Cartel Damages Claims, Passing-On and Passing-Back

Garrod, Luke and Han, Tien-Der Jerry and Harvey, James and Olczak, Matthew

2023

Cartel Damages Claims, Passing-On and Passing-Back*

Luke Garrod,[†] Tien-Der Jerry Han,[‡] James Harvey,[§] and Matthew Olczak[¶]

February 21, 2023

Abstract

Firms can mitigate the harm of an input cartel by passing on some of the overcharge to their customers through raising their own prices. Recent claims for damages have highlighted that firms may also respond by negotiating lower prices with their suppliers of other complementary inputs, thereby passing back some of the harm upstream. By analysing a model where downstream supply requires two inputs, we derive the equilibrium 'passing-on' and 'passing-back' effects when one input is cartelised. We show that the cartel causes a larger passing-back effect when there is greater market power in the complementary input sector. This reduces the passing-on effect. We find that the passing-back effect can inflict substantial harm on the complementary input suppliers and can reduce the harm inflicted on direct and/or indirect purchasers.

JEL classifications: D43, K21, L13, L40

Key words: damages, cartel overcharge, pass-on, complements, negotiation

---

*We are grateful for comments from Anna Rita Bennato and Chris M. Wilson. Any mistakes are our own. A motivating case used in this paper is a damages claim in the UK relating to the EU trucks cartel for which James Harvey acted as the economic expert for the claimants. The theoretical model presented in this paper has evolved from a simpler framework developed by Luke Garrod.

[†]Loughborough University and Economic Insight Ltd. Corresponding author, email: l.garrod@lboro.ac.uk

[‡]Loughborough University

[§]Economic Insight Ltd

[¶]Aston University

1 Introduction

Cartel victims can sue for damages to compensate them for the harm they have suffered, but calculating the harm can be fraught with difficulties. Many of the difficulties arise from estimating the overcharge (i.e., how much more a buyer paid for each unit), which is necessary because the price that buyers would have paid absent the infringement is hypothetical. Other difficulties arise due to the "passing-on effect", where the harm of an input cartel on downstream firms depends not only on the overcharge but also on whether they raised their own prices. Further complications relating to a "passing-back effect" have been highlighted in recent claims for damages in the UK (reviewed in section 2). Such effects can arise when downstream firms mitigate the harm of an input cartel by negotiating lower prices with their suppliers of other complementary inputs. These passing-back effects have yet to be investigated in the literature, so many questions remain unanswered. For example, how does a passing-back effect relate to the overcharge and passing-on effect? How are these relationships affected by market conditions? What are the implications for the harm inflicted on direct purchasers, indirect purchasers and complementary input suppliers?

The aim of this paper is to theoretically analyse the effects of an input cartel when the cartel's direct purchasers can i) pass on the price increase to their consumers by raising their own prices and ii) pass back the price increase by lowering the prices paid to their suppliers of other complementary inputs. We achieve this by first developing a successive oligopolies model in which differentiated downstream firms must source two inputs to produce their final products. Competition downstream is modelled generally using the conjectural variations approach and we also capture the whole competition spectrum in the complementary input sector by varying the number of homogeneous input suppliers that compete in quantities. We derive the equilibrium passing-on and passing-back effects associated with an exogenous increase in the other input price. We show how the harm inflicted on the market participants changes with concentration in the complementary input sector and the downstream cost pass-through rate. Following this, we extend our analysis to a situation where the complementary input prices are determined by negotiation. This extension shows that our successive oligopolies model may be used to quickly determine the results of market settings with different contractual arrangements.

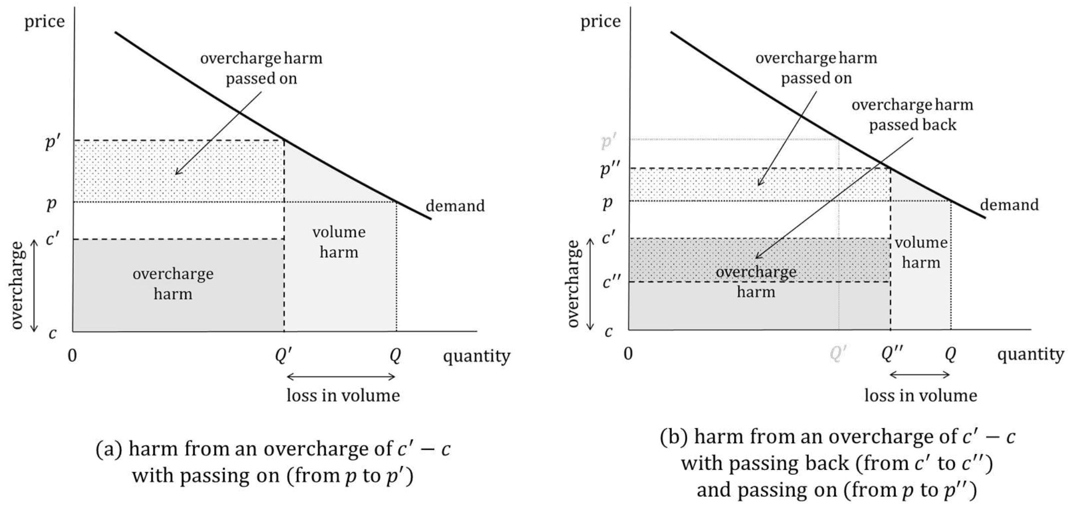

Figure 1: The harm caused by an input cartel

To help summarise our main results, Figure 1 illustrates the harm of an input cartel. Figure 1(a) shows the well-known case where there is only a passing-on effect: the input cartel raises the costs of direct purchasers from c to c' and in response their prices rise from p to p', lowering the industry quantity from Q to Q'. The overcharge harm is caused by the direct purchasers incurring an overcharge of $(c'-c)$ on each of the Q' units sold, yet they pass on $(p'-p)$ of this harm per unit to their buyers (i.e., the indirect purchasers). The volume harm is associated with the loss in volume, $Q-Q'$, where direct purchasers incur additional harm of $(p-c)(Q-Q')$ and indirect purchasers also incur the loss in consumer surplus. This case is equivalent to our setting when the complementary input suppliers have no market power, so that the price equals marginal cost and hence is unresponsive to the cartel price increase. Figure 1(b) depicts the case where there is market power in the complementary input sector. Here, the complementary input suppliers expect direct purchasers to pass on a proportion of the overcharge to indirect purchasers, so they anticipate a fall in the quantity demanded of the final products. This in turn will lead to a decrease in demand for the complementary input, so its price falls. This passing-back effect limits the increase in downstream marginal cost to $(c''-c)$, so the rise in the downstream price is limited to $(p''-p)$ and there is a smaller loss in volume, $Q-Q''$. Consequently, the existence of the passing-back effect reduces the passing-on effect.

Clearly, the passing-back effect will influence how the harm is divided among the market participants. In particular, the complementary input suppliers will incur some of the harm that would have otherwise fallen entirely on the direct and/or indirect purchasers and hence it is a potential source of inaccuracy in damages claims. We show that as the complementary input sector becomes more concentrated, the proportion of the overcharge harm and the total harm (i.e., the sum of the overcharge and volume harm) incurred by the complementary input suppliers strictly increases and the proportions of the direct and indirect purchasers strictly decrease. The reason is that the magnitude of the passing-back effect is larger and the magnitude of the passing-on effect is smaller. We also show that a higher downstream cost pass-through rate decreases direct purchasers' share of the overcharge harm and total harm at the expense of the complementary input suppliers and/or indirect purchasers. This arises because the pass-through rate (and hence its determinants, such as product substitutability) only affects the passing-on effect and does not influence the passing-back effect.

In the extension, we analyse a setting where two downstream firms each have an exclusive supplier of the complementary input. The input price of each pair is determined by negotiation as modelled by the symmetric Nash bargaining solution (Dobson and Waterson, 1997 and 2007). We show that increased bargaining power in the complementary input sector leads to a greater passing-back effect and a smaller passing-on effect. Indeed, by writing the equilibrium prices in a similar form as in the successive oligopolies model, we quickly derive all of the results relating to this setting. This indicates that by following a similar approach, our model can be used to generate results relating other market settings, which may be beneficial to parties involved in future damages claims. One difference with the successive oligopolies model is that the passing-back effect in the extension is smaller when products are more substitutable downstream. The reason is that greater product substitutability strengthens the resolve of each downstream firm in the negotiation with its supplier, so input prices are closer to marginal cost and therefore less responsive to the changes in demand associated with the cartel price increase.

Our results have two important implications for damages claims in practice. First, the passing-back effect can inflict significant harm on the suppliers of complementary inputs. Hence, there is an argument that they should be encouraged (or even allowed) to sue for compensation as well as direct and indirect purchasers. This is especially the case when they have greater market power. Second, since the harm inflicted on direct and/or indirect purchasers can be reduced by a passing-back effect, estimating the harm of an input cartel on purchasers should also involve estimating the size of any passing-back effect. This may involve developing an understanding of competition in the complementary input sector, even though the complementary input suppliers may not be part of the trial.

Our paper is related to a literature on the accuracy of estimates of the harm caused by input cartels. Hellwig (2007) analyses how an input cartel affects downstream profits. Kosicki and Cahill (2006) focusses on the harm incurred by indirect purchasers. Han *et al.* (2008) considers a vertical industry with many different sectors and analyses the total harm associated with a cartel in one sector on its purchasers and its suppliers. Verboven and van Dijk (2009) shows how the total harm inflicted on direct purchasers' profits can be estimated as a discount on the overcharge in a range of models of imperfect competition. Basso and Ross (2010) shows how the total harm of an input cartel on direct purchasers differs to the overcharge harm. Boone and Muller (2012) shows how the share of the total harm between direct and indirect purchasers can be estimated when firms have general cost functions. Bet *et al.* (2021) analyses how the harm of an input cartel inflicted on one downstream firm depends upon whether its rival is vertically integrated or not. All of these papers focus on passing-on effects and do not consider passing-back effects.

More broadly, our paper is related to the literature on cost pass-through under imperfect competition (e.g., Bulow and Pfleiderer, 1983; Seade, 1985; Anderson *et al.*, 2001; Weyl and Fabinger, 2013; and Ritz, 2022). The main aim of this literature is to uncover the determinants of the cost pass-through rate, defined by the derivative of the equilibrium price with respect to marginal cost. The passing-on effect in our model differs to this literature in that the aggregate increase in the downstream equilibrium price is determined by the direct effect of an increase in marginal cost and an additional indirect effect associated with the decrease in the equilibrium price of an input. Finally, our paper is distinct to Adachi and Ebina (2014a; 2014b) and Gaudin (2016) that analyse cost pass-through in vertically-related markets. Their focus is on how an exogeneous upstream cost increase is passed through the upstream and downstream sectors. In contrast, the exogenous increase in marginal cost arises downstream in our model and we analyse the resultant equilibrium effects on the downstream and upstream prices.

The rest of the paper is structured as follows. In section 2, we review the recent experience of damages claims in the EU and UK.[1] In section 3, we present the successive

[1] For discussion of the US experience, see Verboven and van Dijk (2009) and Boone and Muller (2012).

oligopolies model and solve for the equilibrium. In section 4, we investigate the equilibrium passing-on and passing-back effects of an input cartel. In section 5, we analyse an extension where the price of the complementary input is determined by negotiation. We offer concluding remarks in section 6.

2 Recent Experience in the EU and UK

In Europe, it has now "become normal for the victims of cartels to bring 'follow-on' actions for damages after the European Commission or an NCA [National Competition Authority] has adopted a decision finding an infringement of Article 101 or its domestic equivalent" (Whish and Bailey, 2021, p.311). This represents a significant change from the situation 10 years ago when such private actions were rare. Part of the reason for this change is due to the adoption of the Damages Directive by the EU in November 2014,[2] with its key features later being incorporated into UK law following the UK's withdrawal from the EU.[3]

The Damages Directive states that "...any natural or legal person who has suffered harm caused by an infringement of competition law is able to claim and to obtain full compensation for that harm."[4] Thus, claimants are not required to be direct purchasers of the infringer and there is nothing to stop suppliers from being claimants. Full compensation is defined as placing "... a person who has suffered harm in the position in which that person would have been had the infringement of competition law not been committed. It shall therefore cover the right to compensation for actual loss and for loss of profit, plus the payment of interest".[5] This implies that compensation can be claimed not just for the harm associated with paying too much for an input (i.e., the overcharge harm) but also for the loss of profit associated with the loss in volume (i.e., the volume harm). However, in practice the latter is more difficult to prove, so estimating the overcharge harm is often an important part of damages claims. The Directive also states that full compensation "... shall not lead to overcompensation, whether by means of punitive, multiple or other types of damages."[6] This 'single damages' approach contrasts with the

[2]Directive 2014/104/EU of the European Parliament and of the Council on certain rules governing actions for damages under national law for infringements of the competition law provisions of the Member States and of the European Union, *Official Journal of the European Union*, L 349, 5 November, 2014.

[3]For further details, see Coulson and Blacklock (2021).

[4]*Sup.* note 2, Article 3, paragraph 1.

[5]*Id.*, Article 3, paragraph 2.

[6]*Id.*,, Article 3, paragraph 3.

US approach, where 'treble damages' are available to successful claimants.

While in principle the quantification of harm should avoid over- or under-compensation, it has been recognised that achieving full compensation in practice is difficult due to issues relating to estimating the overcharge and passing-on effects. With these challenges in mind, the European Commission has published two guidance documents to assist practitioners and national courts. First, European Commission (2013) is a "Practical Guide" to quantifying harm in actions for damages, which primarily discusses the methods to quantify the overcharge. Second, European Commission (2019) provides "Guidelines" to quantify how much of an overcharge has been passed on. Neither of these guidance documents address the quantification challenges relating to the passing-back effects, because such issues have only recently come to light as the case law in the UK has evolved.

The passing-back effect was first mentioned in the UK in a follow-on claim for damages brought by Sainsbury's against Mastercard.[7] Sainsbury's claimed that it had been overcharged by Mastercard for "Merchant Services". In response, Mastercard argued, amongst other things, that Sainsbury's would have passed-on any overcharge and so was not harmed. In its June 2020 judgment, the Supreme Court of the UK outlined the various ways in which a buyer – in this case, Sainsbury's as the merchant – could respond to an overcharge (bold emphasis added):

> There are four principal options: (i) a merchant can do nothing in response to the increased cost and thereby suffer a corresponding reduction of profits or an enhanced loss; or (ii) the merchant can respond by reducing discretionary expenditure on its business such as by reducing its marketing and advertising budget or restricting its capital expenditure; or **(iii) the merchant can seek to reduce its costs by negotiation with its many suppliers**; or (iv) the merchant can pass on the costs by increasing the prices which it charges its customers.[8]

It concluded:

> In our view the merchants are entitled to claim the overcharge on the MSC [Merchant Service Charges] as the *prima facie* measure of their loss. **But if there is evidence that they have adopted either option (iii) or (iv)**

[7] Judgement in Sainsbury's Supermarkets Ltd (Respondent) v Visa Europe Services LLC and others (Appellants) Sainsbury's Supermarkets Ltd and others (Respondents) v Mastercard Incorporated and others (Appellants), Supreme Court, UKSC 24, 17 June, 2020

[8] *Id.*, paragraph 205.

> **or a combination of both to any extent, the compensatory principle mandates the court to take account of their effect and there will be a question of mitigation of loss**[9]

This indicates that applying the compensatory principle can require one to take account of: the size of the overcharge, the size of any passing-on effect (option iv) and the size of any passing-back effect (option iii). The implication is that the quantification of harm could in some cases involve developing an understanding of the markets in which the infringement took place, the market(s) in which the direct purchasers operated, and the market(s) that supplied the direct purchasers.

The importance of the passing-back effect for complementary inputs is further demonstrated in the private damages claim brought by Royal Mail and BT against DAF.[10] Royal Mail and BT claimed that they had been overcharged for the trucks purchased from DAF between 1997 and 2011 due to, amongst other things, DAF price fixing with other truck manufacturers MAN, Volvo/Renault, Daimler, and Iveco. DAF sought permission from the UK Competition Appeal Tribunal (CAT) to bring two defences.[11] The first defence was concerned with the passing-back effect on any other input purchased by the claimants. The CAT refused DAF permission to bring this defence, concluding that (bold emphasis added):

> for a defendant to be permitted to raise a plea of mitigation in this way **in general terms**, there must be something more than broad economic or business theory to support a reasonable inference that the claimant would in the particular case have sought to mitigate its loss and that the steps taken by it **were triggered by, or at least causally connected to, the overcharge** in the direct manner required by the British Westinghouse principle.[12]

The second defence was concerned with the passing-back effect on the claimants' purchases of bodies and trailers, which are complements to trucks because they attach to trucks and contain the cargo. The CAT granted DAF permission to bring the second de-

[9] *Id.*, paragraph 206.
[10] Royal Mail Group Limited v DAF Trucks Limited & Others, BT Group PLC & Others v DAF Trucks Limited & Others, Judgment: Expert Evidence and Amendment, Competition Appeal Tribunal, 1284-1290/5/7/18 (T), 13 May 2021
[11] *Id.*, paragraph 20.
[12] *Id.*, paragraph 36.

fence[13] and, in doing so, the CAT appeared to consider the complementary relationship between the trucks and bodies/trailers to represent the "something more" that supported the "reasonable inference that the claimant would [...] have sought to mitigate its loss".

For the trial, substantial time and expense was spent both in preparation of the evidence and at the hearing to address the passing-back effects.[14] DAF argued that the passing-back effects allowed Royal Mail and BT to recover 6% and 25% of the overcharge on trucks, respectively,[15] but this was disputed by Royal Mail and BT who argued no such effects existed.[16] In its Judgment, the CAT accepted that the existence of a passing-back effect was theoretically plausible because "trailer/body suppliers might have responded to the fall in demand for their products [...] by reducing their selling prices and margins".[17] Furthermore, it was also noted that any such effect would imply that the trailer/body suppliers "would consequently have a potential damages claim against the truck suppliers who had engaged in the cartel infringement".[18] However, ultimately the CAT dismissed the defence, because DAF's evidence failed to establish the existence of passing-back effects in these cases.[19]

The implications of passing-back effects are currently under researched because, in addition to such effects not being addressed in the guidance documents mentioned above, they have not yet been investigated in the academic literature either. Consequently, there is no guidance to practitioners regarding when it is likely to be important to consider the possibility of a passing-back effect; the market conditions affecting the size of any passing-back effect; how the passing-back effect is related to the overcharge and the passing-on effect; or how it might be quantified in practice. The model that is developed in the remainder of the paper goes some way to address this gap. This model should be of interest for scholars and practioners outside of the UK, because it is important to understand potential sources of inaccuracy in damages estimates within jurisdictions where the case law currently differs to the UK regarding passing-back effects.

[13] *Id.*, paragraph 21.
[14] Royal Mail Group Limited v DAF Trucks Limited & Others, BT Group PLC & Others v DAF Trucks Limited & Others, Judgment, Competition Appeal Tribunal, 1284-1290/5/7/18 (T), 7 February 2023, paragraph 504.
[15] *Id.*, paragraph 491
[16] *Id.*, paragraph 492.
[17] *Id.*, paragraph 508.
[18] *Id.*, paragraph 490.
[19] *Id.*, paragraph 509.

3 Model

3.1 Basic Assumptions

Suppose there is a market in which $n \geq 1$ downstream firms (henceforth retailers) wish to sell differentiated products to final consumers. To produce one unit of a product, a retailer must source two complementary inputs, denoted as A and B. For a given input $S = \{A, B\}$, let $\varphi_S > 0$ represent the units of input S needed to produce one unit of the downstream products and let w_{Si} be the price that retailer $i = \{1, ..., n\}$ pays for each unit of input S. Thus, letting κ denote the marginal costs associated with retailing, the marginal cost of retailer i for the final good is $c_i \equiv \kappa + \Sigma_S \varphi_S w_{Si}$.

Without loss of generality, suppose a cartel fixes the price of input A. Following the damages literature, the cartel is modelled as an exogenous price increase and we wish to analyse the equilibrium effects on the downstream and input B sectors. To determine the equilibrium prices of input B and the final products, we analyse a successive oligopoly model in which the quantities and prices of input B are determined first, and then the quantities and prices of the final products are determined. To model downstream competition as generally as possible, we use the conjectural variations approach to nest different forms of competition that span from monopoly to no market power, including differentiated Cournot and Bertrand competition. In the input B sector, we assume that there are $m \geq 1$ homogeneous suppliers that compete in quantities. Since the input B suppliers are undifferentiated, we capture the full spectrum from monopoly ($m = 1$) to no market power ($m \to \infty$), without the need for conjectural variations.

Let q_i represent the quantity of retailer i and $\boldsymbol{q} = (q_1, \ldots, q_i, \ldots, q_n)$ denote the vector of quantities for all retailers. Let the inverse demand function of retailer i's product be

$$p_i(\boldsymbol{q}) \quad = \quad v - \frac{1}{\beta}\left[(1-\sigma)\, n q_i + \sigma\left(q_i + \sum_{j \neq i} q_j\right)\right] \tag{1}$$

where $v > 0$ and $\beta > 0$, and $\sigma \in [0, 1]$ represents the degree of substitutability between the products. The products are independent when $\sigma = 0$ and are increasingly substitutable as σ rises; they are perfect substitutes when $\sigma = 1$. As per Shubik and Levitan (1980), this inverse demand function can be derived by maximising the following net surplus function of a representative consumer with respect to q_i:

$$CS(\boldsymbol{q}) = \sum_{i=1}^{n} (v - p_i)\, q_i - \frac{n}{2\beta}\left[(1-\sigma)\sum_{i=1}^{n} q_i^2 + \frac{\sigma}{n}\left(\sum_{i=1}^{n} q_i\right)^2\right]$$

An advantage of this demand system is that it isolates the competition effect of product differentiation, because there is no market expansion effect. To see this, note that if $q_i = q$ for all i, then demand is independent of σ, since $p_i(\boldsymbol{q}) = v - \frac{nq}{\beta}$ for all i.[20]

Finally, let $\varphi_S x_{Sk}$ represent the quantity of input S sold by supplier k, where $X_S \equiv \varphi_S \Sigma_k x_{Sk}$. Given the demand for each input is derived from the demand for the final good, in equilibrium we must have $X_S = \varphi_S Q$ for any S, where $Q \equiv \Sigma_i q_i$. Let $c_B \geq 0$ represent the marginal cost of input B. All fixed costs are normalised to zero. We restrict attention to symmetric subgame perfect equilibria, where $x_{Bk} = x_B$ for all k, and $w_{Si} = w_S$ and $q_i = q$ for all i. We drop subscripts when there is no ambiguity.

3.2 Equilibrium Analysis

We begin the analysis by solving for the downstream equilibrium. Retailer i's profit function is $\pi_{Ri}(\boldsymbol{q}) = (p_i(\boldsymbol{q}) - c_i)\, q_i$. Assume that when retailer i changes its quantity by a small amount it conjectures that its rivals will change their quantities by $\frac{\partial q_j}{\partial q_i} = \frac{\theta-1}{n-1}$ for all $j \neq i$, so that conduct parameter is $\sum_{j\neq i} \frac{\partial q_j}{\partial q_i} = \theta - 1$. With this conjecture, the first-order condition of retailer i is

$$\frac{\partial \pi_{Ri}(\boldsymbol{q})}{\partial q_i} = p_i(\boldsymbol{q}) - c_i + \left(\frac{\partial p_i(\boldsymbol{q})}{\partial q_i} + (\theta - 1)\sum_{j\neq i} \frac{\partial p_i(\boldsymbol{q})}{\partial q_j}\right) q_i = 0 \tag{2}$$

where $\frac{\partial p_i(\boldsymbol{q})}{\partial q_i} = -\frac{(1-\sigma)n+\sigma}{\beta}$ and $\sum_{j\neq i} \frac{\partial p_i(\boldsymbol{q})}{\partial q_j} = -\frac{\sigma(n-1)}{\beta}$ from (1). Substituting into (2) and imposing symmetry derives each retailer's symmetric equilibrium quantity

$$q^*(c) = \frac{\beta(v-c)}{n\left[2 - \sigma\left(1 - \frac{\theta}{n}\right)\right]} \tag{3}$$

Clearly, (2) yields the Cournot outcome when $\theta = 1 \equiv \theta^c$ and the local monopoly outcome when $\theta = n \equiv \theta^m$ for all i. The latter is often referred to as the perfect collusion outcome but it could also be interpreted as when retailers have exclusive territories. The perfectly competitive outcome (where price equals marginal cost) requires $\theta = -\frac{n(1-\sigma)}{\sigma} \equiv$

[20] For more details on the demand system, see Choné and Linnemer (2020).

$\theta^p < 0$. This can be intrepreted as a setting where each differentiated product is sold by at least two homogeneous retailers that compete in prices. The Bertrand outcome can be derived by setting $\theta = \frac{1}{1+\frac{\sigma}{1-\sigma}\frac{n-1}{n}} \equiv \theta^b \in (0,1)$.[21] Note that θ^p and θ^b converge to zero when the products are homogeneous, $\lim_{\sigma\to 1}\theta^b = \theta^p = 0$. Furthermore, θ^b converges on θ^c when products are independent, $\lim_{\sigma\to 0}\theta^b = 1$, in which case both yield the monopoly outcome despite $\theta^m > 1$, because $\lim_{\sigma\to 0}\sum_{j\neq i}\frac{\partial p_i(\boldsymbol{q})}{\partial q_j} = 0$.

Now consider the input B sector. Given in total $Q^*(c) = nq^*(c)$ units of the final products will be bought in equilibrium, it follows that $X_B = \varphi_B nq^*(c)$ units of input B will be demanded. Substituting $c = \kappa + \varphi_A w_A + \varphi_B w_B$ into $X_B = \varphi_B nq^*(c)$ and rearranging yields the inverse demand curve for input B

$$w_B(w_A, X_B) = \frac{1}{\varphi_B}\left[v - \kappa - \varphi_A w_A - \frac{X_B}{\varphi_B}\left(\frac{2-\sigma\left(1-\frac{\theta}{n}\right)}{\beta}\right)\right] \tag{4}$$

Thus, the profit function of input B supplier k is $\pi_{Bk}(X_B) = (w_B(w_A, X_B) - c_B)\,\varphi_B x_{Bk}$.

Proposition 1 derives the equilibrium prices in both sectors for a given w_A.

Proposition 1. *For all $v > \kappa + \varphi_A w_A + \varphi_B c_B$ and $\theta \in \left[-\frac{n(1-\sigma)}{\sigma}, n\right]$, the equilibrium price of input B is*

$$w_B^*(w_A, m) = c_B + \frac{v - \kappa - \varphi_A w_A - \varphi_B c_B}{\varphi_B(m+1)} \in \left[c_B, \frac{v - \kappa - \varphi_A w_A + \varphi_B c_B}{2\varphi_B}\right] \tag{5}$$

The marginal cost of each retailer is $c(w_A, w_B^*(w_A, m)) = \kappa + \varphi_A w_A + \varphi_B w_B^*(w_A, m)$ and the downstream equilibrium price is:

$$p^*(w_A, w_B^*(w_A, m), \sigma, n, \theta) = v - \frac{v - c(w_A, w_B^*(.))}{2-\sigma\left(1-\frac{\theta}{n}\right)} \in \left[c(w_A, w_B^*(.)), \frac{v + c(w_A, w_B^*(.))}{2}\right] \tag{6}$$

Both prices are strictly decreasing in the number of input B suppliers, $\frac{\partial w_B^*}{\partial m} < 0$ and $\frac{dp^*}{dm} < 0$. The downstream price is also strictly decreasing in the number of retailers, $\frac{\partial p^*}{\partial n} < 0$. The effect of a change in the degree of product substitutability on the downstream price depends upon the form of competition downstream.

The downstream equilibrium price exhibits the usual properties. It equals marginal cost when the downstream sector is perfectly competitive, $p^*(., \theta^p) = c(w_A, w_B^*(.))$, and

[21] See Appendix B for details.

it takes the usual form under local monopoly, $p^*\left(.,\theta^m\right) = \frac{v+c(w_A,w_B^*(.))}{2} > c(w_A, w_B^*(.))$. It is between these extremes under Cournot and Bertrand competition, where $p^*\left(.,\theta^b\right) < p^*\left(.,\theta^c\right)$ for all $\sigma > 0$ and $n < \infty$, because competition in prices is more intense than competition in quantities. Similarly, the equilibrium price of input B equals marginal cost when the number of input B suppliers tends to infinity, $w_B^*(w_A, \infty) = c_B$. As the number of input B suppliers falls from infinity, the price increases away from c_B; it equals the monopoly level when $m = 1$.

4 The Effects of an Input Cartel

In this section, we investigate the equilibrium effects of an input cartel that raises the unit price of input A from w_A to $w'_A = w_A + \Delta$, so that $\Delta > 0$ represents the overcharge. Henceforth, we refer to retailers as the "direct purchasers" of input A, final consumers as "indirect purchasers" of input A, and the suppliers of input B as "complementary input suppliers". To simplify notation, we write all expressions as a function of w_A only, so that for example $w_B^*(w_A, m) \equiv w_B^*(w_A)$ and $p^*(w_A, w_B^*(w_A), \sigma, n, \theta) \equiv p^*(w_A)$.

4.1 Passing-On and Passing-Back

We first analyse the effects of the cartel on the equilibrium prices. This follows immediately from Proposition 1.

Corollary 1. *For all $v > \kappa + \varphi_A w'_A + \varphi_B c_B$ and $\theta \in \left[-\frac{n(1-\sigma)}{\sigma}, n\right]$, an increase in the price of input A from w_A to $w'_A = w_A + \Delta$ decreases the equilibrium price of input B,*

$$w_B^*(w'_A) - w_B^*(w_A) = -\frac{\varphi_A \Delta}{\varphi_B (m+1)} \leq 0, \tag{7}$$

where the inequality is strict $\forall m < \infty$, and strictly increases the downstream equilibrium price,

$$p^*(w'_A) - p^*(w_A) = \frac{m}{m+1} \frac{\varphi_A \Delta}{2 - \sigma\left(1 - \frac{\theta}{n}\right)} > 0. \tag{8}$$

To understand the intuition, first consider the case where there is no market power in the input B sector ($m \to \infty$). Given the equilibrium price of input B then equals marginal cost, c_B, there is no passing-back effect and so the passing-on effect is equivalent to that analysed in the literature. The magnitude of the downstream price rise is determined by the multiple of the retail marginal cost increase, $\varphi_A \Delta$, and the downstream cost pass-through rate, which from (6) is:

$$\frac{\partial p^*}{\partial c} = \frac{1}{2 - \sigma\left(1 - \frac{\theta}{n}\right)} \equiv \tau \in \left[\frac{1}{2}, 1\right] \quad \forall \theta \in \left[-\frac{n\,(1-\sigma)}{\sigma}, n\right] \tag{9}$$

Given this case is well understood, our focus henchforth is on when the suppliers of input B do have market power ($m < \infty$), where an increase in w_A can affect the equilibrium price of input B. Specifically, when the input B suppliers observe (or correctly anticipate) the level of w'_A before they set quantities, they know that a proportion of τ of any cost increase will be passed on. Furthermore, they also expect that a higher downstream price will reduce the quantity demanded of the final products and hence reduce the demand of input B. Consequently, if input B suppliers were to hold their quantities constant, an increase in the price of input A will lead to a decrease in the price of input B; using (4), this decrease equals $-\frac{\varphi_A \Delta}{\varphi_B}$. However, the reduction in demand incentivises input B suppliers to reduce their quantities, which has the opposite effect of raising the price of input B by $\frac{\varphi_A \Delta}{\varphi_B}\frac{m}{m+1}$. Summing these two effects yields (7), implying that the former effect dominates the latter, so the equilibrium price of input B falls. This is the passing-back effect.

The fall in the equilibrium price of input B will limit the resultant rise in the downstream equilibrium price because, while the pass-through rate is the same as in (9), the associated rise in retail marginal costs is reduced. In more detail, the change in marginal cost downstream is given by

$$c(w'_A) - c(w_A) = \varphi_A \Delta - \varphi_B \left(\frac{\varphi_A \Delta}{\varphi_B\,(m+1)}\right) \in \left[\frac{\varphi_A \Delta}{2}, \varphi_A \Delta\right] \tag{10}$$

The first term on the right-hand side of the equality is the increase associated with the rise in the price of input A and the second term is the decrease associated with the fall in the equilibrium price of input B in (7). Thus, the increase in the downstream equilibrium price in (8) is given by the multiple of the total increase in marginal costs, in (10), and the downstream cost pass-through rate, in (9). This is the passing-on effect.

Having understood the passing-on and passing-back effects, let us next consider how they vary with the concentration in the input B sector.

Proposition 2. *For any given downstream pass-through rate, $\tau \in \left[\frac{1}{2}, 1\right]$, as the number of input B suppliers decreases from $m \to \infty$ towards 1:*
i) the magnitude of the passing-back effect strictly increases, from 0 to $\frac{\varphi_A}{\varphi_B}\frac{\Delta}{2} > 0$; and
ii) the magnitude of the passing-on effect strictly decreases, from $\varphi_A \Delta \tau$ to $\frac{\varphi_A \Delta \tau}{2} > 0$.

Intuitively, when the input B sector is more concentrated, the total quantity of input B supplied falls to a smaller extent following the rise in w_A. The reason is that while each input B supplier reduces their quantity by more, there are fewer of them. Consequently, $w_B^*(w_A)$ falls to a greater extent and the passing-back effect is larger. This in turn implies that the increase in marginal cost downstream is smaller, because the rise in w_A being offset to a greater extent by the larger fall in $w_B^*(w_A)$. Consequently, $p^*(w_A)$ rises to a smaller extent, so the passing-on effect is smaller.

We next consider the effects of the other parameters that capture the intensity of downstream competition (n, σ, and θ). These only work through the downstream pass-through rate, τ, so to avoid unnecessary duplication, we present the results in terms of τ and then describe how τ is changes with n, σ and θ below.

Proposition 3. *For any $m \in [1, \infty)$, as the downstream cost pass-through rate increases from $\tau = \frac{1}{2}$ towards 1:*
i) the magnitude of the passing-back effect remains constant at $\frac{\varphi_A \Delta}{\varphi_B(m+1)}$; and
ii) the magnitude of the passing-on effect strictly increases, from $\frac{m\varphi_A \Delta}{2(m+1)}$ to $\frac{m\varphi_A\Delta}{m+1}$.

While it is easy to check that τ does not influence the passing-back effect, the magnitude of the passing-on effect is larger for a higher τ, because a given cost increase is passed on at a higher rate. This implies that any change to the downstream sector that raises τ will also increase the passing-on effect but will not change the passing-back effect. This includes moving towards more competitive forms of competition downstream (i.e., raising θ). Alternatively, under Cournot ($\theta^c = 1$) and Bertrand competition ($\theta^b = \frac{1}{1+\frac{\sigma}{1-\sigma}\frac{n-1}{n}}$), a higher τ can result from more downstream firms, n, or greater product substitutability, σ.

4.2 Overcharge Harm

In the remainder of this section, we investigate the ways in which the passing-on and passing-back effects influence how the harm is shared among the market participants.

We start with the overcharge harm as this is often an important estimate of the harm in damages claims. Note that an input A price increase of Δ implies that the overcharge harm of the industry is $\varphi_A \Delta Q^* (w_A)$, where $\varphi_A \Delta$ represents the overcharge harm per unit of the final good. We begin by finding expressions of the overcharge harm per unit of the final good for each market participant and then by dividing these expressions by $\varphi_A \Delta$ we derive the proportions of the overcharge harm *per unit of input A*.

The overcharge harm per unit of the final good incurred by direct purchasers is given by the change to the downstream price-cost margin, $p^*(w_A) - c(w_A) - [p^*(w'_A) - c(w'_A)]$. Substituting in for $c(.)$ from Proposition 1 and then manipulating yields

$$\varphi_A(w'_A - w_A) - [p^*(w'_A) - p^*(w_A)] + \varphi_B[w^*_B(w'_A) - w^*_B(w_A)] \tag{11}$$

The first term is the overcharge that direct purchasers pay on φ_A units of input A, where $\varphi_A(w'_A - w_A) = \varphi_A \Delta$. The second term is the extent to which the overcharge is offset by the passing-on effect for each unit of the final good, where from (8)

$$[p^*(w'_A) - p^*(w_A)] = \varphi_A \Delta \left[\frac{m}{m+1}\tau\right] \equiv \varphi_A \Delta \left[\Omega_F (m, \tau)\right] > 0 \tag{12}$$

The third term is the extent to which the overcharge is offset by the passing-back effect for φ_B units of input B, where from (7)

$$\varphi_B[w^*_B(w'_A) - w^*_B(w_A)] = -\varphi_A \Delta \left[\frac{1}{m+1}\right] \equiv -\varphi_A \Delta \left[\Omega_B (m)\right] < 0, \ \forall m < \infty \tag{13}$$

Substituting (12) and (13) into (11) yields

$$\begin{aligned} \varphi_A \Delta \Omega_R (m, \tau) &\equiv \varphi_A \Delta \left[1 - \Omega_B (m) - \Omega_F (m, \tau)\right] \\ &= \varphi_A \Delta \left[\frac{m}{m+1}(1-\tau)\right] \geq 0 \end{aligned} \tag{14}$$

Note that the term in square brackets in (14), denoted $\Omega_R (m, \tau)$, represents the proportion of the overcharge harm per unit of input A incurred by direct purchasers. Similarly, $\Omega_B (m)$ and $\Omega_F (m, \tau)$ in (12) and (13) represent the proportions of the overcharge harm per unit of input A incurred by the complementary input suppliers and indirect purchasers, respectively.

We next consider how the the market charateristics affect this harm. To begin, note that the overcharge harm, $\varphi_A \Delta Q^* (w_A)$, is greater when there is a higher downstream pass-through rate, τ , and when there are more input B suppliers, m. The reason is that $Q^* (w_A)$ is higher because the downstream equilibrium price is lower. For τ, it is due to more intense downstream competition, and for m it is due to the downstream marginal cost being lower. We next show how the division of this overcharge harm varies with the market characteristics.

Proposition 4. *For any given $\tau \in \left[\frac{1}{2}, 1\right]$, as the number of input B suppliers decreases from $m \to \infty$ towards 1, the proportion of the overcharge harm:*
i) decreases for direct purchasers, $\frac{\partial \Omega_R}{\partial m} \geq 0$, from $1 - \tau \leq \frac{1}{2}$ to $\frac{1-\tau}{2}$,
ii) strictly decreases for indirect purchasers, $\frac{\partial \Omega_F}{\partial m} > 0$, from $\tau \geq \frac{1}{2}$ to $\frac{\tau}{2}$; and
iii) strictly increases for complementary input suppliers, $\frac{\partial \Omega_B}{\partial m} < 0$, from 0 to $\frac{1}{2}$.

Intuitively, when the input B sector is more concentrated, the passing-back effect is larger so the complementary input suppliers get a larger share of the (smaller) overcharge harm. Furthermore, the passing-on effect is smaller, because the marginal cost of direct purchasers increases to a smaller extent. Thus, indirect buyers incur a smaller proportion of the (smaller) overcharge harm. Similarly, direct purchasers also incur a smaller proportion of the (smaller) overcharge harm due to the benefit from the larger passing-back effect dominating the cost relating to the smaller passing-on effect.

Proposition 5. *For any $m \in [1, \infty)$, as the downstream cost pass-through rate increases from $\tau = \frac{1}{2}$ towards 1, the proportion of the overcharge harm:*
i) strictly decreases for direct purchasers, $\frac{\partial \Omega_R}{\partial \tau} < 0$, from $\frac{m}{2(m+1)}$ to 0,
ii) strictly increases for indirect purchasers, $\frac{\partial \Omega_F}{\partial \tau} > 0$, from $\frac{m}{2(m+1)}$ to $\frac{m}{(m+1)} \geq \frac{1}{2}$; and
iii) remains unchanged for complementary input suppliers, $\frac{\partial \Omega_F}{\partial \tau} = 0$, at $\frac{1}{m+1} \leq \frac{1}{2}$.

As τ rises, the passing-on effect increases so direct purchasers get a smaller share of the (larger) overcharge harm and indirect buyers get a greater share. The passing-back effect is unaffected by τ, so the complementary input suppliers get the same share of the (larger) overcharge harm. Thus, even when $\tau = 1$, where direct purchasers will experience no harm because they pass on any harm to indirect buyers, the indirect purchasers will still share the overcharge harm with the complementary input suppliers, for any $m < \infty$, due to the passing-back effect.

Before moving on, let us emphasise that the input B suppliers' share of the overcharge harm can be quite large and so the shares of the direct and indirect purchasers can be smaller than absent the passing-back effect. Table 1 shows how the shares of the overcharge harm vary with the number of input B suppliers (m) and the downstream pass-through rate (τ). It includes three downstream pass-through rates: i) full pass-through ($\tau = 1$), which arises when the downstream sector is perfectly competitive; ii) monopoly pass-through ($\tau = \frac{1}{2}$); and iii) an intermediate pass-through ($\tau = \frac{3}{4}$) that is consistent with, for example, either Bertrand competition downstream with $n = 2$ and $\sigma = \frac{4}{5}$ or Cournot competition downstream with $n = 3$ and $\sigma = 1$.

	$\forall\tau$	$\tau = \frac{1}{2}$		$\tau = \frac{3}{4}$		$\tau = 1$	
m	input B sellers	direct buyers	indirect buyers	direct buyers	indirect buyers	direct buyers	indirect buyers
∞	0.000	0.500	0.500	0.250	0.750	0.000	1.000
$\vdots$	$\vdots$	$\vdots$	$\vdots$	$\vdots$	$\vdots$	$\vdots$	$\vdots$
10	0.091	0.455	0.455	0.227	0.682	0.000	0.909
$\vdots$	$\vdots$	$\vdots$	$\vdots$	$\vdots$	$\vdots$	$\vdots$	$\vdots$
5	0.167	0.417	0.417	0.208	0.625	0.000	0.833
4	0.200	0.400	0.400	0.200	0.600	0.000	0.800
3	0.250	0.375	0.375	0.188	0.563	0.000	0.750
2	0.333	0.333	0.333	0.167	0.500	0.000	0.667
1	0.500	0.250	0.250	0.125	0.375	0.000	0.500

Table 1: Division of the overcharge harm

Table 1 shows that when the input B suppliers have no market power ($m \to \infty$), all of the overcharge harm is incurred by the direct and indirect purchasers. As the input B sector becomes concentrated, the complementary input suppliers get a greater share and the direct and indirect purchasers get smaller shares, especially as the input B sector gets closer to monopoly. When there is a monopoly input B supplier ($m = 1$), it gets a share of 50% of the overcharge harm. Furthermore, when there is the monopoly pass-through rate downstream ($\tau = \frac{1}{2}$) and a monopoly input B supplier ($m = 1$), direct purchasers incur only $\frac{1}{4}$ of the overcharge harm whereas they would incur $\frac{1}{2}$ absent the passing-back effect (equivalent to $m \to \infty$). This indicates that estimating the size of any passing-back effect will be important in calculating the appropriate compensation for direct and indirect purchasers, especially when the complementary input sector is concentrated. Moreover, the size of the overcharge harm incurred by the complementary input suppliers adds weight to the argument that they should be able to sue for compensation.

4.3 Total Harm

In this subsection, we analyse how the passing-on and pass-backing effects influence the total harm. This is important because the overcharge harm only captures a proportion of the harm inflicted on the market participants as it does not account for the harm from the loss in volume. Thus, we first derive expressions of the volume harm and then sum this with the overcharge harm to find expressions of the total harm.

For an increase in marginal cost from $c(w_A)$ to $c(w'_A)$, it follows from (3) that the loss in volume of the downstream good is

$$Q^*(w_A) - Q^*\left(w'_A\right) = \frac{\beta\left[c\left(w'_A\right) - c\left(w_A\right)\right]}{2 - \sigma\left(1 - \frac{\theta}{n}\right)} > 0 \tag{15}$$

The volume harm inflicted on retailers is the loss in volume multiplied by the retail price-cost margin before the cost increase, $[p^*(w_A) - c(w_A)]\,[Q^*(w_A) - Q^*(w'_A)]$. The volume harm incurred by the complementary input suppliers can be calculated in a similar way. In contrast, the volume harm experienced by indirect purchasers is $\frac{1}{2}[p^*(w'_A) - p^*(w_A)]\,[Q^*(w_A) - Q^*(w'_A)]$, which amounts to the lost consumer surplus. Denoting

$$\Psi(\Delta) \equiv \left(2 + \frac{\varphi_A \Delta}{v - \kappa - \varphi_A w'_A - \varphi_B c_B}\right),$$

where $\Psi(\Delta) > 2$ for all $v > \kappa + \varphi_A w'_A + \varphi_B c_B$, we next derive expressions of the total harm for each market participant.

Proposition 6. *For all $v > \kappa + \varphi_A w'_A + \varphi_B c_B$ and $\theta \in \left[-\frac{n(1-\sigma)}{\sigma}, n\right]$, an increase in the price of input A from w_A to $w'_A = w_A + \Delta$ inflicts:*
i) on direct purchasers a total harm of

$$n\left[\pi^*_R(w_A) - \pi^*_R\left(w'_A\right)\right] = \Psi(\Delta)\,\Omega_R(m,\tau)\,\varphi_A \Delta Q^*\left(w'_A\right), \tag{16}$$

ii) on the complementary input suppliers a total harm of

$$m\left[\pi^*_B(w_A) - \pi^*_B\left(w'_A\right)\right] = \Psi(\Delta)\,\Omega_B(m)\,\varphi_A \Delta Q^*\left(w'_A\right), \tag{17}$$

iii) on the indirect purchasers a total harm of

$$CS^*(w_A) - CS^*\left(w'_A\right) = \frac{\Psi(\Delta)}{2}\Omega_F(m,\tau)\,\varphi_A \Delta Q^*\left(w'_A\right), \tag{18}$$

iv) a total industry harm of

$$\mathbb{H} \equiv \Psi(\Delta)\left[1 - \frac{1}{2}\Omega_F(m,\tau)\right]\varphi_A \Delta Q^*\left(w_A'\right) > 0 \tag{19}$$

Proposition 6 shows that the total harm inflicted on each market participant can be written as a function of the overcharge harm they incur. The reason is that the volume harm for each is proportionate to their overcharge harm. In particular, $\Psi(\Delta)$ represents the ratio to which the total harm for direct and indirect purchasers exceeds their overcharge harm. Thus, it represents the extent to which damages for direct purchasers and complementary input suppliers would be underestimated if just the overcharge harm was used to quantify them. In contrast, for indirect purchasers, the ratio to which the total harm exceeds the overcharge charge harm is $\frac{1}{2}\Psi(\Delta)$. Summing the total harm inflicted on all market participants yields the industry total harm, $\mathbb{H}$.

Proposition 6 implies that the shares of the total industry harm incurred by direct purchasers, indirect purchasers and complementary input suppliers are, respectively,

$$\frac{n\left[\pi_R^*(w_A) - \pi_R^*(w_A')\right]}{\mathbb{H}} = \frac{\Omega_R(m,\tau)}{1 - \frac{1}{2}\Omega_F(m,\tau)} \equiv \Phi_R(m,\tau) > \Omega_R(m,\tau), \tag{20}$$

$$\frac{CS^*(w_A) - CS^*(w_A')}{\mathbb{H}} = \frac{\frac{1}{2}\Omega_F(m,\tau)}{1 - \frac{1}{2}\Omega_F(m,\tau)} \equiv \Phi_F(m,\tau) < \Omega_F(m,\tau), \tag{21}$$

$$\frac{m\left[\pi_B^*(w_A) - \pi_B^*(w_A')\right]}{\mathbb{H}} = \frac{\Omega_B(m)}{1 - \frac{1}{2}\Omega_F(m,\tau)} \equiv \Phi_B(m,\tau) > \Omega_B(m) \tag{22}$$

This shows that the direct purchasers and the complementary input suppliers incur a greater share of the total industry harm than the overcharge harm. In contrast, the indirect purchasers incur a smaller share of the total industry harm. The reason is that the volume harm is relatively less costly for the indirect purchasers because, since demand is downward sloping, each extra unit lost is valued less by indirect purchasers but it is valued the same by the direct purchasers and complementary input suppliers.

We first consider how the total industry harm varies with the market charateristics.

Proposition 7. *The total industry harm is greater when the input B sector is less concentrated,* $\frac{\partial \mathbb{H}}{\partial m} > 0$*, or when the downstream pass-through rate is higher,* $\frac{\partial \mathbb{H}}{\partial \tau} > 0$*.*

We can describe the intuition in terms of changes to the overcharge harm, $\varphi_A \Delta Q^*(w_A')$, and volume harm, $\left[\frac{1}{2}\left(p^*(w_A') + p^*(w_A)\right) - \kappa - \varphi_A w_A - \varphi_B c_B\right]\left[Q^*(w_A) - Q^*(w_A')\right]$. As explained in section 4.2, the overcharge harm is larger when there are more input B suppliers and when there is a higher downstream pass-through rate. In contrast, the volume harm can be larger or smaller when the input B sector is more concentrated or the downstream pass-through rate is higher. The reason is that while there is a larger loss in volume, $[Q^*(w_A) - Q^*(w'_A)]$, due to the greater passing-on effect, there is the opposing effect of a smaller welfare loss per unit caused by lower $p^*(w'_A)$ and $p^*(w_A)$. Nevertheless, the larger overcharge harm always dominates any counteracting decrease in the volume harm, so the total industry harm increases.

We next show how the division of the total industry harm varies with the market characteristics.

Proposition 8. *For any given $\tau \in \left[\frac{1}{2}, 1\right]$, as the number of input B suppliers decreases from $m \to \infty$ towards 1, the proportion of the total industry harm:*
i) decreases for direct purchasers, $\frac{\partial \Phi_R}{\partial m} \geq 0$, from $\frac{2(1-\tau)}{2-\tau} \leq \frac{2}{3}$ to $\frac{2(1-\tau)}{4-\tau}$,
ii) strictly decreases for indirect purchasers, $\frac{\partial \Phi_F}{\partial m} > 0$, from $\frac{\tau}{2-\tau} \geq \frac{1}{3}$ to $\frac{\tau}{4-\tau}$; and
iii) strictly increases for complementary input suppliers, $\frac{\partial \Phi_B}{\partial m} < 0$, from 0 to $\frac{2}{4-\tau}$.

As concentration in the input B sector rises, the larger passing-back effect and the smaller passing-on effect decreases the overcharge harm and volume harm inflicted on direct and indirect purchasers. The reason is that the overcharge harm is smaller, the direct and indirect purchasers get smaller shares of the overcharge harm, and the volume harm they incur is proportionate to their overcharge harm. Together with Proposition 7, Proposition 8 implies that the total harm inflicted on the direct and indirect purchasers decreases at a faster rate than the total industry harm. Thus, they get a smaller share of the (smaller) total industry harm. In contrast, the overcharge harm and volume harm inflicted on the complementary input suppliers increases, because their larger share of the overcharge harm dominates the reduction in the overcharge harm. Consequently, they get a larger share of the (smaller) total industry harm.

Proposition 9. *For any $m \in [1, \infty)$, as the downstream cost pass-through rate rises from $\tau = \frac{1}{2}$ towards 1, the proportion of the total industry harm:*
i) strictly decreases for direct purchasers, $\frac{\partial \Phi_R}{\partial \tau} < 0$, from $\frac{2m}{4+3m}$ to 0,
ii) strictly increases for indirect purchasers, $\frac{\partial \Phi_F}{\partial \tau} > 0$, from $\frac{m}{4+3m}$ to $\frac{m}{2+m} \geq \frac{1}{3}$; and

iii) strictly increases for complementary input suppliers, $\frac{\partial \Phi_B}{\partial \tau} > 0$*, from* $\frac{4}{4+3m}$ *to* $\frac{2}{2+m} \leq \frac{2}{3}$.

As the downstream cost pass-through rate rises, the constant passing-back effect and the larger passing-on effect increases the overcharge harm and volume harm inflicted on the indirect purchasers and complementary input suppliers. The reason is that the overcharge harm is larger, indirect purchasers get a larger share of the overcharge harm while the complementary input suppliers' share is constant, and the volume harm they incur is proportionate to their overcharge harm. Together with Proposition 7, Proposition 9 implies that the total harm inflicted on the indirect purchasers and complementary input suppliers increases at a faster rate than the total industry harm, so they both get a greater share of the (larger) total industry harm. In contrast, the total harm inflicted on the direct purchasers decreases, because their reduced share of the overcharge harm dominates the increase in the overcharge harm. Consequently, they get a smaller share of the (larger) total industry harm.

Let us end this section by noting that the share of the total industry harm incurred by input B suppliers can be quite large and that this can affect the harm inflicted on direct and/or indirect purchasers. Table 2 shows how the shares of the total industry harm vary with the number of input B suppliers (m) for the same three downstream pass-through rates (τ) as in Table 1.

	$\tau = \frac{1}{2}$			$\tau = \frac{3}{4}$			$\tau = 1$		
m	input B sellers	direct buyers	indirect buyers	input B sellers	direct buyers	indirect buyers	input B sellers	direct buyers	indirect buyers
∞	0.000	0.667	0.333	0.000	0.400	0.600	0.000	0.000	1.000
$\vdots$	$\vdots$	$\vdots$	$\vdots$	$\vdots$	$\vdots$	$\vdots$	$\vdots$	$\vdots$	$\vdots$
10	0.118	0.588	0.294	0.138	0.345	0.517	0.167	0.000	0.833
$\vdots$	$\vdots$	$\vdots$	$\vdots$	$\vdots$	$\vdots$	$\vdots$	$\vdots$	$\vdots$	$\vdots$
5	0.211	0.526	0.263	0.242	0.303	0.455	0.286	0.000	0.714
4	0.250	0.500	0.250	0.286	0.286	0.429	0.333	0.000	0.667
3	0.308	0.462	0.231	0.348	0.261	0.391	0.400	0.000	0.600
2	0.400	0.400	0.200	0.444	0.222	0.333	0.500	0.000	0.500
1	0.571	0.286	0.143	0.615	0.154	0.231	0.667	0.000	0.333

Table 2: Division of the total industry harm

Table 2 shows that when the input B suppliers having no market power ($m \to \infty$), all of the total industry harm is incurred by the direct and indirect purchasers. As the complementary input sector becomes concentrated, the share of input B suppliers rises especially as the complementary input sector tends to monopoly. When there is a monopoly input B supplier ($m = 1$), it incurs over $\frac{1}{2}$ and up to $\frac{2}{3}$ of the total industry harm depending on the downstream pass-through rate. Furthermore, when there is the monopoly pass-through rate downstream ($\tau = \frac{1}{2}$) and a monopoly input B supplier ($m = 1$), direct and indirect purchasers would incur $\frac{2}{7}$ and $\frac{1}{7}$ of the total industry harm, respectively, as a result of the passing-back effect; whereas they would incur $\frac{2}{3}$ and $\frac{1}{3}$ of the total industry harm, respectively, absent the passing-back effect (equivalent to $m \to \infty$). This again demonstrates that the size of a passing-back effect can be an important factor in determining the appropriate compensation for direct and indirect purchasers and that the complementary input suppliers can also have a claim for compensation.

5 Extension: Passing-Back by Negotiation

In this section, we extend our analysis to a situation where the prices of input B are determined by negotiation. We do so for two reasons. First, reducing input prices "by negotiation" was the language used in the UK Supreme Court's judgment in Sainsbury's v Mastercard, so we want to explore how negotiations affect the passing-back and passing-on effects. Second, we wish to show how our successive oligopolies model can help to quickly determine the results of other market settings.

5.1 Basic Assumptions

Assume that there are two retailers, $n = 2$, and two input B suppliers, $m = 2$. Suppose each supplier exclusively supplies a retailer and that the retailer-supplier pairs bargain over input B prices simultaneously. Then, after observing the outcomes from the bargains, each retailer determines the quantities and prices of their final products. Given their exclusive relationships, all firms' outside options equal zero. All other assumptions are the same as in section 2.1. Despite possible asymmetries in downstream marginal costs, the conduct parameter has the same values for Cournot and Bertrand competition, so we restrict attention to those in this section ($\theta^c = 1$ and $\theta^b = \frac{1}{1+\frac{\sigma}{2(1-\sigma)}}$).[22]

[22]Again, see Appendix B for details regarding the Bertrand conduct parameter.

5.2 Equilibrium Analysis

Using (2) for $i = \{1, 2\}$, we obtain retailer i's equilibrium quantity for close c_i and c_j

$$q_i^*\left(c_i, c_j\right) = \frac{\beta}{2\left(2 - \sigma\left(1 - \frac{\theta}{2}\right)\right)} \left[v - c_i + \left(c_j - c_i\right)\left(\frac{\frac{\sigma}{2(1-\sigma)}}{2 + \frac{\sigma\theta}{2(1-\sigma)}}\right)\right], \tag{23}$$

where (23) collapses to (3) with $n = 2$ when $c_i = c_j$. Denoting $\mathbf{q}^* \equiv \{q_i^*(c_i, c_j), q_j^*(c_j, c_i)\}$, we can write, for a given w_{Bi} and w_{Bj}, the equilibrium profits of retailer i and its supplier as $\pi_{Ri}^*\left(w_{Bi}, w_{Bj}\right) = \left(p_i\left(\mathbf{q}^*\right) - c_i\right) q_i^*\left(c_i, c_j\right)$ and $\pi_{Si}^*\left(w_{Bi}, w_{Bj}\right) = \left(w_{Bi} - c_B\right)\varphi_B q_i^*\left(c_i, c_j\right)$, respectively.

Now consider the negotiations over input B prices. Given the negotiations are conducted simultaneously, each retailer-supplier pair takes their rival's input B price as given during the negotiations. The negotiated input B prices are obtained using the symmetric Nash bargaining solution. For the negotiation between retailer i and its supplier over the price w_{Bi}, the solution is characterised by:

$$w_{Bi}^* = \arg\max_{w_{Bi}} \left[\pi_{Si}^*\left(w_{Bi}, w_{Bj}^*\right)\right]^{\gamma} \left[\pi_{Ri}^*\left(w_{Bi}, w_{Bj}^*\right)\right]^{1-\gamma} \tag{24}$$

where $\gamma \in (0, 1]$ represents the supplier's bargaining power relative to its retailer. When $\gamma \to 0$, retailer i has all of the bargaining power and when $\gamma = 1$, the supplier has all of the bargaining power. The first-order condition of (24) can be expressed as:

$$\frac{\gamma}{\pi_{Si}^*(w_{Bi}, w_{Bj}^*)} \frac{\partial \pi_{Si}^*(w_{Bi}, w_{Bj}^*)}{\partial w_{Bi}} = -\frac{1-\gamma}{\pi_{Ri}^*(w_{Bi}, w_{Bj}^*)} \frac{\partial \pi_{Ri}^*(w_{Bi}, w_{Bj}^*)}{\partial w_{Bi}} \tag{25}$$

Proposition 10. *For all $v > \kappa + \varphi_A w_A + \varphi_B c_B$ and $\theta \in \left\{\frac{1}{1+\frac{\sigma}{2(1-\sigma)}}, 1\right\}$, the symmetric Nash bargaining solution yields a unique symmetric equilibrium input B price of*

$$w_B^N(w_A, \gamma, \sigma, \theta) = c_B + \frac{v - \kappa - \varphi_A w_A - \varphi_B c_B}{\varphi_B\left[\mu^*\left(\gamma, \sigma, \theta\right) + 1\right]} \in \left[c_B, \frac{v - \kappa - \varphi_A w_A + \varphi_B c_B}{2\varphi_B}\right], \tag{26}$$

where $\mu^*\left(\gamma, \sigma, \theta\right) \equiv \left(\frac{2-\gamma}{\gamma}\right)\left(1 + \frac{\frac{\sigma}{2(1-\sigma)}}{2+\frac{\sigma\theta}{2(1-\sigma)}}\right) \geq 1$. The retailers' marginal cost is $c(w_A, w_B^N(.)) = \kappa + \varphi_A w_A + \varphi_B w_B^N(.)$ and the downstream equilibrium price is:

$$p^*(w_A, w_B^N(.), \sigma, \theta) = v - \frac{v - c(w_A, w_B^N(.))}{2 - \sigma\left(1 - \frac{\theta}{2}\right)} \in \left[c(w_A, w_B^N(.)), \frac{v + c(w_A, w_B^N(.))}{2}\right] \tag{27}$$

Both prices are strictly increasing in the supplier's bargaining power, $\frac{\partial w_B^N}{\partial \gamma} > 0$ *and* $\frac{dp^*}{d\gamma} > 0$*, and are strictly decreasing in the degree of product substitutability under Cournot* ($\theta^c = 1$) *and Bertrand competition* ($\theta^b = \frac{1}{1+\frac{\sigma}{2(1-\sigma)}}$)*,* $\frac{\partial w_B^N}{\partial \sigma} < 0$ *and* $\frac{dp^*}{d\sigma} < 0$*.*

The Nash bargaining price of input B equates the supplier's and retailer's weighted concession costs measured as a proportion of the gains from agreement, given by the left-hand side of (25) and the right-hand side, respectively. The intuition is that when, say, the left-hand side is smaller than the right-hand side, it is relatively less costly for the supplier to concede to a lower unit price than it is for the retailer to concede to a higher unit price. Consequently, the retailer bargains more aggressively than the supplier, which reduces the unit price until (25) is balanced.

When the supplier has no bargaining power, the retailer extracts all of the surplus, $\lim_{\gamma\to 0} w_B^N(.) = c_B$. As the supplier's bargaining power rises (and the retailer's falls), it becomes relatively less costly for the retailer to concede to a higher price, so $w_B^N(.)$ rises. When the supplier has all of the bargaining power, $\gamma = 1$, $w_B^N(.)$ maximises the supplier's profits. Furthermore, greater product substitutability endogenously strengthens the retailer's bargaining position relative to the supplier's, so $w^N(.)$ falls. The reason is that a cost disadvantage relative to its rival is more costly for a retailer when products are more substitutable and this strengthens each retailer's resolve in the negotiations with its supplier. For similar reasons, the negotiated price is lower under Bertrand competition compared to Cournot. Finally, given $p^*(.)$ is increasing in marginal cost, the positve relationship between $w_B^N(.)$ and supplier's bargaining power, γ, ensures that $p^*(.)$ also strictly increases with γ; and the negative relationship with $w_B^N(.)$ and product substitutability, σ, implies there is a negative indirect effect on $p^*(.)$ in addition to the standard negative direct effect of product substitutability.

5.3 The Effects of an Input Cartel

Consider the effects of an input cartel. This can be achieved quickly by drawing upon our previous analysis, because the equilibrium prices in (26) and (27) are the same as in (5) and (6), respectively, except that $\mu^*(\gamma, \sigma, \theta)$ replaces m. Thus, substituting $\mu^*(\gamma, \sigma, \theta)$ for m throughout section 4 yields the necessary expressions and results. In particular, consider how the passing-on and passing-back effects vary with the supplier's bargaining power, γ. Given $\lim_{\gamma\to 0} w_B^N(.) = c_B$ (since $\lim_{\gamma\to 0}\mu^*(\gamma, \sigma, \theta) = \infty$), it follows that when the input B suppliers have no bargaining power, there is no passing-back effect and input B suppliers incur none of the overcharge harm or total harm. However, noting that $\frac{\partial \mu^*}{\partial \gamma} < 0$ and using Proposition 2, we can state the following for $\gamma > 0$.

Proposition 11. *For any given downstream pass-through rate, $\tau \in \left[\frac{1}{2}, 1\right]$, as the suppliers' bargaining power rises from $\gamma = 0$ towards 1:*
i) the magnitude of the passing-back effect strictly increases away from 0; and
ii) the magnitude of the passing-on effect strictly decreases away from $\varphi_A \Delta\tau > 0$.

When each input B supplier has greater bargaining power, the negotiated input B price is closer to their profit-maximising price than the retailer's, so it is more responsive to changes in demand (since the retailer's profit-maximising price of c_B is independent of demand). Thus, the decrease in the demand of input B that results from the rise in w_A leads to a greater fall in $w_B^N(.)$ and hence the passing-back effect is larger. This in turn makes the increase in marginal cost downstream smaller, because the rise in w_A is offset by a larger fall in $w_B^N(.)$, so the passing-on effect is smaller.

By using $\frac{\partial \mu^*}{\partial \gamma} < 0$, we can also understand how bargaining power affects the harm of the cartel. First, as each supplier's bargaining power rises, the overcharge harm is smaller because the industry quantity is lower and the downstream price is higher, $\frac{dp^*}{d\gamma} > 0$. Next, from Proposition 4, the direct and indirect purchasers get smaller shares of the overcharge harm, $\frac{d\Omega_R}{d\gamma} = \frac{\partial \mu^*}{\partial \gamma}\frac{\partial \Omega_R}{\partial m} < 0$ and $\frac{d\Omega_F}{d\gamma} = \frac{\partial \mu^*}{\partial \gamma}\frac{\partial \Omega_F}{\partial m} < 0$, and the input B suppliers' share is larger, $\frac{d\Omega_B}{d\gamma} = \frac{\partial \mu^*}{\partial \gamma}\frac{\partial \Omega_B}{\partial m} > 0$. This follows from a larger passing-back effect and a smaller passing-on effect. Similarly, regarding the total harm, Proposition 7 implies the total industry harm is smaller when each input B supplier has greater bargaining power, $\frac{d\mathbb{H}}{d\gamma} = \frac{\partial \mu^*}{\partial \gamma}\frac{\partial \mathbb{H}}{\partial m} < 0$. Moreover, from Propositon 8, the direct and indirect purchasers' shares of the total harm are smaller, $\frac{d\Phi_R}{d\gamma} = \frac{\partial \mu^*}{\partial \gamma}\frac{\partial \Phi_R}{\partial m} < 0$ and $\frac{d\Phi_F}{d\gamma} = \frac{\partial \mu^*}{\partial \gamma}\frac{\partial \Phi_F}{\partial m} < 0$, and the complementary input suppliers' share is larger, $\frac{d\Phi_B}{d\gamma} = \frac{\partial \mu^*}{\partial \gamma}\frac{\partial \Phi_B}{\partial m} > 0$.

One difference compared to the successive oligopolies model is that the passing-back effect is now a function of product substitutability, σ, and the form of competition downstream, θ. The reason is that these parameters endogenously affect the bargaining positions of the negotiating retailer and supplier, where $\frac{\partial \mu^*}{\partial \sigma} > 0$ and $\frac{\partial \mu^*}{\partial \theta} < 0$. This in turn will influence the passing-on effect. To understand the similarities and differences, we consider the effect of product substitutability as an example. We begin by showing that product substitutability affects the passing-back and passing-on effects in a similar way as in the successive oligopolies model.

Proposition 12. *For any $\gamma > 0$, under Cournot ($\theta^c = 1$) and Bertrand competition ($\theta^b = \frac{1}{1+\frac{\sigma}{2(1-\sigma)}}$), as the degree of product substitutability increases from $\sigma = 0$ towards 1:*
i) the magnitude of the passing-back effect strictly decreases; and
ii) the magnitude of the passing-on effect strictly increases.

Intuitively, when the products are closer substitutes the negotiated input B price is closer to c_B because retailers bargain more aggressively. It is therefore less responsive to changes in demand, so a rise in w_A leads to a smaller passing-back effect. As a result, the increase in downstream marginal cost is larger, because the rise in w_A is offset less by the smaller fall in $w_B^N(.)$. Thus, the passing-on effect is larger because, in addition to a higher downstream pass-through rate when products are closer substitutes, $\frac{\partial \tau}{\partial \sigma} > 0$, there is also a greater cost increase to pass on.

Given product substitutability now affects τ and $\mu^*(\gamma, \sigma, \theta)$, the relationships between product substitutability and the various harms can be different. For instance, as products become closer substitutes, the complementary input suppliers' share of the overcharge harm is now smaller, $\frac{d\Omega_B}{d\sigma} = \frac{\partial \mu^*}{\partial \sigma}\frac{\partial \Omega_B}{\partial m} < 0$, due to the smaller passing-back effect, yet the indirect purchasers' share is still larger, due to a greater cost increase being passed on at a higher rate, $\frac{d\Omega_F}{d\sigma} = \frac{\partial \mu^*}{\partial \sigma}\frac{\partial \Omega_F}{\partial m} + \frac{\partial \tau}{\partial \sigma}\frac{\partial \Omega_F}{\partial \tau} < 0$. However, the effect on the direct purchasers' share now depends upon how much they lose from the smaller passing-back effect and how much they gain from the larger passing-on effect, $\frac{d\Omega_R}{d\sigma} = \frac{\partial \mu^*}{\partial \sigma}\frac{\partial \Omega_R}{\partial m} + \frac{\partial \tau}{\partial \sigma}\frac{\partial \Omega_R}{\partial \tau}$. When each supplier's bargaining power is low, the passing-back effect is close to zero, so the direct purchasers' share of the overcharge harm falls with product substitutability, due to the greater passing-on effect. When each supplier's bargaining power is high, there is a u-shaped relationship between the direct purchaser's share and product substitutability. This implies that when products are close substitutes and the indirect purchasers' share rises, the loss from the smaller passing-back effect dominates the benefits from greater passing-on effect. Similar results apply for the division of the total harm (i.e., $\frac{d\Phi_B}{d\sigma} < 0$, $\frac{d\Phi_F}{d\sigma} < 0$ and $\frac{d\Phi_F}{d\sigma}$ is non-monotonic).

6 Concluding Remarks

By developing a model where downstream firms must source two inputs to produce their products, we have analysed the equilibrium effects of an input cartel when the cartel's direct purchasers can pass on and pass back some of the overcharge to their customers and other complementary input suppliers. In a successive oligopolies framework, we showed that the passing-back effect is larger when the complementary input sector is more concentrated and as a result the passing-on effect is smaller. An implication of this is that the complementary input suppliers incur a proportion of the overcharge harm and total harm, and so the proportions of the overcharge harm and total harm incurred by the direct and indirect purchasers are smaller. We also showed that a higher downstream cost pass-through rate leads to a larger passing-on effect and does not affect the passing-back effect. This reduces the direct purchasers' share of the overcharge harm and total harm at the expense of the complementary input suppliers and/or indirect purchasers. In an extension, we used the results of the successive oligopolies model to quickly derive the results of a setting in which the complementary input price is determined by negotiation. We showed that the passing-back effect is larger and the passing-on effect is smaller when the complementary input suppliers have greater bargaining power.

Our results have two important implications for damages claims in practice. First, the complementary input suppliers can incur significant harm as a consequence of the passing-back effect. Hence, there is a case for them to be able to sue for compensation as well as direct and indirect purchasers. Our model predicts that this case is strongest when the complementary input suppliers have market power. Second, deriving the true harm inflicted on direct and/or indirect buyers is also likely to involve estimating the size of any passing-back effect in many cases. This will involve developing an understanding of competition in the complementary input sector, even when the complementary input suppliers are not part of the trial. Our analysis suggests that estimating the passing-back effects will be most important when the complementary input sector is concentrated or when the complementary input suppliers have substantial bargaining power.

References

[1] Adachi, T. and Ebina, T. (2014a) "Double Marginalization and Cost Pass-Through: Weyl–Fabinger and Cowan meet Spengler and Bresnahan–Reiss" *Economics Letters*,

122, 170-175

[2] Adachi, T. and Ebina, T. (2014b) "Cost Pass-Through and Inverse Demand Curvature in Vertical Relationships with Upstream and Downstream Competition," *Economics Letters*, 124, 465-468

[3] Anderson, S., de Palma, A. and Kreider, B. (2001) "Tax Incidence in Differentiated Product Oligopoly," *Journal of Public Economics*, 81, 173-192

[4] Basso, L. and Ross, T. (2010) "Measuring the True Harm from Price-Fixing to Both Direct and Indirect Purchasers," *Journal of Industrial Economics*, 58(4), 895-927

[5] Bulow, J. and Pfleiderer, P. (1983) "A Note on the Effect of Cost Changes on Prices," *Journal of Political Economy*, 91, 182-185

[6] Bet, G., Shana, C. and Sappington, D. (2021) "The Impact of Vertical Integration on Losses from Collusion," *International Journal of Industrial Organization*, 77, 102756

[7] Boone and Muller (2012) "The Distribution of Harm in Price-Fixing Cases," *International Journal of Industrial Organization*, 30: 265-276

[8] Choné, P. and Linnemer, L. (2020) "Linear Demand Systems for Differentiated Goods: Overview and User's Guide" *International Journal of Industrial Organization*, 73, 102663

[9] Coulson, E. and Blacklock, B. (2021) "Brexit and its Impact on Competition Litigation in the UK," in Private Litigation Guide - Third Edition, Global Competition Review, available at: https://globalcompetitionreview.com/guide/private-litigation-guide/third-edition/article/brexit-and-its-impact-competition-litigation-in-the-uk

[10] Dobson, P., and Waterson, M. (1997) "Countervailing Power and Consumer Prices," *Economic Journal*, 107(441), 418–430

[11] Dobson, P., and Waterson, M (2007) "The Competition Effects of Industry-Wide Vertical Price Fixing in Bilateral Oligopoly," *International Journal of Industrial Organization*, 25(5), 935–962

[12] European Commission (2013), Quantifying Harm in Actions for Damages Based on Breaches of Article 101 or 102 of the Treaty on the Functioning of the European Union, available at: https://ec.europa.eu/competition/antitrust/actionsdamages/quantification_guide_en.pdf

[13] European Commission (2019), Guidelines for National Courts on How to Estimate the Share of Overcharge which was Passed on to the Indirect Purchaser, available at: https://eur-lex.europa.eu/legal-content/EN/TXT/PDF/?uri=CELEX:52019XC0809(01)&from=EN

[14] Gaudin, G. (2016) "Pass-Through, Vertical Contracts, and Bargains, " Economics Letters, 139, 1-4

[15] Han, M. Schinkel, M.P. and Tuinstra, J. (2008) "The Overcharge as a Measure for Antitrust Damages," Amsterdam Center for Law & Economics Working Paper No. 2008-08

[16] Hellwig, M. (2007) "Private Damage Claims and the Passing-On Defense in Horizontal Price-Fixing Cases: An Economist's Perspective," in Basedow, J. (ed.), *Private Enforcement of EC Competition Law*, Kluwer Law Aspen Publishers, 121-159

[17] Kosicki, G. and Cahill, M. (2006) "Economics of Cost Pass Through and Damages in Indirect Purchaser Antitrust Cases," *Antitrust Bulletin*, 51 (3), 599-630

[18] Ritz, R. (2022) "Does Competition Increase Pass-Through?" *RAND Journal of Economics*, forthcoming

[19] Seade, J. (1985) "Profitable Cost Increases and the Shifting of Taxation: Equilibrium Response of Markets in Oligopoly," The Warwick Economics Research Paper Series (TWERPS) 260, University of Warwick, Department of Economics

[20] Shubik, M. and Levitan, R. (1980) *Market Structure and Behavior*, Harvard University Press

[21] Verboven, F. and van Dijk, T. (2009) "Cartel Damages Claims and the Passing-On Defense," *Journal of Industrial Economics*, 57(3), 457-491

[22] Weyl, E. G. and Fabinger, M. (2013) "Pass-Through as an Economic Tool: Principles of Incidence under Imperfect Competition," *Journal of Political Economy*, 121(3), 528-583

[23] Whish, R. and Bailey, D. (2021) *Competition Law*, Tenth Edition, Oxford University Press

Appendix

Proof of Proposition 1. Let $\varphi_B X^B_{-k}$ denote the quantity produced by all of supplier k's $m-1$ rivals. Substitute $X^B = \varphi_B x^B_k + \varphi_B Q^B_{-k}$ and (4) into $\pi_{Bk}(X_B) = (w_B(w_A, X_B) - c_B)\varphi_B x_{Bk}$ and maximise with respect to x^B_k to find supplier k's best response function:

$$\varphi_B x^B_k \left(X^B_{-k}\right) = \varphi_B \left[\frac{\beta}{2-\sigma\left(1-\frac{\theta}{n}\right)}\left(\frac{v-\kappa-\varphi_A w_A - \varphi_B c_B}{2}\right) - \frac{X^B_{-k}}{2}\right] \tag{28}$$

In a symmetric equilibrium, where $\varphi_B x^B_k = \varphi_B x^*_B$ for all k, it follows that input B supplier k's rivals will produce $\varphi_B X^B_{-k} = (m-1)\varphi_B x^*_B$. Substituting $\varphi_B x^B_k = \varphi_B x^*_B$ and $\varphi_B Q^B_{-k} = \varphi_B (m-1) x^*_B$ into (28) and then rearranging yields

$$\varphi_B x^*_B(w_A) = \varphi_B \frac{\beta}{2-\sigma\left(1-\frac{\theta}{n}\right)}\left(\frac{v-\kappa-\varphi_A w_A-\varphi_B c_B}{m+1}\right)$$

The total quantity of input B produced is $X^*_B = \varphi_B m x^*_B(w_A)$. In equilibrium, we require $X^*_B = \varphi_B n q^*(c)$, so that the retailers demand φ_B units of input B for each unit of the final good they sell. Substituting in for $\varphi_B m x^*_B(w_A) = \varphi_B n q^*(c)$ and rearranging shows that $w^*_B(w_A, m)$ is as claimed. Substituting $q^*\left(c\left(w_A, w^*_B(.)\right)\right)$ into (1) shows $p^*\left(w_A, \sigma, n, \theta\right)$ is as claimed.

Differentiating (5) and (6) with respect to m yields $\frac{\partial w^*_B}{\partial m} = -\frac{v-\kappa-\varphi_A w_A - \varphi_B c_B}{\varphi_B (m+1)^2} < 0$ and $\frac{dp^*}{dm} = \frac{\partial p^*}{\partial w_B}\frac{\partial w^*_B}{\partial m} < 0$, respectively. Furthermore, differentiating (6) with respect to σ and n shows sign$\left\{\frac{dp^*}{d\sigma}\right\}$ = sign$\left\{\frac{\sigma}{n}\frac{\partial\theta}{\partial\sigma} - \left(1-\frac{\theta}{n}\right)\right\}$ and sign$\left\{\frac{dp^*}{dn}\right\}$ = sign$\left\{\frac{\sigma}{n}\left(\frac{\partial\theta}{\partial n} - \frac{\theta}{n}\right)\right\}$, respectively. Thus, $\frac{dp^*}{d\sigma} < 0$ and $\frac{dp^*}{dn} < 0$ for $\theta^c = 1$ and $\theta^b = \frac{1}{1+\frac{\sigma}{1-\sigma}\frac{n-1}{n}}$ since $\frac{\partial\theta}{\partial n} \leq 0$ and $\frac{\partial\theta}{\partial\sigma} \leq 0$. Substituting $\theta^m = n$ and $\theta^p = -\frac{n(1-\sigma)}{\sigma}$ into (6) shows that $p^*\left(w_A, \sigma, n, \theta\right)$ is independent of σ and n in those cases. Finally, $w^*_B(w_A, m)$ is independent of σ, n and θ. ■

Proof of Proposition 2. Differentiating (7) and (8) with respect to m yields $\frac{\partial\left(w^*_B(w'_A) - w^*_B(w_A)\right)}{\partial m} = \frac{\varphi_A \Delta}{\varphi_B (m+1)^2} > 0$ and $\frac{\partial\left(p^*(w'_A) - p^*(w_A)\right)}{\partial m} = \frac{\varphi_A\Delta}{(m+1)^2}\tau > 0$, respectively. The former implies the price change is less negative, so the magnitude of the passing-back effect is strictly decreasing in m, and the latter implies that the price change is more positive, so again the magnitude of the passing-on effect is strictly increasing in m. ■

Proof of Proposition 3. Differentiating (7) and (8) with respect to τ yields $\frac{\partial\left(w_B^*(w_A')-w_B^*(w_A)\right)}{\partial\tau} = 0$ and $\frac{\partial\left(p^*(w_A')-p^*(w_A)\right)}{\partial\tau} = \frac{\varphi_A \Delta m}{(m+1)} > 0$, respectively. The former implies the magnitude of the passing-back effect is independent of τ, and the latter implies that the magnitude of the passing-on effect is strictly increasing in τ. ■

Proof of Proposition 4. Differentiating $\Omega_R(m,\tau)$, $\Omega_F(m,\tau)$ and $\Omega_B(m)$ with respect to m yields $\frac{\partial\Omega_R}{\partial m} = \frac{1-\tau}{(m+1)^2} > 0$, $\frac{\partial\Omega_F}{\partial m} = \frac{\tau}{(m+1)^2} > 0$ and $\frac{\partial\Omega_B}{\partial m} = -\frac{1}{(m+1)^2} < 0$, respectively. ■

Proof of Proposition 5. Differentiating $\Omega_R(m,\tau)$, $\Omega_F(m,\tau)$ and $\Omega_B(m)$ with respect to τ yields $\frac{\partial\Omega_R}{\partial\tau} = -\frac{m}{(m+1)} < 0$, $\frac{\partial\Omega_F}{\partial\tau} = \frac{m}{(m+1)} > 0$ and $\frac{\partial\Omega_B}{\partial\tau} = 0$, respectively. ■

Proof of Proposition 6. The total harm inflicted on retailers equals

$$\varphi_A \Delta \Omega_R(m,\tau) Q^*\left(w_A'\right) + \left[p^*(w_A) - c(w_A)\right]\left[Q^*(w_A) - Q^*\left(w_A'\right)\right],$$

where the first term is the overcharge harm and the second is the volume harm. Substituting (6) and (15) in for the second term yields

$$\begin{aligned}
\left[p^*(w_A) - c(w_A)\right]\left[Q^*(w_A) - Q^*\left(w_A'\right)\right] &= \left[(1-\tau)(v - c(w_A))\right]\left[\beta\tau\left(c\left(w_A'\right) - c(w_A)\right)\right] \\
&= (1-\tau)\left(\frac{v - c(w_A)}{v - c\left(w_A'\right)}\right) Q^*\left(w_A'\right)\varphi_A\Delta\frac{m}{m+1} \\
&= \varphi_A \Delta \Omega_R(m,\tau) Q^*\left(w_A'\right)\left(\frac{v - c(w_A)}{v - c\left(w_A'\right)}\right)
\end{aligned}$$

since $c\left(w_A'\right) - c(w_A) = \varphi_A \Delta \frac{m}{m+1}$ and recall $\tau = \frac{1}{2-\sigma\left(1-\frac{\theta}{n}\right)}$. Then summing the first term with the second yields

$$\begin{aligned}
n\left[\pi_R^*(w_A) - \pi_R^*\left(w_A'\right)\right] &= \varphi_A \Delta \Omega_R(m,\tau) Q^*\left(w_A'\right)\left[1 + \frac{v - c(w_A)}{v - c\left(w_A'\right)}\right] \\
&= \varphi_A \Delta \Omega_R(m,\tau) Q^*\left(w_A'\right)\Psi(\Delta),
\end{aligned}$$

since $v - c(w_A) = \left(\frac{m}{m+1}\right)(v - \kappa - \varphi_A w_A - \varphi_B c_B)$, so (16) is as claimed.

The total harm on each input B supplier equals

$$\varphi_B \left[w_B^* \left(w_A'\right) - w_B^* \left(w_A\right)\right] Q^* \left(w_A'\right) + \varphi_B \left[w_B^* \left(w_A\right) - c_B\right] \left[Q^* \left(w_A\right) - Q^* \left(w_A'\right)\right],$$

where again the first term is each input B supplier's overcharge harm and the second is the volume harm. Substituting (7) and (15) in for the second term yields

$$\begin{aligned} \varphi_B \left[w_B^* \left(w_A\right) - c_B\right] \left[Q^* \left(w_A\right) - Q^* \left(w_A'\right)\right] &= \left[\frac{v - \kappa - \varphi_A w_A - \varphi_B c_B}{m+1}\right] \left[\beta\tau \left(c\left(w_A'\right) - c\left(w_A\right)\right)\right] \\ &= \frac{\varphi_A \Delta}{m+1} \left(\frac{m}{m+1}\right) \left(\frac{v - \kappa - \varphi_A w_A - \varphi_B c_B}{v - c\left(w_A'\right)}\right) Q^* \left(w_A'\right) \\ &= \varphi_A \Delta \Omega_B \left(m\right) \left(\frac{v - c\left(w_A\right)}{v - c\left(w_A'\right)}\right) Q^* \left(w_A'\right) \end{aligned}$$

Then summing the first term with the second yields

$$\begin{aligned} m \left[\pi_B^* \left(w_A\right) - \pi_B^* \left(w_A'\right)\right] &= \varphi_A \Delta \Omega_B \left(m\right) Q^* \left(w_A'\right) \left[1 + \frac{v - c\left(w_A\right)}{v - c\left(w_A'\right)}\right] \\ &= \varphi_A \Delta \Omega_B \left(m\right) Q^* \left(w_A'\right) \Psi\left(\Delta\right), \end{aligned}$$

so (17) is as claimed.

The total harm on consumers equals

$$\left[p^* \left(w_A',\right) - p^* \left(w_A\right)\right] Q^* \left(w_A'\right) + \frac{1}{2} \left[p^* \left(w_A'\right) - p^* \left(w_A\right)\right] \left[Q^* \left(w_A\right) - Q^* \left(w_A'\right)\right]$$

where the first term is overcharge harm on final consumers and the second is the volume harm. Substituting (8) and (15) in for the second term yields

$$\begin{aligned} \frac{1}{2} \left[p^* \left(w_A'\right) - p^* \left(w_A\right)\right] \left[Q^* \left(w_A\right) - Q^* \left(w_A'\right)\right] &= \frac{\varphi_A \Delta}{2} \Omega_F \left(m, \tau\right) \left[\beta\tau \left(c\left(w_A'\right) - c\left(w_A\right)\right)\right] \\ &= \frac{\varphi_A \Delta}{2} \Omega_F \left(m, \tau\right) \left(\frac{c\left(w_A'\right) - c\left(w_A\right)}{v - c\left(w_A'\right)}\right) Q^* \left(w_A'\right) \end{aligned}$$

Then summing the first term with the second yields

$$\begin{aligned} CS^* \left(w_A\right) - CS^* \left(w_A'\right) &= \varphi_A \Delta \Omega_F \left(m, \tau\right) Q^* \left(w_A'\right) \left[1 + \frac{1}{2} \left(\frac{c\left(w_A'\right) - c\left(w_A\right)}{v - c\left(w_A'\right)}\right)\right] \\ &= \varphi_A \Delta \Omega_F \left(m, \tau\right) Q^* \left(w_A'\right) \frac{1}{2} \Psi\left(\Delta\right), \end{aligned}$$

so (18) is as claimed.

Finally, summing (16), (17), and (18) shows (19) is as claimed. ■

Proof of Proposition 7. Differentiating $\mathbb{H}$ with respect to m and τ yields

$$\frac{\partial \mathbb{H}}{\partial m} = \frac{2\tau}{(m+1)^2}\left(1 - \frac{m}{m+1}\tau\right)\frac{\Psi(\Delta)}{2}\varphi_A \Delta\beta\left(v - \kappa - \varphi_A w_A' - \varphi_B c_B\right) > 0$$

and

$$\frac{\partial \mathbb{H}}{\partial \tau} = \frac{2m}{(m+1)^2}\left(1 - \frac{m}{m+1}\tau\right)\frac{\Psi(\Delta)}{2}\varphi_A \Delta\beta\left(v - \kappa - \varphi_A w_A' - \varphi_B c_B\right) > 0,$$

respectively. ■

Proof of Proposition 8. Differentiating (20), (21) and (22) with respect to τ yields $\frac{\partial \Phi_R}{\partial \tau} = -\frac{2m(m+2)}{[2+m(2-\tau)]^2} < 0$, $\frac{\partial \Phi_F}{\partial \tau} = \frac{2m(m+1)}{[2+m(2-\tau)]^2} > 0$, and $\frac{\partial \Phi_B}{\partial \tau} = \frac{2m}{[2+m(2-\tau)]^2} > 0$, respectively. ■

Proof of Proposition 9. Differentiating (20), (21) and (22) with respect to m yields $\frac{\partial \Phi_R}{\partial m} = \frac{4(1-\tau)}{[2+m(2-\tau)]^2} \geq 0$, $\frac{\partial \Phi_F}{\partial m} = \frac{2\tau}{[2+m(2-\tau)]^2} > 0$ and $\frac{\partial \Phi_B}{\partial m} = -\frac{2(2-\tau)}{[2+m(2-\tau)]^2} < 0$. ■

Proof of Proposition 10. To solve for the symmetric Nash bargaining solution, we must find and substitute in for each of the terms in (25). First, if $w_{Bi} = w_{Bj} \equiv w_B$, such that $c_i = c_j = c$, it follows that for any $v > c$,

$$\pi^*_{Ri}(w_B, w_B) = (v - c)\left(1 - \frac{1}{2 - \sigma\left(1 - \frac{\theta}{2}\right)}\right)\frac{\beta(v-c)}{2\left(2 - \sigma\left(1 - \frac{\theta}{2}\right)\right)}, \tag{29}$$

and

$$\pi^*_{Si}(w_B, w_B) = \varphi_B(w_B - c_B)\frac{\beta(v-c)}{2\left(2 - \sigma\left(1 - \frac{\theta}{2}\right)\right)}. \tag{30}$$

Furthermore, substituting $q^*_i(c_i, c_j)$ and $q^*_j(c_j, c_i)$ into (1) yields

$$p_i(\mathbf{q}^*) = v - \frac{1}{2 - \sigma\left(1 - \frac{\theta}{2}\right)}\left[v - c_i + (c_i - c_j)\frac{\sigma}{2}\left(1 - \frac{1}{2 + \frac{\sigma\theta}{2(1-\sigma)}}\right)\right] \equiv p^*_i(c_i, c_j) \tag{31}$$

Then it follows from $\pi^*_{Ri}(w_{Bi}, w_{Bj}) = (p_i(\mathbf{q}^*) - c_i)\, q^*_i(c_i, c_j)$ that

$$\begin{aligned}
\frac{\partial \pi^*_{Ri}(w_B,w_B)}{\partial w_{Bi}} &= -\frac{\partial c_i}{\partial w_{Bi}}\left[\left(1-\frac{\partial p^*_i}{\partial c_i}\right) q^*_i(c_i,c_j) - (p^*_i(c_i,c_j)-c_i)\frac{\partial q^*_i}{\partial c_i}\right]_{c_i=c_j=c} \\
&= -\frac{\varphi_B \beta (v-c)}{2\left(2-\sigma\left(1-\frac{\theta}{2}\right)\right)^2}\left[\frac{\sigma}{2}\left(1-\frac{1}{2+\frac{\sigma\theta}{2(1-\sigma)}}\right) + \left(1-\sigma\left(1-\frac{\theta}{2}\right)\right)\left(2+\frac{\frac{\sigma}{2(1-\sigma)}}{2+\frac{\sigma\theta}{2(1-\sigma)}}\right)\right]
\end{aligned} \tag{32}$$

and from $\pi^*_{Si}(w_{Bi}, w_{Bj}) = (w_{Bi} - c_B)\, \varphi_B q^*_i(c_i, c_j)$ that

$$\begin{aligned}
\frac{\partial \pi^*_{Si}(w_B,w_B)}{\partial w_{Bi}} &= \left[\varphi_B q^*_i(c_i,c_j) + (w_B - c_B)\varphi_B \frac{\partial c_i}{\partial w_{Bi}}\frac{\partial q^*_i}{\partial c_i}\right]_{c_i=c_j=c} \\
&= \frac{\varphi_B \beta}{2\left(2-\sigma\left(1-\frac{\theta}{2}\right)\right)}\left[v-c-(w_B-c_B)\varphi_B\left(1+\frac{\frac{\sigma}{2(1-\sigma)}}{2+\frac{\sigma\theta}{2(1-\sigma)}}\right)\right]
\end{aligned} \tag{33}$$

given $\frac{\partial c_i}{\partial w_{Bi}} = \varphi_B$, $\frac{\partial p^*_i}{\partial c_i} = \frac{1}{2-\sigma\left(1-\frac{\theta}{2}\right)}\left[1-\frac{\sigma}{2}\left(1-\frac{1}{2+\frac{\sigma\theta}{2(1-\sigma)}}\right)\right]$ and $\frac{\partial q^*_i}{\partial c_i} = -\frac{\beta}{2\left(2-\sigma\left(1-\frac{\theta}{2}\right)\right)}\left[1+\frac{\frac{\sigma}{2(1-\sigma)}}{2+\frac{\sigma\theta}{2(1-\sigma)}}\right]$. Substituting (29), (30), (32) and (33) into (25) and rearranging gives:

$$\varphi_B(w_B - c_B)\left(\left(1+\frac{\sigma}{4(1-\sigma)+\sigma\theta}\right) + (1-\gamma)\left(1+\frac{\sigma\left(1-\frac{2(1-\sigma)}{4(1-\sigma)+\sigma\theta}\right)}{2\left(1-\sigma\left(1-\frac{\theta}{2}\right)\right)}\right)\right) = \gamma(v-c)$$

Finally, substituting in for $c = \kappa + \varphi_A w_A + \varphi_B w_B$ and rearranging shows $w^N_B(w_A, \gamma, \sigma, \theta)$ is as claimed. Substituting $c_i = c_j = c\left(w_A, w^N_B(.)\right)$ into (31) shows $p^*(w_A, w^N_B(.), \sigma, \theta)$ is as claimed. ■

Proof of Proposition 11. Evaluating m at $\mu^*(\gamma, \sigma, \theta)$ in (7) and (8) and totally differentiating with respect to γ yields $\frac{d\left(w^*_B(w'_A) - w^*_B(w_A)\right)}{d\gamma} = \frac{\partial \mu^*}{\partial \gamma}\frac{\partial\left(w^*_B(w'_A) - w^*_B(w_A)\right)}{\partial m} = \frac{\partial \mu^*}{\partial \gamma}\frac{\varphi_A \Delta}{\varphi_B(\mu^*+1)^2} < 0$, and $\frac{d\left(p^*(w'_A) - p^*(w_A)\right)}{d\gamma} = \frac{\partial \mu^*}{\partial \gamma}\frac{\partial\left(p^*(w'_A) - p^*(w_A)\right)}{\partial m} = \frac{\partial \mu^*}{\partial \gamma}\frac{\varphi_A \Delta}{(\mu^*+1)^2}\tau < 0$, respectively, where $\frac{\partial \mu^*}{\partial \gamma} = -\frac{2}{\gamma^2}\left(1+\frac{\sigma}{4(1-\sigma)+\sigma\theta}\right) < 0$. The former implies the price change is more negative, so the magnitude of the passing-back effect strictly increases, and the latter implies that the price change is less positive, so the magnitude of the passing-on effect strictly decreases. ■

Proof of Proposition 12. Evaluating m at $\mu^*(\gamma, \sigma, \theta)$ in (7) and (8) and totally differentiating with respect to σ yields $\frac{d\left(w^*_B(w'_A) - w^*_B(w_A)\right)}{d\sigma} = \frac{\partial \mu^*}{\partial \sigma}\frac{\partial\left(w^*_B(w'_A) - w^*_B(w_A)\right)}{\partial m} = \frac{\partial \mu^*}{\partial \sigma}\frac{\varphi_A \Delta}{\varphi_B(\mu^*+1)^2} > 0$ and $\frac{d\left(p^*_B(w'_A) - p^*_B(w_A)\right)}{d\sigma} = \frac{\partial\left(p^*_B(w'_A) - p^*_B(w_A)\right)}{\partial \sigma} + \frac{\partial \mu^*}{\partial \sigma}\frac{\partial\left(p^*_B(w'_A) - p^*_B(w_A)\right)}{\partial m} = \frac{\partial\left(p^*_B(w'_A) - p^*_B(w_A)\right)}{\partial \sigma} + \frac{\partial \mu^*}{\partial \sigma}\frac{\varphi_A \Delta}{(\mu^*+1)^2}\tau > 0$, respectively, where $\frac{\partial\left(p^*_B(w'_A) - p^*_B(w_A)\right)}{\partial \sigma} = \frac{2-\theta-\sigma\frac{\partial\theta}{\partial\sigma}}{2\left(2-\sigma\left(1-\frac{\theta}{2}\right)\right)^2} > 0$ and $\frac{\partial \mu^*}{\partial \sigma} =$

$\frac{(2-\gamma)\left(4-\sigma\frac{\partial\theta}{\partial\sigma}\right)}{\gamma(4(1-\sigma)+\sigma\theta)^2} > 0\ \forall\gamma > 0$ since $\frac{\partial\theta}{\partial\sigma} \leq 0$ for $\theta^c = 1$ and $\theta^b = \frac{1}{1+\frac{\sigma}{2(1-\sigma)}} \in (0,1)$. The former implies the input B price change is less negative, so the magnitude of the passing-back effect strictly decreases, and the latter implies that the price change is more positive, so the magnitude of the passing-on effect strictly increases. ■

Appendix B: Bertrand Conduct Parameter

In this appendix, we wish to prove that (2) yields the differentiated Bertrand-Nash equilibrium when $\theta^b = \frac{1}{1+\left(\frac{\sigma}{1-\sigma}\right)\left(\frac{n-1}{n}\right)}$. We achieve this by showing that (6) and (31) yield the Bertrand-Nash equilibrium prices when evaluated at θ^b.

B.1 Symmetric Retailers ($n \geq 2$)

Let us solve for the symmetric Bertrand-Nash equilibrium price when $n \geq 2$. When all retailers have positive demand, the direct demand function is

$$q_i\left(p_i, p_{-i}\right) = \frac{\beta}{n}\left[v - p_i + \frac{\sigma}{1-\sigma}\left(\widehat{p} - p_i\right)\right]$$

where $\widehat{p} = \frac{1}{n}\sum_j p_j$. Thus, the associated profits are $\pi_i\left(p_i, p_{-i}\right) = \left(p_i - c_i\right) q_i\left(p_i, p_{-i}\right)$. Differentiating with respect to p_i and setting equal to zero obtains the first-order condition (FOC):

$$\frac{\partial\pi_i}{\partial p_i} = \frac{\beta}{n}\left[v - p_i + \frac{\sigma}{1-\sigma}\left(\widehat{p} - p_i\right)\right] - \left(p_i - c_i\right)\frac{\beta}{n}\left[1 + \left(\frac{\sigma}{1-\sigma}\right)\left(\frac{n-1}{n}\right)\right] = 0$$

When $c_i = c\ \forall i$, it is possible to solve for a symmetric equilibrium by substituting $\widehat{p} = p_i = p^B$ into the FOC. This yields

$$p^B\left(c, n, \sigma\right) = v - \left[1 - \frac{1}{2 + \left(\frac{\sigma}{1-\sigma}\right)\left(\frac{n-1}{n}\right)}\right]\left(v - c\right)$$

To show that (6) yields $p^B\left(c, n, \sigma\right)$ when evaluated at θ^b we need to manipulate the term in square brackets to show that $1 - \frac{1}{2+\left(\frac{\sigma}{1-\sigma}\right)\left(\frac{n-1}{n}\right)} = \frac{1}{2-\sigma\left(1-\frac{\theta^b}{n}\right)}$. Thus,

$$1 - \frac{1}{2 + \frac{\sigma(n-1)}{(1-\sigma)n}} = \frac{1}{1 + \frac{1}{1+\frac{\sigma(n-1)}{(1-\sigma)n}}}$$

$$= \frac{1}{2 - \frac{\frac{\sigma(n-1)}{(1-\sigma)n}}{1+\frac{\sigma(n-1)}{(1-\sigma)n}}}$$

$$= \frac{1}{2 - \sigma\left(\frac{n-1}{n(1-\sigma)+\sigma(n-1)}\right)}$$

$$= \frac{1}{2 - \sigma\left(1 - \frac{1-\sigma}{n(1-\sigma)+\sigma(n-1)}\right)}$$

$$= \frac{1}{2 - \sigma\left(1 - \frac{\theta^b}{n}\right)}$$

Therefore, it follows that (6) yields the Bertrand-Nash equilibrium prices when evaluated at θ^b.

B.2 Asymmetric Retailers ($n = 2$)

For an an asymmetric duopoly, where $c_i \neq c_j$, we can use the FOC for $i = \{1, 2\}$ to obtain the equilibrium price of retailer i for any given c_i and c_j

$$p_i^B(c_i, c_j, \sigma) = v - \left[1 - \frac{1}{2 + \frac{\sigma}{2(1-\sigma)}}\right]\left(v - c_i + (c_i - c_j)\left[\frac{\frac{\sigma}{2(1-\sigma)}}{2 + \frac{3\sigma}{2(1-\sigma)}}\right]\right)$$

We wish to show that (31) yields $p_i^B(c_i, c_j, \sigma)$ when evaluated at θ^b. Clearly, substituting $\theta^b = \frac{1}{1+\frac{\sigma}{2(1-\sigma)}}$ into the term in square brackets shows $\frac{1}{2-\sigma\left(1-\frac{\theta^b}{2}\right)} = 1 - \frac{1}{2+\frac{\sigma}{2(1-\sigma)}}$ from above. Thus, it remains to show that $\frac{\sigma}{2}\left(1 - \frac{1}{2+\frac{\sigma\theta^b}{2(1-\sigma)}}\right) = \frac{\frac{\sigma}{2(1-\sigma)}}{2+\frac{3\sigma}{2(1-\sigma)}}$. Substituting in $\theta^b = \frac{1}{1+\frac{\sigma}{2(1-\sigma)}}$ yields

$$\frac{\sigma}{2}\left(1 - \frac{1}{2 + \frac{\sigma\theta^b}{2(1-\sigma)}}\right) = \frac{\sigma}{2}\left(1 - \frac{1}{2 + \frac{\sigma}{2(1-\sigma)+\sigma}}\right)$$

$$= \frac{\sigma}{2}\left(\frac{1 + \frac{\sigma}{2(1-\sigma)+\sigma}}{2 + \frac{\sigma}{2(1-\sigma)+\sigma}}\right)$$

$$= \frac{\sigma}{2}\left(\frac{2}{4(1-\sigma) + 3\sigma}\right)$$

$$= \frac{\sigma}{2(1-\sigma)}\left(\frac{2}{4 + 3\frac{\sigma}{(1-\sigma)}}\right)$$

$$= \frac{\sigma}{2(1-\sigma)}\left(\frac{1}{2 + \frac{3\sigma}{2(1-\sigma)}}\right)$$

Thus, (31) yields the Bertrand-Nash equilibrium prices when evaluated at θ^b.