# Unification and Division: A Theory of Institutional Choices in Imperial China

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Abstract

Ancient China experienced various rounds of division and unification. Unification was maintained through economic and political institutions such as low tax rates to reduce peasant rebellions and the division of authority among government officials to reduce usurpation of power. A ruler's choice of institutions to maintain unification is studied in a theoretical model. Interactions among external threats, internal rebellions by peasants, and usurpation of power by government officials are established. A higher level of external threats induces the ruler to choose a higher level of autonomy for government officials and a higher tax rate. That is, equilibrium probability of internal rebellions increases endogenously with the level of external threats. When government officials are more likely to usurp power, the ruler will choose a higher tax rate, thus equilibrium probability of peasant rebellions increases. Interestingly, a higher level of state power could induce the ruler to choose a lower tax rate!

## 1. Introduction

Ancient China experienced various rounds of division and unification. According to Ge (2013, p. 65), for the time period between 221 BC-1911, China was unified between periods 221 BC-209 BC, 108 BC-22, 50-184, 280-301, 589-616, 630-755, 1279-1351, 1382-1644, and 1683-1850.[1] The total number of years that China was under unification is 950, or 45% of the time period covered.

There were various factors behind Imperial China's unification and division. Geographical conditions played a significant role in affecting China's division and unification, especially at the early stage of Chinese civilization. First, geographical conditions such as the amount of rain fall affected demarcations between agricultural regions and steppes, and rulers in ancient China were better at controlling agricultural regions rather than steppes (Huang, 1997; Ge, 2013). Second, geographical conditions affected transportation costs. Transportation efficiency determined the

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[1] The establishment of a dynasty does not necessarily mean unification of the country, and the country might already be divided before the end of a dynasty (Ge, 2013).

radius of military expeditions.[2] Transportation efficiency also affected food supply and the selection of capital. For example, while Xi'an was easier to defend than Kaifeng, rulers in Northern Song did not choose Xi'an as the capital because transportation costs of food from southern China to Xi'an would be too high (Qian, 2001). Northern Song ended when its capital Kaifeng was seized by Jin (金).[3] Third, related to geographical conditions, Chi (1963) emphasizes water control and proposes the concept of key economic areas to provide an economic basis of unity and division in ancient China. If the government provides water control, a key economic area is a geographical region with good conditions to produce and transport food. By controlling key economic areas, other subordinate areas can be controlled, and the country would be unified.[4] Economic self-sufficiency of a region was necessary for a region to become independent. After the Tang dynasty, the south became important economically. Since Tang, the north depended on the south economically, and this interdependence helped the unification of China.[5] However, Ge (2013) argues that geographical conditions might not be the most crucial factor in explaining ancient China's division and unification. According to his study, while geographical conditions were the same, in some periods a region could be part of China; in some other periods the same region was not a part of China.

Scholars have different opinions on the importance of population pressure in causing peasant rebellions thus dynastic changes in ancient China. Even though Usher (1989) and Chu and Lee (1994) emphasize population dynamics in dynastic cycles, Ge (2013) states that there were only regional rather than national population pressures until the Ming dynasty. Chen (2015a) argues that natural disasters and climate changes were more important than population pressure in determining whether peasants would rebel.

Since geographic conditions such as the amount of rain fall were beyond the control of rulers, rulers in ancient China adopted various institutions to maintain rule. For example, rulers kept tax rates low to reduce the possibility of peasant rebellion, increased the degree of the division

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[2] If an army travels 40 kilometers per day in a round trip, then the radius of activities of an army is about 640 kilometers (Ge, 2013, pp. 89-91).

[3] Jin could mean 晋 or 金 in Chinese, both were used as names of dynasties in ancient China. A Chinese character will be added after a word if there is potential confusion.

[4] Chi (1963) argues that Guanzhong Plain was the key economic area during the Warring States period. The water project Zhengguoqu (郑国渠) helped Guanzhong to become a key economic area and the unification of China by Qin.

[5] When China was divided, usually Huan River rather than the Yangtze River was the dividing line between the north and the south. The reason is that if the north pushes to the Yangtze River, the south could not achieve self-sufficiency economically (Ge, 2013, p. 131).

of power among government officials to reduce usurpation of power by officials, and controlled military spending to handle external threats.

While there are many studies on Imperial China's division and unification (Ge, 2013), in this paper we illustrate factors affecting Imperial China's division and unification and study a ruler's institutional choices in a mathematical model.[6] A formal model will be helpful to organize our thinking on this important issue and to address the interaction among different institutional choices. For example, if the ruler chooses a lower tax rate to reduce the possibility of peasant rebellion, a lower revenue resulting from a lower tax rate may limit the government's military spending to handle external threats. If the ruler chooses a lower level of autonomy for officials, while this reduces the probability of usurpation of power by government officials, it could reduce the level of discretion of officials useful in handling external threats. A ruler needs to balance different tradeoffs in choosing institutions and the magnitudes of different tradeoffs are affected by various factors, such as the magnitude of external threats, the level of state power, population size, and the level of ability of the ruler.

This paper contributes to the literature by demonstrating that there are interactions among external threats, internal rebellions by peasants, and the usurpation of power by government officials. First, when the level of external threats increases, the ruler chooses a higher level of autonomy for government officials and a higher tax rate. That is, the possibility of usurpation of power by government officials and peasant rebellions will increase endogenously with the level of external threats. Second, when government officials are more likely to usurp power, the ruler chooses a higher tax rate which increases the equilibrium probability of peasant rebellion. Third, a higher sensitivity to tax burden induces the ruler to choose a higher level of autonomy for government officials and a lower level of military spending, thus increasing the equilibrium

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[6] We do not address the question why China was unified while Europe was divided. Geographic differences could have played a vital role in explaining why China was frequently unified while Europe was never unified. For example, Hicks (1969, pp. 38-39) writes "The fact that European civilization has passed through a city-state phase is the principal key to the divergence between the history of Europe and the history of Asia. The reason why it has done so is mainly geographical. The city state of Europe is a gift of the Mediterranean. In the technical conditions that have obtained through the greater part of recorded history, the Mediterranean has been outstanding as a highway of contact, between countries of widely different productive capacities; further, it is rich in pockets and crannies, islands, promontories, and valleys, which in the same conditions have been readily defensible. Asia has little to offer that is at all comparable." Hui (2005) provides a comparative study on why ancient China could achieve unification while Europe was not. She argues that the degree of monetization explains the difference between ancient China and early modern Europe. With a standing army and the county system, Qin unified China. Compared with Qin, Europe had a higher level of monetization which led to reliance on mercenary army, indirect rule, and tax farming. With deformation of state capacity, early modern Europe remained divided.

possibility of usurpation of power by government officials and reducing the effectiveness in handling external threats. Interaction among external threats and internal rebellions happened in Chinese history such as at the end of the Ming dynasty when external threats from Manchu interacted with peasant rebellions.

Our model sheds light on some historical observations. For example, tax rate in Imperial China was lower than that in Europe. While ancient China and Europe faced different levels of external threats which could induce different tax rates, we show that high state power in Imperial China could contribute to this difference in tax rates.

This paper is related to the literature on the number and size of nations. In their book, Alesina and Spolaore (2003) have compared the number and size of nations chosen by a median voter with the socially optimal size and number of nations. In their study, one key tradeoff is the following: A larger nation brings benefits in providing public goods because of the existence of increasing returns in production. However, with the existence of population heterogeneity, a larger nation means that an individual is on average farther away from his preferred choice. One significant difference between their approach and this one is that we focus on the role of institutions behind the division and unification of nations, rather than the tradeoff between increasing returns in production and population heterogeneity. In our model, the key choices are the tax rate, the degree of autonomy for officials, and the level of military spending.

For formal models on choices of tax rate and institutions in ancient China, Zhou (2012) studies a ruler's choice between feudalism and commandery-county form in ancient China. One important difference between this paper and Zhou (2012) is that this paper incorporates the ruler's choice of economic institutions through the choice of the tax rate and personal consumption into the model. Chan and Laffargue (2016) have built a stochastic growth model in which the ruler invests in state capacity. Like their model, ability of the ruler affects the choice of institutions in this model. One difference between their model and this one is that we emphasize the division of power among officials which is not addressed in their model. Zhou (2018) addresses institutional complementarities among commandery-county government organizational form, division of power among government officials, and imperial examination system in Imperial China. One significant difference between this paper and Zhou (2018) is that Zhou (2018) does not study peasant rebellion and the choice of tax rates. Ma and Rubin (2019) establish a model in which the ruler hires agent to collect taxes, subject to the participation constraint of the agent who chooses

effort level. In their model, the ruler chooses not to invest in administrative capacity to ensure that no predation becomes credible. One crucial difference between their model and this one is that external threats and division of power among government officials are not addressed in their model. To illustrate the state of Qin's unification of China in 221 BC, Zhou (2023) analyses the mutual dependence between national integration and institution building. He shows that a decrease in transportation costs leads to a decrease in the equilibrium number of states and the adoption of rule-based institutions. The unification process can feed on itself when transportation costs or population size become endogenous. Peasant rebellion and external threats are not addressed in Zhou (2023).

The plan of the paper is as follows. Section 2 illustrates Imperial China's division and unification and a ruler' institutional choices to motivate the model. Section 3 specifies the model and establishes equilibrium conditions. Section 4 conducts comparative statics to address how a ruler's institutional choices are affected by key parameters, such as the level of external threats and population size. Section 5 concludes.

## 2. Unification and division and institutional choices in Imperial China

In this section, we first illustrate factors contributing to unification and division in Imperial China. Then we discuss how rulers designed institutions to prevent division and maintain political unification.

### 2.1. Unification and division in Imperial China

Qin unified China for the first time in 221 BC. [7] Qin Shihuang promoted Qin's commandary-county system, language, measure, and behavioral code to the rest of the country. This unity decreased population heterogeneity and plowed the seed of China's future unification when the country became divided. Qin treated citizens harshly and Qin's unification did not last long. Various factors contributed to Qin's fall. First, Qin had wars with Xiongnu and engaged in many large construction projects. Second, Qin Shihuang enjoyed personal consumption through

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[7] With the Reforms of Shang Yang, Qin adopted Legalist institutions to increase and consolidate resources for military purposes (Zhao, 2015). The elimination of inherited positions created many positions attracting talented individuals from other states (Zhou, 2011, 2021). Qin surpassed significant hurdles such as integration of occupied territories before unifying China. The commandary-county system helped the integration of occupied territories (Hui, 2005).

building palaces and tomb. Peasant rebellion led by Chen Sheng and Wu Guang started the process of the collapse of the Qin dynasty (Huang, 1997).

Among the rebellion leaders at the end of the Qin dynasty, Liu Bang, founder of the Western Han dynasty (202 BC – 8), had superb leadership (Wen and Zhou, 2009). Even though Xiang Yu was the best fighter at his time, he was defeated by Liu Bang (Sima, 1988). The Western Han dynasty ended when Wang Mang usurped power. Wang was killed when a peasant rebellion army occupied the capital, and the country became divided. Liu Xiu (5 BC-57), founder of the Eastern Han dynasty (25 - 220), unified China after years of wars. With poor harvests due to natural disasters, the Yellow Turban Rebellion erupted. To handle rebellions, the central government asked local officials to raise their own armies and Liu Yan (刘焉) proposed to the emperor Liu Hong (刘宏) for the establishment of the position of Zhoumu (州牧). This position led to the consolidation of military and financial powers into the same person. Thus, Zhoumu had great autonomy in managing local affairs and this institutional change planted the seed for the division of China in the Period of Three Kingdoms. Liu Yan became Zhoumu of Yizhou himself and later declined orders from the central government (Chen, 2006). With regional officials fighting for power, the country became divided.

With Cao Cao's attempt to unify China failed, China was divided into three political entities: Cao Cao in the north, Sun Quan in the south with the help of the Yangtze River for defense, and Liu Bei in the Sichuan Basin.[8] As a key economic area (Chi, 1963), Sichuan's geographical conditions were helpful for its defense. All three entities had capable advisors and generals to support them.[9] The size and population of the region controlled by Cao Wei (曹魏) was larger than that of Sun Wu (孙吴) or Shu Han (蜀汉). A larger size and a larger population mean a larger tax base and a larger army. Cao Wei eliminated Shu Han in 263. Sima Yan usurped Cao Wei and established Western Jin (西晋). Ruling the combined area of Cao Wei and Shu Han, Western Jin conquered Sun Wu in 280 and unified China. Western Jin deviated from the commandary-county

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[8] Cao Cao defeated Yuan Xiao and occupied much of the north of China. Rather than taking time to integrate newly occupied regions, Cao Cao was too eager to unify the country. In the Battle of Chibi in AD 208, Cao's army from north was not good at fighting in water. With the spread of diseases, Cao Cao lost to the united army formed by Sun Quan and Liu Bei even though Cao had a larger army. This example demonstrates one difficulty in unifying China: marching into a different region means being exposed to new diseases.

[9] People such as Zuge Liang might have hoped Liu Bei could repeat the achievement of Liu Xiu in unifying the country.

system and internal wars among kings weakened the regime. Also, the selection of Sima Zhong as the emperor affected the decline of the Western Jin dynasty. He might be an incompetent emperor controlled by his wife. Western Jin was eliminated by minority groups (Sima et al., 1084). This was the period of the Uprising of the Five Barbarians.

In the Battle of Feishui in AD 383, while Fu Jian had a larger army, his army was less organized and trained than that in the south.[10] After Fu Jian lost the war, some nomadic groups in the north rebelled against him and he was killed two years later (Sima et al., 1084). The north at that time had high degrees of population heterogeneity and faced internal cohesion issue when different nomadic groups and Han people learned how to coexist. In the south, powerful clans occupied large pieces of land and hoarded population, and this reduced the central government's tax revenue. Eastern Jin (东晋) rulers needed the support of powerful clans to maintain rule. Also, rulers needed to handle the relationship between indigenous residents in the south and immigrants from the north (Tian, 1996). Eastern Jin rulers were not strong enough to unify the country.

During the Southern and Northern dynasties, China was divided between the north and the south.[11] Yang Jian, founder of the Sui dynasty (AD 581 - 618), usurped power from the Northern Zhou. Chen Shubao, the ruler of Southern Chen, indulged himself and did not provide strong leadership in defending his state. Sui conquered Southern Chen and unified China. Sui introduced the imperial examination system, which led to high level of mobility among government officials. This mobility increased social integration and reduced population heterogeneity. Emperor Yang Guang was ambitious and engaged in various wars. He controlled generals fighting with Korea tightly, and generals were required to get his directions even though they could be hundreds of miles away (Yuan, 2001, p. 550). Yang Guang also built huge projects such as the Grand Canal. While the Grand Canal would benefit future generations for centuries to come, its construction

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[10] Even though his control in the north was not firm, Fu Jian was determined to unify China by invading the south while his advisor Wang Meng suggested him not to do so before Wang's death. Fu Jian's proposal to invade the south was also opposed by his crown prince. By balancing the interests of powerful clans and behaving confidently, Xie An provided good leadership in the south.

[11] In the south, Eastern Jin ended when Liu Yu established Liu Song (刘宋) in AD 420. Liu Song was replaced by Southern Qi (AD 479-502), which was replaced by Southern Liang (AD 502-560), which again was replaced by Southern Chen (AD 557-589). In all cases, a general from the previous political regime usurped power and established a new regime. In the north, Northern Wei (AD 386-534) unified north again. The emperor Yuan Hong moved the capital from Pingcheng to Luoyang. With the new capital and the original capital acting as two power centers, rebellion erupted, and Northern Wei became divided into Eastern Wei (AD 534-550) and Western Wei (AD 535-556). Eastern Wei was controlled by the powerful general Gao Huan. Eventually, the Gao family founded Northern Qi, ended the rule of Eastern Wei. Western Wei was controlled by the powerful general Yuwen Tai. The Yuwen family founded Northern Zhou, ended the rule of Western Wei (Huang, 1997).

was a huge burden to citizens. Rebellions by peasants and by government officials led to the collapse of the Sui dynasty.

Li Shimin was outstanding at attracting capable advisors and brave generals around him. He helped his father Li Yuan in the founding of the Tang dynasty (AD 618-960). Under Emperor Li Longji, to deal with external threats, generals were given high levels of autonomy. With the concentration of military and financial power of several regions, general An Lushan rebelled against the Tang government. To defeat this rebellion, regional leaders gained power. While the central government tried, the Tang dynasty could not control some regional leaders. In the resulting Period of Five Dynasties and Ten Kingdoms, China was divided for several decades.

By usurping power from the child ruler of Later Zhou, general Zhao Kuangyin founded the Song dynasty in AD 960.[12] Song's second emperor (Zhao Guangyi)'s three attempts to conquer the north failed, then Song signed a treaty with Liao to gain peace. Neither Northern Song nor Liao could conquer the other side, thus peace lasted for decades. Jin (金) eliminated Liao.[13] With Northern Song also eliminated by Jin, Southern Song was established with temporary capital in Hangzhou in southern China.

After Yuan (AD 1271 - 1368) conquered Jin (金), the Kingdom of Dali, and the Southern Song, China became unified again.[14] As a nomadic group, the Mongol Empire initially did not have a formal tax system. Yuan practiced a system like feudalism. Under this system, generals had strong incentives to conquer. However, the central government had limited control of vassals. Thus, this system led to fast rise and decline of Yuan. Yuan rulers were busy in various wars, internally and externally. With heavy military spending, fiscal management was a very demanding issue (Li, 2014). Peasant rebellions erupted in a water project to control flooding in the Yellow River. The country became divided with peasant rebellions in the south and Mongol generals fighting among themselves in the north.

Zhu Yuanzhang, founder of the Ming dynasty (AD 1368 – 1644), was born into a poor family and self-educated. He defeated other rebellion leaders and pushed Mongols back to the

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[12] Song might be viewed as a period of division rather than unification because there were competing political regimes in the north (Ge, 2013).
[13] Following Liao, Jin initially had a dual-track system in political institutions: one system handling nomadic people and the other handling Han Chinese engaging in agricultural production. During the rule of Emperor Xizong (熙宗), Jin switched to a unitary Han style system.
[14] Yuan institutions were heavily influenced by Jin (金).

north. Zhu was energetic and designed institutions to maintain long-run rule. With high population and poor harvest partially caused by bad weather, peasant rebellions exacerbated the threats from Manchu. The Ming dynasty collapsed when peasant army led by Li Zicheng occupied Beijing, main capital of the Ming dynasty (Huang, 1997).

In the Qing dynasty (AD 1636 -1912), following the Ming dynasty, initially power was divided in the local government. To put down the Taiping Rebellion (1851-1864), heavy military spending led to sale of government positions. The central government asked officials to train militia and Zeng Guofan instead decided to raise and train an army. With insufficient funds from the central government, Zeng Guofan established agencies to collect taxes to finance the war. Giving autonomy to an official does not necessarily lead to the fall of the regime.[15] However, the power of regional officials was much higher after the Taiping Rebellion than that before the rebellion. During the Boxer Rebellion in 1900, the central government declared war with foreign governments but provincial leaders in the south refused to follow the order of the central government. While provincial governments revoked their independence after the Boxer Rebellion was over, this autonomy of provincial governments eventually led to the collapse of the Qing dynasty in 1911. At that time, soldiers in the new army rebelled in Wuhan and provinces declared independence rather than tried to help the central government to put down the rebellion (Huang, 1997).

From the above illustration, unification and division were affected by multiple factors. Overall, external threats, internal rebellions by peasants, and usurpation of power by government officials could lead the country to division. Racial conflicts and the existence of powerful clans would increase government spending or reduce revenue and were harmful for unification. In addition, leaders played important roles in the process of division and unification. A capable and open-minded leader like Li Shimin would be able to attract many capable followers and unify the country, while an incompetent leader such as Sima Zhong would lead the country to disaster and division. Personal consumption of the ruler such as building palaces and tombs could be a significant percentage of government revenue. Indulgence in personal consumption is frequently cited in China as a reason leading to political disasters.

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[15] As a loyal Confucian, Zeng Guofan did not intend to rebel, and he disarmed most of his soldiers after he put down the Taiping Rebellion.

## 2.2. Institutions to maintain unity in Imperial China

In this subsection, we illustrate how rulers in Imperial China established economic and political institutions to prevent division and maintain political unity.

One economic institution to maintain political unity is low tax rate. In ancient China, in additional to taxes, peasants provided corvee. Huge corvee levy from the government contributed to the fall of the Qin and Sui dynasties. Rulers in ancient China such as Li Shimin were highly concerned with the impact of tax on the possibility of peasant rebellions. To prevent peasant rebellion, rulers kept tax rates low.[16] In the Qing dynasty, Emperor Kangxi announced publicly that the government would fix the amount of tax in the future even though population in the Qing dynasty increased significantly over the years (Brandt, Ma, and Rawski, 2014).[17] Compared with European countries, official tax rates in ancient China were low (Rosenthal and Wong, 2011, p. 184).[18] Since rulers in ancient China had unlimited power and thus could have imposed high tax rates theoretically, the low tax rate in ancient China is an interesting phenomenon. Fear of peasant rebellion could be the underlying explanation. As we are going to demonstrate in Proposition 4, a higher state power could induce the ruler to choose a lower tax rate.

For political institutions, when the commandary-county system replaced feudalism, the central government began to appoint local government officials. With the introduction of the imperial examination system, influence of clans on the selection of government officials declined. Overall, the commandary-county system and the imperial examination system almost eliminated the accumulation of power over generations by officials. Those institutions reduced the possibility of usurpation of power by government officials and helped to keep the country unified (Zhou, 2018). Also, rulers established institutions to reduce the level of autonomy of officials. For example, the Sui dynasty established the institution of the Three Departments and Six Ministries. For the three departments, the Secretariat would draft an order, the Chancellery would review the order, and the Department of State Affairs would implement the order. The Department of State Affairs was further divided into six ministries. With multiple prime ministers and the check of

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[16] Tax rates in Imperial China such as in the Ming dynasty were low (Huang, 1974, Ch. 3). Huang (1974) provides a detailed study of taxes in the Ming dynasty, and he shows that military expenditure was the largest item of expenditure for many counties. Liang (1980) consulted different sources of information and provided authoritative data on land taxation in ancient China through hundreds of tables.

[17] With increased expenditures to put down rebellions such as the Taiping Rebellion, various surcharges arose.

[18] While China's tax rate was around 5% to 10% of national output (Deng, 1999, Chap. 3), tax rates in Britain in the 18th century was higher than 10%. In 1780, the tax rate in Britain was 23% (O' Brien, 1988, p. 15).

power among them, the authority of any prime minister decreased significantly. With the lessons of the fall of the Tang dynasty in mind, the Song dynasty used the division of authority to control military officials. Zhao Kuangyin eliminated some important military positions and split each remaining one into several positions to reduce the control of any officer. Prime ministers were not allowed to control the military and finance. While the three offices were managing the armies, the Chief Military Commission (枢密院) had the authority of deploying armies. Those measures weakened the military strength of Song (Wang, 1983). The Ming dynasty used division of authority among officials to maintain unity. In the Ming dynasty, at the province level, there were three offices: the Provincial surveillance Office (按察司), the Provincial Administration Office (布政司), and the Regional Military Commission (都指挥使司) (Zhang, 1996).[19] With the division of power among the three offices, no individual had the authority of a provincial governor. Over time, this division of power among government officials functioned relatively well in reducing usurpation of power. As shown in Ming and Qing, usurpation of power by government officials disappeared and China was able to unify a large territory for hundreds of years.

The central government also reduced autonomy of provinces through reducing geographic independence of regions. Rather than having boundaries of provinces determined by geographical conditions, the central government changed boundaries of provinces to make provinces less independent so that local autonomy would not develop (Zhou, 1998; Ge, 2013).[20]

Overall, a ruler needs tax revenue for military spending and personal consumption, but a higher tax rate leads to a higher probability of peasant rebellion. Highlighted by the experiences of Han and Tang, consolidation of military and financial power into the same official frequently led to the division of the country. Reducing the level of autonomy of government officials through the division of power among officials helped maintaining political stability and unity. However, government officials would be less effective in handling external threats and peasant rebellions. In the following, we establish a formal model to capture those tradeoffs faced by a ruler in choosing institutions for political unity.

## 3. The model

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[19] Huang (1974) provides illustrations on the division of authority in handling economic affairs in the Ming dynasty.
[20] In the Yuan dynasty, the government assigned Hanzhong Basin to Shanxi, rather than to Sichuan even though based on geographical conditions it would be better for Sichuan to manage Hanzhong.

There is a ruler. To maintain political unity, the ruler chooses military spending to handle external threats, the level of autonomy for government officials to reduce the usurpation of power by government officials, and the tax rate to control peasant rebellions. The ruler can enjoy his consumption if he can handle internal rebellions and external threats successfully. The ruler's payoff is zero if he could not handle either internal rebellions or external threats.

A divided regime can further divide, and a unified country can further integrate regions previously not integrated. Regardless of unification or division, a ruler needs to handle peasant rebellions, usurpation of power, and external threats. Thus in this model, we do not differentiate decision-making under division and unification.

The ruler faces various tradeoffs. First, if the ruler chooses a higher level of military spending to handle external threats, this may require a higher tax rate and thus a higher possibility of peasant rebellion. Second, a ruler faces the following tradeoff when deciding the level of autonomy for officials. If the ruler does not give much autonomy to an official, this official needs to consult others in handling important issues. Thus, this official would be less likely to rebel against the central government. If the ruler gives autonomy to an official, this official has monopoly power in deciding critical issues. Giving autonomy to an official has costs and benefits. On the one hand, the benefit is that decision delay is avoided, and an official can handle peasant rebellions and external threats more effectively. On the other hand, the cost is that an official with autonomy may rebel against the central government.[21] Third, when the ruler chooses a higher tax rate, while this increased government revenue could be used by the ruler for personal consumption or military spending, the possibility of internal rebellions increases.

We now specify the ruler's objective function. First, we examine the probability that the ruler handles external threats successfully. The level of exogenously given ability of the leader is $a$, a positive number. This parameter captures the influence of capable leaders such as Li Shimin in the unification of China. The level of autonomy of government officials is $z$. When the level of exogenously given external threats is $\Omega$ and the level of military spending is $m$, the probability that the ruler handles external threats successfully is $\frac{az^\sigma m^\rho}{az^\sigma m^\rho + \Omega}$, where $\sigma$ and $\rho$ are positive constants. With this specification of the contest function, a higher level of ability of the ruler, a

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[21] One example is Li Yuan in the Sui Dynasty. Li Yuan was a general stationed in Taiyuan to defend the country from the invasion of Xiongnu. With peasant rebellions weakening the rule of Sui, Li Yuan revolted against Sui and founded the Tang dynasty.

higher level of military spending, or a lower level of external threats increases the probability that the ruler handles external threats successfully.

Second, we establish the probability that a government official may usurp power. Let $\theta$ denote a positive constant. A higher value of $\theta$ means that a higher level of autonomy is more likely to cause the usurpation of power. The probability of usurpation of power by government officials is $1 - e^{-\frac{\theta z}{a}}$. That is, a higher level of autonomy of officials increases the probability of usurpation of power by government officials.

Third, we specify the probability of peasant rebellions. The probability of peasant rebellions is $1 - e^{-\alpha t}$, and $\alpha$ denotes a positive constant. That is, a higher tax rate increases the possibility of peasant rebellions. A higher value of $\alpha$ means that for the same tax rate peasants are more likely to rebel.

The ruler chooses the level of autonomy for officials, the tax rate, the level of military spending, and the level of personal consumption $c$ to maximize his expected payoff:

$$e^{-\frac{\theta z}{a}}e^{-\alpha}\frac{az^{\sigma}m^{\rho}}{az^{\sigma}m^{\rho}+\Omega}lnc. \tag{1}$$

The size of the population is $L$. The total amount of land is $T$. For $\beta \in (0,1)$, the level of total output produced is $T^{1-\beta}L^{\beta}$. To capture the division of power between state and society, only a percentage $b$ of the population is controlled by the central government and is liable for taxes and military services. A lower value of $b$ means that the power of the central government is lower. The ruler's tax revenue is thus $tbT^{1-\beta}L^{\beta}$. Regardless of whether a person is controlled by the central government, the government needs to provide public goods for this person. The per capita cost of providing public goods is $\gamma$, which is a positive constant. The constraint faced by the ruler is that the level of consumption, military spending, and expenditure on public goods should not be higher than and actually equal to tax revenue:

$$c + m + \gamma L = tbT^{1-\beta}L^{\beta}. \tag{2}$$

Plugging the value of consumption from the constraint (2) into the objective function (1) yields the following objective function for the ruler:

$$e^{-\frac{\theta z}{a}}e^{-\alpha t}\frac{az^{\sigma}m^{\rho}}{az^{\sigma}m^{\rho}+\Omega}ln(tbT^{1-\beta}L^{\beta}-\gamma L - m). \tag{3}$$

Assuming interior solutions,[22] the first order conditions for the ruler's optimal choices of $z$, $t$, and $m$ are[23]

$$\frac{\Omega\sigma}{az^{\sigma}m^{\rho}+\Omega} - \frac{\theta z}{a} = 0, \tag{4}$$

$$\frac{bT^{1-\beta}L^{\beta}}{tbT^{1-\beta}L^{\beta}-\gamma L-m} - \alpha ln\left(tbT^{1-\beta}L^{\beta} - \gamma L - m\right) = 0, \tag{5}$$

$$\frac{\Omega}{az^{\sigma}m^{\rho}+\Omega}ln\left(tbT^{1-\beta}L^{\beta} - \gamma L - m\right) - \frac{m^{\rho}}{tbT^{1-\beta}L^{\beta}-\gamma L-m} = 0. \tag{6}$$

The interpretations of equations (4)-(6) is as follows. For equation (4), the first term in the left-hand side is the marginal benefit and the second term is the marginal cost of a higher autonomy. For equation (5), the first term in the left-hand side is the marginal benefit and the second term is the marginal cost of a higher tax rate. For equation (6), the first term in the left-hand side is the marginal benefit and the second term is the marginal cost of a higher military spending.

## 4. Comparative statics

From the first order conditions, we can establish the following three equations defining three endogenous variables $z$, $t$, and $m$ as functions of exogenous parameters:[24]

$$\Gamma_1 \equiv \frac{\Omega}{az^{\sigma}m^{\rho}+\Omega} - \frac{\theta z}{a\sigma} = 0, \tag{7a}$$

$$\Gamma_2 \equiv \frac{bT^{1-\beta}L^{\beta}}{tbT^{1-\beta}L^{\beta}-\gamma L-m} - \alpha ln\left(tbT^{1-\beta}L^{\beta} - \gamma L - m\right) = 0, \tag{7b}$$

$$\Gamma_3 \equiv \theta zbT^{1-\beta}L^{\beta} - a\alpha\sigma m^{\rho} = 0. \tag{7c}$$

Partial differentiation of equations (7a)-(7c) with respect to $z$, $t$, $m$, $\Omega$, $a$, $\gamma$, $b$, $\alpha$, $\theta$, $T$, and $L$ yields

$$\begin{pmatrix} \frac{\partial\Gamma_1}{\partial z} & 0 & \frac{\partial\Gamma_1}{\partial m} \\ 0 & \frac{\partial\Gamma_2}{\partial t} & \frac{\partial\Gamma_2}{\partial m} \\ \frac{\partial\Gamma_3}{\partial z} & 0 & \frac{\partial\Gamma_3}{\partial m} \end{pmatrix}\begin{pmatrix} dz \\ dt \\ dm \end{pmatrix} = -\begin{pmatrix} \frac{\partial\Gamma_1}{\partial\Omega} \\ 0 \\ 0 \end{pmatrix}d\Omega - \begin{pmatrix} \frac{\partial\Gamma_1}{\partial a} \\ 0 \\ \frac{\partial\Gamma_3}{\partial a} \end{pmatrix}da - \begin{pmatrix} 0 \\ \frac{\partial\Gamma_2}{\partial\gamma} \\ 0 \end{pmatrix}d\gamma$$

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[22] In a corner solution, the ruler could choose a variable such as the level of authority for government officials to a level so low such that the usurpation of power by a government official becomes unlikely, which might have happened in the Ming and Qing dynasties.
[23] From equations (4)-(6), the second order derivatives with respect to $z$, $t$, and $m$ respectively are all negative, consistent with the second order conditions for the ruler's maximization problem.
[24] Equation (7c) is derived from combining (5) and (6) and then (4).

$$-\begin{pmatrix} 0 \\ \frac{\partial \Gamma_2}{\partial b} \\ \frac{\partial \Gamma_3}{\partial b} \end{pmatrix} db - \begin{pmatrix} 0 \\ \frac{\partial \Gamma_2}{\partial \alpha} \\ \frac{\partial \Gamma_3}{\partial \alpha} \end{pmatrix} d\alpha - \begin{pmatrix} \frac{\partial \Gamma_1}{\partial \theta} \\ 0 \\ \frac{\partial \Gamma_3}{\partial \theta} \end{pmatrix} d\theta - \begin{pmatrix} 0 \\ \frac{\partial \Gamma_2}{\partial T} \\ \frac{\partial \Gamma_3}{\partial T} \end{pmatrix} dT - \begin{pmatrix} 0 \\ \frac{\partial \Gamma_2}{\partial L} \\ \frac{\partial \Gamma_3}{\partial L} \end{pmatrix} dL. \qquad (8)$$

Since $\frac{\partial \Gamma_1}{\partial z} < 0$, $\frac{\partial \Gamma_1}{\partial m} < 0$, $\frac{\partial \Gamma_2}{\partial t} < 0$, $\frac{\partial \Gamma_2}{\partial m} > 0$, $\frac{\partial \Gamma_3}{\partial z} > 0$, and $\frac{\partial \Gamma_3}{\partial m} < 0$, the determinant of the coefficient matrix of endogenous variables of (8) is negative: $\Delta \equiv \frac{\partial \Gamma_2}{\partial t}\left(\frac{\partial \Gamma_1}{\partial z}\frac{\partial \Gamma_3}{\partial m} - \frac{\partial \Gamma_1}{\partial m}\frac{\partial \Gamma_3}{\partial z}\right) < 0$. With $\Delta$ nonsingular, a unique equilibrium exists.

The level of external threats could be affected by climate changes. A harsh weather could kill thousands of animals. Without enough food to survive, nomadic peoples would be more likely to invade China (Huang, 1997; Chen, 2015b). The following proposition studies the impact of a change in the level of external threats on the ruler's choices.

Proposition 1: An increase in the level of external threats induces the ruler to choose a higher level of autonomy for officials, a higher tax rate, and a higher level of military spending.

Proof: Applying Cramer's rule on (8) yields

$$\frac{dz}{d\Omega} = -\frac{\partial \Gamma_1}{\partial \Omega}\frac{\partial \Gamma_2}{\partial t}\frac{\partial \Gamma_3}{\partial m}/\Delta > 0,$$

$$\frac{dt}{d\Omega} = -\frac{\partial \Gamma_1}{\partial \Omega}\frac{\partial \Gamma_2}{\partial m}\frac{\partial \Gamma_3}{\partial z}/\Delta > 0,$$

$$\frac{dm}{d\Omega} = \frac{\partial \Gamma_1}{\partial \Omega}\frac{\partial \Gamma_2}{\partial t}\frac{\partial \Gamma_3}{\partial z}/\Delta > 0. \blacksquare$$

When the ruler chooses a higher level of autonomy for government officials, equilibrium probability of usurpation of power increases. When the ruler chooses a higher tax rate, equilibrium probability of peasant rebellions increases. Thus, Proposition 1 establishes the interaction between external threats and internal rebellions. The intuition behind Proposition 1 is as follows. From equation (4) which is the first order condition for the ruler's optimal choice of the level of autonomy, a higher level of external threats increases the marginal benefit of autonomy and does not change marginal cost, thus the ruler chooses a higher level of autonomy for government officials. From equation (6), with a higher level of external threats, marginal benefit of military spending increases while marginal cost does not change, thus the ruler chooses a higher level of military spending. With a higher level of military spending, from equation (5), both marginal

benefit and marginal cost of taxation increases. With $\frac{\partial \Gamma_2}{\partial m} > 0$, the increase in marginal benefit dominates the increase in marginal cost. Thus, the ruler chooses a higher tax rate.

Proposition 1 can be used to understand different tax rates between Imperial China and Europe. Hicks (1969, pp. 38-39) has argued that difference in geographical conditions can be used to explain China's unification and Europe's fragmentation. With frequent international conflicts, the levels of external threats faced by European countries were higher than those faced by China (Rosenthal and Wong, 2011, chap. 6). Military expense was a significant part of British government expenditure in the 18th century (O'Brien, 1988). It is not strange that the tax rate in ancient China was lower than that in European countries.

Because leaders and followers did not use formal contracts to determine rewards to followers, the rewards to followers were heavily influenced by the personalities of leaders. Rulers with strong abilities would be good at attracting talents, judging and adopting advice from officials.[25] While it might be difficult to measure ruler abilities precisely, there were some emperors in China's history such as Li Shimin generally agreed as rulers with strong abilities and superb performance. The following proposition studies the impact of a change in the level of ability on the ruler's choices.

Proposition 2: A ruler with a higher level of ability chooses a lower tax rate, a lower level of military spending, and a higher level of autonomy for officials.

Proof: Partial differentiation of (7a) and (7c) yields

$$\frac{\partial \Gamma_1}{\partial a} = \frac{\theta z \Omega}{a^2 \sigma (a z^\sigma m^\rho + \Omega)} > 0, \tag{9}$$

$$\frac{\partial \Gamma_1}{\partial z} \frac{\partial \Gamma_3}{\partial a} - \frac{\partial \Gamma_1}{\partial a} \frac{\partial \Gamma_3}{\partial z} = \frac{\theta \alpha (1+\sigma) z^\sigma m^{2\rho}}{a z^\sigma m^\rho + \Omega} > 0. \tag{10}$$

Applying Cramer's rule on (8) yields

$$\frac{dt}{da} = \frac{\partial \Gamma_2}{\partial m} \left( \frac{\partial \Gamma_1}{\partial z} \frac{\partial \Gamma_3}{\partial a} - \frac{\partial \Gamma_1}{\partial a} \frac{\partial \Gamma_3}{\partial z} \right) / \Delta < 0,$$

$$\frac{dm}{da} = \frac{\partial \Gamma_2}{\partial t} \left( \frac{\partial \Gamma_1}{\partial a} \frac{\partial \Gamma_3}{\partial z} - \frac{\partial \Gamma_1}{\partial z} \frac{\partial \Gamma_3}{\partial a} \right) / \Delta < 0,$$

$$\frac{dz}{da} = \frac{\partial \Gamma_2}{\partial t} \left( \frac{\partial \Gamma_1}{\partial m} \frac{\partial \Gamma_3}{\partial a} - \frac{\partial \Gamma_1}{\partial a} \frac{\partial \Gamma_3}{\partial m} \right) / \Delta > 0. \ \blacksquare$$

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[25] One example of ruler with strong abilities is Emperor Liu Bang who founded the Han dynasty. He could supervise capable individuals such as Xiao He, Han Xin, Zhang Liang, and Chen Ping. While Han Xin and Chen Ping initially worked for Xiang Yu, Xiang Yu could not employ their skills effectively. With the help of Xiao He, Liu Bang could recruit and support more soldiers to win the war against Xiang Yu (Wen and Zhou, 2009).

Since a higher ruler ability leads to a lower tax rate, the equilibrium probability of peasant rebellions is lower. Thus, a higher ability of the ruler increases the probability of political unity of the country. The intuition behind Proposition 2 is as follows. When the ruler has a higher level of ability, from equation (4), marginal cost of autonomy decreases. Also, marginal benefit of autonomy decreases. In this model, with the validity of (9), the first effect always dominates the second one. Thus, the ruler chooses a higher level of autonomy in equilibrium. From equation (6), when the ruler has a higher level of ability, marginal benefit of military spending decreases and marginal cost does not change. Thus, the ruler chooses a lower level of military spending. With a lower military spending, from equation (5), marginal benefit of tax decreases and marginal cost also decreases. With $\frac{\partial \Gamma_2}{\partial m} > 0$, the decrease in marginal cost of tax dominates and the ruler chooses a lower level of tax rate.

The cost of providing public goods could be affected by geographical conditions and the degree of population heterogeneity. The following proposition studies the impact of a change in the cost of providing public goods on the ruler's choice of institutions.

Proposition 3: A higher cost of public goods does not change the degree of autonomy and the level of military spending. The ruler chooses a higher tax rate.

Proof: Applying Cramer's rule on (8) yields

$$\frac{dz}{d\gamma} = 0,$$

$$\frac{dm}{d\gamma} = 0,$$

$$\frac{dt}{d\gamma} = \frac{\partial \Gamma_2}{\partial \gamma}\left(\frac{\partial \Gamma_1}{\partial m}\frac{\partial \Gamma_3}{\partial z} - \frac{\partial \Gamma_1}{\partial z}\frac{\partial \Gamma_3}{\partial m}\right)/\Delta > 0. \ \blacksquare$$

The intuition behind Proposition 3 is as follows. From equation (5) which is the first order condition for the ruler's optimal choice of the tax rate, a higher cost of public goods increases the marginal benefit of tax and reduces the marginal cost. Thus, the ruler chooses a higher tax rate. Since marginal cost and marginal benefit of autonomy and military spending are not affected by a change in the cost of providing public goods, the level of autonomy and military spending remain the same.

The distribution of power between the state and society varied over time in ancient China.[26] Wars led to the elimination of many powerful clans, such as what happened at the end of the Tang dynasty when Huang Chao's army killed many powerful clans. Frequently the central government could not control powerful clans from hoarding population which neither pay taxes nor provide military services to the central government. For example, in the Eastern Han dynasty, emperor Liu Xiu's efforts to reduce the number of servants of powerful clans failed. One distinct case is the Eastern Jin dynasty. Power of the central government in Eastern Jin was weak from the beginning: only with the support of powerful clans, the government could survive. Thus, the Eastern Jin government failed to tax powerful clans. In addition to clans, temples might also own large amounts of land and hoard population. The following proposition studies the impact of a change in state power on the ruler's choices.

Proposition 4: An increase in state power leads the ruler to choose a lower level of autonomy for officials and a higher military spending. The impact on the tax rate is ambiguous, and a sufficient condition for $\frac{dt}{db} < 0$ is that $\rho \geq 1$.

Proof: Applying Cramer's rule on (8) yields

$$\frac{dz}{db} = \frac{\partial \Gamma_1}{\partial m}\frac{\partial \Gamma_2}{\partial t}\frac{\partial \Gamma_3}{\partial b}/\Delta < 0,$$

$$\frac{dm}{db} = -\frac{\partial \Gamma_1}{\partial z}\frac{\partial \Gamma_2}{\partial t}\frac{\partial \Gamma_3}{\partial b}/\Delta > 0,$$

$$\frac{dt}{db} = \left(\frac{\partial \Gamma_1}{\partial z}\frac{\partial \Gamma_2}{\partial m}\frac{\partial \Gamma_3}{\partial b} + \frac{\partial \Gamma_1}{\partial m}\frac{\partial \Gamma_2}{\partial b}\frac{\partial \Gamma_3}{\partial z} - \frac{\partial \Gamma_1}{\partial z}\frac{\partial \Gamma_2}{\partial b}\frac{\partial \Gamma_3}{\partial m}\right)/\Delta.$$

In general, the sign of $\frac{\partial \Gamma_1}{\partial z}\frac{\partial \Gamma_2}{\partial m}\frac{\partial \Gamma_3}{\partial b} + \frac{\partial \Gamma_1}{\partial m}\frac{\partial \Gamma_2}{\partial b}\frac{\partial \Gamma_3}{\partial z} - \frac{\partial \Gamma_1}{\partial z}\frac{\partial \Gamma_2}{\partial b}\frac{\partial \Gamma_3}{\partial m}$ is ambiguous and thus the sign

of $\frac{dt}{db}$ is ambiguous. When $\rho \geq 1$, $\frac{\partial \Gamma_2}{\partial m}\frac{\partial \Gamma_3}{\partial b} - \frac{\partial \Gamma_2}{\partial b}\frac{\partial \Gamma_3}{\partial m} = \frac{a\alpha\sigma T^{1-\beta}L^{\beta}(m^{\rho}-\rho m^{\rho}-\gamma L\rho m^{\rho-1})}{(tbT^{1-\beta}L^{\beta}-\gamma L-m)^2} -$

$\frac{a\alpha^2\sigma m^{\rho-1}(tbT^{1-\beta}L^{\beta}\rho-m)}{b(tb^{\ 1-\beta}L^{\beta}-\gamma L-m)} < 0.$ Thus, $\frac{\partial \Gamma_1}{\partial z}\frac{\partial \Gamma_2}{\partial m}\frac{\partial \Gamma_3}{\partial b} + \frac{\partial \Gamma_1}{\partial m}\frac{\partial \Gamma_2}{\partial b}\frac{\partial \Gamma_3}{\partial z} - \frac{\partial \Gamma_1}{\partial z}\frac{\partial \Gamma_2}{\partial b}\frac{\partial \Gamma_3}{\partial m} > 0$ and $\frac{dt}{db} < 0.$ ∎

The intuition behind Proposition 4 is as follows. From equation (6) which is the first order condition for the ruler's optimal choice of military spending, a higher state power increases marginal benefit and reduces marginal cost of military spending. Thus, the ruler chooses a higher

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[26] In his stimulating analysis of the rise and decline of nations, Olson (1982) emphasizes the building up of interest groups during peace time.

military spending. From equation (4) which is the first order condition for the ruler's optimal choice of the level of autonomy, with a higher military spending, marginal benefit of autonomy decreases while marginal cost does not change. Thus, the ruler chooses a lower level of autonomy for officials. From equation (5) which is the first order condition for the ruler's optimal choice of the tax rate, the direct effect from an increase in the level of state power is that a higher state power reduces the marginal benefit and increases the marginal cost of tax. However, there is an indirect effect that a higher military spending increases the marginal benefit and reduces marginal cost of tax. The two effects work on opposite directions and the impact on the tax rate in general is ambiguous. With $\rho$ increases, from equation (6), marginal benefit of military spending decreases while marginal cost increases. Since the increase in military spending is smaller, the indirect effect from the change in military spending on marginal cost and marginal benefit of taxation is dominated by the direct effect of a change in the level of state power. Thus, the ruler chooses a lower tax rate.

Interestingly, Proposition 4 shows that the ruler could choose a lower rather than a higher tax rate when state power increases! The tax rate in the Song dynasty was higher than that in the Qing dynasty while the level of state power in Qing was higher than that of Song. This observation is consistent with Proposition 4. In addition, we need to control other factors when comparing tax rates. Song dynasty faced significantly higher level of external threats than those faced by Qing. As shown in Proposition 1, a higher level of external threats could induce the ruler to choose a higher tax rate. That is, Song's higher tax rate could have resulted from several factors, such as lower state power and higher external threats.

With natural disasters leading to poor harvests which could be caused by significant weather changes, peasants would be more likely to rebel even though the tax rate does not change. A higher sensitivity to tax can be captured by an increase in $\alpha$. The following proposition studies the impact of a change in the sensitivity to tax burden on the ruler's choices.

Proposition 5: A higher sensitivity to tax burden leads the ruler to choose a higher level of autonomy for government officials, a lower tax rate, and a lower level of military spending.

Proof: Since $\frac{\partial \Gamma_1}{\partial z} < 0, \frac{\partial \Gamma_1}{\partial m} < 0, \frac{\partial \Gamma_2}{\partial m} > 0, \frac{\partial \Gamma_2}{\partial \alpha} < 0, \frac{\partial \Gamma_3}{\partial \alpha} < 0, \frac{\partial \Gamma_3}{\partial z} > 0$, and $\frac{\partial \Gamma_3}{\partial m} < 0$, then

$$\frac{\partial \Gamma_1}{\partial m}\frac{\partial \Gamma_2}{\partial \alpha}\frac{\partial \Gamma_3}{\partial z} + \frac{\partial \Gamma_1}{\partial z}\frac{\partial \Gamma_2}{\partial m}\frac{\partial \Gamma_3}{\partial \alpha} - \frac{\partial \Gamma_1}{\partial z}\frac{\partial \Gamma_2}{\partial \alpha}\frac{\partial \Gamma_3}{\partial m} > 0.$$

Applying Cramer's rule on (8) yields

$$\frac{dz}{d\alpha} = \frac{\partial \Gamma_1}{\partial m}\frac{\partial \Gamma_2}{\partial t}\frac{\partial \Gamma_3}{\partial \alpha}/\Delta > 0,$$

$$\frac{dt}{d\alpha} = \left(\frac{\partial \Gamma_1}{\partial m}\frac{\partial \Gamma_2}{\partial \alpha}\frac{\partial \Gamma_3}{\partial z} + \frac{\partial \Gamma_1}{\partial z}\frac{\partial \Gamma_2}{\partial m}\frac{\partial \Gamma_3}{\partial \alpha} - \frac{\partial \Gamma_1}{\partial z}\frac{\partial \Gamma_2}{\partial \alpha}\frac{\partial \Gamma_3}{\partial m}\right)/\Delta < 0,$$

$$\frac{dm}{d\alpha} = -\frac{\partial \Gamma_1}{\partial z}\frac{\partial \Gamma_2}{\partial t}\frac{\partial \Gamma_3}{\partial \alpha}/\Delta < 0. \ \blacksquare$$

With a higher sensitivity to tax burden, since the ruler chooses a lower level of military spending, the country will be less successful in handling external threats. With a higher autonomy, government officials are more likely to usurp power. The intuition behind Proposition 5 is as follows. From equation (6) which is the first order condition for the ruler's optimal choice of military spending, marginal benefit of military spending does not change while marginal cost increases. Thus, the ruler chooses a lower level of military spending. From equation (4) which is the first order condition for the ruler's optimal choice of the level of autonomy, with a lower military spending, marginal benefit of autonomy increases while marginal cost does not change. This leads the ruler to choose a higher level of autonomy for officials. From equation (5) which is the first order condition for the ruler's optimal choice of the tax rate, a higher sensitivity to tax burden increases the marginal cost of tax and does not change the marginal benefit. Thus, the ruler chooses a lower tax rate.

The adoption of commandary-county system and imperial examination system reduced probability of usurpation of power by government officials. This kind of change can be captured by a decrease in $\theta$. The following proposition studies the impact of a change in this parameter on the ruler's choices.

Proposition 6: When government officials are more likely to usurp power, a ruler chooses a lower level of autonomy for officials, a higher tax rate, and a higher level of military spending.

Proof: Partial differentiation of (7a) and (7c) yields

$$\frac{\partial \Gamma_1}{\partial z}\frac{\partial \Gamma_3}{\partial \theta} - \frac{\partial \Gamma_1}{\partial \theta}\frac{\partial \Gamma_3}{\partial z} = -\frac{\Omega \sigma ab \ ^{1-\beta}L^\beta z^\sigma m^\rho}{(az^\sigma m^\rho + \Omega)^2} < 0.$$

Applying Cramer's rule on (8) yields

$$\frac{dt}{d\theta} = \frac{\partial \Gamma_2}{\partial m}\left(\frac{\partial \Gamma_1}{\partial z}\frac{\partial \Gamma_3}{\partial \theta} - \frac{\partial \Gamma_1}{\partial \theta}\frac{\partial \Gamma_3}{\partial z}\right)/\Delta > 0,$$

$$\frac{dm}{d\theta} = \frac{\partial \Gamma_2}{\partial t}\left(\frac{\partial \Gamma_1}{\partial \theta}\frac{\partial \Gamma_3}{\partial z} - \frac{\partial \Gamma_1}{\partial z}\frac{\partial \Gamma_3}{\partial \theta}\right)/\Delta > 0,$$

$$\frac{dz}{d\theta} = \frac{\partial \Gamma_2}{\partial t}\left(\frac{\partial \Gamma_1}{\partial m}\frac{\partial \Gamma_3}{\partial \theta} - \frac{\partial \Gamma_1}{\partial \theta}\frac{\partial \Gamma_3}{\partial m}\right)/\Delta < 0. \blacksquare$$

From Proposition 6, when government officials are more likely to usurp power, equilibrium probability of peasant rebellions increases because the equilibrium tax rate is higher. The intuition behind Proposition 6 is as follows. From equation (4) which is the first order condition for the ruler's optimal choice of the level of autonomy, a higher value of $\theta$ increases the marginal cost of autonomy and marginal benefit does not change. Thus, the ruler chooses a lower level of autonomy when $\theta$ increases. From equation (6), with a lower level of autonomy, marginal benefit of military spending increases while marginal cost does not change. Thus, the ruler chooses a higher level of military spending. From equation (5), with a higher military spending, marginal benefit of tax increases and marginal cost also increases. With $\frac{\partial \Gamma_2}{\partial m} > 0$, the impact from marginal benefit change dominates and the ruler chooses a higher tax rate.

Land can be increased through reclaiming unused land. The adoption of crops such as potato could make land previously useless now productive. The following proposition studies the impact of a change in the amount of land on the ruler's choices.

Proposition 7: A higher amount of land leads the ruler to choose a higher level of military spending and a lower level of autonomy for government officials. The impact on the tax rate is ambiguous, and a sufficient condition for $\frac{dt}{dT} < 0$ is that $\rho \geq 1$.

Proof: Applying Cramer's rule on (8) yields

$$\frac{dz}{dT} = \frac{\partial \Gamma_1}{\partial m}\frac{\partial \Gamma_2}{\partial t}\frac{\partial \Gamma_3}{\partial T}/\Delta < 0,$$

$$\frac{dm}{dT} = -\frac{\partial \Gamma_1}{\partial z}\frac{\partial \Gamma_2}{\partial t}\frac{\partial \Gamma_3}{\partial T}/\Delta > 0,$$

$$\frac{dt}{dT} = \left(\frac{\partial \Gamma_1}{\partial m}\frac{\partial \Gamma_2}{\partial T}\frac{\partial \Gamma_3}{\partial z} + \frac{\partial \Gamma_1}{\partial z}\frac{\partial \Gamma_2}{\partial m}\frac{\partial \Gamma_3}{\partial T} - \frac{\partial \Gamma_1}{\partial z}\frac{\partial \Gamma_2}{\partial T}\frac{\partial \Gamma_3}{\partial m}\right)/\Delta.$$

In general, the sign of $\frac{\partial \Gamma_1}{\partial m}\frac{\partial \Gamma_2}{\partial T}\frac{\partial \Gamma_3}{\partial z} + \frac{\partial \Gamma_1}{\partial z}\frac{\partial \Gamma_2}{\partial m}\frac{\partial \Gamma_3}{\partial T} - \frac{\partial \Gamma_1}{\partial z}\frac{\partial \Gamma_2}{\partial T}\frac{\partial \Gamma_3}{\partial m}$ is ambiguous and thus the sign of $\frac{dt}{dT}$ is ambiguous. When $\rho \geq 1$, $\frac{\partial \Gamma_2}{\partial m}\frac{\partial \Gamma_3}{\partial T} - \frac{\partial \Gamma_2}{\partial T}\frac{\partial \Gamma_3}{\partial m}$

$$= -\frac{\alpha a b \sigma (1-\beta) T^{-\beta} L^\beta (m^\rho - \rho m^\rho - \gamma L \rho m^{\rho-1})}{(tb^{\ 1-\beta}L^\beta - \gamma L - m)^2} + \frac{a\alpha^2(1-\beta)\sigma m^{\rho-1}(tb^{\ 1-\beta}L^\beta \rho - m)}{(tbT^{1-\beta}L^\beta - \gamma L - m)T} < 0.$$

Thus, when $\rho \geq 1$, $\frac{\partial \Gamma_1}{\partial m}\frac{\partial \Gamma_2}{\partial T}\frac{\partial \Gamma_3}{\partial z} + \frac{\partial \Gamma_1}{\partial z}\frac{\partial \Gamma_2}{\partial m}\frac{\partial \Gamma_3}{\partial T} - \frac{\partial \Gamma_1}{\partial z}\frac{\partial \Gamma_2}{\partial T}\frac{\partial \Gamma_3}{\partial m} > 0$ and $\frac{dt}{dT} < 0. \blacksquare$

A higher amount of land can be interpreted as resource abundance. If the amount of land is positively related to country size, then Proposition 7 shows that the level of autonomy of government officials may not increase when the country becomes larger. Proposition 7 shows that resource abundance helps reducing internal rebellions and handling external threats. The intuition behind Proposition 7 is as follows. From equation (6) which is the first order condition for the ruler's optimal choice of military spending, a higher amount of land increases marginal benefit and reduces marginal cost of military spending. Thus, the ruler chooses a higher military spending. From equation (4) which is the first order condition for the ruler's optimal choice of the level of autonomy, with a higher military spending, marginal benefit of autonomy decreases while marginal cost does not change. Thus, the ruler chooses a lower level of autonomy for officials. From equation (5) which is the first order condition for the ruler's optimal choice of the tax rate, a higher amount of land reduces the marginal benefit and increases the marginal cost of tax. However, a higher military spending increases the marginal benefit and reduces marginal cost of tax. Thus, in general the impact on the tax rate is ambiguous. With $\rho$ increases, from equation (6), marginal benefit of military spending decreases while marginal cost increases. Since the increase in military spending is smaller, the indirect effect from the change in military spending on marginal cost and marginal benefit of taxation is dominated by the direct effect of a change in the amount of land. Thus, the ruler chooses a lower tax rate.

While population grew significantly during peace time, data from Ge (1991) show that a substantial percentage of population could be destroyed during a war. One example is that only about one third of the population survived during the period of Three Kingdoms. If the amount of land is given, a higher population means a lower land-labor ratio. With diminishing marginal product of labor, a higher population means a lower per capita output (Zhou, 2009). When per capita consumption is lower than the subsistence level, peasant rebellions would result. Wars could lead to population reduction and a lower population pressure. The following proposition studies the impact of a change in population size on the ruler's choices.

Proposition 8: A higher population induces the ruler to choose a higher level of military spending and a lower level of autonomy for government officials. The sign of $\frac{dt}{dL}$ is ambiguous.

Proof: Applying Cramer's rule on (8) yields

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$$\frac{dz}{dL} = \frac{\partial\Gamma_1}{\partial m}\frac{\partial\Gamma_2}{\partial t}\frac{\partial\Gamma_3}{\partial L}/\Delta < 0,$$

$$\frac{dm}{dL} = -\frac{\partial\Gamma_1}{\partial z}\frac{\partial\Gamma_2}{\partial t}\frac{\partial\Gamma_3}{\partial L}/\Delta > 0,$$

$$\frac{dt}{dL} = \left(\frac{\partial\Gamma_1}{\partial m}\frac{\partial\Gamma_2}{\partial L}\frac{\partial\Gamma_3}{\partial z} + \frac{\partial\Gamma_1}{\partial z}\frac{\partial\Gamma_2}{\partial m}\frac{\partial\Gamma_3}{\partial L} - \frac{\partial\Gamma_1}{\partial z}\frac{\partial\Gamma_2}{\partial L}\frac{\partial\Gamma_3}{\partial m}\right)/\Delta.$$

Since the sign of $\frac{\partial\Gamma_1}{\partial m}\frac{\partial\Gamma_2}{\partial L}\frac{\partial\Gamma_3}{\partial z} + \frac{\partial\Gamma_1}{\partial z}\frac{\partial\Gamma_2}{\partial m}\frac{\partial\Gamma_3}{\partial L} - \frac{\partial\Gamma_1}{\partial z}\frac{\partial\Gamma_2}{\partial L}\frac{\partial\Gamma_3}{\partial m}$ is ambiguous, the sign of $\frac{dt}{dL}$ is ambiguous. ∎

While a higher population increases total output, the cost of providing public goods also increases. Thus, the impact of population change on the ruler's choice on the tax rate could be different from that of a change in the amount of land. From Proposition 8, there is no monotonic relationship between population size and tax rate. If $\frac{\partial\Gamma_2}{\partial L} > 0$, $\frac{\partial\Gamma_1}{\partial m}\frac{\partial\Gamma_2}{\partial L}\frac{\partial\Gamma_3}{\partial z} + \frac{\partial\Gamma_1}{\partial z}\frac{\partial\Gamma_2}{\partial m}\frac{\partial\Gamma_3}{\partial L} - \frac{\partial\Gamma_1}{\partial z}\frac{\partial\Gamma_2}{\partial L}\frac{\partial\Gamma_3}{\partial m} < 0$, then an increase in population size leads to a higher tax rate.

Frequently, when a parameter changes, the impact on the ruler's personal consumption is ambiguous. For example, when the level of external threats increases, while tax revenue increases, military spending also increases, thus the impact on the ruler's consumption is ambiguous.

## 5. Conclusion

Imperial China experienced unifications and divisions. Rulers kept tax rates low to reduce peasant rebellion, reduced the level of autonomy for government officials to lessen usurpation of power, and increased military spending to handle higher level of external threats. In this paper, we have used a mathematical model to study a ruler's institutional choices to maintain unification of China. We have established the following analytical results. First, we show that the ruler will choose a higher level of autonomy for officials and a higher tax rate if the level of external threats increases. That is, when external threats increase, the usurpation of power by government officials and peasant rebellions will increase endogenously. Second, when government officials are more likely to usurp power, the equilibrium probability of peasant rebellions also increase endogenously. Third, when peasants are more likely to rebel, the ruler chooses a higher level of autonomy for officials which would increase the possibility of usurpation and a lower level of military spending which would make handling external threats less successfully. Fourth, a higher level of ruler ability reduces the equilibrium probability of peasant rebellions and thus helps

maintaining political unity. Finally, an increase in state power may induce the ruler to choose a lower tax rate.

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