Reversal of Bertrand-Cournot Ranking for Optimal Privatization Level

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15 September 2022

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Abstract

We consider a vertically related differentiated product mixed duopoly market where a public and private firm compete in the downstream market. The public firm is partially privatized and a welfare maximizing regulator chooses the privatization level. The production of the final commodity requires a key input that is supplied by a foreign monopolist who in the upstream market can practice either uniform or discriminatory pricing. We show that with uniform pricing regime the privatization is always larger under Cournot competition while in case of discriminatory pricing regime, the privatization level under Bertrand competition is always larger. We also find that under discriminatory pricing regime, the Cournot-Bertrand ranking of other relevant variables are sensitive to the degree of substitutability.

Keywords: partially private firm, price (Bertrand) competition, quantity (Cournot) competition, optimal privatization, vertical market

JEL classification code: D4, D6, H4, L1,L2

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‡The authors are grateful to Amrita Dhillon, Arghya Ghosh, Manipushpak Mitra, Sukanta Bhattacharyya, Sugata Marjit and Santanu Roy for their comments and suggestions. The authors also thankfully acknowledge the constructive comments received from seminar participants in ACEGD 2019 and PET 2022.

1 Introduction

The privatization is often a debatable issue in developing countries like India and China. Disputes related to the privatization policy of the government is very common in news-papers, news-channels and social media. Supporters of privatization generally have the following perspective that goes in favor of privatization.

(F-1) Public firms are generally inefficient than the private firm. Therefore, privatization may reduce inefficiency in terms of cost reduction.

(F-2) Government can raise the fund to finance its deficits through the privatization policy.

On the other hand, opposition of the privatization have the following views against privatization:

(A-1) The privatization policy shift the objective of public firms from more welfare oriented to less. Consequently, we have the detrimental effect on the welfare of society.

(A-2) Further, people generally have larger faith on the government than private organization therefore privatization policies may lead to trust issues thereby further destabilize the economy.

Generally, macro-economists are more interested in verifying the validity of the statement (A-2) where as the validity of the statement (F-2) is mainly the research area of the public finance. The industrial economists are generally interested in the trade off involved in statements (F-1) and (A-1). This paper is primarily a study of industrial organization hence our focus is also on the related trade off as indicated.

The firms competing with strategic substitute or strategic complement is again another important issue of study in the industrial organization. In this context the comparison of Cournot and Bertrand competition with differentiated products comprise a large part of the literature. The former implies firms compete in quantities and the latter implies the firms compete in prices. If the level of privatization is captured as a continuous variable then the concept of optimum privatization is of utmost relevance. When the regulator behaves optimally the level of privatization under Cournot competition (when firms compete with strategic substitutes) may differ from that under Bertrand competition (firms competes with strategic complements). Fujiwara [12] shows if firms compete under Cournot competition and there is no inefficiency gap between the public and private firms then the public firm becomes partially privatized. On the other hand keeping the framework fixed, Ohnishi [24] shows that the public firm becomes fully public under Bertrand competition.[1] Therefore, by comparing these two existing analysis we may conclude that with no inefficiency gap, the optimum privatization level when the firms compete with strategic substitute is larger than that when the firms compete with strategic compliment.

Our main objective is to verify whether this existing ranking of optimum privatization between Cournot and Bertrand competition is robust enough under the vertical market structure or not. The structure of the model as analyzed by both Fujiwara [12] and Ohnishi [24] are similar in terms of stages through which the sequence of events occurred. For both the model in the first stage the optimum privatization was determined and then the market competition had taken place. Whereas we allow for a vertical structure by introducing a new stage between the privatization level determining stage and the market competition. In this new stage a foreign monopolist charges the optimum input price. Assuming the vertical structure of the model we show that the ranking of optimum privatization between Cournot and Bertrand competition depends on the regime of input pricing. The optimum privatization under Cournot competition is larger than Bertrand competition when we have the uniform input pricing. However, the optimum privatization under Bertrand competition is larger than Cournot competition when we have the discriminatory input pricing.

We also consider the Cournot-Bertrand rankings for other relevant market variables (for example prices, quantities, profits and welfare) separately for uniform and discriminatory pricing regime. Our results indicate that assuming uniform input pricing, the Cournot-Bertrand rankings for all most all the relevant equilibrium market outcomes are identical to the rankings that we

[1]A recent study by Mitra.et.al [20] establishes this conclusion for a wide class of demand functions. Moreover, Mitra.et.al [20] shows that if firms are allowed to endogenize the strategic variable then in equilibrium the price competition emerges and in equilibrium the partially private firm becomes fully public one.

have in Ghosh and Mitra [13]. However, assuming discriminatory pricing regime, the Cournot-Bertrand rankings for most of the market variables are sensitive to the degree of product substitution. Specifically, the Cournot-Bertrand rankings corresponding to market outcomes relating to the private firm and the welfare of society gets reversed with respect to the rankings that we have in Ghosh and Mitra [13] for low degree of product substitution.

These results are very important in the context of privatization literature. Our result explains a new channel through which the privatization is desirable for the society. Specifically the channel is the existence of foreign monopolist who only supply the input. If we have the vertical structure then the monopoly power of the input supplier will increase the amount of distortion to the society. Further, if the monopolist can discriminate then it will exploit the private firm larger than the public firm. Therefore, to protect the private firm, the government can use privatization as the infant industry policy instrument. Our result is also important in terms of the Cournot-Bertrand comparison. Not only the existing Cournot-Bertrand ranking of optimum privatization without vertical structure gets reversed under vertical structure with price discrimination but also for lower degree of product substitution the Cournot-Bertrand rankings of all the market outcomes relating to the private firm and welfare of the society differ between uniform and discriminating pricing regime.

We also consider different extension of our results. Firstly, we consider the extension through the introduction of operational inefficiency of the public firm. Secondly, we consider the extension through the mixed oligopoly structure. However we get the same Cournot-Bertrand ranking in terms of optimal privatization along both the line of extension.

Our study is applicable to various mixed oligopoly industries where the foreign input is used as key input of that industry; specifically the Banking and Insurance industries where foreign capital is one very important key input. It is evident from the study of Chen.et.al [8] that privatization level indeed increased in case of the Chinese banking industry after allowing for the foreign equity. We can apply our model to the health sector as well. In most of the developing countries like India and China the health sector follows the mixed oligopoly structure and the specialized machineries are imported and often the public health sector follows P-P-P kind of structure for its functioning.

The paper is organized as follows. We conclude this section with a brief discussion on the related literature. In section 2, the readers are introduced to the basic framework and the Game structure. In section 3, we present all our results. section 4, introduces two possible extensions of our main result. Finally in section 5, we conclude.

1.1 Related literature

Industrial economists are often interested in comparing different market structures based on respective market outcomes and then try to determine the best market structure considering either the society's welfare or the firm's profit and sometimes considering both. In this context, the Cournot-Bertrand comparison is one such important criterion that has often been analyzed in the literature of industrial economics. The first study with differentiated products was by Singh and Vives [28]. They conclude that under Cournot duopoly each firm in the industry produces less, charges more and earns higher profit than under Bertrand duopoly. Further, they argued that the latter is efficient than the former in terms of welfare ranking. We refer to these rankings as the standard rankings. Subsequent studies are mainly classified in two categories. One branch of literature focuses on verifying the robustness of the standard ranking (see Amir and Jin [2], Vives [31], Okuguchi [26], Hsu and Wang [16]). Other branch of the literature is interested in analyzing the circumstances where these standard rankings are either partially reversed or fully reversed. Moreover, we can classify the second branch in two different sub-categories. First sub-category focuses on private oligopoly market. In this direction, studies by Hackner [15] with quality differences; Mukherjee [23] and Cellini.et.al [7] with free entry; Symeonidis [29] and Lin and Saggi [18] with endogenous Research & Development expenditure; López and Naylor [19] with the wage bargaining provided evidence on partial reversal of the standard rankings. Arya.et.al [4] and Alipranti.et.al [1] have shown the complete reversal of the standard rankings with a vertically related producer. The second sub-category which is relatively younger than previous one focuses on the mixed oligopoly market. The literature of mixed oligopoly became popular after Bȯs [6]. Then the study by Matsumura [21] open up a new direction under mixed oligopoly with privatization. However, the Bertrand-Cournot comparison with mixed oligopoly was rather unexplored. Ghosh and Mitra [14] made the first attempts to introduce Cournot-Bertrand comparison in this context. Ghosh and Mitra [14] found the complete reversal of the standard ranking in the context of mixed oligopoly.

Further, we can classify the literature of mixed oligopoly in two broad categories. The first category includes studies in which issue other than the privatization gets important (see Choi [9], Choi [10], Dong and Wang [11], Mitra.et.al [20], Matsumura and Sunada [22], Scrimitore [27], Nakamura and Takami [30] and many more). The second category includes studies where privatization gets the ultimate importance. Finally, the literature on privatization can be classified into two broad categories. First category studies privatization as a discrete variable (See Anderson et al. [3], Barcena-Ruiz and Garzon [5].). Second category studies privatization as a continuous variable (See Matsumura [21], Fujiwara [12], Ohnishi [24], Ohnishi [25], Wang and Chen [32], Wang and Chiou [33], Wang and Chiou [34], Wen and Yuan [35].).

2 The Framework

Consider a simple economy consisting of two sectors, namely: a competitive sector that produces a nueméraire commodity (money) and an imperfectly competitive sector that produces commodities that are not perfect substitutes. Further, the imperfectly competitive sector consists of two firms: one publicly regulated firm (Firm 0) and one private firm (Firm 1).

2.1 Demand Side

In this subsection we describe the demand side of the economy. The utility of the representative consumer is quasi-linear in the competitive sector's output and is given by $\mathcal{U}(q_0, q_1, y) =$

$U(q_0, q_1) + y$ where for all $i = 0, 1$, q_i be the consumption of quantity of output of Firm i and y be the consumption of quantity of output of the competitive sector. The sub-utility that depends on the commodity bundle purchased from imperfectly competitive sector is assumed to be quadratic and summarized by the following equation,

$$U(q_0, q_1) = a(q_0 + q_1) - \frac{1}{2}\left[q_0^2 + q_1^2 + 2sq_0q_1\right],\ a > 0,\ s \in (0, 1),$$

where a and s respectively represent the test parameter and the parameter of degree of product substitution.[2] Therefore the representative consumer's problem is to maximize $\mathcal{U}(q_0, q_1, y) = U(q_0, q_1) + y$ by choosing (q_0, q_1, y) subject to $p_0q_0 + p_1q_1 + y \leq I$ where for all $i = 0, 1$, p_i be the price of good i charged by Firm i and I be the income of the consumer. Given the quasi-linear specification of the utility function, all income effects are captured in the demand of the nuemerarior good therefore the consumers' problem can be reduced to maximize $G(q_0, q_1) = U(q_0, q_1) - p_0q_0 - p_1q_1$ by choosing (q_0, q_1). Therefore from the first order condition of the consumer's optimization we have the inverse demand function that Firm i faces

$$P_i(q_0, q_1) = \frac{\partial U(q_0, q_1)}{\partial q_i} = a - q_i - sq_j \quad \forall i, j = 0, 1 \ \&\ i \neq j. \tag{1}$$

Given $s \in (0, 1)$ the inverse demand function is invertible and we can solve for q_i to obtained the direct demand function that Firm i faces

$$D_i(p_0, p_1) = \frac{a}{1+s} - \frac{p_i}{1-s^2} + \frac{sp_j}{1-s^2} \quad \forall i, j = 0, 1 \ \&\ i \neq j. \tag{2}$$

Therefore the consumer surplus in term of prices is

$$\overline{CS}(p_0, p_1) = U(D_0(p_0, p_1), D_1(p_0, p_1)) - p_0D_0(p_0, p_1) - p_1D_1(p_0, p_1). \tag{3}$$

[2]Note that $s = 0$ and $s = 1$ respectively represent the cases respectively of the goods that are independent to each other and the goods that are perfectly substitutes. Therefore $s \in (0, 1)$ represents the case when goods are imperfect substitute to each other.

Using the price quantity duality, the consumer surplus in terms of quantities is

$$CS(q_0, q_1) = U(q_0, q_1) - P_0(q_0, q_1)q_0 - P_1(q_0, q_1)q_1. \tag{4}$$

2.2 Supply side

We assume the production of final commodity requires a key input on one-to-one basis. The key input is supplied by a foreign monopolist (Firm U) which applies the liner pricing rule. Suppose w_i denotes the price that Firm U charges to Firm i. Further, there is no other cost of production. Therefore, the cost of production of Firm i to produce q_i unit of quantity is $C_i(q_i; w_i) = w_i q_i$ for all $i = 1, 2$.[3] Therefore, the profit of Firm i in terms of quantities is

$$\pi_i(q_0, q_1; w_i) = P_i(q_0, q_1)q_i - C_i(q_i; w_i). \tag{5}$$

and using the price quantity duality the profit of firm i in terms of prices is

$$\overline{\pi}_i(p_0, p_1; w_i) = p_i D_i(p_0, p_1) - C_i\left(D_i(p_0, p_1); w_i\right). \tag{6}$$

Suppose there is no cost of input production. Therefore the profit of the input monopolist is

$$\pi_u(q_0, q_1, w_0, w_1) = w_0 q_0 + w_1 q_1 \tag{7}$$

[3] One can consider some general type of technology such that to produce one unit of final commodity α unit of the key input required along with some others input required that cost β. The cost function of Firm $i = 1, 2$ corresponds to these general techniques is $C_i(q_i) = (\alpha w_i + \beta)q_i$. However these generalization will have no qualitative change in our results. Hence we consider the simple one to one technology.

2.3 Welfare of the Society

The welfare of society is the sum of consumer surplus and total profit. Therefore the welfare in terms of quantities is

$$\begin{aligned} W(q_0, q_1; w_0, w_1) &= CS(q_1, q_2) + \pi_1(q_0, q_1; w_0) + \pi_2(q_0, q_1; w_1) \\ &= U(q_0, q_1) - w_0 q_0 - w_1 q_1. \end{aligned} \tag{8}$$

Similarly the welfare in terms of prices is

$$\begin{aligned} \overline{W}(p_0, p_1; w_0, w_1) &= \overline{CS}(p_0, p_1) + \overline{\pi}_0(p_0, p_1, w_0) + \overline{\pi}_1(p_0, p_1, w_1) \\ &= U(D_0(p_0, p_1), D_1(p_0, p_1)) - w_0 D_0(p_0, p_1) - w_1 D_1(p_0, p_1). \end{aligned} \tag{9}$$

2.4 Game Structure

The sequence of events are given by the following three stage game.

- **Stage-I** Regulator or planner chooses the optimal privatization ratio $\theta \in [0, 1]$ to maximize social welfare.
- **Stage-II** Firm U chooses input price/ prices to maximize its profit. We consider two regimes of input choice: uniform pricing and discriminatory pricing.
- **Stage-III** Firm 0 and Firm 1 compete in the market. We allow for the two usual modes of competition, namely, Cournot competition and Bertrand competition. In case of former, firms compete in quantity and for the latter firms compete in price.

We use backward induction method to solve this three stage game separately for different regime (uniform pricing and discriminatory pricing) of input pricing and different mode of competitions (Cournot and Bertrand). Our objective is to rank between the Cournot and Bertrand competition for different regimes of input pricing and check how the rankings change between the different input pricing regime. Note that the Firm 0 is a publicly regulated firm where the instrument of regulation is the level of privatization. Given the choice of θ by the social planer at Stage-I, the payoff of Firm 0 is the weighted average of his own profit and the society's welfare where the weight attached to its profit is the privatization ratio.

3 Results

Here in this section we list all our results with corresponding intuitive explanation.[4]

3.1 Uniform Pricing

Suppose that the foreign monopolist is not able to discriminate the input price between the publicly regulated and private firm then it forced to charge same input price, that is, $w_1 = w_2 = w$.

Observation 1 Suppose the input monopolist practices uniform pricing in upstream market then we have the following key outcomes.

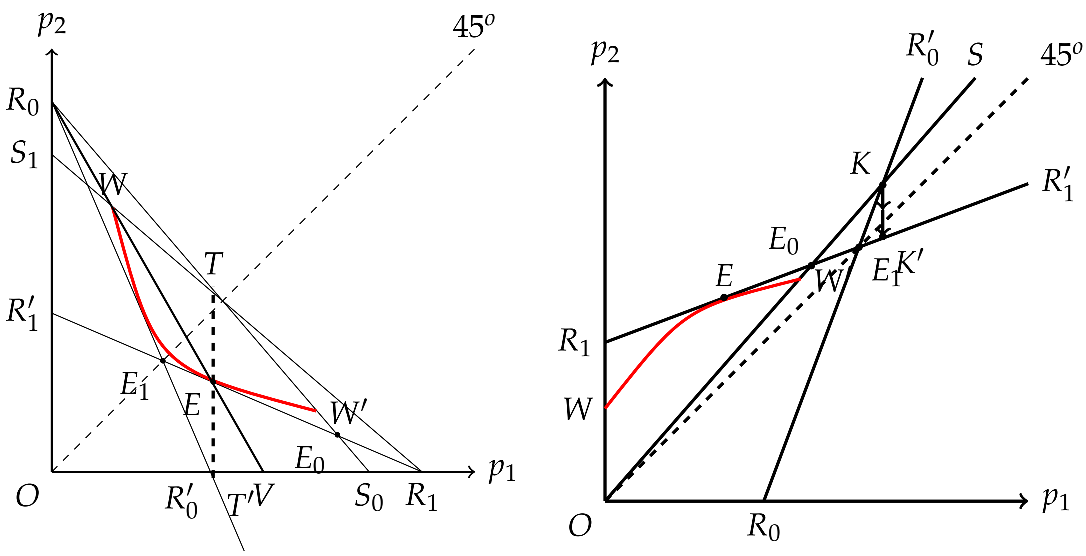
(i) In both Cournot and Bertrand the input monopolist charges half of the market size.

(ii) The social planner always partially privatizes the publicly regulated firm under Cournot competition. However, the publicly regulating firm becomes completely nationalized under Bertrand competition.

(iii) The Bertrand Cournot ranking of all the other market outcomes (such price, quantity and profit of both the firms, social welfare and Consumer surplus) remains unaltered in relation to the findings of Ghosh and Mitra [13]. However, for very high degree of product differentiation the ranking of profit of Firm 0 and Consumer surplus get changed.

We all know that in vertical structure the optimum input price is determined via double marginalization and the source of double marginalization is the ratio of total input demand to marginal input demand. For Cournot and Bertrand competition with linear demand this ratio becomes

[4] All proves are available in the Appendix.



(a) Cournot (b) Bertrand

Figure 1: Optimum privatization determination with uniform pricing.

independent of privatization and hence, the input price. Therefore, the monopolist optimally charge half of the market size. The optimum privatization ratio here is same as Fujiwara [12] for Cournot competition and Ohnishi [24] for Bertrand competition. Since under Cournot competition the society demands larger price cut from full nationalization and smaller price cut than full privatization therefore we have partial privatization (see Figure 1a). However, under Bertrand competition, given any privatization ratio, the society always demands lesser price cut therefore, social planner can do this at best possible way by assigning the role of full public firm to the Firm 0 (see figure 1b). Finally, in comparison to Ghosh and Mitra [13] here under Cournot competition the Firm 0 is partially privatized but that privatization ratio is not sufficient enough to alter the Bertrand and Cournot ranking.

3.2 Discriminatory pricing regime:

Suppose the foreign monopolist is able to discriminate the input price. Hence it can charge different prices to different downstream firms. Unlike the uniform pricing regime here privatization has interesting role in determining the input prices. The role of privatization in determining the input price is stated in the next Lemma 1.

Lemma 1 The followings are true about the Stage-II choice of the foreign monopolist

(i) In case of Cournot competition, foreign monopolist input supplier does not discriminate in terms of input prices and like uniform pricing here also optimally charges half of the market size.

(ii) In case of Bertrand competition, given any $\theta \in [0,1)$ set by the regulator, the foreign input monopolist discriminate the input prices in the upstream market. Moreover, if $w_0^{BD}(\theta)$ and $w_1^{BD}(\theta)$ denotes the input price for Firm 0 and Firm 1 respectively then for all $\theta \in [0,1]$ we have $w_0^{BD}(\theta) < w^{BI} < w_1^{BD}(\theta)$ where $w^{BI} = a/2$ be the optimum input price charged by the monopolist under uniform pricing.

(iii) Finally, the ratio $\omega(\theta) = (w_1^{BD}(\theta) - w^{BI})/(w^{BI} - w_0^{BD}(\theta))$ is decreasing in $\theta \in (0,1)$.

If the monopolist is able to practice the price discrimination then it takes the public and private firm as different source of its demand. Given the one-to-one technology, the final stage quantities are nothing but the demands that the monopolist faces from different its sources. As the final commodities are imperfect substitutes, the input demands from different sources are also imperfect substitutes to each other. Therefore, if the foreign input monopolist practices price discrimination then it becomes a multi-product monopolist with demands that are imperfectly substitute to each other.

If we assume the Cournot competition in Stage-III then the demand functions faced by the monopolist in Stage-II are given by the conditions (27) and (28). Observe, that the cross effects are equal, that is, $(\partial q_0^{CD}/\partial w_1) = (\partial q_1^{CD}/\partial w_0) = s/[2(1+\theta) - s^2]$.[5] One can show that the symmetry of the cross effect implies both the marginal profits with respect to the input prices individually become zero at optimum uniform pricing. Hence the monopolist does not benefit from input-price discrimination.

[5]This imply that the negative externality generated by the input price of private firm on the demand from the public firm is uniformly equal to the negative externality generated by the input price of public firm on the demand from the private firm.

On the other hand, if we have the Bertrand competition in Stage-III, then the demand functions faced by the monopolist in Stage-II are given by the conditions (32) and (33). Unlike Cournot in case of Bertrand the cross effects are not equal, that is, $(\partial q_0^{BD}/\partial w_1) \neq (\partial q_1^{BD}/\partial w_0)$[6]. Moreover, the difference in the cross effect under Bertrand leads to different values of the marginal profit with respect to w_1 and w_2 at the optimum uniform pricing where the sum of marginal profits with respect to the input prices must be zero, hence the price discrimination. Finally at optimum uniform pricing of Bertrand competition the marginal profit of w_2 change is positive and that of w_1 is negative. Hence, monopolist charges a price larger (lesser) than uniform optimum price to the private (public) firm.

Now let us identify the source of cross effect difference (that led to price discrimination) under Bertrand. The answer will be obtained directly by comparing Cournot and Bertrand reaction equations of the public firm. One can derive

$$(p_0 - w_0) + \theta \frac{D_0(p_0, p_1)}{\frac{\partial D_0}{\partial p_0}(p_0, p_1)} + (1-\theta)(p_1 - w_1)\frac{\frac{\partial D_1}{\partial p_0}(p_0, p_1)}{\frac{\partial D_0}{\partial p_0}(p_0, p_1)} = 0 \tag{10}$$

from the Bertrand reaction equation of the public firm given by the condition (30). Now in the left hand side of the condition (10) we have three terms. Notice that the sum of the last two terms is the infra-marginal term corresponding to the optimum behaviour of the publicly regulated firm and the first term is capturing only the direct marginal effect. Moreover the first and the second components of the infra-marginal term are respectively due to the profit maximizing and the welfare maximizing motives of the publicly regulated firm. The comparison of condition (10) with the Cournot reaction condition (25) reveals that (unlike Bertrand in case of Cournot) the welfare maximizing motive of the public firm will not be able to affect the infra-marginal term. Therefore, under Bertrand due to the existence of this extra component of the infra-marginal term, we have an additional direct effect of input price of the private firm on the input demand of the

[6] In case of the Bertrand competition, the negative externality generated by the input price of private firm on the demand from the public firm is not same to the negative externality generated by the input price of public firm on the demand from the private firm.

public firm. Hence, we have the difference in cross effect.

Note that $\omega(\theta)$ as defined in Lemma 1(iii), is a measure of the degree of price discrimination. As the privatization ratio chosen by the regulator increases, the ideological difference between the two firms in competition reduces and as a result the component of the infra-marginal term originating due to the welfare motive weakens and finally as a consequence the cross effect difference between two sources of input demands reduces. Thus the degree of price discrimination decreases.

Proposition 1 We have partial privatization under Cournot and Bertrand competition. Further the following are true about the optimal privatization level.

(i) Given any $s \in [0,1]$ and $a > 0$, the optimum privatization under Cournot is $\theta^{CD} = \theta^{CI}$.

(ii) Given any $s \in (0,1)$ and $a > 0$, there exist an unique optimum privatization $\theta^{BD} \in (0,1)$ under Bertrand competition.

(iii) Under Bertrand competition the public firm is privatized more compared to Cournot competition.

It is quite evident from Lemma 1(i), for any ratio of privatization chosen by the regulator, the monopolist does not gain from price discrimination if the two downstream firms compete in quantity. Therefore the optimum privatization ratio is independent of the input pricing regime. It has already been noted in Lemma 1 (iii) that as the privatization ratio increases, the degree of input price discrimination decreases. Therefore unlike uniform price regime, the optimal privatization ratio under Bertrand (which is 'zero' as shown earlier) does not generate optimal result since the higher input price discrimination would lead to increased drainage in terms of foreign monopolist's profit. And as the distortion due to input price discrimination is severe, we end up with a higher privatization ratio under Bertrand (compared to the Cournot case) in order to control that negative effect. Therefore, given input monopolist practices the price discrimination and firms competing with prices in the downstream market then the privatization policy is used not only to regulate the public firm but also used to protect the private firm from the monopolist input price distortion.

Proposition 2 The followings are true while we compare the uniform input-pricing regime with the discriminating pricing regime:

(i) The Bertrand and Cournot rankings of all the market outcomes for the publicly regulated firm remain same.

(ii) If goods are sufficiently differentiated then the Bertrand and Cournot rankings of all the market outcomes related to private firm get altered.

(iii) If goods are sufficiently differentiated then the Bertrand and Cournot rankings of social welfare get altered.

The explanation of the Proposition 2 comes from the comparison of the optimum privatization between the Cournot and Bertrand competition. These comparison is shown in Figure 2. It is quite evident from Figure 2 that for low value of the degree of substitutability, the difference between the optimal privatization under Bertrand and that under Cournot is not very large. However, the difference is quite significant when the degree of substitutability is large enough. Therefore, at low degree of substitutability, the privatization as an infant industry support for the private firm is not very effective under Bertrand competition (as compared to Cournot competition). Due to price discrimination, the behavior of the private firm drastically changes under Bertrand competition and the same can not be compensated by the privatization policy at low degree of substitutability. Hence, we have the ranking reversal for private firm variables. Observe that the privatization has direct bearing over the public firm and the input-price discrimination takes the public firm in relatively advantageous position than private firm and the behavior of the public firm remains unchanged under Bertrand competition leading to same ranking for the public firm's variables. The ranking reversal of the private firm affects the society comparatively stronger than the ranking reversal of the public firm. As a consequence we observe the reversal of the social welfare ranking.

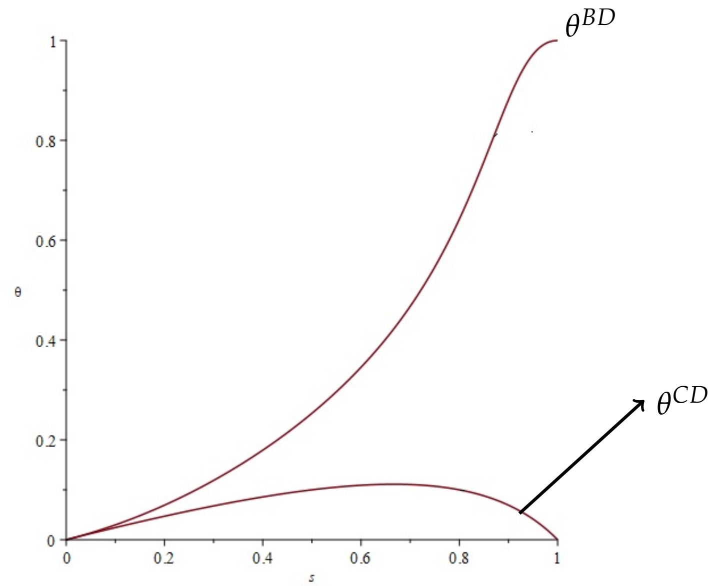


Figure 2: Optimum Privatization Cournot vs Bertrand.

4 Extension and Discussion

The key difference of the our analysis and the existing literature is under discriminating pricing regime the optimum privatization is more under Bertrand competition than under Cournot competition. Here in this section we check the robustness of this fact under different plausible extension of our analysis.

4.1 Inefficient Public Firm

Suppose the demand structure remains same but consider a general type of asymmetric technology such that produce one unit of final commodity Firm $i \in \{0, 1\}$ requires α_i unit of key input supplied by foreign monopolist along with some other inputs which cost β_i per unit of output to Firm i. Therefore, the cost function of Firm i is $C_i(q_i) = (\alpha_i w_i + \beta_i)q_i$. We assume Firm 0 is inefficient firm. Here we can classify three types of inefficiency of Firm 0 (for detail see Yoshida [36]).

(i) **Only α-inefficiency:** In this case we have $\alpha_0 \geq \alpha_1$ and $\beta_0 = \beta_1$.

(ii) **Only β-inefficiency:** In this case we have $\alpha_0 = \alpha_1$ and $\beta_0 > \beta_1$.

(iii) **Only $\alpha\beta$-inefficiency:** In this case we have $\alpha_0 \geq \alpha_1$ and $\beta_0 \geq \beta_1$.

To keep the analysis simple we us the following normalization, $\alpha_1 = 1$ and $\beta_1 = 0$. Further to keep the notation simple we denote $\alpha_0 = \alpha \geq 1$ and $\beta_0 = \beta \geq 0$. Therefore we assume the cost function of Firm 0 is $C_0(q_0) = (\alpha w_0 + \beta)q_0$ and that Firm 1 is $C_1(q_1) = w_1 q_1$. Let us introduce $w_i^{CD}(\theta)$ and $w_i^{BD}(\theta)$ for all $i \in \{0, 1\}$ that respectively denote the Stage-II input price under discriminating pricing that the monopolist will charged to Firm i under Cournot and Bertrand competition. Now under discriminating pricing the Stage-III choices depends on the effective input prices αw_0 and w_1 respectively for Firm 0 and 1. Further, the monopolist's profit in the Stage-II also depends on the effective input prices. Therefore in the Stage-II the monopolist determine the effective input prices. Hence in the determination of the Stage-I optimum privatization the effective input prices only matters and not the individual input prices. Hence one can ignore the α-inefficiency and focus on the β-inefficiency only to check the ranking between the Cournot and Bertrand ranking in terms of optimum privatization. Using only β-inefficiency we do the simulation to check the ranking. The simulation results is summarized in the Table 1 and 2 (see the Appendix). The comparison of Table 1 and 2 reveals that the privatization is always higher under Bertrand that the Cournot.

4.2 Mixed Oligopoly

Consider the following extension of our model in case of mixed oligopoly structure. Assume that in the imperfectly competitive sector there are total $N + 1$ number of firms with $N \geq 1$ and S denotes set of all firms. Further, assume that the Firm 0 is the only publicly regulated firm and any Firm $i \in S \setminus \{0\}$ is the private firm. Therefore, the utility function of the representative consumer changes to $V(\boldsymbol{q}, y) = U(\boldsymbol{q}) + y$ where $\boldsymbol{q} = (q_0, q_1, \ldots, q_N)$ be the vector of imperfectly competitive sector. The function $U(\boldsymbol{q})$ is a quadratic and given by

$$U(\boldsymbol{q}) = a\sum_{j\in S} q_j - \frac{1}{2}\left[\sum_{j\in S} q_j^2 + s\sum_{j\in S}\sum_{j'\in S\setminus\{j\}} q_j q_{j'}\right].$$

The inverse demand function that Firm $i \in S$ faces is $P_i(\boldsymbol{q}) = a - q_i - s\sum_{j\in S\setminus\{i\}} q_j$. We assume that the own effect dominates the sum of the cross effects in absolute terms that is $1 - Ns > 0$. Given $Ns < 1$ and $s \in (0,1)$ one can show that under discriminating regime the privatization ratio under Bertrand competition is always larger than that under Cournot competition. This happens due to the fact that when we have N private firms then the part of the infra-marginal term of the reaction equation of the public firm, that is originated due to presence of the welfare maximizing character of the public firm, increases with the number of private firm.[7] These leads to the larger price discrimination. Therefore, to protect the private firms from the negative effect of price discrimination, we have the larger privatization under Bertrand competition.

5 Conclusion

Our study simultaneously contributed to the literature of Cournot-Bertrand comparison and privatization policy. In terms of privatization policy it suggest that privatization is a desirable policy to the society when we have vertical structure and the practice of price discrimination in the input market. Further, the optimal privatization policy differ in terms of the mode of competition that prevailing in the market. The most catchy point of our analysis is the difference of the privatization policy between different of downstream competition when regime of input pricing changes. These fact leads to the changes in the Cournot-Bertrand ranking of some of the important market outcomes for high degree of product differentiation. Only limitation of our analysis is that it is very much depends on the linear nature of the demand function. However, when the demand system varies our results can only be violated only under the condition that the aggregate effect

[7] That is, the counterpart of the term $(1-\theta)(p_1 - w_1)\frac{\partial D_1(p_0,p_1)}{\partial p_0} / \frac{\partial D_0(p_0,p_1)}{\partial p_0}$ becomes $(1-\theta)\sum_{i\in S\setminus\{0\}}(p_i - w_i)\frac{\partial D_i(p_0,p_1)}{\partial p_0} / \frac{\partial D_0(p_0,p_1)}{\partial p_0}$ which is increasing in N since in Stage-II choices $w_i = w$ and $p_i = p$ for all $i \in S \setminus \{0\}$.

of the second order terms works in opposite to the aggregate effect of the first order terms and the formal will dominate the latter.

A Appendix

Proof of the Observation 1:
Proof of Observation 1 (i):
Derivation of the input price under Cournot: If in Stage-III firms competing with Cournot competition, then given the quantity of Firm 1, q_1, Firm 0 will maximize

$$V^I(q_0, q_1; w, \theta) = V(q_0, q_1; w, w, \theta) = \theta\pi_0(q_0, q_1; w) + (1-\theta)W(q_0, q_1; w, w),$$

by choosing its own output q_0 and given quantity of Firm 0, q_0, Firm 1 will maximize $\pi_1(q_0, q_1; w)$, by choosing its output q_1. Given any $\theta \in [0,1]$ of Stage-I and w of Stage-II, if $(q_0^{CI}(w,\theta), q_1^{CI}(w,\theta))$ be the Stage-III choice vector then $(q_0^{CI}(w,\theta), q_1^{CI}(w,\theta))$ simultaneously satisfy the following reaction equations

$$\frac{\partial V^I}{\partial q_0)}(q_0, q_1, w, \theta) = P_0(q_0, q_1) - w + \theta\frac{\partial P_0}{\partial q_0}(q_0, q_1) = 0 \tag{11}$$

and

$$\frac{\partial \pi_1}{\partial q_1}(q_0, q_1, w) = P_1(q_0, q_1) - w + q_1\frac{\partial P_1}{\partial q_1}(q_0, q_1) = 0. \tag{12}$$

Evaluating condition (11) and (12) at $(q_0^{CI}(w,\theta), q_1^{CI}(w,\theta))$ we get the system of equations involving the Stage-III choices are

$$\begin{aligned}(1+\theta)q_0^{CI}(w,\theta) + sq_1^{CI}(w,\theta) &= a - w\\ sq_0^{CI}(w,\theta) + 2q_1^{CI}(w,\theta) &= a - w\end{aligned}$$

Solving for quantities we have

$$q_0^{CI}(w,\theta) = \frac{(2-s)(a-w)}{2(1+\theta)-s^2} \tag{13}$$

and

$$q_1^{CI}(w,\theta) = \frac{(1+\theta-s)(a-w)}{2(1+\theta)-s^2}. \tag{14}$$

Therefore, substituting in the equation (7) we get

$$\pi_u^{CI}(w,\theta) = \pi_u(q_0^{CI}(w;\theta), q_1^{CI}(w;\theta); w, w) = \frac{w(a-w)(3+\theta-2s)}{2(1+\theta)-s^2} \tag{15}$$

Differentiating the condition (15) with respect to w and evaluating at $w = w^{CI}$ then setting equal to zero we get $\left(\partial \pi_u^{CI}\left(w = w^{CI}, \theta\right)/\partial w\right) = \left(a - 2w^{CI}\right)(3+\theta-2s)/[2(1+\theta)-s^2] = 0$. Solving for w^{CI} we have $w^{CI} = a/2$. Note that $\partial^2 \pi_u^{CI}/\partial w^2 = -2(3+\theta-2s)/[2(1+\theta)-s^2] < 0$. Hence the second order condition also satisfied for Stage-II.

Derivation of the input price under Bertrand: If in Stage-III firms competing with Bertrand competition, then given the price of Firm 1, p_1, Firm 0 will maximize

$$\overline{V}^I(p_0, p_1, w, \theta) = \overline{V}(p_0, p_1; w, w, \theta) = \theta \overline{\pi}_0(p_0, p_1; w) + (1-\theta)\overline{W}(p_0, p_1; w, w),$$

by choosing its price p_0 and given price of Firm 0, p_0, Firm 1 will maximize $\overline{\pi}_1(p_0, p_1; w)$, by choosing its own price p_1. Given any $\theta \in [0,1]$ of Stage-I and w of Stage-II, if $(p_0^{BI}(w,\theta), p_1^{BI}(w,\theta,))$ be the Stage-III choice vector then $(p_0^{BI}(w,\theta), p_1^{BI}(w,\theta))$ simultaneously satisfy the following reaction equations

$$\frac{\partial \overline{V}^I}{\partial p_0}(p_0, p_1, w, \theta) = \theta D_0(p_0, p_1) + (p_0 - w)\frac{\partial D_0}{\partial p_0}(p_0, p_1) + (1-\theta)(p_1 - w)\frac{\partial D_1}{\partial p_0}(p_0, p_1) = 0 \tag{16}$$

and

$$\frac{\partial \overline{\pi}_1}{\partial p_1}(p_0, p_1, w) = D_1(p_0, p_1) + (p_1 - w)\frac{\partial D_1}{\partial p_0}(p_0, p_1) = 0 \tag{17}$$

Evaluating condition (16) and (17) at $(p_0^{BI}(w,\theta), p_1^{BI}(w,\theta))$ we get the system of equations involving the Stage-III choices are

$$\begin{aligned} (1+\theta)p_0^{BI}(w,\theta) - s p_1^{BI}(w,\theta) &= \theta(1-s)a + (1-s+\theta s)w \\ -s p_0^{BI}(w,\theta) + 2p_1^{BI}(w,\theta) &= (1-s)a + w \end{aligned}$$

Solving for prices we have

$$p_0^{BI}(w,\theta) = w + \frac{(1-s)(2\theta+s)(a-w)}{2(1+\theta)-s^2} \tag{18}$$

and

$$p_1^{BI}(w,\theta) = w + \frac{(1-s)(1+\theta+\theta s)(a-w)}{2(1+\theta)-s^2}. \tag{19}$$

Therefore the resulting quantities are

$$q_0^{BI}(w,\theta) = D_0\left(p_0^{BI}(w,\theta), p_1^{BI}(w,\theta)\right) = \frac{\left[2-(1-\theta)s^2+\theta s\right](a-w)}{(1+s)\left[2(1+\theta)-s^2\right]} \tag{20}$$

and

$$q_1^{BI}(w,\theta) = D_1\left(p_0^{BI}(w,\theta), p_1^{BI}(w,\theta)\right) = \frac{(1+\theta+s\theta)(a-w)}{(1+s)\left[2(1+\theta)-s^2\right]}. \tag{21}$$

Therefore, substituting in the equation (7) we get,

$$\pi_u^{BI}(w,\theta) = \pi_u(q_0^{BI}(w;\theta), q_1^{BI}(w;\theta); w, w) = \frac{w(a-w)\left[3+\theta+2s\theta-(1-\theta)s^2\right]}{(1+s)\left[2(1+\theta)-s^2\right]}. \tag{22}$$

Differentiating the condition (22), with respect to w and evaluating at $w = w^{BI}$ then setting equal to zero we get $\left(\partial\pi_u^{BI}\left(w = w^{BI},\theta\right)/\partial w\right) = \left(a-2w^{BI}\right)\left[3+\theta+2s\theta-(1-\theta)s^2\right]/(1+s)\left[2(1+\theta)-s^2\right] = 0$. Solving for w^{BI}, we have $w^{BI} = a/2$. Note that $(\partial^2\pi_u^{BI}/\partial w^2) = -2\left[3+\theta+2s\theta-(1-\theta)s^2\right]/(1+s)\left[2(1+\theta)-s^2\right] < 0$. Hence, the second order condition also satisfied for Stage-II.
Hence, Observation 1 (i)

Proof of Observation 1 (ii):

Optimum privatization under Cournot competition: Substituting the optimum input price in the condition (13) and (14), we will respectively have the resulting quantities $q_0^{CI}(w^{CI},\theta) = (2-s)a/2\left[2(1+\theta)-s^2\right]$ and $q_1^{CI}(w^{CI},\theta) = (1+\theta-s)a/2\left[2(1+\theta)-s^2\right]$. Substituting in the equation (8) we will get the resulting welfare

$$\hat{W}^{CI}(\theta) = \frac{\left[3\theta^2+2(7-5s)\theta+(7-6s-2s^2+2s^3)\right]a^2}{8\left[2(1+\theta)-s^2\right]^2}. \tag{23}$$

Differentiation with respect to θ and evaluating at $\theta = \theta^{CI}$, then setting equals to zero, we get $\left(\partial\hat{W}^{CI}\left(\theta = \theta^{CI}\right)/\partial\theta\right) = (2-s)\left[s(1-s)-(4-3s)\theta\right]/4\left[2(1+\theta)-s^2\right]^3 = 0$. Solving for θ^{CI}, we get $\theta^{CI} = s(1-s)/(4-3s)$.

Optimum privatization under Bertrand competition: Substituting the optimum input price w^{BI} in the condition (20) and (21) we will have the resulting quantities respectively $q_0^{BI}(w^{BI},\theta) = \frac{\left[2-(1-\theta)s^2+\theta s\right]a}{2(1+s)\left[2(1+\theta)-s^2\right]}$ and $q_1^{BI}(w^{BI},\theta) = \frac{(1+\theta+s\theta)a}{2(1+s)\left[2(1+\theta)-s^2\right]}$. Substituting in the equation (8), we will get the resulting welfare

$$\hat{W}^{BI}(\theta) = \frac{\left[(1+s)(3+4s-3s^2)\theta^2+(14-4s-3s^2)\theta+(7+s-7s^2-s^3+2s^4)\right]a^2}{8(1+s)\left[2(1+\theta)-s^2\right]^2}. \tag{24}$$

Differentiation with respect to θ we get $\left(\partial\hat{W}^{BI}(\theta)/\partial\theta\right) = -(2+s)(1-s)^3\left[s(1+s)+(4+3s)\theta\right]/4(1+$

$s)\left[2(1+\theta)-s^2\right]^3<0$. Therefore, $\theta^{BI}=0$.
Hence, Observation 1 (ii)

Proof of the Observation 1 (iii):

The equilibrium quantities produced by the publicly regulated firm are $\tilde{q}_0^{CI}=q_0^{CI}\left(w^{CI},\theta^{CI}\right)=(2-s)(4-3s)a/2\left[8-4s-6s^2+3s^3\right]$ and $\tilde{q}_0^{BI}=q_0^{BI}\left(w^{BI},\theta^{BI}\right)=a/2(1+s)$ respectively for Cournot and Bertrand competition. Therefore, for all $s\in(0,1)$, we have $\tilde{q}_0^{BI}-\tilde{q}_0^{CI}=-sa/2(4+4s-3s^2-3s^3)<0$. Hence, proved.

The equilibrium prices charged by the publicly regulated firm under Cournot competition is $\tilde{p}_0^{CI}=P_0\left(\tilde{q}_0^{CI},\tilde{q}_1^{CI}\right)=\left(8-2s-9s^2+4\right)a/2\left[8-4s-6s^2+3s^3\right]$ and that under Bertrand competition is $\left(\tilde{p}_0^{BI}\right)=P_0^{BI}\left(w^{BI},\theta^{BI}\right)=\left(2+s-2s^2\right)a/2(2-s^2)$. Therefore, for all $s\in(0,1)$, we have $\tilde{p}_0^{BI}-\tilde{p}_0^{CI}=s(2-3s-s^2+3s^3-s^4)a/(2-s^2)(8-4s-6s^2+3s^3)>0$. Hence, proved.

The equilibrium profits earned by the publicly regulated firm are $\tilde{\pi}_0^{CI}=\pi_0\left(\tilde{q}_0^{CI},\tilde{q}_1^{CI};w^{CI}\right)=s(1-s)(4-3s)a^2/4\left[4-3s^2\right]^2$ and $\tilde{\pi}_0^{BI}=\pi_0\left(\tilde{q}_0^{BI},\tilde{q}_1^{BI};w^{BI}\right)=s(1-s)a^2/4(1+s)(2-s^2)$ respectively for Cournot and Bertrand competition. Therefore, we have $\tilde{\pi}_0^{BI}-\tilde{\pi}_0^{CI}=s(1-s)(8-2s-14s^2+s^3+6s^4)a^2/4(1+s)(2-s^2)(4-3s^2)^2\gtreqless 0$ for all $s\lesseqgtr\hat{s}_1$ where one can show $\hat{s}_1\in(0.8359,0.8438)$. Hence proved.

The equilibrium quantities produced by the private firm are $\tilde{q}_1^{CI}=q_1^{CI}\left(w^{CI},\theta^{CI}\right)=(2-3s+s^2)a/2\left[8-4s-6s^2+3s^3\right]$ and $\tilde{q}_1^{BI}=q_1^{BI}\left(w^{BI},\theta^{BI}\right)=a/2(1+s)(2-s^2)$ respectively for Cournot and Bertrand competition. Therefore, we have for all $s\in(0,1)$, $\tilde{q}_1^{BI}-\tilde{q}_1^{CI}=s^2(3-2s^2)a/2(1+s)(2-s^2)(4-3s^2)>0$. Hence, proved.

The equilibrium prices charged by the private firm under Cournot competition is $\tilde{p}_1^{CI}=P_1\left(\tilde{q}_0^{CI},\tilde{q}_1^{CI}\right)=\left(12-10s-4s^2+3\right)a/2\left[8-4s-6s^2+3s^3\right]$ and that under Bertrand competition is $\tilde{p}_1^{BI}=p_1^{BI}\left(w^{BI},\theta^{BI}\right)=\left(3-s-s^2\right)a/2(2-s^2)$. Therefore, we have for all $s\in(0,1)$, $\tilde{p}_1^{BI}-\tilde{p}_1^{CI}=-s^2(2-3s+s^2)a/2(2-s)(2-s^2)(4-3s^2)<0$. Hence, proved.

The equilibrium profits earned by the private firm are $\tilde{\pi}_1^{CI}=\pi_1\left(\tilde{q}_0^{CI},\tilde{q}_1^{CI};w^{CI}\right)=(2-3s+s^2)^2a^2/\left[8-4s-6s^2+3s^3\right]^2$ and $\tilde{\pi}_1^{BI}=\pi_1\left(\tilde{q}_0^{BI},\tilde{q}_1^{BI};w^{BI}\right)=(1-s)a^2/4(1+s)(2-s^2)^2$ respectively for Cournot and Bertrand competition. Therefore, we have for all $s\in(0,1)$, $\tilde{\pi}_1^{BI}-\tilde{\pi}_1^{CI}=s^2(1-s)(8-11s^2+4s^4)a^2/4(1+s)(2-s^2)^2(4-3s^2)^2>0$. Hence proved.

The equilibrium values of the consumer surplus are $\left(\tilde{CS}^{CI}\right)=CS\left(\tilde{q}_0^{CI},\tilde{q}_1^{CI}\right)=(5-4s)a^2/8(4-3s^2)$ and $\left(\tilde{CS}^{BI}\right)=CS\left(\tilde{q}_0^{BI},\tilde{q}_1^{BI}\right)=(5-s-3s^2+s^3)a^2/8(1+s)(2-s^2)^2$ respectively for Cournot and Bertrand competition. Therefore, we have $\tilde{CS}^{BI}-\tilde{CS}^{CI}=s(1-s)(8-s-12s^2+4s^3)a^2/8(1+s)(2-s^2)^2(4-3s^2)\lesseqgtr 0$ for all $s\lesseqgtr\hat{s}_2$ where one can show that $\hat{s}_2\in(0.8984,0.9063)$. Hence proved.

The equilibrium values of the welfare are $\left(\tilde{W}^{CI}\right)=W\left(\tilde{q}_0^{CI},\tilde{q}_1^{CI},w^{CI},w^{CI}\right)=(7-6s)a^2/8(4-3s^2)$ and $\tilde{W}^{BI}=W\left(\tilde{q}_0^{BI},\tilde{q}_1^{BI},w^{BI},w^{BI}\right)=(7+s-7s^2-s^3+2s^4)a^2/8(1+s)(2-s^2)^2$ respectively for

Cournot and Bertrand competition. Therefore, we have for all $s \in (0,1)$,

$$\tilde{W}^{BI} - \tilde{W}^{CI} = \frac{s^2(1-s)(3-2s^2)a^2}{8(1+s)(2-s^2)^2(4-3s^2)} > 0.$$

Hence proved.

Proof of the Lemma 1:

Proof of Lemma 1 (i) If in Stage-III firms competing with Cournot competition, then assuming the quantity of Firm 1 (q_1) is fixed, Firm 0 will maximize

$$V(q_0, q_1; w_0, w_1, \theta) = \theta\pi_0(q_0, q_1; w_0) + (1-\theta)W(q_0, q_1; w_0, w_1)$$

by choosing its own output q_0 and similarly assuming quantity of Firm 0 (q_0) is fixed, Firm 1 will maximize $\pi_1(q_0, q_1; w_1)$ by choosing its output q_1. Given any $\theta \in [0,1]$ of Stage-I and (w_0, w_1) of Stage-II, if the Stage-III choice vector is $(q_0^{CD}(w_0, w_1, \theta), q_1^{CD}(w_0, w_1, \theta))$ then $(q_0^{CD}(w_0, w_1, \theta), q_1^{CD}(w_0.w_1, \theta))$ simultaneously satisfy the following reaction equations

$$\frac{\partial V}{\partial q_0}(q_0, q_1, w_0, w_1, \theta) = P_0(q_0, q_1) - w_0 + \theta q_0 \frac{\partial P_0}{\partial q_0}(q_0, q_1) = 0 \tag{25}$$

and

$$\frac{\partial \pi_1}{\partial q_1}(q_0, q_1, w_1) = P_1(q_0, q_1) - w_1 + \theta q_1 \frac{\partial P_1}{\partial q_1}(q_0, q_1) = 0. \tag{26}$$

Evaluating condition (25) and (26) at $(q_0^{CD}(w_0, w_1, \theta), q_1^{CD}(w_0, w_1, \theta))$ we get the system of equations involving the Stage-III choices are

$$\begin{aligned}(1+\theta)q_0^{CD}(w_0, w_1, \theta) + sq_1^{CD}(w_0, w_1, \theta) &= a - w_0 \\ sq_0^{CD}(w_0, w_1, \theta) + 2q_1^{CD}(w_0, w_1, \theta) &= a - w_1.\end{aligned}$$

Solving for quantities we have

$$q_0^{CD}(w_0, w_1, \theta) = \frac{2(a-w_0) - s(a-w_1)}{2(1+\theta) - s^2} \tag{27}$$

and

$$q_1^{CD}(w_0, w_1, \theta) = \frac{(1+\theta)(a-w_1) - s(a-w_0)}{2(1+\theta) - s^2}. \tag{28}$$

Therefore, substituting $q_0 = q_0^{CD}(w_0, w_1, \theta)$ and $q_1 = q_1^{CD}(w_0, w_1, \theta)$ in the equation (7) we get

$$\pi_u^{CD}(w_0, w_1, \theta) = \frac{w_0\left[2(a - w_0) - s(a - w_1)\right] + w_1\left[(1+\theta)(a - w_1) - s(a - w_0)\right]}{2(1+\theta) - s^2} \tag{29}$$

If w_0^{CD} and w_1^{CD} are respectively the optimum input price for publicly regulated and private firm then w_0^{CD} and w_1^{CD} satisfy the first order conditions $\partial \pi_u^{CD}\left(w_0 = w_0^{CD}, w_1 = w_1^{CD}, \theta\right)/\partial w_0 = 0$ and $\partial \pi_u^{CD}\left(w_0 = w_0^{CD}, w_1 = w_1^{CD}, \theta\right)/\partial w_1 = 0$. Therefore, the system of equation involving the Stage-II choices are $2(a - 2w_0^{CD}) - s(a - 2w_1^{CD}) = 0$ and $(1+\theta)(a - 2w_1^{CD}) - s(a - 2w_0^{CD}) = 0$. Solving for w_0^{CD} and w_1^{CD} we have $w_0^{CD} = w_1^{CD} = a/2$. Note that $\partial^2 \pi_u^{CD}/\partial w_0^2 = -4/[2(1+\theta) - s^2] < 0$, $\partial^2 \pi_u^{CD}/\partial w_1^2 = -2(1+\theta)/[2(1+\theta) - s^2] < 0$, and $H^{CD} = \left(\partial^2 \pi_u^{CD}/\partial w_0^2\right)\left(\partial^2 \pi_u^{CD}/\partial w_1^2\right) - \left(\partial^2 \pi_u^{CD}/\partial w_0 \partial w_1\right) = 4/[2(1+\theta) - s^2] > 0$ therefore, the second order condition also satisfied for Stage-II. Hence proved.

Proof of Lemma 1 (ii): If in Stage-III firms competing with Bertrand competition, then assuming the price of Firm 1 (p_1) is fixed Firm 0 will maximize

$$\overline{V}(p_0, p_1; w_0, w_1, \theta) = \theta \overline{\pi}_0(p_0, p_1; w_0) + (1-\theta)\overline{W}(p_0, p_1; w_0, w_1)$$

by choosing its own price, p_0 and assuming the price of Firm 0 (p_0) is fixed Firm 1 will maximize $\overline{\pi}_1(p_0, p_1; w_1)$ by choosing its own price, p_1. Given any $\theta \in [0,1]$ of Stage-I and (w_0, w_1) of Stage-II, if the Stage-III choice vector is $(p_0^{BD}(w_0, w_1, \theta), p_1^{BD}(w_0, w_1, \theta))$ then $(p_0^{BD}(w_0, w_1, \theta), p_1^{BD}(w_0.w_1, \theta))$ simultaneously satisfy the following reaction equations

$$\frac{\partial \overline{V}}{\partial p_0}(p_0, p_1, w_0, w_1, \theta) = (p_0 - w_0)\frac{\partial D_0}{\partial p_0}(p_0, p_1) + \theta D_0(p_0, p_1) + (1-\theta)(p_1 - w_1)\frac{\partial D_1}{\partial p_0}(p_0, p_1) = 0 \tag{30}$$

and

$$\frac{\partial \overline{\pi}}{\partial p_1}(p_0, p_1, w_1) = (p_1 - w_1)\frac{\partial D_1}{\partial p_1}(p_0, p_1) + D_1(p_0, p_1) = 0. \tag{31}$$

Evaluating condition (30) and (31) at $(p_0^{BD}(w_0, w_1, \theta), p_1^{BD}(w_0, w_1, \theta))$ we get the system of equations involving the Stage-III choices are

$$\begin{aligned}(1+\theta)p_0^{BD}(w_0, w_1, \theta) - s p_1^{BD}(w_0, w_1, \theta) &= \theta(1-s)a + w_0 - s(1-\theta)w_1 \\ -s p_0^{BD}(w_0, w_1, \theta) + 2p_1^{BD}(w_0, w_1, \theta) &= (1-s)a + w_1\end{aligned}$$

Solving for prices we have

$$p_0^{BD}(w_0, w_1, \theta) = w_0 + \frac{(2\theta - s^2)(a - w_0) - s(2\theta - 1)(a - w_1)}{2(1+\theta) - s^2}$$

and

$$p_1^{BD}(w_0, w_1, \theta) = w_1 + \frac{[1 + \theta(1 - s^2)](a - w_1) - s(a - w_0)}{2(1 + \theta) - s^2}.$$

Therefore the resulting quantities are

$$q_0^{BD}(w_0, w_1, \theta) = D_0\left(p_0^{BD}(w_0, w_1, \theta), p_1^{BD}(w_0, w_1, \theta)\right) = \frac{(2 - s)(a - w_0) - s(1 + (1 - \theta)(1 - s^2))(a - w_1)}{(1 - s^2)[2(1 + \theta) - s^2]} \tag{32}$$

and

$$q_1^{BD}(w_0, w_1, \theta) = D_1\left(p_0^{BD}(w_0, w_1, \theta), p_1^{BD}(w_0, w_1, \theta)\right) = \frac{[1 + \theta(1 - s^2)](a - w_1) - s(a - w_0)}{(1 - s^2)[2(1 + \theta) - s^2]}. \tag{33}$$

Therefore, substituting $q_0 = q_0^{BD}(w_0, w_1, \theta)$ and $q_1 = q_1^{BD}(w_0, w_1, \theta)$ in the equation (7) we get

$$\pi_u^{BD}(w_0, w_1, \theta) = \frac{\begin{bmatrix} w_0\left[(2 - s)(a - w_0) - s(1 + (1 - \theta)(1 - s^2))(a - w_1)\right] \\ +w_1\left[[1 + \theta(1 - s^2)](a - w_1) - s(a - w_0)\right] \end{bmatrix}}{(1 - s^2)\left[2(1 + \theta) - s^2\right]} \tag{34}$$

If w_0^{BD} and w_1^{BD} are respectively the optimum input price for publicly regulated and private firm then w_0^{BD} and w_1^{BD} satisfy the first order conditions $\partial\pi_u^{BD}\left(w_0 = w_0^{BD}, w_1 = w_1^{BD}, \theta\right)/\partial w_0 = 0$ and $\partial\pi_u^{BD}\left(w_0 = w_0^{BD}, w_1 = w_1^{BD}, \theta\right)/\partial w_1 = 0$. Therefore, the system of equation involving the Stage-II choices are $2(2 - s^2)w_0^{BD} - s(2 + (1 - \theta)(1 - s^2))w_1^{BD} = (1 - s)(2 + s\theta - s^2 + \theta s^2)a$ and $-s[2 + (1 - \theta)(1 - s^2)]w_0^{BD} + 2[1 + \theta - \theta s^2]w_1^{BD} = (1 - s)(1 + \theta + s\theta)a$. Solving for w_0^{BD} and w_1^{BD} we have $w_0^{BD}(\theta) = [s(1 - s^2)\theta^2 + (4 - s^2 - s^3)\theta + (4 - s - s^2)]a/[-s^2(1 - s^2)\theta^2 + 2(4 + s^2 - s^4)\theta + (8 - 5s^2 + s^4)]$ and $w_1^{BD}(\theta) = [-s^2(1 - s^2)\theta^2 + (4 - 2s + 3s^2 + s^3 - 2s^4)\theta + (2 - s)(2 - s^2)(1 + s)]a/[-s^2(1 - s^2)\theta^2 + 2(4 + s^2 - s^4)\theta + (8 - 5s^2 + s^4)]$. Note that $\partial^2\pi_u^{BD}/\partial w_0^2 = -2(2 - s^2)/(1 - s^2)[2(1 + \theta) - s^2] < 0$, $\partial^2\pi_u^{BD}/\partial w_1^2 = -2(1 + \theta - \theta s^2)/(1 - s^2)[2(1 + \theta) - s^2] < 0$, and $H^{BD} = \left(\partial^2\pi_u^{BD}/\partial w_0^2\right)\left(\partial^2\pi_u^{BD}/\partial w_1^2\right) - \left(\partial^2\pi_u^{BD}/\partial w_0\partial w_1\right) = [(8 - 5s^2 + s^4) + 2(4 + s^2 - s^4)\theta - s^2(1 - s^2)\theta^2]/(1 - s^2)[2(1 + \theta) - s^2]^2 > 0$, therefore, the second order condition for Stage-II also satisfied.

Moreover, we have

$$w^{BI} - w_0^{BD}(\theta) = \frac{s(1 - s)(1 - \theta)(2 + s)[1 + \theta - s(1 - \theta)]a}{2\left[8(1 + \theta) - \{4 + (1 - \theta)^2\}s^2 + (1 - \theta)^2 s^4\right]} > 0 \tag{35}$$

and

$$w_1^{BD}(\theta) - w^{BI} = \frac{s(1 - s)(1 - \theta)[4 + (1 + \theta)s - (1 - \theta)s^2]a}{[8(1 + \theta) - \{4 + (1 - \theta)^2\}s^2 + (1 - \theta)^2 s^4]} > 0 \tag{36}$$

Combining condition (35) and (36) we have $w_1^{BD}(\theta) > a/2 > w_0^{BD}(\theta)$. Hence proved.

Proof of Lemma 1 (iii): Finally, the ratio $\omega(\theta) = (w_1^{BD}(\theta) - w_1^{BI})/(w_0^{BI} - w_0^{BD}(\theta)) = [4 + (1 +$

$\theta)s-(1-\theta)s^2]/(2+s)[1+\theta-s(1-\theta)]$. Hence, differentiating $\omega(\theta)$ with respect to θ we get $\partial\omega/\partial\theta = -4(1+s)/(2+s)[1+\theta-s(1-\theta)]^2 < 0$. Hence, the result.

Proof of Proposition 1:

Proof of Proposition 1 (i): Substituting $w_0 = w_0^{CD}$ and $w_1 = w_1^{CD}$ in the condition (27) and (28) we will respectively have the resulting quantities $\hat{q}_0^{CD}(\theta) = q_0^{CD}(w_0^{CD}, w_1^{CD}, \theta) = (2-s)a/2\left[2(1+\theta)-s^2\right]$ and $\hat{q}_1^{CD}(\theta) = q_1^{CD}(w_0^{CD}, w_1^{CD}, \theta) = (1+\theta-s)a/2\left[2(1+\theta)-s^2\right]$. Substituting $q_0 = \hat{q}_0^{CD}(\theta)$, $q_1 = \hat{q}_1^{CD}(\theta)$, $w_0 = w_0^{CD}$ and $w_1 = w_1^{CD}$ in (8) we have the resulting welfare

$$\hat{W}^{CD}(\theta) = \left[3\theta^2 + 2(7-5s)\theta + (7-6s-2s^2+2s^3)\right]a^2/8\left[2(1+\theta)-s^2\right]^2.$$

Therefore if θ^{CD} be the Stage-I optimum privatization then θ^{CD} satisfy the first order condition $\partial\hat{W}^{CD}\left(\theta=\theta^{CD}\right)/\partial\theta = (2-s)\left[s(1-s)-(4-3s)\theta^{CD}\right]/4\left[2(1+\theta^{CD})-s^2\right]^3 = 0$. Solving for θ^{CD} we get $\theta^{CD} = \theta^{CI} = s(1-s)/(4-3s)$.

Proof of Proposition 1 (ii) Substituting $w_0 = w_0^{BD}(\theta)$ and $w_1 = w_1^{BD}(\theta)$ in the condition (32) and (33) we will respectively have the resulting quantities $\hat{q}_0^{BD}(\theta) = q_0^{BD}\left(w_0^{BD}(\theta), w_1^{BD}(\theta), \theta\right) = \left[(4+s-s^2)+s(1+s)\theta\right]a/(1+s)\left[(8-5s^2+s^4)+2(4+s^2-s^4)\theta-s^2(1-s^2)\theta^2\right]$ and $\hat{q}_1^{BD}(\theta) = q_1^{BD}\left(w_0^{BD}(\theta), w_1^{BD}(\theta), \theta\right) = (2+s)\left[(1-s)+(1+s)\theta\right]a/(1+s)\left[(8-5s^2+s^4)+2(4+s^2-s^4)\theta-s^2(1-s^2)\theta^2\right]$. Substituting $q_0^{BD} = \hat{q}_0^{BD}(\theta)$, $q_1 = \hat{q}_1^{BD}(\theta)$, $w_0 = w_0^{BD}(\theta)$ and $w_1 = w_1^{BD}(\theta)$ in condition (8) we get the resulting welfare is $\hat{W}^{BD}(\theta) = W(\hat{q}_0^{BD}(\theta), \hat{q}_1^{BD}(\theta), w_0^{BD}(\theta), w_1^{BD}(\theta))$. In the Stage-I the social planner will maximize $\hat{W}^{BD}(\theta)$ by choosing $\theta \in [0,1]$. Differentiating $\hat{W}^{BD}(\theta)$ with respect to θ we have

$$\frac{\partial\hat{W}^{BD}}{\partial\theta} = \frac{(1-s)\mathcal{P}_1(\theta,s)a^2}{(1+s)\left[(8-5s^2+s^4)+2(4+s^2-s^4)\theta-s^2(1-s^2)\theta^2\right]^3}. \tag{37}$$

where $\mathcal{P}_1(\theta^{BD}, s) = 0$ where for any $\theta \in [0,1]$ we have $\mathcal{P}_3(\theta,s) = C_4(s)\theta^4 + C_3(s)\theta^3 + C_2(s)\theta^2 + C_1(s)\theta + C_0(s)$ in which we have $C_0(s) = s(1+s)(2+s)(16+2s-19s^2+10s^3+2s^4-4s^5+s^6)$; $C_1(s) = -128+32s+44s^2-36s^3+30s^4-6s^5-2s^6+22s^7-4s^9$; $C_2(s) = 6s^2(1-s^2)(10+4s+2s^3-s^4-s^5)$; $C_3(s) = 2s^2(1+s)(1-s^2)(2+4s-s^2+2s^3+s^4+s^5)$; and $C_4(s) = -s^4(1-s^2)(1+s)$. Given for all $\theta \in [0,1]$ and $s \in (0,1)$, $(8-5s^2+s^4)+2(4+s^2-s^4)\theta-s^2(1-s^2)\theta^2 > 0$, therefore, form condition (37) we have $sign(\partial\hat{W}^{BD}/\partial\theta) = sign(\mathcal{P}_1(\theta,s))$. Now, for all $s \in (0,1)$, $\mathcal{P}_1(\theta=0,s) = C_{10}(s) = s(1+s)(2+s)(16+2s-19s^2+10s^3+2s^4-4s^5+s^6) > 0$ implies at $\theta = 0$, $\partial\hat{W}^{BD}/\partial\theta > 0$. Therefore, full nationalization is not optimal. Further, for all $s \in (0,1)$, $\mathcal{P}_1(\theta=1,s) = \sum_{i=0}^{4} C_{1i}(s) = -16(1-s)^2(2+s)^2 < 0$ implies at $\theta = 1$, $\partial\hat{W}^{BD}/\partial\theta < 0$. Therefore, full privatization is not optimal. Hence, we have the partial privatization. Therefore, given any $s \in (0,1)$, if θ^{BD} be any optimal privatization ratio then $\mathcal{P}_1(\theta^{BD},s) = 0$. Further, we have $\partial\mathcal{P}_1(\theta,s)/\partial\theta = 4C_{14}(s)\theta^3 + 3C_{13}(s)\theta^2 + 2C_{12}(s)\theta + C_{11}(s)$. Since, for all $s \in (0,1)$ we have $C_{12}(s) = 6s^2(1-s^2)(10+4s+2s^3-s^4-s^5) > 0$ and $C_{13}(s) = 2s^2(1+s)(1-s^2)(2+4s-s^2+$

$2s^3+s^4+s^5) > 0$ therefore, $\mathcal{P}_2(\theta,s) = 3C_{13}(s)\theta^2 + 2C_{12}(s)\theta + C_{11}(s)$ is strictly increasing in θ. Further, for all $s \in (0,1)$, $\mathcal{P}_2(\theta=1,s) = -4s^9 - 12s^8 - 8s^7 - 8s^6 - 60s^5 - 84s^4 + 48s^3 + 176s^2 + 32s - 128 < 0$, therefore, for all $s \in (0,1)$ and $\theta \in [0,1]$ we have $\mathcal{P}_2(\theta,s) < 0$. Hence, given $C_{14}(s) = -s^4(1-s^2)(1+s) < 0$ for all $s \in (0,1)$ $(\partial\mathcal{P}_1/\partial\theta) = 4C_{14}(s)\theta^4 + \mathcal{P}_2(\theta,s) < 0$ implies $\mathcal{P}_3(\theta,s)$ is monotonically decreasing in $\theta \in (0,1)$. Therefore, there exist an unique $\theta^{BD} \in (0,1)$ that maximizes $\hat{W}^{BD}(\theta)$ such that $\mathcal{P}_1(\theta^{BD},s) = 0$.

Proof of Proposition 1 (iii): One can show that $\mathcal{P}_1\left(\theta^{CD},s\right) = s^2\mathcal{P}_3(s)/(4-3s)^4$ where $\mathcal{P}_3(s) = (1-s)^3\mathcal{P}_4(s) + \mathcal{P}_5(s)$ in which

$$\mathcal{P}_4(s) = \left[\begin{array}{c} 6402 + 6914(1-s) + 2523(1-s)^2 - 1398(1-s)^3 + 711(1-s)^4 - 1596(1-s)^5 \\ +438(1-s)^6 - 204(1-s)^7 + 72(1-s)^8 + 10(1-s)^9 - (1-s)^{10} + 2(1-s)^{11} - (1-s)^{12} \end{array}\right] > 0$$

and $\mathcal{P}_5(s) = 2880s^2 - 6368s + 3536 \geq (716/45)$. Therefore, $\mathcal{P}_3(s) > 0$ implies $\mathcal{P}_1\left(\theta^{CD},s\right) > 0$. Given $\mathcal{P}_1\left(\theta,s\right)$ is decreasing in θ hence $\theta^{BD} > \theta^{CD}$.

Proof of the Proposition 2:

Proof of Proposition 2 (i): We will derive the Cournot Bertrand ranking corresponding to equilibrium output, price and profit of the Firm 0 one after another sequentially.

Cournot Bertrand ranking corresponding to equilibrium output of Firm 0: The equilibrium quantity that publicly regulated firm produces under Cournot competition is $\tilde{q}_0^{CD} = (4-3s)a/2(4-3s^2)$ and the resulting quantity of output that public firm produces under Bertrand competition evaluated at Stage-II optimum input prices is $\hat{q}_0^{BD}(\theta) = [4+(1+\theta)s-(1-\theta)s^2]a/(1+s)[8+8\theta-\{4+(1-\theta)^2\}s^2+(1-\theta)^2s^4]$. Consider the difference $\hat{q}_0^{BD}(\theta) - \tilde{q}_0^{CD} = \mathcal{P}_6(\theta,s)a/2(1+s)(4-3s^2)[8+8\theta-\{4+(1-\theta)^2\}s^2+(1-\theta)^2s^4]$ where $\mathcal{P}_6(\theta,s) = C_{26}(s)\theta^2 + C_{16}(s)\theta + C_{06}(s)$ where $C_{26}(s) = s^2(1-s)(4-3s)(1+s)^2 > 0$, $C_{16}(s) = -2(1+s)(16-16s+4s^2-4s^4+3s^5) < 0$ and $C_{06}(s) = s^2(1-s)(12+11s-2s^2-3s^3) > 0$. Therefore, $\hat{q}_0^{BD}(\theta) - \tilde{q}_0^{CD} \gtreqless 0$ if and only if $\mathcal{P}_6(\theta,s) = C_{26}(s)(\theta-\theta_1^{q_0}(s))(\theta-\theta_2^{q_0}(s)) \gtreqless 0$ where $\theta_1^{q_0}(s) = \frac{-C_{16}(s)-\sqrt{(C_{16}(s))^2-4C_{26}(s)C_{06}(s)}}{2C_{26}(s)}$ and $\theta_2^{q_0}(s) = \frac{-C_{16}(s)+\sqrt{(C_{16}(s))^2-4C_{26}(s)C_{06}(s)}}{2C_{26}(s)}$. We have the following about $\theta_1^{q_0}(s)$ and $\theta_2^{q_0}(s)$.

(i) One can show that $(C_{16}(s))^2 - 4C_{26}(s)C_{06}(s) > 0$ therefore both $\theta_1^{q_0}(s)$ and $\theta_2^{q_0}(s)$ are real.

(ii) Given $\theta_1^{q_0}(s)\theta_2^{q_0}(s) = C_{06}(s)/C_{26}(s) > 0$ and $\theta_1(s)+\theta_2(s) = -C_{16}(s)/C_{26}(s) > 0$ we have $\theta_1^{q_0}(s), \theta_2^{q_0}(s) > 0$

(iii) Given $C_{16}(s) < 0$ we have $\theta_2^{q_0}(s) > \theta_1^{q_0}(s) > 0$.

(iv) Given $C_{26}(s) - C_{16}(s) + C_{06}(s) > 0$ therefore, $\theta_1^{q_0}(s) < 1$. Further, $C_{26}(s) + C_{16}(s) + C_{06}(s) < 0$ implies $\theta_2^{q_0}(s) > 1$.

Hence, we have $\theta_2^{q_0}(s) > 1 > \theta_1^{q_0}(s) > 0$. Therefore, (a) for all $(\theta,s) \in [0,1]\times(0,1)$ if $\theta_1^{q_0}(s) < \theta \leq 1$ then we have $\mathcal{P}_6(\theta,s) < 0$. Further, given any $s \in (0,1)$ there exist an unique $\theta^{BD} \in (0,1)$ such that $\mathcal{P}_1(\theta^{BD}, s) = 0$. Finally, one can show that $\mathcal{P}_1(\theta_1(s), s) > 0$ for all $s \in (0,1)$. Therefore, using $(\partial\mathcal{P}_1/\partial\theta) < 0$ we have $1 > \theta^{BD} > \theta_1^{q_0}(s)$ for all $s \in (0,1)$. Hence, using (a) we have $\mathcal{P}_6(\theta^{BD}, s) < 0$ implies $\tilde{q}_0^{BD} = \hat{q}_0^{BD}(\theta^{BD}) < \hat{q}_0^{CD}$. Hence the ranking.

Cournot Bertrand ranking corresponding to equilibrium price of Firm 0: The equilibrium price that publicly regulated firm will charged under Cournot competition is $\tilde{p}_1^{CD} = (8 - 2s - 9s^2 + 4s^3)a/2(8 - 4s - 6s^2 + 3s^3)$ and the resulting price that publicly regulated firm will charged under Bertrand competition evaluated at Stage-II optimum input prices is $\hat{p}_1^{BD}(\theta) = [(1+s)(4 - 3s - s^2 + s^3) + (8 - 3s + s^2 - 2s^4)\theta - s^2(1 - s^2)\theta^2]a/[8 + 8\theta - \{4 + (1-\theta)^2\}s^2 + (1-\theta)^2 s^4]$. Consider the difference $\hat{p}_1^{BD}(\theta) - \tilde{p}_1^{CD} = -(1-s)\mathcal{P}_7(\theta, s)a/2(4 - 3s^2)[8 + 8\theta - \{4 + (1-\theta)^2\}s^2 + (1-\theta)^2 s^4]$ where $\mathcal{P}_7(\theta, s) = C_{27}(s)\theta^2 + C_{17}(s)\theta + C_{07}(s)$ in which $C_{27}(s) = s^2(1+s)(4 - s - 2s^2) >$, $C_{17}(s) = -2(2+s)(8 - 4s - 2s^2 + s^3 - 2s^4) <$ and $C_{07}(s) = s^2(4 + 5s - 3s^2 - 2s^3) > 0$. Therefore, $\hat{p}_1^{BD}(\theta) - \tilde{p}_1^{CD} \gtreqless 0$ if and only if $\mathcal{P}_7(\theta, s) = C_{27}(s)(\theta - \theta_1^{p_0}(s))(\theta - \theta_2^{p_0}(s)) \lesseqgtr 0$ where $\theta_1^{p_0}(s) = \frac{-C_{17}(s) - \sqrt{(C_{17}(s))^2 - 4C_{27}(s)C_{07}(s)}}{2C_{27}(s)}$ and $\theta_2^{p_0}(s) = \frac{-C_{17}(s) + \sqrt{(C_{17}(s))^2 - 4C_{27}(s)C_{07}(s)}}{2C_{27}(s)}$. We have the following observations about $\theta_1^{p_0}(s)$ and $\theta_2^{p_0}(s)$.

(i) One can show that $(C_{17}(s))^2 - 4C_{27}(s)C_{07}(s) > 0$ therefore both $\theta_1^{p_0}(s)$ and $\theta_2^{p_0}(s)$ are real.

(ii) Given $\theta_1^{p_0}(s)\theta_2^{p_0}(s) = C_{07}(s)/C_{27}(s) > 0$ and $\theta_1^{p_0}(s) + \theta_2^{p_0}(s) = -C_{17}(s)/C_{27}(s) > 0$ we have $\theta_1^{p_0}(s), \theta_2^{p_0}(s) > 0$.

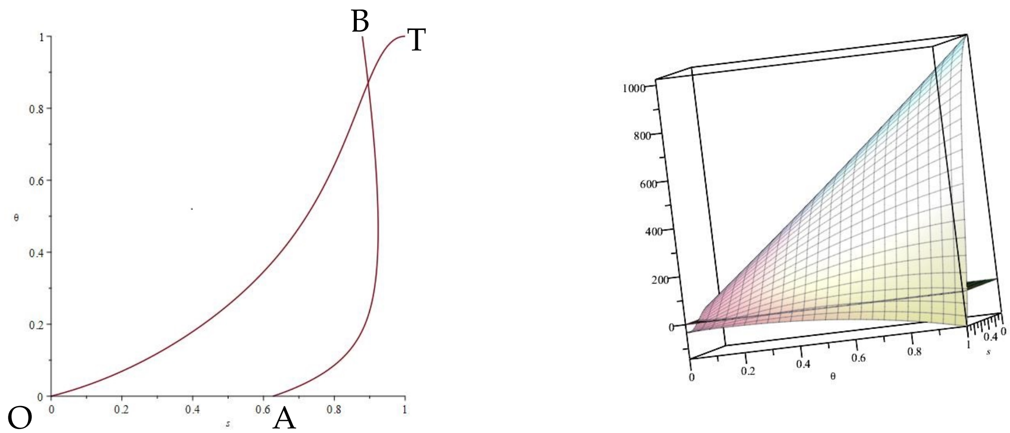
(iii) Given $C_{17}(s) < 0$ we have $\theta_2^{p_0}(s) > \theta_1^{p_0}(s) > 0$.

(iv) Given $C_{27}(s) - C_{17}(s) + C_{07}(s) > 0$ therefore, $\theta_1^{p_0}(s) < 1$. Further, $C_{27}(s) + C_{17}(s) + C_{07}(s) < 0$ implies $\theta_2^{p_0}(s) > 1$.

Hence, we have $\theta_2^{p_0}(s) > 1 > \theta_1^{p_0}(s) > 0$. Therefore, (a_2) for all $(\theta, s) \in [0,1] \times (0,1)$ if $\theta_1^{p_0}(s) < \theta \leq 1$ then we have $\mathcal{P}_7(\theta, s) < 0$. Further, given any $s \in (0,1)$ there exist an unique $\theta^{BD} \in (0,1)$ such that $\mathcal{P}_3(\theta^{BD}, s) = 0$. Finally, one can show that $\mathcal{P}_1(\theta_1^{p_0}(s), s) > 0$ for all $s \in (0,1)$. Therefore, using $(\partial\mathcal{P}_1/\partial\theta) < 0$ we have $\theta^{BD} > \theta_1^{p_0}(s)$ for all $s \in (0,1)$. Hence, using (a_2) we have $\mathcal{P}_7(\theta^{BD}, s) < 0$ implies $\tilde{p}_0^{BD} = \hat{p}_0^{BD}(\theta^{BD}) > \hat{p}_0^{CD}$. Hence the ranking.

Ranking of Profit of Firm 0: The equilibrium profit that public firm will earn under Cournot competition is $\tilde{\pi}_0^{CD} = s(1-s)(4-3s)a^2/4(4-3s^2)^2$ and the profit that public firm will earn at Stage-II optimum input prices under Bertrand competition is $\hat{\pi}_0^{BD}(\theta) = (1-s)[4 + (1+\theta)s - (1-\theta)s^2][4\theta + (2 + \theta - \theta^2)s - (1 - 3\theta + 2\theta^2)s^2 - (1-\theta)^2 s^3]a/(1+s)(4-3s^2)^2[8 + 8\theta - \{4 + (1-\theta)^2\}s^2 + (1-\theta)^2 s^4]^2$. Consider the difference $\hat{\pi}_0^{BD}(\theta) - \tilde{\pi}_0^{CD} = (1-s)\mathcal{P}_8(\theta, s)a^2/4(1+s)(4 - 3s^2)^2[8 + 8\theta - \{4 + (1-\theta)^2\}s^2 + (1-\theta)^2 s^4]^2$ where $\mathcal{P}_8(\theta, s) = C_{48}(s)\theta^4 + C_{38}(s)\theta^3 + C_{28}(s)\theta^2 + C_{18}(s)\theta + C_{08}(s)$ in which $C_{48}(s) = -s^5(1+s)(4-3s)(1-s^2)^2$, $C_{38}(s) = -4s^2(1+s)^2(16 + 4s^2 - 40s^3 + 16s^4 + 10s^5 - 7s^6 + 3s^7)$, $C_{28}(s) = 2s(1+s)(9s^9 - 12s^8 + 24s^7 + 94s^6 - 219s^5 - 140s^4 + 312s^3 + 64s^2 - 32s - 128)$, $C_{18}(s) = -12s^{11} + 4s^10 - 20s^9 - 132s^8 + 480s^7 + 720s^6 - 960s^5 - 624s^4 +$

$512s^3 - 960s^2 + 1024$ and $C_{08} = s(3s^{10} - s^9 + 2s^8 + 10s^7 - 185s^6 - 113s^5 + 620s^4 + 272s^3 - 704s^2 - 192s + 256)$. In Figure 3a we have the implicit plot of the equations $\mathcal{P}_3(\theta, s) = 0$ (the curve OT) and $\mathcal{P}_8(\theta, s) = 0$ (the curve AB). From the 3D plot of $\mathcal{P}_8(\theta, s)$ to the left (right) of the curve AB we have $\mathcal{P}_8(\theta, s) > 0 (< 0)$. Hence using the curve OT one can conclude that for low value of s we have $\tilde{\pi}_0^{BD} = \hat{\pi}_0^{BD}(\theta^{BD}) > \tilde{\pi}_0^{CD}$ and for high value of s we have $\tilde{\pi}_0^{BD} = \hat{\pi}_0^{BD}(\theta^{BD}) < \tilde{\pi}_0^{CD}$.



(a) Implicit plot of $\mathcal{P}_1(\theta, s) = 0$ and $\mathcal{P}_8(\theta, s) = 0$

(b) 3D plot of $\mathcal{P}_8(\theta, s)$

Figure 3

Proof of Proposition 2 (ii): Similar to the Proposition 2 (i) here also we will derive the Cournot Bertrand ranking corresponding to equilibrium output, price and profit of the Firm 1 one after another sequentially.

Cournot Bertrand ranking corresponding to equilibrium output of Firm 1: The equilibrium quantity of Firm 1 under Cournot competition is $\tilde{q}_1^{CD} = (2 - 3s + s^2)a/(8 - 4s - 6s^2 + 3s^3)$ and the resulting quantity of output that Firm 1 produces under Bertrand competition evaluated at Stage-II optimum input price is $\hat{q}_1^{BD}(\theta) = (2+s)[(1+\theta - s(1-\theta))]a/(1+s)[8 + 8\theta - \{4 + (1-\theta)^2\}s^2 + (1-\theta)^2 s^4]$. Consider the difference $\hat{q}_1^{BD}(\theta) - \tilde{q}_1^{CD} = s\mathcal{P}_9(\theta, s)a/2(1+s)(4-3s^2)[8 + 8\theta - \{4 + (1-\theta)^2\}s^2 + (1-\theta)^2 s^4]$ where $\mathcal{P}_9(\theta, s) = C_{29}(s)\theta^2 + C_{19}(s)\theta + C_{09}(s)$ where $C_{29}(s) = s(1-s^2)^2 > 0$, $C_{19}(s) = (1+s)(12 - 8s - s^2 + 2s^3 - 2s^4) > 0$ and $C_{09}(s) = -(1-s)(4 + s - 2s^2 + s^3 + s^4) < 0$. Therefore, $\hat{q}_1^{BD}(\theta) - \tilde{q}_1^{CD} \gtreqless 0$ if and only if $\mathcal{P}_9(\theta, s) = C_{29}(s)(\theta - \theta_1^{q_1}(s))(\theta - \theta_2^{q_1}(s)) \gtreqless 0$ where $\theta_1^{q_1}(s) = \frac{-C_{19}(s) - \sqrt{(C_{19}(s))^2 - 4C_{29}(s)C_{09}(s)}}{2C_{29}(s)}$ and $\theta_2^{q_1}(s) = \frac{-C_{19}(s) + \sqrt{(C_{19}(s))^2 - 4C_{29}(s)C_{09}(s)}}{2C_{29}(s)}$. We have the following about $\theta_1^{q_1}(s)$ and $\theta_2^{q_1}(s)$.

(i) One can show that $(C_{19}(s))^2 - 4C_{29}(s)C_{09}(s) > 0$ therefore both $\theta_1^{q_1}(s)$ and $\theta_2^{q_1}(s)$ are real.

(ii) Given $\theta_1^{q_1}(s)\theta_2^{q_1}(s) = C_{09}(s)/C_{29}(s) < 0$ therefore we have either $\theta_1^{q_1}(s) < 0$ and $\theta_2^{q_1}(s) > 0$ or $\theta_1^{q_1}(s) > 0$ and $\theta_2^{q_1}(s) < 0$.

(iii) Given $C_{16}(s) > 0$ and $C_{29}(s) > 0$ we have $\theta_1^{q_1}(s) < 0$. Therefore, using (ii) we have $\theta_2^{q_1}(s) > 0$.

(iv) Given $C_{26}(s) + C_{16}(s) + C_{06}(s) = 8 + 8s - 6s^2 - 4s^3 > 0$ therefore, $\theta_2^{q_1}(s) < 1$.

Hence, we have $1 > \theta_2^{q_0}(s) > 0 > \theta_1^{q_0}(s)$. Therefore, given $C_{29}(s) > 0$ and $\theta_1^{q_1}(s) < 0$ we can conclude that $\mathcal{P}_9(\theta, s) \gtreqless 0$ if and only if $\theta \gtreqless \theta_2^{q_1}(s)$. One can show that $\mathcal{P}_1(\theta_2^{q_1}(s), s) \gtreqless 0$ if and only if $s \gtreqless \hat{s}_3 \in (0.4255, 0.4256)$ (six decimal estimation of $\hat{s}_3$ is 0.425557.). Therefore, for all $s < \hat{s}_3$ we have $\mathcal{P}_1(\theta_2^{q_1}(s), s) < 0$ implies $\theta_2^{q_1}(s) > \theta^{BD}$ leads to $\mathcal{P}_9(\theta^{BD}, s) < 0$ implies $\tilde{q}_1^{BD} = \hat{q}_1^{BD}(\theta^{BD}) < \tilde{q}_1^{CD}$.

Cournot Bertrand ranking corresponding to equilibrium price of Firm 1: The equilibrium price that private firm will charged under Cournot competition is $\tilde{p}_1^{CD} = (12 - 10s - 4s^2 + 3s^3)a/2(8 - 4s - 6s^2 + 3s^3)$ and the resulting price that private firm will charged under Bertrand competition evaluated at Stage-II optimum input prices is $\hat{p}_1^{BD}(\theta) = [(6-s)(1+\theta) - (4 - \theta + \theta^2)s^2 + (1-\theta)^2 s^4]a/[8 + 8\theta - \{4 + (1-\theta)^2\}s^2 + (1-\theta)^2 s^4]$. Consider the difference $\hat{p}_1^{BD}(\theta) - \tilde{p}_1^{CD} = -s(1-s)\mathcal{P}_{10}(\theta, s)a/2(4 - 3s^2)[8 + 8\theta - \{4 + (1-\theta)^2\}s^2 + (1-\theta)^2 s^4]$ where $\mathcal{P}_{10}(\theta, s) = C_{2,10}(s)\theta^2 + C_{1,10}(s)\theta + C_{0,10}(s)$ in which $C_{2,10}(s) = s(1+s)(2 + 2s - 3s^2)$, $C_{1,10}(s) = -2(4 - 4s + s^2 - s^3 - 3s^4)$ and $C_{0,10}(s) = -(8 - 6s - 10s^2 + s^3 + 3s^4)$. Therefore, $\hat{p}_1^{BD}(\theta) - \tilde{p}_1^{CD} \gtreqless 0$ if and only if $\mathcal{P}_{10}(\theta, s) \lesseqgtr 0$. Let us first note the following.

(i) Given any $s \in (0,1)$, $(\partial \mathcal{P}_{10}(\theta, s)/\partial s) = -12(1-\theta)^2 s^3 - 3(1-\theta)^2 s^2 + 8(1-\theta)^2 s + 12(1+\theta)s + 2(1+\theta)(3+\theta) > 0$ therefore for all $s \in (0,1)$ we have $\mathcal{P}_{10}(\theta, s)$ is increasing in s.

(ii) Further, $D_{10} = (C_{1,10}(s))^2 - 4C_{2,10}(s)C_{0,10}(s) = 4(36s^6 + 60s^5 - 71s^4 - 68s^3 + 44s^2 - 16s + 16)$. One can show that there exist $\hat{s}_4 \in (0.707, 0.708)$ and $\hat{s}_5 \in (0.9893, 0.9894)$ such that we have the following (a) $D_{10} > 0$ for all $s \in (0, \hat{s}_4) \cup (\hat{s}_5, 1)$, and (b) $D_{10} < 0$ for all $s \in (\hat{s}_4, \hat{s}_5)$.

Hence given $C_{2,10}(s) > 0$ for all $s \in (0,1)$ and (ii) (b) one can conclude that for all $s \in (\hat{s}_4, \hat{s}_5)$ we have $\mathcal{P}_{10}(\theta, s) > 0$. Finally using (i) we can conclude that $\mathcal{P}_{10}(\theta, s) > 0$ for all $s \in (\hat{s}_4, 1)$. Observe that we can express $\mathcal{P}_{10}(\theta, s) = C_{2,10}(s)\left(\theta - \theta_1^{p_1}(s)\right)\left(\theta - \theta_2^{p_1}(s)\right)$ where $\theta_1^{p_1}(s) = \frac{-C_{1,10}(s) - \sqrt{D_{10}}}{2C_{2,10}(s)}$ and $\theta_2^{p_1}(s) = \frac{-C_{1,10}(s) + \sqrt{D_{10}}}{2C_{2,10}(s)}$. We have the following about $\theta_1^{p_1}(s)$ and $\theta_2^{p_1}(s)$ for all $s \in (0, \hat{s}_4)$.

(i) Given $D_{10} > 0$ therefore both $\theta_1^{p_1}(s)$ and $\theta_2^{p_1}(s)$ are real.

(ii) For all $s \in (0, \hat{s}_4)$ we have one can show that $C_{1,10}(s) < 0$, therefore $\theta_1^{p_0}(s) < \theta_2^{p_0}(s)$.

Hence, we have $\mathcal{P}_{10}(\theta, s) < 0$ if and only if $\theta \in (\theta_1^{p_1}(s), \theta_2^{p_1}(s))$. One can show that $\mathcal{P}_1(\theta_1^{p_1}(s), s) > 0$ for all $s \in (0, \hat{s}_4)$. Therefore, given $\partial \mathcal{P}_1(\theta, s)/\partial \theta = 0$ and $\mathcal{P}_1(\theta^{BD}, s) = 0$ we have $\theta_1^{p_1}(s) < \theta^{BD}$.

Further, one can show that there exist a $\hat{s}_6 \in (0, \hat{s}_4)$ such that we have $\mathcal{P}_1(\theta_2^{p_1}(s), s) \gtreqless 0$ if and only if $s \gtreqless \hat{s}_6$. Therefore, $\theta^{BD} \lesseqgtr \theta_2^{p_1}(s)$ such that $s \lesseqgtr \hat{s}_6$. Hence, we have $\mathcal{P}_{10}(\theta^{BD}, s) \lesseqgtr 0$ if and only if $s \lesseqgtr \hat{s}_6$. Therefore, $\tilde{p}_1^{BD} = \hat{p}_1^{BD}(\theta^{BD}) \gtreqless \tilde{p}_1^{CD}$ if and only if $s \lesseqgtr \hat{s}_6$. Hence the ranking.

Cournot Bertrand ranking corresponding to equilibrium profit of Firm 1: The equilibrium profit that private firm will earn under Cournot competition is $\tilde{\pi}_1^{CD} = (1-s)^2 a^2/(4-3s^2)^2$ and the profit that private firm will earn at Stage-II optimum input prices under Bertrand competition is $\hat{\pi}_1^{BD}(\theta) = (1-s)(2+s)^2[1+\theta-s(1-\theta)]^2 a^2/(1+s)[8(1+\theta)-(4+(1-\theta)^2)s^2+(1-\theta)^2 s^4]^2$. Consider the difference $\sqrt{\hat{\pi}_1^{BD}(\theta)} - \sqrt{\tilde{\pi}_0^{CD}} = \mathcal{P}_{11}(\theta, s)a/\sqrt{1+s}(4-3s^2)[8(1+\theta)-(4+(1-\theta)^2)s^2+(1-\theta)^2 s^4]$ where $\mathcal{P}_{11}(\theta, s) = C_{2,11}(s)\theta^2 + C_{1,11}(s)\theta + C_{0,11}(s)$ in which $C_{2,11}(s) = s^2(1-s)^2(1+s)^{\frac{3}{2}} > 0$, $C_{1,11}(s) = (8+12s-2s^2-9s^3-3s^4)\sqrt{1-s} - 2(1-s)(4+s^2-s^4)\sqrt{1+s} > 0$ and $C_{0,11}(s) = -(1-s)[(8-5s^2+s^4)\sqrt{1+s} - (8+4s-6s^2-3s^3)\sqrt{1-s}] < 0$. Therefore, $D_{11}(s) = \{C_{1,11}\}^2 - 4C_{2,11}C_{0,11} > 0$ implies $\theta_1^{\pi_1} = (-C_{1,11(s)} + \sqrt{D_{11}(s)})/2C_{2,11}(s)$ and $\theta_2^{\pi_1} = (-C_{1,11}(s) - \sqrt{D_{11}(s)})/2C_{2,11}(s)$ both are real. Further, $C_{1,11}(s) > 0$ implies $\theta_2^{\pi} < 0$. Moreover, $C_{0,11}(s) < 0$ implies $D_{11}(s) > \{C_{1,11}(s)\}^2$ implies $\theta_1^{\pi_1} > 0$. Finally, $C_{0,11}(s) + C_{1,11}(s) + C_{2,11}(s) > 0$ implies $\theta_1^{\pi_1} < 1$. Therefore, we have $1 > \theta_1^{\pi_1} > 0 > \theta_2^{\pi_1}$. Given, $\mathcal{P}_{11}(\theta, s) = C_{2,11}(s)(\theta - \theta_1^{\pi_1})(\theta - \theta_2^{\pi_1})$, $C_{2,11}(s) > 0$ and $1 > \theta_1^{\pi_1} > 0 > \theta_2^{\pi_1}$, one can conclude that $\mathcal{P}_{11}(\theta, s) \gtreqless 0$ if and only if $\theta \gtreqless \theta_1^{\pi_1}$. One can show that $\mathcal{P}_1(\theta_1^{\pi_1}, s) \gtreqless 0$ if and only if $s \gtreqless \hat{s}_7$ with $\hat{s}_7 \in (0.62, 0.63)$. Therefore, for all $s < \hat{s}_7$ we have $\mathcal{P}_1(\theta_1^{\pi_1}, s) < 0$ implies $\theta_1^{\pi_1} > \theta^{BD}$ implies $\mathcal{P}_{11}(\theta^{BD}, s) < 0$ therefore $\tilde{\pi}_1^{BD} = \hat{\pi}_1^{BD}(\theta^{BD}) < \tilde{\pi}_1^{CD}$. Hence, the ranking.

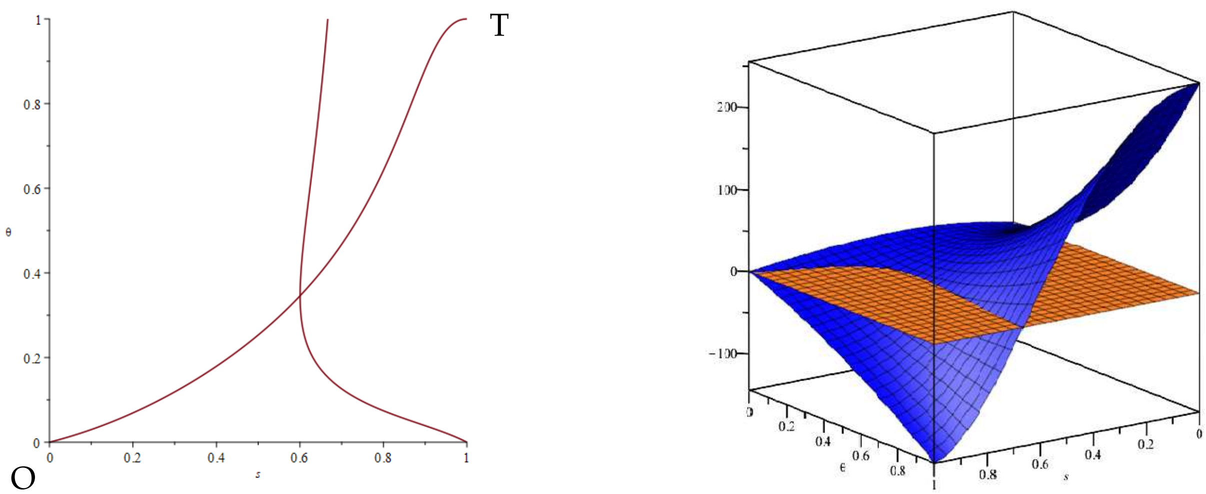
Proof of Proposition 2 (iii): The equilibrium societies welfare under Cournot competition is $\tilde{W}^{CD} = (7-6s)a^2/8(4-3s^2)$ and the society's welfare at Stage-II optimum input prices under Bertrand competition is $\hat{W}^{BD}(\theta) = \mathcal{P}_{12}(\theta, s)a^2/(1+s)[8+8\theta-\{4+(1-\theta)^2\}s^2+(1-\theta)^2 s^4]^2$ where $\mathcal{P}_{12}(\theta, s) = C_{3,12}(s)\theta^3 + C_{2,12}(s)\theta^2 + C_{1,12}(s)\theta + C_{0,12}$ in which $C_{3,12}(s) = -s^2(1-s)(1+s)^3$; $C_{2,12}(s) = (1+s)(6+6s-4s^2+4s^3-s^4-3s^5)$; $C_{1,12}(s) = (28+4s-5s^2-6s^3-10s^4+2s^5+3s^6)$ and $C_{0,12}(s) = (14-16s^2+7s^4-s^6)$. Consider the difference $\hat{W}^{BD}(\theta) - \tilde{W}^{CD} = -(1-s)\mathcal{P}_{13}(\theta, s)a^2/8(1+s)(4-3s^2)^2[8+8\theta-\{4+(1-\theta)^2\}s^2+(1-\theta)^2 s^4]^2$ where $\mathcal{P}_{13}(\theta, s) = C_{4,13}(s)\theta^4 + C_{3,13}(s)\theta^3 + C_{2,13}(s)\theta^2 + C_{1,13}(s)\theta + C_{0,13}(s)$ in which $C_{4,13}(s) = s^4(1-s)(7-6s)(1+s)^3$, $C_{3,13}(s) = -4s^2(1-s)(1+s)^2(20-12s+s^2+s^3-6s^4)$, $C_{2,13}(s) = 2(1+s)(128-160s+32s^2-8s^3-47s^4+55s^5-27s^6-3s^7+18s^8)$, $C_{1,13}(s) = -4s^2(68+8s-51s^2+10s^3-2s^4-20s^5+5s^6+6s^7)$ and $C_{0,13} = s(1-s)(64+32s-80s^2-33s^3+55s^4+27s^5-11s^6-6s^7)$. In Figure 4a we have the implicit plot of the equations $\mathcal{P}_1(\theta, s) = 0$ (the curve OT) and $\mathcal{P}_{13}(\theta, s) = 0$ (the curve AB). Moreover, in the Figure 4b we have the 3D plot of the polynomial $\mathcal{P}_{13}(\theta, s)$ for all $(\theta, s) \in [0,1] \times (0,1)$. From the 3D plot of $\mathcal{P}_{13}(\theta, s)$ to the left (right) of the curve AB we have $\mathcal{P}_{13}(\theta, s) > 0 (< 0)$. Hence using the curve OT one can conclude that for low value of s we have $\tilde{W}^{BD} = \hat{W}^{BD}(\theta^{BD}) < \tilde{w}_0^{CD}$ and for high value of s we have the opposite.

s / m	0.1	0.2	0.3	0.4	0.5	0.6	0.7	0.8	0.9
0	0.03	0.07	0.12	0.18	0.25	0.34	0.46	0.64	0.88
$\frac{(1-s)}{10}$	0.033	0.077	0.133	0.2	0.29	0.41	0.58	0.91	1
$\frac{(1-s)}{9}$	0.0334	0.078	0.135	0.21	0.30	0.42	0.60	0.98	1
$\frac{(1-s)}{8}$	0.034	0.079	0.137	0.21^{+}	0.303	0.43	0.62	1	1
$\frac{(1-s)}{7}$	0.0347	0.081	0.14	0.216	0.31	0.44	0.66	1	1
$\frac{(1-s)}{6}$	0.0357	0.083	0.145	0.224	0.33	0.47	0.72	1	1
$\frac{(1-s)}{5}$	0.0372	0.087	0.15	0.236	0.35	0.51	0.86	1	1
$\frac{(1-s)}{4}$	0.0398	0.093	0.16	0.257	0.38	0.59	1	1	1
$\frac{(1-s)}{3}$	0.0449	0.106	0.19	0.304	0.47	1	1	1	1
$\frac{(1-s)}{2}$	0.0609	0.149	0.28	0.51	1	1	1	1	1

Table 1: Simulation Results for Bertrand Competition

m \ s	0.1	0.2	0.3	0.4	0.5	0.6	0.7	0.8	0.9
0	0.024	0.047	0.067	0.085	0.1	0.109	0.11	0.100	0.069
$\frac{(1-s)}{10}$	0.0272	0.0529	0.076	0.097	0.114	0.125	0.126	0.114	0.0778
$\frac{(1-s)}{9}$	0.0275	0.0537	0.0778	0.0989	0.116	0.1265	0.1281	0.1152	0.0788
$\frac{(1-s)}{8}$	0.028	0.0546	0.0792	0.1008	0.1180	0.129	0.1305	0.1173	0.0801
$\frac{(1-s)}{7}$	0.0286	0.0559	0.0811	0.1032	0.1209	0.1321	0.1336	0.12	0.0817
$\frac{(1-s)}{6}$	0.02951	0.0576	0.0837	0.1066	0.125	0.1365	0.1379	0.1236	0.0839
$\frac{(1-s)}{5}$	0.0308	0.0602	0.0876	0.1117	0.1309	0.1402	0.1442	0.1288	0.0870
$\frac{(1-s)}{4}$	0.0329	0.0646	0.094	0.1200	0.1406	0.1533	0.1542	0.1371	0.0918
$\frac{(1-s)}{3}$	0.0372	0.0731	0.1066	0.136	0.159	0.1728	0.1726	0.152	0.1002
$\frac{(1-s)}{2}$	0.0497	0.0977	0.1420	0.18	0.2083	0.2228	0.2180	0.1867	0.1186

Table 2: Simulation Results for Cournot Competition



(a) Implicit plot of $\mathcal{P}_1(\theta, s) = 0$ and $\mathcal{P}_{11}(\theta, s) = 0$ (b) 3D plot of $\mathcal{P}11(\theta, s)$

Figure 4

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