A Welfare and Pass-Through Effects of Regulations within Imperfect Competition

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Abstract: *This paper furnishes an inclusive framework to examine the welfare effects of the interventions of multiple policies, and other exterior alterations under imperfect competition and an assertion on particular and .the leading case of the ad valorem taxes. In particular, for the tax pass-through, we furnish ''sufficient statistics" equations for measures of the two welfare under the demand of the impartially general class, market competition and production cost. The measures are i) Public fund of the marginal value, ii) Incidence. We start with the status of symmetric firms' face up with both ad valorem taxes and unit tax to derive an empirically pertinent set of formulas and simple. Next, we make a substantial generalization of these results to include firm heterogeneity using the idea of tax revenue defined as a public function defined by a vector of policy instruments including governmental and non-governmental interventions and other non-tax costs.*

Keywords: *Imperfect Competition, Pass-through, Marginal Value of Public, Funds, Incidence, Sufficient Statistics*

1 Introduction

When considering market intervention such as taxes, it is essential to understand how such a policy change distorts economic welfare. In addition, policymakers may also be concerned with the distributional consequences, i.e., how the tax burden is borne by consumers and producers subject to such a change in tax policy. In general, this problem relates to passivity - a measure of how and how the infinitesimal change in the surrounding environment that firms experience affects final prices - a measure of how it affects economic factors differently. Pass-through is important as it is very closely related to (1) the amount of burdens or benefits accruing to society due to the environmental change surrounding firms, and (2) how these burdens or benefits are divided between the demand side (consumers) and the supply side (firms). Examples other than taxes include the change in the exchange rate and technological improvement that leads to lower production costs, to name a few.

The importance of transit has been recognized since, at least, Cournot (1838, ch. 6) in the context of monopoly. A suitable framework extending to imperfect competition, in general, has been introduced by Weyl and Fabinger (2013) who stress the tradition of price theory since Marshall (1890) in recognizing the importance of transit and accidents in a range of economic questions. This generalization is important because many industries are described as oligopolies where a small number of firms dominate. However, Weyl and Fabinger (2013) focus only on the case of one-dimensional transitions such as the continuous change in marginal cost resulting from the introduction of a unit/specific tax.

To extend their framework, this paper presents the generalization of the Weyl and Fabinger (2013) model to include the case of multidimensional interventions, including non-governmental external changes, under general forms of market demand, production cost, and imperfect competition. Since Weyl and Fabinger (2013) consider a particular tax as an example of a one-dimensional intervention, this paper examines a fundamental issue of specific and ad valorem taxation as a leading example of a multidimensional intervention.

Our major contributions to Weyl and Fabinger (2013) are threefold. At the start First, We study the consequences of welfare taxes more widely than others do: in particular, we also consider the "marginal value of public money" (MVPF; see below). Second, as mentioned above, we include ad valorem taxes, while they only take into account specific taxes. We also emphasize (in Appendix C) that our analysis of 2D taxes opens up a methodology to include more general cases of multiple interventions such as tax combinations and non-tax costs such as market regulations. Finally, we allow for any kind of pre-existing (ie, non-zero) taxes, while Weyl and Fabinger (2013) consider specific taxation in previously untaxed markets. Notably, our framework is easily expandable to the case of heterogeneous companies

As noted above, our arguments are best understood in the case of two-dimensional taxes in which identical firms face both unit taxes and value-based taxes, an argument that generalizes Anderson et al. (2001a) and Häckner and Herzing's (2016) taxation analysis beneath imperfect competition to derive 'adequate statistics' equations expressed in terms of perceivable and quantifiable variables such as elasticity (Chetty, 2009; Kleven, 2021).

From taxes to (i) marginal value of public money (MVPF) and (ii) incidence, i.e., the ratio of a marginal change in consumer surplus to marginal change in producer surplus. We also generalize Weyl and Fabinger's (2013) analysis in this dimension because they do not focus on the MVPF aspects of taxation. Here, MVPF is a simple interest/cost ratio which measures the marginal variation in aggregate exceed in the private sector—consumer surplus plus firm surplus which is positive under imperfect competition—relative to the marginal change in a net cost to the government (Hendren, 2016; Hendren and Sprung-Keyser, 2020): In the analysis below, a higher MVPF corresponds to a greater welfare cost imposed on each revenue collected in dollars because we focus only on the revenue side of the government (i.e. taxes) without considering any beneficial effects of government spending on consumers and businesses by The route of public policy or the provision of public goods (e.g. Lockwood, 2003).[1] Additionally, we complement Weyl and Fabinger's (2013) analysis by providing illustrations to facilitate an intuitive understanding of the welfare effects of specific and ad valorem taxes under constant consistency (see subsection 2.2). The taxation welfare aspects have been studied extensively since, at least, Pego (1928). The majority of current studies assume perfect competition (with no pre-existing taxes). As it is widely known, unit tax and ad valorem tax equal the same level of revenue under this condition, and whether consumers or producers bear more is determined by the relative elasticities of demand and supply (Weyl and Fabinger, 2013, p. 534). The assumption of perfect competition was attempted at first by studies of homogeneous oligopoly of products under quantitative competition, i.e., Cournot oligopoly. It should be noted that Delipalla and Keen (1992), Skeath and Trandel (1994), Hamilton (1999); Anderson et al. (2001b) Comparison of unit tax and ad valorem tax in such a situation. [2] Anderson al. (2001a), then, extend these results to the case of differentiated oligopoly under price competition.

They find whether the firms' price after-tax rises including the profit by changing the value tax depending importantly on the ratio of the firm's demand curvature to the elasticity of market demand. Miravete et al (2018) also emphasize the importance of imperfect competition in considering policy recommendations: they find the empirical importance of a firm's strategic pricing responses when assessing the impact of taxes, implying that imperfect competition must be considered for policy evaluation. In contrast to these previous studies, one attractive feature of our framework is - as in Weyl and Fabinger (2013) and Kroft et al. (2020), among others - we introduce the behaviour index, by which we mean the coefficient of behaviour that is not necessarily constant across the production level. The behaviour index measures the degree of market monopoly and therefore includes a variety of market structures.

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[1] Therefore, the net cost to the government in this study is negative for increasing government revenue. On the contrary, for spending or subsidy policies, higher medium-term market value means higher welfare gains for every dollar spent. Note also that, in contrast to the traditional definition of the marginal cost of public funds or MCPF (Stiglitz and Dasgupta, 1971; Atkinson and Stern, 1974; Dahlby, 2008), MVPF focuses squarely on causal effects, not compensatory effects. Therefore, public policy is broadly applicable in guiding cost-benefit analysis in a more systematic manner (Finkelstein and Hendren, 2020; Paradisi 2021). This Hendrenian MVPF is identical to marginal overburden or MEB by Mayshar (1990) and MCPF by Slemrod and Yitzhaki (1996), Slemrod and Yitzhaki (2001) and Kleven and Kreiner (2006). We thank Nathan Hendern for making us realize this point.

[2] More specifically, Delipalla and Keen (1992) have shown for the first time that taxes by value are higher than unit taxes with like-for-quantitative firms. Skeath and Trandel (1994) reinforce the results of Delipalla and Keen (1992) by showing Pareto dominance: for a given level of unit tax under monopoly, there is always an ad valorem tax that produces higher levels of all consumer surplus, firm profits and tax revenue. Under Cournot oligopoly, Skeath and Trandel (1994) show the same result if the required amount of tax revenue is large enough, and this requirement depends on the demand curve and the number of firms in the market.

It allows us to work with a fairly general situation of competition in the market and get acquainted with its complex nature in reality: from a theoretical and empirical point of view, it is desirable to understand the welfare properties of oligopolistic markets of a fairly general category of the competition. 8 In real-world situations, corporate behaviour may not simply be classified into the ideal price or quantity competition, and may also include the possibility of collusive behaviour.

Furthermore, we can provide sufficient statistical equations for measures of well-being useful for the experimental study because we also fit consistent heterogeneity, which cannot be neglected in almost any data. When considered in Section 4, we introduce a firm's pricing strength index and measure the degree of market power of a firm: This concept is related to a behaviour indicator but it is best to work with when firms are no match. Our characterization of the two criteria of well-being can easily be extended to the well-established case of heterogeneity. In this sense, this paper is intended to be a response to a popular view, particularly in the field of public finance, of the following quotes from three representative textbooks (in chronological order; emphasis added): Q1) Unfortunately, there is no well-developed theory of tax occurrence in oligopoly. [...] As economic behaviour under oligopoly becomes better understood, improved models of incidence will be developed" (Rosen and Geyer, 2014, pp. 310-311). Q 2) there is no widely accepted theory of firm behaviour in oligopoly, so it is impossible to make any definitive predictions about the occurrence of taxes in this case" (Stiglitz and Rosengard, 2015, p. 556). Q3) although there are widely accepted models of how competitive and monopolistic markets work, there is much less consensus on models of oligopolistic markets. As a result, economists tend to assume that the same tax limit rules apply in these markets as well, but there is more work to be done to understand the tax burden in oligopolistic markets" (Gruber, 2019, p. 601).

In a similar vein, Kroft et al. (2020) also considered the comparison of value with unit taxes and elicited an adequate statistical equation for the welfare charges of commodity taxes as well as their occurrence under imperfect competition, particularly given the potential for misperceptions by 'behavioural' consumers about whether the price is tax-inclusive. . Specifically, they set standards for the degree to which consumers can accurately attribute a change in a consumer's price to a change in the tax beyond and calibrate the marginal overburden of commodity taxes by maintaining a constant consistency. In turn, we aim to provide general formulas for welfare measures that allow for complete heterogeneity. In this sense, their study and ours are complementary in providing useful structural frameworks for assessing welfare considering a variety of important policy issues under imperfect competition. The remainder of this paper is organized as follows.

In the next section, we build our model of particular and ad valorem taxation under symmetrical imperfect competition and provide general formulas for the public funds' marginal value in addition to the incidence of tax pass-through, the market conduct, and the elasticity of market demand, among others. In Section 3, we conduct a numerical analysis of these formulas by employing three representative classes of market demand. Section 4, our formulas further generalise to contain heterogeneous firms. Finally, Section 5 is the conclusion. Note that some detailed arguments are delegated to the appendices. In particular, Appendix C provides a more general framework, which Section 4 is based on, than simply (two-dimensional) specific and ad valorem taxes to accommodate multi-dimensional interventions including additional changes (such as a change in the exchange rate) that accrue outside of government activities. Lastly, Online Appendix B explains other applications other than taxation such as a sales restriction due to, for instance, tax evasion, and the outbreak of a pandemic.

2 Specific Taxation and Ad valorem under Symmetrical Imperfect Competition

We study in this section the symmetrical oligopoly. Before we start, let us point out that the formulas we derive are not much longer than the corresponding formulas for the special case of monopoly. Keeping our derivations explicit to confirm the logical flow, that generalises beyond sad Valorem and specific taxes and beyond symmetrical firm oligopoly (Appendix C). We use figures as visual anchors to help the reader clearly understand the many welfare component changes and many forces that play a role in the discussion (see Subsection 2.2 below). This section generalizes the results of Anderson et al. 2001a) (ADK) in many important directions. i) We consider a general class of the competition market, the conduct index captures it (see below), including both quantity and price competition. ii) We give a full description of welfare measures which makes one quantitatively compare the burden of consumers and that of producers, while ADKa focuses on prices effective for the consumers' profits and producers. iii) Assumes ADKa fixed marginal cost, and we allow non-constant marginal cost and show how this generalisation burden a difference in our generalized formulas. d) We also generalise the tax level initial. When they analyse the effects of a unit tax, ADKa assumes that ad valorem tax is zero, and vice versa. In contrast, we allow non-zero initial taxes in both dimensions. In general, it appears a generalisation of results of the ADKa of a two-dimensional taxes problem suggests that a much broader range of interventions and taxes describe welfare measures in terms of adequate statistics. The assumption that represents a consumer has a quasi-linear utility

$$U(q,y) = u(q) + y \qquad where\ \ q = q_1, \dots \dots ., q_n$$

is their consumption bundle of (n) single-product firms in the industry, and (y > 0) is an external digital good without taxes. In fact, we assume that all markets outside this industry are perfectly competitive to isolate that particular market from such feedback effects as income effects which may arise under general equilibrium. A full analysis of 'imperfect competition in general equilibrium' awaits further research in this direction (see, for example, d'Aspremont and Dos Santos Ferreira, 2021). Below we use (t) for specific taxes (unit taxes) and (v) for ad valorem taxes. These probably non-negative, in most applications. [2] Then, following Kroft et al. (2020), Häckner and Herzing (2016), and many others, define welfare (W) as $W = CS + PS + R$ where CS, PS and R denote consumer surplus, producer surplus (company profits) and tax revenue respectively. Our task mainly is to describe two substantial measures of welfare that affects taxes on goods:

i) the marginal value of public money $MVPF_T = (-\partial R/\partial T)^{-1}\{(\partial CS/\partial T) + (\partial PS/\partial T)$
ii) the incidence $I_T = (-\partial PS/\partial T)^{-1}(\partial CS/\partial T) \;\; for \;\; T \in (t, v)$

2.1 Setup

Here we study an oligopolistic market with (n) symmetric firms and a general mode of competition and consider the resulting symmetric equilibrium. Formally, the demand for a firm i's product $q = q_i(p_1, \dots \dots, p_n) \equiv q_j(\boldsymbol{p})$ depends on the vector of prices; $\boldsymbol{p} = (p_1, \dots \dots, p_n)$, charged by the individual firms. The demand system is symmetric and the cost function $c(q_i)$ is the same for all firms. We assume that $q_i(.)$ and $c(.)$ are twice differentiable and the conditions for the uniqueness of equilibrium as well as the associated second-order conditions are satisfied. The marginal cost of production is defined by $mc(q_i) \equiv c'(q)$. We denote by $q(p)$ per-firm industry demand under symmetric prices: $q(p) \equiv q_i(p, \dots, p)$. The elasticity of this function, defined as $\epsilon(p) = -pq'(p)/q(p) > 0$ and referred to as the price elasticity of industry demand, should not be confused with the elasticity of the residual demand that any of these firms faces.[3]

We also define by $\eta(q) \equiv 1/\in(p) \mid_{q(p)=q}$ the reciprocal of this elasticity as a function of q. When we do not need to specify explicitly their dependence on either q or p in the following

[2] One may wonder if the welfare distortion in this market can be eliminated if the unit tax is not constrained to be non-negative. This is because, starting from any combination of taxes t and v, it is possible to keep the same level of government revenue but unambiguously lower the deadweight loss by raising v just enough to generate a marginal unit of revenue, and simultaneously lowering t just enough give back that marginal unit of revenue. Extending this reasoning, Myles (1999) finds that the optimal combination entails a positive ad valorem tax and a negative unit tax, although, in reality, the feasibility of this method would be very limited.

[3] Note that Häckner and Herzing (2016) call $(-\partial R/\partial T)^{-1}\{(\partial CS + \partial PS + \partial R)/\partial T\}$ the Marginal Cost of Public Funds. Similarly, Kroft, Laliberté, Leal-Vizcaíno, and Notowidigdo (2020) focus on $\partial W/\partial T$ as the marginal excess burden as a welfare measure. This is equal to $(1 - MVPF_T)(\partial R/\partial T)$. The elasticity ϵ here corresponds to ϵ_D in Weyl and Fabinger (2013, p. 542). Note that $q'(p) = \left\{\frac{\partial q_i(p)}{\partial p_i}\right\} + (n-1)(\frac{\partial q_i(p)}{\partial p_j}) \parallel_{\mathbf{p}=(p,..,p)}$for any two distinct indices i and j. We define the firm's elasticity and other related concepts in Appendix B.

analysis, we use η interchangeably with $1/\in$. Also, the function of industry inverse demand defined $p(q)$ as such the inverse of $q(p)$, which satisfies $\eta(q) = -qp'(q)/p(p)$.[4]

As mentioned above, we introduce two types of taxation: a specific tax (unit tax) t and an ad valorem tax v, with firm i's profit being $\pi_i = (1-v)p_i(\mathbf{q})q_i - tq_i - c(q_i)$. At symmetric output q, the government tax revenue per firm is $R(q) = tq + vp(q)q$, which we can separate into the specific tax part and the ad valorem part: $R(q) = R_t(q) + R_v,\ R_t(q) = tq, R_v(q) = vp(q)q$. We denote by $\tau(q)$ The portion revenue of the firm's pre-tax that the government collects in the form of taxes: τ(q) ≡R(q) / pq = v + t / p (q), because this notation makes many expressions simpler. In the special case of monopoly, the first-order condition for the equilibrium would be $(1-v)mr(q) - t = mc(q)\ with\ mr(q) = p(q) + qp' = p(q) - \eta(q)p(q)\ and\ mc(q) = c'(q)$. This condition can be rearranged as $\{1/\eta(q)p(q)\}\{p(q) - (t + mc(q))/(1-v)\} = 1$. Intuitively, the left-hand side measures a degree of departure from competitive pricing, which would have $p(q) - [\{t + mc(q)\}/(1-v)] = 0$. We use this intuition to write a more general form of the first-order condition that applies to oligopoly. Regarding oligopoly, we introduce θ(q) as the conduct index measures the market degree monopolisation and is determined independently of the cost side. The θ (q) (conduct index) is defined by the condition that the symmetric equilibrium state takes the form

$$\left\{\frac{1}{\eta(q)p(q)}\right\}\left\{p(q) - \frac{t+mc(q)}{1-v}\right\} = \theta(q) \tag{1}$$

Where $mc(q) \equiv c'(q)$ is the marginal cost of production.[5] Perfect competition corresponds to $\theta(q) = 0$ and monopoly to $\theta(q) = 1$.[6] With a little abuse of notation, we denote the equilibrium price by p, and assume that any equilibrium is symmetric. We further impose a condition on the functions in Eq. (1) to ensure that any equilibrium is necessarily unique. We denote by θ the functional value of $\theta(q)$ at the equilibrium quantity. We can think of it as an elasticity-adjusted Lerner index. The Lerner index $\{p - (t + mc)/(1-v)\}/p$ multiplied by the industry demand elasticity $\epsilon = 1/\eta$ equals θ. Here the Learner index is based on an (effective) marginal cost $(t + mc)/(1-v)$. We emphasize that once the conduct index is introduced, it becomes possible to describe oligopoly in a unified manner, without specifying

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[4] In the case of a monopoly, there is no distinction between the industry demand and the demand for the monopolist's good. Then $q(p)$ is the monopolist's demand curve ∈, is its elasticity, and η is the reciprocal of the elasticity.

[5] $\theta(q)$ is a generalization of the conduct parameter in the sense that it is a function of q rather than a constant for any q. Hence, Eq. (1) should not be interpreted as an equation that defines $\theta(q)$. For our analysis, we can just introduce $\theta(q)$ in an implicit manner: $\theta(q)$ is a function independent of the cost side of the problem, in which Eq. (1) is the symmetric first-order condition of the equilibrium. Note that $\theta(q) > 1$ is not necessarily excluded, although in most interesting cases, it lies in (0, 1).

[6] Symmetric Cournot oligopoly also corresponds to a constant conduct index, which in this case takes the value of 1/n, where n is the number of firms. But more generally, $\theta(q)$ depends on q.

whether it is price or quantity setting, or whether it exhibits strategic substitutability or complementarity.[7]

Finally, we define the specific tax pass-through rate p_t and the ad valorem pass-through semi-elasticity p_v as $p_t = (\partial p/\partial t)\ and\ p_v = (1/p)(\partial p/\partial v)$ respectively, where the equilibrium price p is considered as a function of the tax levels. Both p_t and p_v are dimensionless. The reason for considering semi-elasticity for the ad valorem tax becomes clear in the next subsection, where several results take the same form for both taxes and differ just by the presence of p_t or p_v . They are also non-negative because otherwise second-order conditions for the equilibrium would be violated.

2.2. The components of Welfare

We develop how government revenue affected by tax reform , consumer surplus, social welfare, and producer surplus, as the total sum of these. The current derivations with next subsequent supply the building subsections propositions. More accurate, Characteristics of well-being are related to the following four components of well-fare that we study:: (i) consumer surplus per firm $CS = \int_0^q p\,(\tilde{q})d(\tilde{q}) - pq$, (ii) ad valorem tax revenue per firm $R_v = vpq$, (iii) specific tax revenue per firm $R_t = tq$, and (iv) producer surplus per firm $PS = (1-v)pq - tq$. These are depicted in Fig. 1.22 The points $A_0, B_0, C_0, D_0, E_0, F_0$ are at $q = 0$ and the points A, B, C, D, E are at the equilibrium quantity for a given value of the taxes *t* and *v*. Total cost (per firm) $c(q) = \int_0^q mc(\tilde{q})\,d\tilde{q}$ corresponds to B_0BAA_0, producer surplus to C_0CBB_0, specific tax revenue to D_0DCC_0, ad valorem tax revenue to E_0EDD_0, and consumer surplus to the area F_0EE_0. The total welfare (per firm) $W = PS + R_t + R_v + CSC_0$ which represented by F_0FBB_0. While socially optimal quantity is Point O, and area EOB represents deadweight loss.

This figure shows five generally non-linear functions: $mc(q)$, $(1-v)[1-\theta(q)\eta(q)]p(q) - t, and\ p(q)$ that determine the boundaries of the regions. In the special case of monopoly, the figure would look almost the same, except that $(1-v)[1-\theta(q)\eta(q)p(q)] - t$ would be change if we increase the specific tax and the ad valorem tax, respectively, replaced by $(1-v)[1-\eta(q)]\,p(q) - t$. Then, Figs. 2 and 3 indicate how the diagram would

[7] The condition is as follows. Eq. (1) may be rearranged as $\left[1 - \{\eta(q)\theta(q)p(q)\} - \left\{\frac{1}{1-v}\right\}\{t + mc(q)\}\right] = 0$
We require that the left-hand size be a decreasing function q. For a constant marginal cost, this translates to the requirement that $1 - \{\eta(q)\theta(q)p(q)\}$ be a decreasing function of q. In the special case of monopoly, $\theta(q) = 1$, this reduces to the requirement of decreasing marginal revenue.
The tax-adjusted Lerner rule $\{p - (t + mc)/(1-v)\}/p = \eta\theta$ implies the restriction on θ, namely $\theta \leq \in$.

This graphical illustration is helpful for considering the changes in the welfare components if we infinitesimally change the taxes (note that the changes shown in the figures are non-infinitesimal, though). Since taxes change infinitely, t → t + dt, v → v + dv. , the regions corresponding to the wellfare component change due to the horizontal movement of the right boundaries of the regions (points A; B; C; D; E) and due to the vertical movement of the upper and lower boundaries of the regions. We call these ‘‘quantity effects” (↔) and ‘‘value effects” (↕), respectively. For example, tq is the specific tax revenue also corresponding infinitesimal change $d(tq) = tdq + qdt$, consists of a quantity effect tdq and a value effect qdtbecause the right border of the region shifts by dq and the vertical height of the region changes by *dt*. We introduce the following expressions for infinitesimal changes in the welfare components:

$= dPS_{\leftrightarrow} + dPS_{\updownarrow}$; $dR_t = dR_{t\leftrightarrow} + dR_{t\updownarrow}$; $dR_v = dR_{v\leftrightarrow} + dR_{v\updownarrow}$; $dCS = dCS_{\leftrightarrow} +$ dW ; $dW = dW_{\leftrightarrow} + dW_{\updownarrow}$. First, the change in tax revenue $R = tq + vpq$ is given by

$$dR = dR_t + dR_v = dR_{t\leftrightarrow} + dR_{v\leftrightarrow} + dR_{t\updownarrow} + dR_{v\updownarrow} \tag{2}$$

$$dR_{t\leftrightarrow} = tdq\ ,\ dR_{v\leftrightarrow} = vpdq\ \ ,\ dR_{t\updownarrow} = qdt\ ,\ dR_{v\updownarrow} = qvdq + qpdv \tag{3}$$

Note here that the two quantity effects, $dR_{t\leftrightarrow}$ and $dR_{v\leftrightarrow}$, result from the ‘‘behavioural" change in the output: they are presumably negative for a positive change in *t* and *v* because dq < 0 (‘‘fiscal externality”). The value effect for the specific tax change, $dR_{t\updownarrow}$ purely reflects the ‘‘mechanical” change in government revenue with no behavioural response included. In this way, the quantity and value effects for a change in specific tax separately correspond to the behavioural and mechanical changes, respectively. The value effect for the ad valorem tax change, $dR_{v\updownarrow}$, however, includes another behavioural change through the firms’ pricing p that affects the infra-marginal consumers as well.

Next, the change in producer surplus, $PS = (1 - v)qp(q) - tq - c(q), is\ written\ as$

$$dPS = dPS_{\leftrightarrow} + dPS_{\updownarrow} \tag{4}$$

The quality and the value effects are $dPS_{\leftrightarrow} = \{(1 - v)p - (t + mc)dq$, $dPS_{\updownarrow} = (1 - v)qdp - qdt - pqdv$ respectively. Given dq < 0, the first term in the bracket of the quantity effect captures loss from a reduction in production, multiplied by the adjusted price unit$(1 - v)p$, whereas t and mc express the gains from unit tax saving and from cost savings by the output reduction, respectively. In the *dPS (*value effect), the first corresponds term to the (direct) gain from the price increase associated, mitigated by $(1 - v)$, because of the ad valorem tax, multiplied by the output q: the firms’ behavioural response to dt > 0 and dv > 0 contributes positively to their profits. However, the mechanical change, $qdt + pqdv$, has a negative effect: the firms incur the (direct) loss from an increase in unit or ad valorem tax: this is captured by the second and the third terms, respectively. After substituting for

$t + mc = (1 - v)p(1 - \eta\theta)$ from Eq. (1), then alternatively expressed as $dPS_{\leftrightarrow} = \{(1 - v)p\eta\theta\, dq$,

$$dPS_{\leftrightarrow} = \{(1 - v)p\eta\theta\, dq \;\; ; \;\; dPS_{\updownarrow} = (1 - v)q\, dp - q\, dt - pq\, dv \quad (5)$$

Respectively. The quantity effect is equal (0) according to the perfect competition as first expression implies (i.e., $\theta = 0$). For consumer surplus CS, the quantity effect is zero $dCS_{\leftrightarrow} = 0$, and the value effect is $dCS_{\updownarrow} = -q\, dp$, so that

$$dCS = -q\, dp \quad (6)$$

Finally, for W (social welfare), the value effect is zero, $dW_{\updownarrow} = 0$, because the curves mcðqÞ and pðqÞ do not move in response to a tax change, whereas the quantity effect is $dw_{\leftrightarrow} = (p - mc)dq$, implying that $dw = (p - mc)dq$. Substituting for mc using Eq. (1) gives $dW = \{t + vp + (1 - v)p\eta\theta\,\}dq$, or using our definition $\tau = v + t/p$,

$$dW = \{(1 - v)\eta\theta + \tau\}pdg \quad (7)$$

2.3. Changes in equilibrium prices and quantities

It is useful to express infinitesimal price changes and tax changes in terms of infinitesimal quantity changes. In case of a change in (dt) specific tax, the price changes by $dp = \rho_t dt$, and the quantity changes by $dp = -q \in dp/p$. These relationships imply

Fig. 1. Welfare components at tax levels t =0.1 and v =0.1 for a chosen case of oligopoly

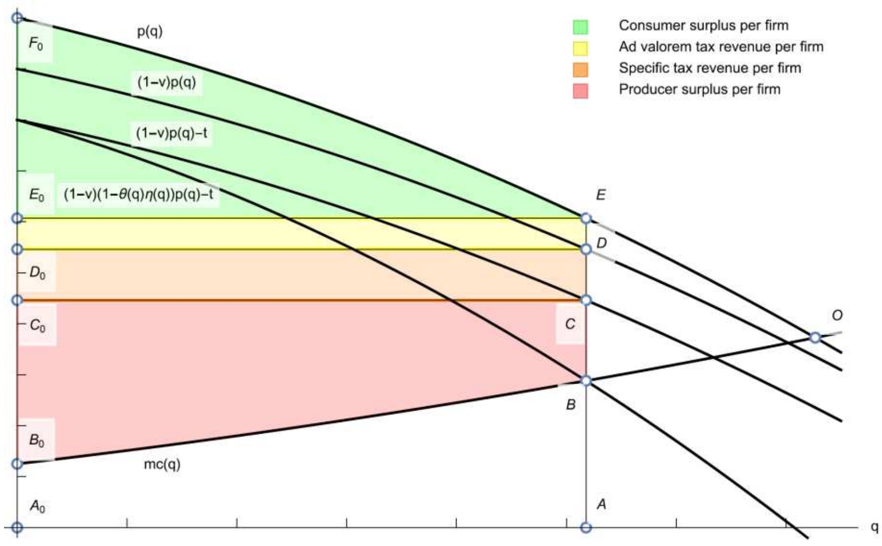


Fig. 2. Oligopoly welfare components Visualisation after an increase of the specific tax from $t = 0.1$ to $\tilde{t} = 0.2$, with v =*0.1* and $p(0) = 1$,

Starting from the situation in Fig. 1. In this figure, PS, R_t, and CS decrease, Whereas R_v increases. See Appendix A.1 for details.

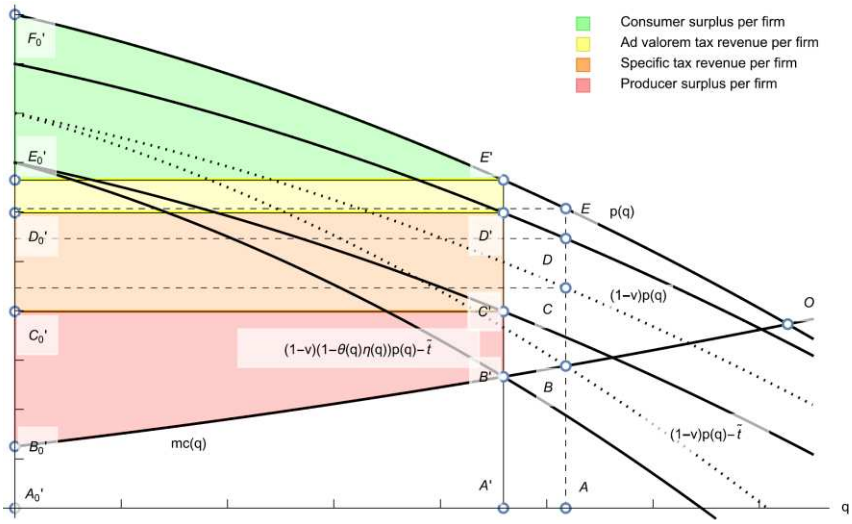
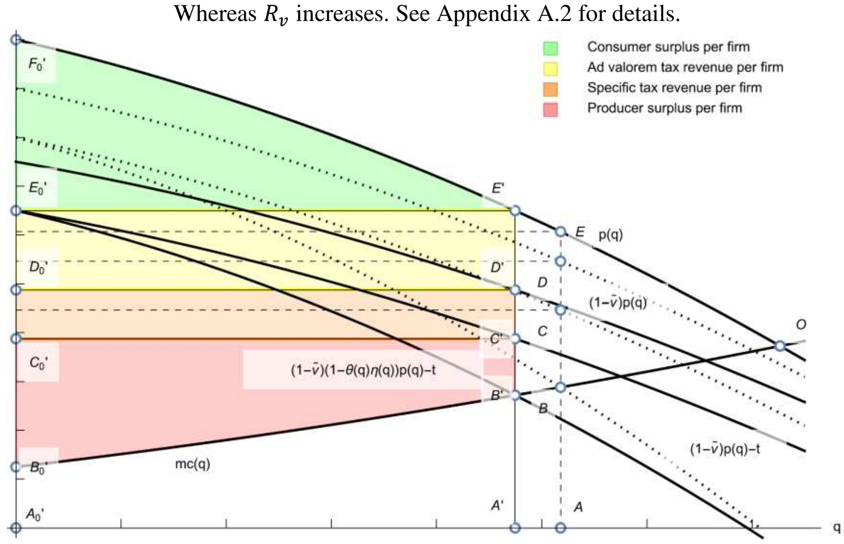


Fig. 3. Oligopoly welfare components Visualization after an increase of the ad-valorem tax from $v = 0.1$ to $\tilde{v} = 0.2$, with t =*0.1* and $p(0) = 1$,

Starting from the situation in Fig. 1. In this figure, PS, R_t, and CS decrease, Whereas R_v increases. See Appendix A.2 for details.



$$dt = (-\eta p/q\rho_t)dg \quad , \quad dp = (-\eta p/q)dg \tag{8}$$

Here, the first relationship states how the mechanical effect, dt, affects the behavioural effect, dp, whereas how the latter is related to the firms' pricing response, dp, is described in the second relationship. In the case of a change in ad valorem tax dv, the price changes by $dp = \rho_v p\ dv$, while the quantity changes by $dq = -q \in dp/p$. Therefore

$$dv = (-\eta/q\rho_v)dg \quad , \quad dp = (-\eta p/q)dg \tag{9}$$

Note the difference between Eqs. (8) and (9): the mechanical changes, *dt* and *dv*, affect the behavioural effect, dq, differently. However, given this behavioural effect, how adjust the firms in pricing is identical.

2.4. Pass-through Tax

We show how two pass-through between the per-unit and ad-valorem tax, ρ_t and ρ_v , are related. This result is interesting for its own sake because it shows that ρ_v is no greater than ρ_t in a general manner.

Proposition 1. Under a symmetric oligopoly and with a possibly of the non-constant marginal cost, the pass-through semi-elasticity qv of an ad valorem tax may be expressed in terms of the unit tax pass-through rate ρ_t , the conduct index θ and the industry demand elasticity $\in$ as

$$\rho_v = \{1 - \theta/\in\}\, \rho_t \tag{10}$$

The proof of the above eq. is provided in Appendix A.3.24. To better understand of this proposition intuitively, to keep quantities and prices constant, $\Delta t \; and \; \Delta v$ must satisfy:

$$(t + \Delta t + mc)/(1 - (v + \Delta v) = (t + mc)/(1 - v)$$

Thus, the relative Δt that must be offset by a reduction $-\Delta v$ is equal to $(t + mc)/(1 - v)$: $\Delta t = -(t + mc)\Delta v/(1 - v)$, which, along with $\rho_t dt + \rho_v p dv = 0$, leads to $(t + mc)\rho_t/\{(1 - v)p\} = \rho_v$. Now, recall (Lerner rule):

$$[1 - \{(t + mc)/(1 - v)p\} = \eta\theta]$$

This indicates that $(1 - \eta\theta)\rho_t = \rho_v$, according to Proposition 1 claims. Here, $\theta/\epsilon = 1 - \rho_v/\rho_t$ implies that $\rho_v \le \rho_t \le (1 - 1/\in)\rho_v$. Next, proposition shows that two pass-through forms are characterized.

Proposition 2. Under the general mode with a symmetric oligopoly of competition and a possibly non-constant marginal cost, the unit tax pass-through is characterized by:

$$\rho_t = \frac{1}{1 - v} \cdot \frac{1}{\left\{1 + \left(\frac{1 - \tau}{1 - v}\right) \epsilon\chi - (\eta + \chi)\, \theta + \epsilon q\, (\theta\eta)'\right\}}$$

Where the derivative is taken concerning q and $\chi \equiv mc'q/mc$ is the elasticity of the marginal cost concerning quantity. Similarly, the ad valorem tax pass-through is characterized by:

$$\rho_t = \frac{\in - \theta}{(1 - v)\epsilon} \cdot \frac{1}{\left\{1 + \left(\frac{1 - \tau}{1 - v}\right) \epsilon\chi - (\eta + \chi)\, \theta + \epsilon q\, (\theta\eta)'\right\}}$$

Moreover, in Weyl and Fabinger's (2013) notation, they are expressed as:

$$\rho_t = \frac{1}{(1-v)} \cdot \frac{1}{\left\{1 + \left\{\left(\frac{1-\tau}{1-v}\right)\epsilon - \theta\right\}\chi + \frac{\theta}{\epsilon_\theta} + \frac{\theta}{\epsilon_{ms}}\right\}}$$

$$and \qquad \rho_t = \frac{\epsilon-\theta}{(1-v)\epsilon} \cdot \frac{1}{\left\{1 + \left\{\left(\frac{1-\tau}{1-v}\right)\epsilon - \theta\right\}\chi + \frac{\theta}{\epsilon_\theta} + \frac{\theta}{\epsilon_{ms}}\right\}}$$

where $\in_\theta \equiv \theta/(\theta')q$ and $\in_{ms} \equiv ms/(ms'q)$ are the inverses of the quantity elasticities of θ, and $ms \equiv -p'q$, which is the ‘‘negative of marginal consumer surplus” (Weyl and Fabinger 2013, p. 538), respectively. Proposition provided in Appendix A.4.25.

As a brief result discussion. In the case of perfect competition $\theta = 0$ with zero initial taxes (t = 0 and v = 0), the pass-through is given by $\rho_t = 1/(1+\epsilon\chi)$ (see Weyl and Fabinger 2013, p. 534) $\rho_v = 1/(1+\epsilon\chi)$. With non-zero initial taxes, $(t, v) \geq 0$), there are adjustment factors, but the nature of the formulas is similar. More specifically, as in Weyl and Fabinger’s (2013, p. 549) explanation, assume that ‘‘h is invariant to be changes in q,” i.e., $1/\epsilon_\theta = 0$, and ‘‘costs are linear,” i.e., χ = 0 (the case of constant marginal cost).

Then, the only difference $\rho_t = \frac{1}{(1-v)} \cdot \frac{1}{1+\frac{\theta}{\epsilon_{ms}}}$ $and\ weyl\ and\ Fabinger's(2013)$ $\rho_t = \frac{1}{1+\frac{\theta}{\epsilon_{ms}}}$. is pass-through should be larger to adjusted to the deducted price $(1-v)p$ due to a positive ad valorem tax v > 0. Positive unit tax $t > 0$ works separately, addition to the function cost: once the non-constant marginal cost is allowed $(\chi \neq 0)$, the adjustment term $(1-\tau)/(1-v)$ in our formula.

More interesting interpretations can be provided by considering our expressions of (the first equation in Proposition 2). With imperfect competition, the term in the denominator $-\eta\theta$ is negative and leads to higher pass-through. This is intuitive because, in less competitive markets, firms can reflect higher costs in their prices to a larger extent. The term in the denominator $-\chi\theta$ has a sign opposite to that of $\chi = mc'q/mc$. For marginal costs increasing, χ is positive and $-\chi\theta$ negative, which leads the pass-through to be higher, especially if θ is large.

Further, with the imperfect competition, term in the denominator $\epsilon q(\eta\theta)'$ may be split into two parts: $\epsilon q(\eta\theta)' = q\theta' + q\epsilon\theta\eta'$. If at lower quantities the market is less competitive, then $\theta' < 0$ and $q\theta' < 0$, which leads to higher pass-through. Intuitively, in such situations, increasing taxes decreases the quantity provided, which in turn makes the market less competitive, leading to an even larger increase in prices than in the case of $\theta' = 0$. Similarly, if at lower quantities the industry demand elasticity, ∈, is lower, then $\eta' < 0\ and\ q\epsilon\theta\eta' < 0$, which leads to higher pass-through. Intuitively, in such situations, increasing taxes decreases the quantity provided, which in turn makes the industry demand more inelastic, leading to an even larger increase in prices than in the case of $\eta' = 0$. This effect is larger for a larger, θ, which is consistent with the fact that in these situations the firms are more sensitive to the properties of the overall industry demand.

We extended these results on pass-through in several directions.

In Online Appendix A, we show how our framework applies to the case of multi-product firms if intra-firm symmetry is guaranteed. In Online Appendix B, we present generalizations that go beyond the case taxation and include other market changes.

2.5. Marginal value of public funds

We now define the marginal value of public funds $MVPF_t$ of the specific tax t and the marginal value of public funds $MVPF_v$ of the ad valorem tax (*v)* as the ratio of the change in consumer and producer surplus to a marginal change in the net cost to the government (which is, in our focus of taxation, the associated change tax revenue induced by an infinitesimal increase the corresponding tax):

$$MVPF_t = (-\partial R/\partial t)^{-1}(\partial CS/\partial t + \partial PS/\partial T)$$

$$MVPF_v = (-\partial R/\partial v)^{-1}(\partial CS/\partial v + \partial PS/\partial v)$$

Note that F_T ; $T \in (t, v)$, in this study measures welfare loss because no beneficial effects of government spending are explicitly modelled: incorporating such effects into our framework is left for future research. We are able now to produce F_T (sufficient statistics formula), in terms from the pass-thorough that we described above, the reciprocity of price elasticity, the behaviour index as well as other observable variables such as *v* and τ.

Proposition 3. Under a symmetric oligopoly with a possibly non-constant marginal cost, the marginal value of public funds (MVPF) associated with a change in the specific tax *t* and the ad valorem tax *v* is characterized by:

$$MVPF_t = \left\{\frac{1}{\rho_t} + v + (1-v)\theta\right\} / \{\frac{1}{\rho_t} + v - \tau\epsilon\}$$

$$MVPF_v = \left\{\frac{1}{\rho_v} + v + (1-v)\theta\right\} / \{\frac{1}{\rho_v} + v - \tau\epsilon\}$$

Proof. Let us consider first the $MVPF_t$ (public funds marginal value) for the specific tax changes $dt \neq 0, dv = 0$. Using Eqs. (2), (3), and (7), we have

$$MVPF_t = \frac{dCS + dPS}{-dR}$$

$$MVPF_t = \frac{[\{(1-v)\eta\theta + \tau\}pdq - (tdq + vpdq + qdt + qvdp)]}{-(tdq + vpdq + qdt + qvdp)}$$

In order to cancel the infinitesimal changes on the right-hand side, we substitute for dp and dt in terms of dq using Eq. (8),

$$MVPF_t = \frac{\big((1-v)\eta\theta + \tau\big)\, pdq\ - (t+vp)\, dq + q\left(\frac{\eta p}{q\rho_t}\, dq\right) + qv\ \left(\frac{\eta p}{q}\ dp\right)}{-\left(tdq + vpdq + q\ \left(-\frac{\eta p}{q\rho_t}\ dq\right) + qv\ \left(-\frac{\eta p}{q} dp\right)\right)}$$

$$MVPF_t = \frac{\big((1-v)\eta\theta + \tau\big)\, p\ - (t+vp) + \left(\frac{\eta p}{\rho_t}\right) + v\eta p}{-\left(t + vp + \left(-\frac{\eta p}{\rho_t}\right) + v\eta p\right)}$$

Dividing the numerator and denominator by p and using $\eta = 1/\in$ yields:

$$MVPF_t = \frac{\frac{1}{\rho_t} + v + (1-v)\,\theta}{\frac{1}{\rho_t} + v + \tau\epsilon}$$

We proceed in a similar fashion for changes in the ad valorem tax, $dv \neq o$; $dt = 0$. The marginal value of public funds $MVPF_v$ is

$$MVPF_v = \frac{dCS + dPS}{-dR}$$

$$MVPF_v = \frac{\big((1-v)\eta\theta + \tau\big)\, pdq\ - (tdq + vpdq + qvdp + qpdv)}{-(tdq + vpdq + q\ v\ dp + qp\ dv)}$$

We substitute for dp and dv in terms of dq using Eq. (9),

$$MVPF_v = \frac{\big((1-v)\eta\theta + \tau\big)\, pdq\ - (t+vp)\, dq + qv\left(\frac{\eta p}{q}\ dq\right) + qp\ \left(\frac{\eta}{q\rho_v}\ dp\right)}{-\left(tdq + vpdq + qv\ \left(-\frac{\eta p}{q}\ dq\right) + qp\ \left(-\frac{\eta}{q\rho_v} dp\right)\right)}$$

$$MVPF_v = \frac{\big((1-v)\eta\theta + \tau\big)\, p\ - (t+vp) + \left(\frac{\eta p}{\rho_v}\right) + v\eta p}{-\left(t + vp + \left(-\frac{\eta p}{\rho_v}\right) + v\eta p\right)}$$

Dividing the numerator and denominator by p and using $\eta = 1/\in$ yield:

$$MVPF_v = \frac{\frac{1}{\rho_v} + v + (1-v)\,\theta}{\frac{1}{\rho_v} + v + \tau\epsilon}$$

Which completes the proof. The intuition that behind Proposition 3 to the case of unit taxation which be explained as: The argument to the ad valorem taxation is be analogous. First, the revenue tax increase by a tax reform dt > 0 has the mechanical change given by the current output q. However, it is also associated with behavioural change with respect to pricing (dp > 0) as well as production/consumption (dq < 0): from Eqs. (2) and (3), it is expressed as

$$dR = \underbrace{q\,dt}_{(+)} + \underbrace{vq\,dp}_{(+)} + \underbrace{(t+vp)\,dq}_{(-)},$$

Where the first term of the right-hand side expresses direct (mechanical) gains, multiplied by the output q, the second term shows indirect (behavioural) gains, due to the associated price is increase, multiplied by vq, and the third term is the part that exhibits another indirect (behavioural) effect that is a loss in government revenue due to the output reduction. Owing to Eq. (8), the government net cost is given by

$$-dR = -(-(p\eta/\rho_t)dq \ - (vpn)dq + (t+vp)dq)$$

$$= ((1/\rho\_t \ + v)\eta \ - \tau) \ \ p\,dq$$

Where the first term in the bracket exhibits gains in the government revenue, and the second term the loss. Now, for the denominator, dCS + dPS, we make use of the relationship, $dCS + dPS = dW - dR$, to treat $dCS + dPS$ as the whole private surplus: next subsection studies $dCS \text{ and } dPS$ are affected by the tax reform differently. As in last part of Subsection 2.2, the effects of an increase in unit tax, $dt > 0$, on the social welfare under imperfect competition can be written as $dW = -(p - mc)(-dq)$, which implies that the firm's per-output profit margin serves as a measure of welfare change.

Then, the firm's per-output profit margin is decomposed into two parts: (a) surplus from imperfect competition, $(1 - v)p\eta\theta$, and (b) tax payment, $t + vp = p\tau$, as Eq. (7) indicates. Therefore,

$$dCS + dPS = dW - dR$$

$$= \big((1-v)\eta\theta + \tau\big)(p\,dq) + \left\{\left(\frac{1}{\rho_t} + v\right)\eta - \tau\right\}\ (p\,dq)$$

$$= \left(\left(\frac{1}{\rho_t} + v\right)\eta + (1-v)\eta\theta\right)(p\,dq)$$

Which implies that the ratio of the loss incurred in the private sector to the total gain for the government, revenue is given by

$$MVPF_t = \frac{\underbrace{\left(\frac{1}{\rho_t} + v\right)\eta + (1-v)\eta\theta}_{\text{private welfare loss}}}{\underbrace{\left(\frac{1}{\rho_t} + v\right)\eta}_{\text{gov. revenue gain}} + \underbrace{(-\tau)}_{\text{gov. revenue loss}}}$$

$$MVPF_t = \frac{\left(\frac{1}{\rho_t} + v\right) + (1-v)\,\theta}{\left(\frac{1}{\rho_t} + v\right) - \left(v + \frac{t}{p}\right)}$$

This latter expression for $MVPF_t$ has some intuitive properties. If we think of $MVPF_t$ as a function of t, keeping all other variables in the expression fixed, we see that it is an increasing function of t. That is intuitive: The tax is more distortionary on the margin if the initial tax level is already high. Since t in the expression is multiplied by $\in/p$ the dependence of MVPFt on t will be stronger if $\in/p$ is large. This is also intuitive: (a) low price p; t is sizable price relative, and (b) $\in$ large elasticity for industry demand, any increase of t may have the larger effect on the supplied quantity. In both cases, we would expect the initial tax level t to strongly influence how distortionary the tax is on the margin. Similarly, if thinking of $MVPF_t$ a function of θ, which keeping all the other variables fixed in the expression, we see that it is an increasing function of θ, the conduct index. This is consistent with the intuition that when the market is very competitive, with a small θ, the tax should not be as distortionary on the margin as when the market is non-competitive. For $MVPF_v$, the expression is the same, except that $\rho_t$is replaced by ρ_t. The intuition regarding pass-through and market competitiveness applies to $MVPF_v$ as well. The dependence on v is more complicated than the dependence on t.

Next we combining both Propositions 1 and 3, result that $MVPF_t$ and $MVPF_v$ can be expressed in terms of estimable elasticities without the conduct index θ. The reasoning is simple: Proposition 1 allows us to express the conduct index θ as $\theta = (1 - \rho_v/\rho_t)\,\epsilon$. Substituting this into the relationships in Proposition 3 then gives the desired result.

Corollary 1. Under a symmetric oligopoly and with a possibly marginal cost non-constant, the unit pass-through rate ρ_t, the ad valorem pass-through semi-elasticity ρ_v, and the elasticity of industry demand ϵ (along with the tax rates and the fraction τ of the firm's pre-tax revenue collected by the government in the form of taxes) serve as sufficient statistics for the marginal value of public funds both with respect to unit taxes and ad valorem taxes. Specifically,

$$MVPF_t = \frac{1 + v\rho_t + (1 - v)(\rho_t - \rho_v)\,\epsilon}{1 + (v - \epsilon\tau)\,\rho_t}$$

$$and \qquad MVPF_v = \frac{1 + v\rho_v + (1 - v)(\rho_t - \rho_v)(\rho_t/\rho_v)\,\epsilon}{1 + (v - \epsilon\tau)\,\rho_v}$$

This corollary is consistent with the well-known result that unit tax and ad valorem tax are equivalent in the welfare effects under perfect competition: if $\theta = 0$, then $\rho_t = \rho_v$, and under imperfect competition, $\rho_t > \rho_v$, and $MVPF_t > MVPF_v$. This provides another look of the result of Anderson et al. (2001b) (ADKb) that specific taxes are welfare-inferior to ad valorem taxes.

2.6. Incidence

Having considered the welfare consequences of specific and ad-valorem taxation by comparing the economy's surplus, CS + PS, to the government revenue, R, we now study the distributional aspects: how the consumers and the firms are differently affected by tax reform. To be done, we provide our second measure of welfare, the I_t incidence of a specific tax t and the I_v incidence of the v ad valorem tax as the ratio of (a) the consumer surplus induced change by an infinitesimal corresponding tax increase, and (b) the associated change in producer surplus, i.e.

“”One could also define social incidence by $SI_T \equiv dW/dPS$ in association with a small change in $T \in (t, v)$ (see Weyl and Fabinger 2013, p. 538). In this paper, we focus on $MVPF_T$ as a measure of welfare burden in society, and I_T as a measure of loss in consumer welfare because once $MVPF_T \equiv (dW - dR)/dR$ and $I_T \equiv dCS/dPS$ are obtained, $SI_T = (dCS + dPS + dR)/dPS = -(1 + I_T)/(1 - 1/MVPF_T)$ can be readily calculated.””

$$I_t = (\partial PS/\partial t)^{-1}\ (\partial CS/\partial t) \quad and \qquad I_v = (\partial PS/\partial v)^{-1}\ (\partial CS/\partial v)$$

The following proposition shows how the incidence is characterized in terms the sufficient statistics such as market conduct and pass-through.

Proposition 4. With symmetric oligopoly and the general competition type and with a possibly cost of non-constant marginal, the specific tax t incidence and the v ad valorem tax is specified by:

$$\frac{1}{I_t} = \frac{1}{\rho_t} - (1 - v)(1 - \theta) \quad and \quad \frac{1}{I_v} = \frac{1}{\rho_v} - (1 - v)(1 - \theta) \quad \text{respectively.}$$

Proof. For a specific tax change dt = 0; dv = 0, we get, using Eqs. (6), (4) and (5),

$$I_t = \frac{dCS}{dPS} = \frac{-q\,dp}{\{(1 - v)p\eta\theta\,dq + (1 - v)q\,dp - q\,dt\}} - q(-\frac{\eta p}{q}\,dq)$$

$$I_t = \frac{dCS}{dPS} = \frac{-q(-\frac{\eta p}{q}\ dq)}{\left\{(1-v)p\eta\theta\ dq + (1-v)q\ \left(\frac{\eta p}{q}\ dq\right) - q(\frac{\eta p}{q\rho_t}\ dq)\right\}}$$

Where we eliminated dp and dt using Eq. (8). After a simplification,

$$I_t = \frac{1}{\frac{1}{\rho_t} - (1-v)(1-\theta)}$$

For an ad valorem tax change, $dv \neq 0,\ dt = 0$, we obtain, again using Eqs. (6), (4) and (5),

$$I_v = \frac{dCS}{dPS} = -q\frac{dp}{(1-v)p\eta\theta\ \ dq + (1-v)q\ dp - pq\ dv}$$

$$I_v = \frac{dCS}{dPS} = \frac{-q(-\frac{p\eta}{q}\ dq)}{(1-v)p\eta\theta\ \ dq + (1-v)q\ (-\frac{p\eta}{q}\ dq) - pq\ (-\frac{\eta}{q\rho_v}\ dq)}$$

Where we substituted for *dp* and *dv* from Eq. (9). This simplifies to

$$I_v = \frac{1}{\frac{1}{\rho_v} - (1-v)(1-\theta)} \qquad\qquad \boldsymbol{which\ completes\ the\ proof.}$$

Note that in the zero ad valorem tax case, the formula for I_t provide to Fabinger's and Weyl (2013, p. 548) Principle of Incidence 3, which states $1/I\_t\ = 1/\rho\_t\ - (1-\theta)$. In this way, we are able to generalize Weyl and Fabinger's (2013) formula for incidence, and respond to the statements by Rosen and Gayer (2014), Stiglitz and Rosengard (2015), and Gruber (2019) mentioned in the Introduction. To provide intuitive reasoning behind Proposition 4, recall from Eq. (5), that

$$dPS = \underbrace{-q\,dt + (1-v)q\,dp}_{\text{value effect}} + \underbrace{(1-v)p\eta\theta\,dq}_{\text{quantity effect}},$$

and from Eq. (9) that the behavioural responses (pricing and production) are expressed in terms of the mechanical change, *dt*, by using the tax pass-through, ρ_t:

$$dp = \rho_t dt \qquad , \qquad dq = -\frac{q\rho_t}{\eta p}\ dt$$

Respectively. Therefore, the above equation can also be interpreted as

$$dPS = \underbrace{-q\,dt+}_{\text{mechnical}} + \underbrace{(1-v)q\,dp + (1-v)p\eta\theta\,dq}_{\text{behavioral}}$$
$$= [\underbrace{-1}_{\text{mechnical}} + \underbrace{(1-v)\rho_t - (1-v)\theta\rho_t}_{\text{behavioral}}](q\,dt),$$

Which implies that the per-unit loss in producer surplus due to specific tax reform is $-1 + (1 - v)(1 - \theta)\rho_t$. Interestingly, provide be better off ($dPS > 0$) by the tax reform if the second term dominates; this would be more likely if v is small, the market is more competitive, and the specific tax pass-through is large. Similarly, the per-unit loss in consumer surplus is simply the tax pass-through itself because from Eq. (6),

$$dCS = -qdp = -\rho_t\ (q\ dt)$$

which implies that consumers can never be better off by the tax reform. Hence, the specific tax incidence is simply the ratio of ρ_t to $1 - (1 - v)(1 - \theta)\rho_t$. A similar argument can also be developed for ad valorem tax. To conclude this section, we briefly describe how one can go beyond the two-dimensional case of specific and ad valorem taxation, while still preserving the simplicity of general forms of multi-dimensional interventions. First, the specific and ad valorem tax payment of a (symmetric) firm is expressed as $\phi(p, q.T) = tq + vpq$, where T is a vector of (multi-dimensional) interventions, in this case, $T = (t, v)$. To generalize this, the key is to ask what the analogue of such a pair of t and v might be. It turns out that in general, we can write $\phi(p, q, T) = \bar{t}q + \bar{v}pq$, where $\bar{t}$ and $\bar{v}$ are the averages of appropriately defined functions t and m over the ranges $(0, q)\ and\ (0, pq)$. In the special case of specific and ad valorem taxes, these simply reduce to constants t and v. [8]

We can achieve this generalization by decomposing $\phi(p, q.T)$ into infinitesimal contributions, each of which resembles specific and ad valorem taxes. Using these functions t and m give rise to a simple way of analysing the welfare consequences of government interventions and non-governmental external changes. The resulting relationships are almost as simple as those in the two-dimensional case of specific and ad valorem taxes. Appendix C formalizes this idea and allows for firm heterogeneity.

3 Numerical Analysis of Parametric Examples

Although our formulas are presented in a general form, it would be illustrative to work through some parametric examples. Below we consider three demand specifications with n symmetric firms and constant marginal cost: $\chi = 0$. We define the own-price elasticity $\epsilon_{own}\ (p)$ of the firm's direct demand and the own quantity elasticity $\eta_{own}(q)$ of the firm's inverse demand by

$$\epsilon_{own}\ (p) \equiv -\frac{p}{q(p)} . \frac{\partial q_i(p)}{\partial p_i}\ |_{p=(p,..,p)} \quad and \quad \eta_{own}(q) \equiv -\frac{p}{q(q)} . \frac{\partial q_i(q)}{\partial q_i}\ |_{q=(q,..,q)}$$

8 Note here that economic agents in the private sector (i.e., consumers and producers) as a whole can never be better off becaused $CS + dPS = -\rho_t - 1 + (1 - v)(1 - \theta)\rho_t = -1 - \big(1 - (1 - v)(1 - \theta)\big)\,\rho_t < 0$; while $\big(1 - (1 - v)(1 - \theta)\big) > 0$; Similar to the Corollary above, the incidence of a unit tax is expressed as $\frac{1}{I_t} = \frac{1}{\rho_t} - (1 - v)(1 - \epsilon) + \frac{\rho_v}{\rho_t}\ \epsilon)\}$, and analogously for the case of an ad valorem tax.

Respectively. Similarly, the curvature of the industry's direct demand $\alpha(p)$ and the curvature of the industry's inverse demand $\sigma(q)$ are defined as follows:

$$\chi(p) = -\frac{pq''(p)}{q'(p)} \qquad and \qquad \alpha(q) = -\frac{qp''(q)}{p'(q)}$$

Then, the results derived in Appendix B indicate that in this case, the pass-through expressions become

$$\rho_t = \frac{1}{(1-v)\left\{1+\left(1-\frac{\alpha}{\epsilon_{own}}\right)\theta\right\}}$$

$$\rho_v = \frac{\epsilon_{own}-1}{\epsilon_{own}\left[(1-v)\left\{1+\left(1-\frac{\alpha}{\epsilon_{own}}\right)\theta\right\}\right]}$$

under price competition, where $\theta = \epsilon/\epsilon_{own}$ own, and

$$\rho_t = \frac{1}{(1-v)\left\{1+\left(1-\frac{\sigma}{\theta}\right)\theta\right\}} \quad and \quad \rho_v = \frac{1-\eta_{own}}{(1-v)\left\{1+\left(1-\frac{\sigma}{\theta}\right)\theta\right\}}$$

Under quantity competition, where $\theta = \eta_{own}/\eta$. Below, we consider three classes of demand specification: linear, constant elasticity of substitution (CES), and multinomial logit, and we assume that the marginal cost is constant.

3.1. Linear demand

The first one is the case wherein each firm faces the following linear demand,

$q_i(\mathbf{p}) = b - \boldsymbol{\lambda}\rho_i + \mu\sum_{i'\neq i} p_{i'}$, where $\lambda > (n-1)\mu$ and $0 \leq mc < b/[\lambda - (n-1)\mu]$,

implying that all firms produce substitutes and l measures the degree of substitutability (firms are effectively monopolists when $\mu = 0$. [9]

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[9] This linear demand is derived by maximizing the representative consumer's net utility, $(q_1, \dots, q_n - \sum_{i'\neq i} pq_1$, with respect to $q_1, \dots and\ q_n$. See Vives (2000, pp. 145–6) for details.

In our notation below, the demand in symmetric equilibrium is given by $q_i(p_i,\ p_{-1}) = b - \lambda p_i + \mu(n-1)p_{i-1}$, whereas it is written as

$$q_i(p_i\,, p_{-i}) = \frac{\alpha}{1+\gamma(n-1)} - \frac{1+\gamma(n-2)}{\left(1-\gamma\left(1+\gamma(n-1)\right)\right)}p_i + \frac{\gamma(n-1)}{(1-\gamma)\{1+\gamma(n-1)\}}\,p_{-i}$$

in Häckner and Herzing's (2016)notation, in which $\gamma \in [0,1]$ is the parameter that measures substitutability between (symmetric) products. Thus, if our (b, λ, μ) is determined by $b = \alpha/\{1+\gamma(n-1)\}$, and $\lambda = \{1 + \gamma(n-2\}/\{(1-\gamma)[1+\gamma(n-1)]\}$, given Häckner and Herzing's (2016) $(\alpha.\gamma)$, then our results below can be expressed by Häckner and Herzing's (2016) notation as well. Note here that our formulation is more flexible in the sense that the number of parameters is three. This is because the coefficient for the own price is normalized to one: $p_i(q_i\,, q_{-i}) = \alpha - q_i - \gamma(n-1)q_{-i}$, which is analytically innocuous, and Häckner and Herzing's (2016) c is the normalized parameter (see also Häckner and Herzing, 2022).

Under symmetric pricing, the industry's demand is thus given by $q(p) = b - [\lambda - (n-1)\mu]p$. The inverse demand system is given by

$$p_i(\mathbf{p}) = \frac{\lambda - (n-2)\mu}{(\lambda+\mu)\{\lambda-(n-1)\mu\}}\,(b - q_j + \frac{\mu}{(\lambda+\mu)\{\lambda-(n-1)\mu\}}\left[\sum_{i'\neq i}(b - q_{i'})\right]$$

Implying that $p(q) = (b-q)/\{\lambda-(n-1)\mu\}$ under symmetric production. Obviously, both the direct and the indirect demand curvatures are zero: $\alpha = 0\,, \sigma = 0$. The pass-through expressions under price competition, the pass-through expressions are

$$\rho_t = 1/((1-v)(1+\theta)) \qquad and \qquad \rho_v = (\epsilon_{own} - 1)/(\epsilon_{own}(1-v)(1+\theta))$$

Where $\theta = \{\lambda-(n-1)\mu\}\lambda\,,\ and\ \epsilon\_own = \lambda(p/q).\ under\ quantity\ competition,$

$$\rho_t = \frac{1}{(1-v)(1+\theta)} \qquad and \qquad \rho_v = \frac{1 - \eta\epsilon_{own}}{\epsilon_{own}(1-v)(1+\theta)}$$

Where $\theta = \{\lambda-(n-1)\mu\}/(\lambda+\mu)$ and $\eta_{own} = [\{\lambda-(n-2)\mu\}(q/p)]/[(\lambda+\mu)\{\lambda-(n-1)\mu\}]$.

Under competition price, the public funds marginal value and the incidence, discussed in both Propositions 3 & 4, respectively, become by

$$MVPF_t = \frac{1+2(1-v)\theta}{1+(1-v)\,\theta - \epsilon\tau} \qquad and \qquad MVPF_v = \frac{v + (1-v)\left(\theta + \frac{1+\theta}{\epsilon_{own}-1}\right)}{\frac{(1-v)(1+\theta)}{\epsilon_{own}} + v - \epsilon\tau}$$

$$I_t = \frac{1}{2(1-v)\left\{1-(n-1)\left(\frac{\mu}{\lambda}\right)\right\}} \qquad and \qquad I_v = \frac{\epsilon_{own}-1}{(1-v)\{2-\epsilon_{own}(1-\theta)\}}$$

with. $\in = \{\lambda-(n-1)\mu\}(p/q)$ Under quantity competition, they are

$$MVPF_t = \frac{1+2(1-v)\theta}{1+(1-v)\,\theta - \left(\frac{1}{\eta}\right)\tau} \qquad and \qquad MVPF_v = \frac{v+(1-v)\left(\theta + \frac{1+\theta}{1-\eta_{own}}\right)}{\frac{(1-v)(1+\theta)}{1-\eta_{own}} + v - \left(\frac{1}{\eta}\right)\tau}$$

$$I_t = \frac{\lambda+\mu}{2(1-v)\{\lambda-(n-2)\mu\}} \qquad and \qquad I_v = \frac{1-\eta_{own}}{(1-v)\{\eta_{own} + (2-\eta_{own})\theta\}}$$

with $1/\eta = \{\lambda-(n-1)\mu\}(p/q)$. Thus, in both cases, it suffices to solve for the equilibrium price and output to compute the pass-through and the marginal value of public funds.

Table 1 (a) summarizes the key variables that determine these values for the case of linear demand. It is verified that under both price and quantity competition, θ is a decreasing function of n and μ. To focus on the role of these two parameters, n and μ, which directly affect the intensity of competition, we employ the following simplification to compute the ratio p/q in equilibrium: b = 1; mc = 0, and $\lambda = 1$. (See Online Appendix H for the expressions of the equilibrium prices and output levels under price and quantity competition).

Table 1: Sufficient Statistics: Elasticities, Conduct Indices, and Curvatures.

(a) Linear Demand

Price setting	Quantity setting
$\varepsilon = [\lambda-(n-1)\mu]\left(\frac{p}{q}\right)$	$\eta = \frac{1}{\lambda-(n-1)\mu}\left(\frac{q}{p}\right)$
$\varepsilon_{own} = \lambda\left(\frac{p}{q}\right)$	$\eta_{own} = \frac{\lambda-(n-2)\mu}{(\lambda+\mu)[\lambda-(n-1)\mu]}\left(\frac{q}{p}\right)$
$\theta = \varepsilon/\varepsilon_{own} = 1-(n-1)\left(\frac{\mu}{\lambda}\right)$	$\theta = \eta_{own}/\eta = \frac{\lambda-(n-2)\mu}{\lambda+\mu}$
$\alpha = 0$	$\sigma = 0$

(b) CES Demand

Price setting	Quantity setting
$\varepsilon = \frac{1}{1-\gamma\xi}$	$\eta = 1-\gamma\xi$
$\varepsilon_{own} = \frac{n-\gamma-(n-1)\gamma\xi}{n(1-\gamma)(1-\gamma\xi)}$	$\eta_{own} = \frac{\gamma(1-\xi)+(1-\gamma)n}{n}$
$\theta = \varepsilon/\varepsilon_{own} = \frac{n(1-\gamma)}{n-\gamma-(n-1)\gamma\xi}$	$\theta = \eta_{own}/\eta = \frac{\gamma(1-\xi)+(1-\gamma)n}{n(1-\gamma\xi)}$
$\alpha = \frac{2-\gamma\xi}{1-\gamma\xi}$	$\sigma = 2-\gamma\xi$

(c) Logit Demand

Price setting	Quantity setting
$\varepsilon = \beta(1-ns)p$	$\eta = \frac{1}{\beta(1-ns)p}$
$\varepsilon_{own} = \beta(1-s)p$	$\eta_{own} = \frac{1-(n-1)s}{\beta(1-ns)p}$
$\theta = \varepsilon/\varepsilon_{own} = \frac{1-ns}{1-s}$	$\theta = \eta_{own}/\eta = 1-(n-1)s$
$\alpha = \frac{(2ns-3)ns}{1-ns}p$	$\sigma = \frac{1-2ns}{1-ns}$

The top two panels in Fig. 4 illustrate how $\rho_t$and ρ_v behave as we increase the number of firms (n, the left side) or the sustainability parameter (μ, the right side). The initial tax levels are t=0:05 and v = 0:05.Wedistinguish price setting and quantity setting by superscripts P and Q, respectively. The middle panels show $MVPF_t$ and $MVPF_v$, while the bottom panels depict $I_t$and I_t. We observe that the ad valorem tax pass-through is close to zero because in this case both $\in_{own}$ *and* η_{own} are close to 1. As competition becomes more intense, both ρ_t^P and ρ_t^Q become larger, and their difference also becomes larger. In the case of linear demand, the difference in the mode of competition does not yield a substantial difference in the three measures. As is verified by ADKb, the ad valorem tax is more efficient on the margin than the specific tax: the dashed lines in the two middle panels lie below the solid lines. This arrangement is inversely related to accidents and pass-through: the incidence or the pass-through, decreases the public funds for marginal value.

3.2. Constant elasticity of substitution (CES) demand

We next consider the market demand with constant elasticity of substitution given by

$$q_i(\mathbf{p}) = (\gamma\xi)^{\frac{1}{1-\gamma\xi}} \left[p_i^{-\frac{-1}{1-\gamma}} \Big/ \left\{ \sum_{i'}^{n} p_{i'}^{-\frac{-\gamma}{1-\gamma}} \right\}^{\frac{1-\xi}{1-\gamma\xi}} \right]$$

where 0 < γ< 1 and 0 < ξ< 1.[10] Hence the direct demand under symmetric pricing is $q(\mathbf{p}) = \left((\gamma\xi)^{\frac{1}{1-\gamma\xi}}\right)\left((n)^{\frac{1-\xi}{1-\gamma\xi}}\right)\left((p)^{\frac{-1}{1-\gamma\xi}}\right)$The elasticity of substitution, $1/(1-\gamma)$, is constant. Table 1 (b) shows the price elasticity of industry demand (e), the ownprice elasticity of a firm's demand (ε_{own}), the conduct index (θ), and the curvature of the industry's direct demand (a) are all independent of the equilibrium price.[11] This feature is in contrast to the linear demand above or the multinomial logit demand below. Similarly, the inverse demand is given by

$$p_i(\mathbf{p}) = (\gamma\xi) \left\{ \sum_{i'=1}^{n} q_{i'}^{\gamma} \right\}^{-(1-\xi)} \left(q_i^{-(1-\gamma)} \right)$$

Hence the inverse demand under symmetric pricing is $p(q) = (\gamma\xi)\big((n)^{1-\xi}\big)\left((p)^{1-\gamma\xi}\right)$. Table 1 (b) indicates that for the case of quantity setting, η , η_{own} , θ , and σ are also independent of the equilibrium output or price.[12] Note that for each tax $T \in (t, v)$, only ρ_T and θ , as well as the initial value of ad valorem tax *v*, are necessary to compute I_T, whereas the equilibrium price is necessary to compute $\tau = v + t/p$. With CES demand and a constant marginal cost mc, the equilibrium price under price, competition is analytically solved as

$$p = \frac{\eta(1-\gamma\xi) - \gamma(1-\xi)}{\gamma\eta(1-\gamma\xi) - \gamma(1-\xi)}\, mc > mc,$$

and the equilibrium price under quantity competition is given by

$$p = \frac{\eta}{\gamma(\eta - (1-\xi)}\, mc > mc,$$

---

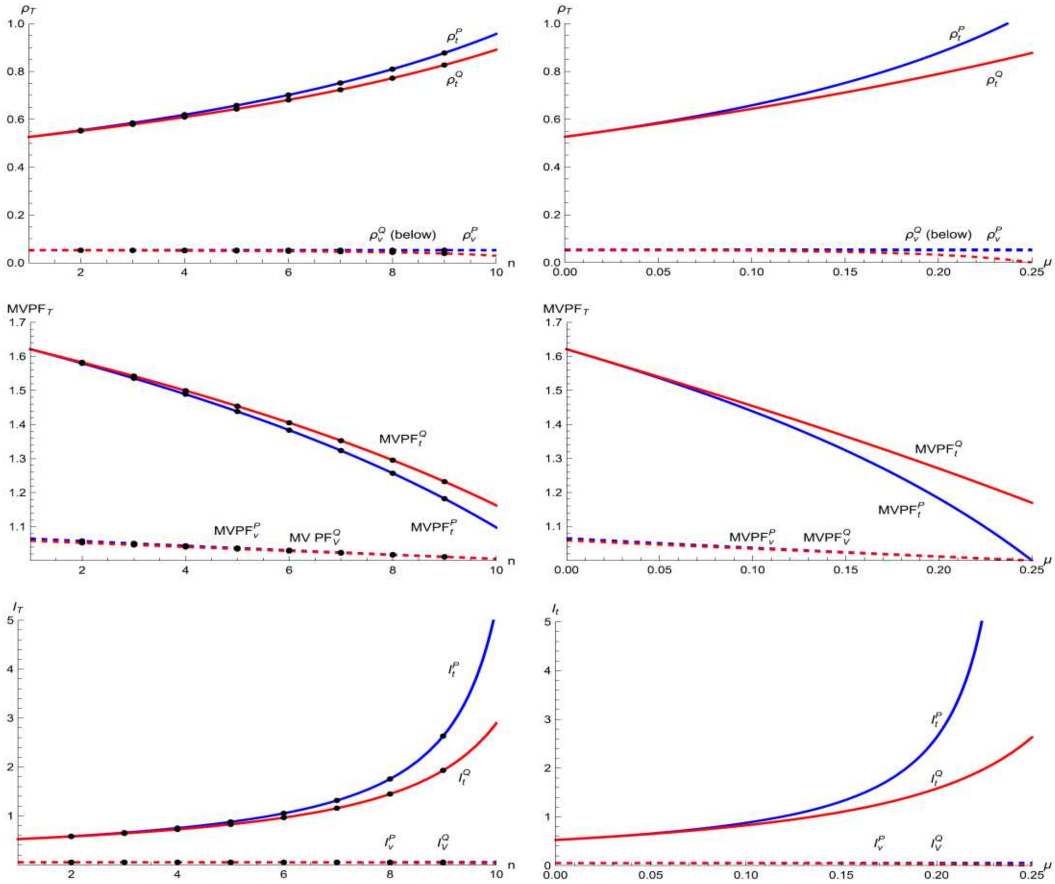
[10] This CES demand is derived from $U(q_1, \ldots, q_n) = \left\{\sum_{i=1}^{n} q_i^{\gamma}\right\}^{\xi}$ as the representative consumer's utility (Vives 2000, pp.147–8), where the elasticity of substitution between the firms is given by $1/(1-\gamma)$.

[11] We use the first-order derivative of $q(p), q'(p) = \left[\left((\gamma\xi)^{\frac{1}{1-\gamma\xi}}\right)\left((n)^{\frac{-(1-\xi)}{1-\gamma\xi}}\right) / (1-\gamma\xi)\right](p)^{\frac{-(2-\gamma\xi)}{1-\gamma\xi}}$ and its second-order derivative, $q''(p) = \left[\left((\gamma\xi)^{\frac{1}{1-\gamma\xi}}\right)\left((n)^{\frac{-(1-\xi)}{1-\gamma\xi}}\right)(2-\gamma\xi)/(1-\gamma\xi)^2\right](p)^{\frac{-(3-2\gamma\xi)}{1-\gamma\xi}}$ for these derivations.

[12] Here, we use the first-order derivative of $q(p), q'(p) = (1-\gamma\xi)(\gamma\xi)(n)^{(1-\xi)}(q)^{(2-\gamma\xi)}$, and its second-order derivative, $p''(q) = (1-\gamma\xi)(2-\gamma\xi)(\gamma\xi)(n)^{(1-\xi)}(q)^{(3-\gamma\xi)}$, for these derivations.

Fig. 4. Pass-through (top),

The marginal value of public funds (middle), and incidence (bottom) with linear demand. The horizontal axes on the left and the right panels correspond to the number of firms (n) with $\mu = 0.1$, and the substitutability parameter (μ) with n = 5, respectively, with the initial tax level, $((t, v) = (0.05\,, 0.05)$.



More details on the equilibrium are included in Online Appendix H. Fig. 5 depicts the differences across the competition-tax pairs regarding the pass-through value (top), the marginal value of public funds (middle), and the incidence (bottom) when mc = 1; $\xi = 0.9$, and $(t, v) = (0.05; 0.05)$. The left panel shows how $p.MVPF$, and I change in response to changes in the number of firms, and the right panel shows such changes in response to changes in γ.[13]

3.3. Multinomial logit demand

The last parametric example is the multinomial logit demand. Each firm $i = 1 \ldots, n$ faces the following demand: $s_i(\mathbf{p}) = exp(\delta - \beta p_i)\,/\{1 + \sum_{i'=1\ldots,n} exp\,(\delta - \beta p_{i'})\}\epsilon(0,1)$, where d is the (symmetric) product-specific utility and b > 0 is responsiveness to the price.37 We define

[13] Here we focus only on the intermediate values of γ $(i.e. \gamma \in (0.3,0.7))$ to ensure that the elasticity of substitution is not close to zero or one.

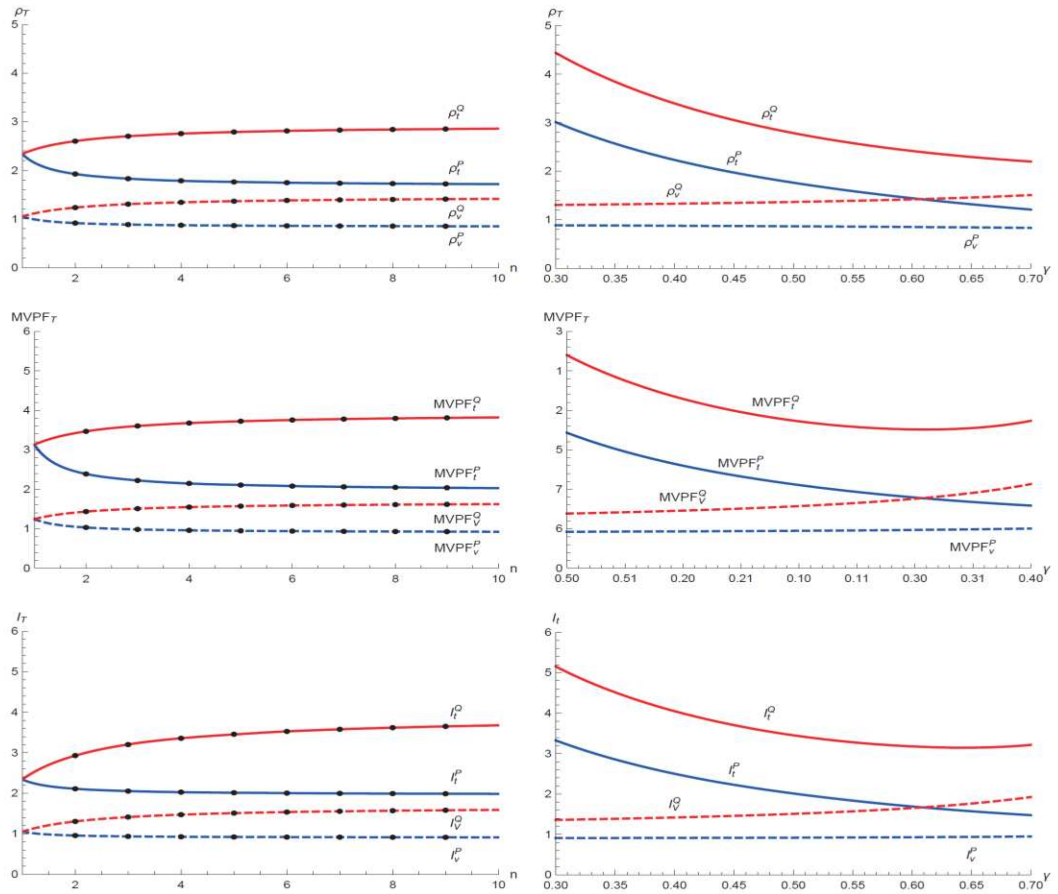
$s_0 = 1 - \sum_{i=1\ldots,n} s_i < 1$ as the share of all outside goods. Table 1 (c) summarizes the key variables that determine the pass-through, the marginal value of public funds, and the incidence.

> *“”Here, $q_i(p_1, \ldots, p_n)$ is derived by aggregating over individuals who choose product i (the total number of individuals is normalized to one): an individual’s net utility from consuming i is given by $\mu_i = \delta - \beta p_i + \bar{\varepsilon}_i$, whereas $\mu_0 = \bar{\varepsilon}_0$ is the net utility from consuming nothing, and $\bar{\varepsilon}_0, \bar{\varepsilon}_1, \ldots, \bar{\varepsilon}_n$are independently and identically distributed according to the Type I extreme value distribution for all individuals. See Anderson et al. (1992, pp.39-45) for details. We work in terms of market share variables s_i and s, instead of q_i and q, which is consistent with the standard notation in the industrial organization literature.””*

We need to numerically solve for the equilibrium price and market share under both settings to compute these values for all four cases. To focus on the two parameters, b and n, we assume that $\delta = 1$ and mc = 0. Because $\delta s_i(\mathbf{p})/\delta p_i \mid_{\mathbf{p}=(\mathbf{p},\ldots,\mathbf{p})} = \beta s(1-s)$, the first-order conditions for the symmetric equilibrium price and the market share satisfy $p - t/(1-v) = 1/\{\beta(1-s)\}$ and $s = \exp(1-\beta p)/\{1 + n\,.\exp(1-\beta pP\}$. If p and s are solved numerically, then $\epsilon, \epsilon_{own}, \theta$ *and* α and a can also be numerically computed. 14

Fig. 5. Pass-through (top),

The marginal value of public funds (middle), and incidence (bottom) with constant elasticity of substitution (CES) demand. The horizontal axes on the left and the right panels are the number of firms (n) with $\gamma = 0.5$, and the substitution parameter (γ with n = 5, respectively (with the initial tax level, $((t, v) = (0.05\,, 0.05)$.



Next, we consider inverse demands under quantitative competition. Then, as in Berry (1994), the inverse demand of firm i is given by $p_i(\mathbf{s}) = \{\delta - \log(s_i/s_0)\}/\beta \; where \; (\mathbf{s}) = (s_0, .., s_n)$, which means that $\partial p_i(\boldsymbol{s})/\partial s_i \mid_{\boldsymbol{s}=(s,..,s)} = -[1-(n-1)s]/\beta s(1-ns)]$. Thus, the conditions of the first order of symmetric equilibrium price and market share satisfy $p - t/(1-v) = [1-(n-1)s]/[\beta(1-ns)] \; and \; p = [1-\log(s/[1-ns])]/\beta$. then, as above, η, η_{own}, $\theta \; and \; \sigma$ are calculated by numerically solving the first-order terms of p and s. Interestingly, it has been verified that in the case of symmetric equilibrium under the determination of quantity, $\partial p/\partial n = 0$: the equilibrium price is the same regardless of the number of firms, while the individual market share decreases in the number of firms: $\partial s/\partial n < 0$. On the other hand, both the equilibrium price and the market share in the price coefficient, β.

Figure 6 shows the pass and the marginal value of public funds and their incidence in comparison with Fig. 4 and 5. The right panels now show the dependence of the variables on the price coefficient β. In general, as in the case of linear demand and CES demand, an increase in value tax has a slight effect on these measures of both n and β, while an increase in unit tax has a large effect.

However, there are two important differences between linear demands and logistical requirements. First, the passage of unit tax under quantitative competition decreases ρ_t^Q in the number of firms. To understand this, compare the difference in the denominators of $\rho_t^p = (1/\{(1-v)[1+(1-\alpha/\epsilon_{own})\theta]\}$ and $\rho_t^Q = (1-v)[1+\theta-\sigma]$. As θ decreases (i.e., as competition becomes more fierce), the second term in the denominator of ρ_t^p decreases, and thus ρ_t^p increases as n increases. However $\theta - \sigma$, increases as θ decreases, and thus ρ_t^Q decreases. This difference in denominators is also reflected in the fact that I_t^Q decreases in n as well. Naturally, $MVPF_t^Q$ decreases in n as in the case of linear demand because $1/\rho_t^Q$ becomes larger (see formulas in Proposition 3). Second, while transit and incidence increase with the increase of β, the marginal value of public money also increases in contrast to the case of linear orders. The reason is that the effect on the MVPF of decreases in θ is weaker than that of increases in $\in$: industry demand becomes elastic rapidly as consumers become more sensitive to price increases.

4. Heterogeneity of firm

In this section, we extend our results to the case of n heterogeneous firms, where each firm is i = 1; 2; . . . ; n controls a strategic variable σ_i, which will be, say, the price or quantity of its product. Appendix C presents the general version of the multidimensional interventions and bases some of the findings on transit and well-being measures. In the following, p_i is the price of the product of firm i, q_i is the quantity of the product sold by firm i. Then, the firm's profit function is written as

$$\pi_i = (1 - v)p_i(\mathbf{q})q_i - tq_i - c_i(q_i) = p_i(\mathbf{q})q_i - c_i(q_i) - R_i(\mathbf{q})$$

Where $R_i(\mathbf{q}) = tq_i + vp_i(\mathbf{q})q_i$ is the (per-firm) tax revenue from firm i. Under this firm Eq. 1 is generalised as

$$\left[\left(1 - \frac{t}{p_i(\mathbf{q})} - v\right) - \psi_i(\mathbf{q})(1 - v)\right] p_i(\mathbf{q}) = mc_i(q_i) \tag{11}$$

for i = 1; 2; . . . ; n, where we call the $\psi_i(\mathbf{q})$ firm i's pricing strength index i. In the case of identical firms, the index of pricing power w is related to the index of behaviour $\theta(q)$ by $\theta = \in \psi$. For clarity of intuition, suppose (t,v)=(0,0). Then, eq. (11) implies $p_i(\mathbf{q}) = mc_i(q_i)/[1 - \psi(\mathbf{q})]$. If it is $\psi(\mathbf{q}) = 0$ for any $\mathbf{q}$ and i, then all firms will adopt marginal cost pricing. If ψ_i is large enough, then p_i can be much higher than the marginal cost. We found that with heterogeneous firms, using the pricing power index is significantly more appropriate than using the behaviour index when we characterize the marginal value of public funds and incidence. Appendix D discusses the relationship between these two concepts.

4.1 pass-through

The pass-through matrix for two-dimensional taxes (t and v) is defined as

$$\bar{\boldsymbol{\rho}} = \begin{pmatrix} \partial p_1/\partial t & \partial p_1/\partial v \\ \vdots & \vdots \\ \partial p_n/\partial t & \partial p_n/\partial v \end{pmatrix}$$

Using the results from Proposition C.3 in Appendix C, the pass-through matrix is described as follows.

Proposition 5. For heterogeneous companies with specific values and value

Taxes, the traffic matrix is equal to

$$\bar{\boldsymbol{\rho}} = \begin{pmatrix} 1 & p_1 & (1 - \psi_1) \\ \vdots & \vdots & \vdots \\ 1 & p_n & (1 - \psi_n) \end{pmatrix}$$

where the *(I,j)* element of the **b** matrix is given by

$$b_{ij} = (1 - v)[1 - \psi_i)\delta_{ij} - \psi_i] + [(1 - \tau_i) - (1 - v)\psi_i]\, \chi_i \in_{ij}$$

Where δ_{ij} is the kronecker delta $\in_{ij} \equiv -\left(\frac{p_i}{q_i}\right)\left(\frac{\partial q_i(\mathbf{p})}{\partial p_j}\right)$, $\psi_{ij} \equiv \left(\frac{p_i}{\psi_i}\right)\left(\frac{\partial \psi_i(\mathbf{q}(\mathbf{p}))}{\partial p_j}\right)$, $\tau_i = \frac{t}{p_i} + v$

As usual, Kronecker delta δ_{ij} is defined as equal to 1 if its two indices are The same and zero otherwise.

Note that if all firms have a fixed marginal cost ($\chi_i = 0\ for\ i$), then the expression for b_{ij} simplifies to $b_{ij} = (1-v)[(1-\psi_i)\delta_{ij} - \psi_i\Psi_{ij}$.

4.2 Characterization of the two measures of welfare

By using the results in Appendix C, we can also obtain the following proposition characterizing the marginal value of public money and the incidence of the case of heterogeneous firms, where we define $\in_i$, an n-dimensional row vector with its *j*-th component equal to $\in_{ij}$ for each *i*, by $\in_i = (\in_{i1}, \dots, \in_{ij}, \dots \in_{in})$.

Proposition 6. Let $\in_{ij}^{\rho} \equiv \in_i \widetilde{\boldsymbol{\rho}}_T/\tilde{\rho}_{iT} = \in_i \boldsymbol{\rho}_T/\rho_{iT}$ for $T\epsilon\{t, v\}$. Then, Marginal value of public money associated with intervention T, $MVPF_{iT} \equiv [(\nabla CS_i)_T + (\nabla PS_i)_T]\,/(-\nabla R_i)_T$ is characterized by:

$$MVPF_{iT} = \frac{\frac{1}{\in_{iT}^{\rho}}\left(\frac{1}{\rho_{iT}} + v\right) + (1-v)\psi_i}{\frac{1}{\in_{iT}^{\rho}}\left(\frac{1}{\rho_{iT}} + v\right) - \tau_{i)}}$$

The occurrence of this interference, $I_{iT} \equiv (\nabla CS_i)_T/(\nabla PS_i)_T$, is characterized by:

$$I_{iT} = \frac{1}{\left(\frac{1}{\rho_{iT}}\right)(1-v)\left(1 - \psi_i \in_{iT}^{\rho}\right)}$$

While Table 2 summarizes our characterization at each stage of publicity

Table 2: Summary of Expressions for the Two Measures of Welfare for T ∈ {t, v} under Imperfect Competition

	Symmetric firms: No pre-existing taxes	Symmetric firms: With pre-existing taxes	Heterogeneous firms
Marginal Value of Public Funds	$\theta\rho_T$	$\frac{(1-v)(\theta/\varepsilon)+\tau}{\frac{\frac{1}{\rho_T}+v}{\varepsilon}-\tau}$	$\frac{(1-v)\,\psi_i+\tau_i}{\frac{\frac{1}{\rho_{iT}}+v}{\varepsilon_{iT}^{\rho}}-\tau_i}$
Incidence	$\frac{1}{\frac{1}{\rho_T}-(1-\theta)}$	$\frac{1}{\frac{1}{\rho_T}-(1-v)(1-\theta)}$	$\frac{1}{\frac{1}{\rho_{iT}}-(1-v)(1-\psi_i\varepsilon_{iT}^{\rho})}$

Figure 7. Pass-through when Company 1 (left) and Company 2 (right)

have identical (linear) demand but different marginal costs (top), the market-wide correlated marginal value of public funds (middle) and incidence (bottom): case price competition.

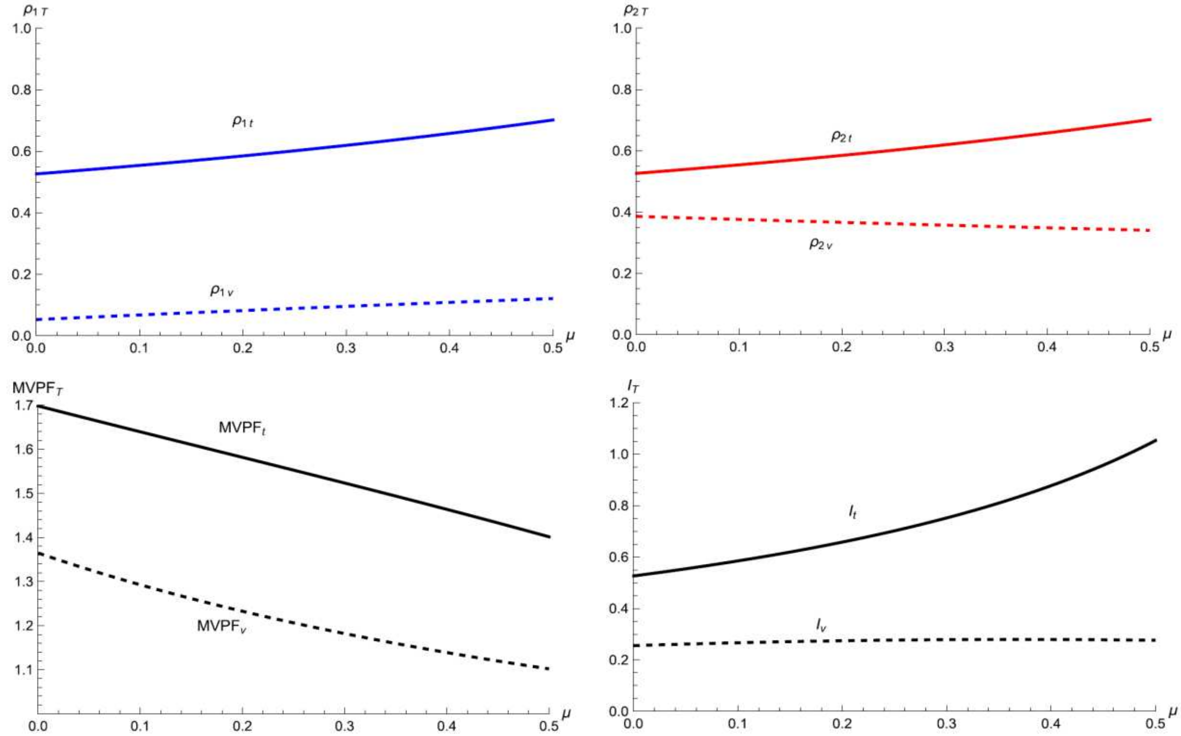
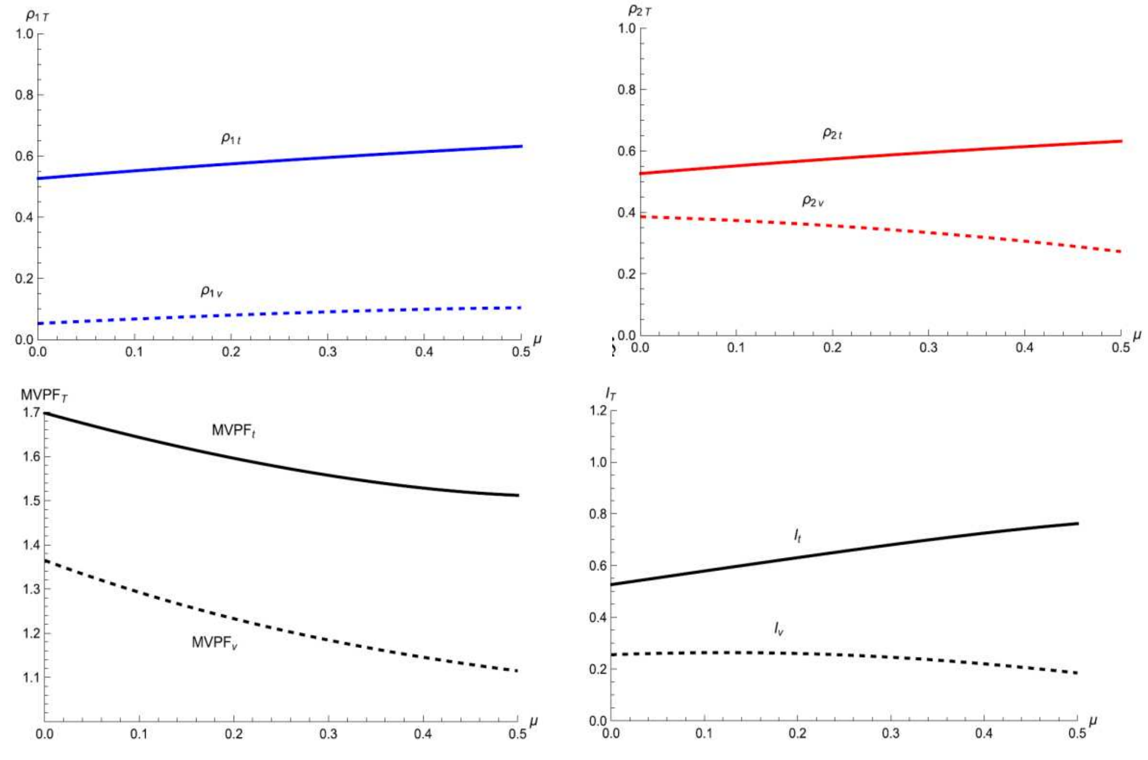


Figure 8. Pass-through when Company 1 (left) and Company 2 (right)

have identical (linear) demand but different marginal costs (top), the market-wide correlated marginal value of public funds (middle) and occurrence (bottom): case Quantitative competition.



4.3 Marginal value of public money and its incidence at the market-level

It is also useful to consider measures of well-being at the market level in light of firm heterogeneity. More specifically, for $T \in (t, v)$, we define $MVPF_T$ at the market and I_T level by

$$MVPF_T = \frac{\left(\frac{\sum_{i=1}^{n} \partial CS_i}{\partial T}\right) + \left(\frac{\sum_{i=1}^{n} \partial CS_i}{\partial T}\right)}{-\frac{\sum_{i=1}^{n} \partial R_i}{\partial T}} \quad and \quad I_T = \frac{\left(\frac{\sum_{i=1}^{n} \partial CS_i}{\partial T}\right)}{\frac{\sum_{i=1}^{n} \partial PS_i}{\partial T}} \quad Respectively$$

$$Where \ \frac{\partial CS_i}{\partial T} = -\tilde{\rho}_{iT} \, . \, q_i \quad , \qquad \frac{\partial PS_i}{\partial T} = (1 - v)\left[\tilde{\rho}_{iT} - \psi_i \, . \sum_{j=1}^{n} \epsilon_{ij}\tilde{\rho}_{jT}\right] . \, q_i - \frac{\partial R_i}{\partial T}$$

$$and \ \frac{\partial R_i}{\partial T} = \left[v - \tau_i \, . \sum_{j=1}^{n} \epsilon_{ij}\right] . \, \tilde{\rho}_{iT} \, . \, q_i + F_{iT}$$

are the marginal changes of each firm in consumer surplus, producer surplus, and government revenue, respectively (they can be derived by applying Proposition C.4.1 to this case from two-dimensional taxation), where $F_{iT} = \begin{cases} q_i & for \ T = t \\ p_i q_i & for \ T = v \end{cases}$

To justify our definitions, remember that we take a representative consumer approach: ignoring y, the net utility, i.e., the total consumer surplus, is $CS = u(\mathbf{q}) - \mathbf{p}^T \, . \, \mathbf{q}$. Under firm symmetry, the change in consumer surplus (per firm) is simply given by $dCS = -q \ dp$ (Equation 6). Now, under constant heterogeneity, note that $CS = u[\mathbf{q}(\mathbf{p}) - p_n q_n(\mathbf{p}) - \cdots - p_n q_n(\mathbf{p})$ so that

$$dCS = \left(\underbrace{\frac{\partial u}{\partial q_1}}_{=p_1} \frac{\partial q_1}{\partial p_1} + \cdots + \underbrace{\frac{\partial u}{\partial q_n}}_{=p_n} \frac{\partial q_n}{\partial p_1}\right) \ldots . . + \left(\underbrace{\frac{\partial u}{\partial q_1}}_{=p_1} \frac{\partial q_1}{\partial p_n} + \cdots + \underbrace{\frac{\partial u}{\partial q_n}}_{=p_n} \frac{\partial q_n}{\partial p_n}\right)$$

Therefore, a change in total consumer surplus is brought about by the following simple sum of contributions of each firm: $dCS_T = \underbrace{-q_1 dp_1}_{\equiv dCS_{1\,T}} - \cdots - \underbrace{-q_n dp_n}_{\equiv dCS_{n\,T}}$, which justifies our definitions above.

4.4 Cost Heterogeneity

To understand how static heterogeneity relates to welfare tax effects, we consider an example where two firms are symmetrically differentiated—and thus face an identical model. Demand - but have different marginal costs. Specifically, $i = 1,2$ faces the linear order, $q_i(p_1, p_2) =$

$b - \lambda p_i + \mu p_i$, $j \neq i$, $j = 1,2$. Assume that the marginal cost of production for either firm is constant, $mc_i \geq 0$, and firm 1 is a low-cost firm: $mc_1 < mc_2$.

In a similar vein, Anderson et al. (2001b) (ADKb) combined with price and quantity competition. However, quantitative competition assumes homogeneous products in the spirit of Cournot's original (1838) formulation. On the other hand, our formula is more general than the ADKb setting because it takes into account the quantitative competition of the homogeneous product and one variable from the Hotelling competition (1929). Instead, our formula allows us to use the demand structure for both price and quantity competition derived from the same utility for the representative consumer. For the sake of presentation, we focus on linearity ordering system. First, suppose that these two firms are complete in price. Then, the first-ranking conditions for firm i are expressed in this pricing game as:

$$\left[\left(1 - \frac{t}{p_i} - v\right) - \frac{q_i}{p_i \cdot \left(-\frac{\partial q_i}{\partial p_i}\right)} (1 - v)\right] p_i = mc_i$$

According to Eq. (12), where $-\partial q_i / \partial p_i = \lambda$. To compute market-level welfare characteristics, we need the values of ψ_i (company i's pricing power index), ρ_{iT} (company i's pass-through), $\in_{iT}^{\rho}$, as well as v (value added tax) and $\tau_i \equiv v + t/p_i$ (revenue Government taxes divided by the company's total revenue i). See Appendix G online for these accounts.

The top panel of Figure 7 shows how the pass-through varies differently across the two firms (left side of firm 1 and right side of firm 2) when the degree of product differentiation changes (higher l indicates lower differentiation), assuming $b = 1$, $(mc_1, mc_2) = (0.05)$, $\lambda = 1$ and $(t, v) = (0.05, 0.05)$ (Here, we consider the restriction, $\mu < \lambda < b/mc_2 + \mu$, for the range of l. In Fig. 7, we highlight $\mu \epsilon [0.0, 0.5]$). As in Section 3, the market-level marginal value of public funds, and market-level incidence are shown in the middle and lower panels, respectively. $MVPF_t$ is observed to be higher than $MVPF_v$, a result consistent with the case of invariant symmetry. Similarly, the first-order conditions of firm i under quantitative competition are given by:

$$\left[\left(1 - \frac{t}{p_i} - v\right) - \frac{q_i}{p_i \cdot \left(-1/(\frac{\partial q_i}{\partial p_i})\right)} (1 - v)\right] p_i = mc_i$$

$in\ accordance\ with\ eq. 12\ where\ -1/((\partial q\_i)/(\partial p\_i\ )) = [(\lambda + \mu)(\lambda - \mu)]/\lambda$

Figure 8 shows the similarity with the case of price competition. In short, it appears that competition between firms' price or quantity does not matter much for determining MVPF and incidence in the general setting of product differentiation. A more detailed analysis is left for future research.

5. Closing remarks

In this paper, we describe welfare measures for taxes. Under the general specification of market demand, cost of production, imperfect competition, which includes a broader category of multiple political interventions and external changes other than taxes.

For symmetric oligopoly, we first show how the unit tax pass rate p_t relates to value tax transit through the quasi-elasticity p_v (i.e. Proposition 1). Passing is also characterized by the generalization of Weyl and Fabinger's (2013) formula (ie, proposition 2). We then derive equations to measure marginal welfare losses due to unit and value taxes, $MVPF_t$ and $MVPF_v$ respectively (Proposition 3) as well as the formulas for the occurrence of the tax, I_t and I_v (Suggestion 4). Section 3 calculates these welfare measures using representative categories of market demand.

We then introduce heterogeneous companies in Section 4 to generalize these formulas which can be understood as a natural extension of those obtained under constant symmetry. Our derivation is based on a general framework, outlined in Appendix C, which uses the idea of tax revenue as a function defined by a vector of tax standards, and thus can allow for a multidimensional pass: a combination of specific and ad valorem taxes is interpreted as a special case of two-dimensional government intervention. In this way, we have provided a comprehensive framework for assessing welfare for taxes under imperfect competition, which could also allow for many applications in a variety of non-tax contexts (see Appendix B online).

In this paper we seek a general analysis of specific and value taxes under imperfect competition, assuming any beneficial effects of government spending. How does the government raise its tax revenue and spend its expenditures in imperfectly competitive product, labour, and capital markets? Admittedly, our framework, as in the analysis by Weyl and Fabinger (2013), is limited to the Cournot-Marshall partial equilibrium model. Our study is a small step towards a more comprehensive understanding of the relationship between imperfectly competitive private markets and the role of the public sector in such A framework as a general equilibrium model (eg, Harberger, 1962; Azar and Vives, 2021).

Appendix A. Proofs and further discussion for Section 2

A.1. Discussion of signs of changes in welfare components for a specific tax increase

Figure 2 shows the effect of a particular tax increase in one case. Here we discuss the signs of changes in the well-being component in general. It is useful to work at the infinite level, where this tax change corresponds to dt > 0 and dv = 0. For producer surplus, the quantity effect is negative $dPS.. = (1 - v)p\eta\theta\ dq < 0$, the effect of value and $dPS_{\updownarrow} = (1 - v)q\ dp - q\ dt = -q(1 - (1 - v)\rho_t)dt$, is negative for $\rho_t < 1/(1 - v)$ and positive for $\rho_t > 1/(1 - v)$. The overall change is

$$dSP = (1 - v)p\eta\theta dq - q[1 - (1 - v)\rho_t]dt = (1/\rho_t - (1 - v)(1 - \theta)\eta p\ dq\ ,$$

It is negative for $(1/\rho_t > (1 - v)(1 - \theta)$ and positive for $= (1/\rho_t < (1 - v)(1 - \theta)$. For a value small enough to cross, the profit of the firms will decrease when *t* is increased. For specific tax revenue, the quantity effect and the value effect have opposite signs: $dR_{t\leftrightarrow} = t\ dp < 0$, $dR_t = q\ dt > 0$. Total change $dR_t = t\ dq + q\ dt = (t - \eta p/\rho_t)\ dq$ is positive for $t > \eta p/\rho_t$ and negative for $t > \eta p/\rho_t$. For value tax revenue, the quantity effect and the value effect again have opposite signs: $dR_{v\leftrightarrow} = vp\ dq < 0$, $dR_{v\updownarrow} = qv\ dp > 0$. The total change $dR = dR_{v\leftrightarrow} + dR_{v\updownarrow} = (1 - \eta)vp\ dq$ is negative, if we assume that $\eta < 1$, As we usually do. Consumer surplus decreases, since $dCS_{\leftrightarrow}$ is zero, and $dCS_{\updownarrow} = q\ dp$p is unambiguously negative for dt > 0.

A.2. Discuss the signs of changes in the components of social welfare in order to increase the tax by value

Figure 3 shows the effect of a specific tax increase in one case over here. We discuss signs of changes in the well-being component in general. It is useful to work at the infinite level where this tax change corresponds to dv> 0 and dt > 0. For producer surplus, the quantity effect is negative $dRS_{\leftrightarrow} = (1 - v)p\eta\ dv$, and the effect of value,

$$dRS_{\updownarrow} = (1 - v)q\ dp - pq\ dv = (1 - (1 - v)\rho_v)pq\ dv$$

is negative for $\rho_t < 1/(1 - v)$ and positive for $\rho_t > 1/(1 - v)$. The overall change is

$$dRS = (1 - v)p\eta\theta\ dp - (1 - (1 - v)\rho_v)pq\ dv = \left[\frac{1}{\rho_v} - (1 - v)(1 - \theta)\right]\eta p dq$$

It is negative for $1/\rho_v > (1 - v)(1 - \theta)$ and positive for $1/\rho_v > (1 - v)(1 - \theta)$. For a value small enough to cross, the profit of the firms will fall as *v* increases. For specific tax revenue, the effect of the quantity $dRS_{t\leftrightarrow} = t\ dq$ is negative, while the effect of $dR_{t\updownarrow}$ is zero because the specific tax rate remains unchanged. So the total change $dR_{\updownarrow} = dR_{t\leftrightarrow} = t\ dq$ is negative.

For VAT revenue, the quantity effect and the value effect again have opposite signs: $dR_{v\leftrightarrow} = vp\ dq < 0$, $dR_{v\updownarrow} = qv\ dp + qp\ dv > 0$. The total change $dR = dR_{v\leftrightarrow} + dR_{v\updownarrow} = (1 - \eta)vp\ dq$ is negative, if we assume that η <1, as we usually do. Consumer surplus decreases, since $dCS_{\leftrightarrow}$ is zero, and $dCS_{\leftrightarrow} = -q\ dp$ is unambiguously negative for dt > 0.

A-3. Proof of Proposition 1

Let us consider an infinitesimal change *dt* and dv in taxes t and v which leaves the equilibrium price (and quantity) unchanged, which would require the "effective" marginal cost $(t + mc)/(1 - v)$ in the equation. (1) To remain as is. This indicates the following comparative statistics relationship:

$$\frac{\partial}{\partial t}\left\{\frac{(t+mc)}{(1-v)}\right\}dt + \frac{\partial}{\partial v}\left(\frac{t+mc}{1-v}\right)dv = 0 \Rightarrow \frac{dt}{1-v} + (\frac{t+mc}{(1-v)^2})dv = 0$$
$$\Rightarrow dt = -\frac{t+mc}{1-v}dv$$

Note here that we do not need to take the mc derivatives even though they depend on *q*, simply assuming the quantity is unchanged. The total induced change in price, generally expressed as $dp = \rho_t dt + \rho_v p\ dv$, should equal zero in this case, implying the desired result:

$$\rho_t dt + \rho_v p\ .dv = 0 \ \Rightarrow \frac{t+mc}{1-v}\rho_t dv + \rho_v p.dv = 0 \Rightarrow \rho_v = (1 - \eta\theta)\rho_t \Rightarrow \rho_v = \frac{\epsilon - \theta}{\epsilon}\rho_t$$

A.4. Proof of Proposition 2

Consider the comparative statistics regarding a small change *dt* in tax per unit t. Then, the learner state becomes:

$$\underbrace{p - \frac{t+mc}{1-v}}_{\text{markup}} = 0.ms\ then, in\ equilibrium\ dp - \frac{dt + dmc}{1-v} = d(\theta.ms)$$

$$\underbrace{(1-v)\left[\underbrace{dp}_{>0} - \underbrace{d(\theta\cdot ms)}_{<0}\right]}_{\text{change in marginal benefit}} = \underbrace{\underbrace{dt}_{>0} + \underbrace{dmc}_{<0}}_{\text{change in specific-tax inclusive marginal cost}},$$

Thus, using $dt = dp/\rho_t$, the equation is rewritten as

$$\rho_t = \frac{1}{\underbrace{(1-v)[dp + (-d(\theta\cdot ms))]}_{\text{(A)>0:revenue increase}} + \underbrace{(-dmc)}_{\text{(B)>0:cost savings}}}dp.$$

Now, consider term (a) above. Note first $d(\theta.ms) = (\theta.ms)'dq$ so that $d(\theta.ms) = q \in (\theta.ms)'(dq/p)$, because by definition $dq = -q \in .(dp/p)$. Here, to slightly increase *dt* > 0,

$$\underbrace{d(\theta \cdot ms)}_{<0} = \underbrace{-q\epsilon}_{>0} (\theta \cdot ms)' \underbrace{\frac{dp}{p}}_{>0}$$

So that $(\theta.ms)' > 0$. By definition $ms = p'q = \eta p$. Thus $d(\theta.ms) = -q \in (\theta\eta p)'(dp/p)$. Now, note that $(\theta\eta p)' = (\theta\eta)'p + (\theta\eta)p'$. Hence $d(\theta.ms) = -q \in [(\theta\eta)'p + (\theta\eta)p']dp/p$

$$\Rightarrow \; d(\theta.ms) = -q \in (\theta\eta)' \, dp + \left\{-q \in (\theta\eta)p'.\left(\frac{dp}{p}\right)\right\} = \{\theta\eta - q\epsilon(\theta\eta)'\}dp > 0$$

Next, consider term (B). The change in marginal cost, *dmc*, is expressed in terms of *dp* by $dmc = -[(1-v)\theta\eta + 1 - \tau]\chi \in .dp < 0$. To see this, first note that $dmc = \chi mc.(dq/p) = -(\chi \in .mc)(dp/p)$. Then, mc can be omitted in this expression by rewriting $-\theta.ms = (mc + t)/(1-v) \Rightarrow mc = (1-v)(p + \theta qp\text{^}') - t = (1-v)(1-\theta\eta)p - t$, which means that $dmc = -[(1-v)(1+\theta\eta) - t/p]\chi \in .dp$. Then, in terms of revenue burden per unit, $\tau \equiv v + t/p\,, that\ is\,,\; dmc = -[(1-v)(1-\theta\eta) - \tau + v]\chi \in dp = -[-(1-v)\theta\eta + 1 - \tau]\chi \in dp$. Finally, using the expressions *dmc* and $d(\theta.ms)$, this is checked

$$\rho_t = dp/\{(1-v)[dp - d(\theta.ms)] - dmc$$

$$= \frac{1}{\underbrace{(1-v)[(1-\theta\eta) + (\theta\eta)'\epsilon q]}_{\text{revenue increase}} + \underbrace{(1-\tau)\epsilon\chi - (1-v)\theta\chi}_{\text{cost savings}}},$$

$$\Longleftrightarrow \rho_t = \frac{1}{1-v} \cdot \frac{1}{\underbrace{[(1-\theta\eta) + (\theta\eta)'\epsilon q]}_{\text{revenue increase}} + \underbrace{\left[-\theta + \frac{1-\tau}{1-v}\epsilon\right]\chi}_{\text{cost savings}}}.$$

Finally, $\rho_v$is obtained from this expression and Eq. (10). Then, to express this formula in terms of Weyl and Fabinger notation (2013, p. 548), remember the equation. (2):

$$\rho = \frac{1}{1 + (\in - \theta)\chi + \dfrac{\theta}{\epsilon_\theta} + \dfrac{\theta}{\epsilon_{ms}}}$$

where $\in_D$and $\in_S$ are replaced by $\in$ and $1/\chi$, respectively. First, the denominator is rewritten in the formula as:

$$1 - (\eta + \chi)\theta + \epsilon q(\theta\eta)' + \frac{1-\tau}{1-v}\epsilon\chi = 1 + \left[\left(\frac{1-\tau}{1-v}\right)\epsilon - \theta\right]\chi + \frac{\theta}{\epsilon_\theta} + \theta.(-\frac{1}{\epsilon} + \eta' \in q$$

Because $(\theta\eta)' \in q = (\theta'\eta + \theta\eta') \in q = \left[\left(\frac{\theta}{q\epsilon_\theta}\right)\eta + \theta\eta'\right] \in q = \frac{\theta}{\epsilon_\theta} + \theta\eta' \in q$

Next, since $\eta = -qp'/p$, it is verified that $\eta' = -\{p'p + qpp' - q(p')^2\}/p^2$, implying that

$$\eta' \in q = \frac{[p' p + qpp'' - q(p')^2]}{p^2} \cdot \frac{p}{p'q} q = \frac{1}{\in} + \left(1 + \frac{p''}{p'} q\right)$$

where $1 + p''q/p$ is replaced by $1/\in_{ms}$ because ms $ms = p'q$ and thus $ms' = -(p''q + p')$. Then, verified that

$$1 - (\eta + \chi)\theta + \epsilon q(\theta\eta)' + \frac{1-\tau}{1-v}\epsilon\chi = 1 + \left[\frac{1-\tau}{1-v}\epsilon - \theta\right]\chi + \frac{\theta}{\epsilon_\theta} + \frac{\theta}{\epsilon_{ms}}$$

In summary, Weyl and Fabinger's (2013, p. 548) original Eq. (2) is generalized to

$$\rho = \frac{1}{1-v} \cdot \frac{1}{1 + \left[\frac{1-\tau}{1-v} \in -\theta\right]\chi + \frac{\theta}{\epsilon_\theta} + \frac{\theta}{\epsilon_{ms}}}$$

With a non-zero initial value tax, which is equivalent to our formula for ρ_t:

$$\rho_t = \frac{1}{1-v} \cdot \frac{1}{1 + \left[\frac{1-\tau}{1-v}\right] \in \chi - (\eta + \chi)\theta + \frac{\theta}{\epsilon_\theta} + \frac{\theta}{\epsilon_{ms}}}$$

And from proposition 1, it is easily seen that qv can also be written in terms of Weyl and Fabinger (2013) notation:

$$\rho_v = \frac{\in -\theta}{(1-v)\epsilon} \cdot \frac{1}{1 + \left(\frac{1-\tau}{1-v} \in -\theta\right)\chi + \frac{\theta}{\epsilon_\theta} + \frac{\theta}{\epsilon_{ms}}}$$

Appendix B: Determination of the imperfect competition method under constant symmetry

In this appendix we show that for a fixed-price game or quantitative competition without anti-competitive behaviour, our general formulas for marginal value of public money and passivity derive expressions in terms of fundamentals of demand such as elasticities, curvatures and marginal cost elasticities χ. The question of whether firms fixing quantity or prices are more appropriate depends on the nature of competition. As Riordan (2008, p. 176) argues, quantitative competition is a more appropriate model if one envisions a situation in which firms determine the necessary production capacity. However, price-fixing firms are more suitable if the firms in focus can quickly adapt to demand by changing their prices.

In this appendix, we assume that corporate behaviour is simply described by a one-shot Nash equilibrium, without any other possibilities such as implicit collusion. As shown below, this assumption enables one to express an indicator of behaviour in terms of demand and the inverse elasticity of demand, using Eq. (1) directly (see subsection B2 below). Online Appendix F looks at the relationship between flexibility and bends.

B 1. Flexibility and bends of the demand system

B.1.1. direct order

We additionally define the cross-price elasticity $\in_{cross}\ (p)$ of the firm's direct demand by

$$\in_{cross}\ (p) \equiv \frac{(n-1)p}{q(p)} \cdot \frac{\partial q_{i'}(\mathbf{p})}{\partial p_i}\ |_{(\mathbf{p}=p,\ldots,p)}$$

where i and $i'$are an arbitrary pair of distinct indices. It is related to the industry demand elasticity $\in_{cross}=\in+\in_{cross}$. We define own curvature $\alpha_{cross}(p)$ of firms cross curvature dirct demand $\alpha_{cross}(p)$ the firm's the direct demand by: [14]

$$\alpha_{cross}(p) \equiv -p.\left(\frac{\partial q_i(\mathbf{p})}{\partial p_i}\right)^{-1}.\left(\frac{\partial^2 q_i(\mathbf{p})}{\partial p_i^2}\right) \ \boldsymbol{and}\ \alpha_{cross}(p) \equiv -(n-1)p.\left(\frac{\partial q_i(\mathbf{p})}{\partial p_i}\right)^{-1}.\left(\frac{\partial^2 q_i(\mathbf{p})}{\partial p_i \partial p_{i'}}\right)$$

[14] Holmes (1989) shows this for two similar firms, but this relationship is generally easy to check. See equation in footnote 13 above. Note that the special equation $\in_{cross}=\in+\in_{cross}$ simply means that the percentage of consumers who stop buying a company's product in response to its price increase decomposes to (1) those who no longer buy from any of the companies ($\in$) and (2) those who switch to (any of) products Other companies ($\in_{cross}$). Thus, the private $\in_{cross}$ measures the competitiveness of the firm, which is expressed in terms of industry flexibility and intensity of competition. In this sense, these three price elasticities characterize "first-order" competitiveness, which determines whether the equilibrium price is high or low, but one is not determined independently of the other two.

respectively, since the derivatives are evaluated again at $(\mathbf{p} = p, \ldots, p)$, i and i' are an arbitrary pair of distinct indices. These curvatures satisfy a value of $\alpha = (\alpha_{cross} + \alpha_{cross})\epsilon_{cross}/\epsilon$ and are related to the $\in_{cross}$ (p) ownership elasticity by $p \in_{cross}{}'$ $(p)/\in_{cross}$ (p) = 1+$\in$ $(p) - \alpha_{cross}(p)$ (see Online Appendix F.1 for a derivation and related discussion).

B 1.2. reverse order

We provide similar definitions for reverse order. Or not, We define the cross elasticity of quantity $\eta_{cross}(q)$ to inverse the firm request like

$$\eta_{cross}(q) \equiv (n-1)\left(\frac{q}{p(q)}\right).\left(\frac{\partial q_{i'}(\mathbf{q})}{\partial q_i}\right) |_{(\boldsymbol{q}=q,\ldots,q)}$$

In order to arbitrarily distinguish i and i0. It is verified that $\eta_{own} = \eta + \eta_{cross}$.[15] Furthermore, we define the bend of the company's reverse order $\sigma_{own}(q)$ and the cross bend R$\sigma_{own}(q)$ of the company's reverse order by:

$$\sigma_{own}(q) \equiv -q.\left(\frac{\partial p_i(\mathbf{q})}{\partial q_i}\right)^{-1}.\frac{\partial^2 p_i(\mathbf{q})}{\partial {q_i}^2} \;\; \boldsymbol{and} \;\; \sigma_{own}(q) \equiv -(n-1)q.\left(\frac{\partial p_i(\mathbf{q})}{\partial q_i}\right)^{-1}.\frac{\partial^2 p_i(\mathbf{q})}{\partial {q_i}^2}$$

respectively, since the derivatives are evaluated again at $(\boldsymbol{q} = q, \ldots, q)$ and the indices i and i' are separate. These bends represent the oligopolistic counterpart of the monopoly $\sigma(q)$ from Aguirre et al. (2010, p. 1603). It satisfies the relationship $\sigma = (\sigma_{own} + \sigma_{cross})(\eta_{own}/\eta)$ and correlates with the elasticity of $\eta_{own}(q)\ by\ q\eta_{own}{}'(q)/\eta_{own}(q) = 1 + \eta(q) - \sigma_{own}(q) - \sigma_{cross}(q)$ (see online Appendix F.2 for derivation and related discussion).

B.2. Scrolling expressions and behaviour index

B.2.1. price competition

In the case of price competition, the indicator of $\theta\ is\ \theta = \epsilon/\epsilon_{own} = 1/(\eta \in_{own})$, which is ascertained by comparing the state of the first-class firm with Eq. (1). The marginal change in maximum gain loss and incidence is obtained by replacing these expressions with those given in Propositions 3 and 4.

[15] The cloak of identity $\eta_{own} = \eta + \eta_{cross}$ means that as a response to an increase in firm's output, the industry as a whole reacts by lowering firm i's price(η). However, every firm (other than i) reacts to the increase in output of that firm by decreasing its output. This nullifies the initial change in price η_{cross} < 0), so the percentage reduction in firm price i η_{own} is smaller thang η, which does not take into account strategic interactions. Note here that $1/\eta\_own\ , not\ \eta\_cross$, measures the competitiveness of the industry. Thus, as in the case of price competition, these three quantity elasticities characterize "first-order" competitiveness, which determines whether the equilibrium quantity is high or low.

Propositions B.2.1. Under oligopoly symmetrical with price competition and at inconsistent marginal cost, the transit of unit tax and ad valorem tax passage are characterized by

$$\rho_t = \frac{1}{(1-v)} \cdot \frac{1}{1 + \left(\frac{(1-\alpha/\epsilon_{own})\epsilon}{\epsilon_{own}}\right) + \left\{\frac{1-\tau}{1-v} - \frac{1}{\epsilon_{own}}\right\}\epsilon\chi}$$

$$and \quad \rho_v = \frac{1}{(1-v)} \cdot \frac{1}{\frac{1}{1-\frac{1}{\epsilon_{own}}} + \left(\frac{(1-\alpha/\epsilon_{own})\epsilon}{\epsilon_{own}-1}\right) + \left\{\frac{1-\tau}{1-v}\cdot\frac{\epsilon_{own}}{\epsilon_{own}-1} - \frac{1}{\epsilon_{own}-1}\right\}\epsilon\chi}$$

Proof. Since if the price is determined $\theta = \epsilon/\epsilon_{own} = 1/(\eta\epsilon_{own})$, we have $(\eta+\chi)\theta = (1+\epsilon\chi)/\epsilon_{own}$ and $(\theta\eta)' \in q(d(\theta\eta)/dq = \in q(d \in_{own}^{-1})/dq = -\in_{own}^{-2} \in q(d\epsilon_{own})/dq = \in_{own}^{-2} p(d\epsilon_{own})/dq = (1+\in -\alpha\epsilon/\epsilon_{own})/\epsilon_{own}$

Equivalent to Eq ρ_t. of price setting $\theta = \epsilon/\epsilon_{own}$in relation with proposition 1 as $\rho_v = ((\in -\theta)\rho_t) \notin= \frac{(\epsilon_{own}-1)\rho_t}{\epsilon_{own}}$, leads to $\rho_t = \frac{1}{1-v} \frac{1}{\underbrace{[(1-\theta\eta)+(\theta\eta)'\epsilon q]}_{\text{revenue increase}} + \underbrace{\left[\frac{1-\tau}{1-v}\epsilon - \theta\right]\chi}_{\text{cost savings}}}$.

Then $= \epsilon/\epsilon_{own}$, $1-\theta\eta = 1 - 1/\epsilon_{own}$, $(\theta\eta)' \in q = (1+\in -\alpha\epsilon/\epsilon_{own})/\epsilon_{own}$ the above became

$$\rho_t = \frac{1}{1-v} \cdot \frac{1}{\underbrace{\left[\left(1-\frac{1}{\epsilon_{own}}\right) + \frac{1+\epsilon-\alpha\epsilon/\epsilon_{own}}{\epsilon_{own}}\right]}_{\text{revenue increase}} + \underbrace{\left[\frac{1-\tau}{1-v} - \frac{1}{\epsilon_{own}}\right]\epsilon\chi}_{\text{cost savings}}}$$

$$= \frac{1}{1-v} \cdot \frac{1}{\underbrace{\left[1 + \frac{(1-\alpha/\epsilon_{own})\epsilon}{\epsilon_{own}}\right]}_{\text{revenue increase}} + \underbrace{\left[\frac{1-\tau}{1-v} - \frac{1}{\epsilon_{own}}\right]\epsilon\chi}_{\text{cost savings}}}.$$

To make it easier to understand the correlation of this result with Proposition 2, consider the case of zero primary taxes $t = v = \tau = 0$. Then, Proposition 2 claims that

$$\rho_t = \frac{1}{1+\in \chi + \theta\chi + \left(-\frac{1}{\epsilon}\cdot\frac{\epsilon}{\epsilon_{own}} + \frac{1+(1-\alpha/\epsilon_{own})\epsilon}{\epsilon_{own}}\right)} = \frac{1}{1+\in \chi + \theta\chi + \left(1 - \frac{\alpha}{\epsilon_{own}}\right)\theta}$$

Because $\theta = \epsilon/\epsilon_{own}$. Here, the direct effect of $-\theta\eta$ cancels out by a part of the indirect effect of $\in q(\theta\eta)'$. The new term, which appears as the fourth term in the denominator, appears. How Industry Curvature Affects Passage: As the curvature of demand becomes larger (i.e., when industry demand becomes more convex), pass-through becomes higher, although this effect is mitigated by the intensity of competition, θ.

B.2.2. Quantitative competition

Then, in the case of quantitative competition, the behaviour index θ is given by $\theta = \eta_{own}/\eta$, which is again verified by comparing the state of the first-order firm with Eq. (1). Again, the marginal change in maximum gain loss and incidence is obtained by replacing these expressions with those given in Propositions 3 and 4.

Proposition B.2.2. Under oligopoly symmetric with quantitative competition and at inconsistent marginal cost, the transit of unit tax and ad valorem tax passage are characterized by

$$\rho_t = \frac{1}{(1-v)} \cdot \frac{1}{1 + \frac{\eta_{own}}{\eta} - \sigma + \left\{\frac{1-\tau}{1-v} - \eta_{own}\right\} \cdot \frac{\chi}{\eta}},$$

$$\rho_v = \frac{1}{(1-v)} \cdot \frac{(1-\eta_F)}{1 + \frac{\eta_{own}}{\eta} - \sigma + \left\{\frac{1-\tau}{1-v} - \eta_{own}\right\} \cdot \frac{\chi}{\eta}}$$

Proof. In the case of specifying the quantity, $\theta = \eta_{own}/\eta$, so $(\eta + \chi)\theta = (1 + \chi/\eta)\eta_{own}$and $(\theta\eta)' \in q = q(\eta_{own})'/\eta = (1 + \eta - \sigma\eta/\eta_{own})\eta_{own}$, where in the last equality we use the expression for the elasticity of the patency $\eta_{own}(q)$and $\sigma_{own} + \sigma_{own} = \sigma\eta/\eta_{own}$from subsection B.1.2 above. Substituting these into the expression for qt in proposal 2 gives

$$\rho_t = \frac{1}{(1-v)} \cdot \frac{1}{1 - \left(1 + \frac{\chi}{\eta}\right)\eta_{own} + \frac{1}{\eta}\left(1 + \eta - \frac{\sigma\eta}{\eta_{own}} + \left\{\frac{1-\tau}{1-v} \cdot \frac{1}{\eta}\right\} \cdot \chi\right.},$$

Which is equivalent to the expression ρ_t in motion. Since $\theta = \eta/\eta_{own}$, Proposition 1 refers to $\rho_v == (\in - \theta)\rho_t/\epsilon = (1/\eta - \eta_{own}/\eta)\rho_t\eta = (1 - \eta_{own})\rho_t$, which can be used to verify the expression of ρ_v. This proposal is similar to Proposition B.2.1 above. I mention it again. This is similar to Proposition B.2.1. Recall again that

$$\rho_t = \frac{1}{1-v} \cdot \frac{1}{\underbrace{[(1-\theta\eta) + (\theta\eta)'\epsilon q]}_{\text{revenue increase}} + \underbrace{\left[\frac{1-\tau}{1-v}\epsilon - \theta\right]\chi}_{\text{cost savings}}}.$$

Then $\theta = \eta_{own}/\eta$ implies $(1/\in_S - \eta)\theta = \{(1/\in_S \eta) - 1\}\eta_{own}$ and $(\theta\eta)'(q/\eta) = q(\eta_{own})'/\eta = (1 + \eta - \sigma_{own} - \sigma_{cross})(\eta_{own}/\eta)$. the above equality become

$$\cdot \frac{1}{\underbrace{\left[(1-\eta_{own}) + \frac{1+\eta-\sigma\eta/\eta_{own}}{\eta}\eta_{own}\right]}_{\text{revenue increase}} + \underbrace{\left[\frac{1-\tau}{1-v} \cdot \frac{1}{\epsilon_S\eta} - \frac{\eta_{own}}{\epsilon_S\eta}\right]}_{\text{cost savings}}}$$

$$\rho_t = \frac{1}{1-v} \qquad = \frac{1}{1-v} \cdot \frac{1}{\underbrace{\left[1 + \frac{\eta_{own} - \sigma\eta}{\eta}\right]}_{\text{revenue increase}} + \underbrace{\left[\frac{1-\tau}{1-v} - \eta_F\right] \frac{1}{\epsilon_S \eta}}_{\text{cost savings}}}.$$

To make it easier to understand the correlation of this result with Proposition 2, consider the case of zero primary taxes (t = v = s = 0) again. Then Proposition B.2.2 makes it clear

$$\rho_t = \frac{1}{1 + \epsilon\chi - \theta\chi + (-\eta \,.\frac{\eta_{own}}{\eta} + \left\{1 + \frac{1}{\eta} - \frac{\sigma}{\eta_{own}}\right\}.\,\eta_{own}} = \frac{1}{1 + \epsilon\chi - \theta\chi + \left(1 - \frac{\sigma}{\theta}\right)\theta}$$

Because $\theta = \eta_{own}/\eta$. Here, the term $(1 - \sigma/\theta)\theta$ shows the effects of industry reverse demand curvature, 333, on the traverse: When the curvature of industry reverse demand becomes larger (that is, when industry reverse demand curvature becomes more convex), the pass-through becomes higher. Interestingly, unlike in the case of price competition, this effect does not dampen competition θ.

Appendix C. Multidimensional scroll frame under

Firm heterogeneity as shown below, it turns out that it is useful to consider a general version of multidimensional interventions because specific and ad valorem taxes can be considered as a special case of a two-dimensional intervention. The main concept is multidimensional scrolling, which is defined as the effect of infinite changes in the $\mathbf{T} \equiv (T_1, \dots, T_d)$ interventions - the d dimensional vector of tax instruments - on the equilibrium price of $\rho_i$for company $i = 1, .., n$. Multidimensional scrolling corresponds to a matrix in the case of heterogeneous companies, which can be simplified as a vector under Symmetric oligopoly. We argue that multidimensional pass-through is an important determinant of the welfare effects of various types of government intervention and external changes, and is not limited to two-dimensional taxation.

C 1. Price sensitivity and quantitative sensitivity to tax

Consider a tax structure under which company i pay taxes is expressed as $\phi_i(p_i, q_i, \mathbf{T})$, so that the profit of the company is written as $\pi_1 = p_i q_i - c_i(q_i) - \phi_i(p_i, q_i, \mathbf{T})$ (To be precise, $\phi_i(p_i, q_i, \mathbf{T})$ represents a simplified notation for a function $\phi_i(p, q,\ T_1, .., T_d)$with d = 2 arguments). Note that the cost of production and, therefore, $mc_i(q_i)$marginal cost of company i is also allowed depending on the identity of a company, and we indicate its elasticity of $\chi_i(q_i) \equiv mc_i'(q_i) q_i / mc_i(q_i)$. In The special case of a unit tax t and a value tax v of $\phi_i(p_i, q_i, \mathbf{T}) = t q_i + v p_i q_i$, where $\mathbf{T} = (t, v)$. Below, we discuss how to generalize our previous framework with two policy tools by defining the unit tax of t and ad valorem tax v even for general interventions that may involve multiple tools, not just two.

We aim to express the decomposition of $\phi_i(p_i, q_i, \mathbf{T})$, is similar to $\phi_i(p_i, q_i, \mathbf{T}) = t q_i + v p_i q_i$,. Specifically, we argue that it is possible to write $\phi_i(p_i, q_i, \mathbf{T}) = \bar{t} q_i + \bar{v} p_i q_i$, where $\bar{t}$ and $\bar{v}$ are the averages of appropriately defined functions t and v across the ranges; $(0, q_i p_i)$ and $(0, q_i)$. In the special case of specific and value taxes, these functions must be reduced to the constants t and v. We check this property by parsing $\phi_i(p_i, q_i, \mathbf{T})$ into micro contributions, each of which is similar to specific and ad valorem taxes, respectively. If we set the tax burden at zero quantities and prices: $\phi_i(0,0, \mathbf{T}) = 0$, we can write the desired relationship $\phi_i(p_i, q_i, \mathbf{T}) = \bar{t} q_i + \bar{v} p_i q_i \ as\ \phi_i(p_i, q_i, \mathbf{T}) = \int_{\mathbf{0}}^{q_i} t(\tilde{p}, \tilde{q},\ \mathbf{T}) d\tilde{q} + \int_{\mathbf{0}}^{q_i p_i} v(\tilde{p}, \tilde{q},\ \mathbf{T}) d\tilde{q}\tilde{p}$, or alternatively

$$\phi_i(p_i, q_i, \mathbf{T}) = \int_{\mathbf{0}}^{\mathbf{1}} \left[\left(\frac{t(\tilde{p}_i(s), \tilde{q}_i(s), \mathbf{T})}{\tilde{p}_i} + v(\tilde{p}_i(s), \tilde{q}_i(s), \mathbf{T}) \right) \tilde{p}_i \frac{d\tilde{q}_i}{ds} + v(\tilde{p}_i(s), \tilde{q}_i(s), \mathbf{T})\ \tilde{q}_i \frac{d\tilde{p}_i}{ds} \right] ds$$

where the integral is on additional parameter s that define the path parameters $\left(\tilde{p}_i(s), \tilde{q}_i(s)\right)$ at the price quantity level as $\left(\tilde{p}_i(0), \tilde{q}_i(0)\right) = (0{,}0)$ and $\left(\tilde{p}_i(1), \tilde{q}_i(1)\right) = (p_i, q_i)$. At the same time, $\phi_i(p_i, q_i, \mathbf{T})$ can be expressed as an integral part of its total difference:

$$\phi_i(p_i, q_i, \mathbf{T}) = \int_{\mathbf{0}}^{\mathbf{1}} \left[\phi_{\tilde{q}_i}(\tilde{p}_i(s), \tilde{q}_i(s), \mathbf{T}) \frac{d\tilde{q}_i}{ds} + \phi_{\tilde{p}_i}(\tilde{p}_i(s), \tilde{q}_i(s), \mathbf{T}) \frac{d\tilde{p}_i}{ds}\right] ds$$

Where low notation is used for partial derivatives. We notice that if we specify

$$\begin{cases} \left(\frac{t\left(\tilde{p}_i(s), \tilde{q}_i(s), \mathbf{T}\right)}{\tilde{p}_i} + v\left(\tilde{p}_i(s), \tilde{q}_i(s), \mathbf{T}\right)\right)\tilde{p}_i = \phi_{\tilde{q}_i}\left(\tilde{p}_i(s), \tilde{q}_i(s), \mathbf{T}\right) \\ v\left(\tilde{p}_i(s), \tilde{q}_i(s), \mathbf{T}\right)\tilde{q}_i = \phi_{\tilde{p}_i}\left(\tilde{p}_i(s), \tilde{q}_i(s), \mathbf{T}\right), \end{cases}$$

then the desired relation $\phi_i(p_i, q_i, \mathbf{T}) = \tilde{t}q_i + \tilde{v}p_iq_i$ is satisfied. Now, we determine the (first-order) price sensitivity of (per company) tax revenue $v_i(p_i, q_i, \mathbf{T}) = \frac{1}{q_i} \cdot \frac{\partial}{\partial p_i} \ \phi_i(p_i, q_i, \mathbf{T})$ and (first-order) sensitive quantity $\tau_i(p_i, q_i, \mathbf{T}) = \frac{1}{q_i} \cdot \frac{\partial}{\partial p_i} \ \phi_i(p_i, q_i, \mathbf{T})$, so $t_i(p_i, q_i, \mathbf{T}) = \tau_i(p_i, q_i, \mathbf{T}) + v_i(p_i, q_i, \mathbf{T})$. Both first and second orders are dimensionless the sensitivity.

C 2. Pricing Strength Index

We now introduce firm i's pricing strength index as a function of $\phi_i(\mathbf{q})$ but independent of the cost side $(\mathbf{q})$ so that firm i's first demand condition is:

$$mc_i(q_i) = \{1 - \tau_i(p_i\ (\mathbf{q}), \mathbf{T}) - \psi_i(\mathbf{q})[1 - v_i(p_i(\mathbf{q}), q_i, \mathbf{T})]\}p_i(\mathbf{q}) \qquad \textbf{(12)}$$

In the special case of identical firms, this pricing power index is expressed by $\psi_i = \eta, \theta$ for all i. Because of this simplicity, oligopolistic analysis in terms of pricing power index is not different from its analysis in terms of behavior index. However, these two approaches may differ for heterogeneous firms. One of the innovations of this paper is to present an oligopoly analysis in terms of the pricing power index. Note here that in the case of specific taxes and value It is verified $\tau_i(p_i, q_i, \mathbf{T}) \equiv \left(\frac{1}{p_i}\right)\left(\frac{\partial \phi_i}{\partial q_i}\right)(p_i, q_i, \mathbf{T}) = \frac{\mathrm{t}}{p_i} + v \ \ and \ v_i(p_i, q_i, \mathbf{T}) \equiv \left(\frac{1}{q_i}\right)\left(\frac{\partial \phi_i}{\partial p_i}\right)(p_i, q_i, \mathbf{T}) = v$ therefore Eq 12 be:

$$mc_i(q_i) = \left\{1 - \frac{1}{(p_i\ (\mathbf{q}))} - \psi_i(\mathbf{q})(\mathbf{1} - \boldsymbol{v})\right\} p_i(\mathbf{q})$$

As in the main text appearanced.

C.3. Pass-through

We express the traffic rate matrix in terms of these rates strength indicators. Specifically, the pass rate is the *n* x *d* matrix of which $\tilde{\boldsymbol{\rho}}$; The (i, T_ℓ) element is $\tilde{\rho}_{iT_\ell} = \partial p_i/\partial T_\ell$. First, we define the following functions:

$$\kappa_i(p_i, q_i, \mathbf{T}) = \frac{\partial^2 \phi_i(p_i, q_i, \mathbf{T})}{\partial p_i \partial q_i} \quad , \qquad v_{(2)i}(p_i, q_i, \mathbf{T}) = \frac{p_i}{q_i} . \frac{\partial^2 \phi_i(p_i, q_i, \mathbf{T})}{\partial p_i^2}$$

$$\tau_{(2)i}(p_i, q_i, \mathbf{T}) = \frac{q_i}{p_i} . \frac{\partial^2 \phi_i(p_i, q_i, \mathbf{T})}{\partial q_i^2} \quad , \quad \in_{ij} = \frac{p_i}{q_i} . \frac{\partial q_i(\mathbf{p})}{\partial p_i} \; , \qquad \mathbf{\Psi}_{ij} = \frac{p_i}{\psi_i} . \frac{\partial \psi_i(\mathbf{q}(\mathbf{p}))}{\partial p_j}$$

Proposition C.3. Pass-Through rate is

$$\underbrace{\tilde{\boldsymbol{\rho}}_{T_\ell}}_{n\times 1} = \underbrace{\mathbf{b}^{-1}}_{n\times n} \underbrace{l_{T_\ell}}_{n\times 1} , \tag{13}$$

b is a matrix n x n, independent of T chose, with element (I , j)

$$b_{ij} = \left[1 - \kappa_i - \left(1 - v_i - v_{(2)i}\right)\psi_i\right]\delta_{ij} - (1 - v_i)\psi_i \mathbf{\Psi}_{ij} + \{\tau_{(2)i} + (v_i - \kappa_i)\psi_i + [1 - \tau_i - (1 - v_i)\psi_i]\chi_{\boldsymbol{i}}\} \in_{ij}$$

δ_{ij} is Kronecker delta , each tax T_ℓ , l_{T_ℓ} is dimensional vector include i-th element therefore:

$$l_{iT_\ell} \equiv p_i . \left(\frac{\partial \tau_i(p_i, q_i, \mathbf{T})}{\partial T_\ell} - \psi_i \frac{\partial v_i(p_i, q_i, \mathbf{T})}{\partial T_\ell} \right)$$

Proof :. Eq. 12

$$\left[p_i . \left(\frac{\partial \tau_i}{\partial p_i} - \psi_i \frac{\partial v_i}{\partial p_i} \right) + (1 - v_i)\psi_i - (1 - \tau_i)\right] dp_i + \left[p_i . \left(\frac{\partial \tau_i}{\partial q_i} - \psi_i \frac{\partial v_i}{\partial q_i} \right) + mc_i'\right] dq_i + p_i . \left(\frac{\partial \tau_i}{\partial \mathrm{T}} - \psi_i \frac{\partial v_i}{\partial \mathrm{T}} \right) d\mathrm{T} + p_i(1 - v_i)d\psi_i = \mathbf{0}$$

$$l_{iT_\ell} dT_\ell = \left[1 - \kappa_i - \left(1 - v_i - v_{(2)i}\right)\psi_i\right]dp_i - (1 - v_i)\psi_i \left(\sum_{\boldsymbol{j=1}}^{\boldsymbol{n}} \mathbf{\Psi}_{ij} dp_j \right) + \{\tau_{(2)i} + (v_i - \kappa_i)\psi_i + [1 - \tau_i - (1 - v_i)\psi_i]\chi_{\boldsymbol{i}}\} \left(\sum_{\boldsymbol{j=1}}^{\boldsymbol{n}} \in_{ij} dp_j \right)$$

As $dp_j = -\left(\frac{q_i}{p_i}\right) \sum_{j=1}^{n} \in_{ij} dp_j$, $d\psi_i = (\psi_i/p_i) \sum_{\boldsymbol{j=1}}^{\boldsymbol{n}} \mathbf{\Psi}_{ij} dp_j$ and $mc_i' = \chi_{\boldsymbol{i}} mc_i/q_i = (p_i/q_i)[1 - \tau_i - (1 - v_i)\psi_i]\chi_{\boldsymbol{i}}$ are used. (to be note $\frac{\partial \tau_i}{\partial q_i} = \frac{\tau_{(2)i}}{q_i}, \frac{\partial v_i}{\partial q_i} = (\kappa_i - v_i)/q_i$, $\frac{\partial \tau_i}{\partial p_i} = (\kappa_i - \tau_i)/p_i$, $\frac{\partial \tau_i}{\partial q_i} = \tau_{(2)i}/q_i$ can be used also).

$$\underbrace{l_{T_\ell}}_{n\times 1} = \underbrace{\begin{pmatrix} 1-\kappa_1-(1-\nu_1-\nu_{(2),1})\psi_1 \\ \vdots \\ 1-\kappa_j-(1-\nu_j-\nu_{(2),j})\psi_j \\ \vdots \\ 1-\kappa_n-(1-\nu_n-\nu_{(2),n})\psi_n \end{pmatrix}}_{n\times 1} \circ \underbrace{\tilde{\boldsymbol{\rho}}_{T_\ell}}_{n\times 1} - \underbrace{\begin{pmatrix} \ddots & & \\ & (1-\nu_i)\psi_i \Psi_{ij} & \\ & & \ddots \end{pmatrix}}_{n\times n} \cdot \boldsymbol{\rho}^{\sim}{}_{T_\ell}$$

$$+ \underbrace{\begin{pmatrix} \ddots & & \\ & \{\tau_{(2),i}+(\nu_i-\kappa_i)\psi_i+[1-\tau_i-(1-\nu_i)\psi_i]\chi_i\}\epsilon_{i,j} & \\ & & \ddots \end{pmatrix}}_{n\times n} \cdot \boldsymbol{\rho}^{\sim}{}_{T_\ell},$$

Assuming **b** is invertible, therefore Eq. 13 holds. On 2 dimensional taxation, it's $v_{(2)i}(p_i, q_i, \mathbf{T}) = 0$, $\tau_{(2)i}(p_i, q_i, \mathbf{T})$=0, and $\kappa_i(p_i, q_i, \mathbf{T}) = v$, also $l_{it} = 1$ and $l_{iv} = p_i$ because $\tau_i/\partial t = 1/p_i$, $\partial v_i/\partial t = 0$, $\partial \tau_i/dv = 1$ and $\partial v_i/\partial v = 1$.

C 4. Welfare changes

So far, we have introduced $\phi_i(p_i, q_i, \mathbf{T})$ as an additional cost in the firm's profit function: $\pi_i = p_i q_i - c_i(q_i) - \phi_i(p_i, q_i, \mathbf{T})$. Here $\mathbf{T}$ is a vector of interventions (in governmental and other external conditions), which may or may not include conventional taxes. To assess welfare changes, we also need to know what part of this cost is being charged by the government in the form of taxes. We now provide code $\hat{\phi}_i(p_i, q_i, \mathbf{T})$ to pay taxes to the company:

In the main text, this corresponds to R_i. The difference $\phi_i(p_i, q_i, \mathbf{T}) - \hat{\phi}_i(p_i, q_i, \mathbf{T})$ corresponds to the additional non-tax costs faced by the company. In the case of pure taxation, $\hat{\phi}_i(p_i, q_i, \mathbf{T}) = \phi_i(p_i, q_i, \mathbf{T})$. (If the additional firm costs come from the production, we have $\phi_i(p_i, q_i, \mathbf{T}) = 0$ (that is, tax payment equals zero). Then we define for each company i

$$\hat{v}_i(p_i, q_i, \mathbf{T}) = \frac{1}{q_i}\left(\frac{\partial}{\partial p_i}\right)\hat{\phi}_i(p_i, q_i, \mathbf{T}) \;\; and \;\; \hat{\tau}_i(p_i, q_i, \mathbf{T}) = \frac{1}{p_i}\left(\frac{\partial}{\partial q_i}\right)\hat{\phi}_i(p_i, q_i, \mathbf{T})$$

Also we write $\mathbf{f}_i = \frac{1}{q_i}\nabla\phi_i(p_i, q_i, \mathbf{T})$, where $\nabla\phi_i$ components are $\phi_{iT_\ell}(p_i, q_i, \mathbf{T}) = \partial\phi_i(p_i, q_i, \mathbf{T})\partial T_\ell$ and $\hat{\mathbf{f}}_i = \frac{1}{q_i}\nabla\hat{\phi}_i(p_i, q_i, \mathbf{T})$ It is also defined as .

Let's say $\in_{ij}$ is an n-dimensional class vector with its j-th component equal to $\in_{ij}$ every $\in_i = (\in_{i1}, \ldots, \in_{ij}, \ldots, \in_{in})$. For convenience we also define $\mathbf{e}_i$ to be an n-dimensional pointer vector with the i-component equal to 1 and the other components being zero: $\mathbf{e}_i = (0, .., 1, ..0)$. Then , the following proposition is obtained.

Proposition C.4.1. The intervention gradients of consumer surplus, producer surplus, tax revenue, and social welfare with respect to taxes are

$$\left(\frac{1}{q_i}\right)\nabla CS_i = \mathbf{e}_i\tilde{\boldsymbol{\rho}}\,, \left(\frac{1}{q_i}\right)\nabla CS_i = (1-v_i)(\mathbf{e}_i - \psi_i\epsilon_i)\tilde{\boldsymbol{\rho}} - \mathbf{f}_i\,, \left(\frac{1}{q_i}\right)\nabla R_i = (\hat{v}_i\mathbf{e}_i - \boldsymbol{\tau}_i\epsilon_i)\tilde{\boldsymbol{\rho}} - \hat{\mathbf{f}}_i$$

$$\left(\frac{1}{q_i}\right)\nabla W_i = -[\hat{\tau}_i + \psi_i(\mathbf{1}-v_i)]\epsilon_i\tilde{\boldsymbol{\rho}} + (\hat{v}_i - v_i)\mathbf{e}_i\tilde{\boldsymbol{\rho}} + \hat{\mathbf{f}}_i - \mathbf{f}_i \text{ respectivly}$$

Proof. The outcome of $(1/q_i)\nabla CS_i$ is straightforward. It is sufficient to provide expressions for $(1/q_i)\nabla PS_i$ and $(1/q_i)\nabla R_i$ because $(1/q_i)\nabla W_i$ is equal to the sum of the other three expressions. Note first that in response to change $T_\ell \to T_\ell + dT_\ell$, we have $dPS_i = d(p_iq_i - c_i(q_i) - (\phi_i(p_i, q_i, \mathbf{T})$ and $d\hat{\phi}_i(p_i, q_i, \mathbf{T}) = p_i\hat{\tau}_i(p_i, q_i, \mathbf{T})dq_i + q_i\hat{v}_i(p_i, q_i, \mathbf{T})dp_i + (\partial\hat{\phi}_i/\partial T_\ell)$. Then using Eq. (12), can be rewritten:

$$dPS_i = [-(\psi_i v_i - \tau_i - \psi_i + 1)p_i - p_i\tau_i + p_i]dq_i + (q_i - v_iq_i)dq_i - (\partial\hat{\phi}_i/\partial T_\ell)dT_\ell.$$

$$= (1-v_i)\left[\left(p_i\psi_i.\sum_{j=1}^{n}\frac{\partial q_i}{\partial p_i}dp_j\right) + q_idp_i\right].\left(\frac{\partial\hat{\phi}_i}{\partial T_\ell}\right)dT_\ell$$

$$= (1-v_i)\left[\left(p_i\psi_i.\sum_{j=1}^{n}\frac{\partial q_i}{\partial p_i}\tilde{\rho}_j\,T_\ell\right) + q_i\tilde{\rho}_jT_\ell\right]dT_\ell - \left(\frac{\partial\hat{\phi}_i}{\partial T_\ell}\right)dT_\ell$$

$$= (1-v_i)q_i\left[\left(\psi_i.\sum_{j=1}^{n}\frac{p_i\partial q_i}{q_i\partial p_i}\tilde{\rho}_j\,T_\ell\right) + \tilde{\rho}_jT_\ell\right]dT_\ell - \left(\frac{\partial\hat{\phi}_i}{\partial T_\ell}\right)dT_\ell$$

which mark: $$= 1/q_i\nabla PS_i\left[\tilde{\rho}_jT_\ell - \left(\psi_i.\sum_{j=1}^{n}\epsilon_{ij}\,\tilde{\rho}_{jT_\ell}\right)\right] - \mathbf{f}_i = (1-v_i)(\mathbf{e}_i - \psi_i\boldsymbol{\epsilon}_i)\tilde{\rho} - \mathbf{f}_i$$

first we note $dR_i = \hat{v}_iq_idp_i + \hat{\tau}_ip_idq_i + (\partial\hat{\phi}_i/\,\partial T_\ell)dT_\ell$ where $\hat{v}_i = (1/q_i)\hat{\phi}_{ip_i}$ and $\hat{\tau}_i = (1/p_i)\hat{\phi}_{iq_i}$ are used. By using $dq_i = \sum_{j=1}^{n}\frac{\partial q_i}{dq_i\partial p_i}$ and $\tilde{\rho}_{jT_\ell} = \partial p_i/\partial T_\ell$ then proceed:

$$\frac{dR_i}{dT_\ell} = \hat{v}_iq_i\,\tilde{\rho}_{jT_\ell} + \hat{\tau}_ip_i\,.\left[\left(\sum_{j=1}^{n}\frac{\partial q_i}{\partial p_j}\tilde{\rho}_{jT_\ell}\right)\right] + \left(\frac{\partial\hat{\phi}_i}{\partial T_\ell}\right)$$

$$= \hat{v}_iq_i\,\tilde{\rho}_{jT_\ell} + \hat{\tau}_ip_i\,.\left[\left(\sum_{j=1}^{n}\frac{q_i}{p_j}\in_{ij}\,\tilde{\rho}_{jT_\ell}\right)\right] + \left(\frac{\partial\hat{\phi}_i}{\partial T_\ell}\right)$$

$$= \hat{v}_i q_i \, \tilde{\rho}_{jT_\ell} + \hat{\tau}_i q_i \, . \left[\left(\sum_{j=1}^{n} \in_{ij} \, \tilde{\rho}_{jT_\ell}\right)\right] + \left(\frac{\partial \hat{\phi}_i}{\partial T_\ell}\right)$$

Which shows that $(1/q_i)\nabla R_i = (\hat{v}_i \mathbf{e}_i - \hat{\tau}_i \in_i)\tilde{\boldsymbol{\rho}} + \hat{\mathbf{f}}_i$ complement proof. Now, we define the matrix $\rho \, as \, an \, n \, \mathrm{x} \, d$ the semi-elastic matrix passed by the elements:$\rho_{jT_\ell} = \tilde{\rho}_{iT_\ell}/f_{iT_\ell}(p_i, q_i, \mathbf{T})$, and with the indicated rows $\tilde{\boldsymbol{\rho}}$ we also define, for each i; 555. Next, for the firm's welfare change ratios, we get the following proposition of using Proposition C.4.1 results .

Proposition C.4.2. Let us $\in_{jT_\ell}^{\rho} = \in_{\boldsymbol{i}} \tilde{\boldsymbol{\rho}}_{T_\ell}/\tilde{\rho}_{iT_\ell} = \in_{\boldsymbol{i}} \tilde{\boldsymbol{\rho}}_{T_\ell}/\tilde{\rho}_{iT_\ell}$. So, the marginal value of public money associated with the intervention $T_\ell, MVPF_{iT_\ell} = [(\nabla CS_i)_{T_\ell} + (\nabla PS_i)_{T_\ell}]/(-\nabla R_i)_{T_\ell}$, is characterized by:

$$MVPF_{iT_\ell} = \frac{\left[\dfrac{\dfrac{1}{\rho_{iT_\ell}} + v_i}{\in_{iT_\ell}^{\rho}} + (1 - v_i)\left(\dfrac{\psi_i \in_{iT_\ell}^{\rho}}{\in_{iT_\ell}^{\rho}}\right)\right]}{\dfrac{\left\{\left(\dfrac{g_{iT_\ell}}{\rho_{iT_\ell}}\right) + \hat{v}_i\right\}}{\in_{iT_\ell}^{\rho}} - \hat{\tau}_i}$$

Incidence for this intervention $I_{iT_\ell} = (\nabla CS_i)_{T_\ell}/(\nabla PS_i)_{T_\ell}$ characterised by:

$$I_{iT_\ell} = \frac{1}{\left[\left(\dfrac{1}{\rho_{iT_\ell}}\right) - (1 - v_i)\left(1 - \psi_i \in_{iT_\ell}^{\rho}\right)\right]}$$

Appendix D. Behaviour and Well-Being Changes Index

For heterogeneous firms, we can also consider the company behaviour index, rather than the pricing strength firm i, so that

$$\theta_i = -\frac{\sum_{j=1}^{n}[p_j\{1 - \tau_j(p_i, q_i, \mathbf{T})\} - mc_j(q_j)]\,(dq_j/d\sigma_i)}{\sum_{j=1}^{n}\{1 - v_j(p_i, q_i, \mathbf{T})\}q_i\left(\dfrac{dq_j}{d\sigma_i}\right)}$$

We carries. In the special case of having a unit tax only, this definition is abbreviated to Weyl and Fabinger's (2013, p. 552) Eq. (4). In the special case of symmetric firms, the definition reduces $[1 - \tau - (1 - v)\eta\theta]p = mc$ by $\theta_i = 0$. Behaviour index θ_i is closely related to the score strength index ψ_i, but not as closely as in the case of symmetric oligopolists. Using the definitions of indicators, it is clear that

$$\theta_i = -\frac{\sum_{j=1}^{n}\{1 - v_j\}\left(\psi_i \boldsymbol{p}_j\right)]\ (dq_j/d\sigma_i)}{\sum_{j=1}^{n}\{1 - v_j\} q_i \left(\frac{dq_j}{d\sigma_i}\right)}$$

For symmetric oligopoly, this equation simply drops to $\theta_i = \in \psi_i$. The behavior index is used to express changes in the welfare component in response to infinitesimal tax changes. The relationships are a bit more complicated than when using the Pricing Strength Index instead. To find out, we define the price response to an infinitesimal change in the strategic variable rj of firm j by $\xi_{ij} = dq_j/d\sigma_i$. Since vectors $\xi_{i1}, \xi_{i2}, \dots, \xi_{in}$ form a basis in the vector space n to which $\tilde{\rho}_{iT_\ell}$ belongs against a given ℓ, we can write $\tilde{\rho}_{iT_\ell}$ as a linear set of them for some coefficients λ_{iT_ℓ}: $\tilde{\rho}_{iT_\ell} = \sum_{j=1}^{n} \lambda_{iT_\ell}\, \xi_{ij}$. For changes in surplus Consumer and producer, we get:

$$\frac{\partial CS_i}{\partial T_\ell} = -\sum_{i=1}^{n} q_i\, \tilde{\rho}_{iT_\ell} = \sum_{j=1}^{n}\left(\sum_{j=1}^{n} q_i\, \xi_{ij}\right)\lambda_{iT_\ell}$$

$$\frac{\partial CS_i}{\partial T_\ell} = -\sum_{i=1}^{n} f_{iT_\ell}\,(p_i, q_i, \mathbf{T}) - \sum_{i=1}^{n} \xi_{ij}\ \left(1 - \theta_j\right)\lambda_{iT_\ell}$$

where we use coding $\xi_j \equiv \sum_{i=1}^{n}\{1 - v_i\ (p_i, q_i, \mathbf{T})\} q_i\, \xi_{ij}$. These redundant change expressions are a generalization of the redundant expressions in Weyl and Fabinger's (2013) section 5. Note, however, that the results in the previous subsections are significantly more straightforward and applicable than those in this subsection.

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