Robust Inference by Sub-sampling

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Abstract

This paper provides a simple technique of carrying out inference robust to serial correlation, heteroskedasticity and spatial correlation on the estimators which follow an asymptotic normal distribution. The idea is based on the fact that the estimates from a larger sample tend to have a smaller variance which can be expressed as a function of the variance of the estimator from smaller subsamples. The major advantage of the technique other than the ease of application and simplicity is its finite sample performance both in terms of the empirical null rejection probability as well as the power of the test. It does not restrict the data in terms of structure in any way and works pretty well for any kind of heteroskedasticity, autocorrelation and spatial correlation in a finite sample. Furthermore, unlike theoretical HAC robust techniques available in the existing literature, it does not require any kernel estimation and hence eliminates the discretion of the analyst to choose a specific kernel and bandwidth. The technique outperforms the Ibragimov and Müller (2010) approach in terms of null rejection probability as well as the local asymptotic power of the test.

Keywords: HAC, Spatial Correlation, Robust, Inference

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1 Introduction

The existing econometrics literature has a long history of estimation procedures for heteroskedasticity and autocorrelation consistent (HAC) variance-covariance estimators and the asymptotic theory regarding the use of these estimators for HAC robust inference. The major contributions include White (1984), Newey and West (1987), Gallant (1987), Andrews (1991), Andrews and Monahan (1992), Robinson (1998), de Jong and Davidson (2000), Jansson (2002) and Kiefer and Vogelsang (2005). There has also been some literature regarding the inference robust to spatial correlation, such as Kelejian and Prucha (2001), Ibragimov and Müller (2010), Driscoll and Kraay (1998), Alan Bester, Conley, Hansen and Vogelsang (2009), Dale and Fortin (2009), Cameron and Miller (2010), Ibragimov and Müller (2010) and Vogelsang (2012), etc.

There are several papers in the literature which give overviews of various aspects of bootstrapping time series. Among them are Hongyi Li and Maddala (1996), Berkowitz and Kilian (2000), Bühlmann (2002), Ruiz and Pascual (2002), Härdle, Horowitz and Kreiss (2003), and Paparoditis and Politis (2009). These papers suggest that even though there are some promising bootstrap methods available for time series data, however, there is a considerable need for further research in the application of the bootstrap to time series. There may be instances where the bootstrap procedures used are not adequate. Although bootstrapping is (under some conditions) asymptotically consistent, it does not provide general finite-sample guarantees.

In Ibragimov and Müller (2010), an approach to robust inference has been developed with efficiency in terms of local asymptotic power. The power of the test varies with the choice of the number of groups, i.e., q and it is not possible to use data dependent methods to determine an appropriate q, which leaves a lot of ambiguity. There certainly are instances, where choosing $q = 2$ would lead to acceptance of the null as the critical value for $q = 2$ with one degree of freedom is 12.706 leading to a pretty wide confidence interval, and a higher value of q with better local asymptotic power (and worse null rejection probability) would lead to rejection of the null.

This paper provides a simple technique of carrying out inference robust to serial correlation, heteroskedasticity and spatial correlation on the estimators which follow an asymptotic normal distribution. The technique outperforms the approach of Ibragimov and Müller (2010) in terms of null rejection probability as well as the local asypmtotic power. The idea is based on the fact that the estimates from a larger sample tend to have a smaller variance which can be expressed as a function of the variance of the estimator from smaller subsamples. The major advantage of the technique other than the ease of application and simplicity is its finite sample performance both in terms of the empirical null rejection probability as well as the power of the test. It does not restrict the data in terms of structure in any way and works pretty well for any kind of heteroskedasticity, autocorrelation and spatial correlation in a finite sample. Furthermore, unlike theoretical HAC robust techniques available in the exisiting literature, it does not require any kernel estimation and hence eliminates the discretion of the analyst to choose a specific kernel and bandwidth. Unlike Ibragimov and Müller (2010), it provides a unique rule for the estimation of standard errors and the confidence intervals irrespective of the structure of the data leaving no room for ambiguity. The remainder of this paper is organized as follows: Section 2 provides the details of the variance estimator. Section 3 discusses the inference procedure and the asymptotic properties of the t-statistics. Section 4 presents the finite sample null rejection probabilities and the power of the test through Monte Carlo simulations. Section 5 concludes.

2 Variance Estimator

Suppose that the sample mean of time series, Y_t follows an asymptotic normal distribution, i.e.,

$$a_T(\overline{Y}_T - \mu) \xrightarrow{d} N(0, V),$$

where a_T is the scaling factor, and is of order T^γ, where $\gamma > 0$. An estimator of variance of $\overline{Y}_T$ can be written as

$$\widehat{Var\left[\overline{Y}_T\right]} = \frac{\widehat{V}_T}{a_T^2}.$$

The estimator of variance of mean of a subsample of size t can be written as

$$\widehat{Var\left[\overline{Y}_t\right]} = \frac{\widehat{V}_t}{a_t^2}.$$

Similarly the variance estimator of mean of a subsample of size τ can be written as

$$\widehat{Var\left[\overline{Y}_\tau\right]} = \frac{\widehat{V}_\tau}{a_\tau^2}.$$

If the ratio T/τ is the same as τ/t, then

$$\frac{P\lim\left[\widehat{Var\left[\overline{Y}_T\right]}\right]}{P\lim\left[\widehat{Var\left[\overline{Y}_\tau\right]}\right]} = \frac{P\lim\left[\widehat{Var\left[\overline{Y}_\tau\right]}\right]}{P\lim\left[\widehat{Var\left[\overline{Y}_t\right]}\right]},$$

which implies that

$$P\lim\left[\widehat{Var\left[\overline{Y}_T\right]}\right] = P\lim\frac{\left[\widehat{Var\left[\overline{Y}_\tau\right]}\right]^2}{\left[\widehat{Var\left[\overline{Y}_t\right]}\right]}. \tag{1}$$

Therefore, if we can construct subsamples of size τ and t, then immediate, empirical estimators of $Var\left[\overline{Y}_\tau\right]$ and $Var\left[\overline{Y}_t\right]$ can be constructed from the many size-τ and t sample means respectively that can be extracted from our data $Y_1, Y_2, ..., Y_T$. Furthermore, in order to use the empirical formulas

$$\widehat{Var\left[\overline{Y}_\tau\right]} = \frac{1}{K-1}\sum_{i=1}^{K}(\overline{Y}_{\tau i} - \overline{\overline{Y}}_\tau)^2,$$
$$\widehat{Var\left[\overline{Y}_t\right]} = \frac{1}{J-1}\sum_{i=1}^{J}(\overline{Y}_{ti} - \overline{\overline{Y}}_t)^2,$$

the sample means from the subsamples must be asymptotically i.i.d (the conditions mentioned in Ibragimov and Müller (2010)), which implies that there cannot be overlapping observations in two subsamples. Also

$$P\lim\left[\widehat{Var\left[\overline{Y}_T\right]}\right] = P\lim\frac{\left[\widehat{Var\left[\overline{Y}_\tau\right]}\right]}{a_T^2/a_\tau^2}. \tag{2}$$

Equations (1) and (2) provide two consistent estimators for $Var\left[\overline{Y}_T\right]$, however, their performance in finite samples remain to be seen, which will be presented later in this paper. For an improved finite sample performance, an average of the two estimators is taken to construct the standard error for $\overline{Y}_T$.

$$se\overline{Y}_T = \frac{1}{2}\left[\frac{\widehat{Var\left[\overline{Y}_\tau\right]}}{\sqrt{\widehat{Var\left[\overline{Y}_t\right]}}} + \frac{\sqrt{\widehat{Var\left[\overline{Y}_\tau\right]}}}{a_T/a_\tau}\right]. \tag{3}$$

For the panel data, suppose that the sample mean of data, Y_{it} follows an asymptotic normal distribution, i.e.,

$$a_{NT}(\overline{Y}_{NT} - \mu) \xrightarrow{d} N(0, V),$$

where a_{NT} is the scaling factor. Under $T \to \infty$, $N \to \infty$, asymptotics, the sample means from the subsamples must be asymptotically i.i.d. The assumption of an asymptotic normal distribution for the sample mean rules out the non-stationarity in either dimension, i.e., T as well as N. An estimator of variance of $\overline{Y}_{NT}$ can be written as

$$\widehat{Var\left[\overline{Y}_{NT}\right]} = \frac{\widehat{V}_{NT}}{a_{NT}^2}.$$

The estimator of variance of mean of a subsample of size $\check{n}t$ can be written as

$$\widehat{Var\left[\overline{Y}_{\check{n}t}\right]} = \frac{\widehat{V}_{\check{n}t}}{a_{\check{n}t}^2}.$$

Similarly the variance estimator of mean of a subsample of size $ŋ\tau$ can be written as

$$\widehat{Var\left[\overline{Y}_{ŋ\tau}\right]} = \frac{\widehat{V}_{ŋ\tau}}{a_{ŋ\tau}^2}.$$

If the ratio $NT/ŋ\tau$ is the same as $ŋ\tau/\check{n}t$, then

$$\frac{P\lim\left[\widehat{Var\left[\overline{Y}_{NT}\right]}\right]}{P\lim\left[\widehat{Var\left[\overline{Y}_{ŋ\tau}\right]}\right]} = \frac{P\lim\left[\widehat{Var\left[\overline{Y}_{ŋ\tau}\right]}\right]}{P\lim\left[\widehat{Var\left[\overline{Y}_{\check{n}t}\right]}\right]},$$

which implies that

$$P\lim\left[\widehat{Var\left[\overline{Y}_{NT}\right]}\right] = P\lim\frac{\left[\widehat{Var\left[\overline{Y}_{ŋ\tau}\right]}\right]^2}{\left[\widehat{Var\left[\overline{Y}_{\check{n}t}\right]}\right]}. \tag{4}$$

Therefore, if we can construct subsamples of size $ŋ\tau$ and $\check{n}t$, then immediate, empirical estimators of $Var\left[\overline{Y}_{ŋ\tau}\right]$ and $Var\left[\overline{Y}_{\check{n}t}\right]$ can be constructed from the many size-$ŋ\tau$ and $\check{n}t$ sample means respectively that can be extracted from our data $Y_{11}, Y_{12}, ...Y_{21}, Y_{22}, ..., Y_{NT}$. Furthermore, in order to use the empirical formulas

$$\widehat{Var\left[\overline{Y}_{ŋ\tau}\right]} = \frac{1}{K-1}\sum_{l=1}^{K}(\overline{Y}_{ŋ\tau l} - \overline{\overline{Y}}_{ŋ\tau})^2,$$

$$\widehat{Var\left[\overline{Y}_{\check{n}t}\right]} = \frac{1}{J-1}\sum_{m=1}^{J}(\overline{Y}_{\check{n}tm} - \overline{\overline{Y}}_{\check{n}t})^2,$$

the sample means from the subsamples must be i.i.d, which implies that there cannot be overlapping observations in two subsamples. Also

$$P\lim\left[\widehat{Var\left[\overline{Y}_{NT}\right]}\right] = P\lim\frac{\left[\widehat{Var\left[\overline{Y}_{ŋ\tau}\right]}\right]}{a_{NT}^2/a_{ŋ\tau}^2}. \tag{5}$$

Equations (4) and (5) provide two consistent estimators for $Var\left[\overline{Y}_{NT}\right]$, however, their performance in finite samples remain to be seen, which will be presented later in this paper. For an improved finite sample performance, an average of the two estimators is taken to construct the standard error for $\overline{Y}_{NT}$.

$$se\overline{Y}_{NT} = \frac{1}{2}\left[\frac{\widehat{Var\left[\overline{Y}_{ŋ\tau}\right]}}{\sqrt{\widehat{Var\left[\overline{Y}_{\check{n}t}\right]}}} + \frac{\sqrt{\widehat{Var\left[\overline{Y}_{ŋ\tau}\right]}}}{a_{NT}/a_{ŋ\tau}}\right]. \tag{6}$$

3 Inference and Asymptotic Results for t-statistic

Suppose we are interested in testing the null hypothesis

$$H_0 : \mu = \mu_0, \tag{7}$$

against the alternative hypothesis

$$H_1 : \mu = \mu_1 \neq \mu_0. \tag{8}$$

It is straightforward to construct the test statistic for time series data as

$$t = \frac{\overline{Y}_T - \mu_0}{se\overline{Y}_T}, \tag{9}$$

and for panel data as

$$t = \frac{\overline{Y}_{NT} - \mu_0}{se\overline{Y}_{NT}}. \tag{10}$$

The alternative value of μ_1 is modeled local to μ_0 as

$$\mu_1 = \mu_0 + a_T^{-1}\overline{\mu}_\Delta, \tag{11}$$

and

$$\mu_1 = \mu_0 + a_{NT}^{-1}\overline{\mu}_\Delta, \tag{12}$$

for time series and panel data respectively.

The parameter $\overline{\mu}_\Delta$ measures the magnitude of the departure from the null under the local alternative. The standard error, i.e., $se\overline{Y}_T$ to be used in expression (9) for time series data is calculated as follows:

Initially subsamples of size K and J are chosen in sequence without any overlapping data maintaining the structure of the data involved, e.g., for $T = 100$, 10 subsamples of size 10 are formed by choosing T from1 to 10, 11 to 20, 21 to 30, 31 to 40, 41 to 50, 51 to 60, 61 to 70, 71 to 80, 81 to 90, and 91 to 100. Sample variance for $T = 10$ is calculated from the ten sample means. As a second draw, the first observation is moved to the 100th place and the 100th observation takes the 99th place, and so on. For the third draw, the second observation takes the 100th place, the first observation takes the 99th place, the 100th observation takes the 98th place, and so on. By rotating the placement of the data observations in this manner, we are able to have one hundred draws, i.e., equal to the number of data points. Through each draw, we calculate sample variance for $T = 10$ from the ten sample means. In this way, we have 100 values of the sample variance for a sample size $T = 10$. Taking an average of these 100 values, we get an estimate of the variance of mean of sample size $T = 10$ (Note: when T/τ is an integer, the number of unique draws is equal to τ instead of T).

By choosing different values of τ and t such that the ratio T/τ is the same as τ/t, we estimate the standard error for the mean of sample size T using eq. (3). For various combinations of τ/t, we get different values of the standard error for the mean of sample size T. By taking an average of those standard errors, we construct a standard error for the mean of sample size T. The following formula has been used for calculating the sequence of standard errors for the mean of sample size T.

$$se_M = \frac{1}{2M}\sum_{i=1}^{M}\left[\frac{\widehat{Var\left[\overline{Y}_{\tau_i}\right]}}{\sqrt{\widehat{Var\left[\overline{Y}_{t_i}\right]}}} + \frac{\sqrt{\widehat{Var\left[\overline{Y}_{\tau_i}\right]}}}{a_T/a_{\tau_i}}\right], \tag{13}$$

where $\tau_1 = 10$, $t_1 = 1$; $\tau_2 = 14$, $t_2 = 2$; $\tau_3 = 17$, $t_3 = 3$; $\tau_4 = 20$, $t_4 = 4$; $\tau_5 = 22$, $t_5 = 5$; $\tau_6 = 24$, $t_6 = 6$; $\tau_7 = 26$, $t_7 = 7$; $\tau_8 = 28$, $t_8 = 8$; $\tau_9 = 30$, $t_9 = 9$; $\tau_{10} = 32$, $t_{10} = 10$; $\tau_{11} = 33$, $t_{11} = 11$; $\tau_{12} = 35$, $t_{12} = 12$; $\tau_{13} = 36$, $t_{13} = 13$; $\tau_{14} = 37$, $t_{14} = 14$; $\tau_{15} = 39$, $t_{15} = 15$; $\tau_{16} = 40$, $t_{16} = 16$; $\tau_{17} = 41$, $t_{17} = 17$; $\tau_{18} = 42$, $t_{18} = 18$; $\tau_{19} = 44$, $t_{19} = 19$; $\tau_{20} = 45$, $t_{20} = 20$; $\tau_{21} = 46$, $t_{21} = 21$; $\tau_{22} = 47$, $t_{22} = 22$; $\tau_{23} = 48$, $t_{23} = 23$; $\tau_{24} = 49$, $t_{24} = 24$; $\tau_{25} = 50$, $t_{25} = 25$ for $T = 100$, and $\tau_1 = 14$, $t_1 = 1$; $\tau_2 = 20$, $t_2 = 2$; $\tau_3 = 24$, $t_3 = 3$; $\tau_4 = 28$, $t_4 = 4$; $\tau_5 = 32$, $t_5 = 5$; $\tau_6 = 35$, $t_6 = 6$; $\tau_7 = 37$, $t_7 = 7$; $\tau_8 = 40$, $t_8 = 8$; $\tau_9 = 42$, $t_9 = 9$; $\tau_{10} = 45$, $t_{10} = 10$; $\tau_{11} = 47$, $t_{11} = 11$; $\tau_{12} = 49$, $t_{12} = 12$; $\tau_{13} = 51$, $t_{13} = 13$; $\tau_{14} = 53$, $t_{14} = 14$; $\tau_{15} = 55$, $t_{15} = 15$; $\tau_{16} = 57$, $t_{16} = 16$; $\tau_{17} = 58$, $t_{17} = 17$; $\tau_{18} = 60$, $t_{18} = 18$; $\tau_{19} = 62$, $t_{19} = 19$; $\tau_{20} = 63$, $t_{20} = 20$; $\tau_{21} = 65$, $t_{21} = 21$; $\tau_{22} = 66$, $t_{22} = 22$; $\tau_{23} = 68$, $t_{23} = 23$; $\tau_{24} = 69$, $t_{24} = 24$; $\tau_{25} = 71$, $t_{25} = 25$; $\tau_{26} = 72$, $t_{26} = 26$; $\tau_{27} = 73$, $t_{27} = 27$; $\tau_{28} = 75$, $t_{28} = 28$; $\tau_{29} = 76$, $t_{29} = 29$; $\tau_{30} = 77$, $t_{30} = 30$; $\tau_{31} = 79$, $t_{31} = 31$; $\tau_{32} = 80$, $t_{32} = 32$; $\tau_{33} = 81$, $t_{33} = 33$; $\tau_{34} = 82$, $t_{34} = 34$; $\tau_{35} = 84$, $t_{35} = 35$; $\tau_{36} = 85$, $t_{36} = 36$; $\tau_{37} = 86$, $t_{37} = 37$; $\tau_{38} = 87$, $t_{38} = 38$; $\tau_{39} = 88$, $t_{39} = 39$; $\tau_{40} = 89$, $t_{40} = 40$; $\tau_{41} = 91$, $t_{41} = 41$; $\tau_{42} = 92$, $t_{42} = 42$; $\tau_{43} = 93$, $t_{43} = 43$; $\tau_{44} = 94$, $t_{44} = 44$; $\tau_{45} = 95$, $t_{45} = 45$; $\tau_{46} = 96$, $t_{46} = 46$; $\tau_{47} = 97$, $t_{47} = 47$; $\tau_{48} = 98$, $t_{48} = 48$; $\tau_{49} = 99$, $t_{49} = 49$; $\tau_{50} = 100$, $t_{50} = 50$ for $T = 200$.

For panel data, in order to calculate the standard error, i.e., $se\overline{Y}_{NT}$, the data is arranged as follows: $Y_{11}, Y_{12}, ...Y_{21}, Y_{22}, ..., Y_{NT}$. The subsamples of size K and J are drawn in a similar manner as described above. By choosing different values of $ŋ\tau$ and $\check{n}t$ such that the ratio $NT/ŋ\tau$ is the same as $ŋ\tau/\check{n}t$, we estimate the standard error for the mean of sample size NT using eq. (6). For various combinations of $ŋ\tau/\check{n}t$, we get different values of the standard error for the mean of sample size NT. By taking an average of those standard errors, we construct a standard error for the mean of sample size NT. The following formula has been used for calculating the sequence of standard errors for the mean of sample size NT.

$$se_M = \frac{1}{2M}\sum_{m=1}^{M}\left[\frac{\widehat{Var\left[\overline{Y}_{(\c{n}\tau)_m}\right]}}{\sqrt{\widehat{Var\left[\overline{Y}_{(\check{n}t)_m}\right]}}} + \frac{\sqrt{\widehat{Var\left[\overline{Y}_{(\c{n}\tau)_m}\right]}}}{a_{NT}/a_{(\c{n}\tau)_m}}\right]. \tag{14}$$

A sequence of t-statistics in eq. (9) can be written as follows:

$$t_M = \frac{a_T(\overline{Y}_T - \mu_1) + \overline{\mu}_\Delta}{\frac{a_T}{2M}\sum_{i=1}^{M}\left[\frac{\widehat{Var[\overline{Y}_{\tau_i}]}}{\sqrt{\widehat{Var[\overline{Y}_{t_i}]}}} + \frac{\sqrt{\widehat{Var[\overline{Y}_{\tau_i}]}}}{a_T/a_{\tau_i}}\right]}. \tag{15}$$

The above expression can be written as

$$t_M = \frac{a_T(\overline{Y}_T - \mu_1)/\sqrt{V_{\mu_1}} + \overline{\mu}_\Delta/\sqrt{V_{\mu_1}}}{\frac{a_T}{2M}\sum_{i=1}^{M}\left[\frac{a_{ti}}{a_{\tau_i}^2}\frac{a_{\tau_i}^2\widehat{Var[\overline{Y}_{\tau_i}]}/V_{\mu_1}}{a_{ti}\sqrt{\widehat{Var[\overline{Y}_{t_i}]}}/\sqrt{V_{\mu_1}}} + \frac{a_{\tau_i}^2\sqrt{\widehat{Var[\overline{Y}_{\tau_i}]}}/\sqrt{V_{\mu_1}}}{a_T}\right]},$$

where

$$a_T(\overline{Y}_T - \mu_1)/\sqrt{V_{\mu_1}} \Rightarrow Z,$$

$$Z \sim N(0,1),$$

$$a_{\tau_i}^2\widehat{Var\left[\overline{Y}_{\tau_i}\right]}/V_{\mu_1} \Rightarrow \varkappa_{K_i-1}^2/(K_i - 1),$$

$$a_{ti}\sqrt{\widehat{Var\left[\overline{Y}_{ti}\right]}}/\sqrt{V_{\mu_1}} \Rightarrow \sqrt{\varkappa_{J_i-1}^2/(J_i - 1)},$$

$$a_T.\frac{a_{ti}}{a_{\tau_i}^2} = 1.$$

The denominator in eq. (15) is a function of random variables having some specific asymptotic distributions. Furthermore, those random variables have the same functional dependence irrespective of the data type involved by virtue of the assumption $a_T(\overline{Y}_T - \mu) \xrightarrow{d} N(0, V)$, e.g., the ratio $\frac{\widehat{Var[\overline{Y}_{\tau_i}]}}{\sqrt{\widehat{Var[\overline{Y}_{t_i}]}}}$ contains random variables $\widehat{Var\left[\overline{Y}_{\tau_i}\right]}$ and $\widehat{Var\left[\overline{Y}_{t_i}\right]}$ which have the same functional dependence irrespective of the data type involved. Similarly the random variables $\frac{\widehat{Var[\overline{Y}_{\tau_i}]}}{\sqrt{\widehat{Var[\overline{Y}_{t_i}]}}}$ and $\frac{\sqrt{\widehat{Var[\overline{Y}_{\tau_i}]}}}{a_T/a_{\tau_i}}$ have the same functional dependence, therefore even though the distribution for the t-statistic is hard to derive, the critical values for the unknown distribution of the t-statistic can be

easily simulated by an i.i.d data generating process provided that the subsampling scheme remains the same. As the chi squared distributions in the denominator capture the sizes of the subsamples chosen by the practitioner, the t-statistic should perform well in finite samples.
When $\overline{\mu}_{\Delta} \neq 0$, in which case we are under the alternative, the t-statistic has an additional term in the limit which pushes the distribution away from the null distribution giving the test's power. The greater the departure from the null, the higher should be the power.
The asymptotic theory for the sample mean also applies to the estimators of regression parameters provided that they satisfy the assumptions stipulated for the sample mean. Consider the regression model

$$y_{it} = x_{it}^{'}\beta + \epsilon_{it},\ i = 1, ..., N;\ t = 1, ..., T,$$

where y_{it} is a scalar, x_{it} is a $(k \times 1)$ vector of regressors, β is a $(k \times 1)$ vector of regression parameters, and ϵ_{it} is the regression error. Suppose that $\widehat{\beta}$ follow an asymptotic normal distribution, i.e.,

$$a_{NT}(\widehat{\beta} - \beta) \xrightarrow{d} N(0, V),$$

where a_{NT} is the scaling factor. Under $T \to \infty$, $N \to \infty$, asymptotics, the estimates of β from the subsamples must be asymptotically i.i.d. The assumption of an asymptotic normal distribution for $\widehat{\beta}$ rules out non-stationarity in regressors as well as the dependent variable in either dimension, i.e., T as well as N.

4 Finite Sample Null Rejection Probabilities and Power

Using DGP's with different data structure, the simulated finite sample null rejection probabilities and power of the t-statistic in comparison with the Ibragimov and Müller (2010) (abbreviated as IM) approach are reported in this section. Tables 1-13 report null rejection probabilities for 5% nominal level tests for testing $H_0 : \mu = \mu_0 = 0$ against the two-sided alternative $H_1 : \mu \neq 0$. Results are reported for $T = 100, 200$, and 1000 replications are used in all these cases. Using eq. (13) for the standard error, the results have been reported for se_1 to se_{10}. The results have also been reported for se_{avg} which is calculated as follows:

$$se_{avg} = \frac{1}{N}\sum_{M=1}^{N} se_M,$$

where N is the maximum possible value of M, e.g., for $T = 100$, the maximum possible value of M is 25, therefore M takes values from 1 to 25 in the above mentioned formula. For a comparison with Ibragimov and Müller (2010) (IM) approach, different values of q (notation in their paper), i.e., the number of groups are chosen. For the panel data, eq. (14) is used for the calculation of standard errors. We simulate the critical values for the test statistic through an i.i.d process by subsampling technique explained in section 3, e.g., for estimating only the intercept in regression, we use the following data generating process:

$$y_i = \mu + \epsilon_i, \epsilon_i \sim N(0, 1).$$

The values of standard errors are reported for different values of M for comparison with the standard deviation of the sample mean. The standard errors are pretty accurate for almost all values of M reported in the tables, however, for inference purposes se_{avg} seems to provide the most reliable and robust formula for the calculation of standard error in finite samples. The results suggest that the performance of se_{avg} to construct a test statistic is better than all other se_M in terms of the empirical null rejection probability as well as the width of the confidence interval. The confidence intervals constructed through se_{avg} have the minimum width in nearly all the cases reported in tables 1-12. Tables 11-12 report the results for heteroskedasticity and the panel data with serial correlation, spatial correlation and heteroskedasticity altogether.

As is evident from the tables that for the IM approach, the null rejection probability is the most accurate for $q = 2$, however, the critical value in this case is 12.706, leading to a pretty wide confidence interval and lower local asymptotic power. From $q = 4$ onwards, the IM approach shows overrejections when the serial correlation is strong and underrejections when the data is heteroskedastic and/or panel.

Table 13 shows the finite sample power performance of the test in comparison with IM approach ($q = 4$) for time series data (AR(1) process) with different magnitudes of serial correlation. The results are reported for $T = 100$, and 1000 replications are used in each case. The power of the test increases rapidly as we move father away from the null, with higher power for lower magnitude of the serial correlation as compared to the higher one, the reason being that the variance increases as the serial correlation becomes stronger. The IM approach has much lower power.

Table 14 presents results for regression parameters and a comparison is also drawn with White (1984), and Newey and West (1987).[1]Table 15 presents results for 1000 data points.

5 Conclusion:

In this paper a simple technique for carrying out inference robust to serial correlation, heteroskedasticity and spatial correlation on the estimators which follow an asymptotic normal distribution has been devised. The standard error of the sample mean from a larger sample size has been expressed as a function of the standard errors of the sample means from smaller subsamples. The Monte Carlo simulation results show that the technique works pretty well in finite samples both in terms

[1]Table 14 is prepared as per suggestions of anonymous referees.

of the empirical null rejection probability as well as the power of the test. The technique is extremely simple and can be programmed in any statistical software for ease of application just like an i.i.d. bootstrap. The technique works pretty well for any kind of data structure in terms of heteroskedasticity, autocorrelation and spatial correlation in a finite sample. For time series data, it does not require any kernel estimation (unlike theoretical HAC robust techniques available in the existing literature) and eliminates the need for bandwidth choice procedures. The technique outperforms the Ibragimov and Müller (2010) approach in terms of null rejection probability as well as the local asypmtotic power of the test.

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Table 1: A Comparison of the standard deviation of the sample mean with the mean value of the standard error calculated by eq. (13) through 1000 replications, and empirical null rejection probabilities for the test statistic and IM approach. Subscript 1 is for $T = 100$ and 2 for $T = 200$. DGP: $y_t = \mu + \phi_1 y_{t-1} + \epsilon_t$, $\epsilon_t \sim N(0, 1)$; $\mu = 0$.

$\phi_1 = 0$	CV_1	St. Dev_1	St. $Error_1$	CI_1	Rej. Pr_1	q	IM_1
se_1	2.4443936	0.0964563	0.0947298	$[-0.2351893, 0.2279247]$	0.042		–
se_2	2.3328373		0.0990655	$[-0.234736, 0.2274714]$	0.045	2	0.054
se_3	2.4233392		0.0968687	$[-0.238378, 0.2311134]$	0.048		–
se_4	2.5100734		0.094756	$[-0.2414769, 0.2342123]$	0.053	4	0.044
se_5	2.5143351		0.0953288	$[-0.2433208, 0.2360562]$	0.047	5	0.041
se_6	2.4522659		0.0971599	$[-0.2418941, 0.2346295]$	0.042		–
se_7	2.5034686		0.0959467	$[-0.2438318, 0.2365672]$	0.045		–
se_8	2.5178804		0.0959016	$[-0.245101, 0.2378364]$	0.046		–
se_9	2.5029468		0.0964672	$[-0.2450845, 0.2378199]$	0.038		–
se_{10}	2.4459803		0.0979359	$[-0.2431817, 0.2359171]$	0.045	10	0.047
se_{avg}	2.4174228		0.098128	$[-0.2408491, 0.2335845]$	0.044		
$\phi_1 = 0$	CV_2	St. Dev_2	St. $Error_2$	CI_2	Rej. Pr_2	q	IM_2
se_1	1.3287834	0.069950	0.0693725	$[-0.1562712, 0.0374605]$	0.053		–
se_2	1.3839202		0.0697918	$[-0.1583042, 0.0394934]$	0.051	2	0.057
se_3	1.371984		0.0695271	$[-0.1575635, 0.0387528]$	0.052		–
se_4	1.355027		0.0699338	$[-0.1600374, 0.0402266]$	0.050	4	0.044
se_5	1.3508825		0.0690563	$[-0.1549379, 0.0361271]$	0.055	5	0.042
se_6	1.3681737		0.0694819	$[-0.1562996, 0.0389489]$	0.052		–
se_7	1.3686779		0.0698397	$[-0.1597813, 0.0399706]$	0.051		–
se_8	1.391217		0.0694178	$[-0.1556333, 0.0388226]$	0.053		–
se_9	1.40414		0.0693352	$[-0.1553368, 0.0387261]$	0.054		–
se_{10}	1.4054723		0.0697124	$[-0.1582514, 0.0394406]$	0.051	10	0.046
se_{avg}	1.397614		0.0697559	$[-0.1505429, 0.0347321]$	0.051		

Table 2: A Comparison of the standard deviation of the sample mean with the mean value of the standard error calculated by eq. (13) through 1000 replications, and empirical null rejection probabilities for the test statistic and IM approach. Subscript 1 is for $T = 100$ and 2 for $T = 200$. DGP: $y_t = \mu + \phi_1 y_{t-1} + \epsilon_t$, $\epsilon_t \sim N(0,1)$; $\mu = 0$.

$\phi_1 = 0.1$	CV_1	St. Dev_1	St. $Error_1$	CI_1	Rej. Pr_1	q	IM_1
se_1	2.4443936	0.1070215	0.1087097	$[-0.2699164, 0.2615422]$	0.046		–
se_2	2.3328373		0.1120459	$[-0.2655721, 0.2571979]$	0.041	2	0.034
se_3	2.4233392		0.1091766	$[-0.2687591, 0.2603849]$	0.046		–
se_4	2.5100734		0.106531	$[-0.2715876, 0.2632134]$	0.048	4	0.046
se_5	2.5143351		0.1069318	$[-0.2730494, 0.2646752]$	0.043	5	0.031
se_6	2.4522659		0.1086605	$[-0.2706515, 0.2622773]$	0.047		–
se_7	2.5034686		0.1072284	$[-0.2726301, 0.2642559]$	0.033		–
se_8	2.5178804		0.1070881	$[-0.2738221, 0.265448]$	0.031		–
se_9	2.5029468		0.1076556	$[-0.2736433, 0.2652692]$	0.037		–
se_{10}	2.4459803		0.1090902	$[-0.2710196, 0.2626454]$	0.042	10	0.057
se_{avg}	2.4174228		0.1092987	$[-0.2684082, 0.260034]$	0.045		
$\phi_1 = 0.1$	CV_2	St. Dev_2	St. $Error_2$	CI_2	Rej. Pr_2	q	IM_2
se_1	1.3287834	0.081369	0.0821973	$[-0.1759334, 0.0349085]$	0.049		–
se_2	1.3839202		0.0815916	$[-0.1740895, 0.0340646]$	0.051	2	0.037
se_3	1.371984		0.0819877	$[-0.1757784, 0.0347535]$	0.052		–
se_4	1.355027		0.0820981	$[-0.1759073, 0.0348824]$	0.049	4	0.040
se_5	1.3508825		0.0820407	$[-0.175831, 0.03488061]$	0.049	5	0.032
se_6	1.3681737		0.0812884	$[-0.1738474, 0.0338225]$	0.051		–
se_7	1.3686779		0.0815873	$[-0.1739925, 0.0339675]$	0.048		–
se_8	1.391217		0.0810504	$[-0.1735589, 0.033434]$	0.052		–
se_9	1.40414		0.0809073	$[-0.1725762, 0.0333513]$	0.053		–
se_{10}	1.4054723		0.0812735	$[-0.173824, 0.0337605]$	0.052	10	0.056
se_{avg}	1.397614		0.0812958	$[-0.1709146, 0.0333497]$	0.051		

Table 3: A Comparison of the standard deviation of the sample mean with the mean value of the standard error calculated by eq. (13) through 1000 replications, and empirical null rejection probabilities for the test statistic and IM approach. Subscript 1 is for $T = 100$ and 2 for $T = 200$. DGP: $y_t = \mu + \phi_1 y_{t-1} + \epsilon_t$, $\epsilon_t \sim N(0,1)$; $\mu = 0$.

$\phi_1 = 0.2$	CV_1	St. Dev_1	St. $Error_1$	CI_1	Rej. Pr_1	q	IM_1
se_1	2.4443936	0.1202334	0.1264811	$[-0.3140503, 0.3042888]$	0.042		–
se_2	2.3328373		0.1285461	$[-0.3047578, 0.2949963]$	0.039	2	0.054
se_3	2.4233392		0.1247986	$[-0.30731, 0.2975486]$	0.040		–
se_4	2.5100734		0.1214474	$[-0.3097226, 0.2999611]$	0.029	4	0.046
se_5	2.5143351		0.1216001	$[-0.3106241, 0.3008626]$	0.031	5	0.051
se_6	2.4522659		0.1231606	$[-0.3069034, 0.2971419]$	0.030		–
se_7	2.5034686		0.1214372	$[-0.3088949, 0.2991334]$	0.034		–
se_8	2.5178804		0.1211618	$[-0.3099517, 0.3001902]$	0.031		–
se_9	2.5029468		0.1217103	$[-0.3095152, 0.2997537]$	0.037		–
se_{10}	2.4459803		0.1230789	$[-0.3059293, 0.2961679]$	0.041	10	0.047
se_{avg}	2.4174228		0.1233528	$[-0.3030766, 0.2933151]$	0.043		
$\phi_1 = 0.2$	CV_2	St. Dev_2	St. $Error_2$	CI_2	Rej. Pr_2	q	IM_2
se_1	1.3287834	0.087232	0.0877668	$[-0.1908837, 0.0423623]$	0.042		–
se_2	1.3839202		0.0828754	$[-0.1889536, 0.0404323]$	0.055	2	0.057
se_3	1.371984		0.0828077	$[-0.1878716, 0.0393502]$	0.056		–
se_4	1.355027		0.0825091	$[-0.1860627, 0.0375413]$	0.057	4	0.040
se_5	1.3508825		0.0821989	$[-0.1853018, 0.0367804]$	0.059	5	0.052
se_6	1.3681737		0.0812049	$[-0.1853631, 0.0368417]$	0.065		–
se_7	1.3686779		0.081414	$[-0.1856902, 0.0371688]$	0.063		–
se_8	1.391217		0.0807199	$[-0.1865595, 0.0380382]$	0.066		–
se_9	1.40414		0.0804914	$[-0.1872819, 0.0387605]$	0.067		–
se_{10}	1.4054723		0.0808347	$[-0.1878717, 0.0393503]$	0.066	10	0.046
se_{avg}	1.397614		0.0847581	$[-0.1847197, 0.0341984]$	0.052		

Table 4: A Comparison of the standard deviation of the sample mean with the mean value of the standard error calculated by eq. (13) through 1000 replications, and empirical null rejection probabilities for the test statistic and IM approach. Subscript 1 is for $T = 100$ and 2 for $T = 200$. DGP: $y_t = \mu + \phi_1 y_{t-1} + \epsilon_t$, $\epsilon_t \sim N(0, 1)$; $\mu = 0$.

$\phi_1 = 0.3$	CV_1	St. Dev_1	St. Error_1	CI_1	Rej. Pr_1	q	IM_1
se_1	2.4443936	0.1372048	0.1496581	$[-0.3716065, 0.3600403]$	0.040		–
se_2	2.3328373		0.1500708	$[-0.3558739, 0.3443077]$	0.038	2	0.044
se_3	2.4233392		0.145173	$[-0.3575865, 0.3460203]$	0.039		–
se_4	2.5100734		0.1408789	$[-0.3593995, 0.3478333]$	0.018	4	0.066
se_5	2.5143351		0.1406734	$[-0.3594831, 0.3479169]$	0.029	5	0.057
se_6	2.4522659		0.141958	$[-0.3539017, 0.3423356]$	0.031		–
se_7	2.5034686		0.139837	$[-0.3558606, 0.3442944]$	0.034		–
se_8	2.5178804		0.1393662	$[-0.3566906, 0.3451245]$	0.033		–
se_9	2.5029468		0.1398613	$[-0.3558485, 0.3442824]$	0.032		–
se_{10}	2.4459803		0.1411056	$[-0.3509246, 0.3393584]$	0.042	10	0.067
se_{avg}	2.4174228		0.1415067	$[-0.3478645, 0.3362984]$	0.044		
$\phi_1 = 0.3$	CV_2	St. Dev_2	St. Error_2	CI_2	Rej. Pr_2	q	IM_2
se_1	1.3287834	0.104423	0.1043189	$[-0.2234573, 0.0537771]$	0.050		–
se_2	1.3839202		0.0977657	$[-0.2201401, 0.0504599]$	0.052	2	0.047
se_3	1.371984		0.0970598	$[-0.2180046, 0.0483244]$	0.053		–
se_4	1.355027		0.0961848	$[-0.215173, 0.0454928]$	0.054	4	0.060
se_5	1.3508825		0.0955149	$[-0.2138695, 0.0441893]$	0.054	5	0.053
se_6	1.3681737		0.0941856	$[-0.2137024, 0.0440222]$	0.055		–
se_7	1.3686779		0.0942575	$[-0.2138483, 0.0441681]$	0.054		–
se_8	1.391217		0.0933438	$[-0.2147015, 0.0450213]$	0.056		–
se_9	1.40414		0.0929922	$[-0.2154141, 0.0457339]$	0.057		–
se_{10}	1.4054723		0.0932936	$[-0.2159617, 0.0462814]$	0.056	10	0.067
se_{avg}	1.397614		0.0973369	$[-0.2208796, 0.0511994]$	0.052		

Table 5: A Comparison of the standard deviation of the sample mean with the mean value of the standard error calculated by eq. (13) through 1000 replications, and empirical null rejection probabilities for the test statistic and IM approach. Subscript 1 is for $T = 100$ and 2 for $T = 200$. DGP: $y_t = \mu + \phi_1 y_{t-1} + \epsilon_t$, $\epsilon_t \sim N(0, 1)$; $\mu = 0$.

$\phi_1 = 0.4$	CV_1	St. Dev_1	St. $Error_1$	CI_1	Rej. Pr_1	q	IM_1
se_1	2.4443936	0.1597824	0.1808641	$[-0.4491099, 0.4350964]$	0.040		–
se_2	2.3328373		0.1790882	$[-0.4247903, 0.4107768]$	0.045	2	0.044
se_3	2.4233392		0.1726673	$[-0.4254383, 0.4114248]$	0.031		–
se_4	2.5100734		0.167095	$[-0.4264274, 0.412414]$	0.025	4	0.068
se_5	2.5143351		0.16637	$[-0.4253167, 0.4113032]$	0.036	5	0.057
se_6	2.4522659		0.1672044	$[-0.4170363, 0.4030229]$	0.030		–
se_7	2.5034686		0.1645263	$[-0.4188931, 0.4048797]$	0.031		–
se_8	2.5178804		0.1637664	$[-0.4193509, 0.4053375]$	0.032		–
se_9	2.5029468		0.1641486	$[-0.4178618, 0.4038484]$	0.035		–
se_{10}	2.4459803		0.1651659	$[-0.4109992, 0.3969857]$	0.042	10	0.047
se_{avg}	2.4174228		0.1657708	$[-0.4077448, 0.3937313]$	0.041		
$\phi_1 = 0.4$	CV_2	St. Dev_2	St. $Error_2$	CI_2	Rej. Pr_2	q	IM_2
se_1	1.3287834	0.115998	0.1269269	$[-0.2675621, 0.0697547]$	0.032		–
se_2	1.3839202		0.1181525	$[-0.2624173, 0.0646099]$	0.045	2	0.043
se_3	1.371984		0.1165477	$[-0.2588053, 0.0609979]$	0.047		–
se_4	1.355027		0.1148399	$[-0.2545148, 0.0567074]$	0.052	4	0.069
se_5	1.3508825		0.1136483	$[-0.2524292, 0.0546218]$	0.053	5	0.059
se_6	1.3681737		0.111842	$[-0.251923, 0.0541156]$	0.054		–
se_7	1.3686779		0.111702	$[-0.2517878, 0.0539804]$	0.055		–
se_8	1.391217		0.1104717	$[-0.2525938, 0.0547864]$	0.056		–
se_9	1.40414		0.1099374	$[-0.2532712, 0.0554638]$	0.061		–
se_{10}	1.4054723		0.110165	$[-0.2537376, 0.0559302]$	0.056	10	0.049
se_{avg}	1.397614		0.1149732	$[-0.2586135, 0.0608061]$	0.052		

Table 6: A Comparison of the standard deviation of the sample mean with the mean value of the standard error calculated by eq. (13) through 1000 replications, and empirical null rejection probabilities for the test statistic and IM approach. Subscript 1 is for $T = 100$ and 2 for $T = 200$. DGP: $y_t = \mu + \phi_1 y_{t-1} + \epsilon_t$, $\epsilon_t \sim N(0,1)$; $\mu = 0$.

$\phi_1 = 0.5$	CV_1	St. Dev_1	St. $Error_1$	CI_1	Rej. Pr_1	q	IM_1
se_1	2.4443936	0.1912783	0.2245867	$[-0.5577294, 0.5402274]$	0.010		–
se_2	2.3328373		0.2198794	$[-0.5216938, 0.5041917]$	0.037	2	0.044
se_3	2.4233392		0.2114198	$[-0.5210929, 0.5035908]$	0.020		–
se_4	2.5100734		0.204084	$[-0.5210169, 0.5035149]$	0.021	4	0.048
se_5	2.5143351		0.2026002	$[-0.5181558, 0.5006537]$	0.026	5	0.047
se_6	2.4522659		0.2027028	$[-0.5058322, 0.4883302]$	0.030		–
se_7	2.5034686		0.1992212	$[-0.5074952, 0.4899931]$	0.031		–
se_8	2.5178804		0.1980233	$[-0.5073501, 0.4898481]$	0.032		–
se_9	2.5029468		0.1981863	$[-0.5048007, 0.4872986]$	0.043		–
se_{10}	2.4459803		0.1987971	$[-0.4950048, 0.4775027]$	0.046	10	0.057
se_{avg}	2.4174228		0.199689	$[-0.4914838, 0.4739817]$	0.042		
$\phi_1 = 0.5$	CV_2	St. Dev_2	St. $Error_2$	CI_2	Rej. Pr_2	q	IM_2
se_1	1.3287834	0.145693	0.1504033	$[-0.3302164, 0.0931427]$	0.027		–
se_2	1.3839202		0.1487447	$[-0.3225917, 0.0855181]$	0.039	2	0.043
se_3	1.371984		0.1485322	$[-0.3168327, 0.079759]$	0.041		–
se_4	1.355027		0.1478271	$[-0.3103852, 0.0733115]$	0.043	4	0.049
se_5	1.3508825		0.1476117	$[-0.3071359, 0.0700622]$	0.044	5	0.049
se_6	1.3681737		0.1471052	$[-0.3061203, 0.0690467]$	0.046		–
se_7	1.3686779		0.1466314	$[-0.3055412, 0.0684675]$	0.047		–
se_8	1.391217		0.1449278	$[-0.3062507, 0.069177]$	0.049		–
se_9	1.40414		0.1441125	$[-0.3068496, 0.069776]$	0.050		–
se_{10}	1.4054723		0.1442104	$[-0.3071659, 0.0700922]$	0.051	10	0.059
se_{avg}	1.397614		0.1428239	$[-0.3017422, 0.0691686]$	0.052		

Table 7: A Comparison of the standard deviation of the sample mean with the mean value of the standard error calculated by eq. (13) through 1000 replications, and empirical null rejection probabilities for the test statistic and IM approach. Subscript 1 is for $T = 100$ and 2 for $T = 200$. DGP: $y_t = \mu + \phi_1 y_{t-1} + \epsilon_t$, $\epsilon_t \sim N(0,1)$; $\mu = 0$.

$\phi_1 = 0.6$	CV_1	St. Dev_1	St. $Error_1$	CI_1	Rej. Pr_1	q	IM_1
se_1	2.4443936	0.2382851	0.2890184	$[-0.7178931, 0.6950561]$	0.010		–
se_2	2.3328373		0.2804254	$[-0.6656053, 0.6427683]$	0.017	2	0.044
se_3	2.4233392		0.269239	$[-0.663876, 0.6410389]$	0.018		–
se_4	2.5100734		0.2594337	$[-0.6626163, 0.6397792]$	0.028	4	0.048
se_5	2.5143351		0.2568362	$[-0.6571908, 0.6343538]$	0.026	5	0.057
se_6	2.4522659		0.2557502	$[-0.638586, 0.615749]$	0.025		–
se_7	2.5034686		0.2510788	$[-0.6399863, 0.6171493]$	0.027		–
se_8	2.5178804		0.2492048	$[-0.6388863, 0.6160493]$	0.028		–
se_9	2.5029468		0.2489527	$[-0.634534, 0.611697]$	0.040		–
se_{10}	2.4459803		0.2488425	$[-0.6200825, 0.5972454]$	0.041	10	0.067
se_{avg}	2.4174228		0.2500641	$[-0.6159291, 0.593092]$	0.041		
$\phi_1 = 0.6$	CV_2	St. Dev_2	St. $Error_2$	CI_2	Rej. Pr_2	q	IM_2
se_1	1.3287834	0.181605	0.2086768	$[-0.425236, 0.1293365]$	0.032		–
se_2	1.3839202		0.1923892	$[-0.414201, 0.1183015]$	0.038	2	0.043
se_3	1.371984		0.1874853	$[-0.4051765, 0.109277]$	0.043		–
se_4	1.355027		0.1826155	$[-0.3953987, 0.0994992]$	0.047	4	0.049
se_5	1.3508825		0.1794332	$[-0.3903429, 0.0944434]$	0.051	5	0.059
se_6	1.3681737		0.17586	$[-0.3885567, 0.0926573]$	0.052		–
se_7	1.3686779		0.1748458	$[-0.3872573, 0.0913578]$	0.057		–
se_8	1.391217		0.1724024	$[-0.3877989, 0.0918994]$	0.058		–
se_9	1.40414		0.1711371	$[-0.3882501, 0.0923507]$	0.062		–
se_{10}	1.4054723		0.1710024	$[-0.3882889, 0.0923894]$	0.065	10	0.069
se_{avg}	1.397614		0.1765728	$[-0.3869352, 0.0910357]$	0.048		

Table 8: A Comparison of the standard deviation of the sample mean with the mean value of the standard error calculated by eq. (13) through 1000 replications, and empirical null rejection probabilities for the test statistic and IM approach. Subscript 1 is for $T = 100$ and 2 for $T = 200$. DGP: $y_t = \mu + \phi_1 y_{t-1} + \epsilon_t$, $\epsilon_t \sim N(0, 1)$; $\mu = 0$.

$\phi_1 = 0.7$	CV_1	St. Dev_1	St. Error_1	CI_1	Rej. Pr_1	q	IM_1
se_1	2.4443936	0.316008	0.3905051	$[-0.9705136, 0.9385828]$	0.012		–
se_2	2.3328373		0.377099	$[-0.895676, 0.8637452]$	0.018	2	0.054
se_3	2.4233392		0.3624281	$[-0.8942516, 0.8623208]$	0.017		–
se_4	2.5100734		0.3491941	$[-0.8924683, 0.8605375]$	0.027	4	0.058
se_5	2.5143351		0.3449971	$[-0.8834036, 0.8514728]$	0.025	5	0.067
se_6	2.4522659		0.3419981	$[-0.8546356, 0.8227048]$	0.021		–
se_7	2.5034686		0.3355449	$[-0.8559914, 0.8240606]$	0.026		–
se_8	2.5178804		0.332624	$[-0.8534727, 0.8215419]$	0.029		–
se_9	2.5029468		0.3315947	$[-0.8459292, 0.8139984]$	0.040		–
se_{10}	2.4459803		0.3302009	$[-0.8236304, 0.7916996]$	0.046	10	0.077
se_{avg}	2.4174228		0.3315832	$[-0.8175423, 0.7856115]$	0.043		
$\phi_1 = 0.7$	CV_2	St. Dev_2	St. Error_2	CI_2	Rej. Pr_2	q	IM_2
se_1	1.3287834	0.241153	0.2707476	$[-0.5834842, 0.189197]$	0.022		–
se_2	1.3839202		0.267998	$[-0.5680315, 0.1737443]$	0.028	2	0.053
se_3	1.371984		0.2599864	$[-0.5538408, 0.1595535]$	0.027		–
se_4	1.355027		0.2520536	$[-0.5386831, 0.1443959]$	0.037	4	0.059
se_5	1.3508825		0.246914	$[-0.5306953, 0.1364081]$	0.035	5	0.069
se_6	1.3681737		0.2416625	$[-0.5277799, 0.1334927]$	0.038		–
se_7	1.3686779		0.2397455	$[-0.525278, 0.1309908]$	0.037		–
se_8	1.391217		0.2360891	$[-0.5255948, 0.1313076]$	0.039		–
se_9	1.40414		0.2340723	$[-0.5258139, 0.1315267]$	0.049		–
se_{10}	1.4054723		0.2335112	$[-0.5253371, 0.1310499]$	0.051	10	0.077
se_{avg}	1.397614		0.2355784	$[-0.5163913, 0.1299041]$	0.048		

Table 9: A Comparison of the standard deviation of the sample mean with the mean value of the standard error calculated by eq. (13) through 1000 replications, and empirical null rejection probabilities for the test statistic and IM approach. Subscript 1 is for $T = 100$ and 2 for $T = 200$. DGP: $y_t = \mu + \phi_1 y_{t-1} + \epsilon_t$, $\epsilon_t \sim N(0, 1)$; $\mu = 0$.

$\phi_1 = 0.8$	CV_1	St. Dev_1	St. $Error_1$	CI_1	Rej. Pr_1	q	IM_1
se_1	2.4443936	0.4686303	0.5664212	$[-1.409759, 1.359353]$	0.012		–
se_2	2.3328373		0.5485023	$[-1.30477, 1.254363]$	0.018	2	0.044
se_3	2.4233392		0.5302791	$[-1.310249, 1.259843]$	0.029		–
se_4	2.5100734		0.5127817	$[-1.312323, 1.261916]$	0.027	4	0.068
se_5	2.5143351		0.5066161	$[-1.299006, 1.248599]$	0.025	5	0.077
se_6	2.4522659		0.5007495	$[-1.253174, 1.202768]$	0.028		–
se_7	2.5034686		0.4917829	$[-1.256366, 1.20596]$	0.032		–
se_8	2.5178804		0.4873622	$[-1.252323, 1.201917]$	0.045		–
se_9	2.5029468		0.4849576	$[-1.239026, 1.18862]$	0.047		–
se_{10}	2.4459803		0.4813079	$[-1.202473, 1.152066]$	0.046	10	0.097
se_{avg}	2.4174228		0.4818551	$[-1.190051, 1.139644]$	0.049		
$\phi_1 = 0.8$	CV_2	St. Dev_2	St. $Error_2$	CI_2	Rej. Pr_2	q	IM_2
se_1	1.3287834	0.359234	0.4689628	$[-0.7353108, 0.5109891]$	0.022		–
se_2	1.3839202		0.4370496	$[-0.7170026, 0.492681]$	0.028	2	0.043
se_3	1.371984		0.4237301	$[-0.6935117, 0.4691901]$	0.039		–
se_4	1.355027		0.4109784	$[-0.6690476, 0.444726]$	0.037	4	0.061
se_5	1.3508825		0.4017554	$[-0.6548852, 04305636]$	0.035	5	0.076
se_6	1.3681737		0.392662	$[-0.6493906, 0.425069]$	0.038		–
se_7	1.3686779		0.3876784	$[-0.6427676, 0.418446]$	0.040		–
se_8	1.391217		0.3799619	$[-0.6407702, 0.4164486]$	0.046		–
se_9	1.40414		0.3753938	$[-0.6392662, 0.4149446]$	0.049		–
se_{10}	1.4054723		0.3734275	$[-0.6370028, 0.4126811]$	0.047	10	0.095
se_{avg}	1.397614		0.3706044	$[-0.6301227, 0.4058011]$	0.051		

Table 10: A Comparison of the standard deviation of the sample mean with the mean value of the standard error calculated by eq. (13) through 1000 replications, and empirical null rejection probabilities for the test statistic and IM approach. Subscript 1 is for $T = 100$ and 2 for $T = 200$. DGP: $y_t = \mu + \phi_1 y_{t-1} + \epsilon_t$, $\epsilon_t \sim N(0,1)$; $\mu = 0$.

$\phi_1 = 0.9$	CV_1	St. Dev_1	St. $Error_1$	CI_1	Rej. Pr_1	q	IM_1
se_1	2.4443936	0.8974888	0.9254669	$[-2.314443, 2.209967]$	0.029		–
se_2	2.3328373		0.9093736	$[-2.173659, 2.069182]$	0.033	2	0.064
se_3	2.4233392		0.8918764	$[-2.213557, 2.109081]$	0.034		–
se_4	2.5100734		0.8723568	$[-2.241918, 2.137441]$	0.038	4	0.088
se_5	2.5143351		0.8659517	$[-2.229531, 2.125055]$	0.045	5	0.109
se_6	2.4522659		0.8574554	$[-2.154947, 2.050471]$	0.042		–
se_7	2.5034686		0.8464618	$[-2.171329, 2.066852]$	0.045		–
se_8	2.5178804		0.841077	$[-2.169969, 2.065493]$	0.046		–
se_9	2.5029468		0.8371613	$[-2.147608, 2.043132]$	0.043		–
se_{10}	2.4459803		0.8302671	$[-2.083055, 1.978579]$	0.066	10	0.201
se_{avg}	2.4174228		0.8264156	$[-2.050034, 1.945558]$	0.062		–
$\phi_1 = 0.9$	CV_2	St. Dev_2	St. $Error_2$	CI_2	Rej. Pr_2	q	IM_2
se_1	1.3287834	0.702983	0.7116937	$[-1.374395, 0.8797364]$	0.032		–
se_2	1.3839202		0.7113736	$[-1.370206, 0.8755471]$	0.045	2	0.063
se_3	1.371984		0.710876	$[-1.336845, 0.842187]$	0.043		–
se_4	1.355027		0.709357	$[-1.299252, 0.8045939]$	0.044	4	0.081
se_5	1.3508825		0.708952	$[-1.277994, 0.7833356]$	0.054	5	0.106
se_6	1.3681737		0.707455	$[-1.274098, 0.7794393]$	0.058		–
se_7	1.3686779		0.706462	$[-1.26102, 0.7694433]$	0.047		–
se_8	1.391217		0.705077	$[-1.264573, 0.7699148]$	0.047		–
se_9	1.40414		0.704161	$[-1.264262, 0.7696036]$	0.045		–
se_{10}	1.4054723		0.703267	$[-1.260107, 0.7654488]$	0.062	10	0.205
se_{avg}	1.397614		0.7024424	$[-1.229073, 0.734414]$	0.052		–

Table 11: A Comparison of the standard deviation of the sample mean with the mean value of the standard error calculated by eq. (14) through 1000 replications, and empirical null rejection probabilities for the test statistic and IM approach. Subscript 1 is used for $T = 100$.
DGP: $y_i = \mu + \exp(\beta x_i).\epsilon_i$, $\epsilon_i \sim N(0,1)$, $x_i \sim N(0,1)$; $\mu = 0$, $\beta = 0.4$.

	CV_1	St. Dev_1	St. $Error_1$	CI_1	Rej. Pr_1	q	IM_1
se_1	2.4443936	0.1162029	0.116314	$[-0.2908727, 0.2777617]$	0.042		–
se_2	2.3328373		0.1213187	$[-0.2895723, 0.2764613]$	0.043	2	0.000
se_3	2.4233392		0.1190771	$[-0.2951196, 0.2820086]$	0.041		–
se_4	2.5100734		0.1169148	$[-0.3000202, 0.2869092]$	0.046	4	0.000
se_5	2.5143351		0.1178461	$[-0.3028601, 0.2897491]$	0.048	5	0.000
se_6	2.4522659		0.1199574	$[-0.3007229, 0.2876119]$	0.040		–
se_7	2.5034686		0.1185563	$[-0.3033575, 0.2902465]$	0.049		–
se_8	2.5178804		0.1185558	$[-0.3050649, 0.2919539]$	0.046		–
se_9	2.5029468		0.1193456	$[-0.3052711, 0.2921601]$	0.044		–
se_{10}	2.4459803		0.1210026	$[-0.3025255, 0.2894145]$	0.056	10	0.000
se_{avg}	2.4174228		0.1214158	$[-0.3000689, 0.2869579]$	0.051		

DGP: $y_{it} = \mu + \phi_1 y_{it-1} + w_i + \exp(\beta x_{it}).\epsilon_{it}$, $\epsilon_{it} \sim N(0,1)$, $x_{it} \sim N(0,1)$; $\mu = 0$, $\phi_1 = 0.2$, $\beta = 0.2$; $w_i = \lambda w_{i-1} + v_i$, $\lambda = 0.3$.

$\phi_1 = 0.1$	CV_1	St. Dev_1	St. $Error_1$	CI_1	Rej. Pr_1	q	IM_1
se_1	2.4443936	0.2059888	0.200824	$[-0.4655353, 0.5162506]$	0.041		–
se_2	2.3328373		0.2086386	$[-0.4613621, 0.5120775]$	0.051	2	0.000
se_3	2.4233392		0.2066676	$[-0.475468, 0.5261834]$	0.055		–
se_4	2.5100734		0.2041653	$[-0.4871121, 0.5378275]$	0.057	4	0.000
se_5	2.5143351		0.2061785	$[-0.4930443, 0.5437596]$	0.056	5	0.000
se_6	2.4522659		0.2086671	$[-0.4863496, 0.537065]$	0.059		–
se_7	2.5034686		0.2066445	$[-0.4919702, 0.5426856]$	0.061		–
se_8	2.5178804		0.2065773	$[-0.4947793, 0.5454947]$	0.063		–
se_9	2.5029468		0.2073685	$[-0.4936746, 0.54439]$	0.066		–
se_{10}	2.4459803		0.2087266	$[-0.4851834, 0.5358988]$	0.062	10	0.000
se_{avg}	2.4174228		0.2071774	$[-0.4754777, 0.5261931]$	0.064		

Table 12: A Comparison of the standard deviation of the sample mean with the mean value of the standard error calculated by eq. (14) through 1000 replications, and empirical null rejection probabilities for the test statistic and IM approach. Subscript 2 is used for $T = 200$.
DGP: $y_i = \mu + \exp(\beta x_i).\epsilon_i$, $\epsilon_i \sim N(0,1)$, $x_i \sim N(0,1)$; $\mu = 0$, $\beta = 0.4$.

	CV_2	St. Dev_2	St. $Error_2$	CI_2	Rej. Pr_2	q	IM_2
se_1	1.3287834	0.069950	0.0647191	$[-0.1122103, 0.098671]$	0.057		–
se_2	1.3839202		0.0673187	$[-0.1146352, 0.099981]$	0.054	2	0.000
se_3	1.371984		0.0680771	$[-0.1156292, 0.104721]$	0.053		–
se_4	1.355027		0.0691489	$[-0.1172963, 0.105392]$	0.052	4	0.000
se_5	1.3508825		0.0678461	$[-0.1155983, 0.104692]$	0.053	5	0.000
se_6	1.3681737		0.0699574	$[-0.1181392, 0.106389]$	0.049		–
se_7	1.3686779		0.0685563	$[-0.1170479, 0.105274]$	0.052		–
se_8	1.391217		0.0685958	$[-0.1171392, 0.105296]$	0.052		–
se_9	1.40414		0.0693456	$[-0.1173128, 0.105429]$	0.051		–
se_{10}	1.4054723		0.0680026	$[-0.1156191, 0.104542]$	0.053	10	0.000
se_{avg}	1.397614		0.0689158	$[-0.1102902, 0.096361]$	0.052		

DGP: $y_{it} = \mu + \phi_1 y_{it-1} + w_i + \exp(\beta x_{it}).\epsilon_{it}$, $\epsilon_{it} \sim N(0,1)$, $x_{it} \sim N(0,1)$; $\mu = 0$, $\phi_1 = 0.2$, $\beta = 0.2$; $w_i = \lambda w_{i-1} + v_i$, $\lambda = 0.3$.

$\phi_1 = 0.1$	CV_2	St. Dev_2	St. $Error_2$	CI_2	Rej. Pr_2	q	IM_2
se_1	1.3287834	0.144922	0.1533778	$[-0.2240488, 0.1835629]$	0.041		–
se_2	1.3839202		0.1457532	$[-0.2219537, 0.1814678]$	0.048	2	0.000
se_3	1.371984		0.1458801	$[-0.2203881, 0.1799023]$	0.047		–
se_4	1.355027		0.1460175	$[-0.2181006, 0.1776147]$	0.046	4	0.000
se_5	1.3508825		0.1461213	$[-0.2176357, 0.1771498]$	0.045	5	0.000
se_6	1.3681737		0.1444437	$[-0.217867, 0.1773811]$	0.052		–
se_7	1.3686779		0.1443676	$[-0.2178357, 0.1773498]$	0.055		–
se_8	1.391217		0.1426047	$[-0.2186369, 0.1781511]$	0.057		–
se_9	1.40414		0.1418511	$[-0.2194218, 0.1789359]$	0.058		–
se_{10}	1.4054723		0.1420772	$[-0.2199285, 0.1794426]$	0.057	10	0.000
se_{avg}	1.397614		0.1487665	$[-0.2181611, 0.1776752]$	0.045		

Table 13: Finite sample power performance of the test statistic for se_{avg} and IM approach for $q = 4$ for 1000 reps. 5% Nominal Level, Two-sided Tests. $H_0 : \mu = \mu_0 = 0$, $H_1 : \mu = \mu_1$. DGP: $y_t = \mu + \phi_1 y_{t-1} + \epsilon_t$, $\epsilon_t \sim N(0, 1)$; $\mu = 0$, $T = 100$, $CV = 2.4174228$.

$\phi_1 = 0$	Rej. Pr	IM	$\phi_1 = 0.1$	Rej. Pr	IM	$\phi_1 = 0.2$	Rej. Pr	IM
$\mu = 0.00$	0.046	0.048	$\mu = 0.00$	0.047	0.044	$\mu = 0.00$	0.057	0.051
$\mu = 0.05$	0.088	0.060	$\mu = 0.05$	0.086	0.063	$\mu = 0.05$	0.085	0.075
$\mu = 0.10$	0.128	0.102	$\mu = 0.10$	0.127	0.101	$\mu = 0.10$	0.125	0.101
$\mu = 0.15$	0.230	0.199	$\mu = 0.15$	0.228	0.179	$\mu = 0.15$	0.227	0.180
$\mu = 0.20$	0.369	0.276	$\mu = 0.20$	0.368	0.297	$\mu = 0.20$	0.366	0.278
$\mu = 0.25$	0.589	0.412	$\mu = 0.25$	0.587	0.429	$\mu = 0.25$	0.567	0.431
$\mu = 0.30$	0.658	0.555	$\mu = 0.30$	0.651	0.592	$\mu = 0.30$	0.647	0.529
$\mu = 0.35$	0.887	0.706	$\mu = 0.35$	0.819	0.641	$\mu = 0.35$	0.817	0.677
$\mu = 0.40$	0.928	0.734	$\mu = 0.40$	0.926	0.763	$\mu = 0.40$	0.922	0.744
$\mu = 0.45$	0.992	0.851	$\mu = 0.45$	0.987	0.843	$\mu = 0.45$	0.985	0.837
$\phi_1 = 0.3$	Rej. Pr	IM	$\phi_1 = 0.4$	Rej. Pr	IM	$\phi_1 = 0.5$	Rej. Pr	IM
$\mu = 0.00$	0.055	0.048	$\mu = 0.00$	0.062	0.050	$\mu = 0.00$	0.050	0.051
$\mu = 0.05$	0.087	0.061	$\mu = 0.05$	0.088	0.063	$\mu = 0.05$	0.081	0.093
$\mu = 0.10$	0.125	0.118	$\mu = 0.10$	0.126	0.136	$\mu = 0.10$	0.124	0.119
$\mu = 0.15$	0.226	0.184	$\mu = 0.15$	0.225	0.201	$\mu = 0.15$	0.224	0.202
$\mu = 0.20$	0.365	0.273	$\mu = 0.20$	0.346	0.288	$\mu = 0.20$	0.339	0.298
$\mu = 0.25$	0.508	0.412	$\mu = 0.25$	0.499	0.425	$\mu = 0.25$	0.492	0.442
$\mu = 0.30$	0.639	0.563	$\mu = 0.30$	0.627	0.551	$\mu = 0.30$	0.619	0.551
$\mu = 0.35$	0.815	0.665	$\mu = 0.35$	0.796	0.657	$\mu = 0.35$	0.788	0.643
$\mu = 0.40$	0.919	0.767	$\mu = 0.40$	0.917	0.782	$\mu = 0.40$	0.915	0.785
$\mu = 0.45$	0.982	0.824	$\mu = 0.45$	0.975	0.857	$\mu = 0.45$	0.967	0.835

Table 14: A Comparison of the standard deviation of the sample mean with the mean value of the standard error calculated by eq. (14) and eq. (13) respectively through 1000 replications, and empirical null rejection probabilities for the test statistic, White (1984) and Newey and West (1987) approach. $T = 100$.
DGP: $y_i = x_i\beta + \epsilon_i$, $\epsilon_i \sim N(0, \sigma_i^2)$, $x_i \sim N(0,1)$; σ_i^2 takes values from 0.01 to 1, $\beta = 0$.

	CV	St. Dev	St. Error	CI	Rej. Pr	White (1984)
se_2	1.4376333	0.0752358	0.1561381	$[-0.2247313, 0.2242074]$	0.040	0.081
se_3	1.9541209		0.1006337	$[-0.1969123, 0.1963885]$	0.043	
se_4	2.2493941		0.0831105	$[-0.1872101, 0.1866863]$	0.041	
se_5	2.3671093		0.0769950	$[-0.1825175, 0.1819937]$	0.046	
se_6	2.2528766		0.0769994	$[-0.1737321, 0.1732083]$	0.042	
se_7	2.3821085		0.0728513	$[-0.1738015, 0.1732777]$	0.035	
se_8	2.4516591		0.0710890	$[-0.1745479, 0.1740241]$	0.038	
se_9	2.4504642		0.0708547	$[-0.1738889, 0.1733651]$	0.040	
se_{10}	2.2882882		0.0735069	$[-0.1684669, 0.1679431]$	0.044	
se_{avg}	2.1288568		0.0800460	$[-0.1703876, 0.1713001]$	0.043	

DGP: $y_t = \phi y_{t-1} + \epsilon_t$, $\epsilon_t \sim N(0,1)$, $\phi = 0.5$.
For Newey and West (1987), $Kernel = Bartlett, Bandwidth = 10$.

	CV	St. Dev	St. Error	CI	Rej. Pr	Newey&West (1987)
se_3	3.7796679	0.0880726	0.0692434	$[-0.0654307, 0.4580036]$	0.057	0.105
se_4	3.7065494		0.0761618	$[-0.0860111, 0.478584]$	0.056	
se_5	3.4024051		0.0846737	$[-0.0918078, 0.4843806]$	0.051	
se_6	2.8266524		0.0948328	$[-0.0717728, 0.4643457]$	0.054	
se_8	2.864615		0.0969039	$[-0.081306, 0.4738788]$	0.049	
se_9	2.7945549		0.0998083	$[-0.0826334, 0.4752062]$	0.059	
se_{10}	2.4958118		0.1054803	$[-0.0669724, 04595453]$	0.055	
se_{avg}	2.7289522		0.1003769	$[-0.0656373, 0.4602101]$	0.052	

Table 15: A Comparison of the standard deviation of the sample mean with the mean value of the standard error calculated by eq. (13) through 1000 replications, and empirical null rejection probabilities for the test statistic and IM approach. $T = 1000$.
DGP: $y_t = \mu + \phi_1 y_{t-1} + \epsilon_t$, $\epsilon_t \sim N(0,1)$; $\mu = 0$.

$\phi_1 = 0.5$	CV	St. Dev	St. Error	CI	Rej. Pr	q	IM
se_1	2.138258	0.0654725	0.084693	$[-0.1808599, 0.1813311]$	0.014		–
se_2	2.1455735		0.0802065	$[-0.1718532, 0.1723245]$	0.026	2	0.045
se_3	2.1502745		0.0777197	$[-0.1668832, 0.1673544]$	0.030		–
se_4	2.1553015		0.0754233	$[-0.1623243, 0.1627955]$	0.032	4	0.047
se_5	2.1652035		0.074209	$[-0.160442, 0.1609133]$	0.033	5	0.048
se_6	2.172101		0.0728887	$[-0.1580859, 0.1585572]$	0.035		–
se_7	2.163528		0.0718671	$[-0.1552509, 0.1557221]$	0.036		–
se_8	2.1648155		0.0715329	$[-0.1546199, 0.1550912]$	0.038		–
se_9	2.1893815		0.0709835	$[-0.1551743, 0.1556456]$	0.040		–
se_{10}	2.19799		0.0703676	$[-0.1544316, 0.1549029]$	0.041	10	0.056
se_{avg}	2.12701		0.0711599	$[-0.1545033, 0.1549745]$	0.043		