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The Nexus between Public Debt and the Government Spending Multiplier: Fiscal Adjustments Matter

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Abstract

This paper studies the evolution of government spending multipliers in the post-war U.S. using a time-varying parameter VAR model. We achieve identification by imposing sign and zero restrictions on the systematic component of policy rules and impulse responses. Our results show that the U.S. multipliers in the post-OBRA93 period are smaller than those in the 1970s. The multipliers are found to be more strongly correlated with the estimated coefficients of the debt-stabilizing rule than the debt-to-GDP ratios. The increased magnitude of fiscal adjustments appears to be the major driving force behind the decline in multipliers rather than debt accumulation itself.

JEL classification numbers

E32, E62, H60.

Keywords

Bayesian VARs; Time-varying parameters; Fiscal multipliers; Fiscal policy.

I Introduction

The past decade has witnessed increased attention to the size of government spending multipliers and their heterogeneity over time and across countries. Investigating the sources of heterogeneity in multipliers across countries, the literature has provided ample evidence that government spending multipliers are large in countries with low public debt (e.g., Ilzetzki et al. (2013); Nickel and Tudyka (2014); Huidrom et al. (2020)). The role of public debt in affecting the size of multipliers has also become a very relevant issue for the United States. As illustrated in Figure 1, the public debt-to-GDP ratio in the U.S. has been following an upward trajectory since the 1980s. After the Global Financial Crisis, it soared to a level above the thresholds used to define high-debt countries in previous studies.

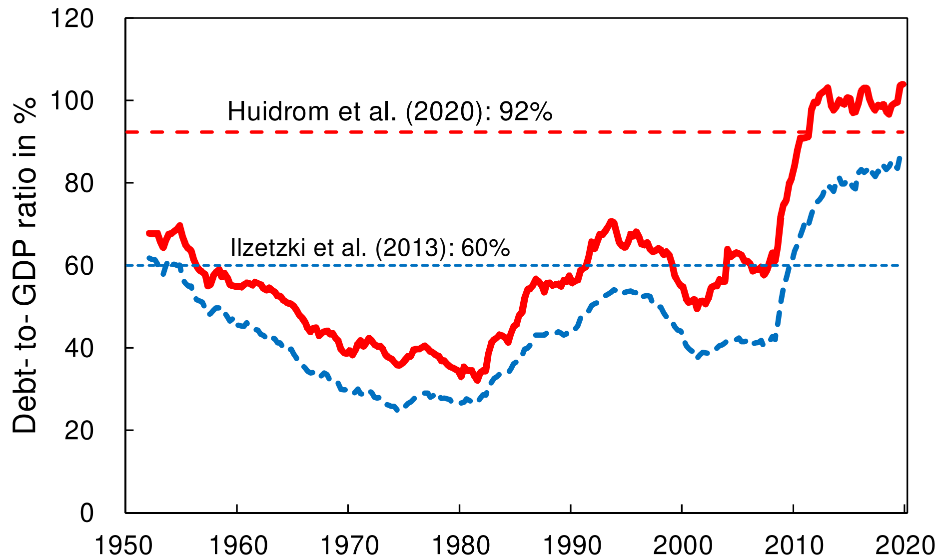

Figure 1. U.S. debt-to-GDP ratio.
Notes: The red solid line and the blue dashed line represent the debt-to-GDP ratio of the general government and that of the federal government, respectively. The horizontal lines indicate the threshold debt-to-GDP ratios used to define high-debt countries in the studies by Ilzetzki et al. (2013) and Huidrom et al. (2020). The thresholds used by Ilzetzki et al. (2013) and Huidrom et al. (2020) are those for the federal government and the general government, respectively.

However, the public debt dependency of government spending multipliers in the U.S. time-series data remains somewhat neglected in the literature. Although time variation in

U.S. multipliers is an area of active research, existing studies focus on its state-dependent nature across business cycles, relying on a regime-switching framework. The growing body of empirical evidence suggests that multipliers are larger in recessions than in expansions (e.g., Auerbach and Gorodnichenko (2012); Candelon and Lieb (2013); Caggiano et al. (2015)).[1] Aside from business cycle dependency, Bilbiie et al. (2008) find smaller multipliers in the post-1980 period than in the preceding period. However, they attribute the cause to changes in the conduct of monetary policy after Volcker's appointment as Fed Chairman.[2]

The nexus between public debt and fiscal policy effects has been studied since Giavazzi and Pagano (1990)'s report of cases of expansionary fiscal adjustments from Danish and Irish experiences in the 1980s.[3] The transmission, through which households reduce their consumption in anticipation of future fiscal adjustments, is examined in the literature (e.g., Blanchard (1990); Sutherland (1997); Perotti (1999)). Huidrom et al. (2020) call the transmission a *Ricardian channel* and consider it to be the underlying cause of the debt-dependent multipliers. In providing evidence of the U.S. government's reaction to debt accumulation, Bohn (1998) finds a positive correlation between the magnitude of fiscal

[1]For a theoretical account of the business cycle dependency of government spending multipliers, see Canzoneri et al. (2016), Shen and Yang (2018), and Ghassibe and Zanetti (2022). Ghassibe and Zanetti (2022) develop a general theory of state-dependent fiscal multipliers that accounts for the business cycle dependency of tax cut multipliers in addition to that of spending multipliers. Whereas Ferraro and Fiori (2022) suggest that the stimulative effect on employment of a tax cut is larger in a recession than in an expansion using a heterogeneous-agent model, recent empirical studies point to the procyclical nature of the U.S. tax multipliers (e.g., Eskandari (2019); Ziegenbein (2021)). Ghassibe and Zanetti (2022) show that the source of business cycle fluctuations explains the cyclicality of spending and tax cut multipliers in a consistent manner with the empirical evidence.

[2]Bilbiie et al. (2008) also suggested that smaller multipliers in the post-Volcker period can be attributed to increased asset market participation as well as the more active monetary policy of the period. In theory, asset market participation allows households to save or borrow to smooth their consumption in anticipation of future fiscal adjustments. Therefore, its increase could also represent strengthening of the *Ricardian channel* through which households reduce their consumption in expectation of larger fiscal adjustments.

[3]The general theory of state-dependent fiscal multipliers recently developed by Ghassibe and Zanetti (2022) provides a theoretical account of the expansionary effects of fiscal adjustments (the 'expansionary austerity hypothesis'). For an overview of empirical findings on the hypothesis, see Alesina et al. (2019).

adjustments and the debt-to-GDP ratio. Based on this finding, the empirical literature investigating the size of U.S. multipliers documents the importance of capturing the dynamics of fiscal adjustments in structural vector autoregressive (SVAR) models (e.g., Chung and Leeper (2007); Corsetti et al. (2012); Favero and Giavazzi (2012)). However, empirical evidence for the debt dependency of U.S. multipliers is not yet established. Caggiano et al. (2015) and Bernardini and Peersman (2018) control for public debt levels in their regime-switching models, but they do not find major differences between the sizes of multipliers across different debt regimes. The empirical difficulty in isolating the debt-dependent government spending effects is also addressed by Bi et al. (2016). To provide theoretical grounds for debt-dependent multipliers, they first show that a larger magnitude of fiscal adjustments induces stronger negative effects on consumption, thus leading to smaller multipliers when the debt levels are high. They then perform VAR estimations using the simulated data from their neoclassical model and conclude that debt-dependent effects are difficult to recover even if the effects exist in the data, because of various factors such as monetary policy.

Against this background, this paper aims to provide time series evidence of debt-dependent multipliers from the U.S. data and to investigate the transmission, paying particular attention to the role of fiscal adjustments. For these purposes, we employ the time-varying parameter vector autoregressive (TVP-VAR) model with stochastic volatility developed by Primiceri (2005).[4] The TVP-VAR model allows parameters to vary continuously over time in a stochastic manner and therefore does not require researchers to have particular knowledge of regimes. Although the TVP-VAR model has drawbacks

[4]The importance of a stochastic volatility assumption is revisited in the recent research by Alessandri and Mumtaz (2017) and Alessandri and Mumtaz (2019).

in flexibility,[5] it is suitable for capturing permanent and gradual changes in the transmission mechanism and hence may well describe the possible changes in household behaviour and in the magnitude of fiscal adjustments.[6] Furthermore, the abovementioned findings of Bohn (1998) indicate the necessity of estimating a debt-stabilizing rule with time-varying coefficients. While Caldara and Kamps (2017) address the deterministic role of the coefficients of fiscal rules in the size of multipliers estimated in SVAR models, Arias et al. (2019)'s recent work shows that restrictions on the systematic component of the monetary policy equation help in identifying monetary policy shocks. In this paper, we apply the SVAR methodology of Arias et al. (2019) to a TVP-VAR framework and extend it to restrict the systematic component of the fiscal policy equation. Our algorithm draws heavily from Rubio-Ramírez et al. (2010) and Arias et al. (2018). The application allows us to estimate permanent and gradual changes in the coefficients of the fiscal policy rule as well as in household behaviour so that we can address the role of fiscal adjustments in the size of multipliers and the relevance of the Ricardian channel. While fiscal adjustments can be achieved either by cutting spending or by raising tax, Corsetti et al. (2012) argue that spending-based adjustments are more relevant to the U.S. fiscal policy. We hence consider a debt-stabilizing spending rule in our TVP-VAR model. Because monetary policy is considered to be an important factor in determining the size of the multipliers (e.g., Bilbiie et al. (2008); Bi et al. (2016)), we also impose restrictions on the coefficients of the monetary policy rule.

[5] Because the TVP-VAR model has a large number of parameters relative to the number of observations, it faces a higher computational burden and greater concerns about overparameterization and overfitting than other VARs. To avoid the risk of imprecise estimates due to overfitting, researchers typically set tight priors for hyperparameters in the TVP-VAR model (e.g., Koop (2012); Prüser and Schlösser (2020)).

[6] Primiceri (2005) provides a succinct discussion of the advantages and disadvantages of the TVP-VAR model over regime-switching models.

The analysis provides evidence of a decline in the multipliers between the 1970s and the early 1990s. We then examine the underlying cause of the decline. While the results do not support a strong offsetting response of monetary policy during the period, we find that the magnitude of spending-based fiscal adjustments increased in the 1990s. The multipliers are more strongly correlated with the estimated coefficients of the debt-stabilizing spending rule than the debt-to-GDP ratios. The result leads us to conjecture that the increased magnitude of fiscal adjustments could be the major driving force behind the decline in the multipliers. Furthermore, we show the relevance of the Ricardian channel by augmenting the baseline model with private consumption.

From a methodological standpoint, this paper is related to those of Canova and Gambetti (2009) and Belongia and Ireland (2016), both of which report the evolution of coefficients of the monetary policy rule in a TVP-VAR model with stochastic volatility. However, they impose sign restrictions only on impulse responses. This paper's contribution is to be the first to extend the identification procedure of Arias et al. (2019) to allow for time variation both in the coefficients and in the covariance matrix. In addition, we apply the procedure to restrict the systematic component of fiscal policy as well as monetary policy. With regard to the time-varying effects of government spending, Kirchner et al. (2010) present the only study that we are aware of that explores the debt dependency of government spending multipliers using time series data. By conducting regression analysis on the posterior mean multipliers calculated from their recursively identified TVP-VAR model for the euro area and the possible explanatory factors, they conclude that an increase in the debt-to-GDP ratio has a negative impact on the multipliers.[7] Our study differs from theirs in that it identifies the U.S. government spending

[7]The application of the TVP-VAR framework to fiscal policy analysis is relatively limited when comparing it with its application to monetary policy. Although there has been a growing interest in applying the TVP-VAR framework to fiscal policy analysis, studies such as those by Rafiq (2012),

shocks by imposing sign and zero restrictions on both the impulse responses and the structural parameters in monetary and fiscal policy rules. Accordingly, it considers that the Ricardian channel operates in response to the increased magnitude of fiscal adjustments rather than debt accumulation itself, while Kirchner et al. (2010) focus on the direct relationship between the multipliers and the debt-to-GDP ratio.

The remainder of this paper is organized as follows. Section II discusses the empirical methodology. Section III reports the results. Section IV investigates the sources for the debt-dependent multipliers. Section V concludes.

II Empirical methodology

To implement the identification procedure of Arias et al. (2019) in a TVP-VAR model, we exploit the algorithms developed by Rubio-Ramírez et al. (2010) and Arias et al. (2018). This section describes a way to extend the SVAR model identified with sign and zero restrictions to allow for time variation both in the coefficients and in the covariance matrix following Primiceri (2005).

The TVP-VAR model identified with sign and zero restrictions

Let us consider the following TVP-VAR model with stochastic volatility[8]:

$$\mathbf{y}_t'\mathbf{A}_{0,t} = \sum_{l=1}^{P} \mathbf{y}_{t-l}'\mathbf{A}_{l,t} + \varepsilon_t'\mathbf{H}_t^{1/2} \tag{1}$$

Pereira and Lopes (2014), and Glocker et al. (2019), do not find any evidence to support debt-dependent multipliers using data from Japan, the U.S., and the U.K., respectively.

[8] The TVP-VAR model with stochastic volatility is broadly used in analysing the behaviour of macroeconomic time series. For applications that study inflation and monetary policy, the effects of globalization, and labor market dynamics, see Benati and Surico (2008), Bianchi and Civelli (2015), and Mumtaz and Zanetti (2015), respectively.

for $t = P+1, ..., T$, where $\mathbf{y}_t$ is a $k \times 1$ vector of observed variables, $\mathbf{A}_{l,t}$, $l = 0, ..., P$, are $k \times k$ matrices of time-varying parameters, and the structural impact matrix $\mathbf{A}_{0,t}$ is invertible. ε_t is Gaussian with mean zero and covariance matrix $\mathbf{I}_k$, the $k \times k$ identity matrix, and $\mathbf{H}_t^{1/2}$ is a diagonal matrix of time-varying standard deviations. Let $\mathbf{A}'_{+,t} = [\mathbf{A}'_{1,t} \ \cdots \ \mathbf{A}'_{P,t}]$ and $\mathbf{x}'_t = [\mathbf{y}'_{t-1} \ \cdots \ \mathbf{y}'_{t-P}]$, then, the model (1) can be written compactly as follows:

$$\mathbf{y}'_t \mathbf{A}_{0,t} = \mathbf{x}'_t \mathbf{A}_{+,t} + \varepsilon'_t \mathbf{H}_t^{1/2}. \tag{2}$$

The reduced-form representation is given by the following equation:

$$\mathbf{y}'_t = \mathbf{x}'_t \mathbf{B}_t + \mathbf{u}'_t, \tag{3}$$

where $\mathbf{B}_t = \mathbf{A}_{+,t}\mathbf{A}_{0,t}^{-1}$. Like Mumtaz and Zanetti (2015), we factor the covariance matrix of the innovations $\mathbf{u_t}$ as $VAR(\mathbf{u_t}) \equiv \mathbf{\Sigma}_t = (\mathbf{A}_t^{-1})'\mathbf{H}_t\mathbf{A}_t^{-1}$, where $\mathbf{H}_t$ is a diagonal matrix with variances of structural shocks defined as $\mathbf{H}_t = \text{diag}\left[h_{1,t}, \ldots, h_{k,t}\right]'$ and $\mathbf{A}_t$ is a $k \times k$ upper triangular matrix with all the diagonals equal to one.

Let $\widetilde{\mathbf{A}}_{0,t}$ be a matrix that satisfies $\mathbf{\Sigma}_t = \widetilde{\mathbf{A}}_{0,t}{}'\widetilde{\mathbf{A}}_{0,t}$. Using a $k \times k$ orthogonal matrix, $\mathbf{Q}_t$**,** a candidate structural impact matrix is given as $\mathbf{A}_{0,t} = \widetilde{\mathbf{A}}_{0,t}\mathbf{Q}_t$.[9] Notice that orthogonality among the structural shocks is ensured by design (e.g., Fry and Pagan (2011)). The time-varying VAR model

$$\mathbf{y}'_t = \mathbf{x}'_t \mathbf{B}_t + \varepsilon'_t \widetilde{\mathbf{A}}_{0,t}, \tag{4}$$

represents an observationally-equivalent rotation of the model's equation (3). We impose sign and zero restrictions on some elements of $\mathbf{A}_{0,t}$ and on the impulse responses at the

[9] Although we cannot give an economic interpretation of the orthogonal matrix $\mathbf{Q}_t$, as described by Baumeister and Hamilton (2015), we exploit the structure of the matrix to preserve computational efficiency of our algorithm. As argued by Arias et al. (2018), an alternative approach could become computationally inefficient.

short- and long-run horizons exploiting the algorithms developed by Rubio-Ramírez et al. (2010) and Arias et al. (2018). Following Arias et al. (2015), we let the elements of matrices $\mathbf{IR}_{S,t}(\mathbf{A}_{0,t},\ \mathbf{A}_{+,t})$ and $\mathbf{IR}_{L,t}(\mathbf{A}_{0,t},\ \mathbf{A}_{+,t})$ be the impulse responses of the i-th variable to the j-th structural shock at the short- and long-run horizons[10] and consider a single stacked matrix of dimension $3k \times k$:

$$f(\mathbf{A}_{0,t}, \mathbf{A}_{+,t}) = \underbrace{\begin{bmatrix} \mathbf{A}_{0,t} \\ \mathbf{IR}_{S,t}(\mathbf{A}_{0,t},\ \mathbf{A}_{+,t}) \\ \mathbf{IR}_{L,t}(\mathbf{A}_{0,t},\ \mathbf{A}_{+,t}) \end{bmatrix}}_{3k\times k} = \underbrace{\begin{bmatrix} \widetilde{\mathbf{A}}_{0,t}\mathbf{Q}_t \\ \mathbf{IR}_{S,t}(\widetilde{\mathbf{A}}_{0,t}\mathbf{Q}_t,\ \mathbf{B}_t\widetilde{\mathbf{A}}_{0,t}\mathbf{Q}_t) \\ \mathbf{IR}_{L,t}(\widetilde{\mathbf{A}}_{0,t}\mathbf{Q}_t,\ \mathbf{B}_t\widetilde{\mathbf{A}}_{0,t}\mathbf{Q}_t) \end{bmatrix}}_{3k\times k}. \tag{5}$$

We define the $s_j \times 3k$ matrix $\mathbf{S}_j$ as the sign restrictions on the j-th structural shocks and the $z_j \times 3k$ matrix $\mathbf{Z}_j$ as the zero restrictions on the j-th structural shocks, where s_j and z_j are the numbers of sign and zero restrictions imposed to identify the j-th structural shock for $1 \leq j \leq k$. The structural parameters are assumed to satisfy the identifying restrictions $\mathbf{S}_j f(\mathbf{A}_{0,t},\ \mathbf{A}_{+,t})\mathbf{e}_j > \mathbf{0}$ and $\mathbf{Z}_j f(\mathbf{A}_{0,t},\ \mathbf{A}_{+,t})\mathbf{e}_j = \mathbf{0}$ for $1 \leq j \leq k$, where $\mathbf{e}_j$ is the j-th column of $\mathbf{I}_k$. As indicated in equation (5), we can make inferences on structural parameters from the reduced-form parameters and orthogonal matrices.[11]

The stochastic volatility assumption requires Bayesian inference via Markov Chain Monte Carlo (MCMC) methods. Following Primiceri (2005), we estimate the reduced-form parameters in time-varying matrices $\mathbf{B}_t$, $\mathbf{A}_t$, and $\mathbf{H}_t$. Letting β_t be the stacked $k^2P \times 1$ vector of the elements in the columns of $\mathbf{B}_t$, and a_t be the stacked vector of non-zero and non-one elements in $\mathbf{A}_t$, we assume that these vectors follow a random-walk process: $\beta_{t+1} = \beta_t + u_{\beta,t}$, and $a_{t+1} = a_t + u_{a,t}$. The elements of $\mathbf{H}_t$ are assumed to evolve

[10] The short- and long-run horizons are set to $S = 1$ and $L = 40$, respectively.

[11] We present a more detailed procedure for implementing sign and zero restrictions in Online Appendix A.

as geometric random walks $\ln h_{j,t+1} = \ln h_{j,t} + u^j_{h,t}$, belonging to the class of stochastic volatility models. All the innovations in the model are assumed to be jointly normally distributed with the following assumptions on the covariance matrix:

$$\begin{bmatrix} \varepsilon_t \\ u_{\beta,t} \\ u_{a,t} \\ u_{h,t} \end{bmatrix} \sim N\left(0, \begin{bmatrix} \mathbf{I}_k & \mathbf{O} & \mathbf{O} & \mathbf{O} \\ \mathbf{O} & \mathbf{\Sigma}_\beta & \mathbf{O} & \mathbf{O} \\ \mathbf{O} & \mathbf{O} & \mathbf{\Sigma}_a & \mathbf{O} \\ \mathbf{O} & \mathbf{O} & \mathbf{O} & \mathbf{\Sigma}_h \end{bmatrix}\right), \qquad (6)$$

where $\mathbf{\Sigma}_\beta$, $\mathbf{\Sigma}_a$, and $\mathbf{\Sigma}_h$ are assumed to be block diagonal for simplicity. The TVP-VAR model has a flexible structure that requires researchers to choose only the priors for the hyperparameters of the covariance matrix. The flexibility, however, can lead to a risk of overfitting. Therefore, we set tight priors for the hyperparameters in the estimation.[12] To draw samples of β_t and a_t, we use the simulation smoother of de Jong and Shephard (1995). In drawing samples of h_t, we employ the multi-move sampler of Shephard and Pitt (1997) and Watanabe and Omori (2004) for non-linear and non-Gaussian state-space models.[13] For each draw of reduced-form parameters, we need to find an orthogonal matrix $\mathbf{Q}_t$ that satisfies the identifying restrictions. The structural parameters can be recovered from the reduced-form parameters and the corresponding orthogonal matrix.

Data and the identification scheme

Our baseline model consists of five variables in $\mathbf{y}_t$ ordered as $\mathbf{y}_t = [g_t, y_t, d_t, p_t, r_t]'$, where g_t is the government spending, y_t is the gross domestic product (GDP), d_t is the debt-

[12] Regardless of the tight priors that we set, we find substantial time variation in some impulse responses and structural parameters, as we will show below.

[13] To estimate a model that contains a relatively large number of variables, we rely on the efficient algorithm proposed by Nakajima et al. (2011), which is developed by modifying Primiceri (2005)'s algorithm. See Online Appendix B for an outline of the MCMC algorithm used in this study.

to-GDP ratio, p_t is the GDP deflator, and r_t is the interest rate. We use U.S. quarterly data for the period from 1952:Q1 to 2019:Q4.[14] The government spending and GDP are expressed in real per capita terms. We use the logarithm for all the variables except the debt-to-GDP ratio and the interest rate. The lag length is set to $P = 4$, following Blanchard and Perotti (2002). All the data are extracted from the FRED database of the Federal Reserve Bank of St. Louis. See the Appendix for a detailed description of the data sources.

Our identification scheme consists of three parts. First, we identify multiple fundamental shocks by imposing sign restrictions because the identification of additional shocks helps to identify the structural shock of interest (e.g., Peersman (2005); Mountford and Uhlig (2009)).[15] While we are interested in a government spending shock only, controlling for other shocks that are uncorrelated with a government spending shock allows us to filter out the automatic response of government spending to these shocks. Second, we impose sign and zero restrictions on the systematic component of policy rules following Arias et al. (2019). Third, we impose long-run exclusion restrictions on impulse responses in addition to sign restrictions on short-run impulse responses. The objective is two-fold. First, the set of admissible models can be narrowed down by imposing long-run exclusion restrictions on which most economists can easily agree. As addressed by Kilian and Lütkepohl (2017), it is difficult to find enough short-run restrictions in

[14]The sample period starts at the time when quarterly data series on public debt become available. Our sample covers the zero-interest-rate policy period in light of the findings of Nakajima (2011). Based on the Japanese experience, Nakajima (2011) documents that a zero lower bound on nominal interest rates has negligible effects on impulse responses in a TVP-VAR model with stochastic volatility.

[15]Sign restrictions have become popular as an attractive alternative to the traditional recursive identification. While the recursive identification sometimes produces results at odds with economic theory, sign restrictions allow researchers to impose restrictions in a theoretically consistent manner. See, e.g., Mumtaz and Zanetti (2012), Mumtaz and Zanetti (2013), and Liu et al. (2019). Because sign-identified SVAR models are only set identified, most papers, including ours, rely on the Bayesian approach. See Granziera et al. (2018) for a method of constructing classical confidence intervals in sign-restricted SVARs. In a more recent work, Jarociński and Karadi (2020) propose to combine high-frequency identification and sign restrictions to analyse the case in which the data contradict the standard theory.

practice. Furthermore, imposing long-run exclusion restrictions helps to satisfy the rank condition described by Arias et al. (2018),[16] while short-run exclusion restrictions often lack a theoretical foundation and can lead to implausible results (e.g., Peersman (2005)). To implement those restrictions, we exploit the algorithms developed by Rubio-Ramírez et al. (2010) and Arias et al. (2018).

We identify four structural shocks: an expansionary government spending shock, a contractionary monetary policy, a positive demand shock, and a positive supply shock.[17] Table 1 reports the sign restrictions imposed on the impulse responses for a quarter after the shock.[18] We impose a minimum set of restrictions to make our identification as agnostic as possible. In particular, we leave the responses of output to a monetary policy shock and a government spending shock unrestricted, as suggested by Uhlig (2005) and Mountford and Uhlig (2009), respectively. In addition, we follow previous studies to identify monetary policy and demand and supply shocks (e.g., Canova et al. (2007); Benati and Surico (2008); Belongia and Ireland (2016)). A government spending shock is assumed to increase the debt-to-GDP ratio, which is the key identifying restriction that distinguishes the shock from other shocks. The restriction shares similarities with those in previous studies (e.g., Canova and Pappa (2011); Enders et al. (2011); Bouakez et al. (2014)).

[16] The sufficient condition requires there to be at least $k - j$ zero restrictions and at least one sign restriction on the impulse responses to the j-th structural shock, for $1 \leq j \leq k$. The necessary rank condition requires the number of zero restrictions to be greater than or equal to $k(k-1)/2$.

[17] To compare the results with those of other studies, we restrict our focus in this study to a traditional unanticipated government spending shock.

[18] The choice of the period during which to restrict the responses does not change the basic results. It is also computationally burdensome to estimate impulse responses from a TVP-VAR model that imposes sign restrictions for several periods.

TABLE 1

Sign restrictions on short-run impulse responses

Variables	*Shocks*			
	Monetary policy	*Gov. spending*	*Demand*	*Supply*
Government spending		+		
GDP			+	+
Debt-to-GDP ratio		+		
GDP deflator	−		+	−
Interest rate	+		+	

Notes: The table shows the sign restrictions imposed on the impulse responses of the variables to a contractionary monetary policy shock, an expansionary government spending shock, a positive demand shock, and a positive supply shock. A blank indicates that the variable's response is unrestricted. A positive [negative] sign indicates that the variable's response is restricted to being positive [negative] for a quarter after the shock.

We combine the sign restrictions on impulse responses described above with sign and zero restrictions on the systematic component of monetary and fiscal policy equations.[19] Following Arias et al. (2019), we concentrate on the contemporaneous coefficients of the policy equations as it is more controversial to impose restrictions on lagged coefficients. Without loss of generality, we let the first and second shocks in our TVP-VAR model described in equation (1) be the monetary and fiscal policy shocks, respectively. Thus, we only restrict the contemporaneous coefficients that correspond to the elements in the first and second columns of $\mathbf{A}_{0,t}$. Letting $a_{0,t,ij}$ denote the (i,j) entry of $\mathbf{A}_{0,t}$, the monetary policy equation can be expressed as

$$r_t = \varphi_{g,t} g_t + \varphi_{y,t} y_t + \varphi_{d,t} d_t + \varphi_{p,t} p_t + u_{r,t}, \tag{7}$$

where $\varphi_{g,t} = -a_{0,t,51}^{-1} a_{0,t,11}$, $\varphi_{y,t} = -a_{0,t,51}^{-1} a_{0,t,21}$, $\varphi_{d,t} = -a_{0,t,51}^{-1} a_{0,t,31}$, $\varphi_{p,t} = -a_{0,t,51}^{-1} a_{0,t,41}$, and $u_{r,t} = -a_{0,t,51}^{-1} h_{1,t}^{1/2} \varepsilon_{1,t}$. Note that the lagged variables are abstracted from the equation. In line with the standard specification of the Taylor rule, we assume that the central

[19] Table A.1 in the Online Appendix provides a summary of the sign and zero restrictions imposed on the systematic component of the monetary and fiscal policy equations as well as those on the short- and long-run impulse responses.

bank only reacts contemporaneously to an increase in output and prices by raising the interest rate as in Arias et al. (2019). This assumption implies $\varphi_{g,t} = \varphi_{d,t} = 0$ and $\varphi_{y,t}$, $\varphi_{p,t} > 0$. Abstracting from the lagged variables, the fiscal policy equation is expressed as

$$g_t = \psi_{y,t} y_t + \psi_{d,t} d_t + \psi_{p,t} p_t + \psi_{r,t} r_t + u_{g,t}, \tag{8}$$

where $\psi_{y,t} = -a_{0,t,12}^{-1} a_{0,t,22}$, $\psi_{d,t} = -a_{0,t,12}^{-1} a_{0,t,32}$, $\psi_{p,t} = -a_{0,t,12}^{-1} a_{0,t,42}$, $\psi_{r,t} = -a_{0,t,12}^{-1} a_{0,t,52}$, and $u_{g,t} = -a_{0,t,12}^{-1} h_{2,t}^{1/2} \varepsilon_{2,t}$. Compared with monetary policy, there is much less agreement on the specification of the fiscal rule. Because we are interested in the role of spending-based fiscal adjustments in determining the size of government spending multipliers, we assume that government spending only reacts contemporaneously to output and public debt. This assumption implies $\psi_{p,t} = \psi_{r,t} = 0$. Although Blanchard and Perotti (2002) and Corsetti et al. (2012) suggest $\psi_{y,t} = 0$ and $\psi_{d,t} < 0$, respectively, we leave these coefficients unrestricted to remain agnostic in the estimates.

When implementing the procedure of Arias et al. (2019), we need to add two more zero restrictions on the monetary policy equation and one more zero restriction on the fiscal policy equation to satisfy the rank condition. When we label the third and fourth shocks to be the demand and supply shocks, the rank condition requires us to impose two more and one more zero restrictions on those equations, respectively. Hence, we impose long-run exclusion restrictions on the responses of output to the monetary policy, government spending, and demand shocks because, following the work of Blanchard and Quah (1989), it is widely accepted that those shocks have no long-run impact on output.[20] In addition, assuming that government spending is exogenously determined, we impose long-run exclusion restrictions on the responses of government spending to the other three

[20] Canova et al. (2007) consider government spending shocks as real demand shocks.

shocks.[21]

III Results

Identified macroeconomic shocks

The endogenous variables used in the estimation are shown in Figure 2. The left panels present the raw time series data described in the previous section. We extract trend components of all the variables, except for the interest rate, using the methodology of Hamilton (2018).[22] The procedure suggests that the detrended component of a variable at horizon h can be constructed as the residual of the linear projection of the variable on a constant and its four most recent values.[23] Formally, the regression can be written as

$$z_{t+h} = b_0 + b_1 z_t + b_2 z_{t-1} + b_3 z_{t-2} + b_4 z_{t-3} + \nu_{t+h},$$

where z_{t+h} is the realized value of the variable at time $t + h$. We choose a horizon of $h = 8$ as suggested by Hamilton (2018) for quarterly data. The detrended components

[21] Government spending is typically modelled as an exogenous stochastic process (e.g., Mertens and Ravn (2010)). In the short run, however, Caldara and Kamps (2017) find evidence that government spending responds to inflation negatively, suggesting that it is not fully indexed to inflation within a quarter.

[22] The methodology of Hamilton (2018) is proposed as a better alternative to the Hodrick-Prescott (HP) filter showing that the HP filter introduces spurious dynamic relations. The methodology is employed in the recent works by Richter et al. (2019), Mihai (2020), Montagnoli et al. (2021), and Gabriel et al. (2022). While we obtain similar results to those of our baseline estimation using a linear and quadratic trend, the detrended data appears to put too much emphasis on very low-frequency components. Taking the first difference is another popular method for detrending, but it does not remove high-frequency components that are not of interest in our research. For discussions on high- and low-frequency components of time series and detrending methods, see Baxter and King (1999). The detrended series obtained by taking the first difference and those obtained by removing a linear and quadratic trend are shown in Figure E.1 in the Online Appendix. The evolution of the government spending multipliers estimated using data detrended with a linear and quadratic trend is also presented in Figure E.2.

[23] Extracting a trend component via regression prior to the estimation is popular in the empirical literature on the effects of fiscal policy (e.g., Brückner and Pappa (2012); Ravn et al. (2012); Ilzetzki et al. (2013); Nickel and Tudyka (2014); Caldara and Kamps (2017); Huidrom et al. (2020); Angelini et al. (2022)).

are shown in the right panels of Figure 2.

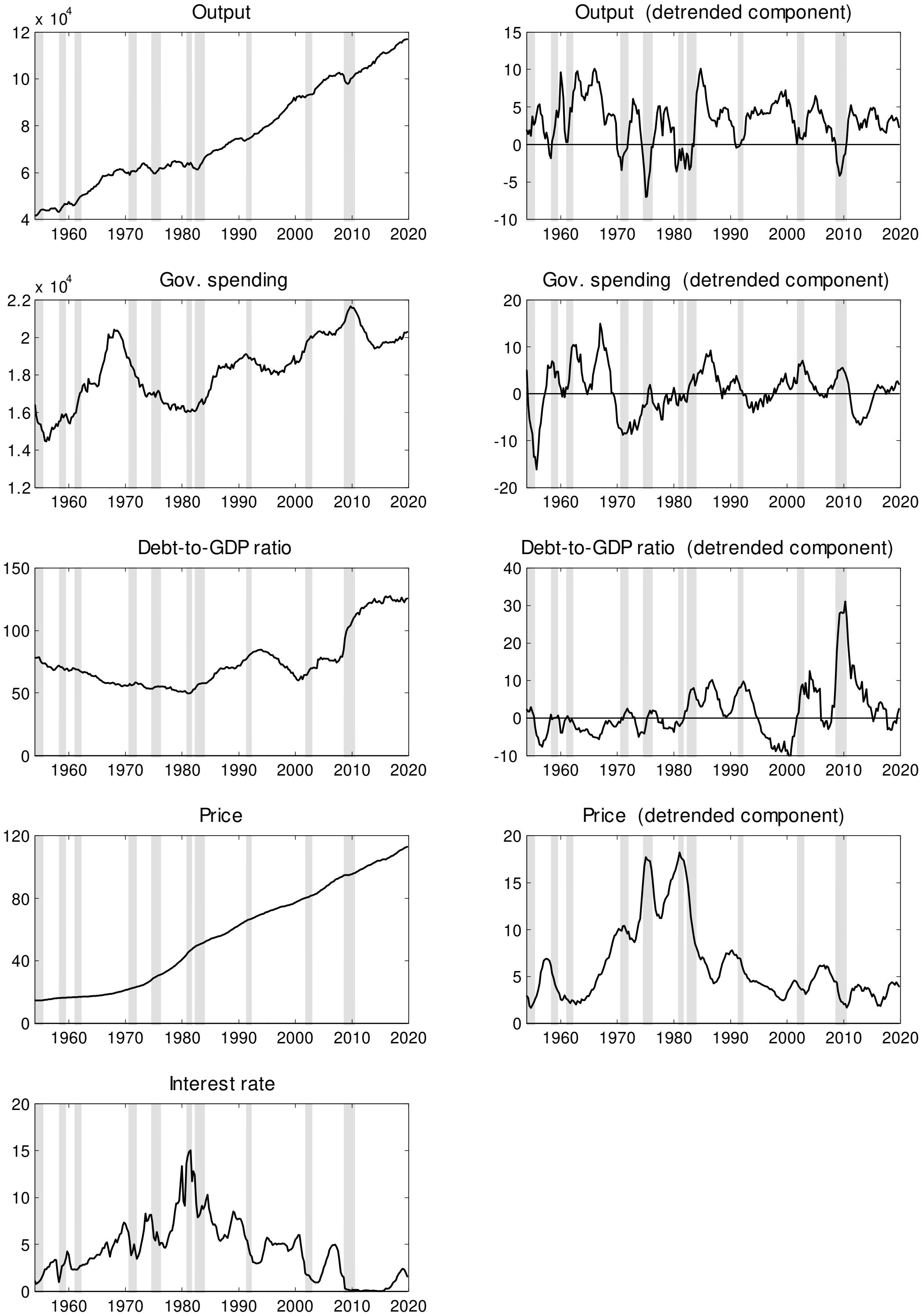

Figure 2. Endogenous variables used in the estimation
Notes: The raw time series data are plotted in the left panels. For the first four variables, we detrend the raw series using the methodology of Hamilton (2018). The cyclical components are plotted in the right panels. The shaded areas represent recessions as defined by the NBER.

We run eight parallel MCMC chains, each chain executing 70,000 replications and discarding the first 20,000 draws. We conduct the convergence diagnostics of Gelman and Rubin (1992) and Geweke (1992), which find no evidence of non-convergence.[24] Figure 3 presents the stochastic volatility of the identified structural shocks. We compute the volatility using the structural impact matrix. The estimated volatility of the monetary policy shock is consistent with that reported in previous studies (e.g., Canova and Gambetti (2009); Mumtaz and Zanetti (2013); Belongia and Ireland (2016)). The volatility increased substantially during the Great Inflation of the mid-1970s and then at the time of Volcker's appointment as Fed Chairman, while showing a large decline in the early 1980s. The observed reduction in the volatility of the government spending shock from the 1960s to the 1990s can also be found in Justiniano and Primiceri (2008). As our sample period covers the Great Recession of 2007-2009, a substantial increase in the volatility during the period is observed reflecting the massive fiscal response of the period. The volatility of the demand and supply shocks increased during the Great Inflation and reached its peak after the oil shock of 1978-79. Similar patterns are reported by Belongia and Ireland (2016). Because demand and supply shocks are associated with the reduced-form innovations of the output growth and inflation equations respectively, similarity can also be found in those reported in previous studies (e.g., Canova and Gambetti (2009); Mumtaz and Zanetti (2013)). As in the case of government spending shocks, the volatility of the demand and supply shocks shows smaller peaks during the Great Recession. Since the estimation results here are largely consistent with those reported in previous studies, we can conclude that the time-varying volatility is captured well in our model. The inclusion of stochastic volatility in the TVP-VAR model appears to be essential to detecting structural changes appropriately in the transmission of government spending shocks.

[24] The results of the diagnostics are provided in Online Appendix C.

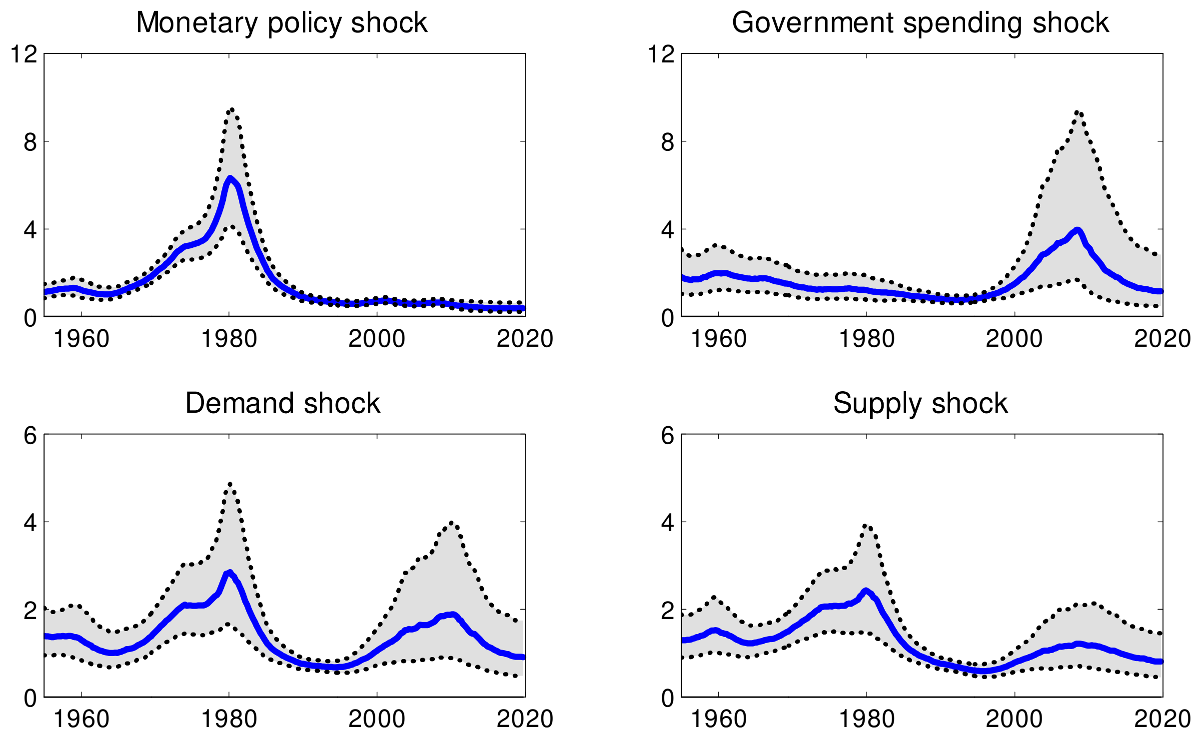

Figure 3. Stochastic volatility of the structural shocks
Notes: The solid lines represent the posterior mean, with the shaded areas representing the 16th-84th percentile ranges.

Before investigating the evolution of the government spending multipliers, it is instructive to consider whether there has been a change in the structure of the U.S. economy. Figure 4 displays the time profile of the posterior mean impulse responses of output to our four identified structural shocks. As we deal with relatively large numbers of variables and restrictions, we compute impulse responses for each period based on the parameters estimated for that period following the standard convention in the literature (e.g., Primiceri (2005); Koop et al. (2009); Korobilis (2013); Belongia and Ireland (2016)). The impulse responses thus indicate how the economy responded to shocks at each point in time.[25]

[25] Given the non-linear nature of our TVP-VAR framework, simulation method of Koop et al. (1996) can be applied in computing impulse responses. The main drawback of the method is its heavy computational burden (e.g., Koop et al. (2009); Korobilis (2013)). Moreover, Koop et al. (2009) document that only a slight difference is expected from using the computationally demanding method. We compared impulse responses for the baseline model computed using the method of Koop et al. (1996) against those obtained following the standard convention and confirmed that the method did not alter our results reported in this paper.

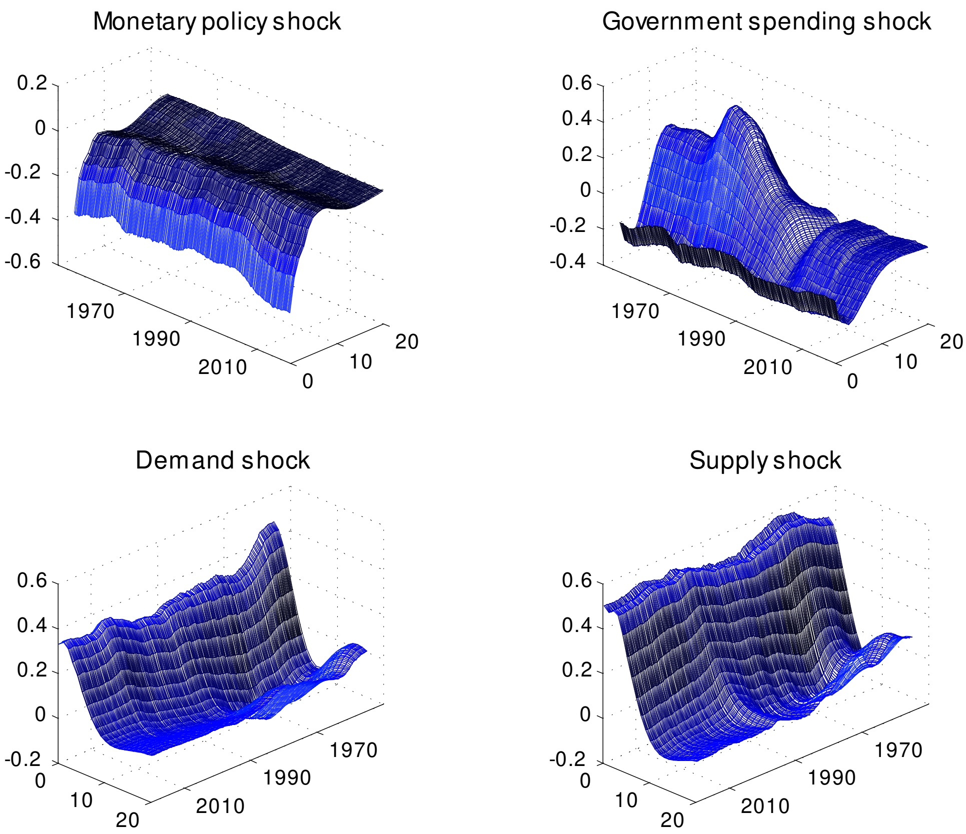

Figure 4. Evolution of output responses to structural shocks
Notes: The figure shows the time profiles of the posterior mean responses of output to a one-standard-deviation innovation of a contractionary monetary policy shock, an expansionary government spending shock, a positive demand shock, and a positive supply shock.

One can easily notice that the shape of the output response to a government spending shock has changed over time while those to the other three shocks has hardly changed. While we will examine the impulse responses to a government spending shock in detail in the next subsection, we would like to mention here that the results are in line with the previous evidence. The output falls immediately after the contractionary monetary policy shock and returns to its pre-shock level within a year. The shape of the response and its time variation are quite similar to those reported in previous studies (Canova and Gambetti (2009); Baumeister and Benati (2013); Belongia and Ireland (2016)). The positive demand and supply shocks increase the output on impact. The output responses show similar shapes while the effect of the demand shock is less persistent than that of the supply shock. The slight difference in the persistence of the output responses can also be seen in previous studies (Canova et al. (2007); Gambetti et al. (2008); Belongia and Ireland (2016)). To conclude, Figure 4 suggests that the overall structure of the U.S. economy has changed very little and that changes in the effects of a government spending shock are not interrelated with those in the transmission mechanism of other fundamental structural shocks.

Evolution of the government spending multipliers

Figure 5 shows the impulse responses of output to government spending shocks in four arbitrarily selected time periods: 1955:Q1, 1975:Q1, 1995:Q1, and 2015Q1. While previous studies tend to find positive output effects on impact (e.g., Blanchard and Perotti (2002); Mountford and Uhlig (2009)), we do not obtain initial increases in output regardless of the time period chosen. The ambiguous impact response can be attributed to our identification scheme. Caldara and Kamps (2017) show that government spending shocks may either increase or decrease output on impact depending on the estimated values of fiscal policy rule coefficients.[26] Although initial responses are statistically insignificant throughout the estimation period, the peak output responses differ substantially across time periods. A comparison of impulse responses in the four representative periods indicates expansionary effects of government spending in 1955:Q1 and 1975:Q1, but they disappear in 1995:Q1 and 2015:Q1. The highest peaks in the former two selected time periods are observed around a three-year horizon. The timing of the peak effect is similar to those reported in previous studies (e.g., Blanchard and Perotti (2002)).

[26] Caldara and Kamps (2017) also find a zero impact response of output to a government spending shock under a simple fiscal rule estimated using the identification scheme of Mountford and Uhlig (2009). While we consider a more general fiscal policy rule than theirs, our estimates of the coefficients are consistent with those reported in previous studies as we will see below. From a different perspective, Ben Zeev and Pappa (2015) obtain a zero impact response of output to a defense spending shock using measurement-error free measures of output and TFP.

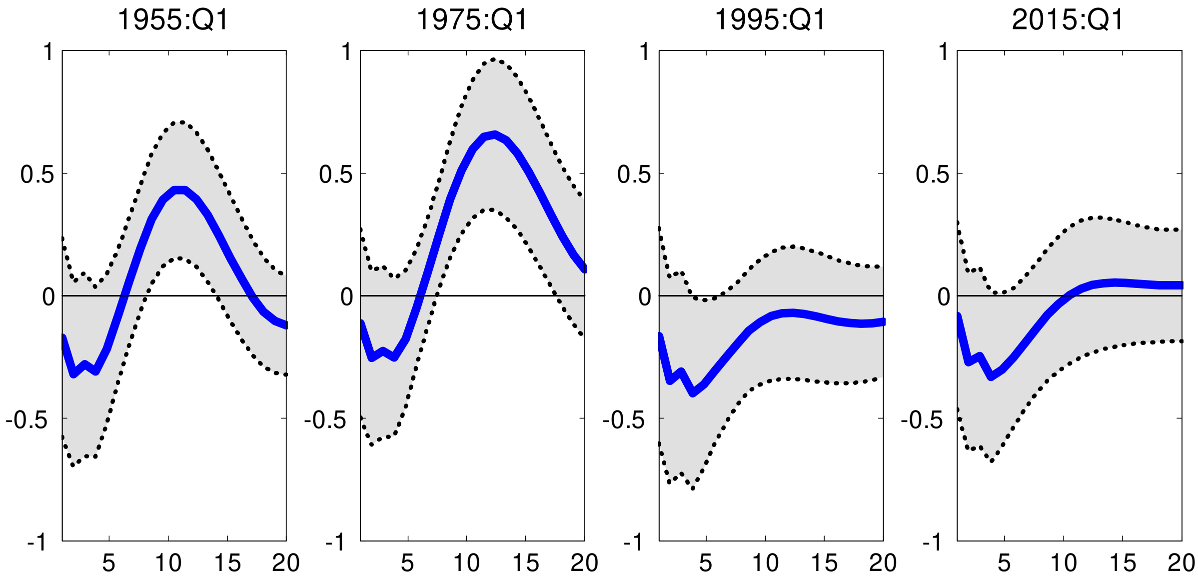

Figure 5. Output responses to a government spending shock
Notes: The solid lines represent posterior mean responses to a 1 percentage point increase in government spending, with the shaded areas representing the 16th-84th percentile ranges.

To illustrate the evolution of the government spending multipliers, we compute the multipliers for each sample period following Blanchard and Perotti (2002).[27] The government spending multiplier, often referred to as the *peak* multiplier, is defined as the ratio of the peak of the output response to the initial government spending shock. Because government spending and output are expressed in logs, we convert the peak response using the sample average ratio of output to government spending.[28] The upper panel of Figure 6 presents the time profile of the posterior mean estimates of the multipliers together with the 16th-84th percentiles. The lower panel indicates the horizon when the output response reaches its peak in each sample period. The multipliers are positive until the late 1980s but not statistically larger than zero in the post-1990 period. A steady downward trend can be seen between the 1970s and the early 1990s. The observed

[27]Auerbach and Gorodnichenko (2013), Leeper et al. (2013), Caldara and Kamps (2017), and others follow Blanchard and Perotti (2002).

[28]Ramey and Zubairy (2018) point out a potential problem arising from the use of the sample average ratio to calculate multipliers by considering the large variation found in their long samples of historical data. Nevertheless, we use the average ratio not only because it is relatively stable in our post-war sample but also because we intend to highlight the time variation in multipliers without interference from changes in the ratio.

decline in multipliers corroborates Bilbiie et al. (2008)'s finding of smaller multipliers in the post-1980 period than in the earlier period.

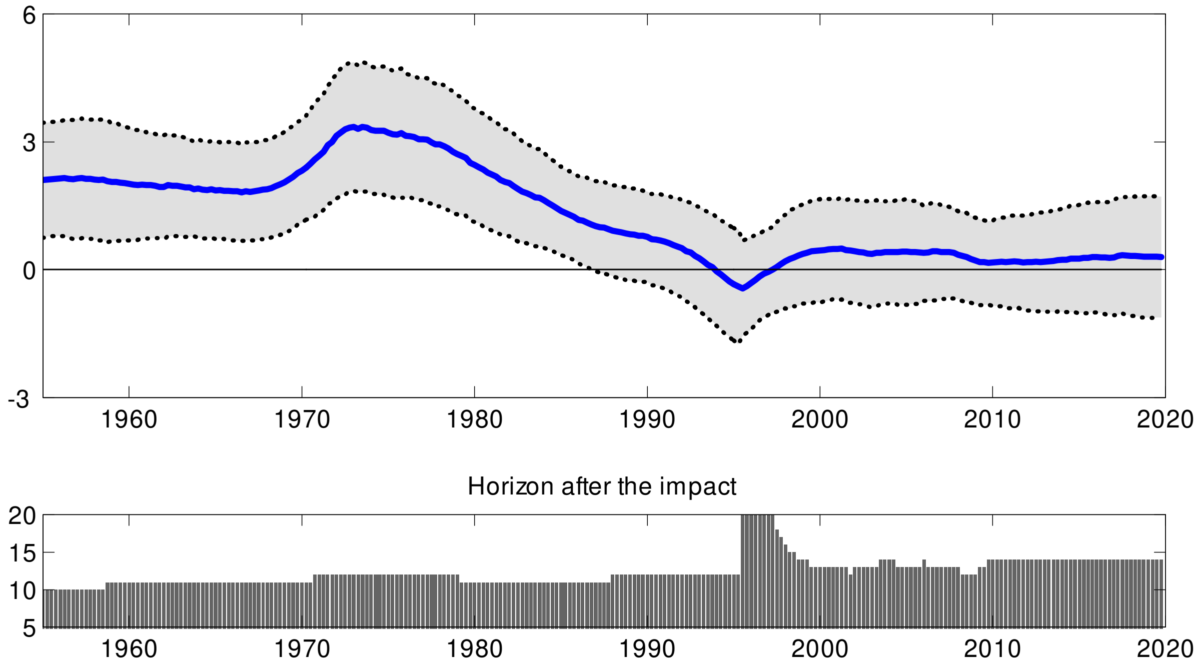

Figure 6. Evolution of the government spending multipliers
Notes: The upper panel presents the time profile of the government spending multipliers. The multipliers are calculated as the maximum impact of a 1 dollar increase in government spending on output for each period. The solid line represents the posterior mean of the multipliers with the shaded area representing the 16th-84th percentile range. The lower panel indicates the horizon when the maximum impact on output is observed after the government spending increase.

Table 2 reports the sample average, maximum, and minimum values of the posterior means of the government spending multipliers together with the averages of the selected subsample periods, such as the post-OBRA93 period. We believe that the choice of the post-OBRA93 period is relevant because the debt-to-GDP ratio was stable for several years after its passage. The sample average of time-varying multipliers lies between 0.8 and 1.5, which is suggested as a reasonable range of the government spending multiplier in the Ramey (2011)'s review of the literature. As we have seen in Figure 6, the confidence intervals of the maximum and minimum multipliers do not overlap. While the estimates do not provide evidence of larger multipliers in the pre-Volcker period than in the later period, we find that the multipliers in the 1970s are statistically larger than those in the post-OBRA93 period on average. Our results point to a decline in the government spending multipliers during the period between the 1970s and the early 1990s.

TABLE 2

Multipliers

Sample avg. (1955:Q1-2019:Q4)	*Max (1973:Q1)*	*Min (1995:Q3)*	
1.27	3.35	-0.44	
[0.05 2.50]	[1.85 4.86]	[-1.55 0.68]	
Pre-Volcker (-1979:Q2)	*Post-OBRA93 (1993:Q3-)*	*1960s*	*1970s*
2.39	0.24	1.94	3.01
[1.07 3.71]	[-0.95 1.43]	[0.75 3.12]	[1.58 4.45]
1980s	*1990s*	*2000s*	*2010s*
1.48	0.18	0.39	0.25
[0.35 2.62]	[-0.97 1.34]	[-0.76 1.53]	[-1.00 1.49]

Notes: The first row of the table reports the sample average, maximum, and minimum values of the posterior means of the government spending multipliers. The second and third rows report the average values of the posterior mean multipliers computed over the subsample periods shown. The 16th-84th percentile ranges reported in square brackets are calculated using the corresponding standard deviations.

IV Explaining the evolution of multipliers

In this section, we turn our attention to the sources of the debt-dependent multipliers. We investigate whether the decline in multipliers can be attributed to changes in the conduct of monetary and fiscal policy. The relevance of the Ricardian channel is also considered.

Monetary policy response

The results from the baseline model show that the government spending multipliers declined between the 1970s and the early 1990s. One explanation for the decline is that the more active monetary policy during the post-Volcker period offsets the stimulative effects of government spending strongly (e.g., Bilbiie et al. (2008)). Figure 7 shows the interest rate responses to government spending shocks in selected time periods. The same time periods are chosen as in Figure 5, so that we can compare the changes in the responses of the output and interest rate. While we have seen that the expansionary effects of government spending disappeared in 1995:Q1 and 2015:Q1 in Figure 5, we do not observe significant increases in the interest rate response in those periods. The initial negative responses of interest observed in 1975:Q1 and 1995:Q1 seem puzzling, but the counterintuitive results can be found in previous studies, such as Mountford and Uhlig (2009).[29]

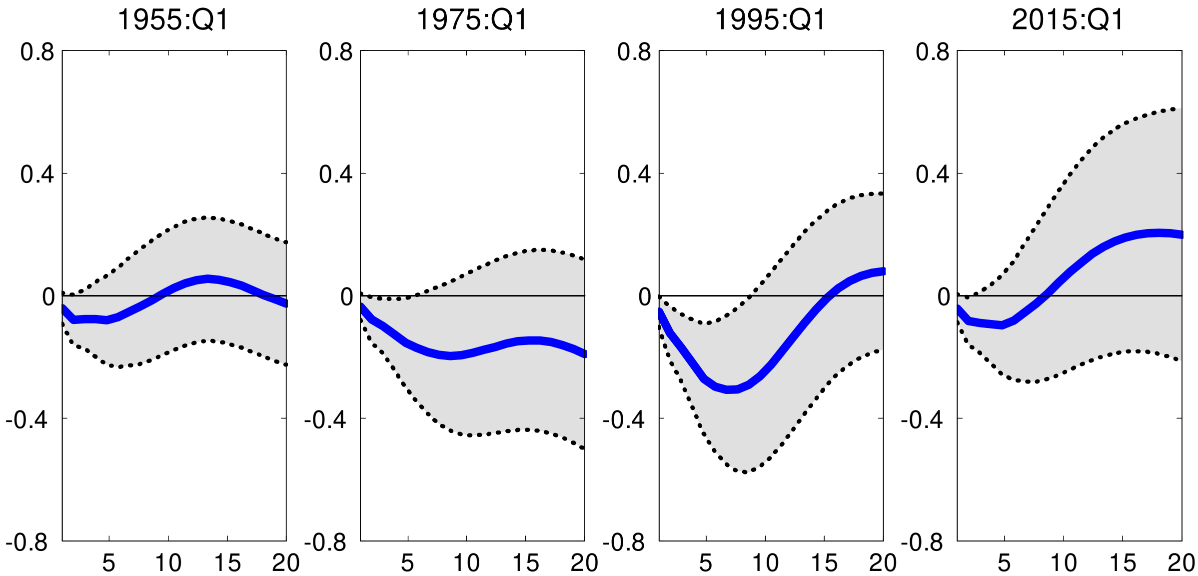

Figure 7. Interest rate responses to a government spending shock
Notes: The solid lines represent posterior mean responses to a 1 percentage point increase in government spending, with the shaded areas representing the 16th-84th percentile ranges.

To see the role of the interest rate response in the evolution of multipliers, we look at the time profile of the interest rate response in the upper panel of Figure 8. Because an

[29]Following Mountford and Uhlig (2009), we do not impose a sign restriction on the response of the interest rate to a government spending shock. Enders et al. (2011) obtain a positive response of the interest rate to a government spending shock while restricting the response to be positive.

increase in the interest rate has an immediate negative impact on the output, as we have seen in Figure 4, we choose the response at the horizon when the output response reaches its peak. The horizons shown in the lower panel are therefore the same as those presented in Figure 6. The evolution of the interest rate response does not show a similar pattern to that of multipliers. In particular, there is no visible trend during the period between the 1970s and the early1990s. It is also worth noting that the interest rate responses are not statistically different from zero throughout the sample period.

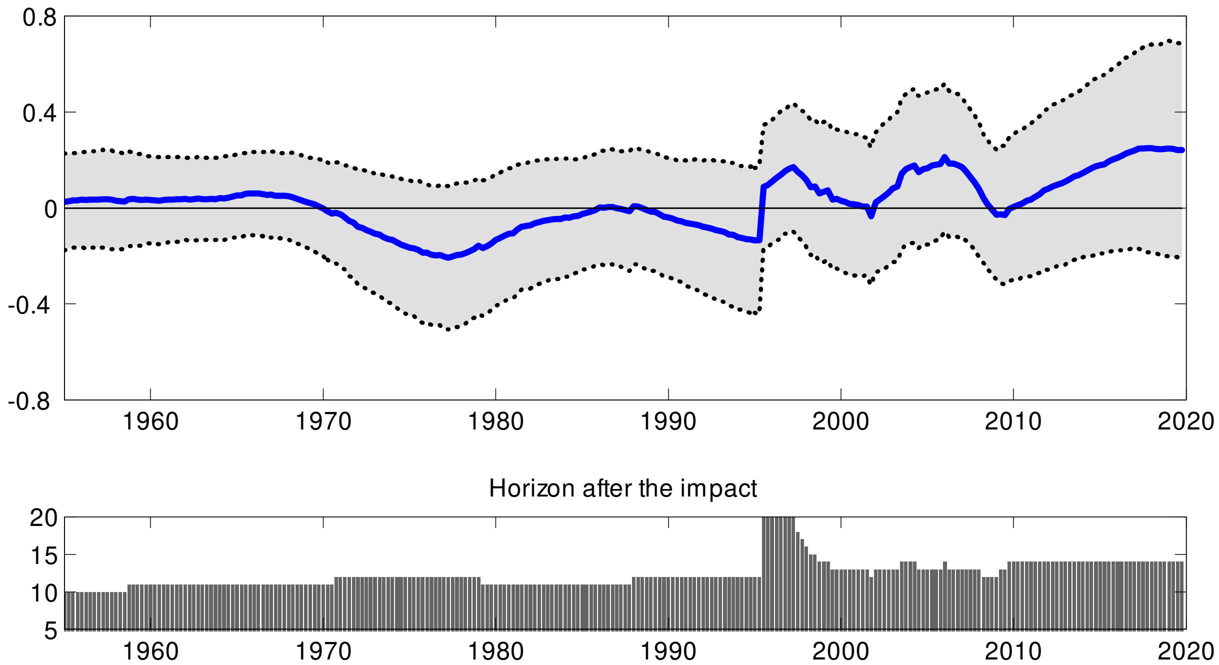

Figure 8. Evolution of the interest rate response
Notes: The upper panel presents the time profile of the interest rate response at the horizon when the maximum impact of a 1 dollar increase in government spending on output is observed for each period. The solid line represents the posterior mean of the interest rate response with the shaded area representing the 16th-84th percentile range. The lower panel indicates the horizon when the maximum impact on output is observed after the government spending increase.

As we employ the identification procedure of Arias et al. (2019) in our TVP-VAR framework, the coefficients of the systematic component of monetary policy can be estimated as time-varying parameters. The evolution of the monetary policy rule coefficients is shown in Figure 9. Both of the coefficients for output, $\varphi_{y,t}$ and inflation, $\varphi_{p,t}$ are relatively stable over the sample period, with the posterior mean estimates close to 0.25

and 1.0, respectively. The similar relative size difference between the two coefficients can be found in the estimates of Arias et al. (2019), although our estimated values are substantially smaller than theirs.[30] One explanation for this result is that our model allows for changes in the volatility of a monetary policy shock. In this regard, perhaps the most comparable study to ours is that of Belongia and Ireland (2016). They consider a reduced-form interest rate equation as a monetary policy rule within their second-order TVP-VAR model with stochastic volatility.[31] Despite the differences in the specification of the monetary policy rule, our estimates are broadly in line with their estimates for the impact coefficients. While small peaks are observed around the time of Volcker's appointment in Belongia and Ireland (2016)'s estimates for the both coefficients, these may be related to the fact that their estimates for the volatility of monetary policy shocks during the periods are much smaller than our estimates.

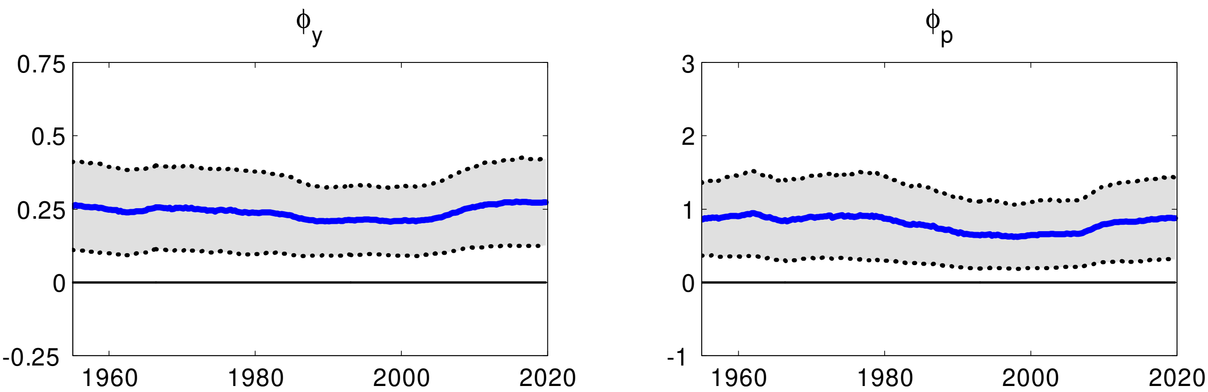

Figure 9. Evolution of monetary policy rule coefficients
Notes: The solid lines represent the posterior mean with the shaded areas representing the 16th-84th percentile ranges.

[30] Arias et al. (2019) report the posterior median estimates for the coefficients for output and inflation as 0.84 and 2.73, respectively.

[31] Belongia and Ireland (2016) estimate the monetary policy equation that includes a time-varying intercept terms, time-varying coefficients for the current and two lagged values of inflation and the output gap and the lags of the interest rate. Using the coeficients for the lagged values, they also report estimates for the long-run coefficients, the median values of which are about twice as large as those of the impact coefficients. Canova and Gambetti (2009) also consider an interest rate equation within their second-order TVP-VAR model with stochastic volatility. Their model consists of intercepts and up to two lags of interest rate, inflation, output growth, and money growth. They only report estimates for long-run coefficients with wide intervals that contain zero for most of the sample period.

Overall, the results do not suggest a strong offsetting response of monetary policy during the period between the 1970s and the early 1990s. The decline in multipliers does not appear to be attributable to the change in the conduct of monetary policy.

Fiscal adjustments

In theory, the smaller multipliers during times of high debt could be attributed to a larger magnitude of fiscal adjustments (Bi et al. (2016)). Regarding the adjustment measures taken in the U.S., Corsetti et al. (2012) provide evidence of spending-based adjustments using data covering the period 1983:Q1 to 2007:Q4.[32] Figure 10 shows the time path of government spending after a government spending shock in selected time periods. The same time periods as those in Figures 5 and 7 are chosen. As predicted, the results confirm the evidence of spending-based fiscal adjustments for the period 1995:Q1, which is the mid-point of the sample period used by Corsetti et al. (2012).

Nevertheless, the impulse response of government spending alone does not provide a definitive answer to the question of whether government spending adjusts in a manner to stabilize debt. Figure 11 illustrates the evolution of the government spending rule coefficients. The mean estimate for the coefficient for output, $\psi_{y,t}$, exhibits a gradual downward trend, although the confidence interval contains zero throughout the sample period. Similar results can be found in the study by Caldara and Kamps (2017), who report a negative median estimate for $\psi_{y,t}$ with the interval that contains zero. Caldara and Kamps (2017) also analytically show the negative relationship between the size of the government spending multiplier and the systematic response to output in the government

[32] Corsetti et al. (2012) also show that the New Keynesian business cycle model can generate crowding-in of consumption by incorporating a debt-stabilizing spending rule. The crowding-in is attributed to the decline in the interest rate caused by the expected reduction in the tax burden. Our results, however, do not support the transmission channel because we do not find unambiguous evidence of a decline in the interest rate after a government spending shock.

spending rule. Thus, the results suggest that the coefficient for output appears to play no role in the decline of the multipliers observed in Figure 6. On the other hand, the coefficient for debt, $\psi_{d,t}$, shows a clear downward trend since the 1970s and turns negative in the 1990s, consistent with what we have seen in Figure 10. Recall that the value of $\psi_{d,t}$ is left unrestricted and that the negative value of $\psi_{d,t}$ represents the degree of fiscal adjustments in response to an increase in debt. Our estimate for the coefficient corroborates the action-based data set of Devries et al. (2011), which suggests that U.S. fiscal consolidation relies more on spending cuts during the post-OBRA93 period than in the preceding period.[33]

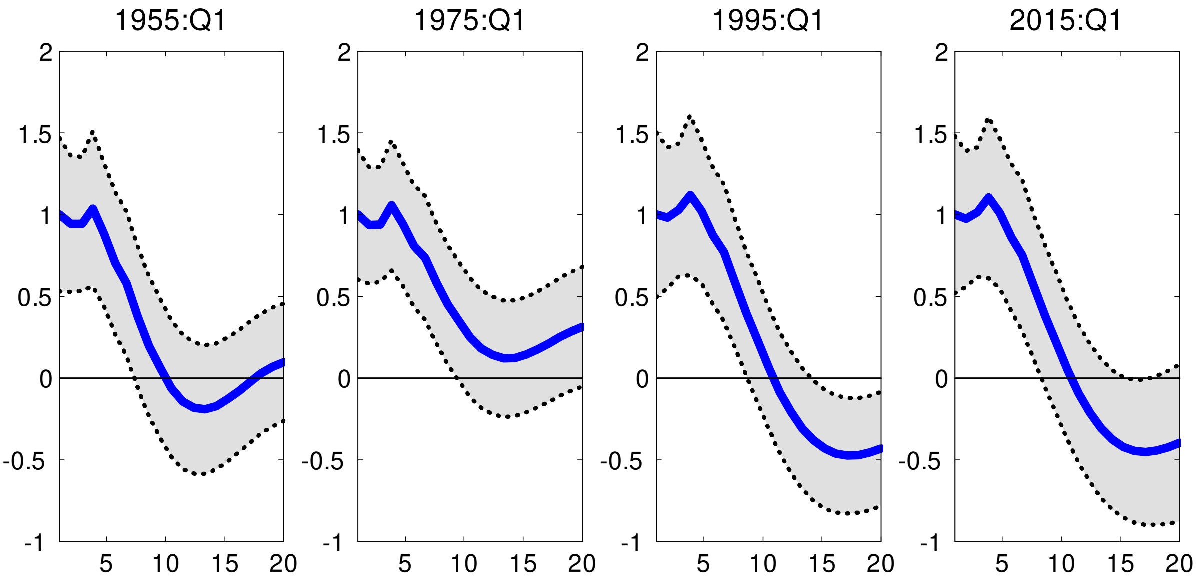

Figure 10. Time paths of government spending after a government spending shock
Notes: The solid lines represent the posterior mean responses to a 1 percentage point increase in government spending, with the shaded areas representing the 16th-84th percentile ranges.

As regards the macroeconomic effects of spending-based fiscal adjustments, two different theoretical predictions have been provided. While neoclassical models typically assume that spending cuts have positive wealth effects associated with a lower future

[33]The data set records fiscal consolidation during the period 1978-2009. While fiscal consolidation based on a tax hike and spending cut amount to 0.8% and 1.7% of the GDP during the post-OBRA period, those occurring during the preceding period account for 1.9% and 1.1% of the GDP, respectively. The data set reports no fiscal consolidation occurring during the period 2000-2009.

tax burden (e.g., Perotti (1999)), traditional Keynesian models and new Keynesian models tend to predict contractionary effects (e.g., DeLong and Summers (2012); Galí et al. (2007)). The importance of wealth effects has been highlighted in the literature in favor of the 'expansionary austerity hypothesis' (e.g., Alesina et al. (2019)). Recent cross-country empirical studies, however, find evidence that fiscal adjustments are contractionary even when they are based on spending cuts (e.g., Guajardo et al. (2014); House et al. (2020)). With regard to the effects of spending-based fiscal adjustments on the U.S. economy, Alesina et al. (2015) present evidence that spending-based fiscal adjustments have negative effects on consumption while they are less harmful than tax-based ones. More recently, Barnichon et al. (2022) document that contractionary effects of negative government spending shocks are larger in absolute terms than expansionary effects of positive government spending shocks. These empirical observations support the view that fiscal adjustments in the U.S. are contractionary even when they are spending-based ones. Therefore, the policy change observed during the post-OBRA93 period in the presence of debt accumulation might have contributed to raising expectations of the future fiscal adjustments, thereby leading to smaller multipliers. It is worth mentioning that the magnitude of fiscal adjustments decreased in the 2010s when the U.S. public debt reached its highest level in the post-war period. The observed timing of the decrease is consistent with D'Erasmo et al. (2016)'s evidence of structural change leading to smaller fiscal adjustments in the post-2008 U.S. data. Despite the high levels of debt, the government spending multipliers do not exhibit a substantial decline in the 2010s, as we have seen in Figure 6.

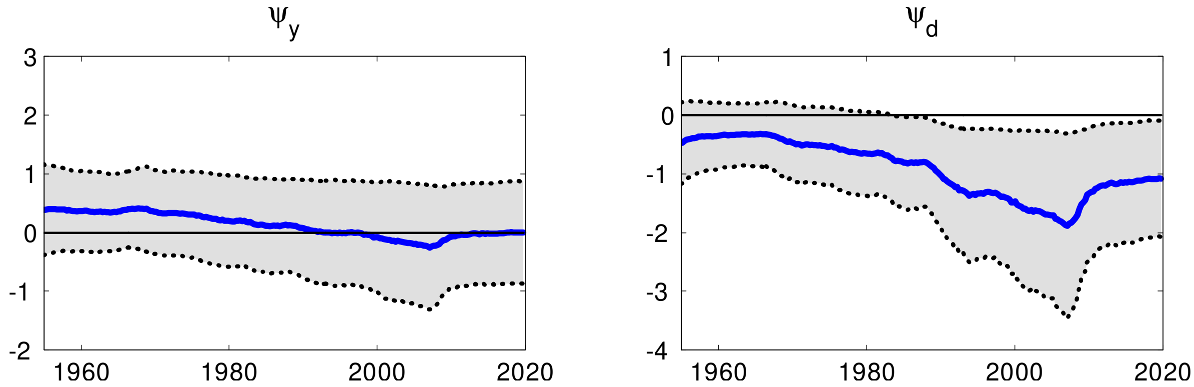

Figure 11. Evolution of the government spending rule coefficients
Notes: The solid lines represent the posterior mean, with the shaded areas representing the 16th-84th percentile ranges.

We now proceed to examine the role of observed spending-based fiscal adjustments in debt-dependent multipliers. The left panel of Figure 12 displays a scatter plot of the mean estimates of the multipliers against historical data on the debt-to-GDP ratio of corresponding periods. The negative correlation provides evidence of the debt-dependent nature of the U.S. multipliers, which is not reported in previous studies to our knowledge. The right panel of Figure 12 shows the relationship between the multipliers and the magnitude of spending-based fiscal adjustments. Interestingly, the multipliers are more strongly correlated with the magnitude of fiscal adjustments than debt. This leads us to conjecture that the increased magnitude of fiscal adjustments in the presence of rising indebtedness could be the major driving force for the decline in multipliers observed between the 1970s and the early 1990s.[34]

[34]We also estimate smooth transition VAR models with the same data set using two different types of transition variables: the debt-to-GDP ratio and the magnitude of fiscal adjustments estimated in our baseline model. In line with the findings of Caggiano et al. (2015) and Bernardini and Peersman (2018), we do not find any significant difference across regimes. However, it is worth mentioning that we obtain positive multipliers during the regime of weak fiscal adjustments while the multipliers in other regimes are not statistically different from zero. We present the impulse responses of output to government spending shocks in high- and low-debt regimes and strong and weak fiscal adjustment regimes in Figure F.2 in the Online Appendix.

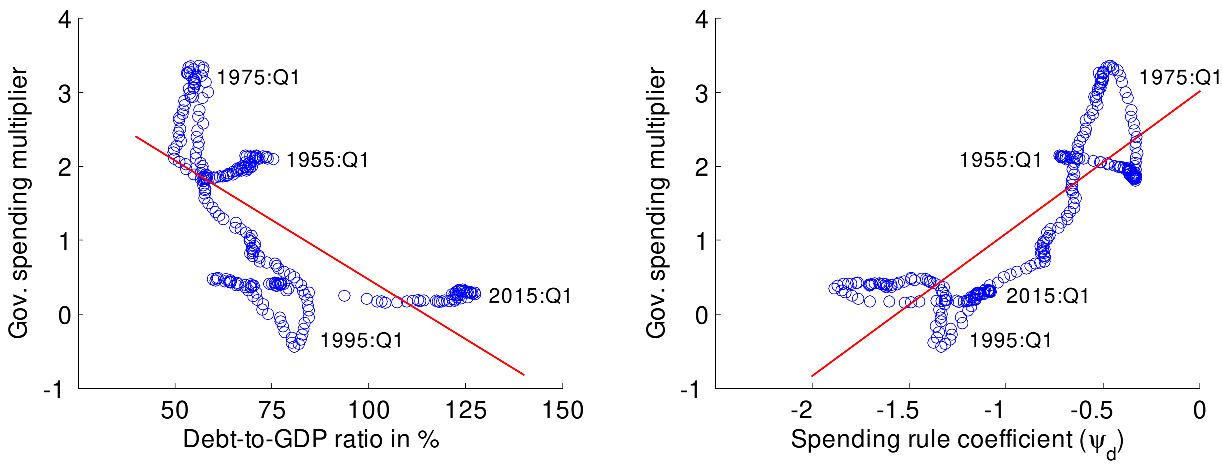

Figure 12. Relationship between debt (left) [fiscal adjustments (right)] and multipliers
Notes: The left [right] panel plots the debt-to-GDP ratio [government spending rule coefficients] and the government spending multipliers. R-squared: 0.46 (left); 0.68 (right).

Extended experiment

Our next question is whether the increased magnitude of spending-based fiscal adjustments caused the decline in multipliers via the Ricardian channel. To examine the relevance of the channel, we augment the baseline model with private consumption following Huidrom et al. (2020). Using cross-country panel data, they establish the relevance by showing negative responses of output and consumption to a government spending shock for countries with weak fiscal positions. As in the case of the government spending and GDP, the private consumption is expressed in the logarithm of real per capita terms and detrended following Hamilton (2018). See the Appendix for a detailed description of the data. Because our augmented model consists of six variables, we need to impose more zero restrictions either on the contemporaneous coefficients or on the impulse responses in addition to the restrictions that we imposed on the baseline model. As we are interested in exploring the changes in household behaviour, we leave the short-run impulse responses of consumption unrestricted. While letting the first and second shocks be the monetary and fiscal policy shocks as in the baseline model, we label the third, fourth, and fifth shocks as the demand, consumption, and supply shocks in the augmented model, respectively. The rank condition then requires us to add one more zero restriction on the monetary policy, government spending, and demand shocks and two zero restrictions on the consumption shock. As we assume that the monetary and fiscal policy do not react to a contemporaneous increase in consumption, the corresponding coefficients for consumption are restricted to be zero. The demand and consumption shocks are assumed to have no long-run impact on consumption. We also impose long-run exclusion restrictions on the response of government spending to a consumption shock, assuming that government spending is exogenously determined.[35]

Figure 13 shows the responses of output and consumption to government spending shocks in selected periods in the augmented model. The periods are chosen to determine whether the augmented model replicates the disappearance of expansionary effects of government spending between the 1970s and the early 1990s observed in Figure 5. The shapes of the output responses shown in the left two panels are basically the same as those in Figure 5. Overall, the inclusion of consumption in the baseline model does not change the results. The responses of consumption, shown in the right two panels, exhibit similar shapes and time variation to those of output. While a crowding-in of consumption can be seen in the 1970s, it disappears in the 1990s when spending-based fiscal adjustments are observed, as shown in Figure 10. The results are consistent with Alesina et al. (2015), who find negative effects of spending-based fiscal adjustments on consumption for U.S. data.

[35] Table A.2 in the Online Appendix provides a summary of the restrictions imposed. The results of the convergence diagnostics of Gelman and Rubin (1992) and Geweke (1992) are provided in Online Appendix C.

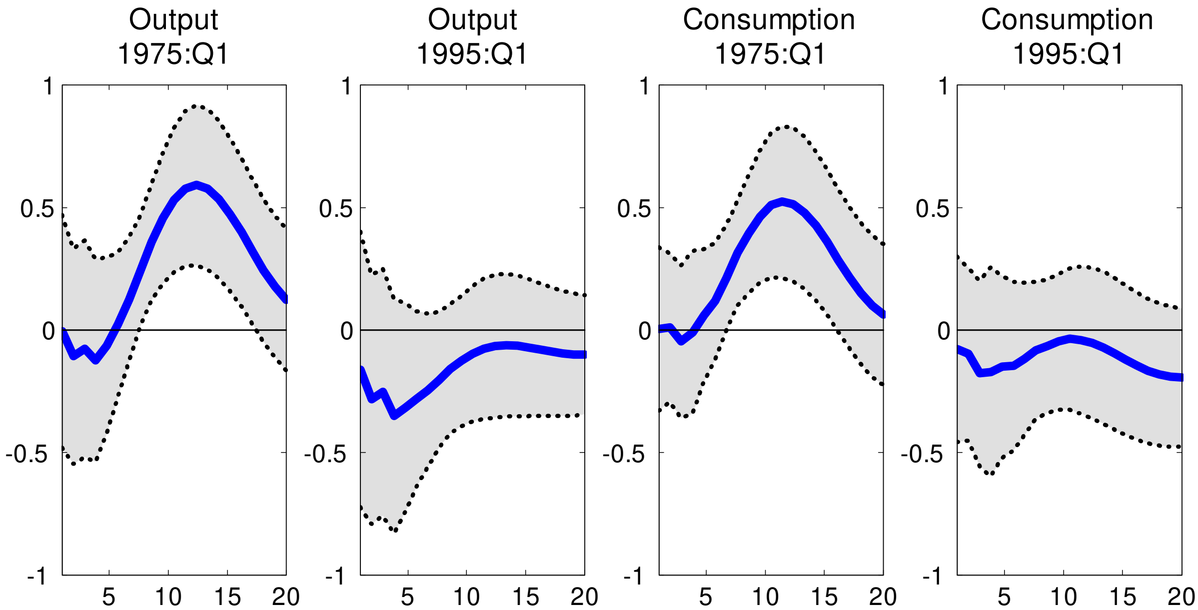

Figure 13. Output and consumption responses to a government spending shock
Notes: The solid lines represent the posterior mean response to a 1 percentage point increase in government spending, with the shaded areas representing the 16th-84th percentile ranges.

Figure 14 illustrates the similarity between the evolution of peak responses of output and that of consumption to government spending shocks. The evolution in the peak response of output shows basically the same pattern as those of the multipliers shown in Figure 6. The peak responses of output and consumption both show a steady downward trend between the 1970s and the early 1990s. Their co-movement indicates that the decline in the output response is mostly led by that in consumption. The results suggest that the Ricardian channel accounts for the decline in multipliers: households reduce the amount of consumption after a government spending shock, expecting a larger magnitude of fiscal adjustments.

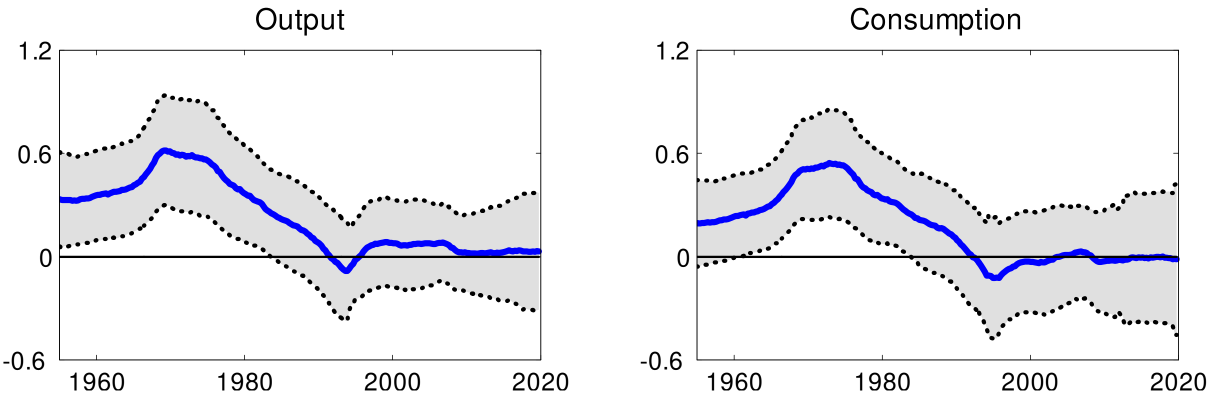

Figure 14. Evolution of the peak responses of output and consumption to a government spending shock
Notes: The peak responses are calculated as the maximum impact of a 1 percentage point increase in government spending on output and consumption in each period. The solid lines represent the posterior mean of the peak responses, with the shaded areas representing the 16th-84th percentile ranges.

V Conclusions

This study provides new time series evidence of government spending multipliers during the post-war period in the United States. We achieve identification by imposing sign and zero restrictions on the systematic component of policy rules as well as impulse response functions. We apply the SVAR methodology of Arias et al. (2019) to a TVP-VAR framework and extend it by imposing restrictions on the fiscal policy equation. The application allows us to observe permanent and gradual time variation in policy coefficients and multipliers in a single framework, enabling us to consider the effects of changes in the conduct of monetary and fiscal policy on the size of the multipliers.

Our results show that the U.S. multipliers declined between the 1970s and the early 1990s. The U.S. public debt grew rapidly over most of this period but was stable for several years after the passage of OBRA93. Accordingly, we find a negative correlation between the debt-to-GDP ratios and the multipliers. The public debt dependency of multipliers is investigated empirically and theoretically in previous studies, but this study differs from them in that we provide the empirical evidence by analysing U.S. time series data. Our findings point to the advantage of the TVP-VAR model in capturing permanent and gradual time variation in multipliers because previous studies find it difficult to isolate the debt-dependent government spending effects by fitting regime-switching models to U.S. data (e.g., Caggiano et al. (2015); Bernardini and Peersman (2018)).

Another contribution of the study is the investigation of the underlying structural changes in the transmission mechanism behind the debt-dependent multipliers. By analysing the evolution of impulse responses to government spending shocks and the coefficients of policy rules, we find that spending-based fiscal adjustments play an important role in determining the size of multipliers while monetary policy has little effect. The multipliers are found to be more strongly correlated with the estimated coefficients of the debt-stabilizing spending rule than the debt-to-GDP ratios. Furthermore, we find that the decline in the output response is mostly led by that in consumption. Households appear to reduce the amount of consumption after a government spending shock, expecting a larger magnitude of fiscal adjustments. Our results suggests that the increased magnitude of fiscal adjustments is the major driving force behind the decline in multipliers rather than debt accumulation itself. This could have major policy implications for fiscal adjustment strategies when fiscal stimulus is necessary.

Nevertheless, there remains much work ahead. Although our atheoretical approach is a flexible way to model the evolution of time series data, it has limitations in explaining the transmission mechanism. Future research could be directed toward developing a theoretical model that accounts for the decline in multipliers and its underlying mechanism reported in this paper. Extending our analysis to investigate the evolution of taxation multipliers would be another interesting avenue. Moreover, while we do not consider the relevance of a sovereign risk channel as the U.S. economy has supposedly not yet reached the fiscal limit, it would be worth exploring the channel as the concerns about the U.S. debt sustainability increase in the future (e.g., Corsetti et al. (2013); Huidrom et al. (2020)).

Appendix: Data Sources

We obtain all quarterly data from the FRED database of the Federal Reserve Bank of St. Louis. The seasonally adjusted series for real government spending, the real gross domestic product, and real private consumption are Real Government Consumption Expenditures and Gross Investment (GCEC1), Real Gross Domestic Product (GDPC1), and Real Personal Consumption Expenditures (PCECC96), respectively. To convert the series into per capita terms, we divide them by the seasonally adjusted Civilian Labor Force (CLF16OV). The ratios of output to government spending used to calculate the multipliers are constructed from seasonally adjusted series for Gross Domestic Product (GDP) and Government Consumption Expenditures and Gross Investment (GCE), respectively. We use the seasonally adjusted GDP Chain-type Price Index (GDPCTPI) as the price and the 3-Month Treasury Bill Secondary Market Rate (TB3MS) as the nominal interest rate. The debt-to-output ratio is calculated by dividing the sum of federal, state, and local government liabilities by the seasonally adjusted Gross Domestic Product (GDP). We use the Liabilities of the Federal Government (FGDSLAQ027S) and those of the State and Local Governments (SLGLIAQ027S) in the calculation.

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Online Appendix to "Fiscal Adjustments and Debt-Dependent Multipliers: Evidence from the U.S. Time Series"

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A Estimation Methods

This section describes the procedures of the econometric method employed in this paper. First, the overall procedure of the Bayesian estimation method with MCMC is presented. Next, the derivation of the structural VAR using the zero-sign constraint method is explained, followed by the Bayesian estimation method for the TVP-VAR model, and finally, the stochastic volatility estimation method is described.

A.1 Bayesian inference and MCMC Algorithm

The MCMC algorithm estimating our model consists of the following nine steps.

1. Initialize parameters: Σ_β, Σ_a, Σ_h, and state variables: a_t, β_t, h_t.

2. Generate the state variables β_t given a_t, h_t, Σ_β, $\mathbf{y_t}$, from the conditional posterior distribution: $f(\beta_t|a_t, h_t, \Sigma_\beta, \mathbf{y_t})$.

3. Generate the parametersΣ_β given β_t, from the conditional posterior distribution: $f(\Sigma_\beta|\beta_t)$.

4. Generate the state variables a_t given β_t, h_t, Σ_a, $\mathbf{y_t}$, from the conditional posterior distribution: $f(a_t|\beta_t, h_t, \Sigma_a, \mathbf{y_t})$.

5. Generate the parametersΣ_a given a_t, from the conditional posterior distribution: $f(\Sigma_\alpha|\alpha_t)$.

6. Generate the state variables h_t given β_t, a_t, Σ_h, $\mathbf{y_t}$, from the conditional posterior distribution: $f(h_t|a_t, \beta_t, \Sigma_\beta, \mathbf{y_t})$.

7. Generate the parameters Σ_h, given h_t, from the conditional posterior distribution: $f(\Sigma_h|h_t)$.

8. Generate the IRFs: $f(A_0, A_+)$, based on the structural parameters: A_0, A_+, identified with zero and sign restrictions, given a_t, β_t, h_t, $\mathbf{y_t}$.

9. Return to step 2 until the required number of draws from the posterior distribution

Here, we remark some points of the above MCMC simulation. In Steps 2 and 4, the simulation smoother of de Jong and Shephard (1995) is used for drawing β_t and a_t . In Step 7, a nonlinear filtering method based on block-sampling method is used for sampling stochastic volatility h_t, following Shephard and Pitt (1997), Watanabe and Omori (2004) and Nakajima et a. (2011). These parts explaining the MCMC procedure generating parameters in reduced-form TVP-VARs are described in the following Section A3 and A4 in more detail. In Step 8, the identification of SVARs and generation of IRFs are implemented from the way described in the following appendix section A2.

The priors of the parameters are specified as:$(\Sigma_\beta)_i^2 \sim IG(20, 10^{-4})$, $(\Sigma_a)_i^2 \sim IG(20, 10^{-4})$, and $(\Sigma_h)_i^2 \sim IG(20, 10^{-4})$, where subscript i denotes the i-th diagonal elements of the covariance matrices and IG an inverse-Gamma distribution. The initial state variables are set as $\beta_0 \sim N(0, 10I)$, $a_0 \sim N(0, 10I)$, and $h_0 \sim N(0, 10I)$.

In the state space model and the impulse response function involved the SVARs, draws generated iteratively from the above conditional posterior distributions of state variables and parameters must tend to convergence to the posterior joint distributions based on the property of Gibbs sampler. We collect 400,000 draws which consists of 50,000 MCMC iterations times 8 chains, after discarding the first 20,000 iterations of each chain to converge to the ergodic distribution, and sampling only draws satisfying the zero and sign restrictions out of these simulations.

A.2 Algorithm of Zero and Sign Restrictions

1. Zero restrictions

We consider how to impose the IRFs from the zero restrictions, using the manner by Arias, Rubio-Ramirez, and Waggoner (2018). Let Z_j denote a matrix in which the number of column is equal to the number of rows in $f(A_0, A_+)$ and j is the j-th structural shock imposing the zero restrictions. Using the orthogonal matrix Q_t, the product of the zero restrictions matrices and the IRF is transformed as below.

$$Z_j\, f(A_0Q,\, A_+Q)\, e_j = Z_j\, f(A_0,\, A_+)\, Q\, e_j = Z_j\, f(A_0,\, A_+)\, q_j,$$

where $q_j = Qe_j$. And, the zero restrictions will hold if and only if

$$Z_j f(A_0, A_+) q_j = 0, \text{ for } 1 \leq j \leq n.$$

where n is number of endogenous variables. From Table A1 for the case of five variables (and Table A2 for the case of six variables), we set up the matrix of zero restrictions of government spending shock, Z_1, as

$$\underbrace{Z_1}_{R_z \times 3n} = \underbrace{\begin{bmatrix} g & \mathbf{y} & dbt & p & \mathbf{int} & \vdots & g & \mathbf{y} & dbt & p & \mathbf{int} & \vdots & g & \mathbf{y} & dbt & p & \mathbf{int} \\ - & - & - & - & - & \vdots & - & - & - & - & - & \vdots & - & - & - & - & - \\ 0 & 0 & 0 & 1 & 0 & \vdots & 0 & 0 & 0 & 0 & 0 & \vdots & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & \vdots & 0 & 0 & 0 & 0 & 0 & \vdots & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & \vdots & 0 & 0 & 0 & 0 & 0 & \vdots & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & \vdots & 0 & 0 & 0 & 0 & 0 & \vdots & 0 & 0 & 0 & 0 & 0 \end{bmatrix}}_{\text{Contemporaneous Block, Short Run, Block Long Run Block}}$$

where elements corresponding to the endogenous variables imposed zero restrictions are set one, otherwise zero. The first n columns block of the zero restriction matrix correspond to contemporaneous matrix A_0 , the second n columns block correspond to the short run restriction; $LR_0(A_0, A_+)$, while the last n columns block of the matrix do to the long run restrictions: $LR_L(A_0, A_+)$. And the number of rows, R_Z, equals the number of the zero restrictions of the corresponding i-th shock shown in Tables A1 and A2. Notice that the the number of the zero restrictions is equal to the number of endogenous variables: n, less the ordinal number i of the i-th structual shock.

2. Sign restrictions

In the similar way to the above zero restrictions, sign restrictions can be implemented using a matrix expression. Let S_j be a matrix in which the number of column is equal to the number of rows in $f(A_0, A_+)$ and j is the j-th structural shock imposed the sign restrictions. Using the orthogonal matrix Q_t, the product of the sign restrictions matrices and the IRF is transformed as below.

$$S_j \, f(A_0 Q, A_+ Q) \, e_j = S_j f(A_0, A_+) Q e_j = S_j f(A_0, A_+) q_j,$$

And then, the sign restrictions will hold if and only if

$$S_j \, f(A_0, A_+) \, q_j > 0, \text{ for } 1 \leq j \leq n.$$

From Table A1 for the case of five variables (and Table A2 for the case of six variables), we set up the matrix of sign restrictions of government spending shock, S_1, as

$$\underbrace{S_1}_{R_S \times 3n} = \underbrace{\left[\begin{array}{ccccc:ccccc:ccccc} g & \mathbf{y} & dbt & p & \mathbf{int} & g & \mathbf{y} & dbt & p & \mathbf{int} & g & \mathbf{y} & dbt & p & \mathbf{int} \\ - & - & - & - & - & - & - & - & - & - & - & - & - & - & - \\ +1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & +1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & +1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \end{array}\right]}_{\text{Contemporaneous Block, Short Run Block, Long Run Block}},$$

where elements corresponding to the endogenous variables imposed the positive and negative sign restrictions are set plus and minus one, respectively, and otherwise zeros. The first n columns block of the sign restriction matrix correspond to contemporaneous matrix A_0 , the second n columns block correspond to the short run restriction; $LR_0(A_0, A_+)$, while the last n columns block of the matrix do to the long run restrictions: $LR_L(A_0, A_+)$. And the number of rows, R_S, indicates the number of the sign restrictions of the corresponding i-th shock shown in Tables A1 and A2.

Table 1: Zeros and Signs Restrictions of SVAR with Five Variables

	variables	shocks			
		gov. spending	demand	supply	monetary policy
systematic component	gov	+			0
	y	?	+		-
	dbt	?			0
	p	0		-	-
	int	0			+
Short Run	gov	+			
	y		+	+	
	dbt	+			
	p		+		-
	int		+		+
Long Run	gov		0	0	0
	y	0	0		0
	dbt				
	p				
	int				
# of zero restrictions		3	2	1	4

Notes:

Table 2: Zeros and Signs Restrictions of SVAR with Six Variables

	variables	shocks				
		gov. spending	demand	supply	monetary policy	consumption
systematic component	gov	+			0	
	y	?	+		-	
	dbt	?			0	
	p	0		-	-	
	int	0			+	
	cons	0			0	
Short Run	gov	+				
	y		+	+		
	dbt	+				
	p		+		-	
	int		+		+	
	cons					
Long Run	gov		0	0	0	
	y	0	0		0	0
	dbt					
	p					
	int					
	cons		0			0
# of zero restrictions		4	3	1	5	2

Notes: see Table A1

3. QR decomposition

Let $X = QR$ be the QR decomposition of a $n \times n$ matrix X. The $n \times n$ random matrix Q has the uniform distribution, i.e., $Q\,Q' = I$. and the $n \times n$ matrix R is a upper triangular matrix.

Let the matrix X be defined as

$$\underbrace{X_j(A_0, A_+)}_{n\times n} = \begin{bmatrix} Z_j f(A_0, A_+) \\ Q'_{j-1} \end{bmatrix}^T,$$

and the orthogonal matrix Q_j given from the QR decomposition of a $n \times n$ matrix $X_j(A_0, A_+)$ satisfies the zero restrictions, or $X_j(A_0, A_+)q_j = 0$ where $q_j = Q_j e_j$. By stacking them such as $Q = [q_1, \cdots, q_n]$, we obtain the rotation matrix Q to identify the SVAR model.

4. Impulse Response Functions (IRFs) with identified with zero and sign restrictions

We consider the derivation of IRFs in a standard VAR with constant structural parameters:A_0, A_+, following Arias, Rubio-Ramirez, and Waggoner (2018). Let $L_h(A_0, A_+)$ denote the IRF of the i-th variable to j-th structural shock at finite horizon h given by a $n \times n$ matrix as below.

$$\underbrace{IR_h(A_0, A_+)}_{n\times n} = (A_0^{-1} J' F^h J)'$$

where $A_+' = [A_1', \cdots, A_p']$,

$$\underbrace{F}_{pn\times pn} = \begin{bmatrix} A_1 A_0^{-1} & I_n & \cdots & 0 \\ \vdots & \vdots & \ddots & \vdots \\ A_{p-1}A_0^{-1} & 0 & \cdots & I_n \\ A_p A_0^{-1} & 0 & \cdots & 0 \end{bmatrix} \text{ and } \underbrace{J}_{pn\times n} = \begin{bmatrix} I_n \\ 0 \\ \vdots \\ 0 \end{bmatrix},$$

where I_n is a $n \times n$ identity matrix. And, we apply them to the IRFs in the TVP-VARs. The IRFs: $L_h(A_0, A_+)$, can be rewritten as

$$\underbrace{IR_h(A_{t,0}, A_{t,+})}_{n\times n} = \left(A_{t,0}^{-1} J' \left(\prod_{i=t}^{t+h} F_i \right) J \right)^T,$$

where $A_{t,+}' = [A_{t,1}', \cdots, A_{t,p}']$,

$$\underbrace{F_t}_{pn\times pn} = \begin{bmatrix} A_{t,1} A_{t,0}^{-1} & I_n & \cdots & 0 \\ \vdots & \vdots & \ddots & \vdots \\ A_{t,p-1}A_{t,0}^{-1} & 0 & \cdots & I_n \\ A_{t,p} A_{t,0}^{-1} & 0 & \cdots & 0 \end{bmatrix}.$$

Notice that the product of time-varying structural parameters: $A_{t,k}A_{t,0}^{-1}$ is equivalent to time-varying reduced-form parameters $B_{t,k}$ for $1 \leq k \leq p$.

Using the orthogonal matrix Q_t, the above IRF, $IR_h(A_0, A_+)$= $IR_h(A_{tr}Q, A_+)$, is transformed to $IR_h(A_{tr}, A_+Q')Q$, for horizons, $0 \leq h \leq \infty$. It indicates that the sets of structural parameters (A_0, A_+) and (A_{tr}, A_+Q') are observationally equivalent so that we can replace A_0 with A_{tr} in the IRF. Accordingly, instead of A_0, the lower triangular matrix A_{tr} derived from Cholesky decomposition is used together with the matrix Q to be convenient to calculate. Let $f(A_0, A_+)$ be combination of contemporaneous matrix A_0 and the stacked IRF at horizon zero and long term: L, given by a $3n \times n$ matrix as below.

$$f(A_0, A_+) = \underbrace{\begin{bmatrix} A_0 \\ IR_0(A_0, A_+) \\ IR_L(A_0, A_+) \end{bmatrix}}_{3n\times n} = \underbrace{\begin{bmatrix} A_{tr}Q \\ IR_0(A_{tr}, A_+Q')Q \\ IR_L(A_{tr}, A_+Q')Q \end{bmatrix}}_{3n\times n}. \tag{1}$$

Using the function $f(A_0, A_+)$, we can identify the SVARs imposed from the zero and sign restrictions of the IRFs to the four structural shocks including monetary and fiscal policy shocks.

5. Algorithm for zero and sign restrictions

Finally, we show algorithm for the two restrictions using the above QR decomposition. The sets of structural parameters are identified based on *Algorithm 4* by Arias et al. (2018) consisting of the following four steps.

1. Draw the sets of reduced-form parameters (B, Ω).
2. Using the QR decomposition mentioned above, draw an orthogonal matrix Q satisfies the zero restrictions, or $Z_j f(A_0, A_+)q_j = 0$, for $1 \leq j \leq n$.
3. Keep the draw if the sign restrictions are satisfied, or $S_j\, f(A_0, A_+)\, q_j > 0$, for $1 \leq j \leq n$, otherwise discard the draw.
4. Return to step 1 until the required number of draws from the posterior distribution conditional on the sign and zero restrictions has been obtained.

Here, we remark as follows. In Step 2 and Step 3, the structural parameters A_0 are observationally equivalent to the lower triangular matrix A_{tr}. So instead of A_0, we use A_{tr} derived from the inverse of Cholesky decomposition of Ω. And A_+ is derived from BA_{tr}.

A.3 MCMC procedure for TVP-VARs

In Section A1, we describe the nine steps of the MCMC algorithm estimating our model. Here, we focus on the steps generating parameters in reduced-form TVP-VARs. This section is described based on Appendix of Nakajima (2011) and Nakajima et al. (2011).

Step 1. Generate the state variables β_t given a_t, h_t, Σ_β, Y_t, from the conditional posterior distribution: $f(\beta_t|a_t, h_t, \Sigma_\beta, Y_t)$. To generate β_t from the conditional posterior distribution: $f(\beta_t|a_t, h_t, \Sigma_\beta, Y_t)$, we introduce the simulation smoother by de Jong and Shephard (1995) and Durbin and Koopman (2002) using the state space model with respect to β_t given by

$$y_t = X_t\beta_t + A_t^{-1}\Sigma_t\varepsilon_t,\ t = s+1, \cdots, n, \tag{2}$$

$$\beta_{t+1} = \beta_t + u_\beta,\ t = s+1, \cdots, n-1,$$

where β_s is set as μ_{β_0}, and $u_{\beta_s} \sim N(0, \Sigma_{\beta_0})$.

Step 2. Generate the state variables a_t given β_t, h_t, Σ_a, Y_t , from the conditional posterior distribution: $f(a_t|\beta_t, h_t, \Sigma_a, Y_t)$. To generate a_t from the conditional posterior distribution: $f(a_t|\beta_t, h_t, \Sigma_a, Y_t)$, the simulation smoother is also adopted from the following state space model,

$$\hat{y}_t = \hat{X}_t a_t + \Sigma_t\varepsilon_t,\ t = s+1, \cdots, n,$$

$$a_{t+1} = a_t + u_{at},\ t = s, \cdots, n-1,$$

where $a_s = \mu_{a0}$, $u_{as} \sim N(0, \Sigma_{a0})$, $\hat{y}_t = y_t - X_t\beta_t$, and

$$\hat{X}_t = \begin{bmatrix} 0 & \cdots & & & & & & 0 \\ -\hat{y}_{1t} & 0 & 0 & \cdots & & & & \vdots \\ 0 & -\hat{y}_{1t} & -\hat{y}_{2t} & 0 & \cdots & & & \\ 0 & 0 & 0 & -\hat{y}_{1t} & \cdots & & & \\ \vdots & & & & \ddots & 0 & \cdots & 0 \\ 0 & \cdots & & & 0 & -\hat{y}_{1t} & \cdots & -\hat{y}_{k-1t} \end{bmatrix},$$

for $t = s+1, \cdots, n$.

Step 3. Generate the state variables h_t given β_t, a_t, Σ_h, Y_t, from the conditional posterior distribution: $f(h_t|a_t, \beta_t, \Sigma_\beta, Y_t)$. To generate the stochastic volatility h_t from the conditional posterior distribution: $f(h_t|a_t, \beta_t, \Sigma_\beta, Y_t)$, we conduct the inference for $h_{jt}{}_{t=s+1}^{n}$ separately for j, because it is assumed that Σ_h and Σ_{h0} are diagonal matrices. Let y_{it}^* denote the i-th element of $A_t y_t$. Then, we can write:

$$y_{it}^* = exp(h_{it}/2)\varepsilon_{it},\ t = s+1, \cdots, n,$$

$$h_{i,t+1} = h_{it} + \eta_{it},\ t = s, \cdots, n-1,$$

$$\begin{pmatrix} \varepsilon_{it} \\ \eta_{it} \end{pmatrix} \sim N\left(0, \begin{pmatrix} 1 & 0 \\ 0 & \nu_i^2 \end{pmatrix}\right),$$

where $\eta_{is} \sim N(0, \nu_{i0}^2)$, and ν_i^2 are the i-th diagonal elements of Σ_h and Σ_{h0}, respectively, and η_{it} is the i-th element of u_{ht}. We sample $h_t = (h_{i,s+1}, \cdots, h_{in})$ using the multi-move sampler developed by Shephard and Pitt (1997) and Watanabe and Omori (2004), the algorithm of which is described in the following subsection.

Step 4. Generate the parameters Σ_α , Σ_β, andΣ_h. To generate the parameter Σ_a given a_t, we draw the sample from the conditional posterior distribution: $\Sigma|a_t \sim IW(\hat{\nu}, \hat{\Omega}^{-1})$, where IW denotes the inverse-Wishart distribution, and $\hat{\nu} = \nu_0 + n - 1$, $\hat{\Omega} = \Omega_0 + \sum_{t=1}^{n-1}(a_{t+1} - a_t)(a_{t+1} - a_t)'$ in which the prior is set as $\Sigma \sim IW(\nu_0, \Omega_0^{-1})$. Sampling the diagonal elememts of Σ_β, Σ_h is also the same way to sample Σ_a.

A.4 Multi-Move Sampler of Stochastic Volatilities

This section is described based on Appendix of Nakajima (2011) and Nakajima et al. (2011). The algoritm of the multi-move sampler proposed by Shephard and Pitt (1997), Watanabe and Omori (2004) is adopted to generate draws of stochastic volatilities in the TVP-VARs from the conditional posterior distributions explainded in Appendix A2. We show the stochastic volatilities model again.

$$y_t^* = exp(h_t/2)\varepsilon_t,\ t = s+1, \cdots, n,$$

$$h_{t+1} = \phi h_t + \eta_t,\ t = s, \cdots, n-1,$$

$$\begin{pmatrix} \varepsilon_t \\ \eta_t \end{pmatrix} \sim N\left(0, \begin{pmatrix} 1 & 0 \\ 0 & \sigma_\eta^2 \end{pmatrix}\right),$$

where y_t^* denote the i-th element of $A_t y_t$ shown in Eq.(**??**). For drawing a typical block such as$(h_r, \cdots, h_{r+d})$, we consider the draw of

$$\begin{aligned}(\eta_{r-1}, \cdots, \eta_{r+d-1}) &\sim \pi(\eta_{r-1}, \cdots, \eta_{r+d-1}|\omega) \\ &\propto \prod \frac{1}{e^{h_t/2}} \exp\left(\frac{y_t^{*2}}{2e^{h_t}}\right) \times \prod f(\eta_t) \times f(h_{r+d}) \qquad (3)\end{aligned}$$

where

$$f(\eta_t) = \begin{cases} \exp\left\{-\frac{(1-\phi^2)\eta_0^2}{2\sigma_\eta^2}\right\} & (\text{if } t = 0), \\ \exp\left(-\frac{\eta_t^2}{2\sigma_\eta^2}\right) & (\text{if } t \geq 1), \end{cases}$$

$$f(h_{r+d}) = \begin{cases} \exp\left\{-\frac{(h_{r+d+1}-\phi h_{r+d})^2}{2\sigma_\eta^2}\right\} & (\text{if } r+d < n), \\ 1 & (\text{if } r+d = n), \end{cases}$$

and $\omega = (h_{r-1}, h_{r+d+1}, \beta, \gamma, \phi,)$. The posterior draw of $(h_r, \cdots, h_{r+d})$ can be obtained by running the state equation with the draw of $(\eta_{r-1}, \cdots, \eta_{r+d-1})$ given h_{r-1}.

We sample $(\eta_{r-1}, \cdots, \eta_{r+d-1})$ from the density (3) using the acceptance-rejection MH algorithm (Tierney, 1994; Chib and Greenberg, 1995) with the following proposal distribution constructed from the second-order Taylor expansion of

$$g(h_t) \equiv -\frac{h_t}{2} - \frac{y_t^{*2}}{2e^{h_t}},$$

around a certain point $\hat{h_t}$ which is given by

$$\begin{aligned} g(h_t) &\doteqdot g(h_t) + g'(\hat{h}_t)(h_t - \hat{h}_t) + \frac{1}{2}g''(\hat{h}_t)(h_t - \hat{h}_t)^2 \\ &\propto \frac{1}{2}g''(\hat{h}_t)\left\{h_t - \left(\hat{h}_t - \frac{g'(\hat{h}_t)}{g''(\hat{h}_t)}\right)\right\}^2, \end{aligned}$$

Here, the first and second derivatives are obtained such that

$$g'(\hat{h}_t) = -\frac{1}{2} + \frac{y_t^{*2}}{2e^{h_t}},\ g''(\hat{h}_t) = -\frac{y_t^{*2}}{2e^{h_t}},$$

And the proposal density of $\pi(\eta_{r-1}, \cdots, \eta_{r+d-1}|\omega)$ is given by

$$q(\eta_{r-1}, \cdots, \eta_{r+d-1}|\omega) \propto \prod \exp\left\{-\frac{(h_t^* - h_t)^2}{2\sigma_t^{*2}}\right\} \times \prod f(\eta_t),$$

where

$$\sigma_t^{*2} = -\frac{1}{g''(\hat{h}_t)},\ h_t^* = h_t + \sigma_t^{*2} g'(\hat{h}_t), \tag{4}$$

for $t = r, \cdots, r+d-1$, and $t = r+d$ in the case that $r+d = n$. Meanwhile, in the case that$r+d \leq n$,

$$\sigma^{*2}_{r+d} = \frac{1}{-g''(\hat{h}_{t+d}) + \phi^2/\sigma^2_\eta} \tag{5}$$

$$h^*_{r+d} = \sigma^{*2}_{r+d} \left\{ g'(h_{r+d}) - g''(h_{r+d})h_{r+d} + h_{r+d}/\sigma^2_\eta \right\}, \tag{6}$$

for $t = r+d$. The proposal density of the AR-MH algorithm is derived from the following state space model,

$$h^*_t = h_t + \varsigma_t,\ t = s+1, \cdots, n,$$

$$h_{,t+1} = h_t + \eta_t,\ t = s, \cdots, n-1, \tag{7}$$

$$\begin{pmatrix} \varsigma_t \\ \eta_t \end{pmatrix} \sim N\left(0, \begin{pmatrix} \sigma^{*2}_t & 0 \\ 0 & \sigma^2_\eta \end{pmatrix}\right),$$

with $\eta_{r-1} \sim N(0, \sigma^2_\eta)$ when $r \geq 2$ and $\eta_s \sim N(0, \sigma^2_\eta/(1-\phi^2))$. Given ω, we draw candidate point of $(\eta_{r-1}, \cdots, \eta_{r+d-1})$ for AR-MH algorithm by running the simulation smoother over the state-space representation (7).

For realizing efficient drawings, we need to calculate the mode of the above posterior density for $(\hat{h}_r, \cdots, \hat{h}_{r+d})$. Numerically, we obtain the mode by iterating the following steps several times,

1. Initialize $(\hat{h}_r, \cdots, \hat{h}_{r+d})$.

2. Compute $(h^*_r, \cdots, h^*_{r+d})$, and $(\sigma^*_r, \cdots, \sigma^*_{r+d})$ by eq.(4) through eq.(6).

3. Run the simulation smoother for state space model eq.(7) with $(h^*_r, \cdots, h^*_{r+d})$, and $(\sigma^*_r, \cdots, \sigma^*_{r+d})$ as obervable variables. And Generate estimations $h^*_t = E(h_t|\omega)$ for $t = r, \cdots, r+d$.

4. Replace $(\hat{h}_r, \cdots, \hat{h}_{r+d})$ with $(h^*_r, \cdots, h^*_{r+d})$.

5. Return to Step 2.

To implement a block sampling for h_t, they are devided into $K+1$ blocks, say, $(h_{k(i-1)}, \cdots, h_{k(i)})$ for $i = 1, \cdots, K+1$. Shephard and Pitt (1997) suggested to adopt stochastic knots for determining the positions of blocks: i, the rule of which is given by

$$k(i) = int\left[\frac{n(j+U_i)}{K+2}\right],$$

for $i = 1, \cdots, K$, where int is a function rounding to an integer value from the insight, and U_i is the random sample from the uniform distribution $U[0,1]$.

B Estimation Results and Convergence Diagnostic

Here we report posterior estimates of the parameters of the reduced forms of our two TVP-VAR models and the results of the convergence diagnostics for the MCMC estimation results. The parameters of the structural model presented in this paper are derived from the posterior estimates of the parameters of the reduce forms. As will be shown below, the only fixed parameters in our model are *hyper-parameters*, i.e., the standard deviations portion of the random walk process that accounts for the time-varying parameters.

Tables B1 and B2 show the posterior estimation results for the five- and six-variable models, respectively. The values of the parameters, all of which are standard deviations, do not differ significantly in the magnitude of the posterior estimates because the set values of the prior distributions are much larger than the likelihood information provided by the data. Again, however, since the values of the hyper-parameters have only a secondary effect, we are confident that stable estimation results are obtained by keeping these parameters within a certain range rather than arbitrarily relaxing these values to larger values in the sense of comparing our study with a SVAR model with the usual fixed coefficients

Table 3: Posterior Estimate of hyper-parameter in the TVP-VAR with Five Variables

parameters	posterior mean	standard deviation	convergence diagnostic
$(\Sigma_\beta)_1$	$0.21788 * 10^{-2}$	$0.11870 * 10^{-3}$	1.000
$(\Sigma_\beta)_2$	$0.21794 * 10^{-2}$	$0.11900 * 10^{-3}$	1.000
$(\Sigma_\beta)_3$	$0.21790 * 10^{-2}$	$0.11911 * 10^{-3}$	1.000
$(\Sigma_\beta)_4$	$0.21792 * 10^{-2}$	$0.11888 * 10^{-3}$	1.000
$(\Sigma_\beta)_5$	$0.21793 * 10^{-2}$	$0.11909 * 10^{-3}$	1.000
$(\Sigma_a)_1$	$0.16978 * 10^{-2}$	$0.7211 * 10^{-4}$	1.000
$(\Sigma_a)_2$	$0.16980 * 10^{-2}$	$0.7209 * 10^{-4}$	1.000
$(\Sigma_a)_3$	$0.16979 * 10^{-2}$	$0.7215 * 10^{-4}$	1.000
$(\Sigma_a)_4$	$0.16980 * 10^{-2}$	$0.7236 * 10^{-4}$	1.000
$(\Sigma_a)_5$	$0.16978 * 10^{-2}$	$0.7218 * 10^{-4}$	1.000
$(\Sigma_h)_1$	$8.4900 * 10^{-2}$	$3.6110 * 10^{-3}$	1.000
$(\Sigma_h)_2$	$8.4895 * 10^{-2}$	$3.6154 * 10^{-3}$	1.000
$(\Sigma_h)_3$	$8.4896 * 10^{-2}$	$3.6054 * 10^{-3}$	1.000
$(\Sigma_h)_4$	$8.4885 * 10^{-2}$	$3.6053 * 10^{-3}$	1.000
$(\Sigma_h)_5$	$8.4889 * 10^{-2}$	$3.6127 * 10^{-3}$	1.000

Table 4: Posterior Estimate of hyper-parameters in the TVP-VAR with Six Variables

parameters	posterior mean	standard deviation	convergence diagnostic
$(\Sigma_\beta)_1$	$0.21788 * 10^{-2}$	$0.11870 * 10^{-3}$	1.000
$(\Sigma_\beta)_2$	$0.21794 * 10^{-2}$	$0.11900 * 10^{-3}$	1.000
$(\Sigma_\beta)_3$	$0.21790 * 10^{-2}$	$0.11911 * 10^{-3}$	1.000
$(\Sigma_\beta)_4$	$0.21792 * 10^{-2}$	$0.11888 * 10^{-3}$	1.000
$(\Sigma_\beta)_5$	$0.21793 * 10^{-2}$	$0.11909 * 10^{-3}$	1.000
$(\Sigma_\beta)_5$	$0.21793 * 10^{-2}$	$0.11909 * 10^{-3}$	1.000
$(\Sigma_a)_1$	$0.16978 * 10^{-2}$	$0.7211 * 10^{-4}$	1.000
$(\Sigma_a)_2$	$0.16980 * 10^{-2}$	$0.7209 * 10^{-4}$	1.000
$(\Sigma_a)_3$	$0.16979 * 10^{-2}$	$0.7215 * 10^{-4}$	1.000
$(\Sigma_a)_4$	$0.16980 * 10^{-2}$	$0.7236 * 10^{-4}$	1.000
$(\Sigma_a)_5$	$0.16978 * 10^{-2}$	$0.7218 * 10^{-4}$	1.000
$(\Sigma_a)_5$	$0.16978 * 10^{-2}$	$0.7218 * 10^{-4}$	1.000
$(\Sigma_h)_1$	$8.4900 * 10^{-2}$	$3.6110 * 10^{-3}$	1.000
$(\Sigma_h)_2$	$8.4895 * 10^{-2}$	$3.6154 * 10^{-3}$	1.000
$(\Sigma_h)_3$	$8.4896 * 10^{-2}$	$3.6054 * 10^{-3}$	1.000
$(\Sigma_h)_4$	$8.4885 * 10^{-2}$	$3.6053 * 10^{-3}$	1.000
$(\Sigma_h)_5$	$8.4889 * 10^{-2}$	$3.6127 * 10^{-3}$	1.000
$(\Sigma_h)_5$	$8.4889 * 10^{-2}$	$3.6127 * 10^{-3}$	1.000

Our estimation uses eight independent chains as sampling for the posterior distribution of the parameters. Thus, as a convergence diagnostic, it seems more efficient to employ the diagnostic proposed by Gelman and Rubin (1992). The Gelman–Rubin convergence diagnostic provides a numerical convergence summary based on multiple chains.[1] The convergence diagnostic itself is $R = \sqrt{\frac{(d+3)\widehat{V}}{(d+1)W}}$ Values substantially above 1 indicate lack of convergence.

[1]Convergence is diagnosed when the chains have 'forgotten' their initial values, and the output from all chains is indistinguishable. There are two ways to estimate the variance of the stationary distribution: the mean of the empirical variance within each chain,W, and the empirical variance from all chains combined, which can be expressed as $\widehat{\sigma}^2 = \frac{(n-1)W}{n} + \frac{B}{n}$ where n is the number of iterations and B/n is the empirical between-chain variance. The convergence diagnostic is based on the assumption that the target distribution is normal. A Bayesian credible interval can be constructed using a t-distribution with mean $\widehat{\mu}$ = Sample mean of all chains combined and variance $\widehat{V} = \widehat{\sigma}^2 + \frac{B}{mn}$ and degrees of freedom estimated by the method of moments $d = \frac{2*\widehat{V}^2}{\mathrm{Var}(\widehat{V})}$ Use of the t-distribution accounts for the fact that the mean and variance of the posterior distribution are estimated.

C Robustness Check for Detrend

Figure 1: Data after Detrend

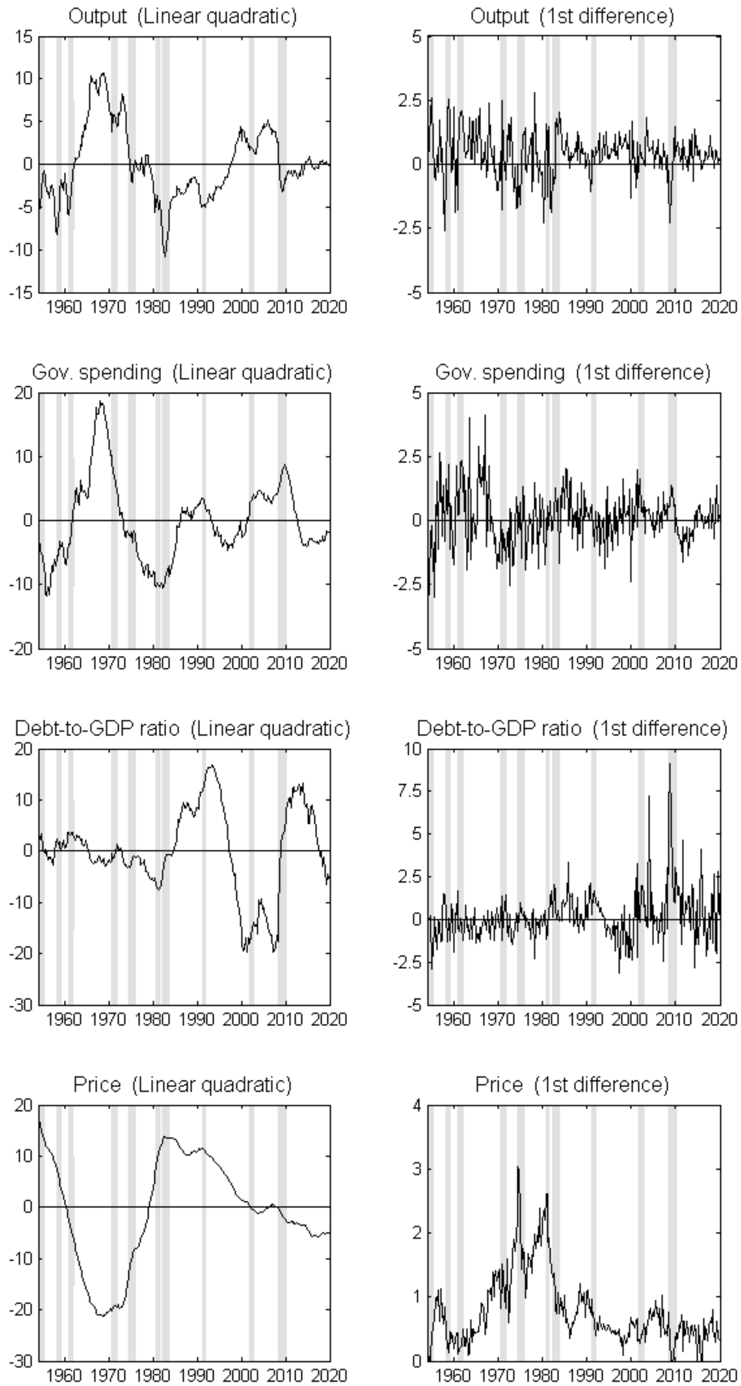
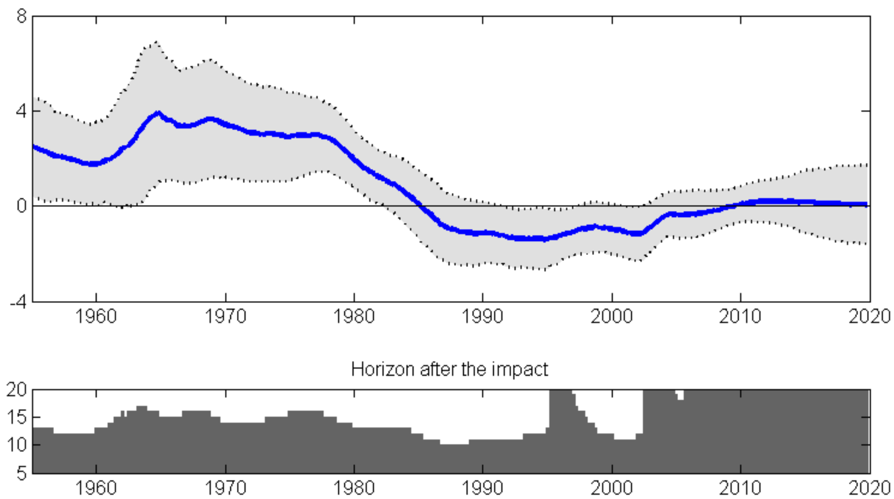


Figure 2: Peak Mulipliers with Detrend of Linear and Quadratic



D Comparison with Case of Smooth Transition VAR model

Our study computes fiscal multipliers by using a VAR model with time-varying parameters to identify fiscal policy shocks by imposing zero and sign restrictions. A model belonging to a similar strand is a structured VAR model with switch coefficients caused by changes in state variables such as government debt. In this section, we discuss the fiscal multipliers estimated by the same strand of models.

In this section, we employs alternative threshold and smooth transition VAR models. Background on these models is provided by Hubrich and Terasvirta (2013) in their survey paper on the threshold and smooth transion VAR model for macroeconomic analysis. Following Threshold VAR models by Alessandri and Mumtaz (2017, 2019), we apply their model to a smooth transition VAR model as

$$Y_t = \left[c_1 + \sum_{j=1}^{P} B_{1,j} Y_{t-j} + \Omega_1^{1/2} e_t \right] S_t + \left[c_2 + \sum_{j=1}^{P} B_{2,j} Y_{t-j} + \Omega_2^{1/2} e_t \right] (1 - S_t)$$

where $S_t = 1$(or 0) denote the period t reime is in Regime 1 (or Regime 2), and we set $S_t = G(\gamma, c; s_t)$ which is a logistic function of s_t given as

$$G(\gamma, c; s_t) = (1 + exp\{-\gamma(s_t - c)\})^{-1}, \quad \gamma > 0.$$

Here we examine two models, each with a Debt regime or an FA regime. Table 5 reports the results of the fiscal multipliers estimated in these models

Table 5: Multipliers in the TwoSTVAR Models

(a) Smooth Transition VAR model shifted by Debt Regime

	Regime 1	Regime 2
Posterior Mean	2.25	1.71
90% Credible Bands	[-0.58, 4.42]	[-0.86, 4.13]

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(b) Smooth Transition VAR model shifted by FA Regime

	Regime 1	Regime 2
Posterior Mean	2.95	1.64
90% Credible Bands	[0.58, 4.83]	[-0.73, 4.17]

Figure 3: Probabilities of Regimes in the Two STVAR Models

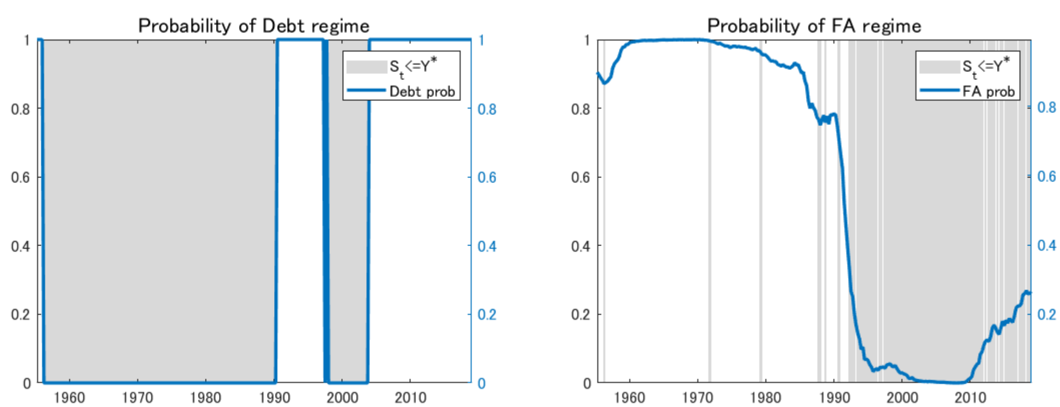
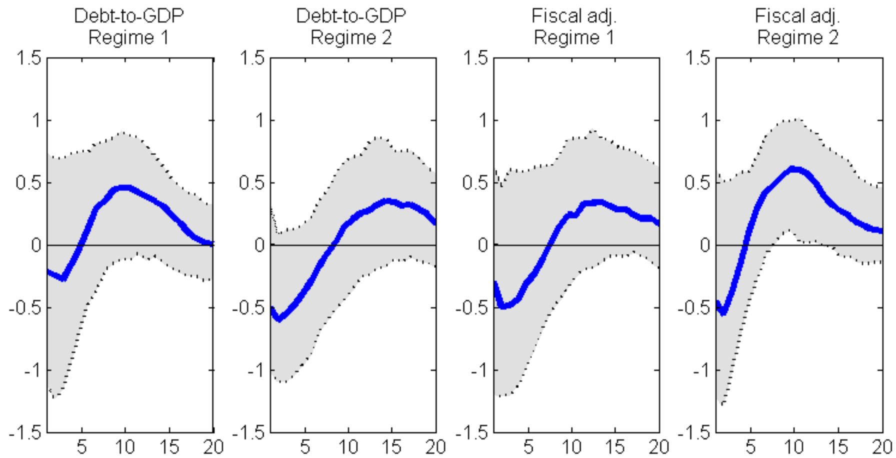


Figure 4: IRF with Mulipliers with Detrend of Linear and Quadratic



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