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The Shine Beneath: Foreign Exchange Intervention in Resource-rich Economies

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Abstract

We propose a dynamic general equilibrium model to study the optimal reaction to terms of trade shocks when international financial markets are imperfect and the composition of capital flows affects the exchange rate determination. These elements allow us to showcase the interactions between commodity prices and international financial market inefficiencies. Positive commodity price shocks will generate a real over-appreciation of the currency and an inefficiently large shift of factors between the tradable and non-tradable sectors. We study the welfare implications of foreign exchange intervention through optimal simple rules and find support for leaning-against-the-wind foreign exchange intervention. Our setup, allows us to rationalize the reserve accumulation episodes commonly observed during periods of high commodity prices in resource-rich economies.

Key words: Open Economy Macroeconomics, Foreign exchange intervention, Terms of trade.

JEL Classification: F41, E32, D58, F31, G15, O24.

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1. Introduction

Small open economies (SOE) have been frequently exposed to large commodity price shocks which explain a considerable share of business cycle fluctuations[1]. Sustained shifts in these prices often generate episodes of real appreciations, fall in unemployment rates, widespread growth, fiscal surpluses, current account deficits, and reserve accumulation.

We present the case of Peru as an example. Peru is a SOE with a large commodity-exporting sector and a central bank that actively intervenes to manage capital flows.[2] In 2020 it was the second largest copper and silver producer. It is also a top 10 producer of gold, lead and zinc. Panels (a), (b), (c), and (d) present the following variables: exports, the trade balance, nominal exchange rate, and copper price[3]. A rise in copper prices leads to an improvement in exports and the trade balance, and a currency appreciation (as a fall in the exchange rate).

In a standard small open economy model such as Schmitt-Grohé and Uribe (2018), higher terms of trade generate an income effect that increases home agents' demand for non-tradable goods and leisure, raising wages and production costs. This, in turn, increases domestic prices producing a real exchange appreciation. We consider that this narrative leaves out three key elements. First, the adjustment in real exchange rates occurs through the nominal exchange rate as non-tradable prices adjust slowly, as shown by Hess and Shin (2010). Second, some imbalances arise during these episodes as shown by Benigno et al. (2015) who study 155 episodes of large capital inflows in seventy countries and find that a significant share of them end in a crisis. Moreover, when the shift of factors between tradable and non-tradable sectors is larger, a crisis becomes more likely. Third, policymakers tend to express their concern with these episodes and accumulate foreign reserves. Aizenman et al. (2012) and Castillo and Medina (2021) show that economies that actively manage their international reserves can insulate better from commodity price shocks affecting the long-run adjustment of the real exchange rate and its volatility. Benigno et al. (2015) also find that countries that accumulate foreign reserves during the boom, reduce the probability of facing a crisis.

In this paper, we propose a simple SOE model that connects the real side effects of terms of trade

[1] See Kose (2002), Fernández et al. (2018), Fernández et al. (2017) for a discussion.
[2] See Castillo (2015).
[3] We use copper price as a proxy for commodity prices.

shocks with the financial channels at play during these episodes. Introducing financial frictions can highlight how these shocks can cause inefficient dynamics. Our mechanism is closely related to Itskhoki and Mukhin (2021), Gabaix and Maggiori (2015), Cavallino (2019), and Montoro and Ortiz (2020) who model financial market imperfections to explain exchange rate dynamics. The last two papers also explore the role of foreign exchange intervention and reserve accumulation as a policy tool to manage large capital inflows and outflows.

In our setup, foreign trade and capital flows are settled in the foreign country baskets.[4] Thus, an increase in commodity prices leads to a higher net foreign assets position in the small open economy. Since these assets are priced in foreign baskets, domestic agents will shift their position to their desired one against financial intermediaries. These risk adverse intermediaries will introduce an endogenous premium, based in the exposure to relative price changes which induces an appreciation beyond the efficient level of the real exchange rate, which in turn affects the optimal allocation of resources between the tradable and non-tradable sectors and amplifies the impact of commodity price shocks in the economy[5]. We complement our setup by introducing a central bank that intervenes by absorbing the short/long position of intermediaries and smoothing the path of the real exchange rate.

Additionally, we study potential policy responses for a central bank. Following Catão and Chang (2013) and Chang and Catão (2013), we study the implementation of optimal simple rules, as in Schmitt-Grohe and Uribe (2007). When international financial markets are segmented, countries do not find instruments to diversify their exposure to international shocks such as terms-of-trade (ToT) shocks, breaking Backus and Smith's condition. A series of dilemmas emerge: How does imperfect risk diversification affect the path of the exchange rate? What is the role of the exchange rate regime? What impact does it have on the allocation of factors between the tradable and non-tradable sectors?

To model the real economy, we follow Ferrero and Seneca (2019) who present a model small open economy model in the spirit of Gali and Monacelli (2005) an add a commodity exporting sector that demands intermediate home economy goods. The authors use this model to evaluate the optimal monetary

[4]In a setup with money, this is equivalent to setting this operations in a hard currency. See Gopinath et al. (2020) for a discussion on dominant currencies.

[5]Other papers emphasizing financial frictions and misallocation of factors are García-Cicco et al. (2015), Akıncı (2013), and Özge Akinci (2021)

policy in the presence of price rigidities and terms of trade shocks.

Findings: Our results show that central banks can improve welfare and international risk-sharing by accumulating/consuming reserves when international financial markets are segmented, allowing countries to take better advantage of higher international prices of their exports. When shocks are transitory and domestic agents can only accumulate foreign assets in a hard currency, central banks can eliminate the effects of the exchange rate pressures through FXI.

Related Literature: Three strands of the literature are relevant for our research. The first one studies the effect of terms of trade in a SOE. This literature indicates that terms of trade (ToT) are an important source of cyclical fluctuation[6]. Mendoza (1991) and Kose (2002) show that ToT shocks drive short-term fluctuations, using a calibrated model. Other authors such as Schmitt-Grohé and Uribe (2018), Fernández et al. (2018) and Shousha (2016), analyze the business cycles of the 1990s in small open economies with a commodity-exporting sector.

On the real effects on the economy, Malakhovskaya and Minabutdinov (2014) estimate a model for Russia accounting explicitly for the increase in the oil export revenues. Fornero et al. (2015) emphasize the investment channel and the spillovers to non-commodity sectors. Medina et al. (2007) stress the impact of commodity price shocks on the current account dynamics in Chile and New Zealand. Fornero and Kirchner (2018) introduce a learning mechanism about the persistence of commodity price shocks to replicate several stylized facts of the commodity-exporting economy like Chile.

From an empirical point of view, Benigno and Fornaro (2014) and Reis (2013) show that there is an established connection between episodes of large capital inflows and movements of productive resources towards less competitive non-tradable sectors. Also, Giavazzi and Spaventa (2010) discuss whether episodes of large capital inflows during stable times set the stage for later crises, known in the literature as sudden stops. Broda (2004) finds that ToT shocks play a more prevalent role in economies with a fixed exchange rate than in the flexible ones at the business cycle frequency.

Second, as mentioned, we explore the financial channels in the transmission of shocks in an open economy model. Aoki et al. (2016) examines the role of the financial sector development and integration

[6]Particularly in the recession phases of emerging countries, a connection is established between episodes of large capital inflows and slowdowns in GDP growth, arguing that these new incomes would trigger a movement of resources towards sectors that are less productive.

to the international financial markets. Itskhoki and Mukhin (2021) study the effect of noise trader portfolio shocks in a segmented financial market as households only access bonds in their home currency and financial intermediaries charge a risk premium to absorb open currency positions. Gabaix and Maggiori (2015) present a similar setup where financial intermediaries face financial constraints, impeding perfect international risk sharing.

Finally, a handful of authors have explored the use of policy tools to deal with external shocks. Cavallino (2019) finds the optimal FXI policy and shows strong interactions between FXI and monetary policy. In a similar vein, Montoro and Ortiz (2020) show that exchange rate interventions can be used as stabilizing exchange rate policy and present a set of optimal simple rules. Additionally, Ostry et al. (2010) analyzes the inclusion of capital controls in their set of macroprudential policy tools recommended for small open economies.

Outline: The present paper is structured as follows: Section 2 presents an estimated model for Peru, a small open and resource-rich economy with a central bank that actively intervenes in FX markets. In Section 3, we present the theoretical model and defines the competitive equilibrium. In Section 4 we perform the model simulations and explore the use of optimal simple rules. Section 5 concludes.

2. Terms of Trade and Foreign Reserves: A SVAR model with block exogeneity

2.1. The setup

Consider a big economy modeled as an independent Vector Autoregressive system (VAR) and also a small open economy modeled as a Vector Autoregressive system with exogenous variables, which are basically the Terms of Trade (TOT). In this context, the big economy is represented for $t = 1, ..., T$ by

$$\mathbf{y}_t^{*\prime}\mathbf{A}_0^* = \sum_{i=1}^{p} \mathbf{y}_{t-i}^{*\prime}\mathbf{A}_i^* + \mathbf{w}_t^{\prime}\mathbf{D}^* + \varepsilon_t^{*\prime}, \tag{1}$$

where y_t^* is $n^* \times 1$ vectors of endogenous variables for the big economy; ε_t^* is $n^* \times 1$ vectors of structural shocks for the big economy ($\varepsilon_t^* \sim N(0, I_{n^*})$); $\widetilde{\mathbf{A}}_i^*$ and $\mathbf{A}_i^*$ are $n^* \times n^*$ matrices of structural parameters for $i = 0, \ldots, p$; $\mathbf{w}_t$ is a $r \times 1$ vector of exogenous variables; $\mathbf{D}^*$ is $r \times n$ matrix of structural parameters; p is the lag length; and, T is the sample size.

The small open economy (Peru) is represented by

$$\mathbf{y}_t'\mathbf{A}_0 = \sum_{i=1}^{p}\mathbf{y}_{t-i}'\mathbf{A}_i + \sum_{i=0}^{p}\mathbf{y}_{t-i}^{*\prime}\widetilde{\mathbf{A}}_i^* + \mathbf{w}_t'\mathbf{D} + \varepsilon_t', \tag{2}$$

where y_t is $n \times 1$ vector of endogenous variables for the small economy; ε_t is $n \times 1$ vector of structural shocks for the domestic economy ($\varepsilon_t \sim N(0, I_n)$ and structural shocks are independent across blocks i.e. $E(\varepsilon_t \varepsilon_t^{*\prime}) = \mathbf{0_{n \times n^*}}$); $\mathbf{A}_i$ are $n \times n$ matrices of structural parameters for $i = 0, \ldots, p$; and, $\mathbf{D}$ is $r \times n$ matrix of structural parameters. The latter model can be expressed in a more compact form, so that

$$\begin{aligned}\begin{bmatrix} \mathbf{y}_t' & \mathbf{y}_t^{*\prime} \end{bmatrix} \begin{bmatrix} \mathbf{A}_0 & \mathbf{0} \\ -\widetilde{\mathbf{A}}_0^* & \mathbf{A}_0^* \end{bmatrix} &= \sum_{i=1}^{p} \begin{bmatrix} \mathbf{y}_{t-i}' & \mathbf{y}_{t-i}^{*\prime} \end{bmatrix} \begin{bmatrix} \mathbf{A}_i & \mathbf{0} \\ \widetilde{\mathbf{A}}_i^* & \mathbf{A}_i^* \end{bmatrix} \\ &\quad + \mathbf{w}_t' \begin{bmatrix} \mathbf{D} \\ \mathbf{D}^* \end{bmatrix} + \begin{bmatrix} \varepsilon_t' & \varepsilon_t^{*\prime} \end{bmatrix} \begin{bmatrix} I_n & \mathbf{0} \\ \mathbf{0} & I_{n^*} \end{bmatrix},\end{aligned}$$

The experiment design for the Peruvian economy is as follows. We include the Terms of Trade (TOT) in the exogenous block, and we consider a similar vector of variables as in Schmitt-Grohé and Uribe (2018), but we also include the Net International Reserves as an additional explanatory variable (see technical details in the appendix). Estimation results are shown in the next section.

2.2. Results

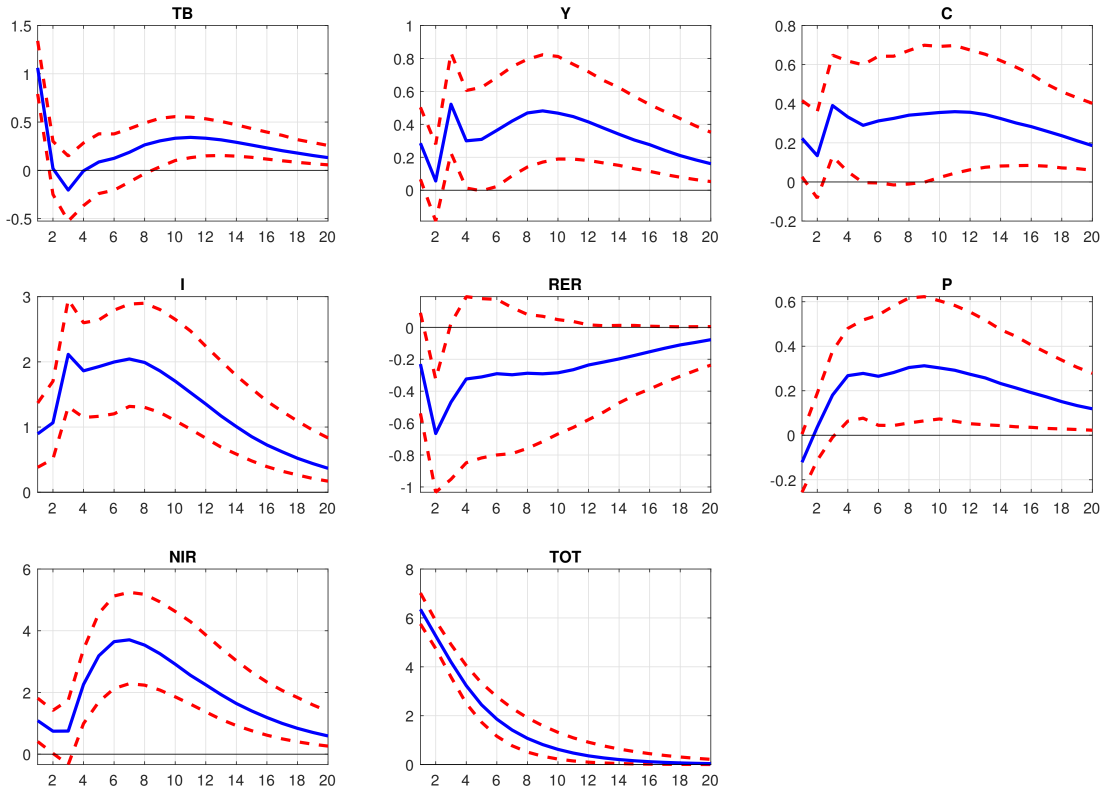

Figure 1: Responses of Peruvian variables after a TOT shock; median value (solid line) and 68% bands (dotted lines)

Results in Figure 1 show that an exogenous Terms of Trade shock produces a natural increase in the Trade Balance because of the price effect. Then, the shock produces a real and persistent appreciation, which triggers the accumulation of Net International Reserves, which is in part associated with Exchange Rate Intervention in the spot market. In the subsequent periods the shock produces the typical increase in the agreggate demand and its components, and also an initial negative effect in inflation. All in all, the empirical exercise for the Peruvian economy shows the crucial and significant role of net international reserves (NIR) accumulation in the transmission mechanism of the terms of trade shock.

3. The Model

We follow closely the model by Gali and Monacelli (2005) and Ferrero and Seneca (2019). There is a continuum of households who derive utility from consumption and leisure. Because of the segmented financial markets assumption, households will only consider the domestic interest rate in their maximization problem. We assume financial and real international markets only operate in bonds that pay one unit of the foreign country's consumption basket.

Figure 6 in the appendix shows a diagram of the model, in which we find FX dealers, the central bank, and households. Domestic agents demand assets in local baskets and own capital, while the central bank and financial intermediaries will hold a position in foreign baskets.[7]

Here we reproduce the main features of the model.

3.1. Households

Households solve:

$$\mathbb{E}_0\left[\sum_{t=0}^{\infty}\beta^t\frac{C_t^{1-\gamma_c}}{1-\gamma_c}-\frac{L_t^{1+\chi}}{1+\chi}\right] \tag{3}$$

Subject to:

$$B_{t+1}=R_{t-1}B_t+W_tL_t-P_tC_t+\Gamma_{dt}+\Psi_{Ct}+\Gamma_{CBt} \tag{4}$$

where B is the level of risk free one period bonds, W is the nominal wage and C represents the consumption basket. Γ_d and Γ_{CB} stand for the profits of the financial intermediaries and the central bank, respectively. Finally, Ψ_C represents the dividends from the ownership of commodity sector.

The consumption basket is given by:

$$C_t\equiv\left[(\gamma)^{1/\varepsilon_H}\left(C_t^H\right)^{\frac{\varepsilon_H-1}{\varepsilon_H}}+(1-\gamma)^{1/\varepsilon_H}\left(C_t^F\right)^{\frac{\varepsilon_H-1}{\varepsilon_H}}\right]^{\frac{\varepsilon_H}{\varepsilon_H-1}}, \tag{5}$$

where supra-indexes H and F stand for home and foreign produced goods, respectively. The parameter γ regulates the home bias. Consumer price index, under these preference assumptions, is determined by the following condition:

$$P_t\equiv\left[\gamma\left(P_t^H\right)^{1-\varepsilon_H}+(1-\gamma)\left(P_t^F\right)^{1-\varepsilon_H}\right]^{\frac{1}{1-\varepsilon_H}} \tag{6}$$

where P_t^H and P_t^F denote the price level of the home-produced and imported goods, respectively. Home

[7] As Itskhoki and Mukhin (2021) points out, this assumption can be relaxed to allow for the home economy agents to hold assets in foreign baskets (currency), as long as the portfolio composition of the financial account funds differs from the desired domestic portfolio, leading to a change in the portfolio of financial intermediaries.

goods index is defined as follows[8]:

$$P_t^H \equiv \left[\int_0^n P_t^H(z)^{1-\varepsilon} dz \right]^{\frac{1}{1-\varepsilon}} \tag{7}$$

Consumption decisions and the supply of labour. The condition characterizing the optimal allocation of domestic consumption is given by the following equation:

$$U_{Ct} = \beta E_t \left\{ U_{C,t+1} R_t \frac{P_t}{P_{t+1}} \right\} \tag{8}$$

The first-order conditions that determine the supply of labour are characterized by the following equation:

$$-\frac{U_{Lt}}{U_{Ct}} = \frac{W_t}{P_t} \tag{9}$$

where $\frac{W_t}{P_t}$ denotes real wages. In a competitive labour market, the marginal rate of substitution equals the real wage.

Demand for each type of good is given by:

$$C_t^H = \gamma \left(\frac{P_t^H}{P_t} \right)^{-\varepsilon_H} C_t \tag{10}$$

$$C_t^F = (1-\gamma) \left(\frac{P_t^F}{P_t} \right)^{-\varepsilon_H} C_t \tag{11}$$

Foreign economy follows a similar pattern for home goods demand:

$$C_t^{*,H} = (1-\gamma^*) \left(\frac{P_t^H}{S_t P_t^*} \right)^{-\varepsilon_F} C_t^* \tag{12}$$

We take the small economy assumption, thus domestic economy demand will not affect international

[8]In the case of unitary elasticity we assume:

$$C_t \equiv \frac{\left(C_t^H\right)^\gamma \left(C_t^F\right)^{1-\gamma}}{\gamma^\gamma (1-\gamma)^{(1-\gamma)}}; \qquad P_t = \left(P_t^H\right)^\gamma \left(P_t^F\right)^{1-\gamma}$$

prices.

$$S_t P_t^* = P_t^F \tag{13}$$

Domestic households will only access domestic bonds and cannot directly trade assets with foreign households. Financial markets will operate through intermediaries who operate as market makers. They provide a service to home and foreign households, absorbing their portfolio positions with their own wealth. For simplicity, we assume they follow a zero-capital carry trade strategy. Intermediaries maximize the following CARA utility of the real return on bond investments (in units of the national consumer good):

$$\max_{d_{t+1}^*} \mathbb{E}_t \left\{ -\frac{1}{\omega} \exp\left(-\omega \frac{\tilde{R}_{t+1}^*}{R_t} d_{t+1}^* \right) \right\}$$

FX intermediaries' return is given by:

$$\tilde{R}_{t+1}^* = R_t^* \frac{S_{t+1}}{S_t} - R_t$$

Where $\mathbb{E}_t$ is the rational expectations operator, $\omega \geq 0$ is the absolute risk aversion coefficient and $\tilde{R}_{t+1}^*$ is the return of the carry trade in domestic baskets. Note that the open position absorbed by each dealer (d^*) will be endogenous, since it will be derived from the domestic demand of households (through current account flows), the carry trade and the central bank's intervention. Finally, S_t is the relative price between foreign and domestic baskets.

Dealers will quote a price for each equilibrium position they absorb. Since trading against all agents occurs simultaneously, the portfolio equation can be used to obtain the exchange rate at which currency providers are willing to mirror the position of other agents. Following Campbell et al. (2002) and Itskhoki and Mukhin (2021), the portfolio solution of financial intermediaries under the CARA utility function, which gives a modified UIP equation:

$$E(s_{t+1}) = s_t + i_t^* - i_t - \omega\sigma^2 d^*$$

where $i_t \equiv logR_t$ and $i_t^* \equiv logR_t^*$ and $s \equiv logS$.

Now we can use the zero capital dealers' position given by $S_{t-1}D_t^* = D_t$ so that the dealers' profits are:

$$\Gamma_t^d = R_{t-1}(S_t - S_{t-1})D_t^* + \left(R_{t-1}^* - R_{t-1}\right) S_t D_t^*$$

Firms. A continuum of z of intermediate firms exists. These firms operate in a perfectly competitive market and use the following linear technology:

$$Y_t^{int}\left(z\right) = A_t L_t\left(z\right) \tag{14}$$

$L_t\left(z\right)$ is the amount of labour demand from households, A_t is the level of technology.

These firms take as given the real wage, W_t/P_t, paid to households and choose their labour demand by minimising costs given the technology. The corresponding first order condition of this problem is:

$$L_t\left(z\right) = \frac{MC_t\left(z\right)}{W_t/P_t} Y_t^{int}\left(z\right)$$

where $MC_t\left(z\right)$ represents the real marginal costs in terms of home prices. After replacing the labour demand in the production function, we can solve for the real marginal cost:

$$MC_t\left(z\right) = (1 - \mu_H)\frac{W_t}{A_t} \tag{15}$$

Given that all intermediate firms face the same constant returns to scale technology, the real marginal cost for each intermediate firm z is the same, that is $MC_t\left(z\right) = MC_t$. Also, given these firms operate in perfect competition, the price of each intermediate good is equal to the marginal cost. Therefore, the relative price $P_t\left(z\right)/P_t$ is equal to the real marginal cost in terms of consumption unit (MC_t).

Price-Setting. Final goods producers purchase intermediate goods and transform them into differentiated final consumption goods. Therefore, the marginal costs of these firms equal the price of intermediate goods. These firms operate in a monopolistic competitive market, where each firm faces a downward-sloping demand function, given below. Furthermore, we assume that each period t final goods producers face an exogenous probability of changing prices given by $(1-\theta)$. Following Calvo (1983), we assume that this probability is independent of the last time the firm set prices and the previous price level. Thus, given a price fixed from period t, the present discounted value of the profits of firm z is given by:

$$\max_{\hat{P}^H} \sum_{k=0}^{\infty} \theta^k E_t \left[\mathcal{Q}_{t,t+k} \left(Y_{t+k}^H(j) \left\{ \hat{P}_{Ht} - (1-\mu_H) P_t MC_{t+k} \right\} \right) \right] \tag{16}$$

where $\mathcal{Q}_{t,t+k} = \beta^k \frac{U_{C,t+k}}{U_{Ct}}$ is the stochastic discount factor and $Y_{t,t+k}^H(j)$ is the demand for good j in $t+k$ conditioned to a fixed price from period t, given by:

$$Y_{t+k}^H(j) = \left(\frac{\hat{P}_t^H}{P_{t+k}^H} \right)^{-\varepsilon} Y_{t+k}^H \tag{17}$$

Regarding the subsidy μ_H, we the policy maker sets it constant to maximize the steady state welfare of the domestic economy as in Ferrero and Seneca (2019):

$$1 + \mu_H = \frac{\varepsilon}{\varepsilon - 1} \frac{s_c}{\xi_t} \tag{18}$$

Where $s_c = \frac{C_H}{Y_H}$ and $\xi_t = \frac{C_H + (1-\nu)^{-1} M}{Y_H}$. Each firm j chooses $\hat{P}_t^H(j)$ to maximise (16). The first order condition of this problem is:

$$\sum_{t=0}^{\infty} \theta^k E_t \left[\mathcal{Q}_{t,t+k} Y_{t+k}^H (P_{Ht+k})^{\varepsilon} \left\{ (1-\varepsilon)(\hat{P}_{Ht})^{-\varepsilon} + \varepsilon(1-\mu_H) P_{t+k} MC_{t+k} (\hat{P}_{Ht})^{-\varepsilon-1} \right\} \right] = 0 \tag{19}$$

We define, $\tilde{p}_t^H \equiv \frac{\hat{P}_t^H}{P_t^H}$

$$\sum_{t=0}^{\infty} P_t C_t \left(\beta\theta\right)^k E_t \left[\left((1-\gamma^*) \frac{C_{t+k}^*}{C_{t+k}} \mathcal{S}_{t+k} + \gamma \mathcal{S}_{t+k}^{1-\gamma} \right) (X_{t,k}^H)^{-\varepsilon} \left\{ t_{t+k}^H \tilde{p}_{Ht} X_{t,k}^H - \frac{\varepsilon}{\varepsilon - 1} (1-\mu_H) MC_{t+k} \right\} \right] = 0 \tag{20}$$

where: and:

$$X_{t,k}^H = X_{t+1,k-1}^H \frac{1}{\pi_{t+1}^H}, \ k \geq 0. \tag{21}$$

Solving for $\tilde{p}^H$ yields:

$$\tilde{p}_t^H = \frac{\sum_{t=0}^{\infty} (\beta\theta)^k E_t \left[(X_{t,k}^H)^{-\varepsilon} \frac{\varepsilon}{\varepsilon-1} (1-\mu_H) MC_{t+k} \right]}{\sum_{t=0}^{\infty} (\beta\theta)^k E_t \left[(X_{t,k}^H)^{1-\varepsilon} t_{t+k}^H \right]} = \frac{K_t}{F_t} \tag{22}$$

Now we set the recursive equations:

$$K_t = \frac{\varepsilon}{\varepsilon - 1} (1-\mu_H) \frac{MC_{t+k}}{t_{t+k}^H} + \beta\theta E_t \left(\frac{1}{\pi_{t+1}^H} \right)^{-\varepsilon} K_{t+1} \tag{23}$$

$$F_t = 1 + \beta\theta E_t \left(\frac{1}{\pi_{t+1}^H} \right)^{1-\varepsilon} K_{t+1} \tag{24}$$

The Calvo pricing assumption allows to write an expression for the home prices index:

$$P_t^H \equiv \left[\theta P_{H,t-1}^{1-\varepsilon} + (1-\theta) \left(\hat{P}_t^H \right)^{1-\varepsilon} \right]^{\frac{1}{1-\varepsilon}} \tag{25}$$

Dividing by P_t^H:

$$1 \equiv \left[\theta \left(\pi_t^H \right)^{\varepsilon-1} + (1-\theta) \left(\tilde{p}_t^H \right)^{1-\varepsilon} \right] \tag{26}$$

Solving for $\tilde{p}_t^H$, we obtain:

$$\tilde{p}_t^H = \left[\frac{1 - \theta \left(\pi_t^H \right)^{\varepsilon-1}}{1-\theta} \right]^{\frac{1}{1-\varepsilon}} \tag{27}$$

Regarding the subsidy, we assume the government can set a constant value for μ_H to fix the distortion caused by the assumption the market power in the steady-state. See appendix **??** for the derivation of the value. Domestic firms will sell at the same local currency price in both economies. This is also known as producer currency pricing (PCP).

From **?** the evolution of the price dispersion is given by:

$$Z_t = (1-\theta)\left[\frac{1-\theta\left(\pi_t^H\right)^{\varepsilon-1}}{1-\theta}\right]^{\frac{\varepsilon}{\varepsilon-1}} + \theta\left(\pi_t^H\right)^{\varepsilon} Z_{t-1} \tag{28}$$

Commodity Producers The production of commodities Y_t^C will use final goods as from the home economy as an input and exhibits a diminishing returns technology.

$$Y_t^C = A_{Ct} M_t^{\nu} \tag{29}$$

where A_{Ct} is the total factor productivity of the commodity sector technology and $\nu \in (0,1)$. Commodity producers take their sell price as given in the world markets, $P_t^C = S_t P_t^{C,*}$. The problem of the commodity producers is given by:

$$\max_{M_t} P_t^C Y_t^C - P_{Ht} M_t \tag{30}$$

subject to (29). The first order condition yields:

$$P_t^C A_{Ct} \nu M_t^{\nu-1} - P_{Ht} = 0 \tag{31}$$

and the profits from the commodity sector are given by:

$$\Psi_t^C = (1-\nu) P_t^C Y_t^C \tag{32}$$

Goods Market Clearing and Current Account. Following Ferrero and Seneca (2019), we assume the domestic economy only exports commodities, thus:

$$Y_{Ht} = C_{Ht} + M_t \tag{33}$$

For the current account, it is convenient to define net foreign assets as:

$$\mathcal{A}_t = S_{t-1}B_t^{cb,*} + S_{t-1}D_t^* - N_t \tag{34}$$

So, the current account is equivalent to:

$$CA_t = \mathcal{A}_{t+1} - \mathcal{A}_t \tag{35}$$

From the budget constraints in the model we obtain:

$$CA_t = NX_t + \left(\frac{S_t}{S_{t-1}}R_{t-1}^* - 1\right)\left(S_{t-1}B_t^{cb,*} + S_{t-1}D_t^*\right) - \left(R_{t-1} - 1\right)N_t \tag{36}$$

where

$$NX_t = P_t^C Y_t^C - P_t C_t \tag{37}$$

3.2. Central Bank and FXI

We assume the central bank follows a zero capital strategy, issuing bonds that pay one unit of the domestic basket and acquiring bonds that pay one unit of the foreign basket. By doing this, the central bank will absorb the relative price risk.

$$B_{t+1}^{cb} + S_t B_{t+1}^{cb,*} = 0 \tag{38}$$

The central bank conducts sterilized interventions in the form

$$B_{t+1}^{cb} + S_t B_{t+1}^{cb,*} + \Gamma_t^{cb} = R_{t-1}B_t^{cb} + S_t R_{t-1}^* B_t^{cb,*}$$

Central bank's profits are transferred back to households. We use the sterilized intervention condition to obtain:

$$\begin{aligned}\Gamma_t^{cb} &= R_{t-1} B_t^{cb} + S_t R_{t-1}^* B_t^{cb,*} \\ &= \left[\left(R_{t-1}^* - R_{t-1} \right) S_t B_t^{cb,*} + \left(S_t - S_{t-1} \right) R_{t-1} B_t^{cb,*} \right]\end{aligned}$$

Thus, the central bank's profits have a valuation component and a return spread component that will be transferred back to households to maintain a zero capital balance.

3.3. Optimal simple rules

Following Gali and Monacelli (2005), we propose optimal simple rules. In this case, we propose an intervention rules that responds to the Backus-Smith wedge deviation, following the LQ results of Cavallino (2019).

$$B_{t+1}^{cb,*} = \phi_{cb}^1(\lambda_t)$$

Where ϕ_{cb}^1 is the rule parameters, which is obtained by maximizing the unconditional welfare of the economy. To assess each type of policy we compute the unconditional household welfare in each policy as:

$$W = \mathbb{E}_0 \sum_{t=0}^{\infty} \beta^t U\left(c_t, l_t\right)$$

Since we emphasize the welfare gains relative to the non-intervention case ($NOFXI$) in percentage of steady-state welfare (W^{SS}), we compute the mean difference ($\Delta\mu(W)^r$) as:

$$\Delta\mu(W^r) = \frac{\mu(W^r) - \mu(W^{NOFXI})}{W^{SS}}$$

where r stands for either a policy responding to commodity prices shocks or to depreciations. We label the former as $PXST$, while the later as LAW, as it is shown in the following sections, figures and tables.

4. Results

4.1. Model Dynamics

The calibrated values are taken from Schmitt-Grohé and Uribe (2018) and Itskhoki and Mukhin (2021) which we present in Table 1. Figure 7 show the reaction of the economy to a commodity price shock. When the central bank does not intervene, we observe a strong real exchange rate appreciation and a strong expansion of consumption. Considering the financial channel, the positive effect in the current account generates a increase in the position of financial intermediaries. This creates a stronger appreciation than the 'efficient' one, reducing the shift of goods to the commodity sector as household agents expand their demand beyond the optimal point.

In a nutshell, the portfolio channel exacerbates the appreciation of the exchange rate, reducing the incentives to shift goods (resources) to the commodity sector. This is the margin the central bank attacks via FX intervention. By absorbing the position of financial intermediaries, the central bank can reduce this effect, allowing for a slower shift of resources and letting the economy take full advantage of the positive shock in commodity prices.

There is an additional channel at play, related to international risk sharing. As Cole (1988) and Cole and Obstfeld (1991) discuss, absent the instruments for agents to ex ante diversify the risk associated with the relative price of their human capital to their consumption basket, idiosyncratic shocks will yield inefficient relative prices in goods. As Cole (1988) shows, the existence of Arrow securities would make home agents enter an implicit pooling agreement through which income effects generated by these shocks would be reduced. Thus, with imperfect risk sharing and home bias in consumption, the income effect generated by the shock inefficiently increases the relative price of home to foreign good. Therefore, when a positive commodity price shock hits the economy, the real exchange rate follows a path that is more appreciated than the optimal, triggering the central bank to purchase foreign bonds.[9]

The dynamics under FX intervention attacks the income effect by engineering a smaller appreciation.

[9] Jermann (2002) also discusses the optimal international risk sharing in production economies under idiosyncratic productivity shocks.

This allows the economy to devote more resources (labour) to take advantage of the temporary price increase. Now, while the price is high, the economy expands significantly, as agents are more willing to offer labour. The higher exports allow the economy to save resources, in particular through NIR accumulation.

5. Conclusions

In this paper, we present a simple small open economy model to analyze how the interaction between commodity price shocks and financial market imperfections can affect the efficient relative prices and optimal factor allocation. Since international financial markets are incomplete and segmented, terms of trade shocks will affect the composition of financial intermediaries' portfolios, pushing relative prices (exchange rate) out of its efficient path.

The central bank can intervene by absorbing the position of financial intermediaries, restoring the relative price and undoing the effects of inefficient risk sharing initial point. This is equivalent to a reserve accumulation/deaccumulation strategy in periods of high/low commodity prices which is a stylized fact observed in several emerging economies.

We provide empirical support for this behaviour by analyzing the impact of higher commodity prices in a small open mineral exporting economy like Peru. In episodes of ToT positive shocks, central banks can intervene to avoid the excessive appreciation of their currencies, this in turn allows the economy to efficiently allocate resources to the exportable sector. In the alternative scenario of no intervention, the real exchange rate distortion diverts resources away from the exportable sector. Thus, optimal simple rules show that central banks reacting to commodity prices by accumulating foreign reserves can improve the welfare of the economy.

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Appendices

A. Data Description

We include quarterly data for the period 2002:01 - 2019:04. Data was taken from the Central Reserve Bank of Peru Website.

A.1. Big economy block variables $\mathbf{y}_t^*$

We only include year-to-year growth rate of the Terms of Trade (TOT) for the exogenous block.

A.2. Peruvian Economy block variables ($\mathbf{y}_t$*)*

We include the following variables from the Peruvian economy:

- Trade Balance (as % of GDP) (TB)
- GDP Index (2007=100) (Y)
- Real Consumption (C)
- Real Investment (I)
- Real Exchange Rate (RER)
- Consumer Price Index (P)
- Net International Reserves in USD Millions (NIR)

All variables with the exception of the Trade Balance are included as year-to-year growth rates.

B. A SVAR model with block exogeneity

B.1. The setup

Consider a big economy modelled as an independent Vector Autoregressive system (VAR) and also a small open economy modelled as a Vector Autoregressive system with exogenous variables, which are basically the Terms of Trade (TOT). In this context, the big economy is represented for $t = 1, ..., T$ by

$$\mathbf{y}_t^{*\prime}\mathbf{A}_0^* = \sum_{i=1}^{p} \mathbf{y}_{t-i}^{*\prime}\mathbf{A}_i^* + \mathbf{w}_t'\mathbf{D}^* + \varepsilon_t^{*\prime}, \tag{39}$$

where y_t^* is $n^* \times 1$ vectors of endogenous variables for the big economy; ε_t^* is $n^* \times 1$ vectors of structural shocks for the big economy ($\varepsilon_t^* \sim N(0, I_{n^*})$); $\widetilde{\mathbf{A}}_i^*$ and $\mathbf{A}_i^*$ are $n^* \times n^*$ matrices of structural parameters for $i = 0, \ldots, p$; $\mathbf{w}_t$ is a $r \times 1$ vector of exogenous variables; $\mathbf{D}^*$ is $r \times n$ matrix of structural parameters; p is the lag length; and, T is the sample size.

The small open economy (Peru) is represented by

$$\mathbf{y}_t'\mathbf{A}_0 = \sum_{i=1}^{p} \mathbf{y}_{t-i}'\mathbf{A}_i + \sum_{i=0}^{p} \mathbf{y}_{t-i}^{*\prime}\widetilde{\mathbf{A}}_i^* + \mathbf{w}_t'\mathbf{D} + \varepsilon_t', \tag{40}$$

where y_t is $n \times 1$ vector of endogenous variables for the small economy; ε_t is $n \times 1$ vector of structural shocks for the domestic economy ($\varepsilon_t \sim N(0, I_n)$ and structural shocks are independent across blocks i.e. $E(\varepsilon_t\varepsilon_t^{*\prime}) = \mathbf{0_{n \times n^*}}$); $\mathbf{A}_i$ are $n \times n$ matrices of structural parameters for $i = 0, \ldots, p$; and, $\mathbf{D}$ is $r \times n$ matrix of structural parameters. The latter model can be expressed in a more compact form, so that

$$\begin{aligned}
\begin{bmatrix} \mathbf{y}_t' & \mathbf{y}_t^{*\prime} \end{bmatrix} \begin{bmatrix} \mathbf{A}_0 & \mathbf{0} \\ -\widetilde{\mathbf{A}}_0^* & \mathbf{A}_0^* \end{bmatrix} &= \sum_{i=1}^{p} \begin{bmatrix} \mathbf{y}_{t-i}' & \mathbf{y}_{t-i}^{*\prime} \end{bmatrix} \begin{bmatrix} \mathbf{A}_i & \mathbf{0} \\ \widetilde{\mathbf{A}}_i^* & \mathbf{A}_i^* \end{bmatrix} \\
&\quad + \mathbf{w}_t' \begin{bmatrix} \mathbf{D} \\ \mathbf{D}^* \end{bmatrix} + \begin{bmatrix} \varepsilon_t' & \varepsilon_t^{*\prime} \end{bmatrix} \begin{bmatrix} I_n & \mathbf{0} \\ \mathbf{0} & I_{n^*} \end{bmatrix},
\end{aligned}$$

or simply

$$\overrightarrow{\mathbf{y}}'_t\overrightarrow{\mathbf{A}}_0 = \sum_{i=1}^{p} \overrightarrow{\mathbf{y}}'_{t-i}\overrightarrow{\mathbf{A}}_i + \mathbf{w}_t'\overrightarrow{\mathbf{D}} + \overrightarrow{\varepsilon}'_t, \tag{41}$$

where $\overrightarrow{\mathbf{y}}'_t \equiv \begin{bmatrix} \mathbf{y}_t' & \mathbf{y}_t^{*\prime} \end{bmatrix}$, $\overrightarrow{\mathbf{A}}_i \equiv \begin{bmatrix} \mathbf{A}_i & \mathbf{0} \\ \widetilde{\mathbf{A}}_i^* & \mathbf{A}_i^* \end{bmatrix}$ for $i = 1, \ldots, p$, $\overrightarrow{\mathbf{D}} \equiv \begin{bmatrix} \mathbf{D} \\ \mathbf{D}^* \end{bmatrix}$ and $\overrightarrow{\varepsilon}'_t \equiv \begin{bmatrix} \varepsilon_t' & \varepsilon_t^{*\prime} \end{bmatrix}$.

The system (40) represents the small open economy in which its dynamic behavior is influenced by the big economy block (39) through the parameters $\widetilde{\mathbf{A}}_i^*, \mathbf{A}_i^*$ and $\mathbf{D}^*$. On the other hand, the big economy evolves independently, i.e. by construction, the small open economy cannot influence the dynamics of the big economy.

Even though block (39) has effects over block (40), we assume that the block (39) is independent of block (40). This type of *Block Exogeneity* has been applied in the context of SVARs by Cushman and Zha (1997), Zha (1999) and Canova (2005), among others. Moreover, it turns out that this is a plausible strategy for representing small open economies such as the Latin American ones, since they are influenced by external shocks such as the Terms of Trade.

B.2. Reduced form estimation

The system (41) is estimated by block separately. We first present a foreign, then a domestic block, and finally introduce a compact form system i.e. stack both blocks into a one system.

B.2.1. Big economy block

The independent SVAR (39) can be written as

$$\mathbf{y}_t^{*\prime}\mathbf{A}_0^* = \mathbf{x}_t^{*\prime}\mathbf{A}_+^* + \varepsilon_t^{*\prime} \text{ for } t = 1, \dots, T;$$

where

$$\mathbf{A}_+^{*\prime} \equiv \begin{bmatrix} \mathbf{A}_1^{*\prime} & \cdots & \mathbf{A}_p^{*\prime} & \mathbf{D}^{*\prime} \end{bmatrix}, \ \mathbf{x}_t^{*\prime} \equiv \begin{bmatrix} \mathbf{y}_{t-1}^{*\prime} & \cdots & \mathbf{y}_{t-p}^{*\prime} & \mathbf{w}_t^{\prime} \end{bmatrix},$$

so that the reduced form representation is

$$\mathbf{y}_t^{*\prime} = \mathbf{x}_t^{*\prime}\mathbf{B}^* + \mathbf{u}_t^{*\prime} \text{ for } t = 1, \dots, T; \tag{42}$$

where $\mathbf{B}^* \equiv \mathbf{A}_+^* \left(\mathbf{A}_0^*\right)^{-1}$, $\mathbf{u}_t^{*\prime} \equiv \varepsilon_t^{*\prime} \left(\mathbf{A}_0^*\right)^{-1}$, and $E\left[\mathbf{u}_t^*\mathbf{u}_t^{*\prime}\right] = \mathbf{\Sigma}^* = \left(\mathbf{A}_0^*\mathbf{A}_0^{*\prime}\right)^{-1}$. Then the coefficients $\mathbf{B}^*$ are estimated from (42) by OLS.

B.2.2. Small open economy block

The SVARX system (40) is written as

$$\mathbf{y}_t^{\prime}\mathbf{A}_0 = \mathbf{x}_t^{\prime}\mathbf{A}_+ + \varepsilon_t^{\prime} \text{ for } t = 1, \dots, T;$$

where

$$\begin{aligned}
\mathbf{A}'_+ &\equiv \begin{bmatrix} \mathbf{A}'_1 & \cdots & \mathbf{A}'_p & \widetilde{\mathbf{A}}^*_0 & \widetilde{\mathbf{A}}^*_1 & \cdots & \widetilde{\mathbf{A}}^*_p & \mathbf{D}' \end{bmatrix} \\
\mathbf{x}'_t &\equiv \begin{bmatrix} \mathbf{y}'_{t-1} & \cdots & \mathbf{y}'_{t-p} & \mathbf{y}^{*\prime}_t & \mathbf{y}^{*\prime}_{t-1} & \cdots & \mathbf{y}^{*\prime}_{t-p} & \mathbf{w}'_t \end{bmatrix}.
\end{aligned}$$

The reduced form is now

$$\mathbf{y}'_t = \mathbf{x}'_t\mathbf{B} + \mathbf{u}'_t \;\text{ for } t = 1, \ldots, T; \tag{43}$$

where $\mathbf{B} \equiv \mathbf{A}_+\mathbf{A}_0^{-1}$, $\mathbf{u}'_t \equiv \varepsilon'_t\mathbf{A}_0^{-1}$, and $E\left[\mathbf{u}_t\mathbf{u}'_t\right] = \mathbf{\Sigma} = \left(\mathbf{A}_0\mathbf{A}'_0\right)^{-1}$. As we can see, foreign variables are treated as predetermined in this block, i.e. it can be considered as a VARX model. In this case, coefficients $\mathbf{B}$ are estimated from (43) by OLS.

B.2.3. A compact form

The reduced form of the two models can be stacked into a single one, so that the SVAR model (41) can be estimated through standard methods. Thus, the model can be written as

$$\overrightarrow{\mathbf{y}}'_t\overrightarrow{\mathbf{A}}_0 = \overrightarrow{\mathbf{x}}'_t\overrightarrow{\mathbf{A}}_+ + \overrightarrow{\varepsilon}'_t \;\text{ for } t = 1, \ldots, T;$$

where

$$\begin{aligned}
\overrightarrow{\mathbf{A}}'_+ &\equiv \begin{bmatrix} \overrightarrow{\mathbf{A}}'_1 & \cdots & \overrightarrow{\mathbf{A}}'_p & \overrightarrow{\mathbf{D}} \end{bmatrix} \\
\overrightarrow{\mathbf{x}}'_t &\equiv \begin{bmatrix} \overrightarrow{\mathbf{y}}'_{t-1} & \cdots & \overrightarrow{\mathbf{y}}'_{t-p} & \mathbf{w}'_t \end{bmatrix}.
\end{aligned}$$

As a result, the reduced form is now

$$\overrightarrow{\mathbf{y}}'_t = \overrightarrow{\mathbf{x}}'_t\overrightarrow{\mathbf{B}} + \overrightarrow{\mathbf{u}}'_t \;\text{ for } t = 1, \ldots, T; \tag{44}$$

where $\overrightarrow{\mathbf{B}} \equiv \overrightarrow{\mathbf{A}}_{+}\left(\overrightarrow{\mathbf{A}}_0\right)^{-1}$, $\overrightarrow{\mathbf{u}}'_t \equiv \overrightarrow{\varepsilon}'_t\left(\overrightarrow{\mathbf{A}}_0\right)^{-1}$, and $E\left[\overrightarrow{\mathbf{u}}_t \overrightarrow{\mathbf{u}}'_t\right] = \overrightarrow{\mathbf{\Sigma}} = \left(\overrightarrow{\mathbf{A}}_0 \overrightarrow{\mathbf{A}}'_0\right)^{-1}$. In this case, if we estimate $\overrightarrow{\mathbf{B}}$ by OLS, this must be performed taking into account the block structure of the system imposed in matrices $\overrightarrow{\mathbf{A}}_i$, i.e. it becomes a restricted OLS estimation. Clearly, it is easier and more transparent to implement the two step procedure described above and, ultimately, since the blocks are independent by assumption, there are no gains from this joint estimation procedure (Zha, 1999). Last but not least, the lag length p is the same for both blocks and it is determined as the maximum obtained from the two blocks using the Schwarz information criterion (SIC).

B.3. SUR representation

Recall the linear model (43) and take the transpose, so that

$$\mathbf{y}_t = B'\mathbf{x}_t + \mathbf{u}_t$$

Then, following Koop and Korobilis (2010) use the $vec(.)$ operator, so that

$$\mathbf{y}_t = \left(\mathbf{x}'_t \otimes I_K\right) vec\left(B'\right) + \mathbf{u}_t$$

$$\mathbf{y}_t = Z_t\beta + \mathbf{u}_t$$

where $Z_t \equiv \left(\mathbf{x}'_t \otimes I_K\right)$ and β is a column vector with all the model coefficients. Then, using the entire sample $t = 1, \ldots, T$ we can write the VARX model as:

$$Y = Z\beta + U$$

such that $U \sim N\left(0, I \otimes \Sigma\right)$. As a result, the VARX system can be rewritten as a Normal linear regression model with a particular variance-covariance matrix for the error term, i.e. the SUR regression problem.

B.4. Priors and Posterior distribution

We adopt natural conjugate priors for the reduced form model parameters. The latter implies that the prior distribution, the likelihood function and the posterior distribution come from the same family of distributions (Koop and Korobilis, 2010). The introduction of priors is desirable, since the number of parameters to be estimated is very high and the number of observations is limited. Therefore, this a plausible strategy for reducing the amount of posterior uncertainty and, at the same time, it is useful for disciplining the data. In this regard, it is important to remark that we introduce priors for the reduced form coefficients, but this does not mean that we impose any prior information about the structural form. The latter is out of the scope of this paper, but more details can be found in Baumeister and Hamilton (2014) and Canova and Pérez Forero (2015).

We assume that the prior distribution of the object $\left(\mathbf{B}, \mathbf{\Sigma}^{-1}\right)$ is Normal-Wishart for each block separately. Since each block is going to be treated symmetrically, we only present the analytical distributions of the domestic block, so that

$$\beta \mid Y, \mathbf{\Sigma} \sim N\left(\underline{\beta}, \mathbf{\Sigma} \otimes \underline{V}\right)$$

$$\mathbf{\Sigma}^{-1} \mid Y \sim W\left(\underline{S}^{-1}, \underline{\nu}\right),$$

where $\underline{\beta} = vec\left(\underline{\mathbf{B}}\right)$ and $\left(\underline{\mathbf{B}}, \underline{V}, \underline{S}^{-1}, \underline{\nu}\right)$ are prior hyper-parameters with $\underline{\nu} = \tau$. In particular, we parametrize:

$$\underline{\beta} = \mathbf{0}, \underline{S} = \underline{h}\Sigma_{\tau}, \quad \underline{V} = \Omega,$$

with $\underline{h} = 1$ being a hyper-parameter, K the number of regressors in the model and Ω is the prior variance, which is calibrated using a Minnesota-style parametrization. As a result, the posterior distribution is

$$\beta \mid Y, \mathbf{\Sigma} \sim N\left(\overline{\beta}, \mathbf{\Sigma} \otimes \overline{V}\right)$$

$$\mathbf{\Sigma}^{-1} \mid Y \sim W\left(\overline{S}^{-1}, \overline{\nu}\right),$$

where

$$\overline{V} = \left[\underline{V}^{-1} + \sum_{t=1}^{T} Z_t' \Sigma^{-1} Z_t\right]^{-1}$$

$$\overline{\beta} = \overline{V}\left[\underline{V}^{-1}\underline{\beta} + \sum_{t=1}^{T} Z_t' \Sigma^{-1} \mathbf{y}_t\right]$$

where $\overline{\beta} = vec\left(\overline{\mathbf{B}}\right)$ and

$$\overline{S} = \underline{S} + \sum_{t=1}^{T} \left(\mathbf{y}_t - Z_t\beta\right)\left(\mathbf{y}_t - Z_t\beta\right)'$$

$$\overline{\nu} = T + \underline{\nu}.$$

Given these analytical forms, we explain the next section how to obtain draws of $(\mathbf{B}, \mathbf{\Sigma})$ from the posterior distribution.

C. Bayesian Estimation

C.1. A Gibbs Sampling routine

Sampling from the posterior distribution of $\left(\overrightarrow{\beta}, \overrightarrow{\mathbf{\Sigma}}\right)$ is always difficult. However, in this case we have an analytical expression for each parameter block. Therefore it is possible to implement a Gibbs sampling routine. In this process, it is useful to divide the parameter set into different blocks.

The routine starts here. Set $k = 1$ and denote K as the total number of draws. Then follow the steps below:

1. Draw coefficients from the exogenous block $p\left(\beta^* \mid \mathbf{\Sigma}^*, \mathbf{y}^{*T}\right)$ and for domestic block $p\left(\beta \mid \mathbf{\Sigma}, \overrightarrow{\mathbf{y}}^T\right)$.

2. Construct $\overrightarrow{\beta} = \{\beta, \beta^*\}$ and compute the associated companion form. If the candidate draw is stable keep it, otherwise discard it.

3. Draw the covariance matrices through $p\left(\mathbf{\Sigma}^* \mid \beta^*, \mathbf{y}^{*T}\right)$ and $p\left(\mathbf{\Sigma} \mid \beta, \overrightarrow{\mathbf{y}}^T\right)$.

4. If $k < K$ set $k = k + 1$ and return to Step 1. Otherwise stop.

C.2. Estimation setup

We run the Gibbs sampler for $K = 100,000$ and discard the first $50,000$ draws in order to minimize the effect of initial values. Moreover, in order to reduce the serial correlation across draws, we set a thinning factor of 10, i.e. given the remaining 50,000 draws, we take 1 every 10 and discard the remaining ones. As a result, we have $5,000$ draws for conducting inference. We estimate the model for the period 2002:01-2019:04. Specific details about the Data Description can be found in Appendix A.

D. Figures and Tables

Figure 2: Copper price, trade balance and exports in Peru, 2000:Q3-2018:Q4.

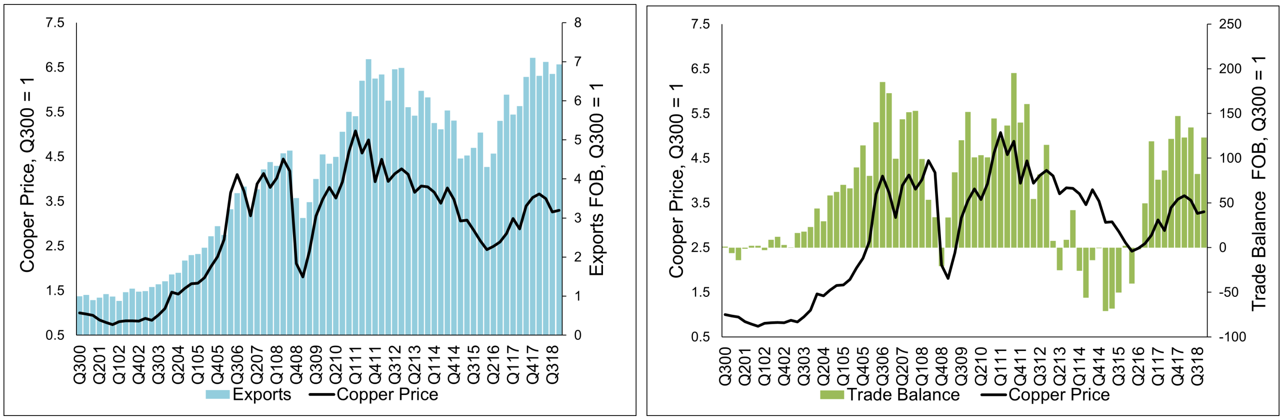
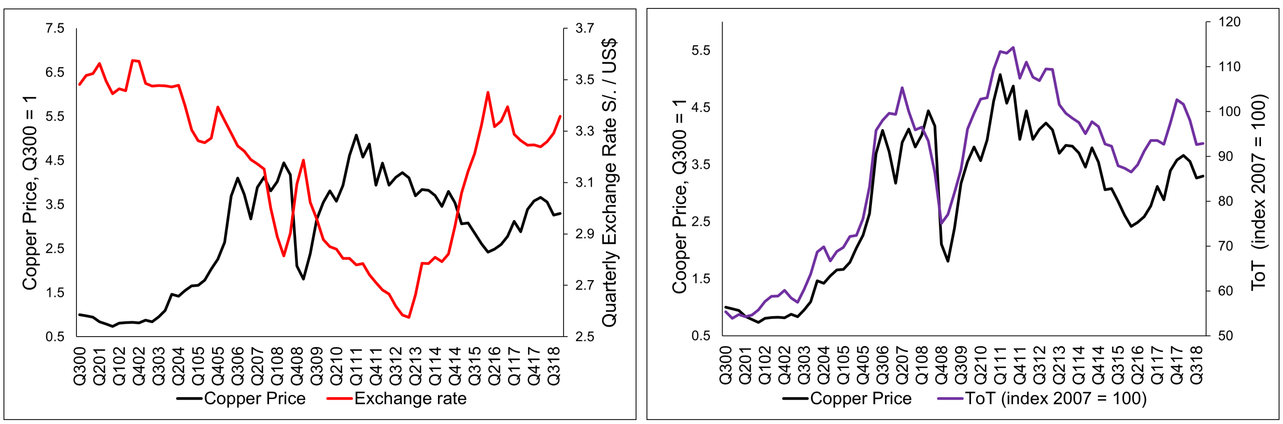
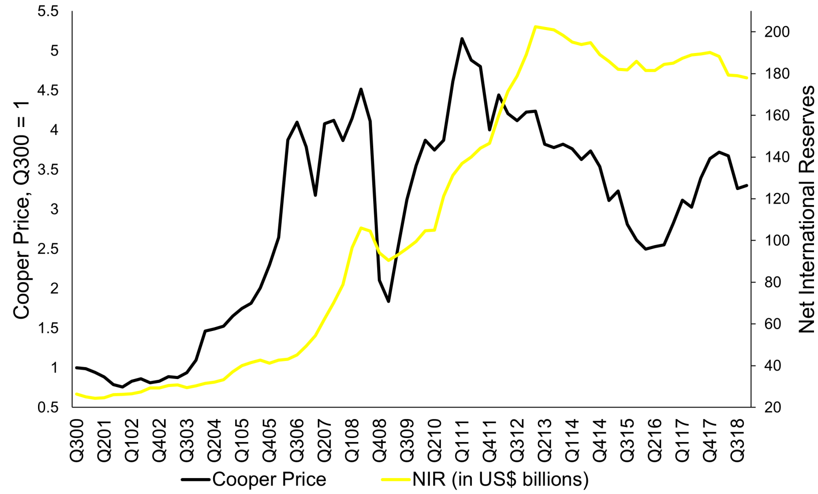


Figure 3: Copper price, exchange rate and terms of trade, 2000:Q3-2018:Q4.



Note: *Exports*, *Trade Balance* and *CU* are defined as normalized FOB exports, FOB trade balance and international copper price (the series are normalized so that their average value in 2000:Q3 is equal to 1), respectively, *ToT*, as the index (2007 = 100) of foreign trade terms of trade, and *NER*, as the exchange rate in \$*USD* Source: BCRP, Bloomberg. Own elaboration.

Figure 4: Net international reserves, Peru, 2000.Q3-2018.Q4



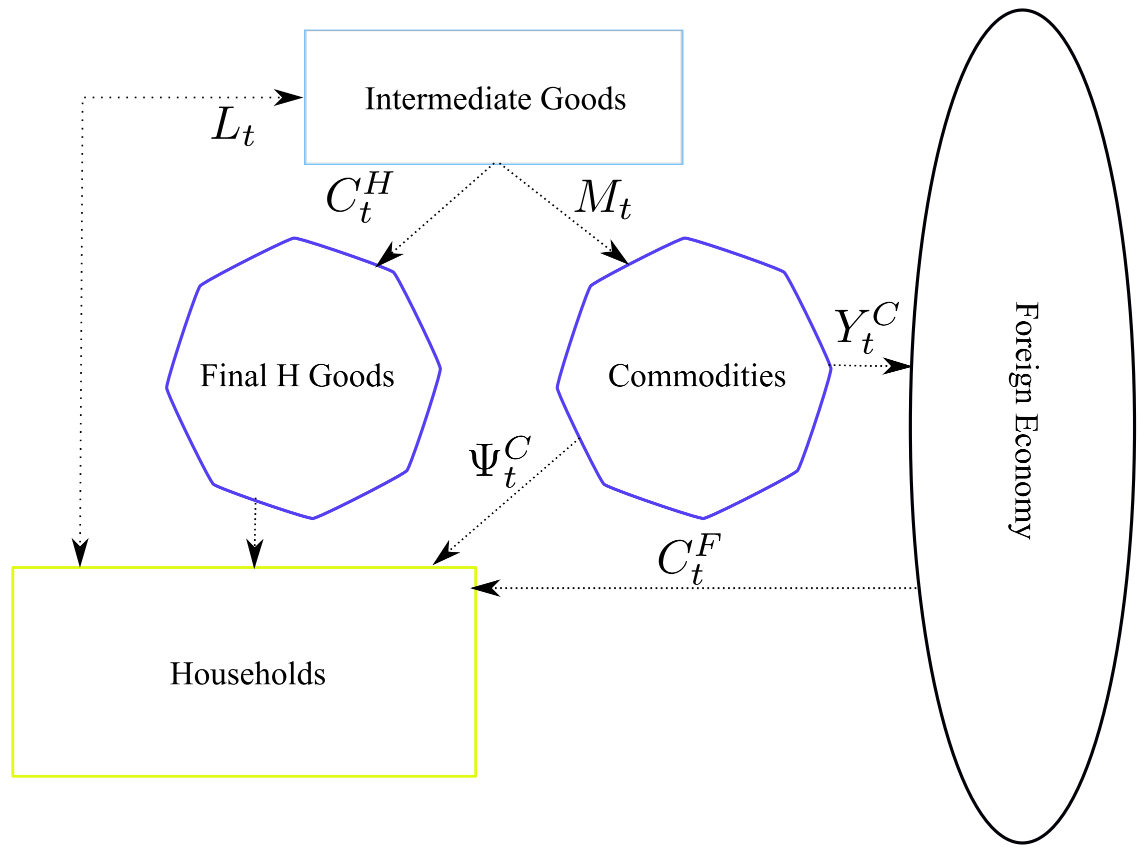
Note: We define the copper price as normalized copper price (we normalize the mean value of 2000:Q3 to 1) and NIR as net international reserves in US$ billions. Source: BCRP.

E. Calibration

Table 1: **Calibration**

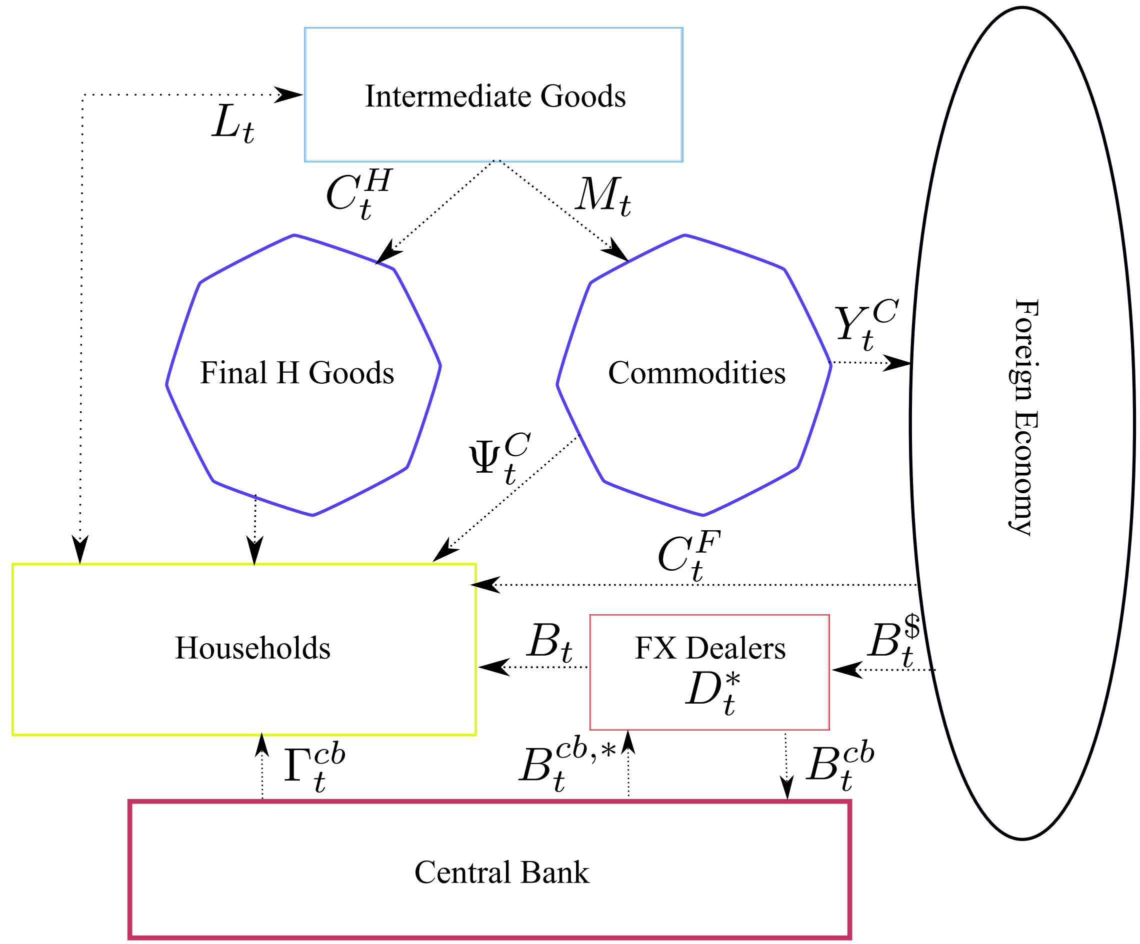
Description	**Parameter**	**Value**	**Source**
Frisch elasticity parameter	χ	1.455	Schmitt-Grohé and Uribe (2018)
Intertemporal discount parameter	β	0.98	Montoro and Ortiz (2020)
Total factor productivity, by firm	A^c, A	1	Schmitt-Grohé and Uribe (2018)
Household risk aversion parameter	γ_c	2	Schmitt-Grohé and Uribe (2018)
Elasticity of substitution between H and F	ε_H	1	Montoro and Ortiz (2016)
Mass of dealers	m	1	Montoro and Ortiz (2020)
Absolut risk aversion parameter for FX dealers	ω	500	Bacchetta and Van Wincoop (2006)

Figure 5: Ferrero & Seneca Model



Note: We define; L_t as labour, C^H as the demand of intermediate goods for domestic consumption, M as the demand of the commodity exporting sector for intermediate goods, Y^C as the commodities exported, C^F as the demand for imported final goods, Ψ^C as the profits from the commodity sector.

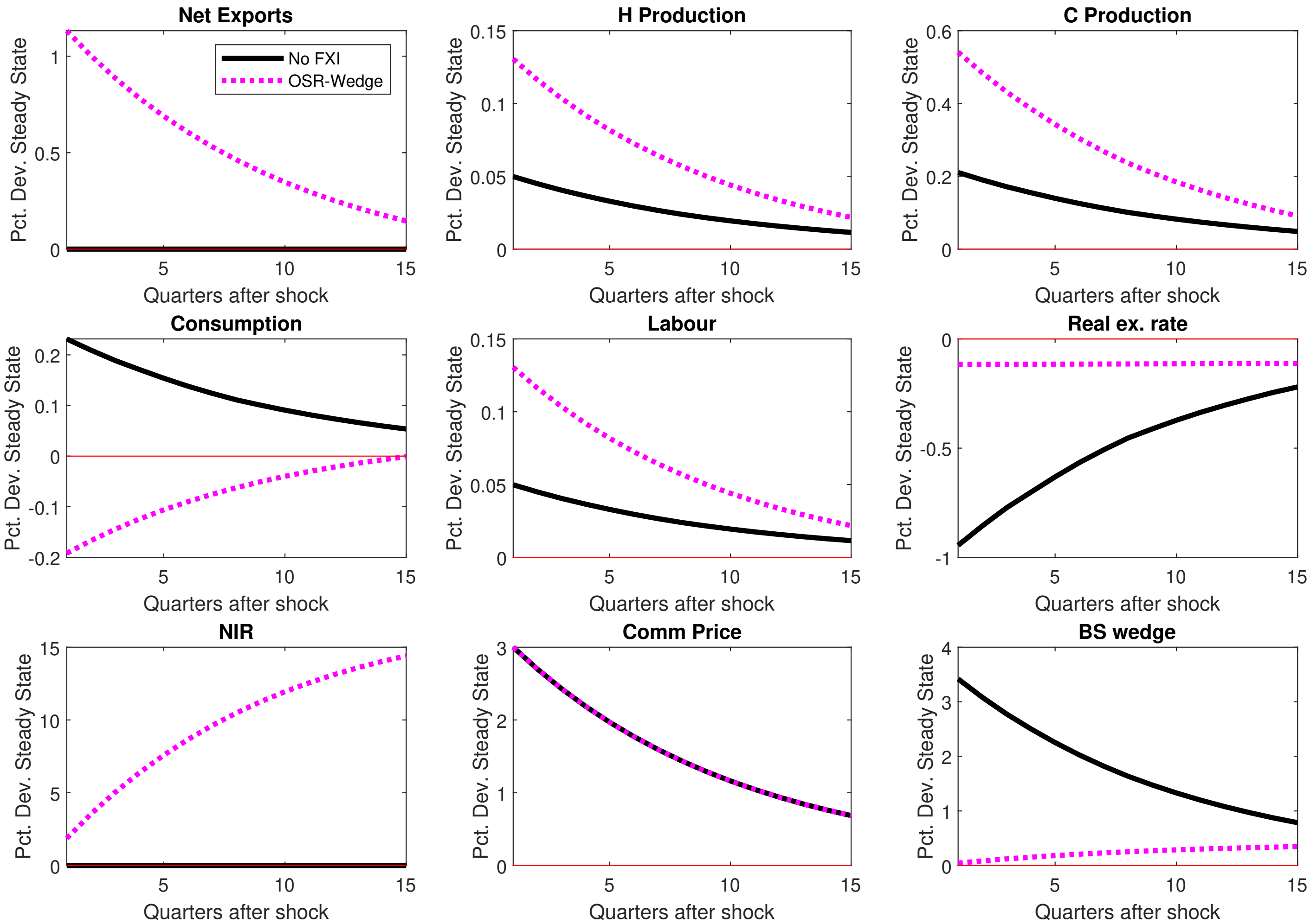
Figure 6: Ferrero & Seneca Augmented Model



Note: We define; L_t as labour, C^H as the demand of intermediate goods for domestic consumption, M as the demand of the commodity exporting sector for intermediate goods, Y^C as the commodities exported, C^F as the demand for imported final goods, Ψ^C as the profits from the commodity sector; B as the local currency savings of domestic households, $B^{\$}$ as the dollar payment for net exports; B^{cb} and B^{cb} as the bond dollar and peso position held by the central bank, respectively. Finally, D_t^* represents the financial intermdiares open position.

F. Model dynamics

Figure 7: Response to a 1% std. dev. shock to commodity prices



Note: The results are generated under the calibration shown in Table 1. The exchange rates are plotted so that an increase corresponds to depreciation.

G. Linear Equilibrium

The first-order approximation of several equations from the equilibrium are:

$$c_t = -(i_t - \mathbb{E}_t\pi_{t+1}) + \mathbb{E}_t c_{t+1} \tag{45}$$

$$c_t = \lambda_t + y_t^* + \gamma\tau_t \tag{46}$$

$$a_{Ht} - (1-\gamma)\tau_t + mc_t = \chi n_t + c_t \tag{47}$$

$$c_{Ht} = (1-\gamma)\tau_t + c_t \tag{48}$$

$$c_{Ft} = -\gamma\tau_t + c_t \tag{49}$$

$$y_{Ht} = a_{Ht} + n_t \tag{50}$$

$$\pi_{Ht} = \beta E_t \pi_{Ht+1} + \kappa mc_t \tag{51}$$

$$mc_t = w_t - p_{Ht} - a_{Ht} \tag{52}$$

$$\pi_{Ht} = p_{Ht} - p_{Ht-1} \tag{53}$$

$$y_{Ct} = a_{Ct} + \nu m_t \tag{54}$$

$$p_{Ht} - p_{Ct} = a_{Ct} + (\nu - 1)m_t \tag{55}$$

$$p_{Ct} = s_t + p_{Ct}^* \tag{56}$$

$$q_t = \gamma\tau_t \tag{57}$$

$$\tau_t = p_{Ft} - p_{Ht} \tag{58}$$

$$y_{Ht} = s_c c_{Ht} + s_m m_t \tag{59}$$

H. Natural Allocation

Is defined as the friction-less equilibrium: flexible price and perfect risk-sharing equilibrium. The labor supply condition (47), using both international imperfect risk-sharing condition and the definition of production function:

$$
\begin{aligned}
a_{Ht} - (1-\gamma)\tau_t^n &= \nu n_t^n + c_t^n \\
a_{Ht} - (1-\gamma)\tau_t^n &= \nu n_t^n + \gamma\tau_t^n + y_t^* \\
a_{Ht} &= \chi n_t^n + \tau_t^n + y_t^* \\
a_{Ht} &= \chi(y_{Ht}^n - a_{Ht}) + \tau_t^n + y_t^* \\
0 &= \chi y_{Ht}^n - (1+\chi)a_{Ht} + \tau_t^n + y_t^* \qquad (60)
\end{aligned}
$$

Independently of price setting, we can use the imperfect risk sharing and demand relation to rewrite the resource constraint:

$$
\begin{aligned}
y_{Ht} &= s_c c_{Ht}^n + s_m m_t^n \\
y_{Ht} &= s_c(\tau_t + y_t^*) + s_m m_t
\end{aligned}
$$

From the (55)

$$
\begin{aligned}
p_{Ht} - p_{Ct} &= a_{Ct} + (\nu-1)m_t \\
p_{Ht} - (s_t + p_{Ct}^*) &= a_{Ct} + (\nu-1)m_t \\
p_{Ht} - s_t - p_t^* + p_t^* - p_{Ct}^*) &= a_{Ct} + (\nu-1)m_t \\
(p_{Ht} - s_t - p_t^*) + p_t^* - p_{Ct}^*) &= a_{Ct} + (\nu-1)m_t \\
(p_{Ht} - p_{Ft}) + p_t^* - p_{Ct}^*) &= a_{Ct} + (\nu-1)m_t \\
-\tau_t + p_t^* - p_{Ct}^*) &= a_{Ct} + (\nu-1)m_t \\
(\nu-1)m_t &= -\tau_t + p_t^* - p_{Ct}^* - a_{Ct} \\
m_t &= \frac{1}{1-\nu}(\tau_t + p_{Ct}^* + a_{Ct} - p_t^*)
\end{aligned}
$$

If $p_t^* = 0$ then

$$m_t = \frac{1}{1-\nu}(\tau_t + p_{Ct}^* + a_{Ct})$$

Using this condition into the resources constraint and the imperfect risk-sharing condition:

$$
\begin{aligned}
y_{Ht} &= s_c(\tau_t + y_t^* + \lambda_t) + s_m\left(\frac{1}{1-\nu}(\tau_t + p_{Ct}^* + a_{Ct} - p_t^*)\right) \\
y_{Ht} &= \left(s_c + \frac{s_m}{1-\nu}\right)\tau_t + s_c\lambda_t + s_c y_t^* + s_m\left(\frac{1}{1-\nu}(p_{Ct}^* + a_{Ct})\right) \\
y_{Ht} &= \left(s_c + \frac{s_m}{1-\nu}\right)\tau_t + s_c\lambda_t + s_c y_t^* + \frac{s_m}{1-\nu}\left(p_{Ct}^* + a_{Ct}\right) \\
y_{Ht} &= \xi_\tau\tau_t + s_c\lambda_t + s_c y_t^* + \frac{s_m}{1-\nu}\left(p_{Ct}^* + a_{Ct}\right) \qquad (61)
\end{aligned}
$$

or in the natural equilibrium:

$$
m_t = \frac{1}{1-\nu}(\tau_t + p_{Ct}^* + a_{Ct})
$$

Using this condition into the resources constraint and the imperfect risk-sharing condition:

$$
y_{Ht}^n = \xi_\tau\tau_t^n + s_c y_t^* + \frac{s_m}{1-\nu}\left(p_{Ct}^* + a_{Ct}\right) \qquad (62)
$$

Reduce (60):

$$
\tau_t^n = -\chi y_{Ht}^n + (1+\chi)a_{Ht} - y_t^*
$$

then replace into (62) to obtain:

$$
\begin{aligned}
y_{Ht}^n &= \xi_\tau\tau_t^n + s_c y_t^* + \frac{s_m}{1-\nu}\left(p_{Ct}^* + a_{Ct}\right) \\
y_{Ht}^n &= \xi_\tau(-\chi y_{Ht}^n + (1+\chi)a_{Ht} - y_t^*) + s_c y_t^* + \frac{s_m}{1-\nu}\left(p_{Ct}^* + a_{Ct}\right) \\
(1+\xi_\tau\chi)y_{Ht}^n &= \xi_\tau(1+\chi)a_{Ht} + (s_c - \xi_\tau)y_t^* + \frac{s_m}{1-\nu}\left(p_{Ct}^* + a_{Ct}\right) \\
(1+\xi_\tau\chi)y_{Ht}^n &= \xi_\tau(1+\chi)a_{Ht} + \left(s_c - s_c - \frac{s_m}{1-\nu}\right)y_t^* + \frac{s_m}{1-\nu}\left(p_{Ct}^* + a_{Ct}\right)
\end{aligned}
$$

That yields:

$$(1+\xi_\tau\chi)y^n_{Ht} = \xi_\tau(1+\chi)a_{Ht} + \frac{s_m}{1-\nu}\left(p^*_{Ct} + a_{Ct} + y^*_t\right) \tag{63}$$

Combine (62) for both natural and price rigidity allocations:

$$y_{Ht} - y^n_{Ht} = \xi_\tau(\tau_t - \tau^n_t) + s_c\lambda_t$$

Marginal cost is

$$mc_t = \chi y_{Ht} - (1+\chi)a_{Ht} + \tau_t + y^*_t + \lambda_t$$
$$0 = \chi y^n_{Ht} - (1+\chi)a_{Ht} + \tau^n_t + y^*_t$$

Gives us:

$$mc_t = \chi(y_{Ht} - y^n_{Ht}) + \tau_t - \tau^n_t + \lambda_t$$
$$mc_t = \chi(y_{Ht} - y^n_{Ht}) + \xi_\tau^{-1}(y_{Ht} - y^n_{Ht} - s_c)\lambda_t + \lambda_t$$
$$mc_t = (\chi + \xi_\tau^{-1})(y_{Ht} - y^n_{Ht}) - s_c\xi_\tau^{-1}\lambda_t + \lambda_t$$
$$mc_t = (\chi + \xi_\tau^{-1})(y_{Ht} - y^n_{Ht}) + (1 - s_c\xi_\tau^{-1})\lambda_t$$

I. Efficient Allocation

The central planner solves:

$$\max_{\{L_t,\mathcal{T}_t} log(\mathcal{T}_t^{\gamma} Y^*_t\}) - \frac{L_t^{1+\chi}}{1+\chi}$$
$$\text{s.t.}\quad A_{Ht}L_t = \gamma\mathcal{T}_t Y^*_t + \left(\nu A_{Ct}\frac{P^*_{Ct}}{P^*_t}\mathcal{T}_t\right)^{\frac{1}{1-\nu}}$$

Replace the restriction into the optimization problem.

$$\max_{\{\mathcal{T}_t\}} log(\mathcal{T}_t^{\gamma} Y_t^*) - \frac{\left[\frac{1}{A_{Ht}}\left(\gamma \mathcal{T}_t Y_t^* + \left(\nu A_{Ct}\frac{P_{Ct}^*}{P_t^*}\mathcal{T}_t\right)^{\frac{1}{1-\nu}}\right)\right]^{1+\chi}}{1+\chi} \tag{64}$$

First order condition yields:

$$\begin{aligned}
\frac{\gamma}{\mathcal{T}_t} &= (L_t^e)^{\chi}\left[\frac{1}{A_{Ht}}\left(\gamma Y_t^* + \nu A_{Ct}\frac{P_{Ct}^*}{P_t^*}(\frac{1}{1-\nu})\mathcal{T}_t^{\frac{1}{1-\nu}-1}\right)\right] \\
\frac{\gamma}{\mathcal{T}_t} &= (L_t^e)^{\chi}\left[\frac{1}{A_{Ht}\mathcal{T}_t}\left(\gamma \mathcal{T}_t Y_t^* + A_{Ct}\frac{P_{Ct}^*}{P_t^*}(\frac{\nu}{1-\nu})\mathcal{T}_t^{\frac{1}{1-\nu}}\right)\right] \\
\gamma &= (L_t^e)^{\chi}\left[\frac{L_t^e}{A_{Ht}L_t^e}\left(C_{Ht} + \frac{1}{1-\nu}M_t\right)\right] \\
\gamma &= (L_t^e)^{\chi}N_t\left[\frac{1}{Y_{Ht}}\left(C_{Ht} + \frac{1}{1-\nu}M_t\right)\right] \\
\gamma &= (L_t^e)^{\chi}N_t\left[\frac{C_{Ht} + \frac{1}{1-\nu}M_t}{Y_{Ht}}\right] \\
\gamma &= (L_t^e)^{1+\chi}\xi_{\tau t} \\
\frac{\gamma}{\xi_{\tau t}^e} &= (L_t^e)^{1+\chi} \qquad (65)
\end{aligned}$$

Where

$$\xi_{\tau t} = \frac{C_{Ht} + \frac{1}{1-\nu}M_t}{Y_{Ht}} = s_c + \frac{s_m}{1-\nu}$$

$$\frac{\gamma}{\xi_{\tau t}^e} = (L_t^e)^{1+\chi} \tag{66}$$

$$\gamma = (L_t^n)^{1+\chi} \tag{67}$$

If $\xi_{\tau t}$ is efficient allocated then a first-order approximation of the efficient allocation of labor (65) yields:

$$(1+\chi)l_t^e = -\xi_\tau^{-1}(s_c c_{Ht}^e + \frac{1}{1-\nu}s_m m_t^e) + y_{Ht}^e$$

Where $s_c = \frac{C_H}{Y_H}$ and $s_m = \frac{M}{Y_H}$. Using the perfect risk-sharing condition we obtain the efficient allocation for output as:

$$\chi\xi_\tau y_{Ht}^e = -\zeta_\tau \tau_t^e + (1+\chi)\xi_\tau a_{Ht} - s_c y_t^* - \frac{s_m}{(1-\nu)^2}(a_{Ct} + p_{Ct}^*) \tag{68}$$

Where

$$\zeta_\tau = s_c + \frac{s_m}{(1-\nu)^2}$$

The resources constraint (61) is given by:

$$y_{Ht} = \xi_\tau \tau_t + s_c \lambda_t + s_c y_t^* + \frac{s_m}{1-\nu}\left(p_{Ct}^* + a_{Ct}\right) \tag{69}$$

Notice that (69) is the same restriction in the efficient case so that we can use the subscript ^{e}.

$$y_{Ht}^e = \xi_\tau \tau_t^e + s_c y_t^* + \frac{s_m}{1-\nu}(a_{Ct} + p_{Ct}^*) \tag{70}$$

Reducing (70) and replace into (68) to obtain:

$$\left(\frac{\zeta_\tau}{\xi_\tau} + \chi\xi_\tau\right) y_{Ht}^e = (1+\chi)\xi_\tau a_{Ht} - \left(1 - \frac{\zeta_\tau}{\xi_\tau}\right) s_c y_t^* - \frac{s_m s_c}{(1-\nu)\xi_\tau}\frac{\nu}{1-\nu}(a_{Ct} + p_{Ct}^*) \tag{71}$$

J. Linear Constraints

Rewriting the Phillips curve as:

$$
\begin{aligned}
&\pi_{Ht} = \beta E_t \pi_{Ht+1} + \kappa mc_t \\
&\pi_{Ht} = \beta E_t \pi_{Ht+1} + \kappa(\chi + \xi_\tau^{-1})(y_{Ht} - y_{Ht}^n) + \kappa(1 - s_c \xi_\tau^{-1})\lambda_t \\
&\pi_{Ht} = \beta E_t \pi_{Ht+1} + \kappa(\chi + \xi_\tau^{-1})y_{Ht} - \kappa(\chi + \xi_\tau^{-1})y_{Ht}^n + \kappa(\chi + \xi_\tau^{-1})y_{Ht}^e - \kappa(\chi + \xi_\tau^{-1})y_{Ht}^e + \kappa(1 - s_c \xi_\tau^{-1})\lambda_t \\
&\pi_{Ht} = \beta E_t \pi_{Ht+1} + \kappa(\chi + \xi_\tau^{-1})(y_{Ht} - y_{Ht}^e) - \kappa(\chi + \xi_\tau^{-1})y_{Ht}^n + \kappa(\chi + \xi_\tau^{-1})y_{Ht}^e + \kappa(1 - s_c \xi_\tau^{-1})\lambda_t \\
&\pi_{Ht} = \beta E_t \pi_{Ht+1} + \kappa(\chi + \xi_\tau^{-1})(y_{Ht} - y_{Ht}^e) + \kappa(1 - s_c \xi_\tau^{-1})\lambda_t + \kappa(\chi + \xi_\tau^{-1})(y_{Ht}^e - y_{Ht}^n)
\end{aligned}
$$

The efficient and natural output are:

$$
\begin{aligned}
&\left(\frac{\zeta_\tau}{\xi_\tau} + \chi\xi_\tau\right) y_{Ht}^e = (1+\chi)\xi_\tau a_{Ht} - \left(1 - \frac{\zeta_\tau}{\xi_\tau}\right) s_c y_t^* - \frac{s_m s_c}{(1-\xi_\tau^{-1})\xi_\tau} \frac{\nu}{1-\nu}(a_{Ct} + p_{Ct}^*) \\
&y_{Ht}^e = \left(\frac{\zeta_\tau}{\xi_\tau} + \chi\xi_\tau\right)^{-1} \left[(1+\chi)\xi_\tau a_{Ht} - \left(1 - \frac{\zeta_\tau}{\xi_\tau}\right) s_c y_t^* - \frac{s_m s_c}{(1-\nu)\xi_\tau} \frac{\nu}{1-\nu}(a_{Ct} + p_{Ct}^*)\right] \\
&(1 + \xi_\tau \chi) y_{Ht}^n = \xi_\tau (1+\chi) a_{Ht} + \frac{s_m}{1-\nu}\left(p_{Ct}^* + a_{Ct} + y_t^*\right) \\
&y_{Ht}^n = (1 + \xi_\tau \chi)^{-1} \left[\xi_\tau (1+\chi) a_{Ht} + \frac{s_m}{1-\nu}\left(p_{Ct}^* + a_{Ct} + y_t^*\right)\right]
\end{aligned}
$$

By subtracting both terms:

$$
\begin{aligned}
y_{Ht}^e - y_{Ht}^n &= \left[\left(\frac{\xi_\tau}{\gamma_\tau} + \chi\gamma_\tau\right)^{-1} \gamma_\tau(1+\chi) - (1 + \gamma_\tau\chi)^{-1}\gamma_\tau(1+\chi)\right] a_{Ht} - ... \\
&= \left[\left(\frac{\xi_\tau}{\gamma_\tau} + \chi\gamma_\tau\right)^{-1} \left(1 - \frac{\xi_\tau}{\gamma_\tau}\right) s_c - (1 + \gamma_\tau\chi)^{-1} \frac{s_m}{1-\eta}\right] y_t^* - ... \\
&= \left[\left(\frac{\xi_\tau}{\gamma_\tau} + \chi\gamma_\tau\right)^{-1} \frac{s_m s_c}{(1-\eta)\gamma_\tau} \frac{\eta}{1-\eta} - (1 + \gamma_\tau\chi)^{-1} \frac{s_m}{1-\eta}\right] (a_{Ct} + p_{Ct}^*)
\end{aligned}
$$

Since it depends on exogenous variables, we can replace it with an u such that

$$
y_{Ht}^e - y_{Ht}^n = (\kappa(\chi + \gamma_\tau^{-1}))^{-1} u_t
$$

Or

$$u_t = (\kappa(\chi + \gamma_\tau^{-1}))(y^e_{Ht} - y^n_{Ht})$$

in the marginal cost

$$\begin{aligned}
\pi_{Ht} &= \beta E_t \pi_{Ht+1} + \kappa(\chi + \gamma_\tau^{-1})(y_{Ht} - y^e_{Ht}) + u_t \\
\pi_{Ht} &= \beta E_t \pi_{Ht+1} + \xi x_{Ht} + u_t \qquad (72)
\end{aligned}$$

Where

$$\xi = \kappa(\chi + \gamma_\tau^{-1}) = \kappa \left(\frac{1 + \chi \gamma_\tau}{\gamma_\tau} \right)$$

The Euler equation

$$c_t = -(i_t - \mathbb{E}_t \pi_{t+1}) + \mathbb{E}_t c_{t+1}$$

Define the efficient interest rate as:

$$\begin{aligned}
r^e_t &= \mathbb{E}_t c^e_{t+1} - c^e_t \\
c^e_t &= -r^e_t + \mathbb{E}_t c^e_{t+1}
\end{aligned}$$

So that

$$c_t - c^e_t = -(i_t - \mathbb{E}_t \pi_{t+1} - r^e_t) + \mathbb{E}_t(c_{t+1} - c^e_{t+1})$$

Imperfect and perfect risk-sharing conditions:

$$\begin{aligned}
c_t &= (1 - \gamma)\tau_t + y^*_t + \lambda_t \\
c^e_t &= (1 - \gamma)\tau^e_t + y^*_t
\end{aligned}$$

So that:

$$c_t - c_t^e = (1-\gamma)(\tau_t - \tau_t^e) + \lambda_t$$

From the resources constraint (69)

$$\begin{aligned} y_{Ht} - y_{Ht}^e &= \gamma_\tau(\tau_t - \tau_t^e) \\ x_{Ht} &= \gamma_\tau(\tau_t - \tau_t^e) \end{aligned} \tag{73}$$

Thus:

$$\begin{aligned} c_t - c_t^e &= (1-\gamma)(\tau_t - \tau_t^e) \\ c_t - c_t^e &= \frac{(1-\gamma)}{\gamma_\tau}\gamma_\tau(\tau_t - \tau_t^e) \\ c_t - c_t^e &= \frac{(1-\gamma)}{\gamma_\tau} y_{Ht} - y_{Ht}^e \end{aligned}$$

Thus:

$$\begin{aligned} & c_t - c_t^e = -(i_t - \mathbb{E}_t\pi_{t+1} - r_t^e) + \mathbb{E}_t(c_{t+1} - c_{t+1}^e) \\ & \frac{(1-\gamma)}{\gamma_\tau} y_{Ht} - y_{Ht}^e = -(i_t - \mathbb{E}_t\pi_{t+1} - r_t^e) + \mathbb{E}_t(\frac{(1-\gamma)}{\gamma_\tau} y_{H,t+1} - y_{H,t+1}^e) \\ & \frac{(1-\gamma)}{\gamma_\tau} x_{Ht} = -(i_t - \mathbb{E}_t\pi_{t+1} - r_t^e) + \frac{(1-\gamma)}{\gamma_\tau}\mathbb{E}_t x_{Ht+1} \\ & x_{Ht} = -\frac{\gamma_\tau}{(1-\gamma)}(i_t - \mathbb{E}_t\pi_{t+1} - r_t^e) + \mathbb{E}_t x_{Ht+1} \\ & x_{Ht} = -\sigma_\gamma^{-1}(i_t - \mathbb{E}_t\pi_{t+1} - r_t^e) + \mathbb{E}_t x_{Ht+1} \end{aligned} \tag{74}$$

Where

$$\sigma_\gamma = \frac{(1-\gamma)}{\gamma_\tau}$$

Finally, inflation:

$$p_t = (1-\gamma)p_{Ht} + \gamma p_{Ft}$$

$$p_t = p_{Ht} + \gamma(p_{Ft} - p_{Ht})$$

$$p_t = p_{Ht} + \gamma\tau_t$$

$$p_{t-1} = p_{Ht-1} + \gamma\tau_{t-1}$$

To get

$$\pi_t = \pi_{Ht} + \gamma(\tau_t - \tau_{t-1}) \tag{75}$$

K. Welfare derivation

The representative household welfare is defined as:

$$W_0 = E_0\left\{\sum_{t=0}^{\infty}\beta^t\left[lnC_t - \frac{(Y_{Ht}/A_{Ht})^{1+\chi}}{1+\chi}exp(z_t)^{1+\chi}\right]\right\}$$

The following second-order approximation:

$$y_{Ht} + \frac{1}{2}y_{Ht}^2 = s_c(c_{Ht} + \frac{1}{2}c_{Ht}^2) + s_m(m_t + \frac{1}{2}m_t^2) \tag{76}$$

$$m_t = \frac{1}{1-\nu}(a_{Ct} + p_{Ct}^* + \tau_t) \tag{77}$$

$$c_t = \lambda_t + y_t^* + (1-\gamma)\tau_t \tag{78}$$

$$c_{Ht} = \gamma\tau_t + c_t \tag{79}$$

Rewrite labor dis-utility as:

$$\frac{Y_{Ht}^{1+\chi}}{(1+\chi)A_t^{1+\chi}}z_t^{1+\chi} \approx \frac{\bar{Y}_H^{1+\chi}}{(1+\chi)\bar{A}^{1+\chi}}\bar{z}^{1+\chi} + \bar{Y}_H^{1+\chi}(y_t - \bar{y}_t) + ...$$
$$\frac{1}{2}(1+\chi)\bar{Y}_H^{1+\chi}(y_t - \bar{y})^2 + \bar{Y}_H^{1+\chi}(log\int_0^1\left(\frac{P_{Ht}(i)}{P_{Ht}}\right)^{-\epsilon}di - log1) + ...$$

Define z_t as:

$$z_t = log \int_0^1 \left(\frac{P_{Ht}(i)}{P_{Ht}} \right)^{-\epsilon} di$$

$$\int_0^1 \left(\frac{P_{Ht}(i)}{P_{Ht}} \right)^{-\epsilon} di = 1 + \frac{\epsilon}{2} var_i p_{Ht}(i)$$

$$\int_0^1 \left(\frac{P_{Ht}(i)}{P_{Ht}} \right)^{-\epsilon} di = 1 + \frac{\epsilon}{2} var_i p_{Ht}(i)$$

$$\Delta_t = var_i p_{Ht}(i)$$

$$\sum_{t=0}^{\infty} \beta^t \Delta_t = \sum_{t=0}^{\infty} \frac{\theta}{(1-\theta)(1-\beta\theta)} \pi_t^2$$

$$-\sum_{t=0}^{\infty} \beta^t \bar{Y}_H^{1+\chi} \left(log \int_0^1 \left(\frac{P_{Ht}(i)}{P_{Ht}} \right)^{-\epsilon} di \right) = -\sum_{t=0}^{\infty} \beta^t \bar{Y}_H^{1+\chi} log \left(1 + \frac{\epsilon}{2} \Delta_t \right)$$

$$log \left(1 + \frac{\epsilon}{2} \Delta_t \right) \approx \frac{\epsilon}{2} \Delta_t$$

$$\sum_{t=0}^{\infty} \beta^t \Delta_t = \sum_{t=0}^{\infty} \frac{\theta}{(1-\theta)(1-\beta\theta)} \pi_t^2$$

$$-\sum_{t=0}^{\infty} \beta^t \bar{Y}_H^{1+\chi} log \left(1 + \frac{\epsilon}{2} \Delta_t \right) = -\sum_{t=0}^{\infty} \bar{Y}_H^{1+\chi} \frac{\epsilon}{2} \beta^t \Delta_t = -\bar{Y}_H^{1+\chi} \frac{\epsilon}{2} \sum_{t=0}^{\infty} \beta^t \Delta_t = -\bar{Y}_H^{1+\chi} \frac{\epsilon}{2} \sum_{t=0}^{\infty} \frac{\theta}{(1-\theta)(1-\beta\theta)} \pi_t^2$$

$$\frac{Y_{Ht}^{1+\chi}}{(1+\chi)A_t^{1+\chi}} z_t^{1+\chi} \approx \frac{\bar{Y}_H^{1+\chi}}{(1+\chi)\bar{A}^{1+\chi}} \bar{z}^{1+\chi} + \bar{Y}_H^{1+\chi}(y_t - \bar{y}_t) + ...$$
$$\frac{1}{2}(1+\chi)\bar{Y}_H^{1+\chi}(y_t - \bar{y})^2 + \bar{Y}_H^{1+\chi} \frac{\epsilon}{2} \frac{\theta}{(1-\theta)(1-\beta\theta)} \pi_t^2 + 2\frac{1}{2} g_{\bar{A}\bar{Y}} \bar{Y}\bar{A}(y_{Ht} - \bar{y})(a_t - \bar{a})...$$

Then the welfare

$$
\begin{aligned}
W_t = & \sum_{t=0}^{\infty} \beta^t \left[lnC_t - \frac{(Y_{Ht}/A_{Ht})^{1+\chi}}{1+\chi} exp(z_t)^{1+\chi} \right] \\
\approx & \sum_{t=0}^{\infty} \beta^t \left[c_t - \left(\bar{Y}_H^{1+\chi}(y_t - \bar{y}_t) - (1+\chi)\bar{Y}_H^{1+\chi} y_{Ht} a_{Ht} + \frac{1}{2}(1+\chi)\bar{Y}_H^{1+\chi}(y_t - \bar{y})^2 + \bar{Y}_H^{1+\chi} \frac{\epsilon}{2} \frac{\theta}{(1-\theta)(1-\beta\theta)} \pi_t^2 \right) \right] + \\
\approx & \sum_{t=0}^{\infty} \beta^t \left[c_t - \left(\bar{Y}_H^{1+\chi} y_{Ht} + \frac{1}{2}(1+\chi)\bar{Y}_H^{1+\chi} y_{Ht}^2 - (1+\chi)\bar{Y}_H^{1+\chi} y_{Ht} a_{Ht} + \bar{Y}_H^{1+\chi} \frac{\epsilon}{2\kappa} \pi_t^2 \right) \right] + ... \\
\approx & \sum_{t=0}^{\infty} \beta^t \left[c_t - \bar{Y}_H^{1+\chi} \left(y_{Ht} + \frac{1}{2}(1+\chi) y_{Ht}^2 + \frac{\epsilon}{2\kappa} \pi_t^2 - (1+\chi) y_{Ht} a_{Ht} \right) \right] + ... \\
\approx & \sum_{t=0}^{\infty} \beta^t \left[c_t - \bar{Y}_H^{1+\chi} \left(y_{Ht} + \frac{1}{2} \left[(1+\chi)(y_{Ht}^2 - 2 y_{Ht} a_{Ht}) + \frac{\epsilon}{\kappa} \pi_t^2 \right] \right) \right] + ...
\end{aligned}
$$

Where $\kappa = \frac{(1-\theta)(1-\beta\theta)}{\theta}$. Since $\bar{L} = \bar{Y_H}$

$$
\begin{aligned}
c_t - L^{1+\chi} y_{Ht} = & c_t - L^{1+\chi} \left(s_c(c_{Ht} + \frac{1}{2} c_{Ht}^2) + s_m(m_t + \frac{1}{2} m_t^2) - \frac{1}{2} y_{Ht}^2 \right) \\
= & c_t - L^{1+\chi} \left(s_c c_{Ht} + s_m m_t + \frac{1}{2}(s_c c_{Ht}^2 + s_m m_t^2 - y_{Ht}^2) \right) \\
= & c_t - L^{1+\chi} \left(s_c(\gamma\tau_t + c_t) + s_m(\frac{1}{1-\nu}(a_{Ct} + p_{Ct}^* + \tau_t)) + \frac{1}{2}(s_c c_{Ht}^2 + s_m m_t^2 - y_{Ht}^2) \right) \\
= & y_t^* + (1-\gamma)\tau_t + \lambda_t - L^{1+\chi} \left(s_c(\gamma\tau_t + c_t) + s_m(\frac{1}{1-\nu}(a_{Ct} + p_{Ct}^* + \tau_t)) + \frac{1}{2}(s_c c_{Ht}^2 + s_m m_t^2 - y_{Ht}^2) \right) \\
= & (1-\gamma)\tau_t + \lambda_t - L^{1+\chi} \left(s_c(\gamma\tau_t + (1-\gamma)\tau_t + \lambda_t) + s_m(\frac{1}{1-\nu}(\tau_t)) + \frac{1}{2}(s_c c_{Ht}^2 + s_m m_t^2 - y_{Ht}^2) \right) + t.i. \\
= & (1-\gamma)\tau_t + (1 - L^{1+\chi} s_c)\lambda_t - L^{1+\chi} \left(s_c\tau_t + s_m(\frac{1}{1-\nu}\tau_t) + \frac{1}{2}(s_c c_{Ht}^2 + s_m m_t^2 - y_{Ht}^2) \right) + t.i.p \\
= & (1-\gamma)\tau_t + (1 - L^{1+\chi} s_c)\lambda_t - L^{1+\chi} \left((s_c + s_m \frac{1}{1-\nu})\tau_t + \frac{1}{2}(s_c c_{Ht}^2 + s_m m_t^2 - y_{Ht}^2) \right) + t.i.p \\
= & (1-\gamma)\tau_t + (1 - L^{1+\chi} s_c)\lambda_t - L^{1+\chi} \left(\xi_\tau \tau_t + \frac{1}{2}(s_c c_{Ht}^2 + s_m m_t^2 - y_{Ht}^2) \right) + t.i.p \\
= & (1-\gamma)\tau_t + (1 - L^{1+\chi} s_c)\lambda_t - L^{1+\chi} \xi_\tau \tau_t - \frac{L^{1+\chi}}{2} \left(s_c c_{Ht}^2 + s_m m_t^2 - y_{Ht}^2 \right) + t.i.p \\
= & ((1-\gamma) - L^{1+\chi} \xi_\tau)\tau_t + (1 - L^{1+\chi} s_c)\lambda_t - \frac{L^{1+\chi}}{2} \left(s_c c_{Ht}^2 + s_m m_t^2 - y_{Ht}^2 \right) + t.i.p
\end{aligned}
$$

where ξ_τ is such that $((1-\gamma) - L^{1+\chi}\xi_\tau) = 0$ so that

$$c_t - L^{1+\chi} y_{Ht} = (1 - L^{1+\chi} s_c)\lambda_t - \frac{L^{1+\chi}}{2}\left(s_c c_{Ht}^2 + s_m m_t^2 - y_{Ht}^2\right) + t.i.p$$

The Welfare then:

$$\begin{aligned} W_t &\approx \sum_{t=0}^{\infty} \beta^t \left[c_t - \bar{Y}_H^{1+\chi}\left(y_{Ht} + \frac{1}{2}\left[(1+\chi)(y_{Ht}^2 - 2y_{Ht}a_{Ht}) + \frac{\epsilon}{\kappa}\pi_t^2\right]\right)\right] + t.i.p \\ &\approx \sum_{t=0}^{\infty} \beta^t \left[(1 - L^{1+\chi} s_c)\lambda_t - \frac{L^{1+\chi}}{2}\left(s_c c_{Ht}^2 + s_m m_t^2 + \chi y_{Ht}^2 - 2(1+\chi) y_{Ht} a_{Ht} + \frac{\epsilon}{\kappa}\pi_t^2\right)\right] + t.i.p \end{aligned}$$

Or in a compact form

$$W_t \approx \sum_{t=0}^{\infty} \beta^t \mathcal{L}_t$$

Where

$$\mathcal{L}_t = (1 - L^{1+\chi} s_c)\lambda_t - \frac{L^{1+\chi}}{2}\left(s_c c_{Ht}^2 + s_m m_t^2 + \chi y_{Ht}^2 - 2(1+\chi) y_{Ht} a_{Ht} + \frac{\epsilon}{\kappa}\pi_t^2\right)$$

We still have the linear term of the wedge, which we must take out. Here we must use the budget constraint, though it is a bit problematic. We define:

$$\begin{aligned} NX_t &= P_t Y_{Ht} - P_t C_t + P_{Ct} Y_{Ct} - P_{Ht} M_t \\ &= P_{Ht}(C_{Ht} + M_t) - P_t C_t + P_{Ct} Y_{Ct} - P_{Ht} M_t \\ &= P_{Ht} C_{Ht} - P_{Ht} C_{Ht} - P_{Ft} C_{Ft} + P_{Ct} Y_{Ct} \\ &= P_{Ct} Y_{Ct} - P_{Ft} C_{Ft} \\ &= S_t P_{Ct}^* Y_{Ct} - S_t P_{Ft}^* C_{Ft} \\ &= S_t (P_{Ct}^* Y_{Ct} - P_{Ft}^* C_{Ft}) \end{aligned}$$

First-order approximation

$$
\begin{aligned}
nx_t &= p^*_{Ct} + y_{Ct} - p^*_{Ft} - c_{Ft} \\
&= \frac{1}{1-\nu}(p^*_{Ct} + \nu a^*_{Ct}) + \frac{\nu}{1-\nu}\tau_t + \lambda_t + y^*_t
\end{aligned}
$$

Now we use the second-order approximation of the budget constraint.

$$
\begin{aligned}
&b_{t+1} + \frac{1}{2}b^2_{t+1} = nx_t + nx^2_t + \frac{1}{\beta}(b_t + \frac{1}{2}b^2_t) \\
&0 = \sum_{t=0}^{\infty} \beta^t (nx_t + nx^2_t)
\end{aligned}
$$

Assuming that

$$\lim_{n \longrightarrow \infty} \beta^n b_{t+n} = 0$$

$$\lim_{n \longrightarrow \infty} \beta^n \frac{1}{2} b^2_{t+n} = 0$$

$$b_0 = 0$$

Where

$$b_t = log\left(\frac{B_t}{Y_H}\right)$$

$$
\begin{aligned}
0 &= \sum_{t=0}^{\infty} \beta^t \left(\frac{\nu}{1-\nu}\tau_t + \lambda_t + \frac{1}{2}\left(\frac{\nu}{1-\nu}\right)^2 \tau_t^2 + \frac{1}{2}\lambda_t^2 \right) + t.i.p \\
nx_t &= (1-\gamma+\gamma\Lambda_t - \Lambda_t) P_t^F C_t^* && (80) \\
-\,b_0 &= \sum_{t=0}^{\infty} \beta^t (1-\gamma+\gamma\Lambda_t-\Lambda_t) P_t^F C_t^* && (81) \\
&\sum_{t=0}^{\infty} \beta^t - (1-\gamma)(\Lambda_t - 1) P_t^F C_t^* = 0 && (82) \\
&\sum_{t=0}^{\infty} \beta^t (-(1-\gamma)\bar{C}^* \bar{P}^F (\Lambda_t - 1) + 0 + \ldots) = 0 && (83) \\
&\sum_{t=0}^{\infty} \beta^t \left(\frac{\Lambda_t - 1}{1} \right) = 0 && (84) \\
&\sum_{t=0}^{\infty} \beta^t \left(\tilde{\Lambda}_t \right) = 0 && (85) \\
&\sum_{t=0}^{\infty} \beta^t \left(\hat{\Lambda}_t + \frac{1}{2}\hat{\Lambda}_t^2 \right) = 0 && (86) \\
&-\sum_{t=0}^{\infty} \beta^t \left(\hat{\Lambda}_t \right) = \sum_{t=0}^{\infty} \beta^t \left(\frac{1}{2}\hat{\Lambda}_t^2 \right) && (87)
\end{aligned}
$$

Using (48) condition

$$c_{Ht} = \tau_t + y_t^* + F$$

$$\mathbb{W} \equiv \sum_{t=0}^{\infty} \beta^t \left[-\frac{1}{2}\left((1-\gamma^2) + (1-\gamma)\gamma^2\right)\hat{\Lambda}_t^2 - \gamma\frac{\varepsilon}{2}\frac{\theta}{(1-\beta\theta)(1-\theta)}(\pi_t^H)^2 - \gamma\frac{(1+\chi)}{2}(x_t^H)^2 \right] + t.i.p$$

$$\mathbb{W} \equiv \sum_{t=0}^{\infty} \beta^t \left[-\frac{1}{2}\left(1-\gamma^3\right)\hat{\Lambda}_t^2 - \gamma\frac{\varepsilon}{2}\frac{\theta}{(1-\beta\theta)(1-\theta)}(\pi_t^H)^2 - \gamma\frac{(1+\chi)}{2}(x_t^H)^2 \right] + t.i.p$$

Finally, we obtain:

$$\mathbb{W} \equiv -\frac{1}{2}\sum_{t=0}^{\infty} \beta^t \left(\phi_\lambda \hat{\Lambda}_t^2 + \phi_\pi (\pi_t^H)^2 + \phi_y (x_t^H)^2 \right) \tag{88}$$

where:

$$\phi_\lambda = 1 - \gamma^3 \tag{89}$$

$$\phi_\pi = \frac{\gamma\varepsilon\theta}{(1-\beta\theta)(1-\theta)} \tag{90}$$

$$\phi_y = \gamma(1+\chi) \tag{91}$$

$$\begin{aligned}
\mathcal{L}_t &= (1 - L^{1+\chi}s_c)\lambda_t - \frac{L^{1+\chi}}{2}\left(s_c c_{Ht}^2 + s_m m_t^2 + \chi y_{Ht}^2 - 2(1+\chi)y_{Ht}a_{Ht} + \frac{\epsilon}{\kappa}\pi_t^2\right) \\
&= (1 - L^{1+\chi}s_c)\lambda_t - \frac{L^{1+\chi}}{2}\left(s_c(\tau_t + y_t^* + \lambda_t)^2 + s_m m_t^2 + \chi y_{Ht}^2 - 2(1+\chi)y_{Ht}a_{Ht} + \frac{\epsilon}{\kappa}\pi_t^2\right) \\
&= (1 - L^{1+\chi}s_c)\lambda_t - \frac{L^{1+\chi}}{2}\left(s_c\tau_t^2 + 2s_c\tau_t y_t^* + s_c(y_t^*)^2 + s_c\lambda_t s_m m_t^2 + \chi y_{Ht}^2 - 2(1+\chi)y_{Ht}a_{Ht} + \frac{\epsilon}{\kappa}\pi_t^2\right) \\
&= (1 - L^{1+\chi}s_c)\lambda_t - \frac{L^{1+\chi}}{2}\left(s_c\tau_t^2 + 2s_c\tau_t y_t^* + s_m m_t^2 + \chi y_{Ht}^2 - 2(1+\chi)y_{Ht}a_{Ht} + \frac{\epsilon}{\kappa}\pi_t^2\right) + t.i.p
\end{aligned}$$

Using the log-linear version of the demand of commodities:

$$m_t = (1-\nu)^{-1}(a_{Ct} + p_{Ct}^* + \tau_t)$$

square both sides:

$$m_t^2 = (1-\nu)^{-2}(a_{Ct} + p_{Ct}^* + \tau_t)^2 = (1-\nu)^{-2}(a_{Ct}^2 + (p_{Ct}^*)^2 + \tau_t^2 + 2(a_{Ct}p_{Ct}^* + \tau_t p_{Ct}^* + a_{Ct}\tau_t))$$

and t.i.p

$$m_t^2 = (1-\nu)^{-2}(\tau_t^2 + 2\tau_t p_{Ct}^* + 2a_{Ct}\tau_t) + t.i.p$$

$$\begin{aligned}
\mathcal{L}_t &= (1 - L^{1+\chi}s_c)\lambda_t - \frac{L^{1+\chi}}{2}\left(s_c\tau_t^2 + 2s_c\tau_t y_t^* + s_m\frac{\tau_t^2 + 2\tau_t p_{Ct}^* + 2a_{Ct}\tau_t}{(1-\nu)^2} + \chi y_{Ht}^2 - 2(1+\chi)y_{Ht}a_{Ht} + \frac{\epsilon}{\kappa}\pi_t^2\right) + t.i.p \\
&= -\frac{L^{1+\chi}}{2}\left\{\left[s_c - \frac{s_m}{(1-\nu)^2}\right]\tau_t^2 + 2\left[s_c y_t^* + \frac{s_m}{(1-\nu)^2}(p_{Ct}^* + a_{Ct})\right]\tau_t + \chi y_{Ht}^2 - 2(1+\chi)y_{Ht}a_{Ht} + \frac{\epsilon}{\kappa}\pi_t^2\right\} + t.i.p
\end{aligned}$$

From the efficient level of output (68):

$$\chi\xi_\tau y^e_{Ht} = -\zeta_\tau \tau^e_t + (1+\chi)\xi_\tau a_{Ht} - s_c y^*_t - \frac{s_m}{(1-\nu)^2}(a_{Ct} + p^*_{Ct})$$
$$s_c y^*_t + \frac{s_m}{(1-\nu)^2}(a_{Ct} + p^*_{Ct}) = -\zeta_\tau \tau^e_t + (1+\chi)\xi_\tau a_{Ht} - \chi\xi_\tau y^e_{Ht}$$

The expression:

$$2\left[s_c y^*_t + \frac{s_m}{(1-\nu)^2}(p^*_{Ct} + a_{Ct})\right]\tau_t = 2\left[-\zeta_\tau \tau^e_t + (1+\chi)\xi_\tau a_{Ht} - \chi\xi_\tau y^e_{Ht}\right]\tau_t$$

Again the loss function:

$$\begin{aligned}
\mathcal{L}_t &= -\frac{L^{1+\chi}}{2}\{\left[s_c - \frac{s_m}{(1-\nu)^2}\right]\tau_t^2 - 2\zeta_\tau \tau^e_t \tau_t + 2(1+\chi)\xi_\tau a_{Ht}\tau_t - 2\chi\xi_\tau y^e_{Ht}\tau_t + ...\\
&+ \chi y^2_{Ht} - 2(1+\chi) y_{Ht} a_{Ht} + \frac{\epsilon}{\kappa}\pi_t^2\} + t.i.p\\
&= -\frac{L^{1+\chi}}{2}\{\left[s_c - \frac{s_m}{(1-\nu)^2}\right](\tau_t^2 - 2\tau^e_t\tau_t) - 2(1+\chi)a_{Ht}(y_{Ht} - \xi_\tau\tau_t) - 2\chi\xi_\tau y^e_{Ht}\tau_t + ...\\
&+ \chi y^2_{Ht} + \frac{\epsilon}{\kappa}\pi_t^2\} + t.i.p\\
&= -\frac{L^{1+\chi}}{2}\{\left[s_c - \frac{s_m}{(1-\nu)^2}\right](\tau_t^2 - 2\tau^e_t\tau_t) + \chi y^2_{Ht} - 2(1+\chi)a_{Ht}(y_{Ht} - \xi_\tau\tau_t) + ...\\
&- 2\chi\xi_\tau y^e_{Ht}\tau_t + \frac{\epsilon}{\kappa}\pi_t^2\} + t.i.p
\end{aligned}$$

From the resource constraint equation (69)

$$y_{Ht} = \xi_\tau\tau_t + s_c y^*_t + \frac{1}{1-\nu}(a_{Ct} + p^*_{Ct})$$
$$y_{Ht} - \xi_\tau\tau_t = s_c y^*_t + \frac{1}{1-\nu}(a_{Ct} + p^*_{Ct})$$
$$y_{Ht} - \xi_\tau\tau_t = t.i.p$$

The expression $\xi_\tau y^e_{Ht} = y^e_{Ht}\xi_\tau\tau_t$

$$\xi_\tau\tau_t = y_{Ht} - s_c y^*_t - \frac{1}{1-\nu}(a_{Ct} + p^*_{Ct})$$

So that:

$$y_{Ht}^e \xi_\tau \tau_t = y_{Ht}^e (y_{Ht} - s_c y_t^* - \frac{1}{1-\nu}(a_{Ct} + p_{Ct}^*)$$

$$y_{Ht}^e \xi_\tau \tau_t = y_{Ht}^e (y_{Ht} - t.i.p)$$

Therefore, rewriting the loss function:

$$\mathcal{L}_t = -\frac{L^{1+\chi}}{2} \left\{ \left[s_c - \frac{s_m}{(1-\nu)^2} \right] (\tau_t^2 - 2\tau_t^e \tau_t) + \chi y_{Ht}^2 - t.i.p - 2\chi y_{Ht}^e (y_{Ht} + t.i.p) + \frac{\epsilon}{\kappa} \pi_t^2 \right\} + t.i.p$$
$$\mathcal{L}_t = -\frac{L^{1+\chi}}{2} \left\{ \zeta_\tau (\tau_t^2 - 2\tau_t^e \tau_t) + \chi y_{Ht}^2 - 2\chi y_{Ht}^e y_{Ht} + \frac{\epsilon}{\kappa} \pi_t^2 \right\} + \zeta_\tau \tau_t^2 + \gamma(y_{Ht}^e) + t.i.p$$
$$\mathcal{L}_t = -\frac{L^{1+\chi}}{2} \left\{ \zeta_\tau (\tau_t - \tau_t^e)^2 + \chi (y_{Ht} - y_{Ht}^e)^2 + \frac{\epsilon}{\kappa} \pi_t^2 \right\} + t.i.p$$

From the resource constraint equation for both equilibrium (69)

$$y_{Ht} = \xi_\tau \tau_t + s_c y_t^* + \frac{1}{1-\nu}(a_{Ct} + p_{Ct}^*)$$
$$y_{Ht}^e = \xi_\tau \tau_t^e + s_c y_t^* + \frac{1}{1-\nu}(a_{Ct} + p_{Ct}^*)$$

Reducing both terms gives us:

$$y_{Ht} - y_{Ht}^e = \xi_\tau (\tau_t - \tau_t^e)$$

Finally, the loss function:

$$\mathcal{L}_t = -\frac{L^{1+\chi}}{2}\left\{\zeta_\tau(\tau_t - \tau_t^e)^2 + \chi(y_{Ht} - y_{Ht}^e)^2 + \frac{\epsilon}{\kappa}\pi_t^2\right\} + t.i.p$$
$$\mathcal{L}_t = -\frac{L^{1+\chi}}{2}\left\{\zeta_\tau\gamma_\tau^{-2}(y_{Ht} - y_{Ht}^e)^2 + \chi(y_{Ht} - y_{Ht}^e)^2 + \frac{\epsilon}{\kappa}\pi_t^2\right\} + t.i.p$$
$$\mathcal{L}_t = -\frac{L^{1+\chi}}{2}\left\{(\zeta_\tau\gamma_\tau^{-2} + \chi)(y_{Ht} - y_{Ht}^e)^2 + \frac{\epsilon}{\kappa}\pi_t^2\right\} + t.i.p$$
$$\mathcal{L}_t = -\frac{(1-\gamma)}{\xi_\tau 2}\left\{\left(\frac{\zeta_\tau}{\gamma_\tau^{-2}} + \chi\right)x_t^2 + \frac{\epsilon}{\kappa}\pi_t^2\right\} + t.i.p$$
$$\mathcal{L}_t = -\frac{(1-\gamma)}{\xi_\tau 2}\frac{\epsilon}{\kappa}\left\{\frac{\kappa}{\epsilon}\left(\frac{\zeta_\tau}{\gamma_\tau^{-2}} + \chi\right)x_t^2 + \pi_t^2\right\} + t.i.p$$
$$\mathcal{L}_t = -\frac{(1-\gamma)}{\xi_\tau 2}\frac{\epsilon}{\kappa}\left\{\lambda_x x_t^2 + \pi_t^2\right\} + t.i.p$$
$$\mathcal{L}_t = -\frac{\Omega}{2}\left\{\pi_t^2 + \lambda_x x_t^2\right\} + t.i.p$$

Where

$$\Omega = \frac{(1-\gamma)\epsilon}{\kappa\xi_\tau}$$

and

$$\lambda_x = \frac{\kappa}{\epsilon}\left(\frac{\zeta_\tau}{\gamma_\tau^2} + \chi\right)$$

L. Non-linear Model - Flexible Prices

Aggregate demand (y_t)

$$Y_t^H = \gamma\left(\mathcal{S}_t\right)^{1-\gamma} C_t + M_t \tag{92}$$

Real exchange rate (rer_t)

$$Q_t = S_t P_t^* \tag{93}$$

Euler equation (c_t)

$$C_t^{-\gamma_c} = \beta E_t\left(C_{t+1}^{-\gamma_c}\frac{1+i_t}{1+\pi_{t+1}}\right) \tag{94}$$

Price Level (p_t)

$$1 = \left(t_t^H\right)^\gamma\left(t_t^F\right)^{1-\gamma} \tag{95}$$

Terms of trade $\mathcal{T}_t$

$$\mathcal{T}_t = \frac{P_t^F}{P_t^H} \tag{96}$$

Labour supply (w_t)

$$L_t^{\chi} C_t^{\gamma_c} = w_t \tag{97}$$

Domestic home goods demand $\left(C_t^H\right)$

$$C_t^H = \gamma \left(t_t^H\right)^{-1} C_t \tag{98}$$

Domestic foreign goods demand $\left(C_t^F\right)$

$$C_t^F = (1-\gamma)\left(t_t^F\right)^{-1} C_t \tag{99}$$

Modified UIP (s_t)

$$S_t = E_t S_{t+1} \frac{(1+i_t^*)}{1+i_t}\left(1+\frac{\omega}{m}\sigma^2 d_{t+1}^*\right) \tag{100}$$

Home goods supply (y_t^H)

$$Y_t^H = A_t L_t \tag{101}$$

Labour demand (l_t)

$$\frac{P_t^H}{P_t} = \frac{\epsilon}{\epsilon - 1} M C_t^H \tag{102}$$

Commodity Tech $\left(Y_t^C\right)$

$$Y_t^C = A_{Ct} M_t^{\nu} \tag{103}$$

Commodity sector demand for H goods (M_t)

$$P_t^C A_{Ct} \nu M_t^{\nu-1} - P_{Ht} = 0 \tag{104}$$

Commodity sector profits $\left(\Psi^C\right)$

$$\Psi_t^C = (1-\nu) P_t^C Y_t^C \tag{105}$$

Marginal cost $\left(mc_t^H\right)$

$$MC_t^H = (1-\mu_H)\frac{w_t}{A_t} \tag{106}$$

Current account LHS (CA_t)

$$\frac{CA_t}{\bar{Y}} = S_t B_{t+1}^{cb,*} + S_t D_{t+1}^{*} - N_{t+1} - S_{t-1} B_t^{cb,*} - S_{t-1} D_t^{*} + N_t \tag{107}$$

Current account RHS (CA_t)

$$\frac{CA_t}{\bar{Y}} = NX_t + \left(\frac{S_t}{S_{t-1}} R_{t-1}^{*} - 1\right)\left(S_{t-1} B_t^{cb,*} + S_{t-1} D_t^{*}\right) - \left(R_{t-1} - 1\right) N_t \tag{108}$$

Net exports (NX_t)

$$NX_t = P_t^C Y_t^C - (1-\gamma) Q_t^{-1} C_t \tag{109}$$

Domestic goods relative price t^H

$$t_t^H = \frac{P_t^H}{P_t} \tag{110}$$

Foreign goods relative price t^F

$$t_t^F = \frac{P_t^F}{P_t} \tag{111}$$

Portfolio shocks (ψ_t)

$$n_t^{*} = \rho_\psi n_{t-1}^{*} + \sigma_\psi \varepsilon_t^{\psi} \tag{112}$$

Productivity shocks (a_t):

$$a_t = \rho_a a_{t-1} + \sigma_a \varepsilon_t^{a} \tag{113}$$

Commodity Prices $\left(P_t^C\right)$

$$P_t^C = \rho^c P_{t-1}^c + \varepsilon_t^C \tag{114}$$