An Application of Smooth Transition Regression Models to Homeless Research

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Abstract

This article discusses the limitations of linear models in explaining certain aspects of homelessness-related data and proposes the use of nonlinear models to allow for state-dependent or regime-switching behavior. The threshold autoregressive (TAR) model and its smooth transition autoregressive (STAR) extensions are introduced as a popular class of nonlinear models. The article explains how these models can be applied to univariate time series data to investigate variations in weather conditions on the flow of homeless shelters over time. The objective is to identify the sensitivity of publicly-funded emergency shelter use to changes in weather conditions and better inform social agencies and government funders of predictable and unpredictable changes in demand for shelter beds. The smooth transition regression (STR) model is proposed as a useful tool for investigating nonlinearities in non-autoregressive contexts using both time series and panel data. The article concludes by highlighting the advantages of STR models and their three-stage modeling procedure: model specification, estimation, and evaluation.

Keywords: Homelessness; nonlinear models; smooth transition regression (STR) model.

JEL classification: C01; C53; I32.

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1. Introduction

Most of the statistical models employed in the homelessness literature are linear models. These models investigate the impact of different factors on homelessness-related variables. For example, Jadidzadeh and Kneebone (2015) identify the impact of weather conditions on emergency shelters using Ordinary Least Square (OLS) regressions with time series data. Although linear regressions remain at the forefront of academic and applied research, it has been found that simple linear models do not explain certain aspects of the data with structural and behavioral changes. Therefore, it is reasonable to employ nonlinear models to explain the empirical data in different states of the world or regimes and to allow the dynamics to be different in different regimes.

There is a wide range of nonlinear models that are usually applied to economic and financial time series data to allow for *state-dependent* or *regime-switching* behaviour. Examples are bilinear models, *k* nearest neighbor methods, neural networks, and regime-switching models – see books written by Tong (1990), Granger and Teräsvirta (1993), and Franses and van Dijk (2000) for a detailed description of nonlinear models. Given the existence of nonlinearity in our variable of interest, y_t, theories or even stylized empirical facts may suggest a specific form of nonlinearity. A basic, but popular, class of regime-switching models is the *threshold autoregressive (TAR)* model which is proposed by Tong (1978) and discussed in detail by Tong (1990). TAR models are autoregressive models of order *p*, AR(P), which allow for the model parameters to change according to the value of *threshold variables*, where there is a structural break at a time point. The threshold variables in TAR models are specific lags of the dependent variable, y_{t-d}, which allows for an *abrupt* or *discontinuous* regime switch when they cross a certain value. TAR models have the flexibility of describing processes that can move from one regime to the other such that the transition is *smooth.* Therefore, if the discontinuity of the thresholds is replaced by a smooth transition function, TAR models can be generalized to *smooth transition autoregressive (STAR)* models. Two main classes of STAR models are logistic STAR (LSTAR) and exponential STAR (ESTAR) models.

This family of nonlinear models is developed for univariate time series data, and I do not think they have considerable implications for homelessness research. However, the next generation of threshold models was developed to address the nonlinearities in non-autoregressive contexts using both time series and even panel data. *Smooth transition regression (STR)* models use exogenous transition variables, in addition to the standard autoregressive lags of the dependent variable, as threshold variables in modeling the regime switching behavior of the dependent variable. For example, Khan and Senhadji (2001) and Espinoza et al. (2012) investigate whether there is a nonlinear relationship between inflation and long-run growth first identified by Fischer (1993). Using panel data from many countries for different years, they show that there is a trade off between lowering inflation and achieving higher growth. At some low levels, inflation may be positively correlated with growth, but at higher levels, inflation is likely to be harmful to growth. In a different study, Fahmy (2014) investigates the nonlinearities in commodity prices using time series data. He shows that two exogenous transition variables are successful in capturing the regime switching behavior of commodity prices: inflation rate and oil price.

STR models have been used extensively in the regime switching literature - see, van Dijk et al. (2002) for a recent survey. In addition to their popularity, STR models possess some appealing features. They are based on a three-stage modeling procedure beginning with model specification, estimation, and ending with diagnostic tests for evaluation. In this report, I propose an application of STR models to show how to investigate variations in weather conditions on the flow of homeless shelters over time. My objective is to propose a method to identify the sensitivity of publicly-funded emergency shelter use to changes in weather conditions. This sensitivity arises due to the influence of weather conditions on so-called *rough sleepers*, people who are experiencing homelessness and normally prefer, for many reasons, to sleep outdoors. The hope is that by identifying the sensitivity of shelter use to weather conditions one can better inform social agencies and government funders of the predictable and unpredictable changes in the demand for shelter beds. To do so, I follow Teräsvirta (1998), van Dijk et al. (2002) and Fahmy (2014), in specifying, estimating, and evaluating nonlinear time series models.

In the next two sections, I provide a detailed description of the smooth transition regression (STR) models and the corresponding modeling strategy (i.e. specification, estimation and evaluation) and discuss the application of STR models on sensitivity of emergency homeless shelter use to weather conditions. The final section concludes the paper.

2. Smooth transition regression (STR)

Consider the following univariate smooth transition model to investigate the nonlinear relationship between overnight stays and weather conditions:

$$y_t = \alpha + \boldsymbol{\phi}'\boldsymbol{z}_t + \boldsymbol{\psi}'\mathbf{z}_t G(\omega_t; \gamma, \mathbf{c}) + \boldsymbol{\theta}'\mathbf{X}_t + \varepsilon_t \tag{1}$$

where y_t is the dependent variable (in this case, number of overnight stays in emergency shelters), α represents intercept, $\mathbf{z}_t = (z_{1t}, \cdots, z_{mt})$ is a vector of weather variables such as temperature, precipitation, wind and windchill which are potential candidates to be a transitions variable (ω_t), $\boldsymbol{\phi} = (\phi_1, \cdots, \phi_m)'$and $\boldsymbol{\psi} = (\psi_1, \cdots, \psi_m)'$ are parameter vectors, $\boldsymbol{X}_t$ is a vector of $(h \times 1)$ control variables with $\boldsymbol{\theta} = (\theta_1, \cdots, \theta_h)'$ parameter vectors, and $\varepsilon_t \sim \mathrm{i.i.d.}(0, \sigma^2)$. The preferred transition variable, ω_t, to be identified by the estimation strategy is meant to explain the transition from rough sleeping to using a bed in an emergency shelter (and vice versa).[1]

$G(\omega_t; \gamma, \mathbf{c})$ is a logistic function with the general form

$$G(\omega_t; \gamma, c) = \frac{1}{1 + \exp(-\gamma \prod_{k=1}^{K} (\omega_t - c_k))}, \gamma > 0 \tag{2}$$

where ω_t is a transition variable (in this case one of the weather variables), $c = (c_1, \cdots, c_K)'$ is a vector of threshold parameters such that $c_1 \leq c_2 \leq \cdots \leq c_K$, and $\gamma > 0$ is the slope of the transition function (2) or the speed of transition from one regime to another. The STR model in (1) with (2) define the logistic STR (LSTR) model.[2] The transition function (2) is a bounded

[1] Model (1) can also be presented for panel data with *i* and *t* dimensions as follows:

$$y_{it} = \alpha + \boldsymbol{\phi}'\boldsymbol{z}_{it} + \boldsymbol{\psi}'\mathbf{z}_{it} G(\omega_{it}; \gamma, \mathbf{c}) + \boldsymbol{\theta}'\mathbf{X}_{it} + \varepsilon_{it}$$

[2] Another alternative for transition function (2) is an exponential function with the general form:

$$G(\omega_t; \gamma, c) = 1 - \exp\left(-\gamma \prod_{k=1}^{K} (\omega_t - c_k)\right), \gamma > 0$$

This specification of transition function produces *Exponential Smooth Transition Regression (ESTR)* model.

(between zero and one), continuous and monotonically increasing function of transition variable, ω_t. The most common choices for K are $\mathrm{K} = 1$ (LSTR(1)) and $\mathrm{K} = 2$ (LSTR(2)). In a LSTR model with $\mathrm{K} = 1$ there is one threshold (two regimes) such that parameter vector $\boldsymbol{\phi} + \boldsymbol{\psi}\mathrm{G}(\omega_t; \gamma, \mathbf{c})$ changes monotonically from $\boldsymbol{\phi}$ to $\boldsymbol{\phi} + \boldsymbol{\psi}$ as a function of ω_t. In this case, LSTR(1) is capable of characterizing asymmetric behavior of overnight stays, y_t, to a weather variable such as windchill. As an example, the sensitivity of overnight stays to temperature is different during extreme cold days (lower regime) than during warm days (upper regime) such that the transition from one extreme regime to the other is smooth -- For more detail refer to Teräsvirta et al. (2010).

The transition function (2) in an LSTR(1) model has the form

$$G(\mathrm{w}_t; \gamma, \mathrm{c}) = \frac{1}{1 + \exp\big(-\gamma(\omega_t - \mathrm{c})\big)} \tag{3}$$

It is plotted in Figure 1 with different values of γ. The function implies that

- when the transition variable, ω_t, is very low (i.e. $\omega_t \to -\infty$), then $\mathrm{G}(\cdot) \to 0$; this defines the *lower* regime in which the effect of $\mathbf{z}_t$ on y_t is represented by $\boldsymbol{\phi}$.
- when ω_t is very high (i.e. $\omega_t \to +\infty$), then $\mathrm{G}(\cdot) \to 1$; this defines the *upper* regime in which the effect of $\mathbf{z_t}$ on y_t is represented by $\boldsymbol{\phi} + \boldsymbol{\psi}$.
- The parameter γ captures the speed of transition from one regime to another. For example the effect of windchill on stays may not be strongly negative when γ is low. On the other hand, the transition function becomes steeper when γ is larger, which means the faster the speed of transition is (see Figure 1).
- when $\gamma = 0$, the LSTR(1) turns to a linear model.
- when $\gamma \to +\infty$, the LSTR(1) turns to a pure threshold model (TR/TAR); see the blue dotted line in Figure 1. The policy implication of a high γ for a transition variable such as windchill would be that after the threshold, shelters have to be ready for, and capable of dealing with, a large inflow of rough sleepers.

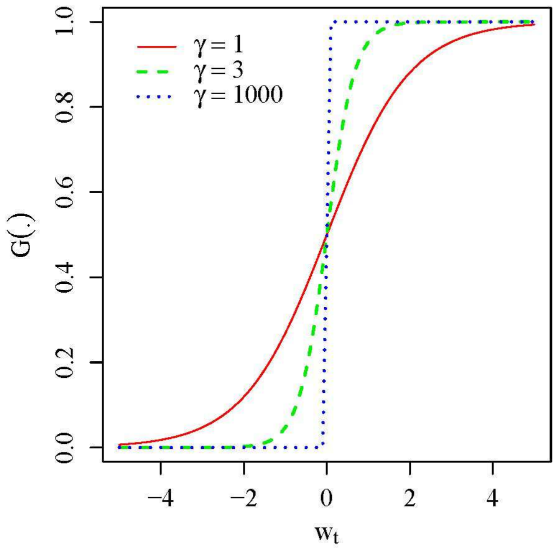

Figure 1. Logistic transition function represented in (3) with c = 0

3. Building the Smooth Transition Regression Model

The application of the STR model represented in (1) with (2) requires a modeling strategy. The STR model building consist of three stages: specification, estimation and evaluation. The specification stage includes testing null hypothesis of linearity, selecting the transition variable (ω_t), and determining the number of thresholds (K). We employ nonlinear least square to estimate parameters. Following González et al. (2005), at the evaluation stage the estimated model is subjected to misspecification tests. The null hypotheses tested at this stage include parameter constancy, no remaining heterogeneity and no autocorrelation in the errors.

3.1. Specification

The sensitivity of the transition from rough sleeping to shelter use due to weather conditions is unknown. Economics and theories of social welfare offer little guidance beyond the obvious suggestion that when it comes to the choice of rough sleeping and shelter use, cold and wet conditions compete against the preference for privacy and concerns for safety. Our strategy is to test alternative specifications of the transition variable. In the initial specification, linearity is tested against an STR model with a predetermined transition variable. The test is repeated for each potential transition variable. The purpose of these tests is twofold. First, they are used to test linearity against alternatives. If no rejections of the null hypothesis are obtained, we accept the linear model and do not proceed with STR models. Second, if the null hypothesis is rejected for at least one of the models, the model against which the rejection, measured in the *p*-value, is strongest is chosen to be the STR model to be estimated. If there are several small *p*-values close to each other, Teräsvirta et al. (2010) suggest that a reasonable way to proceed is to estimate the corresponding STR models and postpone the choice between them to the evaluation stage.

Testing Linearity

The LSTR model (1) with (2) can be reduced to a linear model by imposing $\mathrm{H0}: \gamma = 0$. The classical tests, such as *F*-test, are nonstandard because under the null hypothesis the LSTR model contains unidentified parameters. In particular, the threshold parameters $(\mathbf{c})$ and vector of parameters $\boldsymbol{\psi}$ are not identified under the null hypotheses. The problem of testing linearity against STR has been addressed, for example, in Davies (1987), Luukkonen et al. (1988), Teräsvirta (1994) and Teräsvirta (1998). To circumvent the identification problem, the transition function (2) in Equation (1) is approximated by a Taylor expansion around the null hypothesis, $\mathrm{H0}: \gamma = 0$. As suggested by Teräsvirta (1998), it is customary to assume $\mathrm{K} = 1$ in (2) and use the third-order Taylor approximation. The resulting test has power both against the LSTR(1), i.e. $\mathrm{K} = 1$, and LSTR(2), i.e. $\mathrm{K} = 2$, model. After merging terms and reparametrizing, this approximation yields the following auxiliary regression

$$\mathrm{y_t} = \alpha_0 + \boldsymbol{\beta}_0' \, \mathbf{z}_\mathrm{t} + \boldsymbol{\beta}_1' \, \mathbf{z}_\mathrm{t}\omega_\mathrm{t} + \boldsymbol{\beta}_2' \, \mathbf{z}_\mathrm{t}\omega_\mathrm{t}^2 + \boldsymbol{\beta}_3'\mathbf{z}_\mathrm{t}\omega_\mathrm{t}^3 + \boldsymbol{\theta}_0'\mathbf{X}_\mathrm{t} + \varepsilon_\mathrm{t}^* \tag{4}$$

Where $\varepsilon_t^* = \varepsilon_t + \mathrm{R}_3(\omega_t; \gamma, \mathbf{c})\boldsymbol{\psi}_\mathrm{j}'\mathbf{z}_\mathrm{t}$ with the remainder of the Taylor expansion, $R_3(\omega_t; \gamma, \mathbf{c})$, and $\boldsymbol{\beta}_\mathrm{j}'$, $\mathrm{j} = 1,2,3$, is of the form $\gamma\tilde{\beta}_j$ where $\tilde{\beta}_j \neq 0$ is a function of $\boldsymbol{\psi}$ and $\boldsymbol{c}$. Consequently, testing $\mathrm{H0}: \gamma = 0$ in (1) is equivalent to testing the null hypothesis $\mathrm{H0}: \boldsymbol{\beta}_1 = \boldsymbol{\beta}_2 = \boldsymbol{\beta}_3 = 0$ in (4). Note that under the null hypothesis $\varepsilon_t = \varepsilon_\mathrm{t}^*$, so the Taylor series approximation does not affect the asymptotic distribution theory if an *LM*-type test is used as follows:

1. Estimate model (1) under H0 (i.e. estimate a linear model), and computed the residuals, $\tilde{\varepsilon}_t$, and the sum of squared errors, SSE_0.

2. Regress $\tilde{\varepsilon}_t$ on the control variables $(\mathbf{X}_\mathrm{t})$, $\mathbf{z}_\mathrm{t}$ and $\mathrm{h}_\mathrm{t}^0 = (\mathbf{z}_\mathrm{t}'\omega_\mathrm{t}, \mathbf{z}_\mathrm{t}'\omega_\mathrm{t}^2, \mathbf{z}_\mathrm{t}'\omega_\mathrm{t}^2)'$, and retrieve SSE_1.
3. Compute the χ^2 - or *F*-version of *LM* test.

The χ^2-version of LM test has χ^2 distribution with *3m* degree of freedom and has the form

$$\mathrm{LM}_{\chi^2} = \mathrm{T}\frac{\mathrm{SSE}_0 - \mathrm{SSE}_1}{\mathrm{SSE}_0} \sim \chi^2(3\mathrm{m}) \tag{5}$$

where *m* is the number of variables in $\mathbf{z}_\mathrm{t}$ $(\boldsymbol{m} \times \mathbf{1})$. Further the *F*-version of this test has an approximate *F* distribution with *3m* and *T-4m-1* degree of freedom

$$\mathrm{LM}_\mathrm{F} = \frac{(\mathrm{SSE}_0 - \mathrm{SSE}_1)/3\mathrm{m}}{\mathrm{SSE}_1/(\mathrm{T} - 4\mathrm{m} - 1)} \sim \mathrm{F}(3\mathrm{m}, \mathrm{T} - 4\mathrm{m} - 1) \tag{6}$$

where *T* is the length of the time series for $\mathrm{t} = 1, \cdots, \mathrm{T}$. The *F*-version is preferred to χ^2-version of LM test in small and moderate samples.

As noted by González et al. (2005), this test can be used for selecting the appropriate transition variable ω_t in the LSTR model. The test is carried out for a set of possible transition variables, and the variable that gives rise to the strongest rejection of linearity (if any), i.e. with the smallest *p*-value, is chosen as the transition variable.

Choosing the number of thresholds

If the rejection of linearity hypothesis is obtained and the transition variable is selected in the previous stage, the next step is to choose between two types of LSTR(1) and LSTR(2). That is, we need to choose the number of thresholds. Teräsvirta (1994) shows in the special case $\mathbf{c} = \mathbf{0}$ and then $\boldsymbol{\beta}_2 = 0$, the model is an LSTR(1), whereas $\boldsymbol{\beta}_1 = \boldsymbol{\beta}_3 = 0$ when the model is an LSTR(2). The following *F*-tests sequence was then suggested based on the auxiliary regression in (4):

1. Test $H03$: $\boldsymbol{\beta}_3 = 0$
2. Test $H02$: $\boldsymbol{\beta}_2 = 0$ given that $\boldsymbol{\beta}_3 = 0$
3. Test $H01$: $\boldsymbol{\beta}_1 = 0$ given that $\boldsymbol{\beta}_2 = \boldsymbol{\beta}_3 = 0$

In short, select LSTR(2) if the rejection of $H02$ is the strongest one, measured in *p*-value. Otherwise, select the LSRT(1) model.

3.2. Estimation

If the vector of threshold ($\mathbf{c}$) and the transition speed parameter (γ) are known, the model in (1) reduces to a linear model that can be estimated by ordinary least squares (OLS). Therefore, the first step of estimation is to determine value of $\mathbf{c}$ and γ. To do so, we need to follow Chan (1993) and estimate $\mathbf{c}$ and γ by minimizing the *sum of squared errors* (SSE) for different potential values of $\mathbf{c}$ and γ in (1).

$$(\boldsymbol{c}, \gamma) = \underset{(\boldsymbol{c},\gamma)}{\operatorname{Argmin}} \left\{ \sum_t \hat{\varepsilon}_t^2 \right\} \tag{7}$$

where $\hat{\varepsilon}_t$ are the residuals left by OLS estimation at each iteration of the nonlinear optimization for all combinations of $\mathbf{c}$ and γ. The solution, $(\boldsymbol{c}, \gamma)$, is to find a combination of $\mathbf{c}$ and γ that produces the minimum sum of squared errors (SSR). After having estimated $(\boldsymbol{c}, \gamma)$ from the first step, the LSTR in (1) is estimated by OLS.

3.3. Evaluation

After estimation, the STR model is evaluated to determine whether the assumptions under which it was estimated are valid. We closely follow Teräsvirta (1998) and Teräsvirta et al. (2010), in performing *LM*-type tests of *no error autocorrelation*, *no remaining nonlinearity* and *parameter constancy*.

The null hypothesis of no error autocorrelation of order *q* against the alternative of autocorrelation in (2) is tested. This test can be viewed as a special case of a general test that was first suggested by Godfrey (1988). After estimating the STR model in (1), we need to investigate whether the model adequately characterizes the nonlinearity originally found in the data. We test for no remaining nonlinearity using linearity tests. Finally, before we can be confident the estimated model can be used for forecasting or policy simulation, parameter nonconstancy is tested.

4. Conclusion and Discussion

The smooth transition regression (STR) model is a useful approach to capturing nonlinearities in homeless-related variables. We discussed how the STR model could be employed to identify the sensitivity of emergency homeless shelter use to weather conditions. There are rich data sets describing daily shelter use in the Canadian cities in Alberta, including Edmonton, Calgary, Fort McMurray, Grande Prairie, Lethbridge, Lethbridge, Red Deer, and Lloydminster, which are publicly available in the Government of Alberta open datasets and publications. Large cities, such as Calgary and Edmonton, make for an interesting laboratory to investigate this issue as they are a magnate for job-seekers and have a very small stock of rental accommodations that often force job seekers to use alternative sources of shelter including rough sleeping, couch surfing, and emergency homeless shelters. Calgary and Edmonton also have a climate that is prone to rapid temperature changes that are potentially life-threatening to persons who might choose to sleep outdoors. Therefore, the investigation of the possible non-linear sensitivity of shelter use to combinations of weather-related variables, such as windchill and precipitation, using time series data will be unique. I believe the results using the nonlinear models to be important because assessments of the need for additional shelter beds are, in Alberta and I believe elsewhere, exclusively based on an assumed linear relationship with the temperature that ignores the importance of non-linearities and identifying speeds of transition from one state to another.

In addition, by taking advantage of the panel representation of STR models and individual-level data describing the daily movements in and out of emergency shelters, we can investigate the transition of people from shelters to not-in-shelters or even discover how the transition from being a transitional user of shelters to being an episodic or chronic user. We could be guided by economics and theories of social welfare to choose the transition variables to address these issues. There is considerable evidence not only that the number of people experiencing homelessness is sensitive to rental market tightness (see, for example, Kneebone and Wilkins (2022) and Hanratty (2017)) but also that this relationship may be non-linear – see Glynn and Fox (2019) and Glynn et al. 2021 (2021). Glynn and Fox (2019) suggest that once the ratio of rent to income exceeds 30%, rates of homelessness increase at an increasing rate. This suggests the possibility that the tighter the rental market, the slower may be the transition out of shelters. Therefore, the transition variable which, in this case, could be a measure of rental market tightness such as daily interest (mortgage) rates. As interest rates increase, renting looks more attractive than buying and so increases the demand for rentals. Higher interest rates also encourage landlords to pass along these extra costs. The same reasoning suggests that we could investigate the transition from being a transitional user of shelters to being an episodic or chronic user and use the measure of rental market tightness as the transition variable. The tighter the rental market, the more likely a transitional shelter user transition into chronic homelessness. Specifically, we employ the threshold models to investigate that when the interest rate exceeds a certain threshold level, the shelter users are going to experience longer stays. This structural break between the interest rates and the length of stay helps us to measure the state of homelessness. In addition, we also estimate the speed of transition from one regime (low effect of interest rate on length of stay) to another (high effect of interest rate on length of stay). When the speed of transition is fairly low (high), it implies that interest rates are going to impact the length of stays very smoothly (fast) soon after it exceeds the threshold.

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