Slacktivism

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Abstract

Many countries have introduced e-government petitioning systems, in which a petition that gathers a certain quota of signatures triggers some political outcome. This paper models citizens who choose whether to sign such a petition. Citizens are imperfectly informed about the petition's chance of bringing change. The number of citizens is large, while the cost of signing is positive but low. I show that a petition that can bring change succeeds by a strictly positive margin. Hence, a citizen signing the petition is almost surely not pivotal. On the other hand, a petition that cannot bring change still gathers the required number of signatures when citizens are not very well informed, implying a failure of information aggregation.

Keywords: online petitions, collective action, voting, political participation, threshold public goods.

JEL codes: D72, D83, H41.

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1 Introduction

Online petitions have become a part of the political process in many democracies. Two features of this form of political participation stand out. First, the number of potential participants is very large, essentially including the entire electorate. Second, while there is a cost of participation – the time and effort required to sign an online petition – that cost is usually negligible. Because of the very low cost, political action through online petitions is sometimes referred to, derisively, as slacktivism.

This paper studies participation in this form of political action, under one particular assumption. Specifically, the paper considers settings in which the outcome of the petition depends on whether the number of signatures reaches an exogenous quota.

There are several reasons why exogenous thresholds are relevant to online petitions. First, while a petition can signal preferences of the public to policy makers, they are far more likely to receive the signal if the petition is sufficiently widely reported. Not all petition become newsworthy, and it is reasonable to think that only petitions that attract sufficiently many signatures are reported in the media. Even if all petitions are reported, media reports usually contain coarse information about the number of citizens who sign a petition – for example, a headline may report that a certain petition has gathered "more than a million signatures", without giving the exact number. For these reasons, the signalling value of a petition makes a jump when a certain exogenous number of signatures is reached.

Second, petitions do not only serve as a means of signalling public opinion to policy makers. Instead, in a number of countries, legislatures or governments have committed to act on a petition if it attracts sufficiently many signatures. In these e-government petitioning systems, a petition signed by a certain number of citizens will be debated in the legislature or will trigger some other official response.[1] Although the legislature can still vote to reject the petition, the probability of political change jumps from zero to a positive value if the required number of signatures is reached.

To analyse such settings, the paper adapts the model of threshold public good games to the context of online petitions. A large number of citizens are choosing whether to sign a petition, at some cost. If the petition collects a certain number of signatures, it will be considered by the legislature. Citizens are uncertain about the preferences of members of the legislature. These preferences are represented by a state of the world: in state 0 the legislature will reject the petition, and in the complementary state 1 it will approve it. All citizens receive an imperfect continuous signal about the state. If the petition is approved, it gives each citizen a positive payoff. Hence, citizens have imperfect information about the petition's chance of success, but are fully informed about its value if it succeeds. Citizens' choice is thus driven by the tradeoff between the cost of signing the petition and the benefit of increasing its chance of success.

A crucial feature of the model is that the cost of signing the petition is positive but low. This reflects the nature of online activism, in which the cost of participation – the effort required to open, read, and sign the petition – exists, but is negligible. Specifically, the paper analyses sequences of equilibria when the number of citizens goes to infinity, while the cost of signing falls within certain bounds. In the limit, both the upper and the lower bound converge to zero, but at every point along the sequence there is a realistic interval of small but positive costs that satisfy the bounds. As a consequence, the usual "paradox of not voting" (Downs, 1957) does not emerge: as the size of the electorate approaches infinity, participation in the limit is not universal but is also distinct from zero.

The first result of the paper is that in the limit, aside from a special case, the petition either succeeds or fails by a margin that is strictly positive, both in absolute terms and in relation to the number of citizens. Thus, in the limit, a citizen who signs the petition is almost surely not pivotal, even though along the sequence the probability of being pivotal remains positive. In particular, at a stable equilibrium, in state 1 the petition almost surely succeeds. This outcome is realistic, as the number of signatures that online petitions gather seems to often be far from the quota. At the same time, this prediction contrasts with the standard result in pivotal voter models of elections, where in the limit the outcome is almost surely very close (see, for example, Feddersen and Pesendorfer, 1997). The reason is that in an election, a voter is pivotal when the number of votes on her side matches the number of votes for the other side, and the latter is endogenous and adjusts at the equilibrium. Here, on the other hand, a citizen is pivotal when the number of signatures matches a threshold, which is exogenous.

Second, I analyse the outcomes in state 0, in which the legislature opposes the petition. Because citizens receive informative signals, the petition collects fewer signatures in that state than in state 1. If the outcome was close in state 1, then in state 0 the petition would not reach the required quota of signatures. However, because the petition exceeds the quota by a strictly positive margin in state 1, it can still reach the quota in state 0 too. This happens whenever individual signals are sufficiently imprecise. Hence, a petition can still be forwarded to the legislature and then rejected – an outcome that appears to be fairly common.[2] Such an outcome may impose a social cost – for example, a waste of legislators' time – that is not internalised by citizens signing the petition. Thus, the petitioning procedure does not always attain a social optimum. This again differs from the standard result in pivotal voter models of large elections, in which the election aggregates information to produce an optimal outcome even when individual signals are weak.

Finally, I analyse how the institutional setup of the petitioning procedure affects the willingness of citizens to sign the petition. Even though in equilibrium participation rate does not equal the required quota of signatures, an increase in the quota still induces greater participation. One consequence of this is that the size of the quota does not affect the outcome of the petition, as participation adjusts to the quota. However, since participation is costly, a lower quota reduces the total cost to citizens, increasing welfare.

The rest of this section discusses the related literature. Section 2 presents the model and discusses the assumptions behind it. Section 3 derives equilibria, and characterises political outcomes in each state. Section 4 analyses the effect of the quota of signatures on participation and on social welfare. It also shows that the results of the paper continue to hold when individual costs are heterogeneous. Section 5 concludes. The Appendix contains the proofs, as well as two technical results (Claim 1 and Claim 2) that are used in the proofs of the main results.

Related literature. This paper contributes to the literature on petitions and other forms of political activism. In addition to experimental work on online petitions (Margetts et al., 2011; Ginzburg and Guerra, 2022), a number of papers have developed theoretical models in which citizens choose whether to join a public protest or other type of collective action. Lohmann (1993), Banerjee and Somanathan (2001), and Battaglini and Benabou (2003) model costly participation in a population of finite size. Battaglini (2017) and Battaglini et al. (2021) model participation as costless. Ekmekci and Lauermann (2019) analyse both setups. In these models, citizens have information about the value of a policy, and engaging in a protest serves to signal this information to a decision-maker.[3] At the equilibrium, the decision-maker prefers to change the policy once participation reaches an endogenous cutoff. My paper adds to this literature in two ways. First, I focus on settings in which the cutoff is exogenously determined, either by the rules of the e-government petitioning procedure, or by media reporting policies. Hence, by signing the petition, citizens are not merely sending a signal, but are directly affecting political outcomes. Second, my model incorporates a key feature of online petition – positive but negligible costs of participating, combined with a large number of potential participants. This underlies the main results of the paper, including the strictly positive margin of success, and the ability of a hopeless petition to reach the required quota of signatures. More broadly, the paper adds to the literature studying the role of online platforms in shaping political behaviour (see Pogorelskiy and Shum, 2019; Enikolopov et al., 2020; Denter et al., 2021; Denter and Ginzburg, 2022 for recent work on this topic).

Collective action in which success or failure depends on meeting an exogenous threshold has been the focus of the literature on discrete public goods (see Palfrey and Rosenthal, 1984, for a classic reference). In particular, a number of authors (McBride, 2006; Barbieri and Malueg, 2010; Krasteva and Yildirim, 2013) study discrete public good games with imperfect information (for recent experimental work on discrete public good games, see Cartwright et al., 2019; Lim and Zhang, 2020; Bolle and Spiller, 2021; Ginzburg et al., 2023). Within this literature, the papers that are closest to mine are those that analyse asymptotic properties of the discrete public good game. Nöldeke and Peña (2020) analyse the effect of group size on participation, expected payoffs, and the probability that the public good is provided. They show that all three decrease in group size. In the limit, participation converges to zero while the probability of success remains positive. There are important differences between the model of Nöldeke and Peña and that of this paper. First, they consider a sequence of games with a fixed cost of participation, while in this paper, the cost of participation decreases along the sequence, falling within certain bounds. Second, in the model of Nöldeke and Peña, the threshold remains fixed as the group size increases – consequently, the fraction of group members that need to contribute to ensure success approaches zero, and hence the probability that a given group member participates converges to zero as well. In my paper, however, the magnitude of the threshold remains a constant fraction q of the size of the group – thus, the number of participants required for success increases in absolute terms at the same rate as the group size. These features of the cost and of the threshold imply a different result: in the limit, participation remains strictly positive, even though the probability that a given citizen is pivotal equals zero. Additionally, in my model, citizens are imperfectly informed about the possibility of the public good being provided, which underlies the analysis of information aggregation.

Dziuda et al. (2020) also study asymptotic properties of a discrete public good game. In their model, the value of the public good relative to the cost approaches infinity while the size of the group remains fixed. Because the group size is finite, there is uncertainty about whether the public good is provided. The authors show that both the probability of an individual contributing and the probability of success are increasing in the size of the threshold. The former result also emerges in this paper. However, in my model, while the value of the public good relative to the cost approaches infinity as in Dziuda et al., the size of the group approaches infinity as well. Furthermore, unlike in Dziuda et al., group members are imperfectly informed about their payoffs conditional on reaching the threshold. Because of large group size and uncertain consequences of reaching the threshold, uncertainty enters the results of my model in a different way. Specifically, whether the threshold is reached depends on the state of the world and on the precision of individual signals. However, conditional on these factors, the outcome of the petition is deterministic: in the good state the threshold is almost surely reached irrespective of how restrictive it is, while in the bad state it is reached if signals are sufficiently imprecise.

More broadly, the paper is related to the literature on costly voting (Borgers, 2004; Levine and Palfrey, 2007; Krishna and Morgan, 2012; Ambrus et al., 2017; Myatt, 2015). My paper differs from that literature in two ways. First, I focus on a limit case in which the electorate is large, while the cost is small but positive. Second, in this paper the cost of voting has a very particular form: while signing the petition is costly, voting for the opposite alternative – that is, not signing the petition – is costless. Hence, only voters who prefer to sign the petition face a participation dilemma.

Finally, the paper is also related to the literature on referenda with approval quorums, in which a proposal is adopted only if a certain exogenous share of voters support it. Aguiar-Conraria and Magalhães (2010) analyse such referenda in a model without voting costs. Maniquet and Morelli (2015) examine approval quorums in an electorate of finite size. In Herrera and Mattozzi (2010), voting is costly, but voters are not concerned with being pivotal – instead, each voter receives a payoff from voting for her preferred alternative that depends on the parties' mobilisation efforts. In Aguiar-Conraria et al. (2020), the set of voters is finite, and some voters have negative costs of voting. My paper adds to this literature by analysing a quorum rule in a pivotal voter model with a large electorate and positive, though negligible, cost.

2 Model

There are $n_r + 1$ citizens, who are exposed to an online petition. They simultaneously choose whether to open the petition and sign it. Doing so requires effort, the cost of which is c_r. The pair (c_r, n_r), will be referred to as the *voting environment* indexed by r.

The petition will be considered by the legislature if it gathers strictly more than qn_r signatures, where $q \in (0, 1)$. To simplify notation, throughout the paper I will focus on sequences of voting environments in which qn_r is an integer for all r. If the petition is approved by the legislature, all citizens (both those who signed the petition, and those who did not) receive a payoff that is normalised to 1. A petition that is rejected gives a payoff of 0 to all citizens.

Citizens are uncertain about the legislature's eventual decision[4], but they receive imperfect signals about it upon being exposed to the petition. Formally, there is a state of the world $\theta \in \{0, 1\}$. In state 0 the legislature rejects the petition, while in state 1 it approves the petition. Ex ante, the probability that the state equals 1 is α. At the beginning of the game, each citizen i receives a private signal $s_i \in [\underline{s}, \overline{s}]$. In each state θ, signals of all citizens are drawn independently from cdf F_θ with full support on $s_i \in [\underline{s}, \overline{s}]$, and with density f_θ. Let s denote a generic realisation of the signal. Given a signal s, let $h(s)$ be the posterior probability that the true state is 1. From Bayes rule it follows that $h(s) = \frac{\alpha f_1(s)}{\alpha f_1(s)+(1-\alpha) f_0(s)}$. I will assume that $\frac{f_0(s)}{f_1(s)}$ is strictly increasing in s. This standard monotone likelihood property implies that $h(s)$ is monotone decreasing in s, and hence higher realisations of s indicate a lower probability that $\theta = 1$.

After citizens observe their signals, they simultaneously choose whether to open and sign the petition. Let $\sigma_i(s)$ be citizen i's strategy, that is, her probability of signing the petition after receiving signal s. The paper will focus on symmetric equilibria, in which $\sigma_i(s) = \sigma(s)$, $\forall i$.

Furthermore, the paper will study equilibrium behaviour when the number of citizens is large while the cost is small. Formally, the paper will examine sequences of voting environments such that $\lim_{r\to\infty} n_r = \infty$, while c_r belongs to an interval described by the following assumption, the role of which is discussed further below:

Assumption 1. There exists some $\lambda \in (0, 1)$ such that $\lambda^{n_r} \leq c_r \leq \frac{\sqrt{2\pi}}{e^2} \frac{h(\overline{s})}{\sqrt{q(1-q)n_r}}$ for all r.

The focus of the paper will be the equilibria that emerge in the limit as $r \to \infty$.

Discussion of the modelling choices. One assumption of the model is that citizens make decisions simultaneously. Specifically, they choose whether to pay the cost c_r after observing signals about the petition's chance of succeeding, but without knowing the actions of other citizens. Even when the petitioning system reveals the current number of signatures, this setup often reflects reality. First, many citizens learn about petitions not directly from the petitions website, but via social media or other channels. A citizen who is exposed to the petition through a social media post will, from its headline, have some idea about its chance of success, but is unlikely to know the current tally of signatures prior to clicking on the link and reading the petition – that is, prior to paying the effort cost c_r. Second, even if the citizen is first exposed to the petition through the petition platform, many such platforms require paying the cost prior to observing the current number of signatures. For instance, in the e-petitioning system of the Scottish Parliament, users need to open the petition and scroll until the end to see the number of signatures (Scottish Parliament, 2021).

The model also assumes that all citizens benefit from the petition. This is, in fact, without loss of generality, because acting against the petition – that is, not signing it – is costless. Hence, citizens who oppose the petition do not face a participation dilemma. They can then be omitted from the analysis, and n_r can be interpreted as the number of citizens who receive a positive payoff if the petition succeeds.

A key element of the model is Assumption 1. For each voting environment r, it describes the interval of the values of the cost for which the model is applicable. This ensures the existence of symmetric equilibria with nonzero and nonuniversal participation. Note that the paper focuses on the limit case in which the population size n_r is large, rather than on the effect of increasing n_r. Hence, Assumption 1 should not be understood as describing how an increase in n_r affects the cost. Rather, it states that, while the population is large, the cost of signing the petition is strictly positive but small.

Importantly, Assumption 1 is not "knife-edge". As n_r becomes large, the upper and the lower bound converge to zero in the limit. Nevertheless, along the sequence at every r there exists an interval of costs for which Assumption 1 holds. This interval is realistic. To see this, note that the lower bound is very close to zero for large electorates, even when λ is close to one. The upper bound, on the other hand, can be realistically high. For example, suppose that the electorate consists of 20 million citizens; the minimum number of signatures required for the petition to be forwarded to the legislature is 20 thousand (hence, $q = 0.001$); and $h(\overline{s})$, the probability that the legislature supports the petition conditional on the least favourable signal, is 0.5. Then Assumption 1 says that the cost should not exceed 0.12% of the individual benefit from the petition (which is normalised to one). Since the cost of opening, reading, and signing an online petition is usually very low relative to the potential magnitude of political change in case the petition succeeds, this bound is reasonable.

Finally, the model assumes that the cost of signing is the same for all citizens. In Section 4, I show that the results continue to hold if citizens have heterogeneous costs of signing.

3 Main Results

First, note that a strategy profile in which every citizen signs the petition irrespective of her signal cannot be an equilibrium, because in this case the number of signatures will exceed qn_r with probability 1, and hence every citizen would gain by deviating. Consider instead a strategy profile in which no citizen signs the petition, irrespective of the signal. If $qn_r \geq 1$, no citizen is pivotal under this strategy profile, and hence no citizen gains by deviating. Thus, abstention by all citizens is an equilibrium. For the rest of the analysis, I will focus on equilibria in which each citizen signs the petition with a positive probability. I will refer to these as *positive participation* equilibria.

Take a citizen i who has received signal s_i. If the petition gathers more than qn_r signatures, citizen i will receive a payoff of 1 if $\theta = 1$, and a payoff of 0 if $\theta = 0$. Recall that a higher realisation of s_i is associated with a lower probability that the state is 1. Intuitively, this means that in a given voting environment, a citizen prefers to sign the petition when her signal is sufficiently low, that is, below some cutoff s^r. If that cutoff equals $\underline{s}$, no citizen signs the petition. If it is greater than $\underline{s}$, there exists a positive participation equilibrium. The cutoff must be such that a citizen who receives signal s^r is indifferent between signing and not signing the petition. Hence, the probability of influencing the outcome of the petition conditional on receiving signal s^r must equal the cost of signing. The following result proves the intuition above:

Lemma 1. *Any positive participation equilibrium is characterised by a cutoff $s^r > \underline{s}$ given by the condition*

$$h(s^r) \binom{n_r}{qn_r} F_1(s^r)^{qn_r} \left[1 - F_1(s^r)\right]^{(1-q)n_r} = c_r. \tag{1}$$

Furthermore, in every voting environment r at least one positive participation equilibrium exists.

In the above expression, $F_1(s^r)$ is the probability that a citizen receives a signal below the cutoff in state 1. Hence, it is the probability that a citizen signs the petition in state 1. In turn, $\binom{n_r}{qn_r} F_1(s^r)^{qn_r} \left[1 - F_1(s^r)\right]^{(1-q)n_r}$ is the probability that a citizen is pivotal when the state is 1, while $h(s^r)$ is the probability that the state is 1 conditional on the signal equalling the cutoff. Hence, the left-hand side of (1) is the probability that a citizen's signature changes the outcome of the petition when she receives signal s^r.

Lemma 1 establishes the existence of positive participation equilibria for all finite values of r. At each equilibrium, in state θ a citizen signs a petition with probability $F_\theta(s^r) > 0$. On its own, however, Lemma 1 does not imply that participation remains strictly positive as $r \to \infty$ (i.e. as the electorate becomes arbitrarily large), because s^r may be converging to $\underline{s}$. If that is the case, participation would converge to zero, and the standard paradox of not voting would emerge. We can now show, however, that this does not happen.

Formally, let $r \to \infty$. As each finite r is associated with an equilibrium cutoff s^r, this implies a sequence of such cutoffs. Then the following result characterises equilibria in the limit as $r \to \infty$ (that is, equilibria in large electorates with low cost of participation), and shows that participation in the limit is nonzero and not universal:

Proposition 1. *As $r \to \infty$, there exists a sequence of voting environments along which $c_r^{\frac{1}{n_r}}$ converges to some limit $L \in [\lambda, 1]$, and s^r converges to some limit $\hat{s}$. If $L = 1$, then $\hat{s}$ is given by the equation $F_1(\hat{s}) = q$. If $L < 1$, then $\hat{s}$ can take one of two values $\hat{s}_1, \hat{s}_2$, such that $0 < F_1(\hat{s}_1) < q < F_1(\hat{s}_2) < 1$.*

In words, if the electorate becomes arbitrarily large and the cost stays in the interval described by Assumption 1, in the limit there exist equilibria in which the level of participation is substantial.[5] In the special case when, along the sequence, the relation between the number of citizens and the cost of signing is such that $c_r^{\frac{1}{n_r}}$ converges to one, the fraction of citizens that sign the petition in state 1 converges to q.

In the more general case, when $c_r^{\frac{1}{n_r}}$ does not converge to one, in the limit there exist two positive participation equilibria. The first is a low participation equilibrium, with cutoff $\hat{s}_1$. At this equilibrium the fraction of citizens who sign the petition in state 1 equals $F_1(\hat{s}_1)$, which is strictly below q, and hence the petition almost surely fails. The second is a high participation equilibrium, characterised by cutoff $\hat{s}_2$. At that equilibrium the fraction of signatures in state 1 equals $F_1(\hat{s}_2)$, which is strictly greater than q, and hence the petition almost surely succeeds. In both of these asymptotic equilibria the fraction of citizens signing the petition is distinct from zero and from one. Recall also that there exists, in addition, a zero participation equilibrium, at which no citizen signs the petition (i.e. $\hat{s} = \underline{s}$).

One way to select between the two positive participation asymptotic equilibria is to check equilibrium stability – that is, to check how each equilibrium responds to a small perturbation in strategies. The next proposition describes equilibrium stability:

Proposition 2. *Suppose $L < 1$. Then the high participation asymptotic equilibrium characterised by $F_1(\hat{s}_2) > q$ is stable, while the low participation asymptotic equilibrium characterised by $F_1(\hat{s}_1) < q$ is unstable.*

The fact that $F_1(\hat{s}_2) > q$ implies that at a stable equilibrium, a successful petition gathers strictly more than the required quota of signatures, both in absolute terms and relative to the size of the population. Thus, as $r \to \infty$, in the limit citizens who sign the petition know that they are almost surely not pivotal. Along the sequence, however, at every r the probability of being pivotal is sufficiently large that enough citizens strictly prefer to sign the petition.[6]

We can now turn to analysing the outcomes in state 0, that is, when the petition has no chance of being approved by the legislature. Given an asymptotic equilibrium cutoff $\hat{s}$, the share of signatures in state 0 converges to $F_0(\hat{s})$. This is smaller than $F_1(\hat{s})$, because the monotone likelihood ratio assumption implies that $F_0(s) < F_1(s)$ for all $s \in [\underline{s}, \overline{s}]$.

If $L = 1$, we have $F_0(\hat{s}) < F_1(\hat{s}) = q$, and hence the petition is not sent to the legislature. Consider instead the more general case when $L < 1$. At the low participation asymptotic equilibrium, the share of signatures in state 0 converges to $F_0(\hat{s}_1) < F_1(\hat{s}_1) < q$ – hence, the petition is again not sent to the legislature.

In contrast, at the (stable) high participation equilibrium, the following result emerges:

Proposition 3. *Suppose $L < 1$. In state 0 at the stable positive participation asymptotic equilibrium the petition attains the required quota of signatures if $q < F_0(\hat{s}_2) < F_1(\hat{s}_2)$, and does not attain it if $F_0(\hat{s}_2) < q < F_1(\hat{s}_2)$.*

Hence, at the stable equilibrium, a hopeless petition is forwarded to the legislature if and only if the interval $(F_0(\hat{s}_2), F_1(\hat{s}_2))$ does not cover q. Intuitively, this happens when the distance between F_0 and F_1 is small – that is, when individual signals are not very informative.

It is sometimes reasonable to think that sending a hopeless petition to the legislature is suboptimal. For example, the time that legislators spend on discussing the petition may have some cost, which is not internalised by individual citizens when they sign the petition. Then Proposition 3 suggests that when individual signals are not sufficiently precise, a socially optimal outcome is not achieved even with a large electorate. This differs from the standard result in the literature on information aggregation. The difference comes from the fact that in the standard models, the margin of victory in the limit converges to zero (see, for example, Feddersen and Pesendorfer, 1997). This happens in my setting when $L = 1$, but not in the more general case when $L < 1$.

4 Discussion of Results

Effect of quota on participation. How does participation depend on q? If the group size was finite, then for a sufficiently low cost of participation, a higher threshold would increase the probability that an individual is pivotal, thus raising the incentive to participate (see, for example, Dziuda et al., 2020). In my setting, however, the group size is large. As a consequence, when $L < 1$, the share of citizens signing the petition is almost surely distinct from q, and the probability of a citizen being pivotal is zero, irrespective of the threshold. Nevertheless, the following result shows that in all positive participation equilibria, there is a strictly monotone relationship between the required quota of signatures and participation:

Proposition 4. *In any positive participation asymptotic equilibrium, participation is strictly increasing with q in each state.*

Hence, in a large electorate, the share of citizens who sign the petition at any positive participation equilibrium increases when the petitioning procedure becomes more conservative.

Cost of petition and optimal quota. What value of q is socially optimal? Proposition 1 implies that in state 1, as $r \to \infty$, the petition almost surely succeeds at the stable high participation equilibrium, and almost surely fails at the other two equilibria. Crucially, these outcomes do not depend on the value of q. Hence, out of two values of q, we would prefer the one that achieves the outcome at a lower cost to citizens.

In each state $\theta \in \{0,1\}$, in a given voting environment r, the total cost equals $c_r n_r F_\theta(s^r)$. As $r \to \infty$, this converges to $F_\theta(\hat{s}) \lim_{r\to\infty} c_r n_r$, as long as $\lim_{r\to\infty} c_r n_r$ exists. Assumption 1 implies that for all r we have

$$n_r \lambda^{n_r} \leq c_r n_r \leq \frac{\sqrt{2\pi}}{e^2}\sqrt{\frac{n_r}{q(1-q)}}h(\bar{s}).$$

In the limit, the lower bound on $c_r n_r$ converges to zero, while the upper bound becomes infinite. Hence, $\lim_{r\to\infty} c_r n_r$, if it exists, can take any weakly positive value. If $\lim_{r\to\infty} c_r n_r = 0$, then the total cost converges to zero, and hence all petitioning procedures are welfare-equivalent.

If $c_r n_r$ converges to a finite positive limit, then in the limit the total cost is proportional to $F_\theta(\hat{s})$. By Proposition 4, $F_\theta(\hat{s})$, and hence the total cost, increases with q. This means that a lower quota of signatures is preferable to a higher quota.[7]

Heterogeneous costs. The model assumes that all citizens have the same cost of signing the petition. This section will show that the results continue to hold if costs are heterogeneous.

Suppose that the cost of signing the petition for citizen i is $\mu_i c_r$, where $\mu_i \in [\underline{\mu}, \overline{\mu}] \subset (0,1)$ is an individual-specific cost drawn from some distribution with full support independently of the signal s_i, and c_r is a common cost parameter specific to the voting environment. Let $\delta_i := \frac{h(s_i)}{\mu_i} \in \left[\frac{h(\overline{s})}{\overline{\mu}}, \frac{h(\underline{s})}{\underline{\mu}}\right]$. Then in a voting environment r citizen i signs the petition if and only if $\delta_i \geq \delta^r$ for some cutoff δ^r. Denote by G_θ the cdf of δ_i in state $\theta \in \{0,1\}$. Then the equilibrium is given analogously to (1) as

$$\delta^r \binom{n_r}{q n_r} G_1(\delta^r)^{(1-q)n_r} \left[1 - G_1(\delta^r)\right]^{q n_r} = c_r.$$

Once Assumption 1 is modified with $\frac{h(\overline{s})}{\overline{\mu}}$ replacing $h(\overline{s})$, all of the results of the paper can now be derived in this more general setting. The proofs are the same, except that δ_i replaces $h(s_i)$, and $1 - G_1(\cdot)$ replaces $F_1(\cdot)$. In particular, two positive participation equilibria will emerge, in which the petition will either succeed or fail with a strictly positive margin.

5 Concluding Remarks

This paper looked at online petitions whose outcome depends on whether the number of signatures reaches an exogenous threshold. Two salient features of online petitions are a large number of potential signatories, and a low but positive cost of signing. To analyse this form of political participation, the paper developed a model in which the number of potential participants approaches infinity, while the cost of participation is bounded from above and from below. Furthermore, citizens are imperfectly informed about the value of the petition.

In the limit, participation is nonzero but not universal, and citizens sign the petition even though they are almost surely not pivotal. At a stable equilibrium, a petition that can bring change succeeds by a strictly positive margin. At the same time, a hopeless petition can still gather the required quota of signatures if citizens are insufficiently informed. This contrasts with standard models of voting, in which the margin of victory converges to zero in a large electorate and the election aggregates information. The paper also found a monotone relationship between the required quota of signatures and participation level, which implies that low quotas may be socially preferable.

One feature of the model is that citizens choose whether to pay the cost without knowing the number of other citizens that had signed it earlier. While this is a reasonable assumption in many situations, a different model of online petitions could allow citizens to condition their choice on the number of previous signatories. Future research can analyse this alternative setup.

6 Appendix

For the proofs of the results in the main text, the following two technical results will be useful:

Claim 1. $\frac{\sqrt{2\pi}}{e^2} \leq \binom{n_r}{qn_r} q^{qn_r} (1-q)^{(1-q)n_r} \sqrt{q(1-q)n_r} \leq \frac{e}{2\pi}$ for all r.

Proof. Robbins (1955) shows that $\sqrt{2\pi} k^{k+\frac{1}{2}} e^{-k} e^{\frac{1}{12k+1}} < k! < \sqrt{2\pi} k^{k+\frac{1}{2}} e^{-k} e^{\frac{1}{12k}}$ for all positive integers k. This implies the following weaker result for all positive integers k:

$$\sqrt{2\pi} k^{k+\frac{1}{2}} e^{-k} \leq k! \leq e k^{k+\frac{1}{2}} e^{-k} \tag{2}$$

where the first inequality follows from the fact that $e^{\frac{1}{12k+1}} > 1$; while the second inequality holds trivially for $k = 1$, and for $k \geq 2$ it follows from the fact that $\sqrt{2\pi} e^{\frac{1}{12k}} \leq \sqrt{2\pi} e^{\frac{1}{24}} < e$. Applying (2) to $\binom{n_r}{qn_r} = \frac{n_r!}{(qn_r)!([1-q]n_r)!}$ yields

$$\frac{\sqrt{2\pi} n_r^{n_r+\frac{1}{2}} e^{-n_r}}{\left[e (qn_r)^{qn_r+\frac{1}{2}} e^{-qn_r}\right] \left[e ([1-q] n_r)^{(1-q)n_r+\frac{1}{2}} e^{-(1-q)n_r}\right]} \leq \binom{n_r}{qn_r}$$
$$\leq \frac{e n_r^{n_r+\frac{1}{2}} e^{-n_r}}{\left[\sqrt{2\pi} (qn_r)^{qn_r+\frac{1}{2}} e^{-qn_r}\right] \left[\sqrt{2\pi} ([1-q] n_r)^{(1-q)n_r+\frac{1}{2}} e^{-(1-q)n_r}\right]}.$$

This can be simplified as

$$\frac{\sqrt{2\pi}}{e^2}\frac{1}{q^{qn_r+\frac{1}{2}}\left(1-q\right)^{(1-q)n_r+\frac{1}{2}}n_r^{\frac{1}{2}}}\leq\binom{n_r}{qn_r}\leq\frac{e}{2\pi}\frac{1}{q^{qn_r+\frac{1}{2}}\left([1-q]\right)^{(1-q)n_r+\frac{1}{2}}n_r^{\frac{1}{2}}},$$

which is equivalent to the statement of the lemma. □

Claim 2. The function $A(s) := [F_1(s)]^q [1 - F_1(s)]^{1-q}$ is strictly increasing in s for $F_1(s) < q$, and strictly decreasing in s for $F_1(s) > q$.

Proof. Taking logs and differentiating, we obtain $\frac{d\ln A(s)}{dF_1(s)} = \frac{q}{F_1(s)} - \frac{1-q}{1-F_1(s)}$ which is strictly positive for $F_1(s) < q$ and strictly negative for $F_1(s) > q$. Therefore, $\ln A(s)$, and hence also $A(s)$, is increasing in s for $F_1(s) < q$ and decreasing in s for $F_1(s) > q$. □

Proof of Lemma 1. Let $p_r(k)$ denote the probability that at least k citizens sign the petition conditional on $\theta = 1$. Then citizen i's expected payoff equals $h(s_i)\,p_r(qn_r) - c_r$ if she signs the petition, and $h(s_i)\,p_r(qn_r + 1)$ if she does not. She then signs the petition if and only if $h(s_i)\,[p_r(qn_r) - p_r(qn_r + 1)] \geq c_r$.

Let y_r be the equilibrium probability that a randomly selected citizen signs the petition when $\theta = 1$. Then citizen i will sign the petition if and only if

$$h(s_i)\binom{n_r}{qn_r}y_r^{qn_r}\left(1-y_r\right)^{(1-q)n_r}\geq c_r. \tag{3}$$

Since $h(s_i)$ is strictly decreasing, the equilibrium is of a cutoff form as described in the lemma. Hence, $y_r = F_1(s^r)$, which together with the fact that a citizen is indifferent between signing and not signing when $s_i = s^r$ yields the first statement.

To show existence, note that since the left-hand side of (1) is continuous in s^r, it is sufficient to show that for all r, there exist values of $s^r > \underline{s}$ for which the left-hand side of (1) is smaller than c_r; as well values of $s^r > \underline{s}$ for which it is larger than c_r. For the former, pick any s^r such that $0 < F_1(s^r) < \left[\frac{c_r}{h(\underline{s})\binom{n_r}{qn_r}}\right]^{\frac{1}{qn_r}}$. Then, since $[1 - F_1(s^r)]^{(1-q)n_r} < 1$, the left-hand side of (1) is smaller than

$$h(s^r)\binom{n_r}{qn_r}\left[F_1(s^r)\right]^{qn_r}<\frac{h(s^r)}{h(\underline{s})}c_r<c_r,$$

where the last inequality follows from monotonicity of $h(\cdot)$. For the latter, let $s^r = F_1^{-1}(q)$.

Then the left-hand side of (1) is

$$\begin{aligned}
& h\left(s^r\right)\binom{n_r}{qn_r} q^{qn_r}\left(1-q\right)^{(1-q)n_r} \\
\geq & \frac{\sqrt{2\pi}}{e^2}\frac{h\left(s^r\right)}{\sqrt{q\left(1-q\right)n_r}} \\
> & \frac{\sqrt{2\pi}}{e^2}\frac{h\left(\overline{s}\right)}{\sqrt{q\left(1-q\right)n_r}} \geq c_r,
\end{aligned}$$

where the first inequality follows from Claim 1, the second – monotonicity of $h\left(\cdot\right)$, and the third – from Assumption 1. □

Proof of Proposition 1. To show that $c_r^{\frac{1}{n_r}}$ converges to $L \in [\lambda, 1]$, note that by Assumption 1, $\lambda \leq c_r^{\frac{1}{n_r}} \leq \left(\frac{\sqrt{2\pi}}{e^2}\frac{h(\overline{s})}{\sqrt{q(1-q)n_r}}\right)^{\frac{1}{n_r}}$. Furthermore, when r is sufficiently large, $\frac{\sqrt{2\pi}}{e^2}\frac{h(\overline{s})}{\sqrt{q(1-q)n_r}} \leq 1$, so $\left(\frac{\sqrt{2\pi}}{e^2}\frac{h(\overline{s})}{\sqrt{q(1-q)n_r}}\right)^{\frac{1}{n_r}} \leq 1$. Hence, $\lambda \leq c_r^{\frac{1}{n_r}} \leq 1$ when r is sufficiently large, and the existence of a convergent subsequence follows from Bolzano–Weierstrass theorem.

Now consider a sequence of r at which $c_r^{\frac{1}{n_r}}$ converges. Since s^r is bounded, there exists a subsequence at which s^r converges to some limit $\hat{s}$. To characterise it, we can rewrite (1) as

$$F_1\left(s^r\right)^q\left[1-F_1\left(s^r\right)\right]^{(1-q)} = \left[\frac{c_r}{h\left(s^r\right)\binom{n_r}{qn_r}}\right]^{\frac{1}{n_r}}. \tag{4}$$

At the same time, Claim 1 can be written as

$$\left[\frac{\sqrt{2\pi}}{e^2}\right]^{\frac{1}{n_r}} \leq \left[\binom{n_r}{qn_r}\right]^{\frac{1}{n_r}} q^q\left(1-q\right)^{(1-q)}\left[\sqrt{q\left(1-q\right)}\right]^{\frac{1}{n_r}} n_r^{\frac{1}{2n_r}} \leq \left[\frac{e}{2\pi}\right]^{\frac{1}{n_r}}. \tag{5}$$

Note that $\lim\limits_{r\to\infty}\left(\frac{\sqrt{2\pi}}{e^2}\right)^{\frac{1}{n_r}} = \lim\limits_{r\to\infty}\left(\frac{e}{2\pi}\right)^{\frac{1}{n_r}} = 1$. Hence, (5) implies

$$\lim_{r\to\infty}\left(\left[\binom{n_r}{qn_r}\right]^{\frac{1}{n_r}}\left[\sqrt{q\left(1-q\right)}\right]^{\frac{1}{n_r}} n_r^{\frac{1}{2n_r}}\right) = \frac{1}{q^q\left(1-q\right)^{(1-q)}}.$$

Note that $\lim\limits_{r\to\infty}\left(\sqrt{q\left(1-q\right)}\right)^{\frac{1}{n_r}} = 1$. Also, $\lim\limits_{r\to\infty} n_r^{\frac{1}{2n_r}} = \lim\limits_{n\to\infty} e^{\frac{1}{2n}\ln n} = 1$, where the last equality comes from the fact that $\lim_{n\to\infty}\frac{\ln n}{2n}=\lim_{n\to\infty}\frac{1}{2n}=0$ by L'Hôpital's rule. Hence, we have

$$\lim_{r\to\infty}\left[\binom{n_r}{qn_r}\right]^{\frac{1}{n_r}}=\frac{1}{q^q\,(1-q)^{(1-q)}}.$$

Substituting this into (4), together with the fact that $\lim_{r\to\infty} c_r^{\frac{1}{n_r}}=L$ and $\lim_{r\to\infty}\left[h\left(s^r\right)\right]^{\frac{1}{n_r}}=1$ yields

$$\lim_{r\to\infty}\left[F_1\left(s^r\right)^q\left[1-F_1\left(s^r\right)\right]^{(1-q)}\right]=Lq^q\,(1-q)^{(1-q)}>0.$$

Then $\hat{s}$ is characterised by the condition

$$\left[F_1\left(\hat{s}\right)\right]^q\left[1-F_1\left(\hat{s}\right)\right]^{1-q}=Lq^q\,(1-q)^{(1-q)}\,. \tag{6}$$

If $L=1$, (6) holds if and only if $F_1\left(\hat{s}\right)=q$. Suppose instead that $L<1$, and note that the left-hand side of (6) equals $A\left(\hat{s}\right)$ as defined in Claim 2. Then Claim 2 implies that (6) holds for exactly two values of $\hat{s}$, namely, for some $\hat{s}_1$ such that $F_1\left(\hat{s}_1\right)\in(0,q)$, and for some $\hat{s}_2$ such that $F_1\left(\hat{s}_2\right)\in(q,1)$. □

Proof of Proposition 2. Take a sequence of equilibrium cutoffs $\{s^r\}_{r=0}^{+\infty}$ converging to the high participation asymptotic cutoff $\hat{s}_2$. Now suppose citizens deviate to a sequence $\{\widetilde{s}^r\}_{r=0}^{+\infty}$ that converges to an asymptotic cutoff $\hat{s}_2+\varepsilon$, with $\varepsilon>0$ (that is, they sign the petition with a higher ex ante probability). When r is sufficiently large (so that $\widetilde{s}^r$ is sufficiently close to $\hat{s}_2+\varepsilon$), we have $F_1\left(\widetilde{s}^r\right)>F_1\left(s^r\right)>q$, and so by Claim 2, $A\left(\widetilde{s}^r\right)<A\left(s^r\right)$. Thus, as a result of the deviation, the left-hand side of (1) decreases, and to restore equilibrium, the cutoff must decrease, counteracting the initial deviation. Similarly, if citizens deviate to a sequence converging to a cutoff $\hat{s}_2-\varepsilon$ (where ε is small enough that $F_1\left(\hat{s}_2-\varepsilon\right)>q$), then $A\left(\cdot\right)$ increases, so the cutoff must increase, again counteracting the deviation. Hence, the high participation equilibrium is stable.

On the other hand, consider a sequence of equilibrium cutoffs $\{s^r\}_{r=0}^{+\infty}$ converging to the low participation asymptotic cutoff $\hat{s}_1$. Suppose citizens deviate to a sequence $\left\{\underset{\sim}{s}^r\right\}_{r=0}^{+\infty}$ that converges to an asymptotic cutoff $\hat{s}_1+\varepsilon$ for some $\varepsilon>0$ that is small enough that $F_1\left(\hat{s}_1+\varepsilon\right)<q$. When r is sufficiently large, we have $F_1\left(s^r\right)<F_1\left(\underset{\sim}{s}^r\right)<q$, and by Claim 2, $A\left(\underset{\sim}{s}^r\right)>A\left(s^r\right)$. Thus, the left-hand side of (1) increases, and to restore equilibrium, the cutoff must increase, reinforcing the initial perturbation of s^r. Similarly, if citizens deviate to a sequence converging to a cutoff $\hat{s}_1-\varepsilon$, then $A\left(\cdot\right)^{n_r}$ decreases, and the cutoff must decrease, reinforcing the deviation. Thus, the low participation equilibrium is unstable. □

Proof of Proposition 3. At the asymptotic equilibrium cutoff $\hat{s}_2$, when $\theta = 0$, the share of signatures converges to $F_0(\hat{s}_2) < F_1(\hat{s}_2)$. Additionally, $F_1(\hat{s}_2) > q$, implying the result. □

Proof of Proposition 4. When $L = 1$, $\hat{s} = F_1^{-1}(q)$ and the result follows immediately. Suppose $L < 1$. Then (6) must hold at each of the two positive participation equilibria (see proof of Proposition 1). Taking logs of (6) and differentiating it with respect to q yields

$$\ln[F_1(\hat{s})] + \frac{q}{F_1(\hat{s})}\frac{\partial F_1(\hat{s})}{\partial q} - \ln[1 - F_1(\hat{s})] - \frac{1-q}{1-F_1(\hat{s})}\frac{\partial F_1(\hat{s})}{\partial q} = \ln q + 1 - \ln(1-q) - 1$$

$$\iff \frac{\partial F_1(\hat{s})}{\partial q} = \frac{\ln\left[\frac{q}{F_1(\hat{s})}\right] - \ln\left[\frac{1-q}{1-F_1(\hat{s})}\right]}{\frac{q}{F_1(\hat{s})} - \frac{1-q}{1-F_1(\hat{s})}}$$

Note that when $L < 1$, $F_1(\hat{s}) \neq q$, and hence $\frac{q}{F_1(\hat{s})} - \frac{1-q}{1-F_1(\hat{s})} \neq 0$. Since $\frac{q}{F_1(\hat{s})} > \frac{1-q}{1-F_1(\hat{s})}$ if and only if $\ln\left[\frac{q}{F_1(\hat{s})}\right] > \ln\left[\frac{1-q}{1-F_1(\hat{s})}\right]$, the numerator and the denominator of the above expression have the same sign. Thus, $\frac{\partial F_1(\hat{s})}{\partial q} > 0$ at any equilibrium. Hence, $\hat{s}$ is increasing in q. This implies that $F_0(\hat{s})$ also increases with q. □

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Notes

1. For instance, a petition on the UK parliament website signed by at least 100 thousand citizens will be considered for debate in the parliament (UK Parliament, 2021). A similar system used in Germany requires a petition to collect 50 thousand signatures within four weeks to receive a public hearing in the national parliament (Deutsche Welle, 2017). Similar systems are used by the White House in the US; by parliaments in Canada and Latvia; by regional parliaments in Scotland and Queensland, Australia; as well as by a number of municipalities (see Lindner and Riehm, 2009; Mickoleit, 2014; Grover, 2016). These systems are used frequently: for example, several thousand petitions on the UK Parliament website are currently open, over 500 past petitions have received a response from the government, and almost 100 have been debated in the House of Commons.

2. See, for example, BBC News (2019). While citizens may also be signing a petition for expressive reasons or out of civic duty, this result suggests that a petition that has no chance of succeeding can also reach the required quota of signatures even in the absence of these considerations.

3. See also Salas (2019) for a model that considers protest organiser as a strategic agent.

4. For example, citizens might not know which members of the legislature will be present at the session in which the petition will be considered, or what their positions are on the proposal contained in the petition.

5. These three equilibria also emerge along the sequence for finite r, as long as the c_r is sufficiently low, as Nöldeke and Peña (2020) show.

6. In the context of a finite-sized threshold public good game, a distinction between a stable and un unstable equilibrium can also be made (see Nöldeke and Peña, 2020). However, in a finite-sized game, participants are pivotal with a strictly positive probability.

7. Note that all citizens are assumed to benefit from the petition. However, the aforementioned result does not depend on this assumption: because changing the quota does not affect the outcome of the petition, the only effect of the quota on payoffs is through the total cost incurred by those who sign the petition. Hence, a lower quota is Pareto optimal.