Incentive Contracts and Hedge Fund Management

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10 May 2006

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May 10, 2006

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We would like to thank an anonymous referee, Hank Bessembinder (editor), George Constantinides, Günter Franke, Rick Green, Stewart Hodeges, J. C. Hugonnier, Kostas Iordanidis, Pierre Mella-Barral, Antonio Mello, Paolo Sodini, Fabio Trojani, and Mark Rubinstein for helpful comments. We also thank seminar participants at the 10th Symposium on Finance, Banking, and Insurance, Karlsruhe; the International Conference on Finance, Copenhagen; the 2005 Frontiers of Finance conference, Bonaire; the Conference on Delegated Portfolio Management, Eugene, Oregon; University of Ulm; University Svizzera Italiana, Lugano; University of Zurich; Humbold University, Berlin; Stockholm School of Economics; and the University of Konstanz.

Incentive Contracts and Hedge Fund Management

Abstract

We investigate incentive effects of a typical hedge-fund contract for a manager with power utility. With a one-year horizon, she displays risk-taking that varies dramatically with fund value. We extend the model to multiple yearly evaluation periods and find her risk-taking is rapidly moderated if the fund performs reasonably well. The most realistic approach to modeling fund closure uses an endogenous shutdown barrier where the manager optimally chooses to shut down. The manager increases risk-taking as fund value approaches that barrier, and this boundary behavior persists strongly with multiyear horizons.

I. Introduction

We explore effects of a typical hedge-fund compensation contract on managerial risk-taking using both a one-year evaluation period and a sequence of such evaluation periods. We find that the induced risk-taking is acutely sensitive to the manager's horizon and is often far from the Merton (1969) solution of allocating a constant proportion to the risky asset. Given the trillion dollar size of the hedge fund industry, it is important to understand results from both long and short-horizon settings because some managers will have shorter-term horizons, perhaps for personal reasons. We have an expected-utility maximizing manager who controls the allocation of fund assets between a risky investment and a riskless one. The manager has power utility displaying constant relative risk aversion (CRRA). The manager's compensation includes both a proportional management fee and a performance (incentive) fee based on exceeding a "high-water mark." This corresponds to the typical hedge-fund fee structure. Resetting the high-water mark through time based upon fund performance introduces a path-dependency that can seriously complicate a multiyear analysis; however, we develop a tractable procedure for dealing with this issue. We also allow the manager to have her own capital invested in the fund, which is a realistic possibility.

In our situation, the simple Merton (1969) result is overturned when the manager has incentives other than share ownership – including not only an option-like performance fee but also the possibilities of being fired and of choosing to shut down the fund. Nevertheless, there are circumstances where our fund manager will follow the same constant investment strategy as in Merton. Those circumstances, however, effectively amount to her both owning a proportional share of the fund and being in a state-space location far enough away from triggering point(s) for other compensation incentives that they have no influence on her behavior.

When the fund value is somewhat below the high-water mark (strike price for her incentive option), a manager with a short-term perspective is willing to take added risks in order to increase the probability of her incentive option finishing in-the-money. The resulting region of high risk-taking ("Option Ridge") in the one-year model carries through only partially in a two-year framework and effectively disappears when the manager is far from the terminal date. A one-year manager also dramatically reduces her risk-taking slightly above the high-water mark and only slowly ramps back up to the level of risk-taking that she would have chosen without the incentive option. This pattern of reduced risk-taking slightly above the high-water mark persists strongly over horizons of many years, even though Option Ridge itself does not. Brown, Goetzmann, and Park (2001) provide some limited evidence consistent with such behavior for hedge funds. They find hedge funds that had above average performance during the first half of a year reduce their volatility while those having below average performance tend to increase volatility. However, when they condition on estimated high-water marks, the significance disappears.

In practice, a fund that performs poorly is frequently shut down and liquidated. We include this influence on fund management via incorporating a liquidation boundary into the model. We begin with the simplest case of an exogenous boundary and subsequently extend the model to incorporate a more realistic endogenous shutdown decision by the manager. With the exogenous liquidation barrier, the manager dramatically reduces portfolio risk as fund value declines toward that boundary -- essentially the same result as in Goetzmann, Ingersoll, and Ross (2003). However, this means it is optimal for the manager to avoid fund closure by shifting investment completely into the riskless asset. That outcome is fundamentally inconsistent with the substantial rate of hedge fund closure -- see Getmansky, Lo, and Mei (2004). A more realistic approach, which we develop, is an endogenous shutdown choice. This represents an American-style option where the manager chooses whether or not to liquidate the fund depending on fund value, time, optimal risk-taking, and her outside employment opportunities.

A striking characteristic of the endogenous shutdown situation is that the manager chooses to gamble with high levels of risk just above the shutdown region. Moreover, this behavior is not confined to a one-year horizon but carries through strongly in a multiyear framework. Hu, Kale, and Subramanian (2005) provide evidence in a mutual fund context that a higher probability of termination leads managers to increase portfolio risk. Our situation is somewhat different with the manager choosing to seek other employment; however, the motivation of having little to lose by gambling is presumably similar. With a long horizon and many evaluation periods, such risk-taking on what we call “Decision Ridge” is eventually eliminated by the expected value of continuation; however, this can take decades.

In summary, we find that the industry-standard compensation contract induces widely varying risk-taking if the manager knows she is relatively close to the final evaluation date.[1] Moving to longer horizons with several annual evaluation periods moderates her optimal investment strategy as long as the fund is performing reasonably well. With endogenous shutdown, she begins moving toward much higher risk portfolios as fund value declines toward levels where shutdown is optimal. This behavior carries through strongly with horizons of many years, even decades.

In the next section, we present the basic model and describe the solution methodology. Section III provides numerical results for a standard set of parameters with a one-year evaluation period. Section IV develops the endogenous shutdown version of our one-year model and displays results using that structure. In Section V, we extend our analysis to multiple year-long evaluation periods and discuss how managerial risk-taking is altered. Section VI provides concluding comments.

II. The Basic One-Period Model and Solution Methodology

Here, we describe the basic one-period model beginning with the stochastic process determining the fund's value. Next, we describe the manager's compensation structure. Then, we address optimal control of the fund value process by a manager maximizing her expected utility. Our solution approach utilizes a numerical procedure, with details on implementation available in the Appendix.

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[1] We also relate our one-year results to those from Carpenter (2000), Basak, Pavlova, and Shapiro (2006) as well as Goetzmann, Ingersoll, and Ross (2003). Those papers generate results that correspond to special cases of our one-year model.

A. The Stochastic Process for Fund Value

Assume a single manager controls the allocation of fund value X between a riskless and a risky investment. The risky investment technology has a constant growth rate of μ and a standard deviation of σ. The riskless investment grows at the constant rate r. One should think of the risky investment as a proprietary technology (e.g. convergence trades or macro bets) that can be utilized by the fund manager but is not replicable or contractible by outside investors. Indeed, hedge funds work hard to conceal details of their investment strategies so they cannot be copied by outside investors or competitors. The proportion of the fund value allocated to the risky investment is denoted by κ, which is short for $\kappa(X,t)$. We allow the manager to dynamically alter κ, the risky investment proportion, at discrete points in time. Given a risky investment proportion κ, we assume log returns for the fund value X are normally distributed over each discrete time step of length Δt with mean $\mu_{\kappa,\Delta t} = [\kappa\mu + (1-\kappa)r - \frac{1}{2}\kappa^2\sigma^2]\Delta t$ and volatility $\sigma_{\kappa,\Delta t} = \kappa\sigma\sqrt{\Delta t}$. [2]

We discretize the log fund values onto a grid structure (more details are provided in the Appendix). That grid has equal time increments as well as equal steps in log X. We choose the grid spacing and riskfree interest rate such that a strategy of being fully invested in the riskless asset ($\kappa = 0$) will always end up on a grid point. From each grid point, we allow a multinomial forward move to a relatively large number of subsequent grid points (e.g., 121) at the next time

[2] For illustration purposes, we assume normality for log returns; however, our basic numerical approach can accommodate alternative return distributions such as might be generated by a portfolio including option positions with their highly skewed returns. Furthermore, jumps can also be incorporated with ease.

step. We structure potential forward moves to land on grid points and calculate the associated probabilities by using the discrete normal distribution with a specified value for the control parameter kappa (risky investment proportion).

B. The Manager's Compensation Structure

The manager has no outside wealth but rather owns a fraction of the fund. In practice, a hedge fund manager frequently has a substantial personal investment in the fund.[3] For much of our analysis, we will assume the manager owns $a = 10\%$ of the fund. That level of ownership, or more, is certainly plausible for a medium-sized hedge fund. In a large fund with assets in the billions of dollars, the manager would likely have a smaller (but still non-trivial) percentage ownership. On the remaining $(1\text{-}a)$ of fund assets, the manager earns a management fee of $b = 2\%$ annually plus an incentive fee of $c = 20\%$ on the amount by which the terminal fund value X_T exceeds the "high-water mark" which we denote by H. Such a fee structure is typical for a hedge fund.

Suppose the fund performs reasonably well and is not liquidated prior to time T, the terminal date when the manager is being compensated. Then, the manager's wealth at T equals her compensation and is equivalent to a fractional share plus a fractional call option (incentive option) struck at the high-water mark H:

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[3] This supposedly "inhibits excessive risk-taking" (Fung and Hsieh (1999), p. 316). The absence of outside wealth also induces the manager to be more conservative in that she will exhibit greater aversion to non-systematic risk from the fund investments than if her personal holdings were better diversified. Since much of our concern will be with excessive managerial risk-taking, such a conservative tilt is appropriate.

(1) $$W_T = aX_T + (1-a)bTX_T + (1-a)c(X_T - H)^+$$

A realistic complication is the possibility of liquidation if the fund performs poorly.[4] The simplest approach is to have a prespecified lower boundary -- as in Goetzmann, Ingersoll, and Ross (2003). Our basic valuation procedure uses this approach, with Φ denoting the level of the liquidation boundary. For the time being, we set Φ at 50% of the high-water mark.

Now consider the manager's compensation if the fund value hits the lower (liquidation) boundary at time τ, with $0 \le \tau \le T$. We do not impose a deadweight cost to liquidation but do recognize that the fund value X_τ could have crossed below the liquidation barrier Φ. Our base case assumption will be that the manager recovers her personal investment aX_τ plus a prorated portion of the management fee $\tau(1\text{-}a)b\,\Phi$. This results in:

(2) $$W_\tau = aX_\tau + \tau(1 - a)b(0.5\mathrm{H}) \qquad for\ 0 \le \tau \le T$$

where this value depends on when the fund reaches the boundary and by how much it crosses that boundary. One could also imagine the manager going to work for another organization and achieving greater wealth at time τ than specified in equation (2). Under such circumstances, the manager may seek the fund's liquidation. We explicitly consider this possibility later in the paper

[4] Getmansky, Lo, and Mei (2004) examine the TASS "Graveyard" database, which lists funds that cease to report their performance. Since this reporting is voluntary, there could be a variety of reasons for ceasing to report; however, 913 (out of 1765) funds have a status code indicating they were liquidated. Most of the others are listed as "No Longer Reporting" or their status is unknown.

by allowing the manager to shut down the fund at an endogenously determined liquidation boundary. Allowing an endogenously determined shutdown and liquidation effectively introduces an American-style option into the analysis. Before doing so, we first explore the simpler situation with an exogenously specified boundary.

As we shall see shortly, the lower (liquidation) boundary plays an important role in determining the manager's optimal portfolio allocations over time. Failure to consider such a boundary when modeling managerial behavior leads to very different and potentially seriously misleading results.

C. The Optimization of Expected Utility

We assume the manager seeks to maximize expected utility of terminal wealth W_T and has a utility function that exhibits constant relative risk aversion γ (however, alternative utility functions can be substituted):

$$U(W_T) = \frac{W_T^{\,1-\gamma} - 1}{1-\gamma} \tag{3}$$

For each terminal fund value above the lower boundary, we calculate the manager's wealth and the associated utility. If at any time the fund is shutdown, the manager obtains the utility of W_τ as specified in equation (2). We then step backwards in time to $T\text{-}\Delta t$. At each possible fund value within that time step, we calculate the expected utilities for investment proportions κ in our discrete choice set (κ can be zero or lie at specified steps between 0.2 and 5, details about that set are in the Appendix). We choose the highest of those expected utilities as the optimal indirect utility for that fund value and denote its value as $J_{X,T\text{-}\Delta t}$. We record the optimal indirect utilities and the associated optimal investment proportion for each fund value within that time step. We then loop backward in time, repeating this process through all time steps. This generates the indirect utility surface and optimal investment proportions for our entire grid. Formally:

(4)
$$J_{X,T} = U_{X,T}; \quad J_{X,t} = \max_{\kappa} E_{\kappa}[J_{X,t+\Delta t}]$$
where t takes the values $T - \Delta t, \ldots, 2\Delta t, \Delta t, 0$ one after another.

III. Illustrative One-Period Results

We will frequently refer to a standard set of parameters as displayed in Table 1, which we will use as our reference case. The horizon is one year with portfolio revisions in 50 time steps, roughly once per week.[5] Although a hedge fund's life may be much longer, performance incentives are typically based on one-year evaluation periods. Hence, the one-year horizon is appropriate here.

For our reference case, the starting fund value of 1 equals the current high-water mark. On an unlevered basis, we assume the risky investment has a mean return of 7.78% and a volatility of 5%. The riskless asset yields 5.78%. This combination of mean returns and volatility would be consistent with a market-neutral strategy and implies a Sharpe Ratio of 0.40, which seems reasonable in light of the results reported in Brown, Goetzmann, and Ibbotson

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[5] We have examined daily trading in the one-year model. One or two days prior to maturity, there are a few regions of the state space where the manager takes greater risks; but the basic pattern is the same as with weekly trading. Using a weekly period has effects in those areas similar to trading costs in toning down managerial behavior.

(1999). There are a total of 1800 log steps between the lower and upper boundaries with the initial fund value X_0 located 600 steps above the lower boundary. The risk aversion coefficient of the manager's power utility is $\gamma = 4$.

[Table 1 about here]

Before discussing results for our reference case, it is useful to build some intuition. In Merton (1969), an individual (analogous to our manager) dynamically chooses the optimal allocation of available funds between shares and the riskless asset. In the case where there is no intermediate consumption (between 0 and T), she chooses that investment strategy to maximize her expected utility of terminal wealth W_T. Merton's analysis is in continuous time (as opposed to our discrete-time framework); however, that description otherwise matches the situation of our manager if she had no incentive option and there was no liquidation boundary. In Merton's framework, the optimal proportion allocated to the risky investment would be constant and using our standard parameters implies:

(5) $$\kappa = \frac{(\mu\text{-r})}{\gamma\sigma^2} = 2\,.$$

Our model also generates a flat optimal surface for the risky investment proportion at $\kappa = 2$ when there is no liquidation boundary or incentive option. Thus, our discrete-time analog of Merton's analysis generates the same solution. That is not surprising since optimally allocating a constant proportion to the risky investment does not exploit the rebalancing capability (in either discrete or continuous time). This changes dramatically when we add the liquidation boundary and the incentive option.

The liquidation boundary effectively turns the manager's compensation structure into a knockout call. With our standard parameters, there is a "rebate" equal to equation (2) if the lower boundary is hit. For those parameter values, we depict the manager's optimal risk-taking in Figure 1. The manager clearly wants to avoid hitting the liquidation boundary and dramatically lowers portfolio risk for the fund as she approaches that lower boundary. We have labeled this region the "Valley of Prudence," and one can see that the manager is moving all the way to a purely riskless investment strategy just above the liquidation boundary.

[Figure 1 about here]

Just above the Valley of Prudence, we have a "Merton Flats" area where the manager chooses an optimal risky investment proportion of 2. This represents an area where fund value is far enough from the liquidation boundary (given the time left to her evaluation date) that the possibility of liquidation plays essentially no role in her decision making. In the absence of an incentive option, Merton Flats would stretch across the rest of Figure 1. However, an incentive option introduces new features to the landscape.

There is now a region with high proportions invested in the risky asset, which we term "Option Ridge". This region is centered just below the high-water mark of $H = 1$. The manager is trying to increase the chance of her incentive option finishing substantially in-the-money. She thus increases the risky investment proportion considerably if the fund value is either somewhat below or even slightly above the strike price. This incentive tails off rapidly as the fund value increases since the manager starts having more to lose if her option value drops. This leads to the lock-in style behavior seen above Option Ridge. In what follows, we will use lock-in style behavior to refer generically to reduced risk-taking in the region above Option Ridge, with the manager choosing risky investment proportions below the Merton optimum. Her motivation is to reduce the probability of the option falling out-of-the-money. At still higher fund values, far to the right, there is also another Merton Flats region. To reach that upper Merton Flats, the manager's incentive option has to be sufficiently deep in-the-money that it acts like a fractional share position.

Some recent papers examine effects of incentive compensation on optimal dynamic investment strategies for money managers. Carpenter (2000) and Basak, Pavlova, and Shapiro (2006) focus directly on this issue for mutual funds. Goetzmann, Ingersoll, and Ross (2003) focus primarily on valuing claims (including management fees) on a hedge fund's assets. These three papers all obtain analytic solutions using equivalent martingale frameworks in continuous time. However, they generate seemingly conflicting results regarding the manager's optimal risk-taking behavior. We can shed light on the differing results in the above papers by relating them to our Figure 1. It turns out that these papers have (sometimes rather subtle) differences in how they model the manager's compensation structure. We will now highlight how the compensation structure is being modeled in each of those papers and how those different choices alter risk taking.

Carpenter (2000) has a risk averse money manager whose terminal wealth is composed of a constant amount (external wealth and a fixed wage) plus a fractional call option on the assets under management with a strike price equal to a specified benchmark. That model can be reinterpreted in a hedge fund context with the benchmark corresponding to the high-water mark. One version of the Basak, Pavlova, and Shapiro (2006) model can be viewed as analogous to

Carpenter's model with her standard call replaced by a binary "asset or nothing" option plus the addition of an implicit managerial share position in the fund.

Both Basak, Pavlova, and Shapiro (2006) and Carpenter (2000) find extreme risk-taking near the strike price (our Option Ridge) with a dramatic lowering of risk (lock-in behavior) at somewhat higher fund values followed by gradually increasing risk-taking at still higher fund values leading up to what we would call the upper Merton Flats region. In other words, both these models generate behavior similar to what we see above the high-water mark in Figure 1.

Below the strike price, Basak, Pavlova, and Shapiro (2006) find risk-taking that declines to a constant Merton-style investment strategy when the fund value is far enough below the strike price so that the option effectively plays no role in the manager's decision making. In contrast, Carpenter (2000) finds risk-taking that increases without limit as fund value declines below the option strike price toward zero. This difference is due to the manager's implicit share position in Basak, Pavlova, and Shapiro (2006), while Carpenter's manager has neither a share position nor any other incentive (such as a liquidation boundary) to reduce risk-taking in the lower portion of the state space. Rather, her manager is motivated only by the probability of getting back into the money prior to the evaluation date. The further out-of-the-money and the shorter the time to maturity for her incentive option, the more the manager is willing to gamble.

Neither of these two papers has a lower liquidation boundary. However, such a boundary is incorporated in Goetzmann, Ingersoll, and Ross (2003). One section of that paper has the state space of fund value split into regions and the manager is allowed to choose a different volatility for each region. At their liquidation boundary, fees go to zero. If the objective of the manager is to maximize fees, such a boundary is to be avoided; and this drives their result that volatility should be decreased as asset values approach the boundary. This result is similar to our Valley of Prudence; although, we did not force the manager's compensation to zero.

Comparison of these models highlights the importance of seemingly minor changes in the manager's compensation structure. For example, whether or not the manager has a share position as well as an incentive option can substantially mitigate risk-taking behavior – compare our results and those of Basak, Pavlova, and Shapiro (2006) with the more extreme risk-taking in Carpenter (2000). The nature of the incentive option (e.g. plain vanilla call versus binary asset-or-nothing) also makes a difference, with the binary option inducing more dramatic shifts in risk-taking because of the jump in value at the strike price. On the other hand, both types of options appear to motivate lock-in style behavior slightly above the high-water mark. It is also clear that liquidation barriers can have major effects. Our Figure 1 may not depict the "whole elephant," but it is more general than previous one-period models and illustrates how managerial behavior can vary dramatically in different parts of the state space.

IV. Endogenous Shutdown

Instead of simply using a prespecified liquidation boundary, we adapt the model to include a managerial shutdown option. This is an important and realistic extension which provides a mechanism consistent with a potentially nontrivial number of hedge fund liquidations. Getmansky, Lo, and Mei (2004) report an 8.8% average annual attrition rate for funds in the TASS database during 1994-2003. This rate represents funds dropping from the TASS database for a variety of possible reasons. However, it is clear that a large fraction of the funds were liquidated. In contrast, the Valley of Prudence and the Goetzmann, Ingersoll, and Ross (2003) result of reducing risk-taking to zero approaching the lower boundary imply that liquidation at such an exogenous boundary can be avoided. There are ways to have liquidations with an exogenous boundary such as allowing for negative jumps or introducing transaction costs which deter rapidly reducing risk-taking to zero. However, one would still anticipate a relatively small number of such forced shutdowns. An endogenously chosen shutdown seems more consistent with the rather high liquidation rate.

An endogenous shutdown choice represents an American-style option where the manager can choose to liquidate the fund at asset values above the prespecified lower boundary. Brown, Goetzmann, and Ibbotson (1999) argue that this might happen because it appears unlikely that performance will reach the high-water mark (presumably within a "reasonable" time frame). The problem of covering fixed costs with management fees when fund value is low as well as marketing difficulties associated with a poor track record can also contribute to the exit decision.

Whether the manager will choose to shut down the fund depends on her other opportunities relative to continuing to manage the fund. This would include the possibility of starting a new hedge fund (with a new high-water mark) as well as other alternatives such as accepting outside employment. On the other hand, keeping the fund alive preserves the possibility of earning an incentive fee by exceeding the high-water mark. Also, the manager may desire to invest her own capital using the fund's superior return technology and may not be able to do so unless the fund remains in existence. This could be due to scale economies (e.g. an expensive trading platform) or legal considerations.

We model her outside opportunities in a simple manner, using L to represent an annual compensation rate which is independent of the fund value. If the manager chooses to shut down the fund at time τ with a fund value X_τ, she receives at maturity:

$$W_\tau = aX_\tau + b(1-a)\tau X_\tau + L(T-\tau) \qquad \text{for } 0 \le \tau \le T \tag{6}$$

The first two terms of (6) indicate that the manager recovers her share of the fund aX_τ plus a prorated fraction of the management fee (with no incentive payment). She also earns L prorated over the time remaining until T. As we work backward in time through our grid, we compare the utility of receiving (6) with that from choosing the optimal $\kappa(X,t)$ and continuing to manage the fund. When the utility of (6) dominates, it indicates the manager would voluntarily choose to shut down the fund at that fund value and point in time.

[Figure 2 about here]

In Figure 2, we illustrate a typical situation with endogenous shutdown occurring at fund values well below the lower edge of Option Ridge. In that closure region, the probability of reaching the high-water mark would have been very small. We also see some gambling slightly above the voluntary closure level along what we have labeled "Decision Ridge." Here, the manager is in a situation that could be described as "heads: I win, tails: I don't lose very much" due to the partial floor on her compensation provided by outside opportunities. When the value of her outside opportunities is sufficiently low, the manager will not voluntarily choose to shut down and must be forced to liquidate the fund at the lower boundary.

Note that if a shutdown occurs, outside investors incur a resetting of their high-water mark when switching to another fund. In effect, they are forced to forgo the possibility of gains in the current fund without triggering incentive fees. Moreover, outside investors can experience a pattern of heavy gambling along Option Ridge with fund closure at perhaps only slightly lower asset values – leading to a reset of their high-water mark. To assess the likelihood of such a situation, an outside investor needs to be able to address the manager's optimal actions in an American option framework such as presented here.

V. Managerial Behavior with Multiple Evaluation Periods

Our basic model considers only a single evaluation period, and this may lead to more dramatic swings in managerial risk-taking than would occur with a sequence of performance evaluations. This is also true for Carpenter (2000) and Basak, Pavlova, and Shapiro (2006). In a multiyear framework, the manager would consider not only fund performance to the next evaluation date but also potential subsequent compensation based on fund performance in later periods. This should dampen extreme risk-taking. If the fund performs poorly during the previous year, it reduces expected future compensation since the manager will be starting next year with that next year's incentive option out-of-the-money. Also, the convexity ("gamma") of a European option is a decreasing function of time to maturity and declines rapidly for near-the-money options. Furthermore, options which are substantially out-of-the-money have relatively low convexity. Hence, options maturing in future years will have current managerial behavior effects roughly analogous to additional share positions. Again, this will serve to dampen risk-taking in earlier periods.

Panageas and Westerfield (2005) examine this issue and indeed obtain results that the manager should optimally have a constant risky investment proportion. However, the continuous time and continuous state space structure of their model means that each option is of only infinitesimal size and each evaluation period is instantaneous. So one issue is whether their basic result holds with annual evaluation periods (as in practice) and options of substantial size.

Another important issue is boundary behavior that alters their result. Their fund can be shutdown independently of fund value at a random future date determined by an exogenous Poisson process. In our view, shutdown at relatively low fund values is more likely and the most realistic way to model that is with an endogenous liquidation decision. The high rate of hedge fund closures suggests that we should model this aspect of fund management. Consequently, we use our endogenous shutdown approach as a lower boundary in the following analysis with multiple yearly evaluation periods.

Building a model with multiple evaluation periods is challenging because there is a path dependency in the high-water mark. The last evaluation period is simply our standard setting as in Figure 2 above. In earlier years, the manager's compensation at year end (management and incentive fees for that year) needs to be augmented by the certainty equivalent of the indirect utility of continuation. For year-end fund values below the high-water mark, the indirect utility of continuation is simply the next year's indirect utility measured at the initial fund value equal to the previous year's ending value. For year-end fund values above the high-water mark, the situation is a little more complicated since the high-water mark gets reset. We calculate the certainty equivalent for such a (relatively high) fund value as equal to the continuation certainty equivalent at the high-water mark times the ratio of new (relatively high) fund value to the high water mark. We are able to use this relatively simple procedure due to our assumption of power utility (more details are provided in the Appendix).

[Figure 3 about here]

Figure 3 displays the pattern of optimal managerial risk-taking over the next year, starting 3 years from the terminal date of the analysis. If the fund is not shut down, the manager receives the annual management fee and a potential performance incentive at the end of this evaluation period (2 years from the terminal date). This is followed by another one-year evaluation period with the same compensation structure before the final one-year period which is then identical to what we displayed in Figure 2.

Comparing Figure 3 with Figure 2, we can see that Option Ridge has been pushed downward and is no longer discernable. This result actually takes two years, and a smaller Option Ridge would still be visible in a display for the intervening year. Decision Ridge is still a prominent feature but it has moved downward, with fund closure at lower levels. While there is considerable variation in risk taking across different fund values in Figure 3, there is now much less variation as a function of time until the next evaluation date.

If we were to move backward in time, modeling more and more years until the fund's terminal date, we would see essentially the same pattern as in Figure 3. This continues until Decision Ridge bumps against some hard exogenous liquidation boundary. Then Decision Ridge starts getting pushed downward (over several years) and a Valley of Prudence starts extending upward from the exogenous liquidation boundary. Eventually, the pattern of managerial behavior can be described as a Valley of Prudence starting at the exogenous lower boundary and sloping slowly upward to a Merton Flats area at substantially higher fund values. However, this whole process can take a very long time. For our standard parameters, it takes over 30 years for Decision Ridge to bump up against an exogenous liquidation boundary set at 0.25.

VI. Concluding Comments

A useful way of interpreting our results is to consider the perspective of a hedge fund investor with CRRA utility, who would always prefer a Merton Flats strategy with constant proportional risk-taking. We find that the industry-standard compensation contract induces widely varying risk-taking if the manager knows she is within a year or two of the terminal date.

On the other hand, when she has a longer horizon with several annual evaluation periods, her optimal investment strategy is substantially closer to a constant proportion as long as the fund is performing reasonably well. The much-debated incentive effects stemming from option-like compensation thus die out relatively quickly. If fund value has declined to a level where fund closure is a serious possibility, she begins moving toward much higher risk portfolios along Decision Ridge. This behavior carries through strongly with horizons of many years, even decades. Given the high rate of hedge fund closure, this implication of endogenous shutdown seems particularly important for understanding investment management at hedge funds.

The risk-taking behavior we find is much richer than what can be generated by one-period models such as Carpenter (2000) and Basak, Pavlova, and Shapiro (2006) or models with instantaneous valuation periods such as Goetzmann, Ingersoll, and Ross (2003) as well as Panageas and Westerfield (2005). This results from our ability to solve the American-style option problem needed to implement endogenous shutdown as well as developing a procedure for realistically handling multiple discrete evaluation periods with path dependent high-water marks. We feel this indicates the direction to proceed in analyzing similar incentive issues in the future.

From a policy perspective, it is clear that the typical hedge fund contract plus managerial control of fund investment positions can result in dramatic risk taking as the manager tries to increase the probability of her incentive option finishing substantially in-the-money. This behavior near the high-water mark is much more pronounced when the manager does not own shares (implicitly or explicitly) in the fund. On the other hand, such behavior is diminished when the manager anticipates an extended career with the fund and many evaluation periods – hence, a sequence of incentive options. It is important that fund investors understand this structure. Typically, managerial risk taking that more closely approximates investor preferences can be achieved by using a sequence of incentive options.

Endogenous shutdown also induces a nonlinearity in the manager's compensation. Effectively, the manager has an American put option. Again, it is important for fund investors to understand this phenomenon. If a fund has been performing poorly, the likelihood increases for endogenous shutdown preceded by increased risk taking. Controlling such risk taking could be difficult; however, contractual limits on leverage or verifiable risk management constraints should help.

Appendix: Numerical Procedure

Optimal control of a stochastic process in an investment context is a discrete-time descendent of Merton (1969). Merton's work in turn is based on Markowitz's (1959) dynamic programming approach and Mossin's (1968) implementation of that idea in discrete time. According to Kushner and Dupuis (1992), the models of choice in the stochastic control literature are Markov chain models where the state variable evolves on a finite grid according to transition probabilities from a Markov chain. A state of the art implementation is Jarvis and Kushner (1996) with the drift linear in the control, constant volatility, and controls which are state dependent but constant over time. Our model is also a Markov chain model, albeit on an open grid since we have no upper boundary for our fund value state variable. Our problem is, however, much harder then Jarvis and Kushner (1996) – primarily because our control is both time and state dependent. We also have a log asset value process with drift and volatility being nonlinear and linear in the control, respectively.

The basic structure of our model uses a grid of fund values X and time t, with $\Delta(\log X)$ constant as well as time steps Δt of equal length. The initial fund value X_0 is on the grid, and we choose the grid spacing such that $\Delta(\log X)$ is equal to $r\Delta t$. This choice implies that in the limiting case where $\kappa = 0$ (the manager chooses to only invest in the riskless asset) the value process will still reach a regular grid point.

To calculate expected utilities, we will need the probabilities of moving from one fund value at time t to all possible fund values that can be reached at $t+\Delta t$. The possible $\log X$ moves are $i\Delta(\log X)$. We use i to index the grid points to which we can move. In the current implementation, the range for i is from –60, …, 0, …, 60. The probabilities for those possible moves depend on the choice of kappa (risky investment proportion) which determines the process for X over the next time step. The risky investment technology has a constant growth rate of μ and a standard deviation of σ. The riskless investment grows at the constant rate r. The parameters μ, σ and r can be deterministic functions of (X,t) without generating much additional insight about managerial risk-taking. For a given kappa, the log change in X is normally distributed with mean $\mu_{\kappa,\Delta t} = [\kappa\mu + (1-\kappa)r - \frac{1}{2}\kappa^2\sigma^2]\Delta t$ and volatility $\sigma_{\kappa,\Delta t} = \kappa\sigma\sqrt{\Delta t}$. Note that this mean and variance do not depend on the level of X. They do depend on Δt but not on t itself. Since the normal distribution is characterized by its mean and variance, the probabilites we need are solely functions of κ and not the level of X or time.

We now use the discrete normal distribution. For a given κ, we calculate the probabilities based on the normal density times a normalization constant so that the computed probabilities sum to one:

(A1)
$$p_{i,\kappa,\Delta t} = \frac{\frac{1}{\sqrt{2\pi}\sigma_{\kappa,\Delta t}} EXP\left[-\frac{1}{2}\left(\frac{i\Delta(\log X) - \mu_{\kappa,\Delta t}}{\sigma_{\kappa,\Delta t}}\right)^2\right]}{\sum_{j=-60}^{60} \frac{1}{\sqrt{2\pi}\sigma_{\kappa,\Delta t}} EXP\left[-\frac{1}{2}\left(\frac{j\Delta(\log X) - \mu_{\kappa,\Delta t}}{\sigma_{\kappa,\Delta t}}\right)^2\right]}$$

We keep a lookup table of the probabilities for different choices of κ, which we vary from 0.2 to 5 in steps of 0.01 and where we include 0 in order to allow the risk free investment strategy. However, the ends of this range could be problematic and result in poor approximations to the normal distribution. For low κ values, the approximation suffers from not having fine enough fund value steps. For high κ values, the difficulty arises from potentially not having enough offset range to accommodate the extreme tails of the distribution.

To insure reasonable accuracy, we compare the standardized moments of our approximated normal distribution $\hat{\mu}_j$ with the theoretical moments of the standard normal, $\mu_j = 1 \cdot 3 \cdot \ldots \cdot (j-1)$ for j even and $\mu_j = 0$ for j odd. In particular, we calculate a test statistic based on the differences of the first 10 approximated and theoretical moments scaled by the asymptotic variance of the moment estimation (Stuart and Ord (1987), p. 322):

(A2) $$\frac{1}{10}\sum_{j=1}^{10}\left(\frac{\hat{\mu}_j - \mu_j}{\frac{1}{n}(\mu_{2j} - \mu_j^2 + j^2\mu_2\mu_{j-1}^2 - 2j\mu_{j-1}\mu_{j+1})}\right)^2, \text{ where we set } n = 1 \text{ and } \mu_0 = 0.$$

After some experimentation, we discard distributions with risky investment proportions of less than 0.2. We finally have a matrix of probabilities with a probability vector for each κ value in our remaining choice set.

We now calculate the expected indirect utilities and initialize the indirect utilities at the terminal date J_T to the utility of wealth of our manager $U_T(W_T)$ where her wealth is solely determined by her compensation scheme. Our next task is to calculate the indirect utility function at earlier time steps as an expectation of future indirect utility levels. We commence stepping backwards in time from the terminal date T in steps of Δt. At each fund value within a time step t, we calculate the expected indirect utilities for all κ values using the stored probabilities and record the highest value as our optimal indirect utility, $J_{X,t}$. We continue, looping backward in time through all time steps.

In our situation, using a lookup table for the probabilities associated with the κ's has two advantages compared with using an optimization routine to find the optimal κ value. For one, lookups are faster although coarser than optimizations. Second, a sufficiently fine lookup table is a global optimization method that will find the true maximum even for non-concave indirect utility functions. In such situations, a local optimization routine can get stuck at a local maximum and gradient-based methods might face difficulties due to discontinuous derivatives.

When implementing our backward sweep through the grid, we have to deal with behavior at the boundaries. The terminal step is trivial in that we calculate the terminal utility from the terminal wealth. The lower boundary is also quite straightforward. We stop the process upon reaching or crossing the boundary and calculate the utility of the payment associated with hitting the boundary at that time $U(W_\tau)$. We use these values in calculating the expected indirect utility at earlier time steps.

For the numerical implementation, we also need an upper boundary to approximate indirect utilities associated with high fund values. We use a boundary 1200 steps above the initial X_0 level. For fund values near that boundary, our calculation of the expected indirect utility will try to use indirect utilities associated with fund values above the boundary. We deal with this by keeping a buffer of fund values above the boundary so that the expected indirect utility can be calculated by looking up values from such points. We set the terminal buffer values simply to the utility for the wealth level associated with those fund values. We then step back in time and use as our indirect utility the utility of the following date times a multiplier which is based on the optimal Merton (1969) solution without consumption: $\exp[\Delta t(\mu - r)^2(1-\gamma)/(2\gamma\sigma^2)]$. We do not assume that these values are correct, but they work very well. This approach is potentially suboptimal, which biases the results low. However, the distortion ripples only some 30 – 70 steps below the upper boundary, affecting mainly the early time steps.

Finally, we turn our discussion to the implementation of the multiple-year evaluation structure. We start by considering a the two-year horizon. When the fund value ending the first year X_1 is at or below fund value at the beginning of that year X_0, then the high-water mark is not reset ($H_1 = H_0$). The second year's continuation indirect utility is then simply the indirect utility $J(X_1)$ of starting that second year at X_1 and having a one-year horizon remaining. We convert this $J(X_1)$ to a certainty equivalent value $U^{-1}[J(X_1)]$ and add it to the compensation for year one (W_1), which in this case is just the management fee. We then calculate the total indirect utility $U\left\{W_1 + U^{-1}[J(X_1)]\right\}$ for use in our backward recursion.

Finally, we need to deal with the situation where the high-water mark has been reset. With power utility, managerial behavior for the following period will be the same as simply starting that second period at a high water mark of 1. Here we assume that the lower boundary is also rescaled to the same extent as the high-water mark so that the lower boundary is always half the high-water mark 0.5H. However, the manager's shares (plus management and incentive fees) will be worth more due to the greater fund value. Since we know the indirect utility of starting with a high-water mark of 1, we simply scale the associated certainty equivalent by the ratio of the higher fund value and the old high water mark of 1. The above argument can now be extended by induction to the next earlier evaluation period, and so on.

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Table 1. Standard Parameters

Time to maturity	T	1	Interest rate	r	0.0578
Log value steps below/above X_0		600/1200	Initial fund value	X_0	1.00
Risk aversion coefficient	γ	4	Mean	μ	0.0778
Number of time steps	n	50	Volatility	σ	0.05
Initial high-water mark	H	1.00	Incentive fee rate	c	0.20
Liquidation boundary	Φ	0.50	Basic fee rate	b	0.02
Manager's share ownership	a	0.10			
Future nodes for the Normal approx.		$1+2\times60 = 121$			
Log X step		$(\log(1/0.5))/600 \approx 0.001155$			

Figure 1. Optimal Risky Investment Proportion in One-Period Reference Case

In this figure, the manager receives a management fee (b = 2%), an incentive option (c = 20%), and also has an equity stake (a = 10%). Other parameter values are as specified in Table 1.

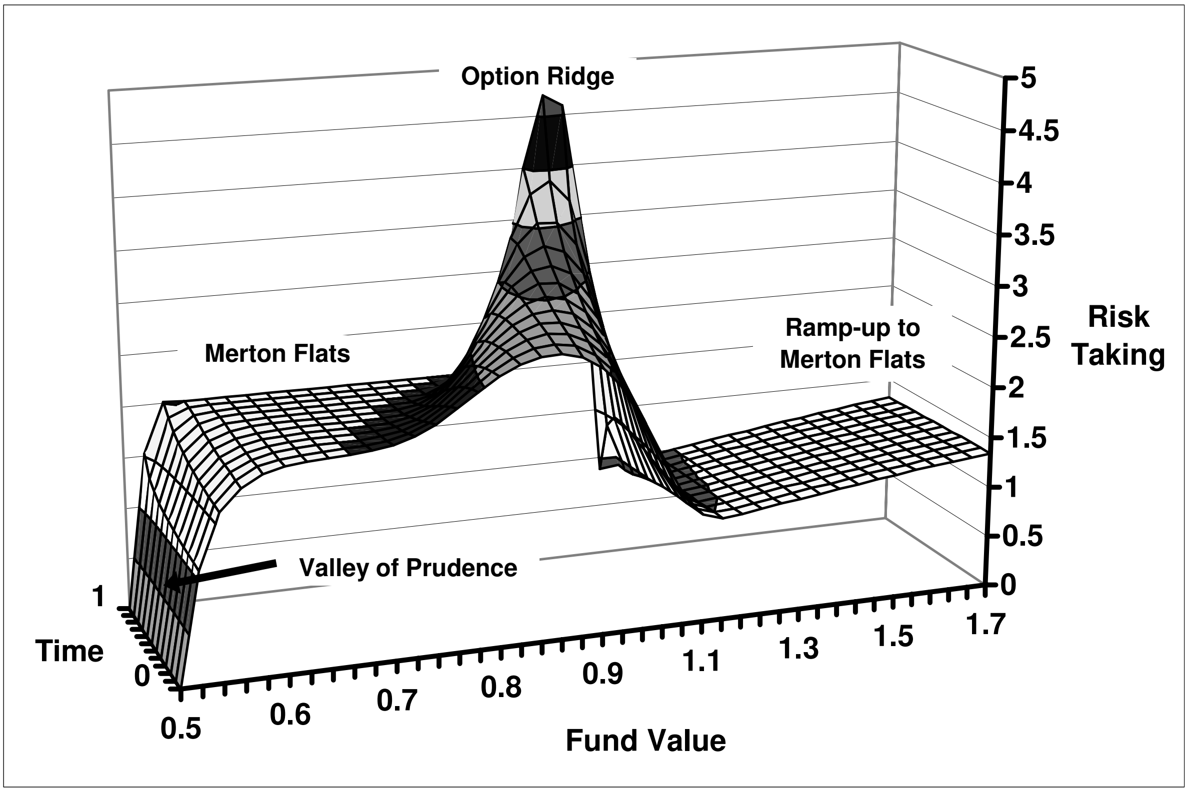

Figure 2. One-Period Optimal Risky Investment with Endogenous Shutdown

In this figure, the manager receives the standard compensation package: a management fee (b = 2%), an incentive option (c = 20%), and also an equity stake (a = 10%). She can also choose to voluntarily shut down the fund to pursue an outside opportunity with an annual compensation of 0.018. Other parameter values are as specified in Table 1.

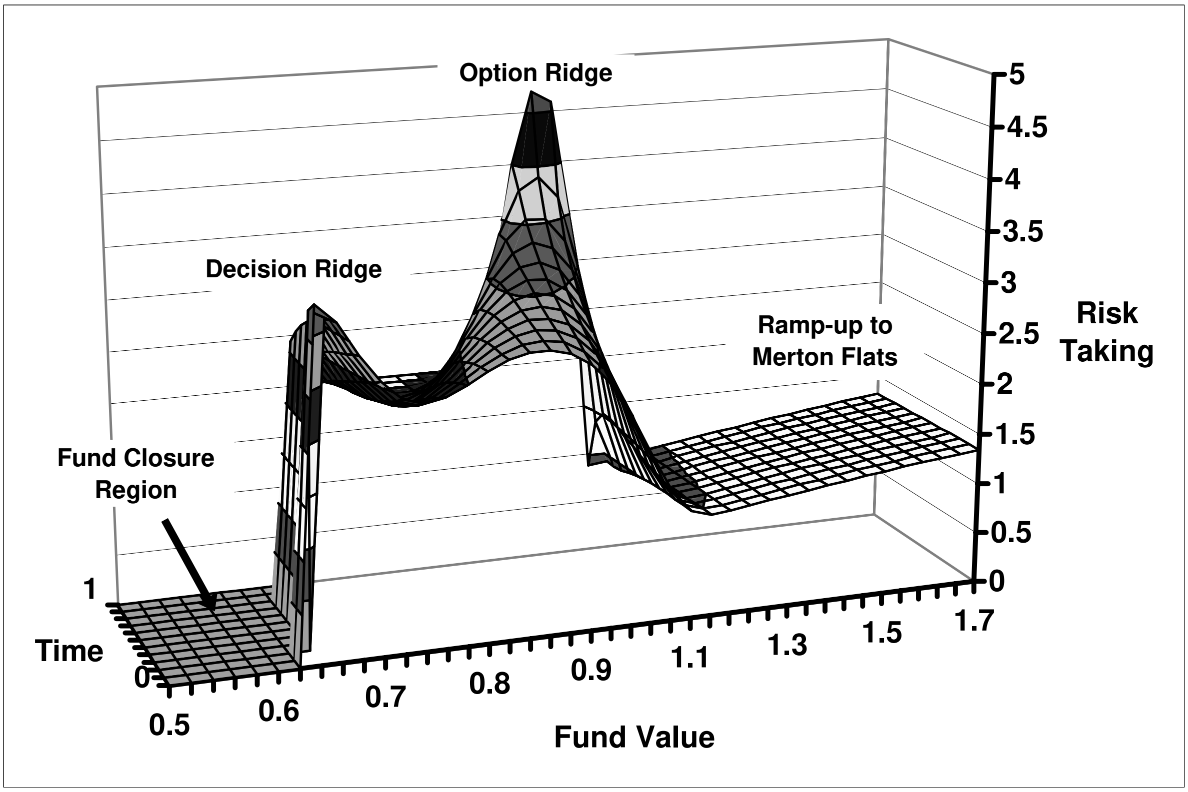

Figure 3. Optimal Risky Investment During an Earlier Evaluation Period

This figure displays risk-taking during a one-year evaluation period starting three years before the final evaluation date. Each year is an evaluation period with the manager having the standard compensation package: a management fee (b = 2%), an incentive option (c = 20%), and also an equity stake (a = 10%). She can also choose to voluntarily shut down the fund to pursue an outside opportunity with an annual compensation of 0.018. Other parameter values are as specified in Table 1.