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Abstract. This paper considers a duopoly with asymmetric costs and demand uncertainty to study the welfare effects of pretend-but-perform regulation (PPR) of Koray and Sertel (1988) under three modes of competition, involving the Cournot, conjectural variations, and supply function competitions. PPR induces a two-stage game where each firm declares in the first stage a cost report and produces in the second stage accordingly. Theoretically characterizing and numerically computing the equilibrium of this game, we show that the consumer surplus increases if PPR is applied under the Cournot competition and it decreases if PPR is applied under the other modes of competition.

Keywords: Duopoly; regulation, Cournot, conjectural variations, supply function equilibrium.

JEL Codes: D43; L13; L51.

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1 Introduction

This paper studies the welfare effects of pretend-but-perform regulation (PPR) in a duopoly with asymmetric costs and demand uncertainty under the comparison of three modes of competition, involving the Cournot, conjectural variations, and supply function competitions. The idea of PPR was first introduced by Sertel (1982) and applied to several industrial problems, such as the principal-agent problem in sharecropping (Alkan and Sertel, 1988), the regulation a Cournot duopoly (Koray and Sertel, 1988, 1989, 2022), and the limit pricing in a Cournot duopoly (Koray and Sertel, 1990).[1] In the context of oligopoly regulation, a single-shot PPR is basically a two-stage strategic game where each firm declares in the first stage a report for its cost parameter and then produces in the second stage in accordance with the cost reports of the firms and the equilibrium predictions of a given mode of competition.

While the idea of PPR appeared in the regulation literature at almost the same time as did the seminal article of Baron and Myerson (1982) that dealt with the optimal regulation of a monopoly with unknown costs, optimal regulatory mechanisms for oligopolies were introduced, much later, by the works of Koray and Sertel (1989, 2022), Gradstein (1995), and Wang (2000). Koray and Sertel (1989, 2022) study delegation games in the context of a symmetric linear Cournotic duopoly and show that as the length of the delegation chain increases unboundedly, the equilibrium output of the industry floor, lying at one end of the delegation chain, converges monotonically to the socially efficient output induced by the marginal-cost pricing. Gradstein (1995) approaches to the oligopolistic regulation problem as an implementation problem under an informational setting where the oligopolistic firms completely know the cost structures of each other (as well as the demand structure). This setting allows him to implement the first-best social outcome (induced by the marginal-cost pricing rule) using the solution concept of Nash (1950, 1951), without appealing to any prior information of the planner (regulator) about the regulated industry. The mechanism proposed to achieve implementation involves transfers among firms as a function of their output choices. Gradstein (1995) shows that these transfers are balanced –requiring no transfers to be made by consumers to the regulated firms– if and only if the demand curve is of a certain polynomial form. To fill the remaining gap in the literature, Wang (2000) studies the problem of regulating an oligopoly with unknown costs, by extending Baron and Myerson's (1982)

[1]For more on this literature, see Koray and Sertel (1983) and Koray (1985).

Bayesian approach for optimal monopoly regulation in a non-trivial way. He considers a simplistic setting where each firm has a constant marginal cost that can be of two types, low and high, with some known (and possibly firm-specific) probability. By the Revelation Principle, Wang (2000) restricts himself to direct revelation mechanisms that require the firms to report their costs and that gives them no incentive to lie. The optimal mechanism he proposes requires that if any firm reports the low cost, all firms that report the low cost produce together the associated first-best output and the other firms produce nothing. If all firms report the high cost, then the ex-ante least efficient firm is entitled to the production of the whole output. As the number of firms becomes higher, the information rents obtained by the regulated firms totally dissipate, rendering the marginal-cost pricing be implemented without any deadweight loss.

The literature on oligopoly regulation involves several other approaches, as well. For example, Liao and Tauman (2004) considers a Cournot oligopoly where a planner offers to the firms a per unit subsidy in return for upfront fees that are determined in a first-price auction to which the participation is voluntary. They show that the socially optimal outcome in this oligopoly can be achieved without the planner running any deficit if and only if at least one firm in the oligopoly is not subsidized. Anton and Gertler (2004) examine the regulation of a differentiated duopoly with incomplete information under spatial competition. They show that the optimal incentive regulation assigns market segments to firms and also determines in each market output distribution across consumers. Earler et al. (2007), Roques and Savva (2009), Grimm and Zöttl (2010), Reynolds and Rietzke (2016), and Okumura (2017) consider the price-cap regulation that are frequently used in Cournot (and/or Stackelberg) oligopolies and show that a reduction of the price-cap level may decrease the social welfare under certain demand and/or cost conditions. Evrenk and Zenginobuz (2010), who study the problem of regulation in a non-linear Cournot duopoly, propose a regulatory mechanism in the form of a revenue contest where the firm with the lower revenue must pay a penalty fee (to the other firm) that is proportional to the difference between their revenues. They show that this mechanism implements the first-best social outcome if the firms are symmetric with respect to their costs, while it may lead to increased social surplus if the firms are asymmetric. More recently, Lan et al. (2013) study the regulation of a multi-firm industry using a yardstick competition where each firm's price is tied to the average of the other firms' expected costs in order to induce similar firms to compete with each other. Their results suggest that the firms have an incentive to lower their marginal costs if their marginal costs exceed the yardstick price .

While the works that deal with the regulation of oligopolies are quite diverse, they all have their limitations. For instance, in Koray and Sertel (1989), the regulated duopoly is linear (and symmetric), and it is an open problem whether their results can be extended to non-linear duopolies as well. In Gradstein (1995) and Evrenk and Zenginobuz (2010), to check whether the mechanism is individually rational for the firms (i.e., their participation constraints are satisfied), the regulator would require complete information about the cost (and demand) structure. This informational assumption is also needed in Liao and Tauman (2004) so that the regulator can correctly calculate the social optimum that she is tasked to implement. Additionally, in Gradstein (1995) the problem of unbalanced transfers may arise unless the demand function is of a particular form. The yardstick competition studied by Lan et al. (2013) and many others can be argued to increase the incentives for collusion by the firms – unless they are designed to be collusion-proof as in Tangerås (2002)– since the firms may quickly realize they are played out against each other. The price-cap regulation studied by Earler et al. (2007) and others is unlikely to yield social optimum, even though it has been extremely popular. Similarly, the studies that deal with oligopoly regulation under private cost information, such as the work of Wang (2000), Anton and Gertler (2004), Lan et al. (2013) among many others, have also their limitations. The proposed optimal incentive-compatible mechanisms in these studies can only ensure an 'incentive-efficient' outcome. Since the informational rents of the firms due to their private information can be optimally limited but never be completely eliminated, the complete information level of the social optimum cannot be ever implemented. Moreover, in studies that inevitably use Bayesian approach because of the presence of incomplete information in the industry, the optimal regulatory mechanisms and their outcomes usually depend on the regulator's prior beliefs about the private costs of the firms. This dependence creates, on the part of the regulator, a moral hazard problem that was conjectured by Crew and Kleindorfer (1986), Vogelsang (1988), Koray and Sertel (1990) among many others, and extensively investigated by Koray and Saglam (1999, 2005).

Returning back to a single-shot PPR, we should notice that it may not implement the social optimum (though it may increase the welfare of consumers), as shown by the earlier work of Koray and Sertel (1988). However, PPR also has a number of strengths in comparison to alternative regulatory schemes. For instance, under PPR one does not require any form of transfers among firms or from consumers to firms; thus the usual problems of the regulation literature, such as the individual rationality of the mechanism or the balancedness of the incentive transfers, do not ever appear. Moreover, PPR does not require the regulator to (even) incompletely know the private cost information of the regulated firms. The regulator is merely tasked under PPR with checking and enforcing that the production strategies of the firms are in accordance with their claimed costs under the equilibrium predictions of the competition mode which the firms are assumed to engage in. Due to the independence of PPR from any subjective assessment (belief) of the regulator about the private cost information of the regulated firms, the regulatory outcome is corruption-proof, i.e., it can never be manipulated by a corrupt regulator for rent seeking.

Although the idea of a PPR is quite old, regulating a duopoly (or an oligopoly) through a single-shot PPR has been so far studied, to the best of our knowledge, by Koray and Sertel (1988) only. They showed that if the duopolistic firms in a linear industry produce a homogenous good under Cournot competition, then a single-shot PPR can yield a higher industry output, a higher consumer surplus, and even a higher social welfare despite the fall in the industry profits. In this paper, we investigate whether the welfare effects of PPR obtained under the Cournot competition can be extended to other competition modes such as the (quantity competition with) conjectural variations and the supply function competition.

Since the equilibria of the three modes of competition under study are entirely different, it is likely that PPR will have different effects on them. Under the Cournot (1838) competition, it is well known that each firm in an oligopoly selects a fixed output that maximizes its profit, given its conjectures about the fixed outputs that will be selected by the other firms in a similar way. The equilibrium arises at an output profile where the actual and conjectured outputs are the same. Quantity competition with conjectural variations additionally takes into account each firm's beliefs or conjectures about the rival firms' reactions to its decision, under the restriction that these conjectures must be consistent with the actual reactions. The origins of the idea of conjectural variations can be traced back to Bowley (1924) and Frisch (1933). This mode of competition is more flexible and general than Cournot competition, which assumes no conjectural variation. Finally, the supply function competition, which was first introduced by Grossman (1981) and developed by Klemperer (1988) for economic applications, allows firms to compete over price-dependent supply functions (instead of merely fixed outputs) taking into account their conjectures about the supply functions of their rivals. The three modes of competition we consider, namely the Cournot competition, the quantity competition with conjectural variations, and the supply function competition, have all been extensively used to model the firms' behavior in power industries.[2] Studying these three modes of competition in comparison, our study will help us to reveal which mode among them is the most desirable in power industries in the absence of regulation or in the presence of the PPR. Moreover, since we consider a duopoly with demand uncertainty and quadratic cost functions, the part of our results dealing with PPR under Cournot competition will allow us to make a (narrow) robustness check for the earlier results of Koray and Sertel (1988) assuming a linear duopoly with a non-stochastic demand curve.

The rest of the paper is organized as follows: Section 2 introduces the basic structures for the duopolistic industry. Section 3 characterizes theoretically the equilibrium outcome of the pretension game associated with PPR under each of our competition modes and also the socially optimal outcome for the industry calculated at the true costs of the firms. Section 4 conducts numerical computations to compare the output and welfare effects of PPR under the studied competition modes. Finally, Section 5 concludes.

2 Basic Structures

We consider a duopoly with two firms indexed by $i = 1, 2$. The firms produce a single homogenous good facing an uncertain demand function

$$D(p) = \alpha - bp, \tag{1}$$

where p is the price of the product, b is a commonly known parameter with a positive real value, and α is a scalar random variable with positive real values representing an ex-ante unobservable demand shock. We denote the mean and the standard deviation of α by $E[\alpha] = \mu$ and σ respectively. The inverse demand function derived from (1) is $P(Q) = (\alpha/b) - Q/b$, where Q

[2] For applications of these three modes of competition to power industries, see Scott and Read (1996) and Ramos et al. (1998) dealing with the Cournot competition, Ventosa et al. (2000), García-Alcalde et al. (2002), and Barquín et al. (2004) dealing with conjectural variations, and Green and Newbery (1992), Rudkevich and Duckworth (1998), Day et al. (2002), Newbery and Greve (2017), Escrihuela-Villar et al. (2020) dealing with the supply function competition.

denotes the industry output. The consumer surplus from consuming Q units of output is $CS(Q) = \int_0^Q P(x)dx - P(Q)Q = Q^2/(2b)$.

Whenever firm i produces $q \geq 0$ units of output, it incurs the cost

$$C_i(q) = \frac{\theta_i}{2} q^2, \tag{2}$$

where $\theta_i \geq 0$ is the marginal cost of producing one unit of output. We assume that θ_i is the private knowledge of firm i, whereas the rest of the above structures is commonly known by both firms as well as by any third party.

We will assume that the firms in the duopoly may jointly engage in one of three competition modes, namely the quantity competition with no conjectural variation, which is also known the Cournot competition (C), the quantity competition with conjectural variation (CV), and the supply function competition (SFC). Hereafter, we let $\gamma \in \{C, CV, SFC\}$ denote the competition mode engaged by the firms. Also, we let $q_i^\gamma(\theta_1, \theta_2, \alpha)$ denote the output produced by firm i under the equilibrium of competition mode γ at the realization α of the demand shock. We will formally present in the next section how the output q_i^γ is calculated.

We assume that a public authority (regulator) uses *Pretend-but-Perform Regulation* (PPR) of Koray and Sertel (1988) to regulate the outputs of firms. The PPR is a two-stage strategic game that requires, in our model, each firm i to publicly report (possibly untruthfully) its private θ_i value, in the first stage, as a function of the realization α of the demand shock and then to choose and declare, in the second stage, its production strategy for the competition mode γ (upon which the firms are assumed to have voluntarily agreed upon prior the starting of the game) under the pretension that the cost reports of the firms reflect their true costs. Thus, from the viewpoints of all parties (including the public authority) the declared production strategy profile of the firms must be an equilibrium profile which can be publicly checked to be consistent with the cost reports of the firms under the competition mode γ. For example, when the mode γ happens to be the Cournot competition, the quantity of outputs declared by the firms must be equal to the Cournot equilibrium quantities that one can calculate using the cost reports of the firms. In the next section, we will deal with the characterization of the equilibrium outcome under the PPR described above.

3 Characterization of Equilibrium

We can solve the regulatory game of pretension described above using backward induction. Thus, we will first consider the second stage game where the firms will choose their production strategies.

3.1 Second Stage: Choosing Production Strategies

For each possible pair of cost reports, the firms will choose, in the second stage of the pretension game, their production strategies depending upon the competition mode γ. Below, we will characterize these strategies for each value of γ in $\{C, CV, SFC\}$ separately.

3.1.1 Cournot Competition

Here, the duopolistic firms determine their (fixed) quantities simultaneously, without observing the realization of the demand shock. Given the output quantities q_1 and q_2 chosen by firms 1 and 2 respectively, the good market clears at the realization α of the demand shock if $D(p) = q_1 + q_2$ implying the equilibrium price

$$p(\alpha, q_1, q_2) = \frac{1}{b}\left(\alpha - q_1 - q_2\right). \tag{3}$$

A pair of quantities of outputs (q_1^*, q_2^*) constitute a Cournot (Nash) equilibrium if for each $i, j \in \{1, 2\}$ with $j \neq i$ the output q_i^* maximizes the expected profit of firm i whenever firm j produces the output q_j^*; i.e., q_i^* solves

$$\max_{q_i \geq 0} E_\alpha\left[\pi_i(\alpha, q_i, q_j^*)\right] = \max_{q_i \geq 0} E_\alpha\left[p(\alpha, q_i, q_j^*)q_i - \frac{\theta_i}{2}(q_i)^2\right], \tag{4}$$

or more explicitly

$$\max_{q_i \geq 0} E_\alpha\left[\frac{1}{b}\left(\alpha - q_i - q_j^*\right)q_i - \frac{\theta_i}{2}(q_i)^2\right]. \tag{5}$$

Proposition 1. *The Cournot competition has always a unique Nash equilibrium such that the output produced by firm $i \in \{1, 2\}$ is equal to*

$$q_i^C(\theta_i, \theta_j) = \frac{(1 + b\theta_j)E[\alpha]}{(2 + b\theta_i)(2 + b\theta_j) - 1} \tag{6}$$

where $j = \{1,2\} \setminus \{i\}$.

Proof of Proposition 1. The first-order necessary conditions for the problem in (5) can be calculated as

$$0 \;=\; \frac{\partial}{\partial q_i} E_\alpha\left[\pi(\alpha, q_i, q_j^*)\right] = E_\alpha\left[\frac{1}{b}\left(\alpha - 2q_i - q_j^*\right) - \theta_i q_i\right], \tag{7}$$

implying

$$q_i = \frac{E[\alpha] - q_j^*}{2 + b\theta_i} \tag{8}$$

for each $i, j \in \{1,2\}$ with $j \neq i$. Writing (8) for $i = 1$ and $i = 2$ separately, and then solving them together yields $q_i^* = q_i^C(\theta_i, \theta_j)$ as given by (6). To check whether the second-order sufficiency condition holds, we differentiate the right-hand side of (7) w.r.t. q_i and obtain

$$\frac{\partial^2}{\partial q_i^2} E_\alpha\big[\pi_i(\alpha, q_i, q_j^*)\big] = -\frac{2}{b} - \theta_i \tag{9}$$

which is always negative. ■

Notice from Proposition 1 that the equilibrium quantities of the firms are ex-ante and ex-post the same, as they depend on the mean of the demand shock α but not on its realization. On the other hand, the ex-post (realized) value of the equilibrium price changes with α, while its ex-ante (expected) value changes with the mean of α. In particular, the expected equilibrium price can be calculated as

$$p^{C,e} = E_\alpha\big[p(\alpha, q_1^C, q_2^C)\big] = \frac{1}{b}\left(\frac{(1+b\theta_1)(1+b\theta_2)}{(2+b\theta_1)(2+b\theta_2) - 1}\right) E[\alpha]. \tag{10}$$

Similarly, the expected equilibrium profit of firm $i \in \{1,2\}$ can be calculated as

$$\pi_i^{C,e} = E_\alpha\big[\pi_i^C(\alpha)\big] = E_\alpha\left[p(\alpha, q_1^C, q_2^C)q_i^C - \frac{\theta_i (q_i^C)^2}{2}\right] = p^{C,e} q_i^C - \frac{\theta_i}{2}(q_i^C)^2. \tag{11}$$

Notice from (6) and (10) that $p^{C,e} = (1 + b\theta_i)q_i^C / b$ for any $i \in \{1,2\}$. Therefore,

$$\pi_i^{C,e} = \left(\frac{1}{b} + \frac{\theta_i}{2}\right)(q_i^C)^2 = \left(\frac{2 + b\theta_i}{2b}\right)\left(\frac{(1+b\theta_j)E[\alpha]}{(2+b\theta_i)(2+b\theta_j) - 1}\right)^2, \tag{12}$$

for any $i \in \{1,2\}$. Finally, we can calculate the expected (equivalently ex-post) equilibrium consumer surplus as

$$CS^{C,e} = \frac{\left(q_1^C + q_2^C\right)^2}{2b} = \left(\frac{1}{2b}\right)\left(\frac{(2+b\theta_1+b\theta_2)E[\alpha]}{(2+b\theta_1)(2+b\theta_2)-1}\right)^2. \tag{13}$$

3.1.2 Quantity Competition with Conjectural Variations

Under this competition mode, the duopolistic firms still compete, as in Cournot competition, by specifying their (fixed) quantities simultaneously, without observing the demand shock. However, each firm, now, also takes into consideration the effect of its output variation on the output of its rival. More formally, we assume that each firm will have a constant conjecture about the marginal reaction of its rival to a marginal change in its output. So, let the conjecture of firm $i \in \{1,2\}$ about the marginal reaction of rival firm $j \neq i$ be given by

$$\frac{dq_j}{dq_i} = r_{ij}. \tag{14}$$

Notice that r_{ij} is always assumed to be zero under Cournot competition. The conjectures of the firms (with nonzero r_{ij}'s) will affect the first-order conditions associated with their profit maximization problems, but the form of these problems are themselves independent of these conjectures. As in the case of Cournot competition, whenever firms 1 and 2 and choose the output levels q_1 and q_2 respectively, the product market will clear at the realization α of the demand shock if equation (3) holds, and given the resulting market-clearing price $p(\alpha, q_1, q_2)$, a pair of outputs (q_1^*, q_2^*) will constitute a Nash equilibrium if for each $i, j \in \{1,2\}$ with $j \neq i$ the quantity q_i^* maximizes the expected profit of firm i expressed in (5).

Following the definition of Possajennikov (2009), we say that the conjecture of firm i is weakly consistent if the conjectured reaction of firm j is equal to the actual slope of the reaction function of firm j at the best responses (q_i^*, q_j^*), i.e. $r_{ij} = dq_j^*(q_i, r_{ji})/dq_i$ at q_i^*. We know that the reaction function of firm j is obtained by the first-order condition associated with the maximization problem of this firm. For any $i, j \in \{1,2\}$ with $j \neq i$, let $F_j(q_j, q_i^*, r_{ji})$ denote the first-order necessary condition associated with the maximization problem of firm j given by (5). Then, the conjecture of firm i is weakly consistent at the best responses (q_i^*, q_j^*) if

$$r_{ij} = \frac{dq_j^*(q_i, r_{ji})}{dq_i}\Big|_{q_i=q_i^*} = -\frac{\partial F_j(q_j^*, q_i^*, r_{ji})/\partial q_i^*}{\partial F_j(q_j, q_i^*, r_{ji})/\partial q_j\Big|_{q_j=q_j^*}} \tag{15}$$

when $\partial F_j(q_j, q_i^*, r_{ji})/\partial q_j\Big|_{q_j=q_j^*} \neq 0$.

Proposition 2. *The quantity competition with conjectural variations has always a unique Nash equilibrium output profile* (q_1^{CV}, q_2^{CV}) *supported by a unique weakly conjectural variation profile* (r_{12}, r_{21}) *such that*

$$q_i^{CV}(\theta_i, \theta_j) = \frac{(1 + r_{ji} + b\theta_j)E[\alpha]}{(2 + r_{ij} + b\theta_i)(2 + r_{ji} + b\theta_j) - 1} \tag{16}$$

and

$$r_{ij} = -\left(1 + \frac{b\theta_i}{2}\right)\left(1 - \sqrt{1 - \frac{4}{(2 + b\theta_i)(2 + b\theta_j)}}\right) \tag{17}$$

for all $i, j \in \{1, 2\}$ *with* $j \neq i$.

Proof of Proposition 2. Under the quantity competition with conjectural variations, the first-order necessary conditions for the problem in (5) can be calculated as

$$0 = \frac{\partial}{\partial q_i} E_\alpha\left[\pi(\alpha, q_i, q_j^*)\right] = E_\alpha\left[\frac{1}{b}\left(\alpha - 2q_i - r_{ij}q_i - q_j^*\right) - \theta_i q_i\right], \tag{18}$$

implying

$$q_i = \frac{E[\alpha] - q_j^*}{2 + r_{ij} + b\theta_i} \tag{19}$$

for each $i, j \in \{1, 2\}$ with $j \neq i$. Writing (19) for $i = 1$ and $i = 2$ separately, and then solving them together yields $q_i^* = q_i^{CV}(\theta_i, \theta_j)$ as given by (16). To check whether the second-order sufficiency condition holds, we have to first calculate the weakly consistent conjectures. Notice that equality (15) implies that

$$r_{ij} = -\frac{-\frac{1}{b}}{-\frac{r_{ji}}{b} - \left(\frac{2}{b} + \theta_j\right)} = -\frac{1}{2 + r_{ji} + b\theta_j} \tag{20}$$

and similarly

$$r_{ji} = -\frac{1}{2 + r_{ij} + b\theta_i}. \tag{21}$$

The two equations above together imply

$$r_{ji} = \left(\frac{2 + b\theta_j}{2 + b\theta_i}\right) r_{ij}. \tag{22}$$

Inserting this into (20) and rearranging the terms, we obtain

$$(r_{ij})^2 + (2 + b\theta_i)r_{ij} + \left(\frac{2 + b\theta_i}{2 + b\theta_j}\right) = 0, \tag{23}$$

which can be solved to obtain (17). Now, to check whether the second-order sufficiency condition associated with (5) is satisfied, we differentiate the right-hand side of (18) w.r.t. q_i and obtain

$$\frac{\partial^2}{\partial q_i^2} E_\alpha\big[\pi_i(\alpha, q_i, q_j^*)\big] = -\frac{2 + b\theta_i + r_{ij}}{b} = -\frac{(2 + b\theta_i)(1 - K)}{b}, \tag{24}$$

where

$$K = \frac{1}{2}\left(1 - \sqrt{1 - \frac{4}{(2 + b\theta_i)(2 + b\theta_j)}}\right) < \frac{1}{2} \tag{25}$$

from (17). Therefore, $\frac{\partial^2}{\partial q_i^2} E_\alpha\big[\pi_i(\alpha, q_i, q_j^*)\big] < 0$ always holds. ■

Notice from Proposition 2 that (as in the case of Proposition 1) the equilibrium quantities of the firms are ex-ante and ex-post the same and independent of the realization of α. However, the ex-post (realized) value of the equilibrium price changes with α, while its ex-ante (expected) value depends on the mean of the distribution of α only. We can calculate the expected price as

$$\begin{aligned} p^{CV,e} &= E_\alpha\big[p(\alpha, q_1^{CV}, q_2^{CV})\big] \\ &= \frac{1}{b}\left(\frac{(1 + r_{12} + b\theta_1)(1 + r_{21} + b\theta_2)}{(2 + r_{12} + b\theta_1)(2 + r_{21} + b\theta_2) - 1}\right) E[\alpha]. \end{aligned} \tag{26}$$

Similarly, we can calculate the expected equilibrium profit of firm $i \in \{1, 2\}$ as

$$\begin{aligned} \pi_i^{CV,e} &= E_\alpha\big[\pi_i^{CV}(\alpha)\big] = E_\alpha\left[p(\alpha, q_1^{CV}, q_2^{CV})q_i^{CV} - \frac{\theta_i(q_i^{CV})^2}{2}\right] \\ &= p^{CV,e}q_i^{CV} - \frac{\theta_i(q_i^{CV})^2}{2}. \end{aligned} \quad (27)$$

Notice from (16) and (26) that $p^{CV,e} = q_i^{CV}(1+r_{ij}+b\theta_i)/b$ for any $i, j \in \{1, 2\}$ with $j \neq i$. Therefore,

$$\begin{aligned} \pi_i^{CV,e} &= \left(\frac{1+r_{ij}}{b} + \frac{\theta_i}{2}\right)(q_i^{CV})^2 \\ &= \left(\frac{1+r_{ij}}{b} + \frac{\theta_i}{2}\right)\left(\frac{(1+r_{ji}+b\theta_j)E[\alpha]}{(2+r_{ij}+b\theta_i)(2+r_{ji}+b\theta_j)-1}\right)^2 \end{aligned} \quad (28)$$

for any $i, j \in \{1, 2\}$ with $j \neq i$. Finally, we can calculate the expected (equivalently ex-post) equilibrium consumer surplus as

$$\begin{aligned} CS^{CV,e} &= \frac{\left(q_1^{CV} + q_2^{CV}\right)^2}{2b} \\ &= \left(\frac{1}{2b}\right)\left(\frac{(2+r_{12}+r_{21}+b\theta_1+b\theta_2)E[\alpha]}{(2+r_{12}+b\theta_1)(2+r_{21}+b\theta_2)-1}\right)^2. \end{aligned} \quad (29)$$

3.1.3 Supply Function Competition

Here, the duopolistic firms compete by specifying their supply functions simultaneously, without observing the realization of the demand shock. We assume that the supply function of each firm is a linear mapping from non-negative prices to non-negative quantities (of output), denoted by $S_i(p) = \eta_i p$, where $\eta_i \geq 0$ denotes the slope parameter. Given the supply functions S_1 and S_2 chosen by firms 1 and 2 respectively, the product market clears at the realization α of the demand shock if $D(p) = S_1(p) + S_2(p)$ implying the equilibrium price

$$p(\alpha, \eta_1, \eta_2) = \frac{\alpha}{b + \eta_1 + \eta_2}. \quad (30)$$

For convenience, let us define $p(\alpha, \eta_i, \eta_j) = p(\alpha, \eta_1, \eta_2)$ for any $i, j \in \{1, 2\}$ with $i \neq j$. A pair of supply functions $(S_1^*(p), S_2^*(p)) = (\eta_1^* p, \eta_2^* p)$ constitute a Nash equilibrium if for each $i, j \in \{1, 2\}$ with $j \neq i$, producing according to $S_i^*(p)$ maximizes the expected profit of firm i when firm j produces according to $S_j^*(p)$. That is, $(\eta_1^* p, \eta_2^* p)$ constitute a Nash equilibrium if for each $i, j \in \{1, 2\}$ with $j \neq i$ the parameter η_i^* solves

$$\max_{\eta_i \geq 0} E_\alpha \left[p\left(\alpha, \eta_i, \eta_j^*\right) S_i^*\left(p\left(\alpha, \eta_i, \eta_j^*\right)\right) - \frac{\theta_i}{2} \left[S_i^*\left(p\left(\eta_i, \eta_j^*, \alpha\right)\right)\right]^2 \right], \quad (31)$$

or more explicitly

$$\max_{\eta_i \geq 0} \left(\eta_i - \frac{\theta_i \eta_i^2}{2}\right) \left(\frac{1}{b + \eta_i + \eta_j^*}\right)^2 E[\alpha^2]. \quad (32)$$

Proposition 3. *The supply function competition has always a unique Nash equilibrium such that the supply function of firm $i \in \{1, 2\}$ is given by $S_i^*(p) = \eta_i^*(\theta_i, \theta_j) p$ where*

$$\eta_i^*(\theta_i, \theta_j) = \frac{-(b + d_i) + \sqrt{(b + d_i)^2 + 4\left(\dfrac{b - d_i}{\theta_j} - b d_i\right)}}{2} \quad (33)$$

with $j \in \{1, 2\} \setminus \{i\}$ and

$$d_i = \frac{b(\theta_i - \theta_j)}{\theta_i + \theta_j + \theta_i \theta_j}. \quad (34)$$

Proof of Proposition 3. If the pair of supply functions $S_1(p) = \nu_1 p$ and $S_2(p) = \nu_2 p$ form a Nash equilibrium, then for each $i, j \in \{1, 2\}$ with $j \neq i$, the market clearing price must solve

$$\max_{p \geq 0} E_\alpha \left[p\left(\alpha - p - S_j(p)\right) - \theta_i (\alpha - p - S_j(p))^2/2 \right]. \quad (35)$$

The corresponding first-order necessary condition is

$$0 = E_\alpha \left[\alpha - bp - S_j(p) + \left(p - \theta_i \left[\alpha - bp - S_j(p)\right]\right) \left(-b - S_j'(p)\right) \right], \quad (36)$$

or

$$\begin{aligned} 0 &= E_\alpha[S_i(p) + (p - \theta_i S_i(p))(-b - \eta_j)] \\ &= [\eta_i + (1 - \theta_i \eta_i)(-b - \eta_j)] E_\alpha[p], \end{aligned} \quad (37)$$

further implying

$$\eta_i(\theta_i, \theta_j) = \frac{b + \eta_j}{1 + \theta_i(b + \eta_j)} \tag{38}$$

for each $i, j \in \{1, 2\}$ with $j \neq i$. Writing (38) for $i = 1$ and $i = 2$ separately, and then solving them together yields (33)-(34). To check whether the second-order sufficiency condition holds, we differentiate the right-hand side of (37) w.r.t. p and obtain

$$\eta_i + \big(1 - \theta_i \eta_i\big)\left(-b - \eta_j\right), \tag{39}$$

which is always equal to zero (hence non-positive) by (38). ■

Notice from Proposition 3 that the equilibrium supply functions of the firms do not depend on the demand shock realization α (or on any of its statistical moments). However, the ex-post (realized) values of the equilibrium price and quantities change with α. That is,

$$p^{SFC}(\theta_1, \theta_2, \alpha) = \frac{\alpha}{b + \eta_1^*(\theta_1, \theta_2) + \eta_2^*(\theta_2, \theta_1)} \tag{40}$$

and

$$q_i^{SFC}(\theta_i, \theta_j, \alpha) = \eta_i^*(\theta_i, \theta_j) p^{SFC}(\theta_1, \theta_2, \alpha). \tag{41}$$

So, we can calculate the expected price as

$$E_\alpha\big[p^{SFC}(\theta_1, \theta_2, \alpha)\big] = \frac{E[\alpha]}{b + \eta_1^*(\theta_1, \theta_2) + \eta_2^*(\theta_2, \theta_1)}. \tag{42}$$

At this price, firm $i \in \{1, 2\}$ will produce the expected equilibrium output

$$\begin{aligned} E_\alpha\big[q_i^{SFC}(\theta_i, \theta_j, \alpha)\big] &= \eta_i^*(\theta_i, \theta_j) E_\alpha\big[p^{SFC}(\theta_1, \theta_2, \alpha)\big] \\ &= \frac{\eta_i^*(\theta_i, \theta_j) E[\alpha]}{b + \eta_1^*(\theta_1, \theta_2) + \eta_2^*(\theta_2, \theta_1)}. \end{aligned} \tag{43}$$

Consequently, the expected equilibrium profit of firm $i \in \{1, 2\}$ will be

$$E_\alpha\big[\pi_i^{SFC}(\theta_i, \theta_j, \alpha)\big] = \left(\eta_i^* - \frac{\theta_i (\eta_i^*)^2}{2}\right)\left(\frac{1}{b + \eta_1^* + \eta_2^*}\right)^2 E\big[\alpha^2\big], \tag{44}$$

where $\eta_i^* = \eta_i^*(\theta_i, \theta_j)$ for each i. Finally, we can calculate the expected equilibrium consumer surplus as

$$\begin{aligned} E_\alpha\big[CS^{SFC}(\theta_1,\theta_2,\alpha)\big] &= E_\alpha\left[\frac{\big(q_1^{SFC}(\theta_1,\theta_2,\alpha)+q_2^{SFC}(\theta_2,\theta_1,\alpha)\big)^2}{2b}\right] \\ &= \frac{(\eta_1^*)^2+(\eta_2^*)^2}{2b(b+\eta_1^*+\eta_2^*)^2}\,E\big[\alpha^2\big]\,. \end{aligned} \tag{45}$$

3.1.4 Social Optimum at the True Costs

Here, we will calculate the socially optimal output levels and the implied welfare distribution in the complete information case where the cost parameters θ_1 and θ_2 of the firms are known by a benevolent regulator. (We will later use these calculations to measure the efficiency of the PPR under the three competition modes dealt with in the previous three subsections.) The aim of the regulator is to choose the output levels that maximize the expected social welfare defined as the sum of the expected consumer surplus and the expected industry profit. Let the regulated quantities of firms 1 and 2 are denoted by q_1 and q_2 respectively. At the realization α of the demand shock, the product market clears if the price satisfies (3). Then, a pair of quantities of outputs (q_1^O, q_2^O) maximize the expected social welfare if for each $i, j \in \{1,2\}$ with $j \neq i$ the quantity q_i^O solves

$$\begin{aligned} \max_{q_i\ge 0} E_\alpha\left[SW(\alpha,q_i,q_j^O)\right] = \max_{q_i\ge 0} E_\alpha &\left[V(q_i+q_j^O) - p(\alpha,q_i,q_j^O)\left(q_i+q_j^O\right)\right] \\ +E_\alpha &\left[p(\alpha,q_i,q_j^O)\left(q_i+q_j^O\right) - \frac{\theta_i}{2}(q_i)^2 - \frac{\theta_j}{2}(q_j^O)^2\right] \\ = \max_{q_i\ge 0} E_\alpha &\left[V(q_i+q_j^O) - \frac{\theta_i}{2}(q_i)^2 - \frac{\theta_j}{2}(q_j^O)^2\right]. \end{aligned} \tag{46}$$

Proposition 4. *The pair of outputs (q_1^O, q_2^O) maximizes the expected social welfare if*

$$q_i^O(\theta_i,\theta_j) = \frac{b\theta_j E[\alpha]}{(1+b\theta_i)(1+b\theta_j)-1} \tag{47}$$

where $j = \{1,2\} \setminus \{i\}$.

Proof of Proposition 4. The first-order necessary condition for the problem in (46) can be calculated as

$$0 = \frac{\partial}{\partial q_i} E_\alpha \left[SW(\alpha, q_i, q_j^O) \right] = E_\alpha \left[\frac{1}{b} \left(\alpha - q_i - q_j^O \right) - \theta_i q_i \right], \qquad (48)$$

implying

$$q_i = \frac{E[\alpha] - q_j^O}{1 + b\theta_i} \qquad (49)$$

for each $i, j \in \{1,2\}$ with $j \neq i$. Writing (49) for $i = 1$ and $i = 2$ separately, and then solving them together yields the solution $q_i^O(\theta_i, \theta_j)$ in (47). To check whether the second-order sufficiency condition holds, we differentiate the right-hand side of (48) w.r.t. q_i and obtain

$$\frac{\partial^2}{\partial q_i^2} E_\alpha \left[SW(\alpha, q_i, q_j^O) \right] = -\frac{1}{b} - \theta_i \qquad (50)$$

which is always negative. ■

Notice from Proposition 4 that the socially optimal outputs of the firms are ex-ante and ex-post the same as these outputs depend on the mean of α but not on its realization. However, the ex-post (realized) value of the equilibrium price changes with α. In particular, the expected value of the socially optimal price can be calculated as

$$p^{O,e} = E_\alpha \left[p(\alpha, q_1^O, q_2^O) \right] = \left(\frac{\theta_1 \theta_2}{(1 + b\theta_1)(1 + b\theta_2) - 1} \right) E[\alpha]. \qquad (51)$$

At the social optimum, the expected profit of firm $i \in \{1,2\}$ can be calculated as

$$\pi_i^{O,e} = E_\alpha \left[\pi_i^O(\alpha) \right] = E_\alpha \left[p(\alpha, q_1^O, q_2^O) q_i^O - \frac{\theta_i (q_i^O)^2}{2} \right] = p^{O,e} q_i^O - \frac{\theta_i (q_i^O)^2}{2}. \qquad (52)$$

Also notice from (47) and (51) that $p^{O,e} = \theta_i q_i^O$ for any $i \in \{1,2\}$. Therefore,

$$\pi_i^{O,e} = \frac{\theta_i}{2} (q_i^O)^2 = \left(\frac{\theta_i}{2} \right) \left(\frac{b\theta_j E[\alpha]}{(1 + b\theta_i)(1 + b\theta_j) - 1} \right)^2 \qquad (53)$$

for any $i \in \{1,2\}$. Finally, we can calculate the expected (equivalently ex-post) equilibrium consumer surplus as

$$CS^{O,e} \quad = \quad \frac{\left(q_1^O + q_2^O\right)^2}{2b} = \left(\frac{1}{2b}\right)\left(\frac{(b\theta_1 + b\theta_2)E[\alpha]}{(1+b\theta_1)(1+b\theta_2)-1}\right)^2. \tag{54}$$

3.2 First Stage: Reporting Cost Parameters

Given any competition mode $\gamma \in \{C, CV, SFC\}$ and any cost report profile $(\hat{\theta}_1, \hat{\theta}_2)$, the firms as well as the public authority can calculate for each α, the equilibrium output of firm i, i.e., $q_i^\gamma(\hat{\theta}_i, \hat{\theta}_j, \alpha)$ with $j \neq i$. For each α, $q_i^\gamma(\hat{\theta}_i, \hat{\theta}_j, \alpha)$ is obtained from equation (6) if $\gamma = C$, obtained from equation (16) if $\gamma = CV$, and obtained from equation (41) if $\gamma = SFC$. Thus, the declared equilibrium outputs cannot differ from the publicly computable outputs under the PPR. Accordingly, using the demand equation (1), all parties in the industry can calculate the induced equilibrium price

$$p^\gamma(\hat{\theta}_1, \hat{\theta}_2, \alpha) = \frac{1}{b}\left(\alpha - q_1^\gamma(\hat{\theta}_1, \hat{\theta}_2, \alpha) - q_2^\gamma(\hat{\theta}_2, \hat{\theta}_1, \alpha)\right). \tag{55}$$

Using the declared cost parameters, the declared equilibrium outputs, and the induced equilibrium price, firm i can then privately calculate its actual profit

$$E_\alpha\left[\pi_i^\gamma\left(\hat{\theta}_i, \hat{\theta}_j, \theta_i, \alpha\right)\right] = E_\alpha\left[p^\gamma(\hat{\theta}_1, \hat{\theta}_2, \alpha) q_i^\gamma(\hat{\theta}_i, \hat{\theta}_j, \alpha) - \frac{\theta_i}{2}\left[q_i^\gamma(\hat{\theta}_i, \hat{\theta}_j, \alpha)\right]^2\right]. \tag{56}$$

Above, we should notice that the unit marginal cost of firm i in the actual cost and profit calculations is θ_i, i.e., its private (and possibly untruthfully reported) cost parameter θ_i. We say that for any realization α of the demand shock a pair of cost reports $(\theta_1^{\gamma,*}(\alpha), \theta_2^{\gamma,*}(\alpha))$ constitute a Nash equilibrium in the first stage of the regulatory game of pretension played by the duopolists under the competition mode γ if $\theta_i^{\gamma,*}$ is a best-response to $\theta_j^{\gamma,*}$ for each $i, j \in \{1,2\}$ with $j \neq i$, i.e.,

$$E_\alpha\left[\pi_i^\gamma\left(\theta_i^{\gamma,*}, \theta_j^{\gamma,*}, \theta_i, \alpha\right)\right] \geq E_\alpha\left[\pi_i^\gamma\left(\hat{\theta}_i, \theta_j^{\gamma,*}, \theta_i, \alpha\right)\right] \text{ for all } \hat{\theta}_i \geq 0. \tag{57}$$

3.3 Bringing the Two-Stages Together

We can combine the two stages of the regulatory game of pretension to characterize the subgame-perfect Nash equilibrium, following the idea of Selten (1965). Let $s_i^\gamma(\hat{\theta}_i, \hat{\theta}_j)$ denote the strategy of firm $i \in \{1, 2\}$ in stage 2 when firm i and firm $j \neq i$ report the cost parameters $\hat{\theta}_i$ and $\hat{\theta}_j$ respectively. Notice that $s_i^\gamma(\hat{\theta}_i, \hat{\theta}_j) = q^C(\hat{\theta}_i, \hat{\theta}_j)$ if $\gamma = C$, $s_i^\gamma(\hat{\theta}_i, \hat{\theta}_j) = q^{CV}(\hat{\theta}_i, \hat{\theta}_j)$ if $\gamma = CV$, and $s_i^\gamma(\hat{\theta}_i, \hat{\theta}_j) = \eta_i^*(\hat{\theta}_i, \hat{\theta}_j)p$ for any $p \geq 0$ if $\gamma = SFC$. We say that for any realization α of the demand shock, a strategy profile involving the two-stage plans $\langle(\theta_1^{\gamma,*}(\alpha), s_1^\gamma(\hat{\theta}_1, \hat{\theta}_2)$ for any $(\hat{\theta}_1, \hat{\theta}_2)\rangle$ for firm 1 and $\langle(\theta_2^{\gamma,*}(\alpha), s_2^\gamma(\hat{\theta}_2, \hat{\theta}_1)$ for any $(\hat{\theta}_2, \hat{\theta}_1)\rangle$ for firm 2 constitute a *subgame-perfect Nash equilibrium* of the regulatory game of pretension under the competition mode γ.

In the next section, we will calculate and analyze the equilibrium of the regulatory game of pretension associated with the PPR using numerical computations, due to the analytical complexity of the two-stage optimization required by the subgame perfection.

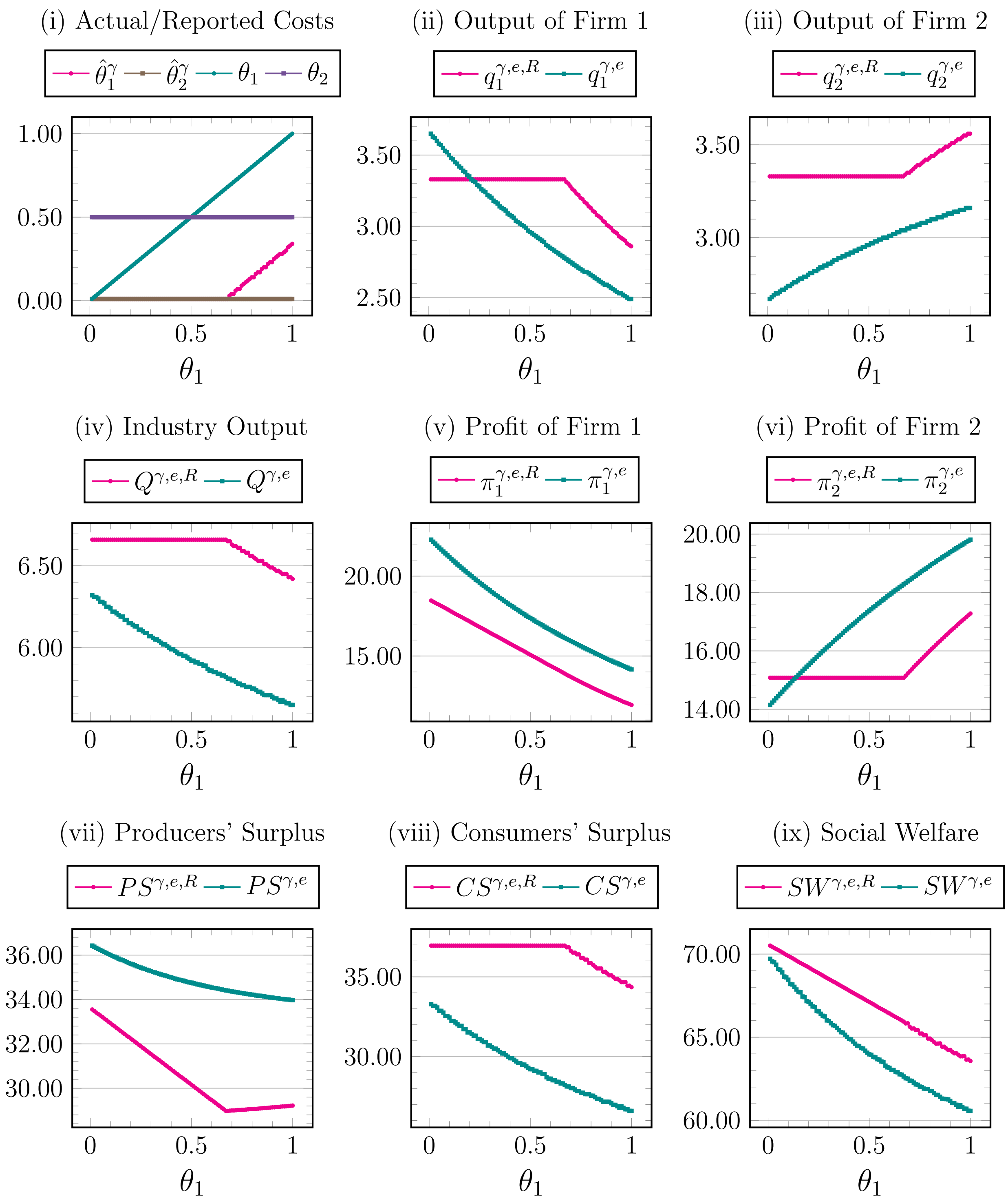
4 Computational Results

All computations in this section are performed using GAUSS, Version 3.2.34. The code and the generated data are available upon request.

For our computations, we fix θ_2 at 0.5, and change θ_1 in $\{0, 0.01, \ldots, 0.99\}$ with increments of 0.01. We should here note that the mean (μ) of the demand shock α and its standard deviation (σ) are only scale parameters in our model. $E[\alpha] = \mu$ scales the expected price and the expected quantities of the firms, whereas $E[\alpha^2] = \mu^2 + \sigma^2$ scales the expected profits of the firms and consumer surplus. Thus, we fix μ at 10 and σ at 5 for simplicity. On the other hand, we vary the demand slope parameter b inside the set $\{0.25, 0.50, 0.75\}$.

We illustrate our computational results in Figures 1-5 and A1-A6. For any model variable in $X \in \{q_1, q_2, Q, \pi_1, \pi_2, PS, CS, SW\}$ presented in Figures 1-5, we let $X^{\gamma,e,R}$ and $X^{\gamma,e}$ respectively denote the PPR (regulated) and unregulated expected values of X under the competition mode γ.

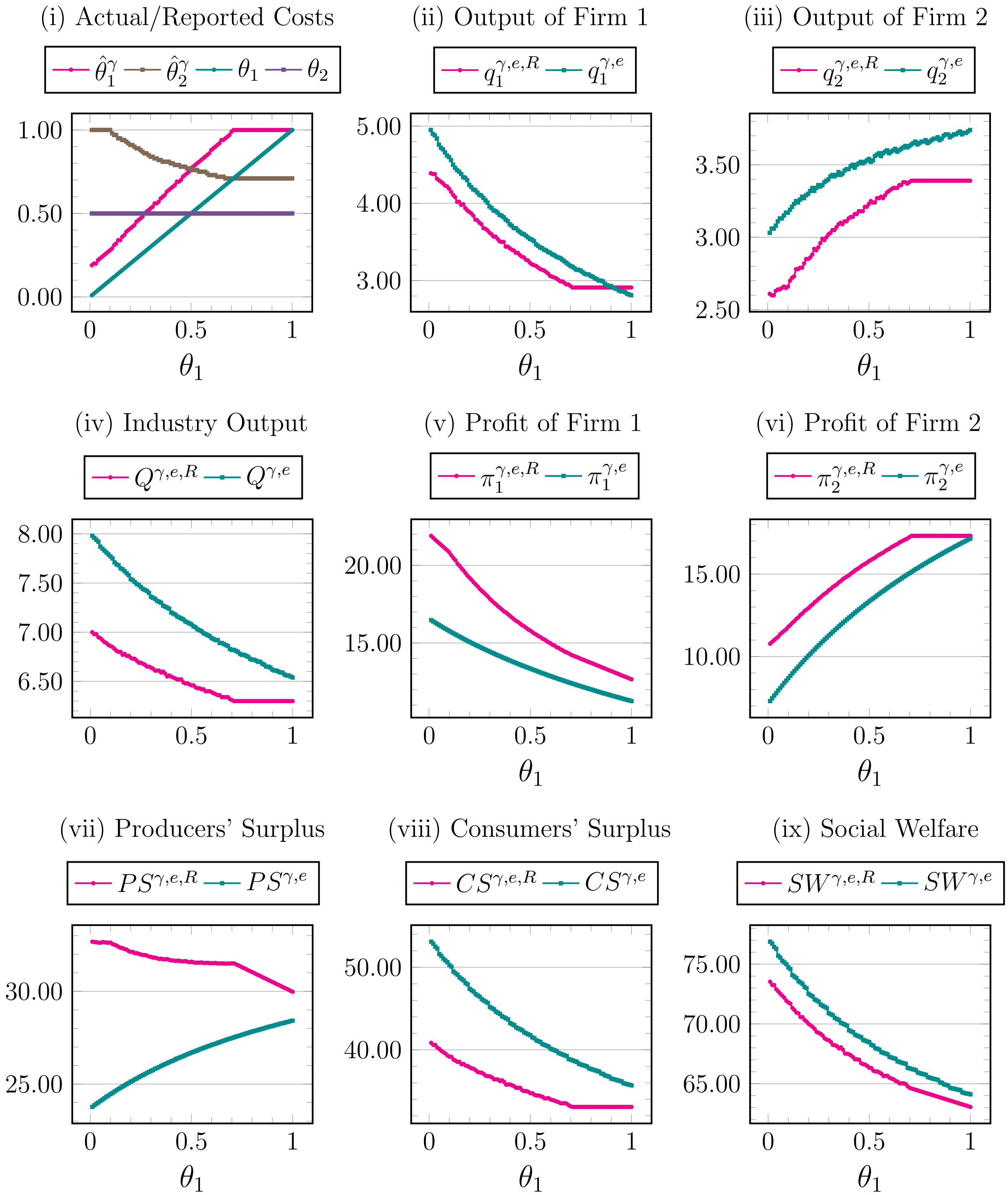
Figure 1. The Effects of PBP Regulation on the Expected Outcomes
$\gamma = C$ (Cournot Competition) & $b = 0.75$



In the next three figures, we fix the demand slope parameter b to 0.75. In Figure 1, we illustrate the effects of PPR on the expected equilibrium outcomes when the firms engage in Cournot competition. Panel (i) shows that each firm understates its private cost parameter under the PPR. Recall that θ_2 is fixed at 0.5 in our simulations. As θ_1 is increased from 0 up to 1, firm 2 always understates its cost parameter at zero. On the other hand, the cost report of firm is zero up to $\theta_1 = 0.7$ and increasing afterwards. As firm 2 understates its cost, it always produces more under the PPR (than under no regulation) and this is true firm 1 only if its unit marginal cost, θ_1, is not extremely small, as shown in panels (ii) and (iii). As expected, the output of firm 1 is nonincreasing and the output of firm 2 is nondecreasing in θ_1, since a rise in θ_1 with θ_2 is kept fixed reduces the competitive power of firm 1 relative to firm 2. Panel (iv) shows that the non-positive effect of θ_1 on the output of firm 1 dominates the non-negative effect on the output of firm 2 both in the presence and absence regulation, and therefore the regulated level of the industry output is always nonincreasing in θ_1. As the firms produce more when they are regulated according to the PPR, they generally obtain lower profits, as illustrated in panels (v) and (vi). Therefore, the producers' surplus (the industry profit) is always lower under the PPR, as portrayed by panel (vii). In accordance with the opposite effects of θ_1 on the outputs of the duopolistic firms, we find that an increase in θ_1 (with θ_2 being fixed) has a non-positive (negative) effect on the profit of firm 1 and a non-negative effect on the profit of firm 2. Of these two effects, the former dominates if firm 1 is not extremely inefficient, i.e., $\theta_1 < 0.7$, and the latter dominates otherwise. Recall that the consumer surplus is proportional to the square of the industry output. Therefore, panel (viii), where we illustrate the variation of consumer surplus, is qualitatively similar to panel (iv) illustrating the variation of the industry output. Panels (vii) and (viii) together show that the PPR has opposite effects on the welfare of consumers and the producers for all values of θ_1: a positive effect on the welfare of consumers and a negative effect on the welfare of the producers. Moreover, as panel (ix) illustrate, the former (positive) welfare effect of regulation is always more dominant; thus the social welfare (the sum of producers' surplus and consumer surplus) is always higher under regulation, while it is also always decreasing in θ_1, as expected.

Figure 2. The Effects of PBP Regulation on the Expected Outcomes

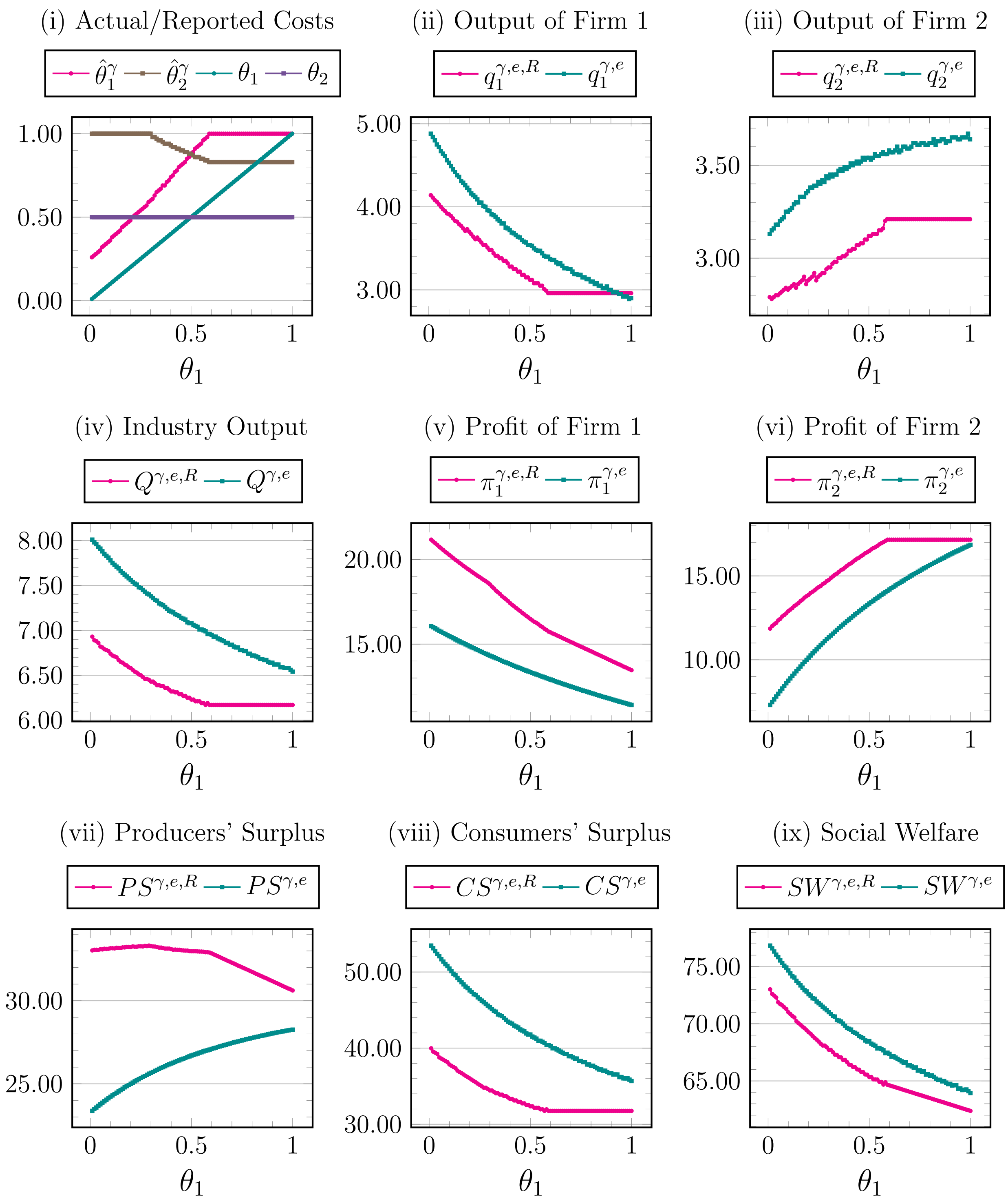
$\gamma = CV$ (Quantity Competition with Conjectural Variations) & $b = 0.75$



In Figure 2, we present the simulation results for the case where the regulated duopolists engage in the quantity competition with conjectural variations. Panel (i) shows that each firm overstates its private cost parameter under the PPR. As θ_1 rises, the cost reported by firm 1 (firm 2) as well as the amount of overstatement increase (decrease), unless θ_1 is above 0.7. As a result of their cost overstatements, both firms in general produce less under the PPR as illustrated in panels (ii) and (iii). Consequently the industry output is always lower under regulation, as shown by panel (iv). This leads to a huge increase in the product price raising the revenue of each firm, despite the reduction in its output. Thus, the profits of the firms, hence the industry profits, are always higher under the PPR as can be seen in panels (v) and (vi). We should also observe that given θ_2 is fixed at 0.5, an increase in θ_1, decreases (increases) the output of firm 1 (firm 2), unless θ_1 is extremely high. In accordance with these output effects, an increase in θ_1 has a negative effect on the profit of firm 1 and a non-negative effect on the profit of firm 2. Of these two effects, the former always dominates and thus the industry profit is always decreasing in θ_1. Also, the reduction in the industry output due to regulation always implies, a reduction in the consumer surplus, as illustrated in panel (viii). This reduction is indeed always higher then the rise in the producers' surplus under regulation, thus the social welfare is always affected negatively by regulation as illustrated in panel (ix). Moreover, the social welfare is always decreasing in θ_1, in accordance with panels (vii) and (viii).

In Figure 3, we illustrate the simulation results for the case where the regulated duopolists engage in the supply function competition. We find that these results are qualitatively similar to those in Figure 2. Both firms overstate their costs, reduce their productions, and obtain higher profits under the PPR, whereas consumers obtain lower surplus and the society as a whole ends up with a lower welfare.

Figure 3. The Effects of PBP Regulation on the Expected Outcomes
$\gamma = SFC$ (Supply Function Competition) & $b = 0.75$



In Appendix Figures A1-A6, we check whether our results in Figures 1-3 are robust to variations in demand. To this aim, we first set the price sensitivity of demand, b, to 0.5 in Figures A1-A3 for the three competition modes of interest and then set it to 0.25 in Figures A4-A6. Notice that as b becomes smaller, the demand curve in equation (1) expands outwards, with the expected total surplus being increased. Figures A1-A6 along with Figures 1-3 reveal that an increase in demand, induced by a fall in the slope parameter b, makes the understatement (overstatement) of costs more visible when the regulated firms engage in Cournot competition (engage in either quantity competition with conjectural variations or supply function competition). Consequently, we observe that all output and welfare effects of regulation are amplified when demand is higher.

Up to now, we have analyzed the effects of regulation for each competition mode in isolation. In the next two figures, we will compare the effects obtained under different competitive modes with each other (both for the case of regulation and the case of no regulation). In these figures, we will also plot, as a reference point, the expected outcomes obtained when the social welfare is maximized under complete information. We start this last analysis with Figure 4, where we illustrate the expected outcomes of all three competition modes under the case of no regulation for our benchmark setting where b is set to 0.75. We observe that the outputs and all welfares are almost the same when γ is equal to CV or SFC. Moreover, under these two competition modes both firms produce always more than what they produce under Cournot competition. We also observe that under all three competition modes firm 1 produces less than the socially optimal level unless θ_1 is extremely large. The output choice of firm 2 is more involved. It produces above the socially optimal level if θ_1 is low and below the socially optimal level if θ_1 is medium or high. As implied by their aggressively high outputs when $\gamma = CV$ or $\gamma = SFC$, the firms always earn less profits than they can do under Cournot competition. We also observe that all three competition modes always reduce the consumer surplus below its optimal level and lead to social welfare losses, while the consumer surplus and social welfare always attain their lowest unregulated levels when the firms engage in Cournot competition.

Figure 4. The Expected Outcomes under No Regulation
($b = 0.75$)

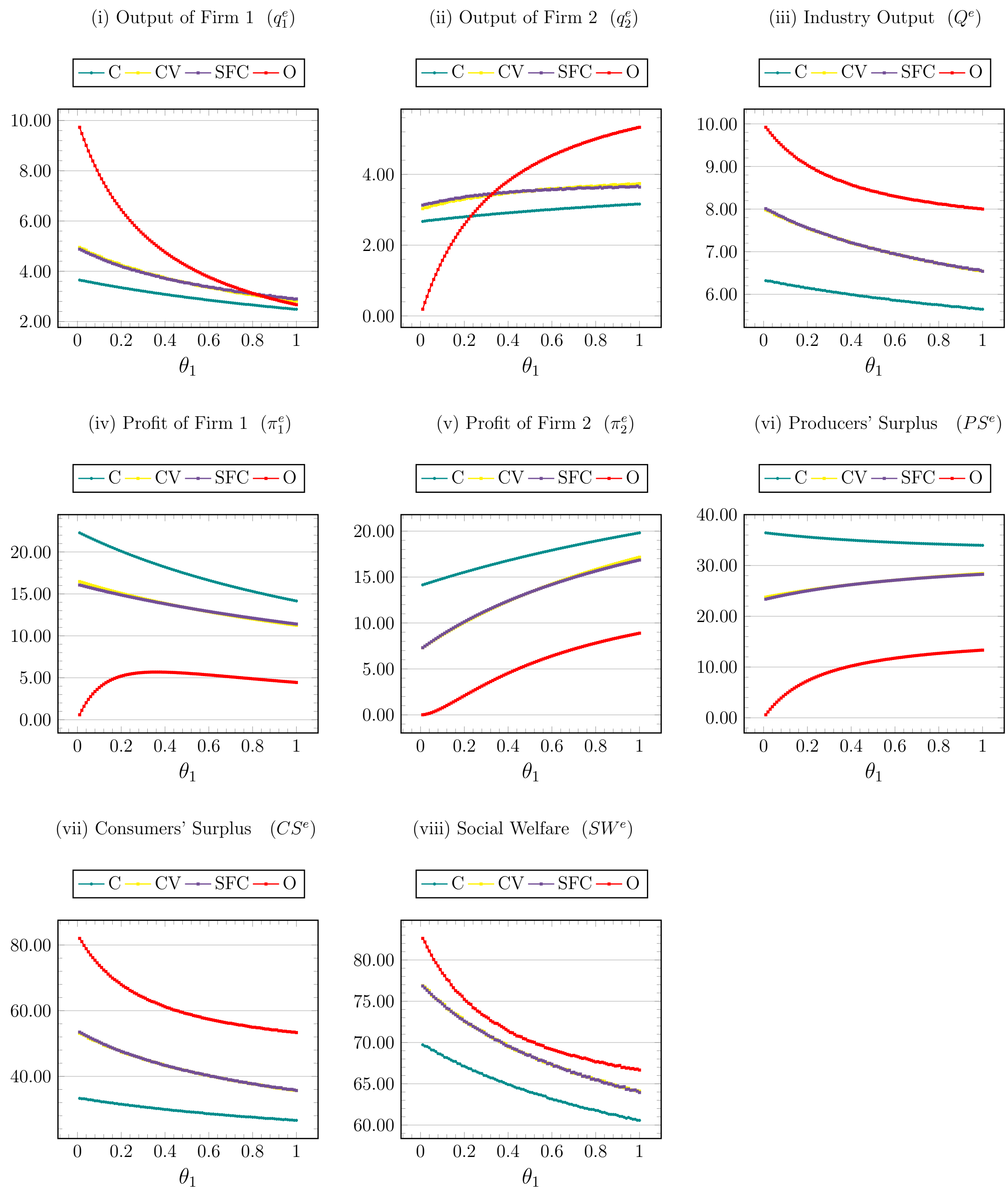
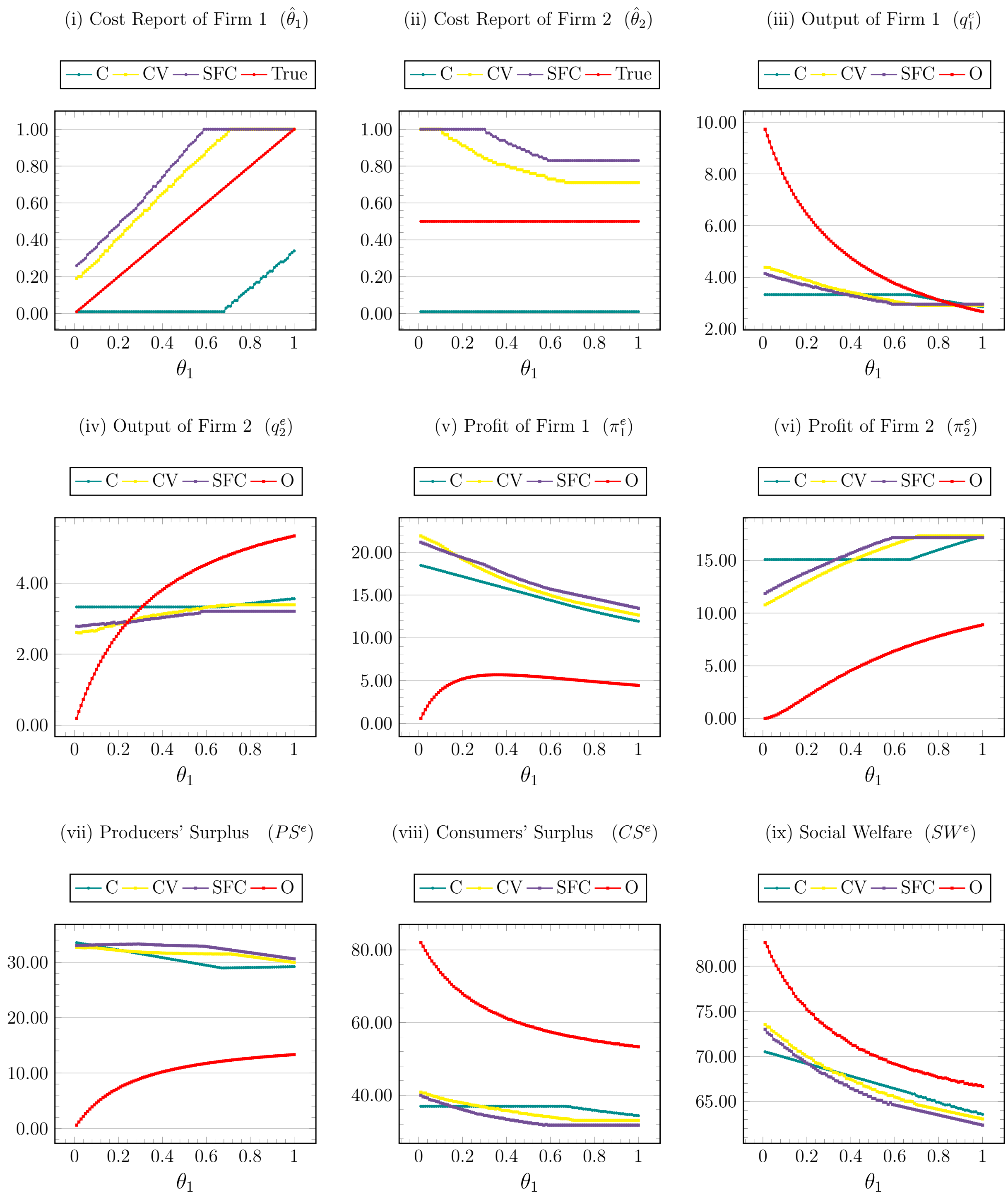


Figure 5. The Expected Outcomes under Regulation
$(b = 0.75)$



In Figure 5, we illustrate the expected outcomes of all three competition modes under the PPR (again when b is set to 0.75). In the first two panels, we present the cost reports of the two firms under regulation in comparison to the true cost parameters plotted with the red colour. As we have established in Figures 1-3, the regulated firms understate their costs when $\gamma = C$ and overstate their costs when $\gamma = CV$ and $\gamma = SFC$. Here, we also observe that the magnitude of the overstatement is in general lower for both firms when $\gamma = CV$ than when $\gamma = SFC$. Thus, each of the regulated firms has generally a higher output and a lower profit under the former ($\gamma = CV$) of these two competition modes. On the other hand, both the consumer surplus and the social welfare are higher under the PPR when $\gamma = CV$ than when $\gamma = SFC$.

As for the Cournot competition, the understatement of costs by the regulated firms makes the results of this competition mode qualitatively distinguished from those of the other two modes. Under the PPR, the second firm almost always produces more when γ is equal to C than when it is either CV or SFC. This is also true for firm 1 if θ_1 is higher than θ_2 (or 0.5). Also, the regulated industry output (the regulated price) in general attains its highest (lowest) level under the Cournot competition. Consequently, when regulated, firm 1 earns its lowest profits if $\gamma = C$, while this is also true for firm 2 whenever θ_1 is higher than θ_2 (or 0.5). In general, the producer's surplus is lower and the consumer surplus and the social welfare are higher under the PPR when γ is equal to C than when it is either CV or SFC.

5 Conclusion

In this paper, we have studied the effects of the PPR in a duopoly with asymmetric costs of quadratic form and demand uncertainty under three possible modes of competition, namely the Cournot competition, the quantity competition with conjectural variations, and the supply function competition. Under the PPR, each firm is allowed to choose and publicly declare a report for its private cost parameter but then produce in accordance with the claimed cost parameters of the firms and the equilibrium predictions of the competition mode which the firms engage in. We have modeled the described problem of regulation as a two-stage strategic game with observed actions. After theoretically characterizing the subgame-perfect Nash equilibrium of this game for each competition mode of interest, we have conducted several numerical computations to obtain comparative static results.

The main finding of this study is that the PPR is not always desirable from the viewpoint of consumers. The desirability of this regulation depends on how competition occurs in the industry. Whereas the consumer surplus is increased by the PPR under the Cournot competition, it is significantly reduced under the quantity competition with conjectural variations as well as under the supply function competition. This difference in results is caused by the drastic effect of the competition mode on the cost reports of the regulated firms. The regulated firms always understate their private cost parameters under the Cournot competition, while they always tend to overstate under the other two modes of competition. Whether the regulated firms understate or overstate their private costs affects their equilibrium outputs drastically. The regulation induces firms to generally produce more and earn less profits under the Cournot competition, while it induces them produce less and earn more profits under the other two modes of competition. The effects on the consumer surplus follow in opposite directions, as mentioned earlier.

One important policy recommendation of our results is that the PPR must not be used in power industries (or in any other industry for the same matter) when the regulated firms compete either in supply functions or in quantities with conjectural variations. Regarding the Cournot competition, the regulator must carefully weigh the benefits and deficiencies of the PPR regulation to those of the alternative forms of regulations. Surely, the PPR increases the consumer surplus, as shown by Koray and Sertel (1988) dealing with a linear and deterministic duopoly as well as by our work dealing with a non-linear duopoly with quadratic costs and demand uncertainty.

A potentially superior rival to the single-shot PBP regulation is the regulatory mechanism proposed by Gradstein (1995), which is non-Bayesian in nature as it can assure the first-best social outcome using the Nash implementation without appealing to the regulator's subjective beliefs. The superiority of Gradstein's mechanism arises when the demand function is linear, as in that particular case the first-best social outcome can be implemented with a balanced mechanism that does not require any transfers to be made by consumers to the regulated firms. In our paper, although the demand function is linear, the first-best social outcome cannot be implemented by the PPR under Cournot competition (or under the other two modes of competition). On the other hand, the superiority of Gradstein's regulation over the PPR is not necessarily universal. The social deadweight loss (due to the welfare distortion of unbalanced transfers) created by Gradstein's regulatory mechanism is already known to be strictly positive when demand is non-linear.

Therefore, future research may profitably compare the welfare implications of the single-shot PBP regulation and Gradstein's regulation in non-linear demand settings under the Cournot competition.

Finally, one may integrate our model with that of Koray and Sertel (1989, 2022) to study whether the socially efficient output obtained by the marginal cost pricing can be obtained under non-linear cost structures and/or non-linear demand structures when the oligopolistic games play a game of delegation under the three competition modes studied in our paper.

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Appendix

Figure A1. The Effects of PBP Regulation on the Expected Outcomes
$\gamma = C$ (Cournot Competition) & $b = 0.5$

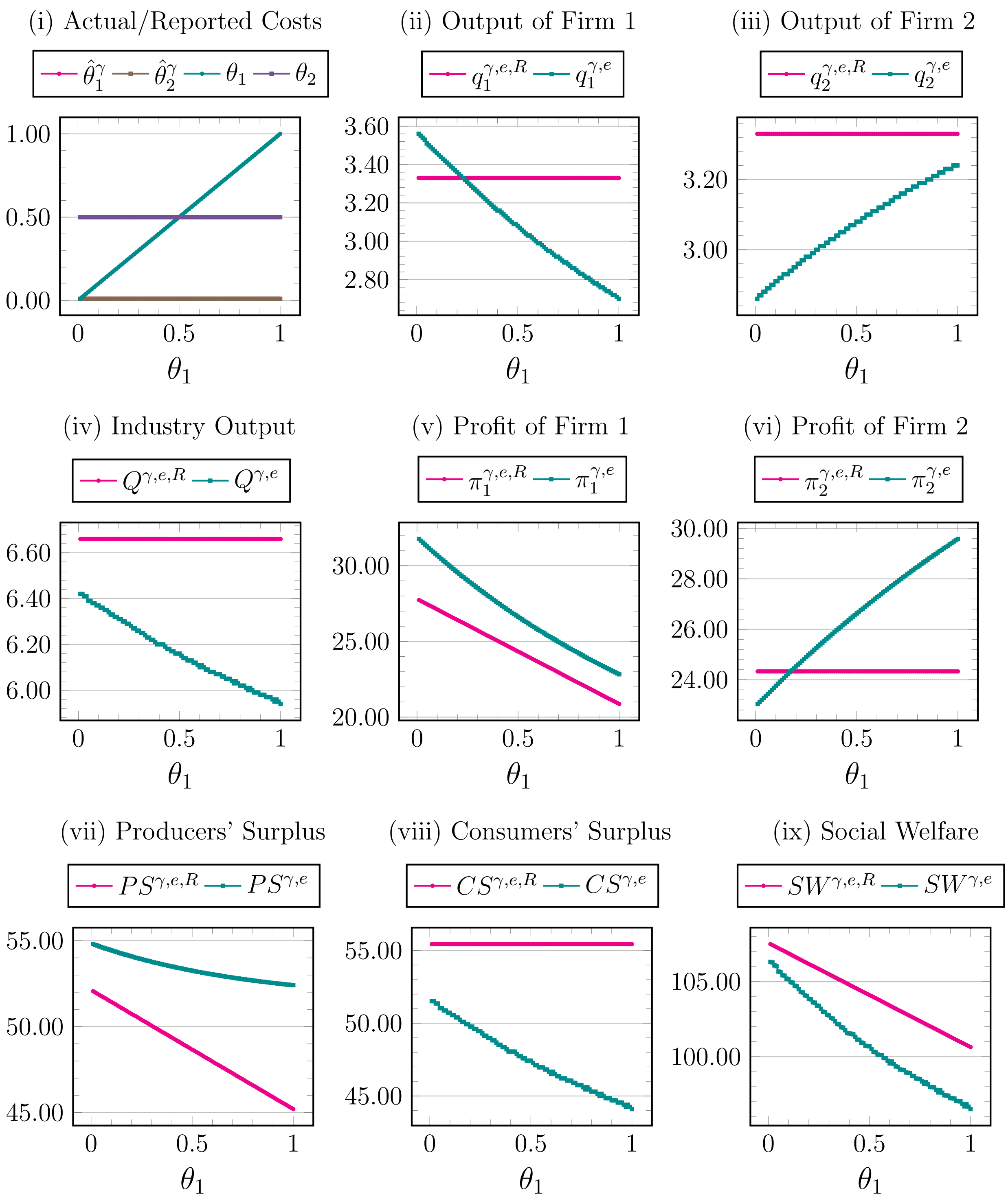

Figure A2. The Effects of PBP Regulation on the Expected Outcomes
$\gamma = CV$ (Quantity Competition with Conjectural Variations) & $b = 0.5$

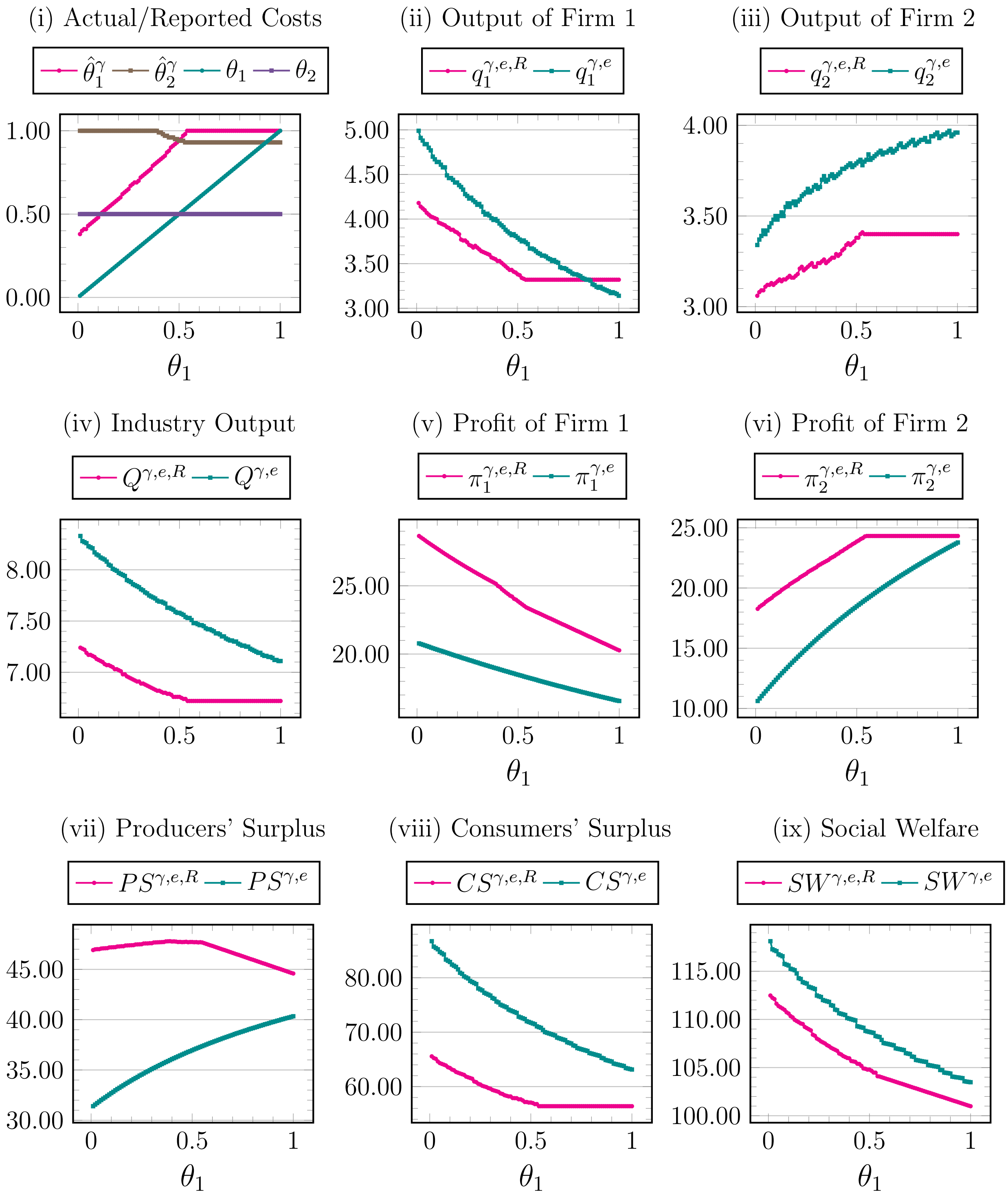

Figure A3. The Effects of PBP Regulation on the Expected Outcomes
$\gamma = SFC$ (Supply Function Competition) & $b = 0.5$

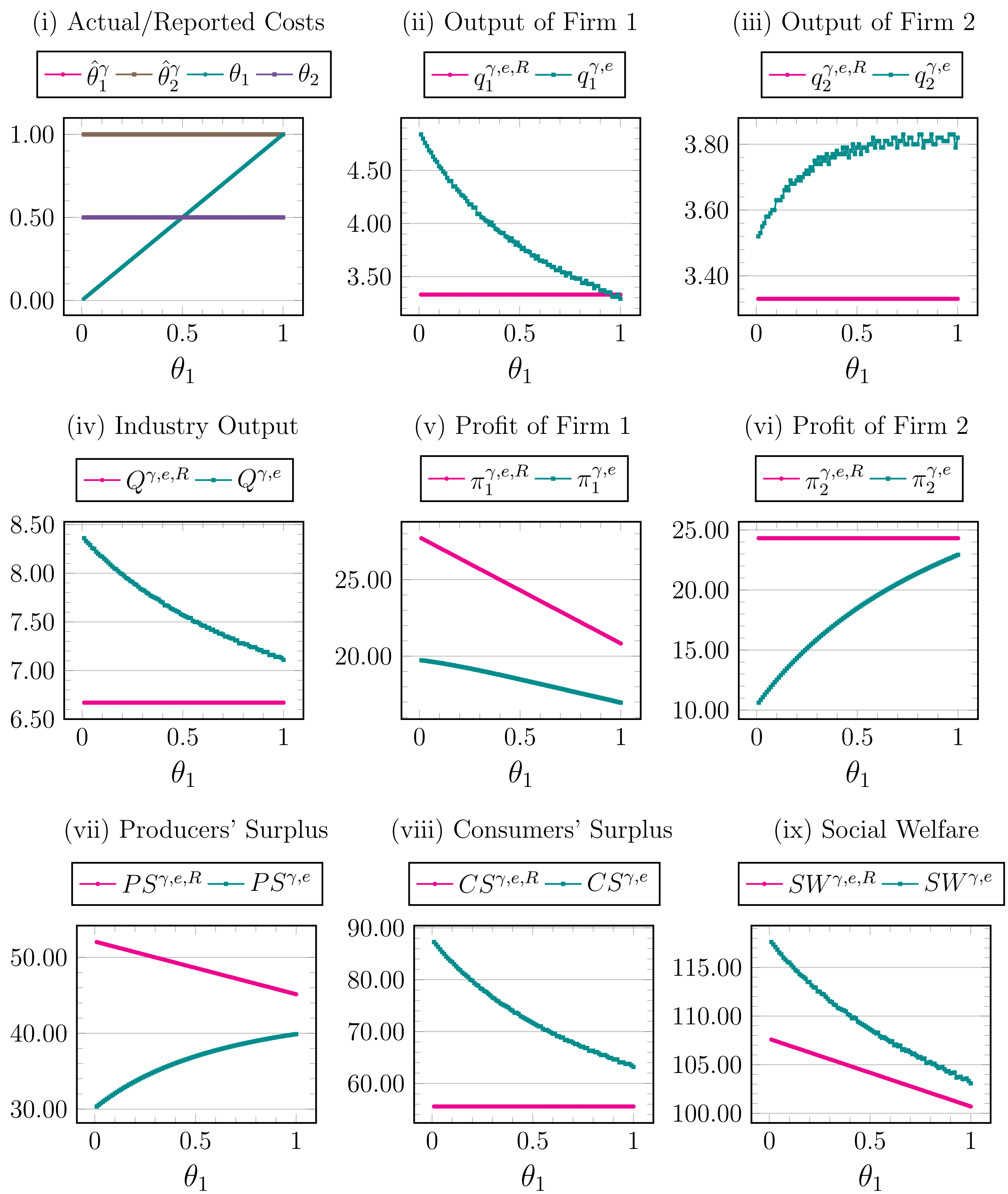

Figure A4. The Effects of PBP Regulation on the Expected Outcomes
$\gamma = C$ (Cournot Competition) & $b = 0.25$

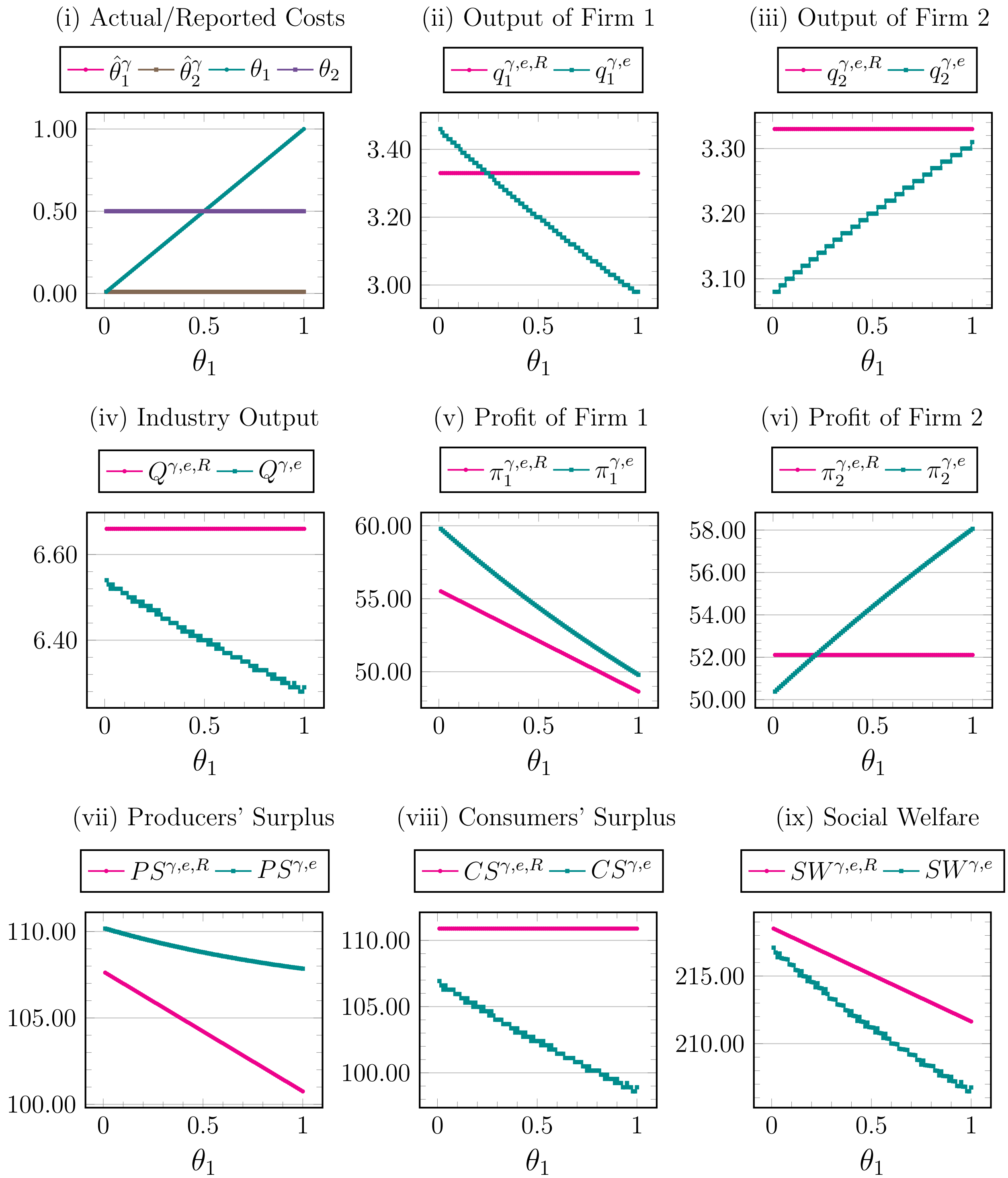

Figure A5. The Effects of PBP Regulation on the Expected Outcomes
$\gamma = CV$ (Quantity Competition with Conjectural Variations) & $b = 0.25$

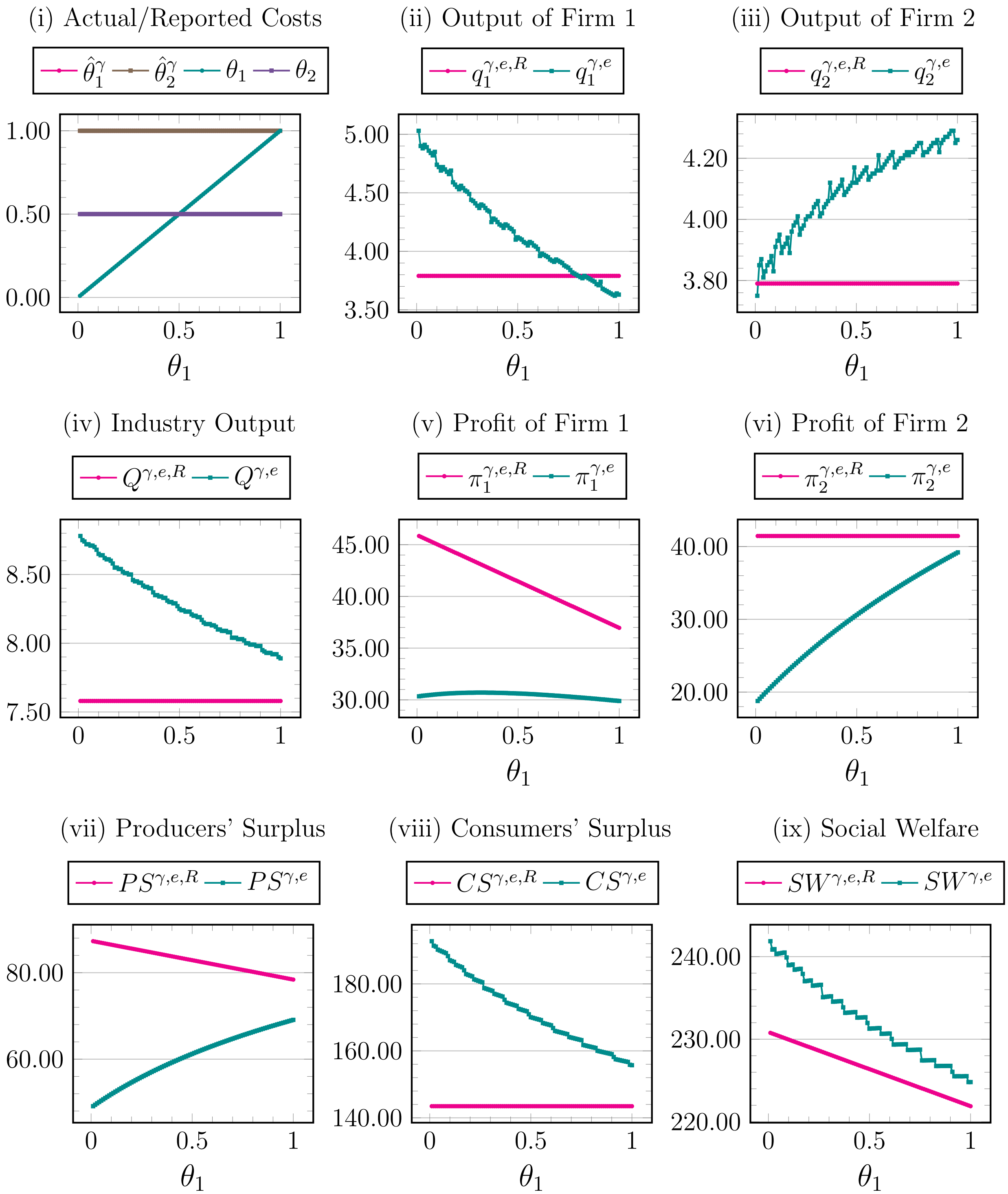

Figure A6. The Effects of PBP Regulation on the Expected Outcomes
$\gamma = SFC$ (Supply Function Competition) & $b = 0.25$