Managerial Responses to Incentives: Control of Firm Risk, Derivative Pricing Implications, and Outside Wealth Management

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by

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Abstract

We model a firm's value process controlled by a manager maximizing expected utility from restricted shares and employee stock options. The manager also dynamically controls allocation of his outside wealth. We explore interactions between those controls as he partially hedges his exposure to firm risk. Conditioning on his optimal behavior, control of firm risk increases the expected time to exercise for his employee stock options. It also reduces the percentage gap between his certainty equivalent and the firm's fair value for his compensation, but that gap remains substantial. Managerial control also causes traded options to exhibit an implied volatility smile.

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In this paper, we have a firm whose value process is dynamically controlled by an individual manager. That manager holds restricted shares in the firm as well as an employee stock option position. However, he is subject to dismissal for poor performance if the firm's value declines to a lower boundary. The manager also has external wealth that he can dynamically allocate between a riskless asset and an index fund which is correlated with firm value. He chooses the optimal joint control positions at the firm and for his outside wealth to maximize his expected utility. We examine how his restricted share and option positions influence that optimal control behavior. Typically, he alters firm risk through time depending on firm value relative to the exercise price of his option and distance to the lower boundary. This makes standard derivative pricing models inappropriate; however once we have identified his optimal dynamic behavior, we can value derivative securities on that controlled process from a market perspective using a risk-neutral pricing procedure.

Our analysis relates to a number of previous papers that addressed the valuation of employee stock options.[1] We also examine issues regarding restricted shares that relate to the analysis in Kahl, Liu, and Longstaff (2003) as well as Ingersoll (2006). The fundamental difference between our model and this previous work is the introduction of dynamic managerial control at the firm level. In some of these papers, there is a "one-shot" control in that managers can alter the firm's initial risk and/or return characteristics in response to the incentive structure of their compensation contracts. However, none of these papers allow the manager to dynamically control the firm through time. Allowing dynamic control has important implications for the manager's behavior through time and for the valuation of his restricted shares and employee stock options.

Not surprisingly, control at the firm level increases his personal certainty equivalent value (CEV) of restricted shares and options relative to the no-control situation. We are able to calculate the value of control and show that for reasonable parameters the value of restricted

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[1] A partial listing of such papers includes Cai and Vijh (2005), Carpenter (1998), Detemple and Sundaresan (1999), Hall and Murphy (2002), Henderson (2005), Huddart (1994), Ingersoll (2006), Ju, Leland, and Senbet (2003), Kulatilaka and Marcus (1994), plus Lambert, Larcker, and Verrecchia (1991).

shares and options is still far below the market value of comparable traded securities. This reinforces results from simpler models, without managerial control, which find that employee stock options are a relatively expensive form of compensation due to the large subjective discount between the employee's CEV and the market value of comparable options. In other words, control has value for the employee but not nearly enough to eliminate the subjective discount on his restricted shares and options.

We are also able to determine optimal exercise behavior for American-style employee stock options with and without managerial control over the firm value process. The existing literature (without control) shows that an expected-utility maximizing manager will optimally exercise early in situations where it would not be optimal to exercise tradable options. This is also true in our model; however, the manager's ability to enhance the value of his option by altering firm risk also induces him to generally delay early exercise. Typically, he will exercise early but some months later than he would without control of the firm value process.

Hedging is an important valuation aspect for restricted shares and options when the employee has access to an external portfolio whose return may be correlated with the firm's return process. If the correlation is non-zero, the employee can use his external wealth position to partially hedge his firm exposure. With dynamic control over the allocation of his external wealth between the riskless asset and the risky external portfolio, the employee can enhance his personal value of restricted shares and options via such dynamic external hedging.[2] Introducing such an external hedging capability allows us to explore both the effects of the external hedging on internal control decisions and vice versa. Internal and external control are partial, but typically very imperfect, substitutes for each other in maximizing managerial expected utility. For example, internal control allows greater risk-taking at the firm level in order to increase the

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[2] Kahl, Liu, and Longstaff (2003) have a similar hedging capability for restricted shares (no employee stock options) in a continuous-time framework but without any control at the firm level. Cai and Vijh (2005) plus Henderson (2005) and Ingersoll (2006) have related papers which also include an employee option position. Henderson has a European option in essentially the same framework as Kahl, Liu, and Longstaff. Again, she only allows manager control of the external portfolio but does explore the comparative static implications for CEV of differing correlations between firm returns and the external portfolio. Ingersoll has both employee option and restricted share positions. His model also includes control over the external wealth allocation but no control (not even one-shot) at the firm level. Cai and Vijh have an American-style option but even less control. In their paper, the manager can only choose his initial external wealth allocation. He has no subsequent external control and no control (ever) at the firm. None of these papers allows for dismissal of poorly performing managers as in our model.

manager's option value; and this tends to result in greater external hedging to reduce the manager's overall portfolio risk.

As a practical matter, managers do take sequences of actions over time to "control" a firm's expected profitability and risk. Moreover, the usual argument for employee stock options and restricted shares is to provide an incentive for firm managers to act in the shareholders' interest. Hence, it makes sense to examine how such securities influence managerial control behavior and how in turn this control changes security values.

Firm managers make decisions regarding real investments (e.g. a new plant or R&D project) as well as financing – both of which alter the firm's risk and return profile. In practice, real investment decisions are typical lumpy and costly to alter after the fact. Capital structure adjustments via new borrowing or loan repayments also tend to be lumpy; although, adjustment cost are generally much smaller than for real investments. An alternative, which we pursue here, is to allow the manager to adjust the firm's value process using forward contracts. Hence, we treat the firm's real investments as fixed – perhaps optimized for scale economies. We also treat its explicit borrowing position as fixed; however, hedging (or reverse hedging) with forward contracts will effectively alter firm leverage.

An issue is how much control a manager can exercise in practice. If the firm produces outputs or uses substantial amounts of inputs that have traded forwards or futures, its risk and expected return can be altered very quickly. There are numerous examples such as utilities hedging (or not) in electricity as well as natural gas markets, airlines with aviation fuel, oil producers as well as refiners, agricultural firms, mining companies, etc. Furthermore, any firm with substantial foreign exchange exposure can effectively have its stock price process altered rapidly using foreign exchange contracts. Typically, there will be limits on a manager's ability to hedge firm operations; and we include such bounds in our model.

The manager's restricted share position forces him to hold an otherwise suboptimal portfolio with more exposure to firm risk than he would like. In our model, with control at the firm level, this share position motivates him to reduce firm risk-taking as well as altering his outside holdings in correlated assets to further hedge his firm exposure. Adding an incentive option can induce him to undertake more risk when firm value is near the option's strike price. The combination of these two effects results in an interesting topography where risk-taking and hence firm volatility vary dramatically depending on the region of the state space.

Putting aside the manager's external hedging capability, this topography of firm risk-taking is related to results from the literature on delegated money management. The closest model to the current paper is in Hodder and Jackwerth (2007). That model has a single state variable and the control is over a hedge fund's value process. The compensation structure involves a management fee plus an incentive fee with a high-water mark, but that can be interpreted as analogous to restricted shares plus an employee stock option position. The focus in that paper is on determining the manager's optimal control rather than valuing option or restricted share positions. Furthermore, the single state variable means the manager can control fund risk but cannot simultaneously control his external wealth allocation so as to partially hedge his personal firm exposure. There are also related models in Basak, Pavlova, and Shapiro (2007), plus Cuoco and Kaniel (2006), as well as Carpenter (2000) which look at optimal control in the delegated money management framework. These papers do not have the lower boundary where the manager gets fired nor are they considering American-style options. Both those features are important, as is the external hedging ability. However, these papers do generate dynamic control behavior where the managed fund volatility can vary as a function of fund value. This is similar to the non-constant volatility we will see resulting from managerial control of the firm value process in the current paper.

Since the controlled process for firm value results in volatility which changes depending on the region of the state space, standard option pricing models (such as Black-Scholes) are inappropriate for valuing traded derivatives on such processes. However, we can condition on the manager's optimal control behavior and determine the appropriate risk-neutral process for firm value at any point in the state space. With that information, we can correctly value derivatives on that controlled process from a market perspective. This includes valuing traded derivatives as well as what the Financial Accounting Standards Board has termed the "Fair Value" for a non-tradable employee stock option. For traded derivatives, the controlled process results in an implied volatility smile that is endogenously generated and consistent with empirical evidence for stock options. This is another important implication of managerial control in our model.

For reasonable parameters, the manager's CEV can easily be 40% below Fair Value. For a lower-level employee, the gap could be even larger due to the lack of control capabilities (by assumption for such an employee) with perhaps higher risk aversion and less outside wealth. As a consequence, our analysis suggests strongly that employee stock options are apt to be a very expensive way to compensate lower-level employees -- particularly when such employees have no control over the firm value process.

In the next section, we describe our modeling approach and basic solution methodology. That methodology involves a dynamic bi-variate optimization on a three-dimensional grid, with more details provided in the Appendix. Section II explores managerial behavior regarding risk-taking at the firm level and his simultaneous investment decisions for external wealth. Section III examines the value of managerial control over firm risk-taking in terms of its effect on CEVs as well as how it influences the manager's early exercise decision for American-style employee stock options. In Section IV, we discuss determining appropriate market values for derivative securities conditional on the manager's optimal control of firm risk-taking. Section V discusses robustness, and Section VI provides concluding comments.

I. The Basic Model and Solution Methodology

We begin by exploring how stock-based compensation incentives influence a manager's dynamic control over a firm's investments as well as his optimal allocation of outside wealth. Exercise of control at the firm in turn affects the value of that firm's derivative securities, including employee stock options. The underlying notion is that managers are granted options as well as restricted stock to provide an incentive for operating the firm in the shareholders' interest. In what follows, we will assume that a single manager controls the stochastic process driving firm value. This is idealized in the sense that few firms are fully controlled by a single individual. However, it does allow us to examine how equity-linked incentives would influence such a manager's behavior. Once we have determined how the manager will control the firm's value process in different regions of the state space, we can value equity derivatives including both employee stock options and traded derivatives.

In formally describing the model, we will first specify stochastic processes for the firm's value and for an external portfolio in which the manager invests his outside wealth. Next, we discuss the manager's compensation conditional on both his pay package and the possibility of dismissal at a lower boundary. We then address how the manager optimally controls the firm value process as well as his outside investment portfolio to maximize his expected utility. Our approach utilizes a numerical procedure, with details on implementation available in the Appendix. Finally, we present and discuss a set of standard parameters that are used in much of our numerical analysis.

A. The Stochastic Process Structure

We allow the manager to have outside wealth Y, with an initial value Y_0, that can be invested in an index fund as well as the riskless asset. The constant riskless interest rate is denoted by r. One could think of the index fund as a proxy for the "market," analogous to the S&P 500 index; however, the expected utility framework we use is partial equilibrium. In that framework, an individual (the manager) will typically seek to place some wealth in both risky securities -- the index fund and the firm's risky technology. Absent constraints on his security holdings, our framework exhibits "two-fund separation," with the manager seeking to divide his wealth between the riskless asset and a portfolio of risky securities. We will use parameter values such that his desired (unconstrained) risky portfolio exactly matches the index fund and he would prefer to have no wealth invested in the firm's risky technology. This can be viewed as approximating a typical firm manager's desire to avoid non-systematic risk.

The index fund exhibits lognormal returns with mean m and standard deviation v. The manager can dynamically control his outside wealth allocation at discrete points in time, with α denoting the fraction invested in the index fund. In general, the manager's optimal α will depend on not only outside wealth but also the value of his restricted shares and employee stock options, which depend on the firm value X. Hence, α is short for $\alpha(X,Y,t)$. For a given proportion α invested in the index fund and a time step of length Δt, the manager's outside wealth Y exhibits normally distributed log returns with mean $m_{\alpha,\Delta t} = [\alpha m + (1-\alpha)r - \frac{1}{2}\alpha^2 v^2]\Delta t$ and volatility $v_{\alpha,\Delta t} = \alpha v\sqrt{\Delta t}$.[3] We introduce realistic lower and upper bounds on the proportion in the index fund α.

Assume the firm's operating assets are invested in a risky technology that generates lognormal returns with mean μ, standard deviation σ, and correlation ρ with the index fund returns. Also assume there exist forward contracts which can be used to hedge (perhaps only partially) the firm's risk from this technology. Let X denote the value of the firm's assets and κ the fraction of those assets which are unhedged. That is, $(1-\kappa)X$ represents hedged assets, which are riskless and earn the riskless interest rate r. Our manager controls κ, which is short for $\kappa(X,Y,t)$. The manager's control of κ does not incur any deadweight costs and hence does

[3] Our approach is not restricted to normal distributions but can accommodate any distributional structure that can be reasonably represented with a bivariate multinomial approximation.

not alter the current value of X. However, that control will affect firm volatility and the value of derivatives where X is the underlying asset.

Typically, some portion of the firm's operations cannot be hedged. We model this situation by having a positive lower bound on κ. We also allow for the possibility that the manager can increase firm risk by choosing κ values greater than one but subject to some upper bound. An alternative way to think about this model structure is to interpret κ values greater than one as representing a levered firm, with the manager able to dynamically adjust the firm's capital structure. To provide a benchmark for comparisons, we assume the firm's shareholders would prefer it operate at $\kappa = 1$.[4] However, those shareholders are willing to tolerate some managerial discretion in the choice of kappa.[5] To model differing degrees of managerial flexibility, we will utilize differing bounds on κ. For a given control value κ, the log returns on the firm value X are normally distributed over each discrete time step of length Δt with mean $\mu_{\kappa,\Delta t} = [\kappa\mu + (1-\kappa)r - \frac{1}{2}\kappa^2\sigma^2]\Delta t$ and volatility $\sigma_{\kappa,\Delta t} = \kappa\sigma\sqrt{\Delta t}$.

We discretize log values for the firm and for outside wealth onto a grid structure (more details are provided in the Appendix). That grid has equal time increments as well as equal steps in the dimensions for firm value (log X) and outside wealth (log Y). From each grid point, we allow a bivariate multinomial forward move to a relatively large number of subsequent grid points (e.g. 31x31) at the next time step. We structure potential forward moves to land on grid points and calculate the associated probabilities by using the discrete bivariate normal distribution with specified values for the control parameters κ and α.

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[4] With full information and costless trading, the shareholders would be indifferent to the manager's actions since they could costlessly undo his hedging behavior by trading in their personal accounts. More realistically, trading frictions and borrowing limitations at the retail level as well as potential differential tax effects will cause shareholders to care about managerial hedging. Rather than introducing such complexities, we simply posit a shareholder preference for $\kappa = 1$.

[5] Presumably, the manager was hired to add value via exercising skill or expending effort in project selection or management. We do not attempt to model such aspects of the manager's employment but rather focus on his ability to adjust the firm's risk-taking via a relatively simple portfolio structure.

B. The Manager's Compensation Structure

The manager owns a non-negative fraction b of the firm's shares, which are restricted and cannot be sold prior to T. He also has employee stock options for a fraction c of the firm's shares.[6] These options have a maturity of T and are issued at-the-money. At the horizon time T, the manager's total wealth includes his outside wealth Y_T plus the value of his firm shares and employee stock options.[7] That is,

$$W_T = bX_T + c(X_T - X_0)^+ + Y_T \tag{1}$$

We allow the firm to dismiss the manager for poor performance. This is modeled by having a lower boundary on X, where the manager is fired. We denote that boundary by Φ, and set it at 50% of the initial firm value X_0. There are a variety of assumptions one could make about the wealth impacts of being fired. Many CEOs and other high-level executives have employment contracts that specify a severance payment in the event of termination. If that payment is large, it is often described as a "Golden Parachute." On the other hand, being fired could negatively impact an individual's human capital due to a loss of reputation and make the manager more cautious near the lower boundary. We utilize a middle ground with only a nominal penalty by assuming the manager's option position (which is out-of-the-money) is cancelled and his restricted shares are immediately liquidated at the prevailing price. If the lower boundary is hit at time τ, the manager's wealth at that time equals the proceeds from the liquidated shares plus his outside wealth:

$$W_\tau = bX_\tau + Y_\tau \tag{2}$$

That wealth can then be managed until the horizon date T by investing optimally in the index fund and the riskless asset, with the fraction invested in the index fund α no longer depending

[6] We will use $b + c \ll 1$ so that his stock and option positions do not represent a significant dilution for outside shareholders. Dilution effects can be accommodated with no qualitative changes in our results.

[7] To simplify the analysis, we assume that the manager's cash salary is matched by his consumption and so does not enter the terminal wealth calculation.

on firm value. This then determines his wealth W_T at the horizon date after having been fired at time τ.

C. The Optimization of Expected Utility

We assume the manager seeks to maximize expected utility of terminal wealth W_T and has a utility function that exhibits constant relative risk aversion γ (an assumption that can be relaxed):

$$U = \frac{W_T^{1-\gamma} - 1}{1-\gamma} \tag{3}$$

For each possible terminal combination of firm value and outside wealth, we calculate the manager's wealth and the associated utility. We then step backwards in time to $T-\Delta t$. At each possible combination of firm value and outside wealth within that time step, we calculate the expected utilities for all control value pairs (κ, α) in a discrete choice set. For example, much of our analysis assumes bounds on the manager's control of firm risk such that κ lies between 0.75 and 1.5, where we use steps of 0.125. For the optimal investment proportion α in the index fund, we also use steps of 0.125 between a minimum value of -0.5 and a maximum of 1.5. We choose the highest of the calculated expected utilities as the optimal indirect utility for that grid point and denote its value as $J_{X,Y,T-\Delta t}$. We record the optimal indirect utilities plus the associated optimal firm level risk-taking and optimal proportion in the index fund for each grid point within that time step and then loop backward in time, repeating this process through all time steps.[8] This generates the indirect utility surface and optimal control values for our entire grid. Formally:

$$J_{X,Y,T} = U_{X,Y,T}; \quad J_{X,Y,t} = \max_{\kappa,\alpha} E[J_{X,Y,t+\Delta t}] \tag{4}$$
where t takes the values $T-\Delta t, \ldots, 2\Delta t, \Delta t, 0$ one after another.

[8] The treatment of boundary layers is described in the Appendix.

D. A Set of Standard Parameters

In the analysis that follows, we will use variations on a standard set of parameters listed in Table 1. A more detailed discussion of robustness to variation of these parameters is in Section V below. There, we investigate changes in the numerical setup, penalties for performing poorly, and comparative statics. Most of the numerical results change only slightly, and the overall economic story remains the same.

Table 1: Standard Parameters

General:					
Time to maturity	T	1	Interest rate	r	0.04
Number of time steps	n	12			
Manager:					
Manager's risk aversion coefficient	γ	2	Manager's shares	b	0.0050
Manager's initial outside wealth	Y_0	0.0050	Manager's options	c	0.0100
Firm:					
Initial firm value	X_0	1.00	Dismissal boundary	Φ	0.50
Firm mean return	μ	0.076	Firm volatility	σ	0.30
Min firm risk-taking	$\underline{\kappa}$	0.75	Max firm risk-taking	$\overline{\kappa}$	1.50
Outside Wealth:					
Fund mean return	m	0.08	Fund volatility	v	0.20
Min fund proportion	$\underline{\alpha}$	-0.50	Max fund proportion	$\overline{\alpha}$	1.50
Correlation with firm risk	ρ	0.60			

The horizon is one year, with monthly adjustments of risk-taking in the firm κ and the manager's investment proportion α in the index fund. The starting firm value of 1 equals the strike price for the manager's incentive options (employee stock options are almost always granted at-the-money). If firm value drops below 0.5, the manager is fired. The risky technology has a mean return of 7.6% and a volatility of 30%. The index fund has a mean return of 8% and a volatility of 20%. The correlation coefficient is 0.6 between these two processes. The riskless asset yields 4%. The risk aversion coefficient of the manager's power utility is $\gamma = 2$. For our standard parameters, we use bounds on risk-taking in the firm κ of 0.75 and 1.5. Thus our standard parameter structure allows the manager to reduce but not eliminate the firm's risk. He can also increase that risk but in a limited manner. For his outside wealth, we use bounds on the fraction α invested in the index fund of -0.5 and 1.5. Thus, he can short as well as take a levered position in the index fund but is restricted from taking extreme positions.

The manager's initial wealth includes his outside wealth of 0.005, a restricted share position that is worth 0.005 (given the initial firm value of 1), and his options on a fraction 0.010 of the firm's shares. To provide a sense of scale, suppose the firm had a value of $\$1$ billion. The manager would have $\$5$ million of restricted firm shares and outside wealth of $\$5$ million. His option position is initially at-the-money and given the other parameters in Table 1 would have a Black-Scholes value of approximately $\$1.375$ million with no managerial control of the firm's risk-taking.[9] As we shall see, the Black-Scholes formula does not correctly value the manager's option position; but it gives us a rough sense of the relative value of that position compared with his restricted shares and outside wealth.

II. Managerial Risk-Taking and External Wealth Management

Before analyzing the manager's behavior in response to an incentive option position, we first consider how he manages firm risk when he has only restricted shares plus his outside wealth. With the parameter values in Table 1, he will immediately reduce firm risk-taking to the lower bound of $\kappa = 0.75$ and keep it there until the horizon date (unless firm value declines to the level where he is fired). If he could further reduce firm risk-taking, he would do so.[10] His

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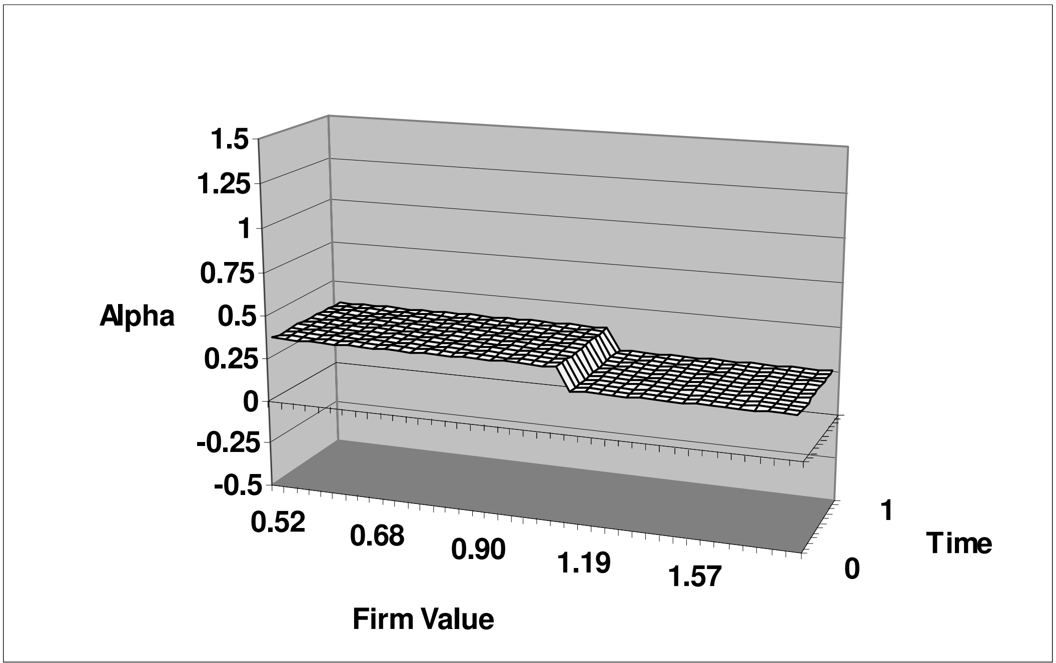
[9] The Black-Scholes calculation is for a one-year, at-the-money call option with 30% volatility (no control) and a riskless rate of 4%. The manager's incentive option is actually a "knock-out call" which becomes worthless if firm value hits the dismissal boundary Φ where he is fired. However with our standard parameters and no managerial control, the probability of hitting the dismissal boundary is very small; and the analytic value for that knock-out call differs from Black-Scholes only in the seventh decimal place.

[10] With just shares (no incentive option), our manager's behavior can be compared to that of a Merton (1969) investor. If the manager could not be fired and had no constraints on his controls, he could achieve a Merton optimal portfolio indirectly by altering the firm's risk-taking. With

restricted share position is forcing him to over-invest in a relatively undesirable asset (the firm's risky technology), and he wants to reduce his exposure to that asset. By reducing firm risk-taking, he effectively lowers his personal exposure to that risky technology.

Figure 1: Managerial Investment in the Index Fund (α)

We depict the optimal alpha surface as a function of firm value X and time to maturity using the standard parameters from Table 1 except that the manager does not have an incentive option, c = 0. Throughout this figure, the manager is holding kappa at its lower bound of 0.75. The alpha levels in this graph are 0.125 apart and can range from -0.5 to 1.5.



With this background, let us now examine Figure 1 which depicts the manager's optimal investment in the index fund when he has some control over firm risk-taking and no incentive option position. With our standard parameters, the manager can only reduce firm risk-taking to $\kappa = 0.75$ and cannot achieve the Merton optimum (indirectly) by eliminating firm risk. This

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our standard parameter values, the Merton investor would optimally invest nothing in the firm's risky technology, place 50% of his wealth in the index fund, and have the other 50% in the riskless asset. Despite holding restricted shares, the manager could achieve that portfolio by reducing firm risk-taking to $\kappa = 0$ which has the effect of transforming his restricted shares into a riskless investment. With half his initial wealth outside the firm and $\alpha = 1$, he could thus attain the Merton optimum.

leads to his optimally investing a much smaller fraction of his outside wealth in the index fund than he would if the constraint on firm risk-taking were not binding. Furthermore, α is decreasing in firm value X. This behavior differs dramatically from the Merton optimum but is relatively easy to understand. The manager reduces his index fund investment to partially hedge his firm risk exposure, which he cannot eliminate because of the lower bound on his control of κ. In Figure 1, his outside wealth Y is held constant. So an increase in firm value X means the manager's fraction of wealth exposed to firm risk increases, and he responds by lowering α to hedge (partially) that increased exposure. In other words, the combination of restricted firm shares and his inability to lower firm risk-taking to $\kappa = 0$ forces the manager to be exposed to the firm's risky technology. That exposure has an interesting spill-over effect on his outside investment behavior as the manager adjusts his index-fund position to partially hedge his firm-risk exposure.[11]

Our standard parameters were chosen such that the manager does not want to invest any of his wealth in the firm's risky technology. This is intended to proxy for a situation where the manager does not want to hold diversifiable risk. The manager's natural response is to reduce firm risk as much as possible. With a lower bound on firm risk-taking at $\kappa = 0.75$, he hedges the firm down to that level and holds it there. If shareholders want the firm to operate at a higher risk level such as $\kappa = 1$, they will have to provide the manager with an incentive in addition to that provided by restricted shares. A typical response to this problem has been to award stock options that provide an incentive for the manager to undertake greater firm risk.[12] We now introduce such an incentive option and examine the manager's optimal control of risk-taking at the firm level as well as his outside wealth allocation.

With an incentive option of sufficient size, our manager is not only more willing to undertake risk on behalf of the firm, he will seek to substantially increase risk beyond $\kappa = 1$ in major portions of the state space. Toward the center of Figure 2, there is a region of high risk-taking which we label "Option Ridge". This region is centered just below the option strike price

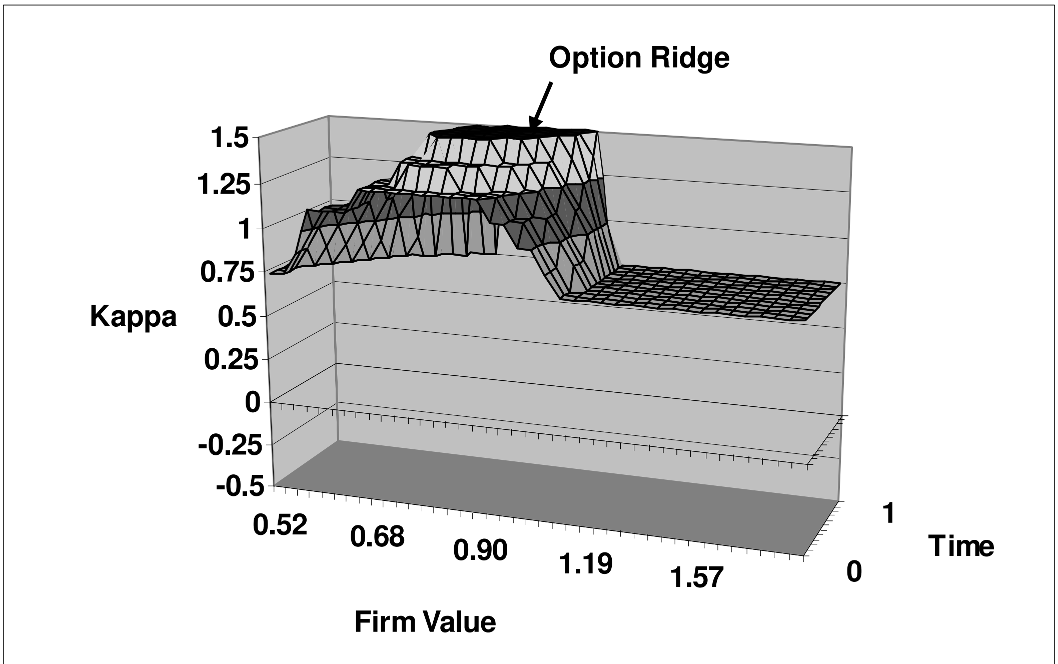
[11] Kahl, Liu, and Longstaff (2003) also find such hedging behavior by their manager, whom they model as having no control over the firm value process. Thus, a lower bound on managerial control that binds throughout the state space leads to qualitatively similar investment behavior regarding his external wealth as when he had no control at the firm. The difference is that partial control over firm risk-taking allows the manager to reduce his firm exposure somewhat; and this results in less hedging with his index-fund position (α value closer to the Merton unconstrained optimum of 1).

[12] See Core, Guay, and Larker (2003) section 3.5 for a discussion of this issue.

of 1. Here, the manager dramatically increases the firm's riskiness in order to increase the chance of finishing with his option substantially in-the-money. At lower firm values, the level of risk-taking declines somewhat but remains at or above $\kappa = 1$ almost all the way to the lower boundary. The height and horizontal spread of Option Ridge depend on the size of the incentive option. The flat top of Option Ridge indicates that the manager would like to increase kappa even further in that area but is limited to the maximum of $\kappa = 1.5$.

Figure 2: Management of Firm Risk (κ) when Holding an Employee Stock Option

We depict the optimal kappa surface as a function of firm value X and time to maturity using the standard parameters from Table 1 with outside wealth at its initial value of 0.005 and alpha (shown in Figure 3 below) optimally chosen. The kappa levels in this graph are 0.125 apart and can range from 0.75 to 1.5.



To the right in Figure 2, Option Ridge drops downward to a flat region with kappa at the minimum level of 0.75. As the manager's incentive option becomes sufficiently deep in-the-money, it starts to behave almost like a share position and the manager returns to minimizing κ as he did with just restricted shares. If he could lower κ further, he would do so. Moving toward the front of Figure 2, we have greater time to maturity and Option Ridge also slopes down toward minimum risk-taking. The convexity of the option payoff is diminished by increased time to maturity and again starts to have incentive effects like a share position. We have examined the situation for options having longer maturities. With our standard parameters, Option Ridge only appears during approximately the last 15 months until maturity. Prior to that, the manager keeps firm risk-taking at the minimum $\kappa = 0.75$, as if he had only restricted shares.

The size of Option Ridge is also influenced by the manager's outside wealth, and he is more willing to take risks at the firm with greater outside wealth. This can result in an Option Ridge which is wider and higher (subject to the maximum risk-taking constraint). A similar effect would be generated if his restricted share position were smaller compared with outside wealth. Thus, the addition of an incentive option can substantially increase managerial risk-taking at the firm level. The size of his option position, value of restricted shares, and outside wealth all influence how he controls firm risk-taking. Figure 2 also illustrates that limits on the manager's control of the firm's value process can play a big role. The fact that his control of kappa reverts back to the lower bound at higher firm values suggests that further incentive options with higher exercise prices (perhaps awarded as a sequence through time) would be needed to induce him to maintain firm risk above $\kappa = 0.75$ as firm value increases.

Figure 3: Manager's Optimal Index Fund Investment (α) when Holding an Employee Stock Option

We depict the manager's optimal fraction of outside wealth to invest in the index fund as a function of firm value X and time to maturity with outside wealth at its initial value of 0.005. This surface is based on the standard parameters from Table 1, with kappa optimally chosen as shown in Figure 2. The alpha levels in this graph are 0.125 apart and can range from -0.50 to 1.5.

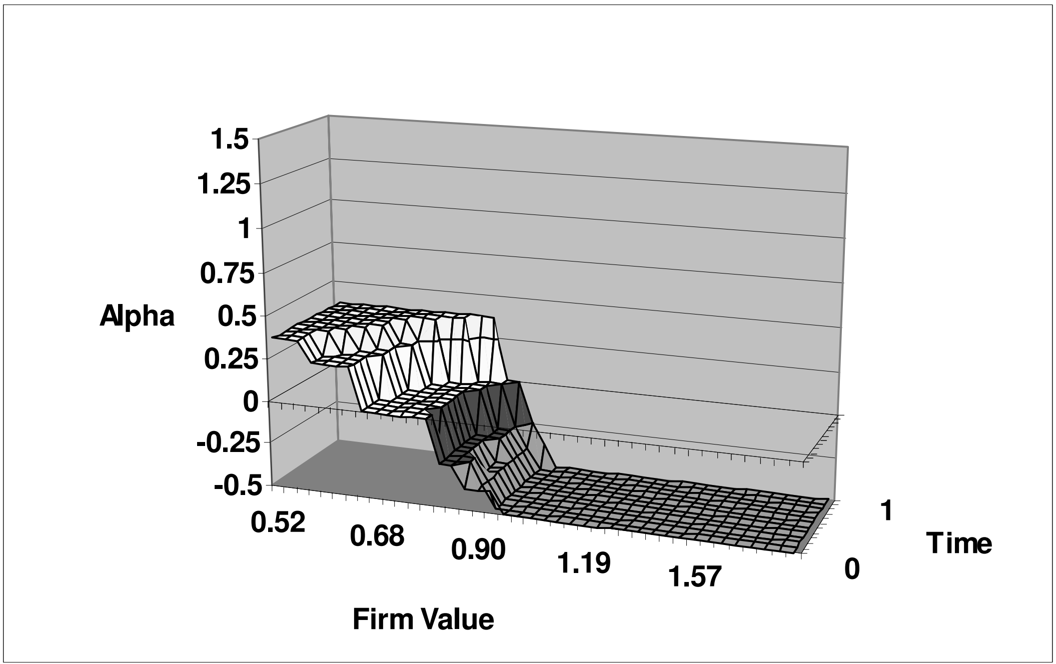
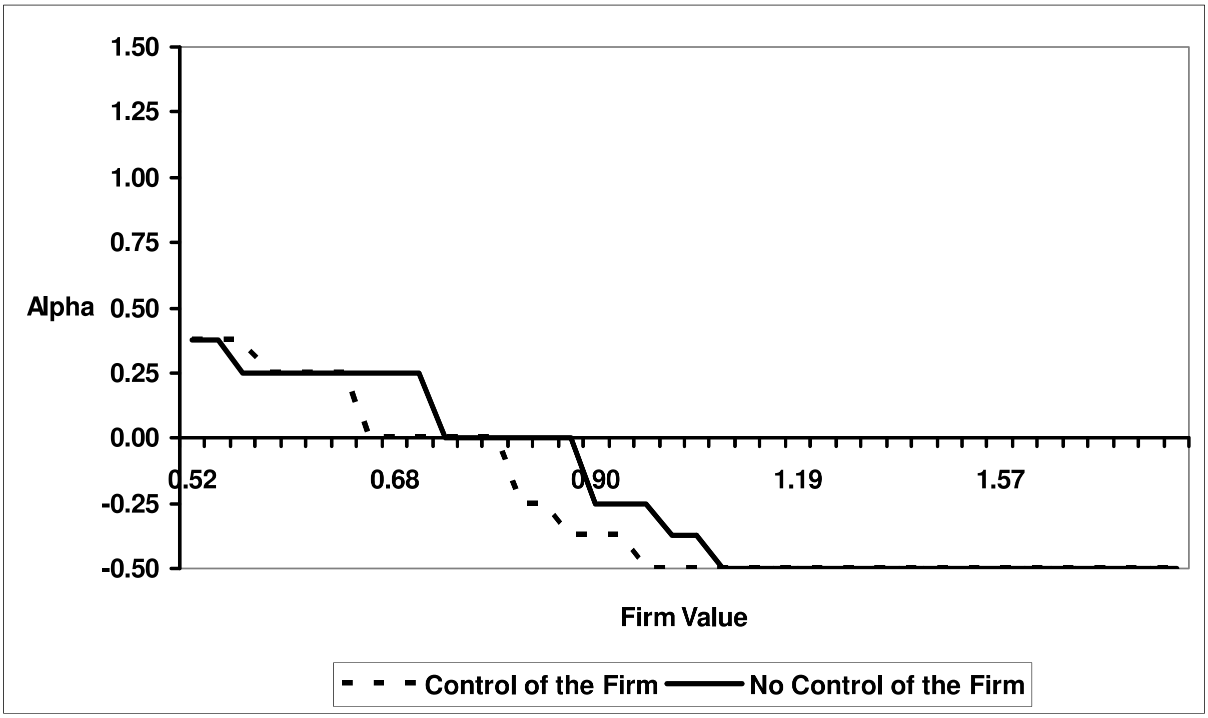


Figure 3 displays the optimal proportion α of the manager's outside wealth Y to invest in the index fund as a function of time to maturity and firm value X. The manager is also optimally controlling firm risk κ as shown in Figure 2, and his control of α takes that into account. Comparing Figure 3 with Figure 1 (where he had no incentive option), we can see that the manager's option position at the firm causes a dramatic difference in his control of outside wealth. His option position has two effects on α that largely reinforce each other.

First, there is the effect of simply owning an option, absent any control ability at the firm. A call option can be dynamically replicated with a levered share position. As X increases, the share component (option delta) increases, and the manager responds by decreasing α to provide a larger hedge to the increasing firm exposure. This effect is illustrated in Figure 4 by the line "No Control of the Firm". Here, firm risk is held constant at $\kappa = 1$; and there is one year to maturity. The downward step in α at the far left of the No Control line from a value of 0.375 to 0.25 is due largely to the increasing value of the manager's share position (the option delta is low at that X value) and is analogous to the downward adjustment near the middle of Figure 1. Note that with $\kappa = 1$, that adjustment occurs at a lower X value in Figure 4 since the firm's shares are riskier than in Figure 1 (where $\kappa = 0.75$). At roughly $X = 0.70$, the increasing option value starts to have an impact and results in α declining relatively rapidly until reaching its lower bound of $\alpha = -0.5$.

Figure 4: Manager's Optimal Index Fund Investment (α) with and without Control at the Firm

We depict the manager's optimal fraction of outside wealth to invest in the index fund as a function of firm value X both with and without control over firm risk-taking. The manager holds both shares and an option position in the firm. Time to maturity is one year, and the manager's outside wealth equals its initial value of 0.005. The analysis uses standard parameters from Table 1 except that when the manager has no control, kappa is fixed at a value of 1. The alpha levels in this graph are 0.125 apart and can range from -0.50 to 1.5.



The second effect of the manager's incentive option on optimal allocation of outside wealth comes from his ability to control firm risk. The option induces the manager to increase firm risk-taking on Option Ridge (shown in Figure 2), which increases the risk of both his restricted shares and the option position due to the higher κ. He offsets that increased risk on his firm securities by decreasing α. That is, increased firm volatility leads to an adjustment of the hedge position implicit in his outside wealth portfolio. This effect results in the substantial divergence between the Control and No Control lines in Figure 4 .

It is clear that employee stock options can have a major impact on optimal management of outside wealth. Particularly when the manager can control firm risk, his optimal allocation of outside wealth changes dramatically with the probability of his option position finishing in-the-money. More generally, when a manager can control firm risk, all option holders should be adjusting their positions in correlated assets as the manager alters firm risk. This issue has received essentially no attention in the literature, perhaps because previous papers did not allow dynamic managerial control over firm risk.

III. Value of Managerial Control and Early Exercise Behavior

Control over firm risk-taking is valuable to the manager, even when he has only restricted shares (no incentive option) and his control is partially constrained. For assessing the value of restricted shares and/or options to the manager, we use a certainty equivalent value based on the amount of cash that optimally invested outside the firm would generate an indirect utility of J_0.

$$CEV = e^{-rT}[1 + J_0(1-\gamma)]^{1/(1-\gamma)} \tag{5}$$

For comparisons later in the paper, we will need the CEV on a per-unit basis, i.e. the CEV of one option or of one share. We estimate this numerically using the sensitivity of the lump-sum amount to a 1% increase in the restricted share position or the option position, as relevant.

We examine the value of control with just restricted shares in Table 2. Not surprisingly, the value of control increases with risk aversion and with the time horizon until his share sale restriction lapses. We also see that the value of control depends on how much of the manager's wealth is tied up in restricted shares. The greater the percentage of wealth he has outside the firm, the less value he derives from control over firm risk-taking.

Table 2: Value of Managerial Control with Just Restricted Shares

We report the certainty equivalent values of a restricted share using the standard parameters from Table 1 except as indicated. For the "No Control" values, the manager's control of firm risk-taking is eliminated so that κ = 1 everywhere. Outside Wealth is reported as a percentage of total wealth.

Outside Wealth (%)	1-Year Restriction Horizon				5-Year Restriction Horizon			
	With Control	No Control	Value of Control Difference	Value of Control Percent	With Control	No Control	Value of Control Difference	Value of Control Percent
		gamma = 2				gamma = 2		
20	0.9792	0.9537	0.0255	2.67%	0.9078	0.8240	0.0839	10.18%
50	0.9858	0.9673	0.0185	1.91%	0.9381	0.8697	0.0684	7.87%
80	0.9979	0.9893	0.0085	0.86%	0.9911	0.9514	0.0397	4.17%
		gamma = 3				gamma = 3		
20	0.9580	0.9167	0.0413	4.50%	0.8275	0.7249	0.1027	14.17%
50	0.9712	0.9451	0.0261	2.76%	0.8772	0.7901	0.0870	11.02%
80	0.9892	0.9731	0.0160	1.65%	0.9500	0.8882	0.0617	6.95%

We now consider the manager's employee stock option position and display in Table 3 the certainty equivalent value for such an option. As we saw with restricted shares, the value of control for our manager increases substantially with maturity because he has longer to exercise that control. With an incentive option, this means he has longer to improve the probability of his option finishing substantially in-the-money. Control is also more valuable as a percentage of the option CEV if the manager is more risk averse. A larger percentage of outside wealth increases the value of control in absolute terms but the percentage effects depend on option maturity. We can also see in Table 3 that increasing the manager's outside wealth improves his option's CEV (both with and without control), whereas increasing relative risk aversion has the opposite effect.

Table 3: Value of Managerial Control

We report the certainty equivalent values of an employee stock option struck at $X_0 = 1$ and using the standard parameters from Table 1 except as indicated. For the "No Control" values, the manager's control of firm risk-taking is eliminated so that $\kappa = 1$ everywhere. The Outside Wealth percentage is measured without including the value of his incentive option.

Outside Wealth (%)	1-Year Option Maturity				5-Year Option Maturity			
	With Control	No Control	Value of Control Difference	Value of Control Percent	With Control	No Control	Value of Control Difference	Value of Control Percent
		gamma = 2				gamma = 2		
20	0.0704	0.0625	0.0079	12.65%	0.1073	0.0745	0.0327	43.91%
50	0.0926	0.0767	0.0159	20.74%	0.1508	0.1098	0.0410	37.32%
80	0.1106	0.0901	0.0205	22.79%	0.1889	0.1472	0.0417	28.36%
		gamma = 3				gamma = 3		
20	0.0452	0.0381	0.0071	18.51%	0.0494	0.0284	0.0210	73.71%
50	0.0666	0.0536	0.0130	24.20%	0.0883	0.0554	0.0329	59.40%
80	0.0889	0.0708	0.0181	25.57%	0.1337	0.0909	0.0427	47.01%

The results for absolute value of control in Table 3 are of roughly similar magnitude to those in Table 2; however in percentage terms, control is substantially more valuable for options compared with restricted shares. For example with standard parameters, the value of control associated with the manager's option position in Table 3 is 0.0159 which represents over 20% of the no-control option CEV. With just restricted shares in Table 2, the comparable value of control is 0.0185; but that is less than 2% of the no-control CEV for that share position.

These are European options, and their value typically increases with maturity. However, there is an interesting counter-example in Table 3 when the manager has No Control, 20% Outside Wealth, and gamma = 3. Then, his CEV drops from 0.0381 with a 1-year option to 0.0284 for a 5-year option. The key here is that he has relatively high risk aversion with most of his wealth concentrated in the firm and no control. This results in a strong aversion to firm risk exposure with no ability to control that risk, and the effect is strong enough to cause the CEV to decline with maturity.

To provide a yardstick for comparison, the Black-Scholes value of a 1-year option struck at-the-money is 0.1375, using a 30% volatility (corresponding to a fixed $\kappa = 1$) and a riskless interest rate of 4%. As we shall discuss in the next section, Black-Scholes is not the appropriate benchmark when managerial control results in firm volatility which varies across the state space;

however, this insight will still be valid with a more appropriate benchmark. The Black-Scholes value for a comparable 5-year option is 0.3396. The CEVs in Table 3 are all substantially below these values, even with managerial control. In several earlier papers, CEVs for employee stock options were found to be much less than the corresponding Black-Scholes values.[13] As indicated in Table 3, managerial control reduces that gap in value but typically does not come close to eliminating it. The added insight here is that considering the value of control to the manager is not apt to overturn concerns that options are a relatively expensive form of compensation.

Most employee stock options are American and can be exercised prior to maturity. Moreover, there is considerable evidence that managers do exercise early even if their firms do not pay dividends.[14] This is illustrated in Table 4, where the CEVs of American options exceed those for their European counterparts – indicating the possibility of early exercise despite the firm not paying dividends. This reflects the manager's desire to reduce exposure to firm-specific risk.

Table 4: Comparison of CEVs for European and American Incentive Options

We report the certainty equivalent values for European and American employee stock options struck at $X_0 = 1$. We use the standard parameters from Table 1 except as indicated. Moneyness is measured as the strike divided by the share price.

	1 Year		**5 Years**	
Moneyness	**European**	**American**	**European**	**American**
0.8	0.1973	0.2114	0.2266	0.2682
0.9	0.1354	0.1441	0.1848	0.2165
1.0	0.0926	0.0983	0.1508	0.1753
1.1	0.0633	0.0671	0.1233	0.1426
1.2	0.0430	0.0457	0.1009	0.1164

We can calculate the expected time to exercise both with and without control over firm risk-taking. That is, we record the time for each early or terminal exercise and calculate the expected time to exercise as we step backward through the grid. At the initial date, we are then

[13] With moderate risk aversion and a substantial portion of his wealth (say 50%) in the firm's securities, the manager's CEV can be less than half the Black-Scholes value. For example, see Hall and Murphy (2002, Table 1) or Ingersoll (2006, Table 1).

[14] Bettis, Bizjak and Lemmon (2005) contains an extensive analysis of early exercise behavior. Other papers examining early exercise behavior include Carpenter (1998) as well as Huddart and Lang (1996).

left with expected time in years that the manager will wait until exercising his option.[15] Figure 5 displays results of such calculations for an American-style employee stock option which is at-the-money and has 5 years remaining to maturity. Notably, the manager tends to exercise that option earlier when he has no control over firm risk. For example with 50% outside wealth and control, his expected time to exercise is 3.42 years; but without control it is only 2.78 years.[16] That difference represents almost 8 months. Having a greater fraction of his wealth in restricted firm shares (smaller outside wealth percentage) increases that difference in expected time to exercise. Increased risk aversion (not shown in Figure 5) reduces the expected time to exercise both with and without control of firm risk-taking.

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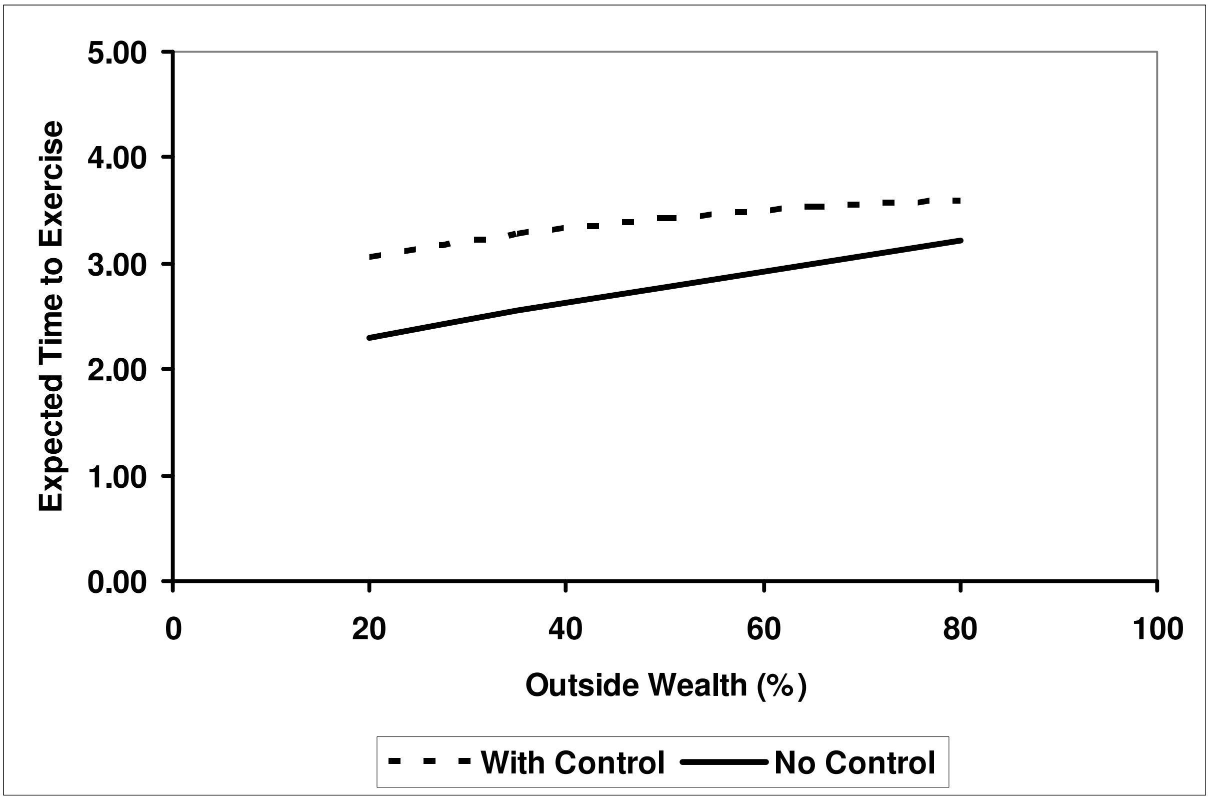
[15] Our approach is similar to Garman (1989) except that we use actual probabilities rather than risk-neutral probabilities in calculating the expected time to exercise.

[16] Interestingly, the manager's ability to control his outside wealth also plays a role in the early exercise decision. If he could only invest outside wealth in the riskless security, his expected time to exercise with control of firm risk-taking would increase from 3.42 to 3.51 years. Investing the proceeds of option exercise only in the riskless asset is less attractive and causes the manager to delay exercise so that the firm stock price can potentially reach a higher level (at a later expected date).

Figure 5: Expected Time to Exercise

We plot the expected time to exercise for an American employee stock option struck at $X_0 = 1$ and 5 years remaining to maturity. We use the standard parameters from Table 1 except as indicated. For the "No Control" values, the manager's control of firm risk-taking is eliminated so that $\kappa = 1$ everywhere. The Outside Wealth percentage is measured without including the value of his incentive option.



There is some empirical evidence consistent with managerial control delaying the exercise date of employee stock options. Bettis, Bizjak, and Lemmon (2005) provide regression estimates that CEOs tend to exercise their options about four months later than other senior executives and six months later than non-management directors. They view these results as due to CEOs tending to have greater outside wealth and possibly less risk aversion. However, many outside (non-management) directors are CEOs of other firms and apt to have a relatively large proportion of their wealth outside the firm under study. Unless they are generally more risk averse, this would suggest observing non-management directors exercising later than the firm's CEO. Our model suggests another explanation based on differing control abilities, with CEOs having the most control and non-management directors the least. One would expect all three factors (outside wealth, risk aversion, and control capability) to play roles in the exercise decision; however, control differences could generate the exercise behavior observed by Bettis, Bizjak, and Lemmon.

IV. Valuing Derivatives on Controlled Processes

Once we have determined the manager's optimal control over firm risk-taking κ, we can then value derivative securities on that controlled process from a market perspective. This simply entails sweeping back through our grid using the manager's optimal κ at each grid point to determine risk-neutral probabilities of moving to subsequent grid points. Since market investors are not restricted in trading firm shares or the market index and are not restricted to a monthly trading frequency, they face a market which is dynamically complete. Thus, we can determine their risk-neutral density of share prices and of payoffs for a specified derivative security. Taking the expectation over that density and discounting at the riskless rate, we obtain that security's market value.

An obvious application of this procedure is determining what the Financial Accounting Standards Board (FASB) has called Fair Value for an employee stock option. The FASB has stated that when possible, Fair Value is to be determined from the market price of the same or a similar option. Absent the availability of such a market price, Fair Value is to be estimated using an option pricing model. For standard option pricing models, the firm's potentially non-constant volatility (as in Figure 2) is a problem; however, that presents no difficulty for our approach.

Using the standard parameters from Table 1, our manager's incentive option would have a Fair Value of 0.1474 if it were European compared with its CEV of 0.0926. That represents a valuation difference of slightly over 59%. If the incentive option were American, its Fair Value would be 0.1414 compared to a CEV of 0.0983, a difference of almost 44%. The lower Fair Value for an American option is caused by the manager's early exercise decision, which is sub-optimal (value destroying) in a risk-neutral pricing framework but optimal for his personal expected utility maximization.

With both European and American incentive options, the difference between the Fair Value and CEV is substantial. In either case, the manager would value his option at considerably less than Fair Value. This is illustrated in Table 5 for European options. American options display similar characteristics. The manager's CEV decreases as a fraction of Fair Value for longer maturity options and with greater managerial risk aversion. Having greater outside wealth and hence less wealth tied up in the firm results in an increased CEV relative to Fair Value. Examining Table 5, it is clear that options are an expensive form of managerial compensation in that their value to the manager (measured by CEV) is substantially below their Fair Value. This is true even though control is of considerable value to the manager.

Table 5: Comparison of the Manager's CEV with Fair Value

We report the certainty equivalent and fair values for European employee stock options using the standard parameters from Table 1 except as indicated. The Outside Wealth percentage is measured without including the value of the manager's incentive option.

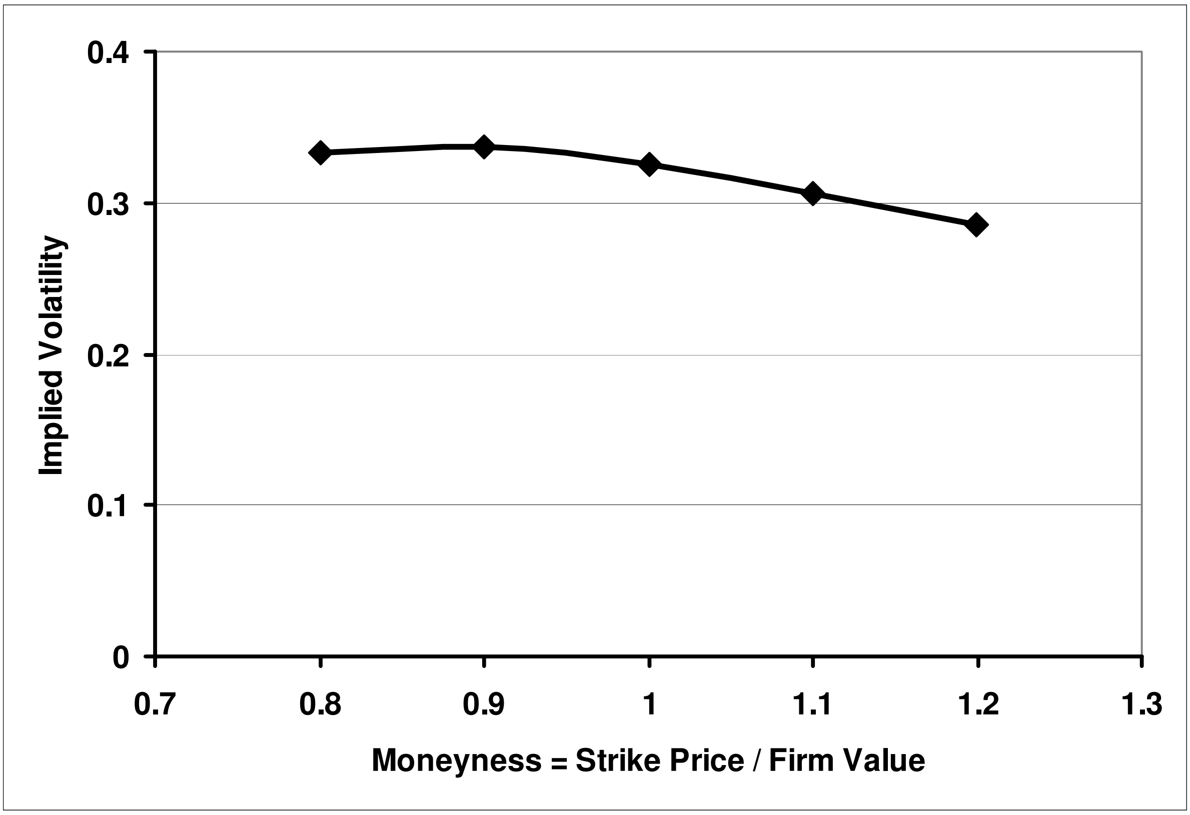
Outside Wealth (%)		**1-Year Option Maturity**			**5-Year Option Maturity**	
	Fair Value	**CEV**	**CEV/FV %**	**Fair Value**	**CEV**	**CEV/FV %**
		gamma = 2			**gamma = 2**	
20	0.1212	0.0704	58.05%	0.2898	0.1073	37.02%
50	0.1474	0.0926	62.77%	0.3073	0.1508	49.09%
80	0.1649	0.1106	67.10%	0.3297	0.1889	57.28%
		gamma = 3			**gamma = 3**	
20	0.1121	0.0452	40.30%	0.2859	0.0494	17.27%
50	0.1302	0.0666	51.13%	0.2963	0.0883	29.78%
80	0.1585	0.0889	56.07%	0.3188	0.1337	41.93%

Our approach could also be used to compare Fair Value with the CEV for other firm employees, who do not control the firm's risk-taking. Fair Value based on our manager's control decisions; and with standard parameters would be 0.1474, as previously calculated. The individual's CEV would be conditioned on our manager's choices regarding firm risk-taking but with that individual's choices regarding investment of their outside wealth and exercise of their option (assuming it is American). It's quite likely that a lower-level employee is more risk averse and has less outside wealth than our top-level manager, which would tend to enlarge the gap between Fair Value and CEV compared with the already large differences we see in Table 5. This suggests that using options as a compensation mechanism will be very inefficient unless they have substantial incentive effects. Hence, the use of options probably does not make sense for lower-level employees due to both less (or no) control and likely a greater personal discount on the option's value.

In the situation we have been analyzing, managerial control results in a Fair Value for a European incentive option which exceeds the comparable Black-Scholes value. However with a differing managerial incentive structure and/or different limits on his control of firm risk-taking, it is possible for Fair Value to be less than the comparable Black-Scholes value. The key is whether the manager's control actions result in probability weighted volatility which is greater or less than the constant value used in the Black-Scholes calculation.

Figure 6: Managerial Control and the Value of Traded Options

We graph the implied volatilities of traded options across moneyness (measured as strike price/firm value) when the manager controls the firm in order to maximize his expected utility. We are using the standard parameters from Table 1.



Once we have determined how the manager will control the firm, we can also use our risk-neutral valuation procedure to price traded derivatives on the firm. The resulting values will generally deviate and may be greater or less than a comparable Black-Scholes value. We illustrate this in Figure 6 using implied volatilities. There we plot Black-Scholes implied volatilities across moneyness (measured as strike price/firm value) for several call options with one-year maturity and differing strike prices. The approach entails first pricing each of these options using our risk-neutral valuation procedure. Then we find the implied volatility for each option that would cause the Black-Scholes model to match our calculated price. That implied volatility varies across moneyness since the different options encounter differing probabilities of being on the high-volatility Option Ridge as opposed to portions of the state space with lower firm risk-taking. For example, the option with a strike price of 1.2 has a lower implied volatility since price paths which wind up in-the-money for that option are likely to spend more time toward the right hand side of Figure 2, where the manager's optimal risk-taking, and thus volatility, is relatively low.

The downward sloping implied volatility smile (skew) in Figure 6 is consistent with empirical results for individual stocks.[17] That shape will be naturally generated by the Option Ridge structure of our manager's optimal control behavior. Hence, such managerial control represents another possible explanation for volatility smiles at the individual firm level. This represents a further piece of empirical evidence supporting the model. Furthermore, managerial control of risk-taking in response to an incentive option provides an endogenous mechanism for generating a volatility smile rather than postulating jumps or some other exogenous approach to explaining the existence of such smiles.

V. Robustness

We investigated the robustness of our results by altering grid size and by changing some of the key parameter values from those shown in Table 1. When we make our grid of states twice as fine (coarse), the CEV and the expected time to exit via hitting the lower boundary change by less than 1% (3%). Making the set of possible kappa levels twice as fine or twice as coarse changes the CEV and time to exit by less than 0.03%. Altering the stochastic process (to be mean-reverting, for example) or using different parameters for the means and volatilities does not change the results fundamentally since the manager can adjust firm and external risk-taking

[17] See for example the discussion in Dennis and Mayhew (2000) and Bakshi, Kapadia, and Madan (2003).

rather frequently. In other words, he can adjust back to his desired risk position at each time step subject only to the upper and lower bounds on the control parameters.

It is possible to induce dramatic shifts in managerial behavior near the lower boundary. For example, the manager could engage in "suicide gambling" if there were a sufficiently large benefit to being fired. However, this kind of gambling does not arise with our standard parameters; and a slight dismissal penalty of 5% on the manager's firm-related wealth does not substantially change any of our results.

Finally, we considered the impact of changes in correlation, option position size, and risk aversion on the CEV and the expected time to exercise for the American option with 1 or 5 years to expiration. Changing the correlation from 0.6 to 0, CEV and the expected time to exercise change by less than 4%. More interesting is an increase in the option position size from 0.01 to 0.10, a ten-fold increase. Here, the CEV for the model with 1 (5) years to expiration is reduced to 46% (43%) of the result with standard parameters. Recall that we are measuring CEV as a marginal value so that it is possible to make comparisons with Fair Value, which is calculated on a per unit basis. Indeed, the manager's larger option position has a greater total value, but the marginal value for the manager declines dramatically with larger and larger option grants. Also, the expected time to exercise is reduced to 72% (63%) of the value with standard parameters, indicating that the manager is also much more inclined to quickly cash out his larger option position. Increasing the manager's risk aversion coefficient from 2 to 10 lowers the CEV with 1 (5) years to expiration to 29% (25%) of the standard result. The option grant is thus worth much less to the more risk averse manager. The expected time to exercise is reduced to 62% (40%) of the standard result, indicating again that the more risk averse manager will try to quickly cash out his option position.

VI. Concluding Comments

Holding restricted shares and/or an employee stock option position has important implications for how our manager exercises control at the firm as well as how he manages his external wealth. When the manager has only restricted shares, there is a significant incentive problem with his seeking to reduce firm risk as much as possible. This illustrates both the importance of potential constraints on managerial control and the role of employee stock options for inducing more willingness to undertake risky firm investments.

We have deliberately chosen parameter values such that firm risk is analogous to non-systematic risk that a typical manager would want to avoid. However, these qualitative results are more general and, as we have demonstrated, robust to changing key parameter values. Even with more attractive parameters for the firm's risky technology, a Merton-optimal investor would only wish to have a modest fraction of wealth tied up in firm securities. For example, adding an extra 2% expected return on the firm's technology increases the desired investment in firm shares to only 17% of wealth for an individual with the same risk aversion as our manager. If the manager's restricted shares constitute a greater fraction of wealth, his natural response is to indirectly reduce personal exposure by altering the firm's risk.

Adding an employee stock option provides an incentive for greater risk-taking in the manager's control of firm investment positions. Our standard parameters impose both upper and lower bounds on that risk-taking, and the managerial behavior in Figure 2 illustrates the significance of both bounds. The manager's attempts to increase risk in the Option Ridge area (near the strike price) are limited by the upper bound on risk-taking. Whereas with higher firm values, the risk-inducing incentive from the option declines; and the manager returns to his risk avoiding behavior, which is limited by the lower bound on firm risk-taking.

As we saw in Section II, there are interesting interactions between the manager's risk-taking within the firm and his control of outside wealth positions. Absent an incentive option, he tries to indirectly reduce the risk of his overall portfolio by decreasing the firm's risk. He then partially hedges his remaining firm exposure by reducing the fraction of outside wealth invested in a correlated asset. Introducing an employee stock option dramatically increases the hedging impact on outside wealth positions. That behavior is further amplified by managerial control of firm risk. We view further exploration of such behavior as an interesting topic for future research.

It is not surprising that control allows the manager to increase his CEV for firm options and for restricted shares. What seems more significant is the potentially substantial delay in early exercise introduced by control. Furthermore as discussed earlier, the results in Bettis, Bizjak, and Lemmon (2005) provide some empirical verification of such behavior. Further empirical testing is warranted, but there will be challenges in controlling for other characteristics which influence the early exercise decision, such as differing risk aversion and wealth distributions.

Once we have determined the manager's optimal control at the firm, we can value derivatives on that controlled process from a market perspective. This allows us to calculate the

Fair Value for his employee stock option position as well as for traded options based on firm value. In addition to showing how to calculate traded option values for a controlled process, we observe that the manager's control behavior endogenously leads to a volatility smile or skew consistent with empirical observations.

Regarding Fair Value, there are a couple of important results. First, control of firm risk-taking has substantial value for the manager; however, the gap between his CEV and Fair Value can be quite large. Our results suggest it can easily be 40% or more. Hence, managerial control does not appear to come anywhere close to eliminating the substantial gap between a firm's cost of employee stock options (as measured by Fair Value) and the manager's personal valuation (as represented by his CEV). Second, that gap is likely to be much larger for lower-level employees who are apt to have less outside wealth and may be more risk averse. Moreover, when such employees have no control over firm risk-taking, granting them options can not influence firm risk. Consequently, our model strongly suggests that option-based compensation is likely to make sense only for employees that can substantially influence the firm value process.

Appendix: Numerical Procedure for Generating the Optimal Control Surface[18]

The basic structure of our model uses a grid of firm values X, outside wealth Y, and time t, with $\Delta(\log X)$ and $\Delta(\log Y)$ constant as well as time steps Δt of equal length. The initial firm value X_0 and initial outside wealth Y_0 are on the grid. To calculate expected utilities, we will need the probabilities of moving from one grid point at time t to all possible grid points that can be reached at $t+\Delta t$. The possible log X moves are $i\Delta(\log X)$, and the possible log Y moves are $j\Delta(\log Y)$. We use i,j to index the grid points to which we can move. In the current implementation, the range for both i and j is from -15, …, 0, …, 15. The probabilities for those possible moves depend on the choice of kappa and alpha which determine the joint process for X and Y over the next time step. For a given kappa, the log change in X is normally distributed with mean $\mu_{\kappa,\Delta t} = [\kappa\mu + (1-\kappa)r - \frac{1}{2}\kappa^2\sigma^2]\Delta t$ and volatility $\sigma_{\kappa,\Delta t} = \kappa\sigma\sqrt{\Delta t}$. Similarly for a given alpha, the log change in Y is normally distributed with mean $m_{\alpha,\Delta t} = [\alpha m + (1-\alpha)r - \frac{1}{2}\alpha^2 v^2]\Delta t$ and volatility $v_{\alpha,\Delta t} = \alpha v\sqrt{\Delta t}$. Note that these means and variances do not depend on the levels of X or Y. They do depend on Δt but not on t itself. Since the bivariate normal distribution is characterized by its means, variances, and correlation coefficient; the probabilities we need are functions of κ and α but not the grid point.

We now use the discrete bivariate normal distribution to generate the move probabilities. For a given κ and α, we calculate the probabilities based on the bivariate normal density times a normalization constant so that the computed probabilities sum to one:

[18] Parts of the following discussion are also covered in Hodder and Jackwerth (2007), where a related model with a single state variable is used to examine the impact of incentive contracts on the behavior of a hedge fund manager. Compare also Judd (1998, ch. 12) and Kushner and Dupuis (2001) for further details on numerical dynamic programming.

$$x_{i,\kappa,\Delta t} = \frac{i\Delta(\log X) - \mu_{\kappa,\Delta t}}{\sigma_{\kappa,\Delta t}}; \quad y_{j,\alpha,\Delta t} = \frac{j\Delta(\log Y) - m_{\alpha,\Delta t}}{v_{\alpha,\Delta t}}$$

$$p_{i,j,\kappa,\alpha,\Delta t} = \frac{\frac{1}{2\pi\sqrt{\sigma^2_{\kappa,\Delta t} v^2_{\alpha,\Delta t}(1-\rho^2)}} EXP\left[-\frac{1}{2}\frac{1}{(1-\rho^2)}\left[x^2_{i,\kappa,\Delta t} - 2\rho x_{i,\kappa,\Delta t} y_{j,\alpha,\Delta t} + y^2_{j,\alpha,\Delta t}\right]\right]}{\sum_{I=-15}^{15}\sum_{J=-15}^{15} \frac{1}{2\pi\sqrt{\sigma^2_{\kappa,\Delta t} v^2_{\alpha,\Delta t}(1-\rho^2)}} EXP\left[-\frac{1}{2}\frac{1}{(1-\rho^2)}\left[x^2_{I,\kappa,\Delta t} - 2\rho x_{I,\kappa,\Delta t} y_{J,\alpha,\Delta t} + y^2_{J,\alpha,\Delta t}\right]\right]} \tag{A1}$$

We keep a lookup table of the probabilities for different choices of κ and α, where we combine each possible κ choice with each possible α choice. In our standard case, we vary κ from 0.75 to 1.5 in steps of 0.125 and α from -0.5 to 1.5 in steps of 0.125. The ends of these ranges are problematic and can result in poor approximations to the normal distribution. For low kappa or alpha values, the approximation suffers from not having fine enough value steps. For high kappa or alpha values, the difficulty arises from potentially not having enough offset range to accommodate the extreme tails of the distribution. We use $\Delta(\log X) = \Delta(\log Y) = [\log(1/0.5)]/40 \approx 0.0173$ in order to achieve sufficiently accurate distributions given the step size of 0.125 in both κ and α. Furthermore, the offset of ± 15 allows moves to 31x31=961 surrounding grid points at each time step and insures that all tail probabilities are reasonably approximated.

We now calculate the expected indirect utilities after initializing the terminal indirect utilities $J_{X,Y,T}$ to the utility of wealth of our manager $U_{X,Y,T}(W_T)$, where his wealth W_T is determined by Y_T as well as X_T and his compensation scheme. Our next task is to calculate the indirect utility function at earlier time steps as an expectation of future indirect utility levels. We commence stepping backwards in time from the terminal date T in steps of Δt. At each grid point within a time step t, we calculate the expected indirect utilities for all combinations of kappa and alpha levels using the stored probabilities and record the highest value as our optimal indirect utility, $J_{X,Y,t}$. We continue, looping backward in time through all time steps.

In our situation, using a lookup table for the probabilities associated with the combinations of kappas and alphas has two advantages compared with using an optimization routine to find the optimal combination of kappa and alpha. For one, lookups are faster although coarser than optimizations. Second, a sufficiently fine lookup table is a global optimization method that will find the true maximum even for non-concave indirect utility functions. In such situations, a local optimization routine can get stuck at a local maximum and gradient-based methods might face difficulties due to discontinuous derivatives.

When implementing our backward sweep through the grid, we have to deal with behavior at the boundaries. The terminal step is trivial in that we calculate the terminal utility from the terminal wealth. The lower boundary in firm value, which we reach in 40 steps from the initial firm value of 1.0, is also quite straightforward. We stop the process upon reaching or crossing the boundary and calculate the utility associated with hitting the boundary at that time τ. In that situation, the manager's wealth has the known value of $W_\tau = bX_\tau + Y_\tau$. As a side calculation we work out the indirect utility of a pure wealth account that can be optimally invested in the index fund and the riskless security until time T. We use the same parameter values as in Table 1. Finally, we look up the interpolated indirect utility of W_τ. For increased accuracy, we interpolate the certainty equivalent values which are almost linear in wealth.

For the numerical implementation, we also need upper boundaries to approximate indirect utilities associated with high firm values and high outside wealth levels. We use a boundary at least 40 steps above the initial X_0 and Y_0 levels. For horizons longer than one year, we increase the number of steps so that we can always accommodate at least a three-standard deviation up-move over that time horizon for the unhedged firm value or the index fund, whichever is greater. For firm values near that boundary, our calculation of the expected indirect utility will try to use indirect utilities associated with firm values above the boundary. We deal with this by keeping a buffer of firm values above the boundary so that the expected indirect utility can be calculated by looking up values from such points. We set the terminal buffer values simply to the utility for the wealth level associated with those firm values. We then step back in time and use as our indirect utility the utility just below the boundary times a multiplier. This multiplier is based on the ratio of indirect utilities above the upper boundary to the indirect utility just below the upper boundary one time step later. Thus, we will always be able to calculate such a multiplier as we solve backward through the state space. We assume that this multiplier does not change from one time step to the next. This approach is potentially suboptimal, which biases the results low. However, the method works very well and the distortions rarely ripple more than a single step below the upper boundary, affecting mainly the early time steps. We use a similar procedure for the boundary layers of outside wealth.

To allow for early exercise, we use another side calculation to construct an optimal indirect utility surface (for all states and time steps) conditional on having exercised at the firm value X_τ and the outside wealth level Y_τ. Then, we work out the optimal final indirect utility surface, determining at each node whether or not it is preferable to exercise. We start at time T with a live option and work backward. We compare the indirect utility of continuing with a live option to the indirect utility of early exercise. The outside wealth conditional on exercising is then $c(X_\tau - X_0) + Y_\tau$ and we find the indirect utility at that outside wealth level on the side calculation surface. Here again, we interpolate linearly in the certainty equivalents across wealth for increased accuracy. If early exercise is preferred, we insert that indirect utility at the relevant node and continue looping backwards to the beginning of the grid.

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