Capital-labor substitution and misallocation

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Abstract

Although the role of the elasticity of substitution between capital and labor (σ) has been emphasized in many areas in macroeconomics, it has been neglected in the misallocation literature. We explore the role of σ in misallocation of resources both analytically and empirically using cross-country firm level survey data. We document that extent of misallocation and aggregate output gain from reallocation of resources are substantially large for low σ (<1) compared to the Cobb-Douglas value of 1 that the extant literature invariably assumes. When σ is low, dispersion of capital-labor ratios generate larger misallocation because marginal product of capital now declines rapidly with increasing capital-labor ratio. Given the overwhelming evidence that σ<1, our findings raise serious concerns about using the Cobb-Douglas production function in the misallocation literature. This is crucial from policy perspectives because σ is influenced by institutional and policy features of a country.

Keywords: Misallocation, Allocative distortions; Elasticity of substitution; CES production function; Total factor productivity
JEL Codes: D24; O11; O14; O47

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1. Introduction

There is a burgeoning literature in development macroeconomics that aims to evaluate the extent of misallocation of resources using micro-level data. This line of research is extremely important to understand the causes of vast differences in income per capita across countries. The difference in income per capita is explained, to a great extent, by the difference in the aggregate total factor productivity (TFP), which depends on TFP at the individual production unit level and allocation of resources across these production units. Misallocation occurs when, because of distortions, resources do not flow to the production units to take advantage of highest marginal returns. Therefore, the aggregate output is lower than the potential output that would have been if marginal returns were equalized across production units.

The extent of distortions in factor markets and the resulting misallocation crucially depends on the specification of the production function. The empirical literature, initiated by the seminal work of Hsieh and Klenow (2009),[1] invariably employs the Cobb-Douglas production function that is characterized by a unitary elasticity of substitution between capital and labor (σ).[2] Although this specification is very convenient for empirical exercise, it imposes an unrealistic restriction on σ, a key parameter of the production function. It is now well accepted that the value of σ differs from unity, more specifically it is much less than 1 (see, Chirinko, 2008 for a survey; Knoblach, Roessler and Zwerschke, 2020 for a meta-analysis).

Elasticity of substitution refers to the ease with which capital can be substituted for labor when their relative price changes. To understand the role of σ in misallocation, first consider the case of the Cobb-Douglas production function. Under the restriction that $\sigma=1$, the ratio of marginal products of capital and labor can be expressed in terms of the capital-labor ratio, which can also be interpreted as *distortions in the capital market* (*relative to the labor market*). If resources are efficiently allocated, the ratio of the marginal products would be the same across all production units and so are their capital-labor ratios. In the absence of efficient allocation, there will be dispersion of capital-labor ratios, which we refer to as *misallocation of capital relative to labor* (MoC). The larger the dispersion of capital-labor ratios, the higher is the MoC. However, if $\sigma\neq1$, distortions in the capital market and MoC also depend on σ. The

[1] Restuccia and Richard (2013) and Hopenhayn (2014) provide nice reviews on the misallocation literature. See Restuccia and Rogerson (2017) for a lucid discussion.

[2] To the best of our knowledge, Whited and Zhao (2021) is the only study that uses the CES production function in calculating misallocation of financial assets in which the real benefit of finance is defined as a CES aggregate of debt and equity.

higher is σ, the greater the similarity between capital and labor; thus, the incremental capital is easily substituted for labor. Consequently, the capital-labor ratio does not substantially increase from the technological point of view, which in turn resists the pull of diminishing returns to capital (Brown, 1968, p. 50). Therefore, heterogeneous production units differing only by their capital-labor ratios will generate smaller (larger) dispersion of marginal products when σ is high (low), and consequently misallocation will be lower (higher).

Extending the above argument, suppose that two otherwise similar firms have differential access to credit (in terms of amount of loan) due to credit market imperfections. The firm with preferential access to credit will have lower marginal product of capital (MPK). However, MPK differential will be larger (lower) between the two firms when σ is low (high) because MPK will decline rapidly (slowly) for the firm receiving preferential credit. Aggregate output (efficiency) gain from reallocation of capital between the two firms will also be larger (lower) for the same reason. If these two firms also differ by their values of σ, then MPK differential will depend not only on their respective values of σ but also on which firm (with higher or lower σ) receives the preferential credit.

Given the importance of σ, this paper revisits the empirical literature on misallocation by introducing the CES production function to allow σ to depart from 1. Specifically, we extend the canonical empirical framework of misallocation accounting (e.g., Hsieh and Klenow, 2009) using the CES production function. The total factor productivity revenue (TFPR) is derived from the firm's optimization problem in terms of *distortions* as a CES aggregate of capital and output wedges, which is also a CES aggregate of marginal revenue products of capital and labor. We refer to dispersion of these allocative distortions as *misallocation of resources* (MoR). We also derive expressions for efficient aggregate productivity and potential aggregate output gain from reallocation of resources. To place our findings in the context of the extant literature, we evaluate the extent of allocative distortions, misallocation and the aggregate output gain for different values of σ relative to the Cobb-Douglas value of 1.

In our baseline framework, we assume the same σ for all firms (homogenous σ). To allow the possibility that capital accumulation by a firm also depends on its σ, we extend the framework to vary σ across firms (heterogeneous σ_i). Misallocation now additionally depends on the dispersion of σ_i's across firms, and the covariance between capital wedges and σ_i's.

Our empirical exercise is based on the World Bank Enterprise Survey (WBES) data. This is a standardized firm level survey of formal businesses in a large number of countries since 2005. The sample firms are in the manufacturing sector classified as ISIC2 codes 15-37 (see Section 3 for construction of our working data). We retain only those countries with at least 30 firms, giving a total of 153 countries. In the case of heterogeneous σ_i's, we vary σ across industry sub-categories (2-digit ISIC codes) but assume the same σ for all firms within an industry sub-category (σ_{ind}). We use industry σ_{ind}'s for the USA estimated by Chirinko and Mallick (2017) for all sample countries.

We find that both distortions (logarithm of TFPR) and aggregate output gain from reallocation of resources are decreasing with the value of σ. For example, relative to σ=1, distortions and output gain are 21 and 28 percent larger, respectively, when σ=0.5, while these are 8 and 6 percent lower, respectively, when σ=1.5. MoR (dispersion of distortions) declines with σ rapidly when σ is low but does not meaningfully depart from 1 for $\sigma \gtrapprox 0.8$ (relative to σ=1). These results suggest that distortions are more responsive than MoR to variation in σ. We find no meaningful difference in MoR for heterogeneous σ_{ind}'s and comparable value of homogeneous σ. For example, MoR is 9.5 percent larger in the case of heterogeneous σ_{ind}'s (with a mean value of 0.34 in our data), which is similar to the extent in the case of homogenous σ=0.34. Importantly, MoC (dispersion of capital wedges) for heterogeneous σ_{ind}'s is more than 300 percent larger than that for σ=1. As a counterfactual exercise, if the mean value of heterogeneous σ_{ind}'s is raised to 1 by rescaling σ_{ind}'s proportionately for all firms, MoC is still approximately 46 percent larger. Given the overwhelming evidence that σ is much less than the Cobb-Douglas value of 1, we conclude that the extent of distortions and misallocation will be considerably underestimated if the Cobb-Douglas production function is employed.

We also find that distortions are negatively related to the economic development of a country for σ<1 but not strongly for σ=1. In contrast, MoR is negatively related to economic development for σ=1 but not strongly for σ<1. These findings further corroborate the importance of specification of the production function in misallocation accounting and also provide mixed support to Inklaar et al. (2017), who observed a lack of relationship between misallocation and economic development using the Cobb-Douglas production function.

Our paper is built on Restuccia and Rogerson (2008) and Hsieh and Klenow (2009) but our novel innovation is the introduction of the CES production function and exploring the role of σ in misallocation.[3] Although this approach abstracts from the origins of misallocation, we

[3] There are several studies that estimate misallocation in the manufacturing sector using the Cobb-Douglas production function that include, among others, Ezra (2013) in Chile, Kalemli-Ozcan and Sørensen (2014) and Cirera, Fattal Jaef and Maemir (2019) in Africa, Inklaar, Lashitew and Timmer (2017) at the cross-country (developing and transition countries) level and Chaudhry, Haseeb and Haroon (2017) in Pakistan. Studies that estimate misallocation in the agricultural sector include, among others, Adamopoulos and Restuccia (2020), Adamopoulos et al. (2022) and Chen et al. (2022). Banerjee and Duflo (2005) summarize microeconomic evidence of misallocation of capital.

review the determinants of σ in the literature and relate them to the origins of misallocation in theoretical models.

Our paper is related to several strands of literature in economics. It identifies σ as an important link between misallocation and economic growth. There is a strand of literature (for example, de La Grandville, 1989; Klump and de La Grandville, 2000; de La Grandville and Solow, 2009) that emphasizes the importance of σ in economic growth. de La Grandville (1989) showed that at any stage of economic development, growth rate of per capita income is increasing with the value of σ.[4] There is even a possibility of perpetual growth without technological progress if $\sigma>1$ (and above a critical value). Conversely, there is a gloomy possibility of perpetual slow-down if $\sigma<1$ (and below a critical value) (Mallick, 2010). The misallocation literature is based on the premise that larger misallocation leads to lower aggregate TFP, which is one of the reasons for economic underdevelopment. We show that low σ is related to larger misallocation thus undermining the prospect for economic growth.

Development accounting by Caselli (2005) shows the sensitivity of the cross-country income differences to the value of σ. When σ is close to 0.5, variation in productive factors accounts for almost all variations in per capita income across countries. The percentage variation is decreasing in σ and drops to 40 percent when σ equals 1. Our paper reinforces the role of σ in development accounting by documenting that the dependence of misallocation on σ.

Our paper also provides a potential link between business-cycles and misallocation. Oberfield (2013) and Sandleris and Wright (2014) document that misallocation increased markedly during prolonged recessions (crises) in Chile in the early 1980s and in Argentina in the early 2000s, respectively, resulting in declines in aggregate TFP. Propagation of business cycles also depend on σ. For example, Cantore et al. (2014) show that the business-cycle fluctuations in employment originating from (factor-augmenting) productivity shocks depend on σ although responses vary between RBC and NK models. Note that in business cycle models, reproducing certain features of macroeconomic data relies, to a certain extent, on capital formation, which is quite sensitive to the choice of σ.

Our analytical and empirical results raise concerns about using the Cobb-Douglas production function in both empirical and theoretical misallocation literature. Similar concerns have also been raised in other branches of literature mentioned above. As σ is also influenced

[4] Mallick (2012) empirically tested this hypothesis and found empirical support.

by institutional and policy frameworks of a country (see Section 5), our findings have important policy implications.

The rest of the paper proceeds as follows. In Section 2, we extend the canonical empirical framework using the CES production function. We describe the data in Section 3. The results are discussed in Section 4. The determinants of σ and their role in misallocation are reviewed in Section 5. Finally, Section 6 concludes.

2. Analytical Framework

In the following, we extend the canonical framework of misallocation accounting by introducing the CES production function at the firm level. We derive expressions for distortions, misallocation and aggregate output gain from reallocation of resources that relate to elasticity of substitution between capital and labor, and compare these with the Cobb-Douglas production function. We first assume that all firms have the same elasticity of substitution (homogenous σ), and later allow σ to vary across firms (heterogeneous σ_i).

2.1 Homogenous σ

Total output is a CES aggregate of firm (i) level output given by

$$Y = \left(\sum_i y_i^{\theta} \right)^{\frac{1}{\theta}}, \qquad \text{---(1)}$$

where $1/\theta$ is the mark-up over price given by $\theta = (\gamma - 1)/\gamma$, and γ is the elasticity of substitution between goods produced by different firms. y_i denotes firm i's real output. Cost minimization gives the firm's demand curve, $p_i y_i = \lambda \theta y_i^{\theta}$; here λ is the Lagrange multiplier. Firm i maximizes the following profit function

$$\Pi_i = p_i y_i (1 - \tau_{yi}) - r k_i (1 + \tau_{ki}) - w l_i, \qquad \text{---(2)}$$

where τ_{yi} and τ_{ki} are output and capital (relative to labor) wedges, respectively. Output wedge is a tax (or subsidy) on final output affecting firm's output price idiosyncratically without altering the capital-labor composition. Capital wedge, for instance, a lower than market interest rate paid by a firm due to political connections, impacts on its capital-labor ratio. The rental and wage rates given by r and w, respectively, are the same across firms.

Output is produced using the CES production function given by

$$y_i = A_i \left[\alpha k_i^{\frac{\sigma-1}{\sigma}} + (1-\alpha) l_i^{\frac{\sigma-1}{\sigma}} \right]^{\frac{\sigma}{\sigma-1}}, \qquad \text{---(3)}$$

where, σ is the elasticity of substitution between capital and labor which is the same for all firms. When σ approaches 1, the CES production function becomes the Cobb-Douglas: $y_i = A_i k_i^{\alpha} l_i^{1-\alpha}$.[5] We assume neutral technology (A_i) to be consistent with the misallocation literature that invariably assumes a Cobb-Douglas production function in which neutral and factor-biased technological changes cannot be differentiated.[6]

Profit maximization gives the following two first-order conditions with respect to capital and labor, in which marginal revenue products of capital and labor are equated to rental rate and wage rate, respectively. Combining with the demand function, $p_i y_i = \lambda \theta y_i^{\theta}$, these can be expressed as:

$$MRPK_i = \alpha A_i^{\frac{\sigma-1}{\sigma}} \theta p_i \left(\frac{y_i}{k_i} \right)^{\frac{1}{\sigma}} = \alpha \left(p_i A_i \right)^{\frac{\sigma-1}{\sigma}} \theta \left(\frac{p_i y_i}{k_i} \right)^{\frac{1}{\sigma}} = r \frac{(1+\tau_{ki})}{(1-\tau_{yi})} \qquad \text{---(4)}$$

$$MRPL_i = (1-\alpha) A_i^{\frac{\sigma-1}{\sigma}} \theta p_i \left(\frac{y_i}{l_i} \right)^{\frac{1}{\sigma}} = (1-\alpha) \left(p_i A_i \right)^{\frac{\sigma-1}{\sigma}} \theta \left(\frac{p_i y_i}{l_i} \right)^{\frac{1}{\sigma}} = \frac{w}{(1-\tau_{yi})} \qquad \text{---(5)}$$

When σ=1, MPRK (MPRL) is expressed as the ratio of revenue to capital (labor). However, when σ≠1, individual marginal revenue products cannot be separated from the total factor productivity revenue (TFPR), which is given by $p_i A_i$.

The marginal (physical) products of capital and labor are given, respectively, by

$$MPK_i = \alpha A_i^{\frac{\sigma-1}{\sigma}} \left(\frac{y_i}{k_i} \right)^{\frac{1}{\sigma}} \text{ and } MPL_i = (1-\alpha) A_i^{\frac{\sigma-1}{\sigma}} \left(\frac{y_i}{l_i} \right)^{\frac{1}{\sigma}}.$$

Combining equations (4) and (5), the ratio of marginal products is expressed in terms of capital wedge that relates to the capital-labor ratio and σ.

[5] In the CES specification, $0 < \alpha < 1$ is the distribution parameter and does not have any direct interpretation. However, α becomes the share of capital in total output when σ=1. In the normalized CES production function (normalized at some baseline values), α is the share of capital in total output at the point of normalization; both A and α become dependent on σ and the baseline values (Klump and de La Grandville, 2000). For simplicity, the baseline values can be set equal to 1 (see, Aquilina, Klump and Pietrobelli, 2006). In empirical works, the baseline values of the variables are usually calculated as their respective (geometric) means; however, given that our empirical exercise is based on cross-section data by country, our approach can be thought of setting the baseline values to 1.

[6] Without this assumption, the ratio of marginal products in equation (6) would also depend on factor-augmenting technological changes.

$$\frac{MRPK_i}{MRPL_i} = \frac{\alpha}{1-\alpha}\left(\frac{k_i}{l_i}\right)^{-1/\sigma} = \frac{r}{w}(1+\tau_{ki}) \qquad \text{---(6)}$$

2.1.1 Allocative distortions (TFPR)

Allocative distortions consisting of capital and output wedges can be expressed in terms of *TFPR* (p_iA_i). In order to do that, we first write the marginal cost of firm *i*:

$$mc_i = \frac{1}{A_i}\left\{\alpha^\sigma r^{1-\sigma}\left[\frac{1+\tau_{ki}}{1-\tau_{yi}}\right]^{1-\sigma} + (1-\alpha)^\sigma w^{1-\sigma}\left[\frac{1}{1-\tau_{yi}}\right]^{1-\sigma}\right\}^{\frac{1}{1-\sigma}}.$$

$$= \frac{1}{A_i(1-\tau_{yi})}\left\{\alpha^\sigma r^{1-\sigma}(1+\tau_{ki})^{1-\sigma} + (1-\alpha)^\sigma w^{1-\sigma}\right\}^{\frac{1}{1-\sigma}}$$

Given that price is a mark-up over marginal cost ($p_i = (1/\theta)*mc_i$), *TFPR* of firm *i* is written as $p_iA_i = (1/\theta)*mc_i*A_i$, so that

$$TFPR_i = \frac{1}{\theta(1-\tau_{yi})}\left\{\alpha^\sigma r^{1-\sigma}(1+\tau_{ki})^{1-\sigma} + (1-\alpha)^\sigma w^{1-\sigma}\right\}^{\frac{1}{1-\sigma}}. \qquad \text{---(7)}$$

Equation (7) is highly non-linear and examination of how σ influences distortions is not obvious. For easier interpretation, we take a second-order Taylor approximation around σ=1.[7]

$$\ln TFPR_i \approx -\ln\theta + \ln\left(\frac{w}{1-\alpha}\right) - \ln(1-\tau_{yi}) + \alpha\left\{\ln\frac{r(1-\alpha)}{w\alpha}(1+\tau_{ki})\right\}$$

$$+\frac{\alpha(1-\alpha)(1-\sigma)}{2}\left\{\ln\frac{r(1-\alpha)}{w\alpha}(1+\tau_{ki})\right\}^2$$

$$= -\ln\theta + \ln\left(\frac{w}{1-\alpha}\right) + \alpha\beta - \ln(1-\tau_{yi}) + \alpha\ln(1+\tau_{ki}) \qquad \text{--(8)}$$

$$+\left\{\frac{\alpha(1-\alpha)(1-\sigma)}{2}\right\}\left[\beta + \ln(1+\tau_{ki})\right]^2$$

$$= -\ln\theta + \ln\left(\frac{w}{1-\alpha}\right) + \alpha\beta + \tau_{yi} + \alpha\tau_{ki} + \left\{\frac{\alpha(1-\alpha)(1-\sigma)}{2}\right\}\left[\beta + \tau_{ki}\right]^2$$

where, $\beta = \ln\left[\frac{r(1-\alpha)}{w\alpha}\right]$, $-\ln(1-\tau_{yi}) \approx \tau_{yi}$ and $\ln(1+\tau_{ki}) \approx \tau_{ki}$

[7] See Kmenta (1967) for a second-order approximation of the CES production function.

It is now clear from equation (8) that, given capital and output wedges, distortions are decreasing in σ, which is shown by the parameters attached to the last term.[8] This is because capital wedge affects the capital-labor ratio by altering the relative input prices, and the extent of the change in the capital-labor ratio depends on σ.

In the Cobb-Douglas case, the TFPR will be given by:

$$TFPR_i^{CD} = \frac{r^\alpha w^{1-\alpha}}{\theta\alpha^\alpha(1-\alpha)^{1-\alpha}}\frac{\left(1+\tau_{ki}\right)^\alpha}{\left(1-\tau_{yi}\right)}$$

$$\begin{aligned}\ln TFPR_i^{CD} &= -\ln\theta + \ln\left(\frac{w}{1-\alpha}\right) + \alpha\beta + \alpha\ln\left(1+\tau_{ki}\right) - \ln\left(1-\tau_{yi}\right) \\ &= -\ln\theta + \ln\left(\frac{w}{1-\alpha}\right) + \alpha\beta + \alpha\tau_{ki} + \tau_{yi}\end{aligned} . \qquad \text{--(9)}$$

This equation is the same as equation (8) after substituting σ=1. The difference in distortions under the CES and Cobb-Douglas production functions is given by the last term in equation (8), which is decreasing in σ.

2.1.2 Misallocation

First, we define *misallocation of capital relative to labor* (MoC) as the dispersion (standard deviation) of marginal revenue products of capital relative to that of labor across firms, which is also the dispersion of their capital wedges. This dispersion will be important to determine the extent of dispersion of TFPR. From equation (6), MoC is given by

$$sd\left[\ln\left(1+\tau_{ki}\right)\right] = \frac{1}{\sigma}sd\left(\ln\frac{k_i}{l_i}\right). \qquad \text{---(10)}$$

When σ=1, MoC is entirely determined by the dispersion of capital-labor ratios across firms. The higher the dispersion of capital-labor ratios, the larger is the MoC. However, for any given dispersion of capital-labor ratios, low σ leads to large MoC. The reason is that with increasing capital-labor ratio, MPK declines rapidly when σ is low, thus firms with different capital-labor ratios will generate larger dispersion of marginal products.

We define *misallocation of resources* (MoR) as the dispersion (standard deviation) of logarithm of TFPR across firms (here, for simplicity, we express in terms of variance rather than standard deviation).

[8] Even in the absence of the wedges ($\tau_{ki}=\tau_{yi}=0$), TFPR is decreasing with σ, which is shown by the last term.

$$\begin{aligned}\operatorname{var}\left(\ln TFPR_i\right) &= \operatorname{var}\left(\tau_{yi}\right) + \alpha^2 \operatorname{var}\left(\tau_{ki}\right) + (1/4)*\alpha^2\left(1-\alpha\right)^2\left(1-\sigma\right)^2 \operatorname{var}\left[\left(\beta+\tau_{ki}\right)^2\right] \\ &+2\alpha \operatorname{cov}\left\{\tau_{yi},\tau_{ki}\right\} + \alpha\left(1-\alpha\right)\left(1-\sigma\right)\operatorname{cov}\left\{\tau_{yi},\left(\beta+\tau_{ki}\right)^2\right\} \\ &+\alpha^2\left(1-\alpha\right)\left(1-\sigma\right)\operatorname{cov}\left\{\tau_{ki},\left(\beta+\tau_{ki}\right)^2\right\}\end{aligned} \qquad \text{--(11)}$$

Dispersions of both capital and output wedges increase MoR as shown by the first three terms. Parameters attached to all variance and covariance terms are decreasing with σ; therefore, misallocation will be larger for lower value of σ. However, the extent of MoR also depends on the signs of the covariances between capital and output wedges, which can be positive, negative or zero. For σ =1,

MoR based on the Cobb-Douglas production function will be the dispersion of ln(TFPR^{CD}) in equation (9), which would also be identical by setting σ =1 in equation (11).

$$\operatorname{var}\left(\ln TFPR_i^{CD}\right) = \operatorname{var}\left(\tau_{yi}\right) + \alpha^2 \operatorname{var}\left(\tau_{ki}\right) + 2\alpha \operatorname{cov}\left\{\tau_{yi},\tau_{ki}\right\} \qquad \text{----(12)}$$

To express TFPR as a CES aggregate of marginal revenue products, substitute equations (4) and (5) into equation (7)

$$TFPR_i = \frac{1}{\theta}\left\{\alpha^\sigma MRPK_i^{1-\sigma} + \left(1-\alpha\right)^\sigma MRPL_i^{1-\sigma}\right\}^{\frac{1}{1-\sigma}}.$$

This equation (as equations (7) and (8)) shows that extent of resource misallocation will be large when there is greater dispersion of marginal products, which is negatively related to σ.

2.1.3 Efficient aggregate output

Assuming that aggregate inputs are fixed so that capital and labor at the firm level sum to their respective aggregate levels, $K = \sum_i k_i$ and $L = \sum_i l_i$, at the efficient level of allocation ($\tau_{ki} = \tau_{yi} = 0$), capital-labor ratios will be the same for all firms and proportional to the aggregate capital-labor ratio.

$$\begin{aligned}&\frac{k_i^e}{l_i^e} = \left(\frac{r}{w}\frac{1-\alpha}{\alpha}\right)^{-\sigma} \\ &\Rightarrow \frac{k_i^e}{l_i^e} = \frac{k_j^e}{l_j^e} = \frac{K}{L}\end{aligned} \qquad \text{---(13)}$$

The efficient level of output for firm *i* is derived as:

$$y_i^e = \frac{A_i^\gamma}{\sum_i A_i^{\gamma-1}}\left[\alpha K^{\frac{\sigma-1}{\sigma}} + (1-\alpha)L^{\frac{\sigma-1}{\sigma}}\right]^{\frac{\sigma}{\sigma-1}}. \qquad \text{---(14)}$$

Summing the efficient output of all firms, the efficient aggregate output is obtained as

$$Y^e = \left(\sum_i A_i^{\gamma-1} \right)^{\frac{1}{\gamma-1}} \left[\alpha K^{\frac{\sigma-1}{\sigma}} + (1-\alpha) L^{\frac{\sigma-1}{\sigma}} \right]^{\frac{\sigma}{\sigma-1}}, \qquad \text{---(15)}$$

where the efficient aggregate TFP is given by $\left(\sum_i A_i^{\gamma-1} \right)^{\frac{1}{\gamma-1}}$.

At the firm level, percentage change in output from decreasing capital wedge by one unit can be derived as (see Appendix A)

$$d\ln y_i = -\gamma \left[\alpha + \frac{\sigma-1}{\sigma} \alpha (1-\alpha) \ln\left(\frac{k_i}{l_i} \right) \right] d\ln(1+\tau_{ki}).$$

Aggregate output gain from reallocation is defined as Y^e/Y in equation (15). It is decreasing with σ, which can be understood intuitively. Given that lower σ is related to larger distortions, aggregate output gain from eliminating distortions will also be larger when σ is low. When capital (other productive resources) is reallocated to equalize marginal returns across firms, for aggregate output to increase the marginal contribution of the firm receiving additional unit of capital (lower capital-labor ratio) would be larger than the marginal decrease in output of the firm giving up capital (higher capital-labor ratio). The difference between the marginal contribution and marginal loss from reallocation of capital between the two firms will be larger (smaller) when σ is low (high).

2.2 Heterogeneous σ_i

To allow σ to vary across firms, rewrite the production function as

$$y_i = A_i \left[\alpha k_i^{\frac{\sigma_i-1}{\sigma_i}} + (1-\alpha) l_i^{\frac{\sigma_i-1}{\sigma_i}} \right]^{\frac{\sigma_i}{\sigma_i-1}}.$$

The expression for capital wedge is an extension of the case for homogeneous σ (previous derivations go through)

$$\frac{MRPK_i}{MRPL_i} = \frac{\alpha}{1-\alpha} \left(\frac{k_i}{l_i} \right)^{-1/\sigma_i} = \frac{r}{w}(1+\tau_{ki}).$$

It can be understood intuitively why dispersion of capital wedges (MoC) will be larger when variation in σ_i's across firms is larger. Suppose, two firms have identical σ but they differed by their marginal products of capital due to the difference in their capital-labor ratios. Also suppose that reallocating one unit of capital from one firm to another would equalize their marginal products. If now σ_i's also differ in the two firms, their marginal products of capital will have larger difference compared to the case of identical σ, and consequently will not equalize from reallocating one unit of capital. MoC is now given by:

$$sd\left[\ln\left(1+\tau_{ki}\right)\right]=sd\left(\frac{1}{\sigma_i}*\ln\frac{k_i}{l_i}\right). \qquad \text{---(16)}$$

Equation (16) shows that MoC is now determined not only by the dispersion of capital-labor ratios, but also by the dispersion of $(1/\sigma_i)$ and their covariance in a highly non-linear manner.[9] The covariance accounts for the possibility that capital accumulation also depends on, in addition to distortions it encounters in the capital market, how easily a firm can substitute its capital for labor. Note that if capital-labor ratio and σ_i are independent but still vary across firms, MoC will be given by $sd(1/\sigma_i)*sd(\ln(k_i/l_i))$.

The expression for TFPR in terms of capital and output wedges is also an extension of the case for homogeneous σ (previous derivations also go through).

$$TFPR_i=\frac{1}{\theta\left(1-\tau_{yi}\right)}\left\{\alpha^{\sigma_i}r^{1-\sigma_i}\left(1+\tau_{ki}\right)^{1-\sigma_i}+\left(1-\alpha\right)^{\sigma_i}w^{1-\sigma_i}\right\}^{\frac{1}{1-\sigma_i}}$$

$$\ln TFPR_i=-\ln\theta+\ln\left(\frac{w}{1-\alpha}\right)+\alpha\beta+\tau_{yi}+\alpha\tau_{ki}+\left\{\frac{\alpha\left(1-\alpha\right)\left(1-\sigma_i\right)}{2}\right\}\left[\beta+\tau_{ki}\right]^2 \qquad \text{---(17)}$$

MoR will be defined by dispersion of logarithm of TFPR in equation (17). Now the variance would be the sum of variances of each individual term in the RHS (except the first two constant terms) and their covariances in a highly non-linear manner. The dependence of capital-labor ratio on σ_i will be accounted for by the covariance terms.

It is important to mention that aggregate efficient output and therefore, output gain from reallocation cannot be derived for heterogeneous σ_i, which can be seen by examining equation (13).

3. Data

3.1 Firm-level data

[9] Denoting $(1/\sigma_i) = x_i$ and $\ln(k_i/l_i) = z_i$, the variance in equation (16) can be written as $\text{var}(x*z)=\mu_x^2\upsilon_z^2+\mu_z^2\upsilon_x^2+2\mu_x\upsilon_{x,z^2}+2\mu_z\upsilon_{x^2,z}+2\mu_x\mu_z\upsilon_{x,z}+\upsilon_{x^2,z^2}-(\upsilon_{x,z})^2$ where μ is the sample mean of x or z, υ and υ^2 are the sample covariance and variance of x or z (and their squared terms), respectively.

The World Bank Enterprise Survey (WBES)[10] is the source of our firm-level data. Since 2002, the WBES has been compiling cross-country standardized survey of formal business establishments employing five or more employees, following stratified random sampling method to ensure representativeness of the sample. The merit of this database is that the data are comparable across countries for a wide range of information at the firm level including, among others, financial and economic transactions, access to finance, obstacles to growth, corruption and competition. Although manufacturing and service sectors are the primary business sectors of interest, the firms in the manufacturing sector dominates in the sample. This corresponds to firms classified by ISIC2 codes 15-37 (Rev.3.1). Only formal (registered) companies are the target sample.

For our analyses, we need firm-level information about value added, capital stock and labor. Value added is calculated by subtracting total costs of intermediate inputs from total sales. Costs of intermediate inputs include costs of raw materials, and other expenditures, such as energy (fuel, electricity and water), communication services and transportation, incurred for production. Capital stock is the sum of book value of machinery, equipment and vehicles, and land and buildings. Number of employees is not adjusted for human capital and also data is missing for some firms. Total wage-bill is used instead (see Inklaar et. al., 2017).

We implement the following steps to prepare the working data: i) only firms in the manufacturing sector are retained,[11] ii) firms having less than five employees are excluded, iii) firms with missing information of sales, intermediate input costs, capital stock, wage bills are excluded; and iv) firms with only positive value added are retained. Outliers are detected by inspecting the capital-output ratio and the capita-labor ratio, and two percent observations from each tail of the distribution of both ratios are deleted. Additionally, two percent observations from each tail of the distribution of the TFPR (for σ=1) are deleted. Finally, countries with at least 30 firms are retained, which gives a total of 153 countries (including some countries that were surveyed in multiple years). A detail list of countries and survey years is provided in Appendix C.

3.2 Elasticity of substitution between capital and labor (σ)

[10] http://www.enterprisesurveys.org (accessed on November 3, 2019).

[11] After following these steps, only two countries with more than 30 firms in the service sector would remain in the sample.

Many studies that attempted to estimate σ mostly focus on the aggregate value in the US context. Although there is no agreement on the precise value of σ, a mounting evidence is in favor of below-unity aggregate value of σ (see, Chirinko, 2008 for a survey; for recent studies see, León-Ledesma, McAdam and Willman, 2010; Chirinko and Mallick, 2017; Knoblach, Roessler and Zwerschke, 2020 for a meta-analysis).[12] For parameterization of heterogeneous σ_i's, we need values of σ for different manufacturing sub-categories (ISIC2 codes) at the cross-country level, which, to the best of our knowledge, are unavailable. We use the industry values of σ for the USA estimated by Chirinko and Mallick (2017; Table 5, p. 248). These authors, using the US KLEM data constructed by Dale Jorgenson, estimated the long-run (low frequency) value of aggregate σ and also disaggregated values at the industry level that are comparable to ISIC2 codes in the WBES codes (see Appendix B for a complete list of the ISIC2 codes in the WBES and values of σ estimated by Chirinko and Mallick (2017)). These estimates are far below unity for the manufacturing sub-categories ranging between 0.078 (Food and Kindred Products) and 0.562 (Primary Metal Industries) with a (unweighted) mean of 0.34 and a standard deviation of 0.14. These magnitudes are broadly in line with Raval (2019) who, using manufacturing plant census, estimated σ for manufacturing sub-categories (two digit SIC codes) and found most estimates concentrated between 0.15 and 0.75.

Recent studies that estimated aggregate σ for the manufacturing sector using micro data also obtained similar ranges. For example, estimates of σ by Raval (2019) and Oberfield and Raval (2021) for the aggregate manufacturing sector using the USA plant level microdata range between 0.3 and 0.7.[13]

Recognizing that the value of σ also varies across countries, we extend our empirical exercise using aggregate value of σ that varies by country (but the same for all manufacturing sub-categories in a country). To the best of our knowledge, Mallick (2012) is the only study that estimated aggregate value of σ at the country level. Mallick (2012) estimated σ for 90 countries using the PWT-6.1 data but the sample countries match with 62 countries in our sample. Given a large variation in the values of σ across countries, we delete 5% extreme values from each tail of the distribution, thus leaving 56 countries. A detail list is provided in Appendix C.

[12] Karabarbounis and Neiman (2014) is an exception, who estimated an aggregate value of σ greater than 1.

[13] Herrendorf, Herrington and Valentinyi (2015) used aggregate data to estimate σ for the US manufacturing sector and obtained a higher value of σ (0.80) but it is still below unity.

4. Results

We evaluate the extent of allocative distortions, misallocation and aggregate output gain resulting from elimination of distortions for different values of σ relative to the Cobb-Douglas value of σ=1 (these statistics are normalized to 1 at σ=1). We consider a plausible range of values between 0.1 and 2; however, given the overwhelming empirical evidence of low σ, we emphasize the results for σ<1. In the case of heterogeneous σ_i's, we assume that σ varies across manufacturing sub-category (ISIC2 codes) but is the same in all firms within a sub-category (σ_{ind}), which is not unrealistic.

In our benchmark evaluation, we set a common $\gamma = 3$,[14] and $\alpha = 0.35$, and retain only those countries with 30 firms. The summary results (graphs and tables) are presented as the ratio of the means; the country averages are calculated for different values of σ, and then divided by the country average for σ=1. The statistics for the range of σ values are displayed in Figures 1-4; Table 1 also reports these for some specific values of σ.

4.1 Distortions and aggregate output gain

The extent of allocative distortions, expressed in terms of logarithm of TFPR, for different values of σ relative to σ=1 is displayed in Figure 1. It monotonically decreases with σ with its value being larger (smaller) than 1 for σ<1 (>1). The decrease is rapid when σ is low; for example, when σ increases from 0.4 to 0.5, distortions decrease from 1.28 to 1.21, while distortions decrease from 1.06 to 1.03 when σ increases from 0.8 to 0.9. There is almost no difference in distortions when comparing heterogeneous σ_{ind}'s and the comparable value of homogenous σ. In our data, the (unweighted) mean value of heterogeneous σ_{ind}'s is 0.34. Both at σ=0.34 and heterogeneous σ_{ind}'s, distortions are approximately 1.32 (shown by the horizontal line in Figure 1).

The aggregate output gain from elimination of distortions, defined by (Y^e/Y) in equation (15) is displayed in Figure 2 (and Table 1). The extent of output gain also monotonically decreases with σ with its relative value being larger (smaller) than 1 for σ<1 (>1). Its extent and pattern of change due to change in σ closely follow that of distortions.

[14] Although the mark-up over price varies across countries (Duarte and Rosa, 2015; Chirinko and Mallick, 2022), following the misallocation literature we use the same value of γ for all countries.

4.2 Misallocation

The extent of MoC is calculated as the standard deviation of the capital wedges in equation (10). Relative to σ=1, it is solely determined by the term (1/σ). MoC decreases rapidly (slowly) with σ when σ is low (high).

Figure-3 displays MoC for different values of σ. MoC for heterogeneous σ_{ind}'s, that accounts for the covariance between capital-labor ratios and σ_{ind}'s, is shown by the horizontal line in the Figure. Its magnitude is 4.32 times larger than that for σ=1, which corresponds to the respective magnitude at homogenous σ≈0.233. For a stricter comparison, if the mean value of heterogeneous σ_{ind}'s is raised from 0.34 to the Cobb-Douglas value of 1 by proportionately increasing all σ_{ind}'s, MoC would be 46 percent larger than that for σ=1. This finding suggests that variation in σ's across firms enormously intensifies MoC.

If we assume that capital-labor ratio in a firm is independent of its σ_{ind}'s, MoC is 2.62 times larger than that for σ=1. This is shown by the dashed horizontal line in the Figure. Although this magnitude is still very large but approximately 65% less than that when capital-labor ratios and σ_{ind}'s are not independent.

Insert Figures 1-4 and Table 1 here

MoR, given by the standard deviation of logarithm of TFPR, is plotted in Figure 4. The extent of MoR decreases with σ but sharply when σ is low. For instance, when σ=0.2, it is 1.16 relative to σ=1, which decreases to approximately 1.05 and 1.01 when σ equals 0.5 and 0.8, respectively. It then stabilizes at around 1 thereafter; indeed, MoR decreases very slowly up to σ ≈1.2 and then increases but not meaningfully (for example, it is 1.006 at σ≈2).

For heterogeneous σ_{ind}'s, MoR is approximately 1.095 suggesting 9.5 percent larger MoR relative to σ=1, which is shown by the horizontal line in the Figure. Approximately the same extent of MoR would occur at σ=0.34, which is the (unweighted) mean value of σ_{ind}'s.

4.3 Addressing measurement errors

Concerns about measurement errors in the data have been raised by several authors including, among others, Bils, Klenow and Ruane (2021). Since our objective is to evaluate distortions and misallocation for different values σ relative to σ =1, measurement errors are less of a concern. The reported statistics are calculated as the ratio of the means; for each value of σ, first the country average is calculated and then is divided by the country average for σ =1 as $\frac{\sum_c X_c(\sigma)}{\sum_c X_c(\sigma=1)}$, where $X_c(\sigma)$ is the respective statistic for country *c* for a specific value of σ.

One might doubt that this approach may not satisfactorily account for measurement errors. To check the robustness, we calculate the mean of the ratios; first calculate the ratios for different values of σ to σ=1 by country, and then take the country average of the ratios as $\frac{1}{N}\sum_c \frac{X_c(\sigma)}{X_c(\sigma=1)}$.[15] The descriptive statistics, presented in Appendix Table A1, are robust. Statistics by country are provided in Appendix C.

4.4 Robustness

We check robustness of our results in a variety of ways that are also standard in the empirical misallocation literature (see, Inklaar et al., 2017). In our first robustness check, we reset $\gamma = 5$ (Appendix Table A2). Note that this parameter enters the formula for only the aggregate output gain through the aggregate efficient TFP (distortions and misallocation are independent of γ). The second robustness check involves deleting 5% of TFPR from each tail of the distribution, which give more conservative estimates (Appendix Table A3). Finally, we retain only the countries for which at least 100 firm observations are available (Appendix Table A4). A larger number of firms is useful for more precise estimation of the dispersion using σ_{ind}'s, as there will be more firms in each industry sub-category. The number of countries now decreases to 64. The results do not qualitatively differ from those obtained in the benchmark estimation. The last exercise also suggests that our results are robust to the different combinations of the sample countries.

4.5 Country level evidence

Our summary results have been presented in terms of averages across countries. Appendix C presents these statistics by country. In the following, we present some highlights. Relative distortions, that is distortions relative to σ=1, are presented for σ=0.5 and 1.5. For all countries, relative distortions are larger than 1 for σ=0.5 and less than 1 for σ=1.5. The five countries with largest relative distortions for σ=0.5 are Kosovo (2.581), Myanmar (1.848), El

[15] Both the ratio of the means and the mean of the ratios are biased estimators (Rao, 2002). The ratio of the means estimator is less dependent on the sample size and has a lower statistical uncertainty. Therefore, our benchmark estimation is based on the ratio of the means. In contrast, the mean of the ratios estimator assigns an equal weight to each ratio and thus more dependent on the sample size.

Salvador (1.706), Mauritania (1.569) and Bosnia and Herzegovina (1.483). The five countries with smallest relative distortions for σ=1.5 are the same five countries with their order reversed.[16]

Aggregate output gain from elimination of distortions for σ=0.5 (relative to σ=1) is also larger than 1 (except for Ethiopia, Kosovo, Kyrgyz Republic, Serbia and Morocco). The five countries with largest aggregate output gain are Kazakhstan (2.395), Israel (1.982), Uruguay (1.931), Madagascar (1.866) and Nicaragua (1.824). MoR for σ=0.5 (relative to σ=1) is also larger than 1 (for 10 countries the ratio is less than 1 but close to 0.99). The five countries with largest relative MoR for σ=0.5 are Kazakhstan (1.209), Guinea-Bissau (1.190), Mali (1.164), Israel (1.157) and Nicaragua (1.150).

As shown in summary statistics (Table 1), MoC differs considerably between heterogeneous σ_{ind}'s and comparable magnitude of homogenous σ. Country ranking based on MoC also differs between heterogeneous σ_{ind}'s and homogenous σ (the same ranking for all values of homogenous σ as it is determined by the dispersion of capital-labor ratios; not reported in Appendix C). For heterogeneous σ_{ind}'s, countries with largest MoC are from Africa and ex-socialist (transition)[17] that include (in order) Angola, Guinea-Bissau, Bosnia and Herzegovina, Lithuania, Hungary, Belarus, Mauritania, Cameroon and Cote d'Ivoire. To see how this ranking would alter in the case of homogenous σ, consider the case of Bosnia and Herzegovina, which now ranks second lowest preceded by Serbia and followed by Zimbabwe and Czech Republic. The five countries with largest MoC are Moldova, Georgia, Yemen, Burundi and Israel.

Hsieh and Klenow (2009) compared misallocation (and efficiency gain) for India and China relative to the USA that has been followed in the subsequent literature as a popular comparison. Note that the direct India-China comparison does not require the USA as the benchmark country (USA is also not in the sample). Average distortions is lower in India than in China. It is approximately 15% lower (the India-China ratio is 0.850) for σ=1, which slightly decreases to approximately 11% for σ=0.5. For heterogeneous σ_{ind}'s, it is also almost the same (approximately 10%). In contrast, dispersion of distortions (MoR) is larger in India than in

[16] This comparison is not strict as different countries were survey in different years.

[17] Bartelsman, Haltiwanger and Scarpetta (2013) also observed larger distortions in transition economies of eastern and central Europe. Easterly and Fischer (1995) argued that σ is low in a socialist economy because these countries are characterized by a narrow range of capital goods. Some forms of the physical or human capital, such as market-oriented entrepreneurial skills, marketing and distribution skills, and information-sensitive physical and human capital were missing in a socialist economy. Our result is interesting since in our evaluation, σ varies only by industry but the same across all countries.

China. It is 7.5 percent larger for σ=1, which modestly increases to 9.6 percent for σ=0.5. Importantly, for heterogeneous σ_{ind}'s, MoR is only approximately 2 percent larger.

A startling result in Inklaar et al. (2017) based on the Cobb-Douglas production function is that misallocation does not vary with the stage of economic development. We replicate this exercise using more recent data and alternative values of σ. Although our sample countries include mostly developing and emerging economics, there is a large variation in per capita real GDP among them. We display a linear fit of distortions and MoR against logarithm of real GDP per capita in 2005 calculated from the PWT-9 data (www.ggdc.net/pwt).[18] We plot fitted lines for two values of homogenous σ: 0.5 and 1, and also for heterogeneous σ_{ind}'s in a single graph. Note that comparison of the slopes of the fitted lines indicate whether the country ranking changes for different values of σ.

Insert Figures 5 and 6 here

The fitted lines for distortions are displayed in Figure 5. For σ=0.5 and heterogeneous σ_{ind}'s, the two lines are parallel to each other with the one for heterogeneous σ_{ind}'s lying above (note that the mean value of σ_{ind}'s is 0.34). The fitted line for σ=1 is slightly flatter than the other two lines and lies at the bottommost. The positions of the three fitted lines corroborate the previous results that distortions are decreasing with the value of σ across countries. All three fitted lines are downward sloping, which imply that average distortions are decreasing with the level of economic development of a country. Both lines for σ=0.5 and heterogeneous σ_{ind}'s have almost identical slopes and robust t-statistics at -0.046 and -2.07, respectively. In contrast, the slope for σ=1 is slightly flatter (-0.036) and the robust t-statistic is -1.66, suggesting a weak negative relationship between distortions and the level of economic development.

Figure 6 displays three linear fitted lines for MoR for the same three values of σ. The relative positions of these fitted lines are similar to those for average distortions corroborating that MoR is decreasing with σ. Furthermore, all three fitted lines are downward sloping. However, the slope is statistically significant only for σ=1, which is -0.024 (with a robust t-statistic of -2.11). For σ=0.5 and heterogeneous σ_{ind}'s, the slopes are even flatter (-0.017 and -0.012, respectively), and both are statistically insignificant at any conventional level. Differential slopes of the fitted lines suggest that country ranking for MoR alters depending on

[18] Non-parametric Lowess fits are also very similar.

the value of σ. Statistical significance of the slope for σ=1 but not for other two values of σ suggest that the negative relationship between MoR and the level of economic development is stronger for σ=1 than lower values of σ. The overall results suggest that the relationship between misallocation and the level of economic development depends on the specification of the production function. Further investigation is required for better understanding of this relationship and its explanation.

4.6 σ varying by country (σ_c)

In this exercise, we use country-specific values of σ (σ_c) estimated by Mallick (2012). Note that σ_c does not vary by industry sub-categories. This exercise is intended to have a better idea about how misallocation would be underestimated, and country ranking would be misconstrued, if a uniform value of σ including 1 is imposed on all countries. Depending on the availability of σ_c estimates, the number of sample countries now reduces to 56. The (unweighted) mean of σ_c is 0.21 and ranges between 0.084 (Nicaragua) and 0.686 (Turkey).

The country ranking now substantially alters compared to uniform σ or σ_{ind}'s for all countries. The five countries with largest MoC (in order), which are Nicaragua, Cameron, Guatemala, Romania and Kenya, have σ_c <0.1. The five countries with smallest MoC, which are Turkey, Nepal, China, Madagascar and Dominican Republic (followed by India), have σ_c>0.5.

The country ranking in terms of MoR is different from that in terms of MoC. The five countries with largest MoR (in order) are Nicaragua, Thailand, Ghana, Nepal and Zimbabwe. Among these countries, Nepal has a high value of σ=0.56, while Cameroon has a low value of σ=0.09. In contrast, countries with lowest MoR are Bangladesh, Dominican Republic, El Salvador, Madagascar, and Cameroon. Both the rankings based on MoC and MoR are different from those using uniform σ for all countries discussed in Section 4.5.

The above results are only suggestive but informative enough to justify that variation in σ, like the dispersion of capital-labor ratios, is crucial for explaining misallocation.

5. Factors determining σ and misallocation

The (indirect) approach that we follow quantifies misallocation and its impact but abstracts from the origins of misallocation. This approach also relies on the structure of the technology to identify misallocation (Hopenhayn, 2014). When Hicks (1932/1963) introduced σ, he realized it as a pure technological parameter. He pointed out the three possible ways in which substitution can take place—intra-sectoral substitution between known methods of production, inter-sectoral substitution of production, and substitution arising from new innovations. If σ is treated as a technology parameter, as Hicks did, then low σ can be interpreted as a restricted choice of technology leading to larger misallocation. When the restrictions on technology vary across firms, misallocation will be magnified.

However, σ is also treated as a general measure of the flexibility of the market system and therefore influenced by institutional settings (Klump and Preissler, 2000; Aquilina, Klump and Pietrobelli, 2006, de La Grandville and Solow, 2009; Solow, 2005). These include, among others, strength of labor unions (Maki and Meredith, 1987), customary and regulatory barriers to large changes in capital–labor ratios (de La Grandville and Solow, 2009), country's monetary and financial system (Klump and Preissler, 2000), openness to trade (Saam, 2008) and inclination to socialist system (Easterly and Fischer, 1995). Structural models are needed to understand the role of specific institutional and policy aspects in misallocation and to establish a causal link. Models that address some of the above features include, among others, the role of labor market regulation (Hopenhayn and Rogerson, 1993), credit market imperfections and financial frictions (Buera et al., 2011; Midrigan and Xu, 2014; Gopinath et al., 2017), trade barriers (Bai, Jin and Lu, 2021), and fluctuations in the interest rate leading to persistence in misallocation (Banerjee and Moll, 2010).

Although our estimation does not attribute any institutional or policy aspects to the extent of misallocation, our results suggest that introducing variation in σ is helpful in capturing their effects on misallocation. In contrast, the Cobb-Douglas production function is rigid in incorporating these effects.

6. Discussions and conclusions

Although the role of the elasticity of substitution between capital and labor (σ) has been emphasized in many areas in macroeconomics, it has been neglected in the misallocation literature. We investigate its role both analytically and empirically using the WBES firm-level survey data at the cross-country level. We derive distortions as a CES aggregate of output and capital wedges that depend on the value of σ. To position our contribution in the extant empirical literature, we compare distortions, misallocation and aggregate output gain from reallocation of resources for different values of σ relative to the Cobb-Douglas value of 1.

We document that the extent of distortions, misallocation and aggregate output gain are large when σ is low (<1) and decreasing with the value of σ. Given the overwhelming evidence that σ is much smaller than 1, we conclude that their magnitudes are considerably underestimated in the empirical literature built on the Cobb-Douglas production function and recommend using the CES production function. Moreover, given that σ is also influenced by a country's institutional and policy features, and that misallocation is an equilibrium outcome of a political process interacting with institutions and the distribution of resources (Acemoglu, Johnson and Robinson, 2005), our results have important policy implications.

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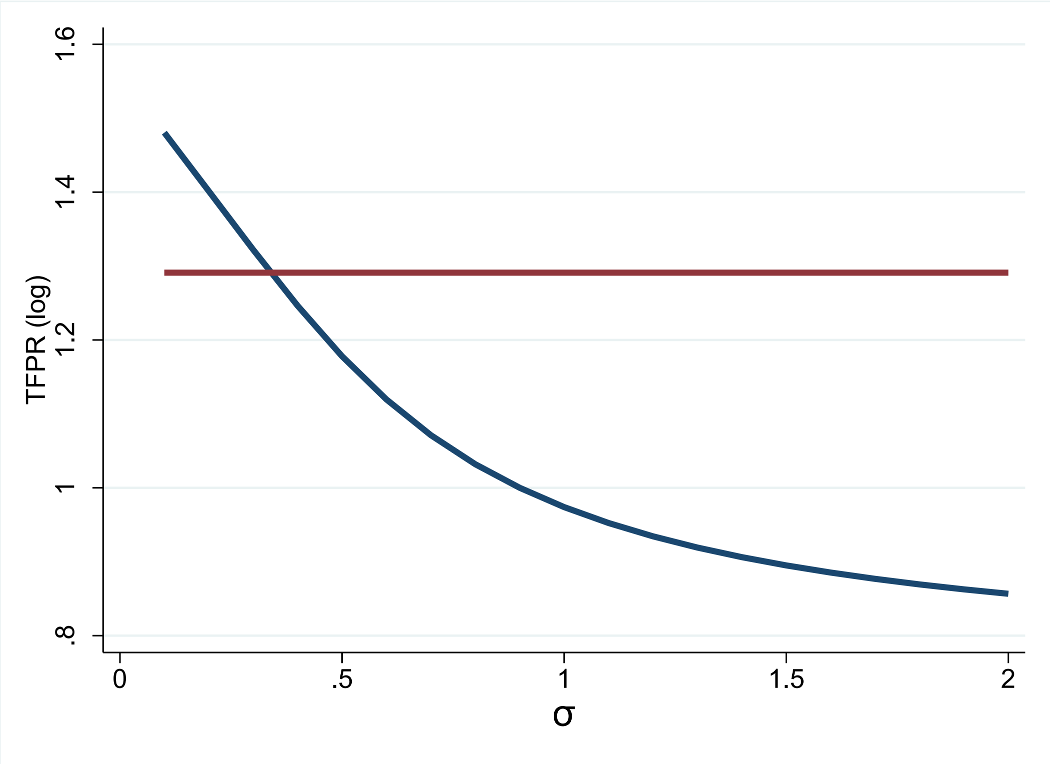
Tables and Graphs

Table 1: Distortions, misallocation and aggregate output gain for different values of σ relative to σ=1.

	MoC	Distortions: ln($TFPR_i$)	MoR	Aggregate Output Gain	Aggregate Efficient TFP
σ=0.2	5.000	1.439	1.137	1.733	1.615
σ ≈ 0.34*	2.982	1.329	1.098	1.522	1.397
σ=0.5	2.000	1.210	1.050	1.282	1.212
σ=1.5	0.666	0.919	1.001	0.940	0.942
Heterogeneous σ_{ind}	4.317	1.326	1.095	-----	---

*Mean value of heterogeneous σ_{ind}'s is 0.3353529 (results compared with homogenous σ=0.3353529). Figures in parentheses are standard deviations.

Figure 1: Allocative distortions (logarithm of TFPR) for different values of σ.



Note: The horizontal line is the TFPR using heterogeneous σ_i relative to $\sigma = 1$ (=1.364), which is (approximately) equal to the TFPR misallocation for homogeneous $\sigma \approx 0.34$.

Figure 2: Aggregate output gain from reallocation for different values of σ.

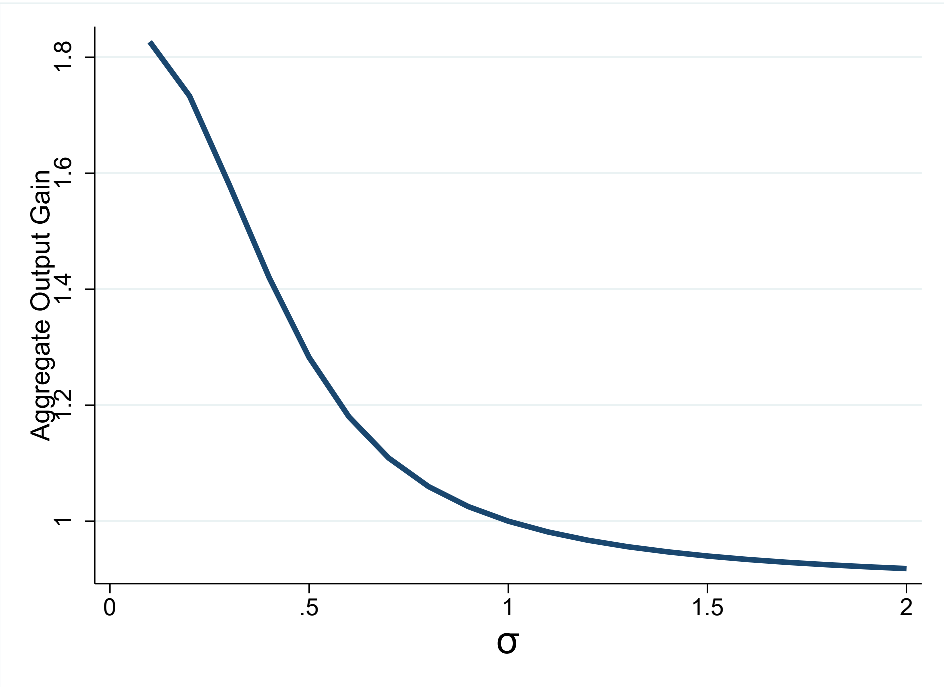
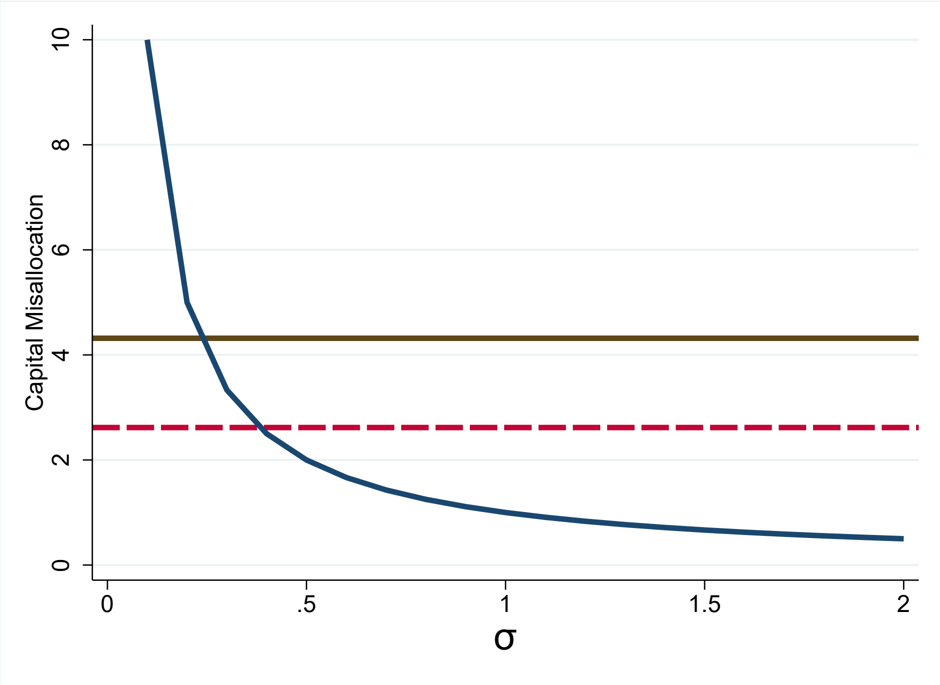
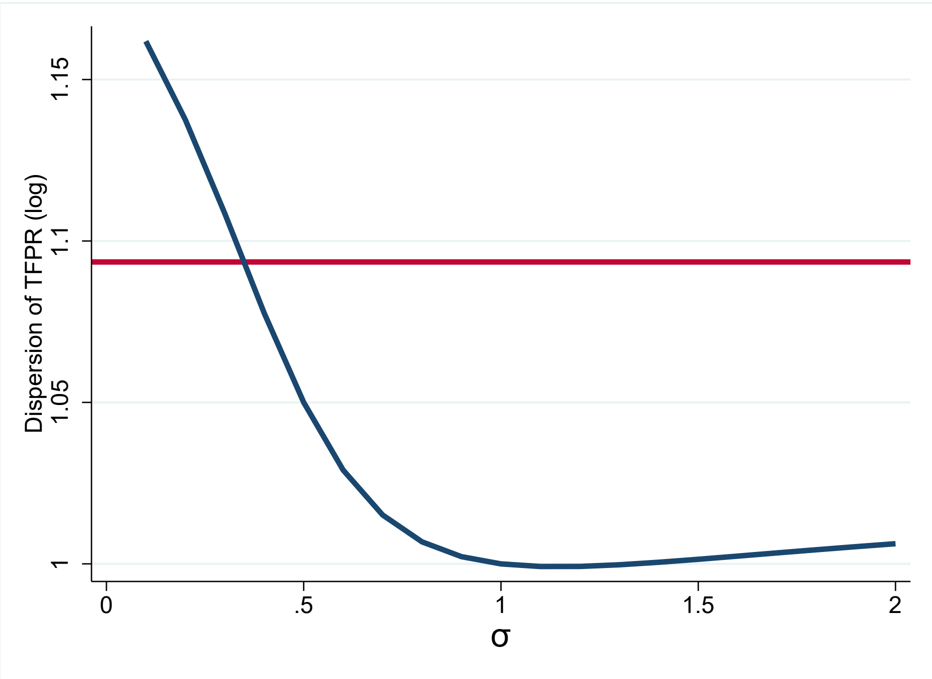


Figure 3: Capital misallocation (dispersion of capital wedges) for different values of σ



Note: The horizontal solid line is the capital misallocation using heterogeneous σ_i relative to $\sigma = 1$ (=4.317), which is (approximately) equal to the capital misallocation for homogeneous $\sigma \approx 0.24$. The horizontal dashed line (at 2.617) is the same but under the assumption that σ and capital-labor ratio are independent.

Figure 4: Dispersion of logarithm of TFPR for different values of σ.



Note: The horizontal line is the misallocation using heterogeneous σ_i relative to $\sigma = 1$ (=1.094), which is (approximately) equal to the misallocation for homogeneous $\sigma \approx 0.34$.

Figure-5: TFPR and income level

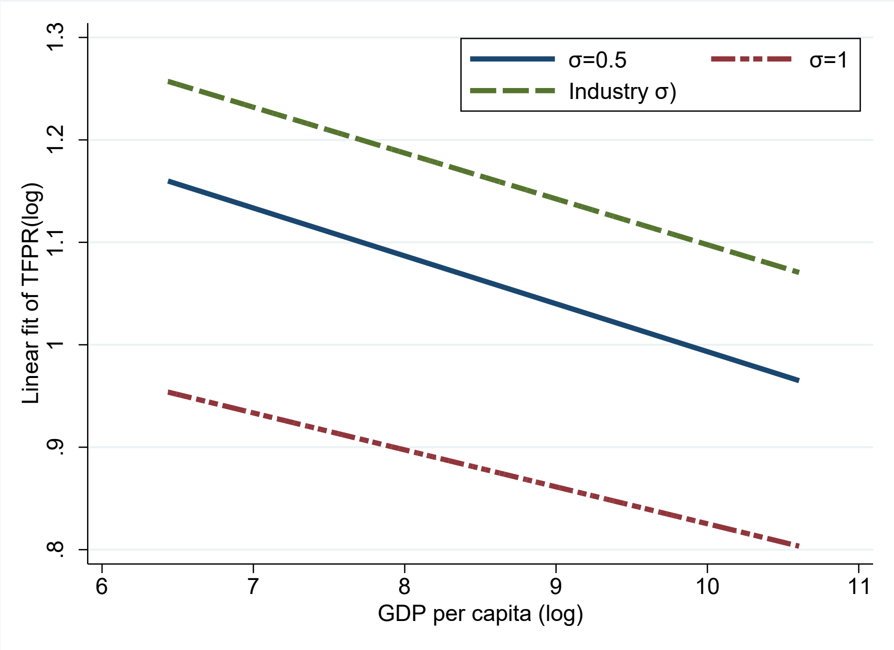
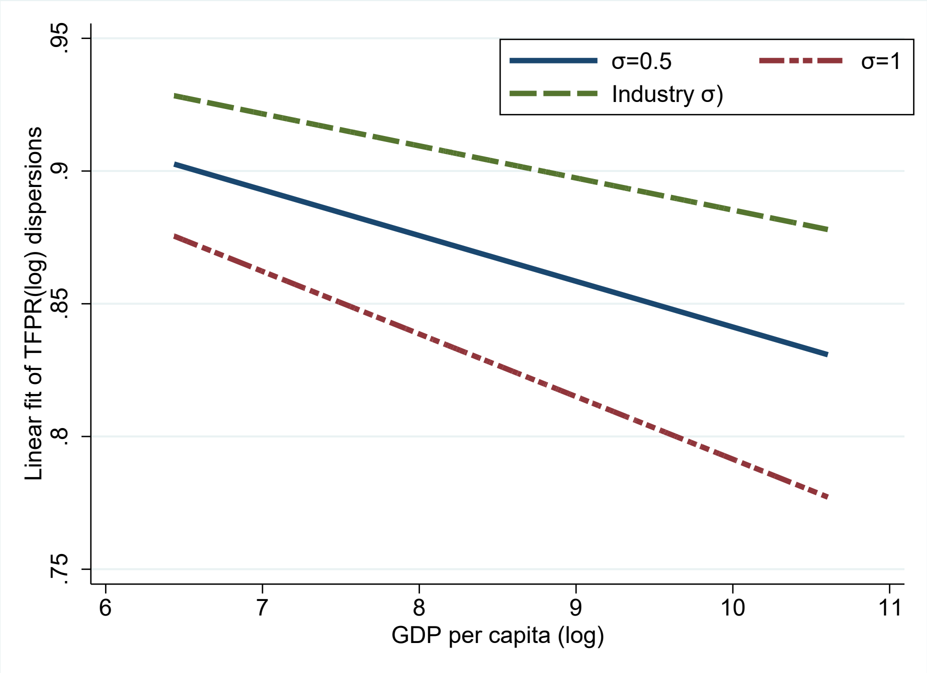


Figure-6: Dispersion of TFPR and income level



Appendix

Appendix A: Derivation of the output gain from removing capital wedge:

Firm-level output derived from cost minimization in equation (1) is given by

Equilibrium demand: $y_i = \left(\frac{p_i}{P}\right)^{-\gamma} Y$

$$MRPL_i = (1-\alpha)A_i^{\rho}\theta p_i \left(\frac{y_i}{l_i}\right)^{1-\rho} = \frac{w}{(1-\tau_{yi})} \qquad \text{----(5)}$$

$$1+\tau_{ki} = \frac{\alpha}{1-\alpha}\frac{w}{r}\left(\frac{k_i}{l_i}\right)^{-1/\sigma} \qquad \text{----(6)}$$

Second-order Taylor approximation of the CES production function in equation (3):

$$\ln\left(\frac{y_i}{l_i}\right) = \alpha \ln\left(\frac{k_i}{l_i}\right) + \frac{1}{2}\frac{\sigma-1}{\sigma}\alpha(1-\alpha)\ln\left(\frac{k_i}{l_i}\right)^2$$

Taking total derivative of the above equations:

$$d\ln y_i = -\gamma d\ln p_i$$

$$d\ln p_i = \frac{1}{\sigma}\left(d\ln y_i - d\ln l_i\right)$$

$$d\ln(1+\tau_{ki}) = -\frac{1}{\sigma} d\ln\left(\frac{k_i}{l_i}\right)$$

$$d\ln y_i - d\ln l_i = \left[\alpha + \frac{\sigma-1}{\sigma}\alpha(1-\alpha)\ln\left(\frac{k_i}{l_i}\right)\right] d\ln\left(\frac{k_i}{l_i}\right)$$

Combining the above 4 equations, we obtain

$$d\ln y_i = -\gamma\left[\alpha + \frac{\sigma-1}{\sigma}\alpha(1-\alpha)\ln\left(\frac{k_i}{l_i}\right)\right] d\ln\left(1+\tau_{ki}\right)$$

Appendix B: List of Manufacturing Industries and Values of σ_{ind}.

ISIC2 Classification Codes	Industry Name	σ_i
15	Construction - General Contractors and Operative Builders	0.410
17	Construction - Special Trade Contractors	0.410
18*	Garments	0.408
19	Other Manufacturing	0.246
20	Food and Kindred Products	0.078
21	Tobacco Products	0.312
22	Textile Mill Products	0.204
24	Lumber and Wood Products, Except Furniture	0.484
25	Furniture and Fixtures	0.203
26	Paper and Allied Products	0.148
27	Printing, Publishing and Allied Industries	0.484
28	Chemicals and Allied Products	0.210
29	Petroleum Refining and Related Industries	0.294
33	Primary Metal Industries	0.562
34	Fabricated Metal Products	0.401
36	Electronic and Other Electrical Equipment and Components	0.486
37	Transportation Equipment	0.419

Note: σ_{ind}'s are from Chirinko and Mallick (2017; Table 5, p. 248). *This ISIC2 code does not exactly match with the industry code in Chirinko and Mallick (2017), so the aggregate value of σ is imputed for this industry.

Appendix C: Misallocation and Output Gain for different values of σ relative to σ=1.

Country	Year	No. of firms	Distortions: $TFPR_i$ (log)		Aggregate output gain		MoR: SD of log($TFPR_i$)		σ_c
			σ=0.5/ σ=1	σ=1.5/ σ=1	σ=0.5/ σ=1	σ=1.5/ σ=1	σ=0.5/ σ=1	σ_c/ σ=1	
Afghanistan	2008	38	1.198	0.931	1.315	0.954	1.033		
Angola	2006	147	1.437	0.837	1.756	0.933	1.144		
Argentina	2006	338	1.139	0.951	1.281	0.956	1.059	1.182	0.112
Argentina	2010	511	1.207	0.931	1.541	0.908	1.118	1.325	0.112
Argentina	2017	45	1.385	0.878	1.097	0.981	0.991	1.376	0.112
Armenia	2009	41	1.207	0.915	1.270	0.953	1.082		
Azerbaijan	2009	68	1.259	0.890	1.108	0.998	1.006		
Bangladesh	2007	992	1.275	0.886	1.252	0.946	1.046	1.190	0.152
Bangladesh	2013	733	1.275	0.886	1.229	0.950	1.035	1.126	0.152
Belarus	2008	43	1.201	0.928	1.392	0.940	1.111		
Belarus	2013	52	1.139	0.950	1.344	0.925	1.064		
Bolivia	2006	159	1.270	0.885	1.020	1.031	1.015		
Bosnia and Herzegovina	2009	58	1.483	0.783	1.038	0.989	0.990		
Bosnia and Herzegovina	2013	44	1.400	0.828	1.536	0.854	1.117		
Botswana	2006	84	1.141	0.943	1.157	0.959	1.009		
Botswana	2010	38	1.207	0.908	1.146	0.959	1.011		
Brazil	2009	444	1.173	0.937	1.345	0.931	1.049	1.145	0.126
Bulgaria	2007	319	1.177	0.937	1.372	0.931	1.068		
Bulgaria	2009	58	1.199	0.919	1.462	0.916	1.065		
Bulgaria	2013	62	1.221	0.921	1.688	0.886	1.086		
Burundi	2006	66	1.309	0.889	1.749	0.903	1.083		
Cameroon	2009	63	1.158	0.938	1.201	1.004	1.044	1.184	0.089
Chile	2006	326	1.214	0.922	1.589	0.899	1.066	1.212	0.100
Chile	2010	534	1.202	0.925	1.226	0.975	1.093	1.252	0.100
China	2012	1195	1.151	0.939	1.391	0.918	1.041	1.313	0.548
Colombia	2006	473	1.160	0.944	1.309	0.937	1.061	1.196	0.147
Colombia	2010	510	1.172	0.938	1.292	0.953	1.059	1.188	0.147
Colombia	2017	100	1.259	0.912	1.115	0.969	1.009	1.647	0.147
Congo, DRC	2006	138	1.208	0.917	1.094	0.986	1.001		
Congo, DRC	2013	102	1.153	0.948	1.375	0.931	1.094		
Costa Rica	2010	163	1.273	0.895	1.283	0.963	1.042	1.173	0.114
Cote d'Ivoire	2009	67	1.126	0.955	1.458	0.948	1.042	1.127	0.144
Croatia	2007	209	1.199	0.925	1.235	0.949	1.033		
Croatia	2013	73	1.259	0.899	1.253	0.966	1.056		
Czech Republic	2009	38	1.184	0.928	1.455	0.843	1.087		
Czech Republic	2013	34	1.149	0.939	1.332	0.891	1.055		

Dominican Republic	2010	81	1.158	0.942	1.293	0.930	1.039	1.380	0.503
Ecuador	2006	204	1.185	0.928	1.521	0.898	1.066	1.178	0.126
Ecuador	2010	78	1.182	0.932	1.757	0.904	1.081	1.234	0.126
Egypt	2013	860	1.347	0.845	1.182	0.969	1.020		
Egypt	2016	153	1.184	0.925	1.215	0.939	1.040		
El Salvador	2006	242	1.245	0.910	1.432	0.924	1.081	1.246	0.191
El Salvador	2010	79	1.310	0.876	1.042	1.034	1.070	1.184	0.191
El Salvador	2016	60	1.706	0.767	1.328	0.943	1.040	1.139	0.191
Estonia	2009	58	1.203	0.923	1.544	0.854	1.119		
Estonia	2013	49	1.185	0.935	1.246	0.943	1.037		
Ethiopia	2011	45	1.202	0.913	1.045	1.002	0.983		
Ethiopia	2015	50	1.227	0.902	0.813	1.127	0.971		
Georgia	2008	52	1.273	0.891	1.456	0.861	1.066		
Georgia	2013	36	1.229	0.918	1.321	0.934	1.037		
Ghana	2007	237	1.206	0.930	1.320	0.950	1.103	1.278	0.141
Ghana	2013	85	1.142	0.940	1.051	1.003	1.024	1.836	0.141
Guatemala	2006	204	1.217	0.919	1.466	0.903	1.056	1.178	0.089
Guatemala	2010	158	1.235	0.912	1.318	0.968	1.049	1.177	0.089
Guinea	2006	89	1.172	0.945	1.650	0.880	1.125		
Guinea-Bissau	2006	32	1.438	0.838	1.611	0.892	1.190		
Honduras	2006	161	1.208	0.914	1.025	1.027	1.001	1.762	0.112
Honduras	2010	47	1.169	0.937	1.164	0.959	0.970	1.357	0.112
Hungary	2009	62	1.207	0.929	1.566	0.894	1.113		
India	2014	3734	1.210	0.911	1.261	0.934	1.010	1.866	0.515
Indonesia	2009	455	1.215	0.917	1.282	0.936	1.053		
Indonesia	2015	165	1.246	0.906	1.469	0.910	1.045		
Iraq	2011	358	1.208	0.909	1.171	0.943	0.999		
Israel	2013	98	1.230	0.915	1.982	0.853	1.157	1.417	0.136
Jamaica	2010	86	1.112	0.955	1.039	1.000	1.073		
Jordan	2013	144	1.185	0.921	1.192	0.982	1.027	1.626	0.331
Kazakhstan	2009	86	1.267	0.906	1.566	0.906	1.123		
Kazakhstan	2013	44	1.219	0.923	2.395	0.774	1.209		
Kenya	2007	318	1.239	0.895	1.067	1.017	1.025	1.918	0.094
Kenya	2013	122	1.166	0.932	1.191	0.987	1.035	1.113	0.094
Kosovo	2009	31	2.581	0.307	0.935	1.038	1.041		
Kyrgyz Republic	2009	34	1.211	0.906	0.954	1.039	1.010		
Kyrgyz Republic	2013	38	1.199	0.928	1.217	0.964	1.028		
Latvia	2009	56	1.150	0.946	1.340	0.929	1.069		
Lebanon	2013	83	1.224	0.906	1.091	0.994	1.008		
Lithuania	2009	57	1.167	0.940	1.389	0.906	1.085		
Lithuania	2013	37	1.218	0.926	1.671	0.881	1.139		
Macedonia, FYR	2009	62	1.266	0.893	1.430	0.939	1.052		
Macedonia, FYR	2013	70	1.242	0.908	1.268	0.948	1.013		

Madagascar	2009	99	1.258	0.900	1.866	0.892	1.084	1.622	0.565
Madagascar	2013	91	1.175	0.936	1.368	0.916	1.031	1.171	0.565
Malaysia	2015	143	1.181	0.919	1.297	0.878	1.048		
Mali	2007	240	1.228	0.925	1.585	0.903	1.164		
Mauritania	2006	67	1.569	0.773	1.197	1.067	1.085	1.292	0.098
Mauritius	2009	84	1.208	0.917	1.331	0.925	1.031		
Mexico	2006	646	1.175	0.930	1.212	0.947	1.043	1.134	0.087
Mexico	2010	881	1.190	0.928	1.405	0.929	1.059	1.195	0.087
Moldova	2009	77	1.167	0.933	1.131	0.982	1.010		
Moldova	2013	31	1.309	0.890	1.420	0.937	1.139		
Mongolia	2009	100	1.419	0.822	1.032	1.030	1.004		
Mongolia	2013	31	1.291	0.906	1.598	0.889	1.117		
Morocco	2013	46	1.153	0.943	0.981	1.030	0.992	1.643	0.102
Mozambique	2007	226	1.266	0.904	1.591	0.889	1.080		
Myanmar	2014	89	1.265	0.890	1.458	0.888	1.048		
Myanmar	2016	31	1.848	0.704	1.308	0.940	1.066		
Namibia	2006	77	1.208	0.911	1.093	0.987	0.988	0.996	0.200
Nepal	2009	66	1.307	0.869	1.139	0.989	1.016	1.836	0.563
Nepal	2013	158	1.130	0.948	1.097	0.989	1.008	1.370	0.563
Nicaragua	2006	164	1.220	0.917	1.363	0.965	1.022	1.119	0.084
Nicaragua	2010	41	1.175	0.944	1.824	0.880	1.150	1.343	0.084
Nigeria	2007	817	1.189	0.933	1.504	0.930	1.110	1.292	0.177
Pakistan	2007	76	1.142	0.954	1.361	0.941	1.080		
Pakistan	2013	89	1.178	0.927	1.123	0.982	1.030		
Panama	2006	87	1.245	0.900	1.497	0.915	1.012	1.618	0.265
Paraguay	2006	89	1.216	0.912	1.160	0.981	1.005		
Paraguay	2010	58	1.242	0.892	1.295	0.895	1.036		
Peru	2006	193	1.170	0.936	1.471	0.908	1.066		
Peru	2010	436	1.152	0.940	1.089	0.991	1.026		
Peru	2017	50	1.219	0.916	1.103	0.975	1.004		
Philippines	2009	305	1.154	0.937	1.133	0.979	1.027		
Philippines	2015	130	1.251	0.906	1.336	0.929	1.030		
Poland	2009	52	1.176	0.930	1.354	0.922	1.076		
Romania	2009	52	1.256	0.906	1.193	0.980	1.089	1.272	0.085
Romania	2013	82	1.229	0.915	1.227	0.970	1.064	1.191	0.085
Russian Federation	2009	235	1.147	0.949	1.299	0.934	1.044		
Russian Federation	2012	374	1.190	0.931	1.375	0.935	1.105		
Rwanda	2006	42	1.256	0.900	1.278	0.940	1.056		
Senegal	2007	204	1.205	0.923	1.371	0.931	1.065		
Serbia	2009	88	1.208	0.912	0.974	1.022	0.988		
Serbia	2013	53	1.155	0.938	1.098	0.972	1.021		
Sierra Leone	2017	42	1.104	0.964	1.355	0.915	1.085		
Slovak Republic	2009	37	1.202	0.918	1.603	0.872	1.109		
Slovenia	2009	71	1.179	0.929	1.100	0.995	1.022		

Slovenia	2013	51	1.287	0.887	1.820	0.866	1.092		
South Africa	2007	574	1.232	0.917	1.490	0.916	1.112		
South Sudan	2014	39	1.105	0.961	1.154	0.968	1.066		
Sri Lanka	2011	181	1.291	0.880	1.355	0.925	1.025	1.366	0.428
Swaziland	2006	57	1.166	0.941	1.386	0.930	1.058		
Sweden	2014	99	1.232	0.926	1.678	0.879	1.148		
Tajikistan	2008	37	1.239	0.898	1.420	0.938	1.011		
Tanzania	2006	168	1.213	0.916	1.090	1.008	1.016		
Tanzania	2013	98	1.229	0.904	1.426	0.901	1.061		
Thailand	2016	171	1.314	0.895	1.332	0.951	1.056	1.130	0.197
Trinidad and Tobago	2010	74	1.179	0.931	1.446	0.912	1.046		
Tunisia	2013	197	1.285	0.884	1.372	0.921	1.019	1.911	0.134
Turkey	2008	341	1.171	0.928	1.218	0.933	1.017	1.572	0.686
Turkey	2013	253	1.178	0.930	1.361	0.926	1.077	1.345	0.686
Uganda	2006	206	1.331	0.869	1.092	1.024	1.073		
Uganda	2013	39	1.105	0.956	1.087	0.993	1.018		
Ukraine	2008	146	1.169	0.937	1.140	0.970	1.025		
Ukraine	2013	188	1.230	0.911	1.344	0.927	1.062		
Uruguay	2006	136	1.175	0.938	1.293	0.950	1.054		
Uruguay	2010	137	1.198	0.931	1.931	0.868	1.090		
Uzbekistan	2008	69	1.171	0.933	1.434	0.904	1.025		
Uzbekistan	2013	71	1.249	0.894	1.264	0.927	1.012		
Vietnam	2009	515	1.227	0.906	1.303	0.917	1.022		
Vietnam	2015	99	1.248	0.902	1.488	0.881	1.041		
West Bank And Gaza	2013	50	1.161	0.936	1.203	0.949	1.003		
Yemen	2010	65	1.255	0.905	1.221	0.959	1.008		
Zambia	2007	239	1.199	0.921	1.132	0.978	1.051	1.178	0.133
Zambia	2013	89	1.114	0.955	1.190	0.959	1.035	1.116	0.133
Zimbabwe	2011	233	1.272	0.875	1.057	0.996	0.996	1.194	0.259
Zimbabwe	2016	46	1.159	0.930	1.183	0.953	1.010	1.357	0.259

Note: σ_c = Aggregate value of σ at the country level (Mallick, 2012); σ_c is the same in all industries but varies across countries.

Appendix Table A1: Distortions, misallocation and aggregate output gain for different values of σ relative to σ=1 (Mean of the Ratios).

	MoC	Distortions: $\ln(TFPR_i)$	MoR	Aggregate Output Gain	Aggregate Efficient TFPQ
σ=0.2		1.491 (0.285)	1.145	1.718	1.741
σ ≈ 0.34*		1.369 (0.221)	1.103	1.512	1.501
σ=0.5		1.235 (0.145)	1.053	1.277	1.270
σ=1.5		0.909 (0.061)	1.001	0.940	0.931
Heterogeneous σ_{ind}	4.328	1.365 (0.213)	1.099	---	-----

*Mean value of heterogeneous σ_{ind}'s is 0.3353529 (results compared with homogenous σ=0.3353529). Figures in parentheses are standard deviations.

Appendix Table A2: Misallocation and aggregate output gain for different values of σ relative to σ=1. (γ = 5).

	Aggregate Output Gain	Aggregate Efficient TFPQ
σ=0.2	1.862	1.862
σ ≈ 0.34*	1.522	1.397
σ=0.5	1.302	1.288
σ=1.5	0.949	0.934

*Mean value of heterogeneous σ_{ind}'s is 0.3353529 (results compared with homogenous σ=0.3353529).

Appendix Table A3: Misallocation and Aggregate Output Gain for different values of σ relative to σ=1. (γ = 3; TFP deleted 5%).

	MoC	Distortions: $\ln(TFPR_i)$	MoR	Aggregate Output Gain	Aggregate Efficient TFPQ
σ=0.2		1.433	1.181	1.793	1.591
σ ≈ 0.34*		1.324	1.130	1.568	1.402
σ=0.5		1.205	1.068	1.308	1.222
σ=1.5		0.921	1.000	0.934	0.938
Heterogeneous σ_{ind}	4.272	1.320	1.127	---	-----

*Mean value of heterogeneous σ_{ind}'s is 0.3353529 (results compared with homogenous σ=0.3353529).

Appendix Table A4: Misallocation and Aggregate Output Gain for different values of σ relative to σ=1 (Number of firms >=100).

	MoC	Distortions: $\ln(TFPR_i)$	MoR	Aggregate Output Gain	Aggregate Efficient TFPQ
σ=0.2		1.439	1.135	1.507	1.382
σ ≈ 0.34*		1.329	1.096	1.277	1.211
σ=0.5		1.209	1.049	0.942	0.939
σ=1.5		0.919	1.002	1.507	1.382
Heterogeneous σ_{ind}	3.996	1.314	1.087	---	-----

*Mean value of heterogeneous σ_{ind}'s is 0.3353529 (results compared with homogenous σ=0.3353529). Number of country-year = 64.