Socially concerned duopolies with lifetime employment as a strategic commitment

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Abstract

This paper considers a two-stage game model with a nonlinear concave demand function where two socially concerned firms compete with each other. Each socially concerned firm maximizes its own profit plus a fraction of consumer surplus. In the first stage, each socially concerned firm decides simultaneously and independently whether to offer lifetime employment as a strategic commitment device. At the beginning of the second stage, each socially concerned firm knows the rival's choice in the first stage, In the second stage, each socially concerned firm chooses simultaneously and independently an actual output level. The paper discusses the equilibrium outcomes of the Cournot duopoly model.

Keywords: Concave demand function; Cournot duopoly model; Lifetime employment; Socially concerned firms

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1. Introduction

Some excellent researchers have studied theoretical economic models that incorporate socially concerned firms (see, for example, Goering, 2007; Kopel and Brand, 2012; Kopel, Lamantia and Szidarovszky, 2014; Kopel, 2015; Lambertini and Tampieri, 2012; Xu, 2014; Cracau, 2015; Flores and García, 2016; Fanti and Buccella, 2018; Planer-Friedrich and Sahm, 2018; García, Leal and Lee, 2019; Han, 2019). Socially concerned firms take both profits and consumer surplus into consideration. Kopel and Brand (2012) and Goering (2007) examine the managerial incentive contract in a mixed duopoly model where a profit maximizing firm and a socially concerned firm compete in output levels. Kopel (2015) investigates the endogenous choice of a price or quantity contract in a mixed duopoly consisting of a profit maximizing firm and a socially concerned firm. Flores and García (2016) examine the output and welfare impacts of a socially concerned firm in mixed duopoly, and show that more social responsibility of the socially concerned firm may reduce welfare. García, Leal and Lee (2019) examine a mixed Cournot duopoly model in which a profit maximizing firm competes with a socially concerned firm by incorporating environmental externality and clean technology. In addition, Han (2019) introduces corporate social responsibility into a mixed oligopoly to investigate effects of socially concerned private firms on privatization of a state-owned public firm. However, these papers consider mixed oligopoly models with linear demand functions.[1]

We consider a two-stage game model with a nonlinear concave demand function where two socially concerned firms compete in quantities. In stage one, each socially concerned

[1] The analysis by Ohnishi (2021) examines a Cournot oligopoly model with a nonlinear demand function where socially concerned firms can offer lifetime employment as a strategic commitment device, and present the reaction functions of socially concerned firms in the Cournot oligopoly model. In this paper, we extent the previous work by Ohnishi (2021) by examining a concrete example.

firm chooses simultaneously and independently whether to offer lifetime employment as a strategic commitment device.[2] In stage two, after observing the rival's choice in stage one, each socially concerned firm chooses simultaneously and independently an actual output level. We discuss the equilibrium outcomes of the Cournot model.

The remainder of the paper is organized in the following manner. In Section two, we describe the model. Section three provides supplementary explanations of the model. Section four presents the equilibrium outcomes of the model. Finally, Section five concludes the paper.

2. Basic setting

Let us consider a model composed of two socially concerned firms: firm 1 and firm 2. There is no possibility of entry or exit. When i and j are used to refer to firms in an expression, they should be understood to represent (firm) 1 and (firm) 2 with $i \neq j$. The price is determined by the inverse demand function: $p = a - Q^2$, where $a \in \left(Q^2, \infty\right)$ is a constant parameter, and $Q = \sum_{i=1}^{2} q_i$ is the industry output.

The two stages of the game are as follows. In the first stage, each firm decides simultaneously and independently whether to offer lifetime employment as a strategic commitment device. If firm i offers lifetime employment, then it chooses an output level $q_i^* \in (0, \infty)$, employs the necessary number of employees to produce q_i^*, and enters into a lifetime employment contract with all of the employees. At the beginning of the second stage, firm i knows firm j's choice in the first stage. In the second stage, each firm i chooses and sells simultaneously and independently an actual output $q_i \in [0, \infty)$.

[2] For details, please see Ohnishi (2001, 2002).

Firm i's profit function is given by

$$\pi_i = \begin{cases} \left(a - Q^2\right) q_i - c q_i^2 - w q_i^2 & \text{if } q_i > q_i^*, \\ \left(a - Q^2\right) q_i - c q_i^2 - w q_i^{*2} & \text{if } q_i \le q_i^*, \end{cases} \tag{1}$$

where $c \in (0, \infty)$ represents the capital cost for each unit of output produced and $w \in (0, \infty)$ is the wage rate.

Firm i's objective function is defined by

$$V_i = \pi_i + \theta_i CS, \tag{2}$$

where CS denotes consumer surplus and $\theta_i \in [0,1]$ is the percentage of the consumer surplus. Hence, (1) can be rewritten as follows:

$$V_i = \begin{cases} \theta_i \left[\int_0^Q \left(a - X^2\right) dX - \left(a - Q^2\right) Q \right] + \left(a - Q^2\right) q_i - c q_i^2 - w q_i^2 & \text{if } q_i > q_i^*, \\ \theta_i \left[\int_0^Q \left(a - X^2\right) dX - \left(a - Q^2\right) Q \right] + \left(a - Q^2\right) q_i - c q_i^2 - w q_i^{*2} & \text{if } q_i \le q_i^*. \end{cases} \tag{3}$$

We adopt subgame perfection as our solution concept. In the next section, we provide supplementary explanations of the model.

3. Supplementary explanations

In this section, we first derive firm i's best reaction function from (3). If $q_i < q_i^*$, then firm i's reaction function is defined by

$$R_i^w(q_j) = \arg\max_{q_i \ge 0} \left\{ \theta_i \left[\int_0^Q \left(a - X^2\right) dX - \left(a - Q^2\right) Q \right] + \left(a - Q^2\right) q_i - c q_i^2 - w q_i^{*2} \right\}, \tag{4}$$

and if $q_i > q_i^*$, then firm i's reaction function is defined by

$$R_i(q_j) = \arg\max_{q_i \ge 0} \left\{ \theta_i \left[\int_0^Q \left(a - X^2\right) dX - \left(a - Q^2\right) Q \right] + \left(a - Q^2\right) q_i - c q_i^2 - w q_i^2 \right\}. \tag{5}$$

Hence, if firm i selects q_i^* and adopts a lifetime employment contract, then its best reply is given by

$$R_i^L(q_j) = \begin{cases} R_i(q_j) & \text{if } q_i > q_i^*, \\ q_i^* & \text{if } q_i = q_i^*, \\ R_i^w(q_j) & \text{if } q_i < q_i^*. \end{cases} \tag{6}$$

Firm i chooses q_i in order to maximize V_i, given q_j. Therefore, if $q_i > q_i^*$, the first-order condition for firm i is

$$a + 2\theta_i \left(q_i + q_j\right)^2 - 3q_i^2 - 2cq_i - 2wq_i - 4q_iq_j - q_j^2 = 0, \tag{7}$$

and the second-order condition is

$$\left(2\theta_i - 3\right)q_i + \left(2\theta_i - 2\right)q_j - c - w < 0. \tag{8}$$

On the other hand, if $q_i < q_i^*$, the first-order condition for firm i is

$$a + 2\theta_i \left(q_i + q_j\right)^2 - 3q_i^2 - 2cq_i - 4q_iq_j - q_j^2 = 0, \tag{9}$$

and the second-order condition is

$$\left(2\theta_i - 3\right)q_i + \left(2\theta_i - 2\right)q_j - c < 0. \tag{10}$$

Therefore, we obtain

$$R_i'(q_j) = -\frac{\left(2\theta_i - 2\right)q_i + \left(2\theta_i - 1\right)q_j}{\left(2\theta_i - 3\right)q_i + \left(2\theta_i - 2\right)q_j - c - w} \tag{11}$$

and

$$R_i^{w\prime}(q_j) = -\frac{\left(2\theta_i - 2\right)q_i + \left(2\theta_i - 1\right)q_j}{\left(2\theta_i - 3\right)q_i + \left(2\theta_i - 2\right)q_j - c}. \tag{12}$$

We now state the following lemma.

Lemma 1: (i) If $\theta_i < \left(2q_1 + q_2\right)/2\left(q_1 + q_2\right)$, then $R_i(q_j)$ and $R_i^w(q_j)$ are downward-sloping.

(ii) If $\theta_i > \left(2q_1 + q_2\right)/2\left(q_1 + q_2\right)$, then $R_i(q_j)$ and $R_i^w(q_j)$ are upward-sloping.

Next, we prove the following two lemmas, which provide characterizations of lifetime employment as a strategic commitment device.

Lemma 2: If firm i enters into a lifetime employment contract with all of the employees necessary to achieve q_i^*, then at equilibrium its actual output q_i coincides with q_i^*.

Proof: We first consider the possibility that $q_i < q_i^*$ at equilibrium when firm i offers a lifetime employment contract. Firm i's objective function V_i is $\theta_i\left[\int_0^Q (a-X^2)dX-(a-Q^2)Q\right]+(a-Q^2)q_i-cq_i^2-w(q_i^*-q_i)^2$. Then, firm i employs the extra employees necessary to produce $q_i^*-q_i$. Therefore, firm i can improve V_i by reducing q_i^*, and the equilibrium point does not change in $q_i \le q_i^*$. Hence, $q_i < q_i^*$ does not result in an equilibrium solution.

Next, we consider the possibility that $q_i > q_i^*$ at equilibrium. In this case, firm i's marginal cost is identical to that when firm i does not offer lifetime employment. It is impossible for the firm to change its output because such a strategy is not credible. Therefore, the lifetime employment contract does not function as a strategic commitment. Q.E.D.

Lemma 3: Firm i's payoff maximizing output when it offers lifetime employment is higher than that when it does not.

Proof: From (3), we see that lifetime employment will never increase the marginal cost of firm i. The first order condition for firm i when $q_i < q_i^*$ is (9):

$$a+2\theta_i\left(q_i+q_j\right)^2-3q_i^2-2cq_i-4q_iq_j-q_j^2=0 .$$

On the other hand, the first order condition for firm i when $q_i > q_i^*$ is (7):

$$a+2\theta_i\left(q_i+q_j\right)^2-3q_i^2-2cq_i-2wq_i-4q_iq_j-q_j^2=0 ,$$

where w_i is positive. To satisfy (7), $a+2\theta_i\left(q_i+q_j\right)^2-3q_i^2-2cq_i-4q_iq_j-q_j^2$ must be positive. Thus, this lemma is proved. Q.E.D.

4. Equilibrium outcomes

In this section, we examine the following three types.

Type 1: $\theta_1 < \left(2q_1+q_2\right)/2\left(q_1+q_2\right)$ and $\theta_2 < \left(2q_1+q_2\right)/2\left(q_1+q_2\right)$

Type 2: $\theta_1 > \left(2q_1+q_2\right)/2\left(q_1+q_2\right)$ and $\theta_2 < \left(2q_1+q_2\right)/2\left(q_1+q_2\right)$

Type 3: $\theta_1 > \left(2q_1+q_2\right)/2\left(q_1+q_2\right)$ and $\theta_2 > \left(2q_1+q_2\right)/2\left(q_1+q_2\right)$

We discuss these types in order.

4.1. Type 1

Each firm i aims to maximize its objective function V_i. Therefore, firm i will adopt lifetime employment if V_i increases by doing so, while it will not adopt lifetime employment if V_i decreases by doing so.

This type is depicted in Figure 1, where R_i represents firm *i*'s reaction curve without lifetime employment and V_i^N is firm *i*'s iso-payoff curve extending through N. For explanation, the figure is drawn simply. In this type, R_i slopes downwards. Point N is the equilibrium solution without lifetime employment as a strategic commitment device.

However, if firm 1 offers a lifetime employment contract, its marginal cost of production decreases and thus it increases its output (Lemma 3). In Figure 1, if firm 1 chooses q_1^* and offers lifetime employment, then its reaction curve shifts to the right for $q_1 < q_1^*$ and becomes the kinked bold lines. Therefore, firm 1's unilateral solution can occur at a point like A. In addition, if firm 2 chooses q_2^* and offers lifetime employment, then its reaction curve shifts upwards for $q_2 < q_2^*$ and becomes the kinked bold broken lines. Hence, the bilateral lifetime employment solution can become a point like B.

We now state the following lemma.

Lemma 4: Suppose that firm i unilaterally offers lifetime employment. Then in equilibrium firm *i*'s objective function V_i is larger than in the game with no lifetime employment.

Proof: From Lemma 3, we know that firm *i*'s payoff maximizing output when it offers lifetime employment is higher than that when it does not. Furthermore, from Lemma 2, we see that $q_i = q_i^*$ in equilibrium. We consider firm *i*'s Stackelberg leader output when each firm does not offer lifetime employment. Firm i selects q_i, and firm j selects q_j after observing q_i. If firm i is the Stackelberg leader, then it maximizes $V_i(q_i, R_j(q_i))$ with respect to q_i. Therefore, the Stackelberg leader output satisfies the first-order condition:

$$\frac{\partial V_i}{\partial q_i} + \frac{\partial V_i}{\partial q_j}\frac{\partial R_j}{\partial q_i} = 0.$$

Since Type 1 is the case of strategic substitutes in which goods are perfect substitutes, $\partial V_i / \partial q_j$ and $\partial R_j / \partial q_i$ are both negative. To satisfy the first-order condition, $\partial V_i / \partial q_i$ must be negative. Hence, firm *i*'s Stackelberg leader output exceeds its Cournot output. Furthermore, V_i is continuous and concave. In R_j, V_i is highest at firm *i*'s Stackelberg leader point, and the further the point on R_j gets from firm *i*'s Stackelberg leader point, the more V_i decreases. Thus, the lemma follows. Q.E.D.

The equilibrium of this type is stated in the following proposition.

Proposition 1: If $\theta_1 < (2q_1 + q_2)/2(q_1 + q_2)$ and $\theta_2 < (2q_1 + q_2)/2(q_1 + q_2)$, then there exists an equilibrium solution in which at least one firm offers lifetime employment as a strategic commitment.

Proof: From Lemma 4, we know that if firm i unilaterally offers lifetime employment, then in equilibrium firm *i*'s objective function is higher than in the game with no lifetime employment. Hence, there is an equilibrium solution in which one of them adopts lifetime employment because cycling of choices is impossible. Furthermore, in equilibrium both firms offer lifetime employment only if that is more profitable than when only one firm adopts lifetime employment. Q.E.D.

4.2. Type 2

This type is depicted in Figure 2. Point N is the equilibrium solution with no lifetime employment contract offered. If firm 1 offers lifetime employment, its marginal cost of production decreases and thus it increases its output. In Figure 2, if firm 1 chooses q_1^*

and adopts a lifetime employment contract, then its reaction curve shifts to the right for $q_1 < q_1^*$ and becomes the kinked bold lines. Therefore, firm 1's unilateral solution can be a point like D. Furthermore, if firm 2 chooses q_2^* and adopts lifetime employment, then its reaction curve is the kinked bold broken lines. Therefore, the bilateral lifetime employment solution can occur at a point like F.

We now state the following lemma.

Lemma 5: Suppose that firm i offers lifetime employment, given firm *j*'s strategy. Then in equilibrium firm *i*'s objective function V_i is lower than at the equilibrium where firm i does not offer lifetime employment.

Proof: We prove the case of firm 1. Lemma 3 states that firm 1's payoff maximizing output when it offers lifetime employment is higher than that when it does not. We consider firm 1's Stackelberg leader output when each firm does not offer lifetime employment. The Stackelberg leader (firm 1) maximizes $V_1(q_1, R_2(q_1))$ with respect to q_1, and the first-order condition is

$$\frac{\partial V_1}{\partial q_1} + \frac{\partial V_1}{\partial q_2}\frac{\partial R_2}{\partial q_1} = 0.$$

Since $\theta_1 > (2q_1 + q_2)/2(q_1 + q_2)$ and $\theta_2 < (2q_1 + q_2)/2(q_1 + q_2)$, $\partial V_1 / \partial q_2$ is positive and $\partial R_2 / \partial q_1$ is negative. To satisfy the first-order condition, $\partial V_1 / \partial q_1$ must be positive. Hence, firm 1's Stackelberg leader output is lower than its Cournot output. Furthermore, V_1 is continuous and concave. In R_2, V_1 is highest at firm 1's Stackelberg leader point, and the further the point on R_2 gets from firm 1's Stackelberg leader point, the more V_1 decreases. Furthermore, $R_1(q_2)$ gives firm 1's payoff maximizing output for each output of firm 2. In Type 2, $R_1(q_2)$ slopes upwards. In $R_1(q_2)$, increasing firm 1's output increases its objective function. Firm 1's payoff maximizing output when it offers lifetime employment is higher than that when it does not (Lemma 3). However, increasing firm 1's output does not increase firm 2's amount of demand because of (6) and Lemma 1 (i).

Since the proof of firm 2 is essentially identical to that of firm 1, it is omitted. Q.E.D.

The equilibrium of Type 2 is stated in the following proposition.

Proposition 2: If $\theta_1 > (2q_1 + q_2)/2(q_1 + q_2)$ and $\theta_2 < (2q_1 + q_2)/2(q_1 + q_2)$, then there is an equilibrium point in which neither firm offers lifetime employment as a strategic commitment.

This proposition follows easily from Lemma 5.

4.3. Type 3

This type is depicted in Figure 3. If firm 1 offers lifetime employment, its marginal cost of production decreases and thus it increases its output. In Figure 3, if firm 1 chooses q_1^* and adopts a lifetime employment contract, then its reaction curve becomes the kinked bold lines. In addition, if firm 2 chooses q_2^* and offers a lifetime employment contract, then its reaction curve is the kinked bold broken lines.

Lemma 6: Suppose that firm i unilaterally offers lifetime employment. Then in equilibrium firm i's objective function V_i is larger than in the game with no lifetime employment contract offered.

This proof is essentially identical to that of Lemma 4 and thus is omitted.

The equilibrium of Type 3 is stated in the following proposition.

Proposition 3: If $\theta_1 > (2q_1 + q_2)/2(q_1 + q_2)$ and $\theta_2 > (2q_1 + q_2)/2(q_1 + q_2)$, then there is an equilibrium point in which at least one firm adopts lifetime employment as a strategic commitment device.

Proof: Lemma 6 states that if firm i offers a unilateral lifetime employment contract, then in equilibrium firm *i*'s objective function is higher than in the game with no lifetime employment. Thus, there is an equilibrium solution in which one of them offers lifetime employment because cycling of choices is impossible. Moreover, at equilibrium both firms adopt lifetime employment only if that is more profitable than when one firm unilaterally offers lifetime employment. Q.E.D.

5. Conclusion

We have considered a two-stage game model with a concave demand function where socially concerned firms compete with each other and have presented the equilibrium outcomes of the model. In this paper, we have considered a duopoly model composed of two socially concerned firms. In the near future, we will investigate various mixed oligopoly models consisting of state-owned, socially concerned and profit-maximizing firms.

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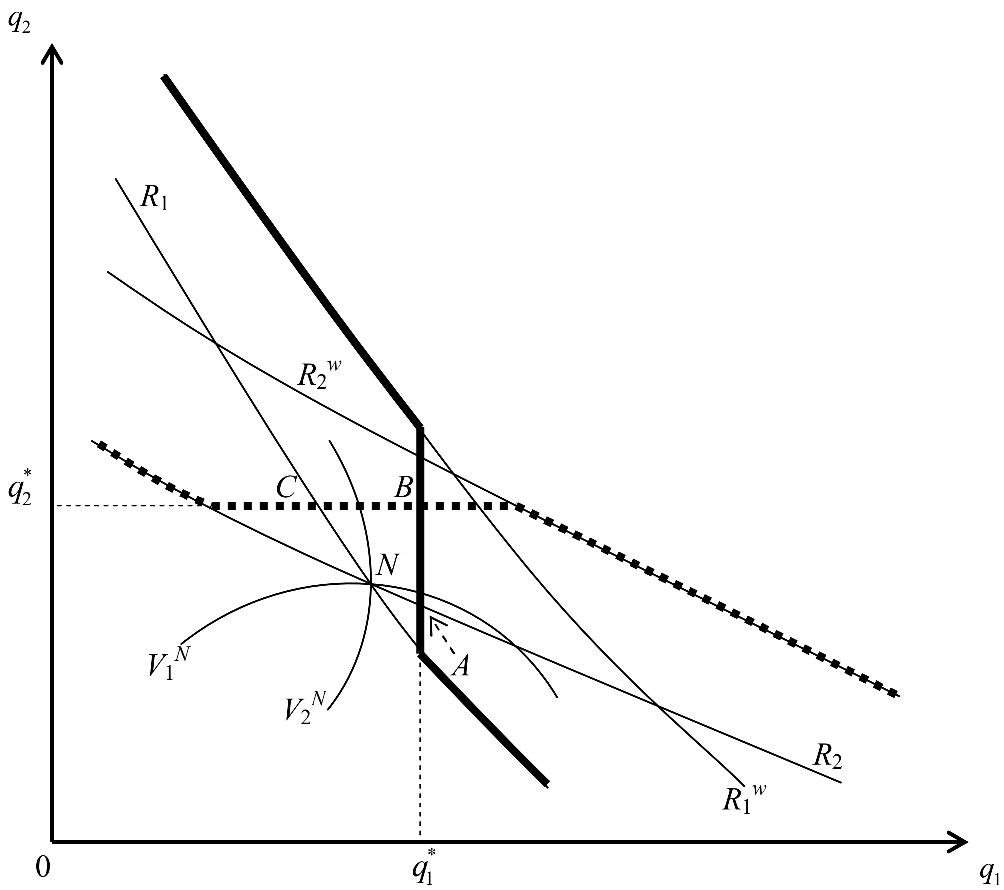

Figure 1: $\theta_1 < (2q_1 + q_2)/2(q_1 + q_2)$ and $\theta_2 < (2q_1 + q_2)/2(q_1 + q_2)$

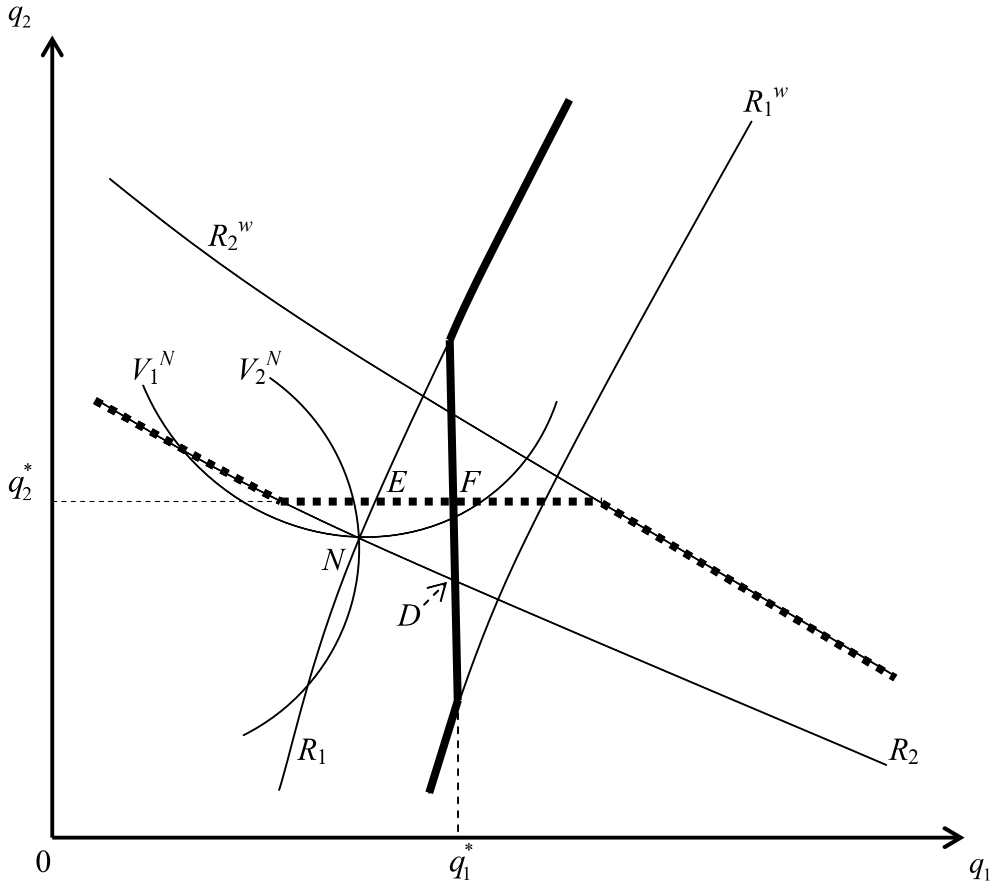

Figure 2: $\theta_1 > (2q_1 + q_2)/2(q_1 + q_2)$ and $\theta_2 < (2q_1 + q_2)/2(q_1 + q_2)$

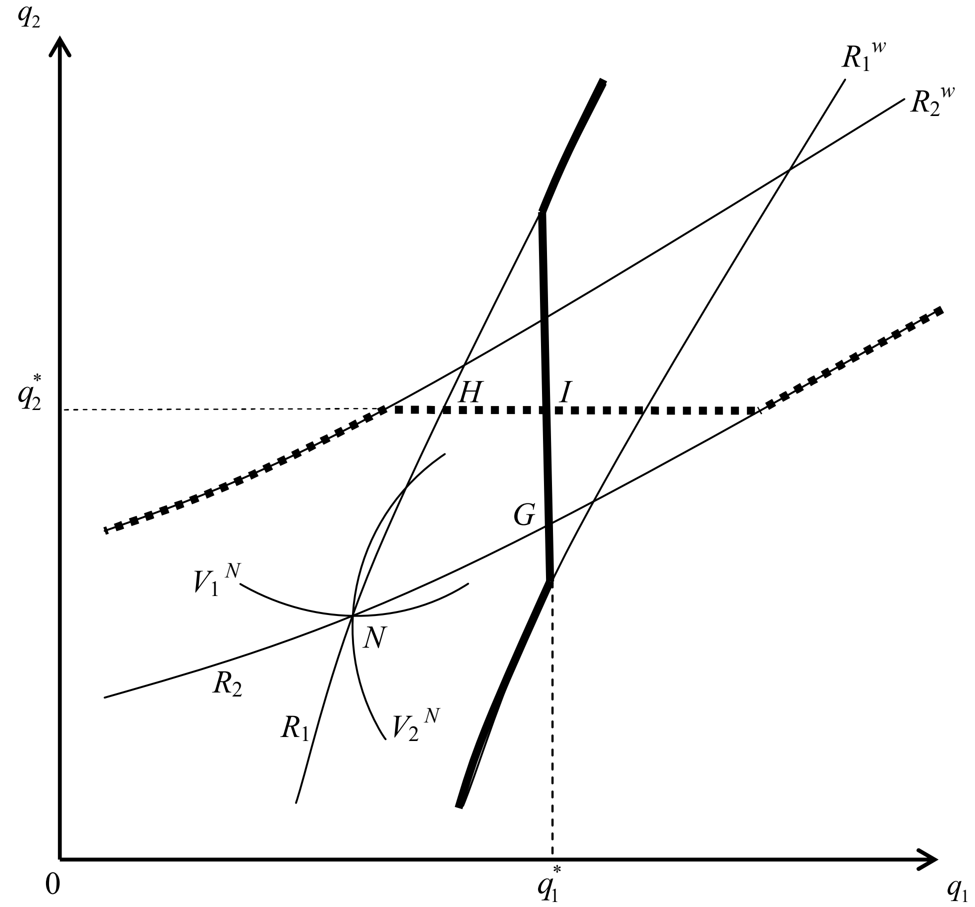

Figure 3: $\theta_1 > (2q_1 + q_2)/2(q_1 + q_2)$ and $\theta_2 > (2q_1 + q_2)/2(q_1 + q_2)$