \left[1 - \frac{yL'(z_1+q^*)}{(w-p_a)(z_2-z_1)-y\rho}\right] - \frac{(1-\alpha)(1+\tau)p_a}{aw (1-\alpha)p_a} = 0, \quad (22b)\]

\[\Omega_3 \equiv \frac{afw}{\sigma(1-\beta w)^2} - (1 - \alpha)p_a\left[L + \tau L - \frac{f}{\sigma(1-\beta w)^2}\right] = 0. \quad (22c)\]

Partial differentiation of equation (6) and an application of the implicit function theorem reveal that $\frac{\partial E}{\partial \rho} < 0$. If $\rho < \rho^*$, from equations (6) and (6*), then $E > E^*$ since other variables in the two equations are the same for the two countries. That is, a country with more patient citizens has a higher percentage of workers employed in the manufacturing sector. This is possible through the adjustment of the number of workers employed in the agricultural sector in the two countries. That is, when two countries have the same population size, the country with more patient citizens will have a lower number of workers employed in the agricultural sector. Since all manufacturing firms have the same level of output and the same technology, a country with a higher employment in the

---

\(^9\) The derivation of the three equations is as follows. First, equation (21a) is the same as (14a). Second, plugging the value of $E$ from equation (6) and the value of $E^*$ from equation (6*) into equation (17) yields equation (21b). Third, plugging the value of $E$ from equation (6), the value of $E^*$ from equation (6*), and the value of $x$ from equation (12) into equation (19) yields equation (21c).

\(^10\) If countries differ only in cost of effort, the analysis will be similar.
manufacturing sector has a higher number of manufacturing firms: \( m > m^* \). That is, a country with a lower discount rate will have a higher number of manufacturing firms.

Partial differentiation of equations (22a) - (22c) with respect to \( n, w, p_a, \) and \( \tau \) yields

\[
\begin{bmatrix}
\frac{\partial \Omega_1}{\partial n} & \frac{\partial \Omega_1}{\partial w} & 0 \\
0 & \frac{\partial \Omega_2}{\partial w} & \frac{\partial \Omega_2}{\partial p_a} \\
\frac{\partial \Omega_3}{\partial n} & \frac{\partial \Omega_3}{\partial w} & \frac{\partial \Omega_3}{\partial p_a}
\end{bmatrix}
\begin{bmatrix}
\frac{dn}{dt} \\
\frac{dw}{dt} \\
\frac{dp_a}{dt}
\end{bmatrix}
= -\begin{bmatrix}
0 \\
\frac{\partial \Omega_2}{\partial \tau} \\
\frac{\partial \Omega_3}{\partial \tau}
\end{bmatrix}
\frac{d\tau}{dt}.
\]

(23)

Let \( \Delta_\Omega \) denote the determinant of the coefficient matrix of endogenous variables of (23). Stability requires that \( \Delta_\Omega < 0 \).

A comparison of equations (14a) -(14c) with equations (22a) -(22c) reveals the following. The opening of international trade can be captured by an increase in \( \tau \): without international trade, \( \tau = 0 \); with international trade, \( \tau = 1 \). The following proposition studies the impact of the opening of international trade.

Proposition 6: With the opening of international trade, manufacturing firms adopt more advanced technologies, and the wage rate in the manufacturing sector and the price of the agricultural good increase.

Proof: By using (8), it can be shown that \( \frac{d}{dn} \frac{f}{(1-\beta w)^3} < 0 \). That is, \( \frac{\partial \Omega_3}{\partial n} < 0 \).

Applying Cramer’s rule on (23) yields

\[
\frac{dn}{d\tau} = \frac{\frac{\partial \Omega_1}{\partial w} \left( \frac{\partial \Omega_2}{\partial \tau} \frac{\partial \Omega_3}{\partial p_a} - \frac{\partial \Omega_3}{\partial \tau} \frac{\partial \Omega_2}{\partial p_a} \right) / \Delta_\Omega > 0,
\]

\[
\frac{dw}{d\tau} = \frac{\frac{\partial \Omega_1}{\partial n} \left( \frac{\partial \Omega_2}{\partial \tau} \frac{\partial \Omega_3}{\partial p_a} - \frac{\partial \Omega_3}{\partial \tau} \frac{\partial \Omega_2}{\partial p_a} \right) / \Delta_\Omega > 0,
\]

\[
\frac{dp_a}{d\tau} = \left( \frac{\partial \Omega_1}{\partial n} \frac{\partial \Omega_2}{\partial \tau} \frac{\partial \Omega_3}{\partial w} - \frac{\partial \Omega_1}{\partial n} \frac{\partial \Omega_2}{\partial \tau} \frac{\partial \Omega_3}{\partial w} - \frac{\partial \Omega_1}{\partial n} \frac{\partial \Omega_2}{\partial \tau} \frac{\partial \Omega_3}{\partial w} \right) / \Delta_\Omega > 0.
\]

The intuition behind Proposition 6 is as follows. The source of the gains from trade is the existence of increasing returns in the manufacturing sector. The opening of international trade increases market size for manufacturing firms. A larger market size leads to a higher level of output and manufacturing firms adopt more advanced technologies. Thus, average cost of producing a manufactured good decreases and the wage rate in the manufacturing sector is higher. When the average cost of producing a manufactured good decreases and the cost of producing the agricultural
good does not change, the relative cost of producing the agricultural good increases and the relative price of the agricultural good increases.

While a higher wage rate in the manufacturing sector increases income and thus welfare, a higher price of the agricultural good can reduce welfare because it could induce a reduction in real purchasing power. Proposition 6 shows that the opening of international trade can have an ambiguous effect on a country’s welfare. In addition to the effects on prices and the wage rate studied in Proposition 6, one extra effect from international trade is how international trade affects the distribution of workers between the two sectors. Overall, the opening of international trade on a country’s national welfare can be ambiguous, as in Bulow and Summers (1986). In the special case that countries are identical in all aspects, the opening of international trade always increases social welfare. The reasoning is as follows. When countries are identical, by comparing equations (22a)-(22c) with $\rho = \rho^*$ with equations (14a)-(14c), the impact of international trade is the same as domestic population increase. From Proposition 1, we know that social welfare increases with population size. If countries are not identical but similar sufficiently, by continuity, they still benefit from trade.

5. Conclusion

The interaction between technology choice and the opening of international trade affects a country’s unemployment rate. In this paper, we have studied the impact of the opening of international trade in a general equilibrium model with the presence of dual labor markets. We have established the following results analytically. First, in a closed economy, an increase in the level of labor market efficiency or an increase in population size induces firms to adopt more advanced technologies and the wage rate in the manufacturing sector increases. Second, with the opening of international trade, countries will have the same wage rate, firm size, technology in the manufacturing sector even though they differ in some aspects such as the degree of patience of citizens. With the presence of a continuum of technologies to produce each manufactured good, technology choice will not lead to firm heterogeneity in the sense that firms choose different technologies with different marginal costs of production. Finally, the opening of international trade on social welfare may be ambiguous because both the wage rate in the manufacturing sector and the price of the agricultural good increase.
In this model, different varieties of manufactured goods exhibit constant elasticity of substitution. This assumption may not be supported by empirical evidence and one question is whether the results here may be sensitive to this assumption. We do not believe so and the reasoning is as follows. First, Bulow and Summers (1986) do not employ this assumption and get similar results for the impact of international trade. Second, while there is a continuum of manufactured goods in our model, since all goods are symmetric, only one manufacturing good is needed. The reason of having a continuum of goods in this model is to eliminate a manufacturing firm’s market power in the labor market. Otherwise, it can be viewed there is only one manufactured good in this model.

There are some possible generalizations and extensions of the model. First, in this model, labor is the only factor of production. Additional factors of production such as capital and land may be incorporated into the model. Capital can be modeled as fixed costs in the manufacturing sector and land can be employed in the agricultural sector. How factor endowments affect the pattern of international trade can be addressed and country’s comparative advantage in producing manufactured goods could be affected by its endowment of capital. Second, technological spillovers may be introduced into the model. With technological spillovers, market failure may result. The incorporation of technological spillovers could be used to address issues like infant industry protection. Finally, in this model, a manufacturing chooses technologies from a continuum of existing technologies. Incorporation of endogenous development of new technologies may be an interesting avenue for future research.

References


