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Survey of Non-Walrasian Disequilibrium Economic Theory

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Abstract

In this study, we present a survey of (non-Walrasian) disequilibrium economics in which the gap between expressed demand and supply and between desired and realized transactions is allowed. We see a brief history of disequilibrium theory and its characteristics, such as temporary equilibria with quantity adjustment and the discontinuity of dynamics due to regime switching. We redefine disequilibrium economics by comparing it to equilibrium economics and find that its core is transaction inconsistency, which Robert Clower emphasized as "dual-decision."

Keywords: Non-Walrasian; Disequilibrium Economics; Survey; Macroeconomics; Economic Theory

1 Introduction

Equilibrium has been a central concept in economic analysis, as Dixon (1990) points out. The Walrasian general equilibrium model, which was modernized in Debreu (1959) and Arrow and Hahn (1971), is not only the core of microeconomics in textbooks but also the base of current macroeconomic models, such as the Dynamic (Stochastic) General Equilibrium models. In model analyses, the term "market-clearing" is often used to simplify the descriptions of the economic system, efficiency problem, and consistency.

Disequilibrium economics is a term currently used by macroeconomics researchers (see Mankiw and Weinzierl (2011), Michaillat and Saez (2015, 2019), Schoder (2017, 2020), van Aarle (2017), Dupor et al. (2019), and Eggertsson et al. (2019)). These studies address macroeconomic problems, such as involuntary unemployment and secular stagnation, and sometimes utilize

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the rationing scheme by referring to disequilibrium economics. For instance, unemployment is depicted as a disequilibrium in the labor market, and labor supply is "rationed" when labor demand is smaller than supply. However, as Backhouse and Boianovsky (2012) argues, disequilibrium research has not been active for a long time, and modern textbooks rarely refer to it. This implies that we need a summary of disequilibrium economics and its tools to fully utilize its potential in economic research.

This study investigates disequilibrium economics from both a historical and theoretical perspective. We find that the spillover effect between markets plays a significant role in disequilibrium economics, although rationing is usually emphasized. We also find that the inconsistency of trades and plans that cause spillover is at the core of disequilibrium. This effect usually makes macroeconomic models complicated that they are often omitted from working models. However, it may be useful to tackle economic issues. One important spillover effect in a disequilibrium economy, emphasized in Patinkin (1956), is the effect of goods demand on labor demand. In short, a firm's intention to employ disequilibrium is influenced by its perceived quantity constraint on sales. In disequilibrium economics, this is known as Keynesian unemployment, and it differs from other types of unemployment rigidity. This study examines how disequilibrium economics has treated this feature to derive ideas for future theoretical analysis.

The remainder of this paper is organized as follows. Section 2 provides a brief history of disequilibrium economics (also known as non-Walrasian or neo-Keynesian economics). The birth of disequilibrium theory was influenced by Keynes's economic views, although the theme was wider. In Section 3, we discuss several properties of the disequilibrium theories. The analytical tools have become interesting but challenging to employ, as we deal with excess demand or supply situations. Why has disequilibrium economics been neglected for so long? In Section 4, we discuss inherent difficulties and unsolved problems. In Section 5, we redefine the disequilibrium model and compare it with the rigidity models. Section 6 summarizes our research findings and discusses future issues.

2 The dawn of disequilibrium economics

This section provides a quick overview of the disequilibrium economics from Keynes until the 1970s reevaluation of Keynes. See Cuddington et al. (1984, Chapter 3) and Backhouse and Boianovsky (2012) for additional information about the history.

2.1 After Keynes

Keynes's *General Theory* (Keynes, 1936) is seminal work in disequilibrium economics. Using a unique consumption function, liquidity preference, and effective demand principle, he demonstrated that involuntary unemployment can persist in the current economy. After its publication, the book was interpreted using mathematical models. "Keynesian" economists such as Hicks (1937) and Modigliani (1944) proposed the IS-LM model, in which the Keynesian situation could be interpreted as an extended classical model with sluggish price adjustment. This neoclassical synthesis has presented many tractable models, but it has been criticized by both (neo-)classical economists and (post-) Keynesian economists; for its lack of consistency and under evaluation of the Keynesian situation.[1]

Patinkin (1956) also intended to merge Walrasian economics with Keynes's theory, and he discovered that Keynes's involuntary unemployment could be shown as the situation where employment and labor supply detach; in other words, the labor market is in disequilibrium. Patinkin specified the spillover effect from goods to labor demand, which has been the core of disequilibrium economics.

In his seminal work (Clower, 1965), Robert Clower demonstrated that Keynes's model is illustrated by market disequilibrium. He presents a simple model in which transactions occur even if the set prices do not reach an equilibrium level. Because the demand and supply quantities of markets could differ in terms of current prices, the realized transaction quantity is different on at least one side; therefore, the demand or supply is rationed. Furthermore, he introduced the dual-decision hypothesis, a key concept in disequilibrium economics. This hypothesis states that individuals who face rationing (the gap between the planned transaction and the realized transaction) will reconsider their other demands and supplies to revise their decisions. The revised demand and supply are called *effective*, as opposed to *notional*, which is derived from a typical decision problem, such as utility maximization and profit maximization without any quantity constraints. Clower demonstrated that consumer demand is a function of realized transactions (realized income), implying that the Keynesian consumption demand function is derived from a microeconomic optimization problem. Clower's novel idea immediately spread in the 1970s macroeconomics, with strong support from the book Leijonhufvud (1968). Leijonhufvud argued that Keynes' economics should be interpreted as an economy in which the Walrasian auctioneer is absent. In the absence of an auctioneer, a transaction must be executed even if prices

[1] Leijonhufvud (1967) criticized "Keynesian" economics since it views Keynes's economics as a special case.

are not in equilibrium. He argues that Keynes assumed that the speed of quantity adjustment was higher than that of price adjustment.

2.2 Disequilibrium macroeconomics in the 1970s

Macroeconomists have developed several mathematical models that tackle quantity rationings after Clower and Leijonhufvud's basic works on disequilibrium interpretations of Keynes's *General Theory*. Most seem to rely on Leijonhufvud's quantity adjustment theory; prices are fixed in the short term or in a static model, and they gradually change in dynamics. However, we should note that Clower, Leijonhufvud, and Patinkin were not satisfied with the mathematical models below; see Backhouse and Boianovsky (2012). The following models are interpretations of their work on disequilibrium.

The first model was Solow and Stiglitz (1968), which described labor market dynamics. Solow and Stiglitz introduced a simple rationing scheme, $Y = \min\{Y^d, Y^s\}$, where Y represents output, superscript d is demand, and s is supply. They demonstrated that under gradual real wage adjustment, which depends on excess demand for goods and labor, there is a possibility of persistent (involuntary) under-employment.

Barro and Grossman (1971) presented the "general" disequilibrium model, in which the spillover effects between goods and labor markets are fully described. They synthesized Patinkin's labor market model and Clower's goods market model. We now describe the abstract of the model. Let x_i denote the realized transactions of good i and let P be the price vector. The Walrasian (equilibrium) model explores the following situation:

$$x_i = x_i^{d*}(P^*) = x_i^{s*}(P^*), \ \forall i. \tag{1}$$

Demand and supply are functions of P (and other parameters) and transactions occur at the equilibrium price P^*. This is because the prices are adjusted to the equilibrium level before the actual trade occurs, and individuals are supposed to know that. The notional demand and supply x^{d*}, x^{s*} are derived from the usual optimization problems without transaction rationing. Next, we considered a simple disequilibrium model. The transaction occurs under the short-side rule.

$$x_i = \min\{\tilde{x}_i^d(P, X), \tilde{x}_i^s(P, X)\}, \ \forall i, \tag{2}$$

where P is the prevailing price vector, $X = \{x_i\}_i$ is the set of realized transactions, and˜(tilde) indicates effective demand and supply. The dual-decision hypothesis emerges as X in each demand and supply; that is, the *realized*

transactions affect demand and supply. It is assumed that quantity adjustment derives this transaction.

Barro and Grossman (1971) organized and compared the notional and the effective demand and supply by presenting two disequilibrium cases: excess supplies in labor and goods markets and excess demands in both. These disequilibrium regimes were sorted and labeled by Malinvaud (1977), and the names are used in subsequent disequilibrium studies. For example:

Keynesian unemployment (KU) : Excess supplies exist in both labor and goods markets so that involuntary unemployment occurs due to the shortage of goods demand.

Classical unemployment (CU) : The labor market is in excess supply, but the goods market is in excess demand, creating involuntary unemployment even though the demand of the goods is sufficient.

Repressed inflation (RI) : Excess demands exist in both labor and goods markets and therefore the price and the wage would rise during the adjustment process.

Underconsumption (UC) The labor market is in excess demand but the goods market is in excess supply, which means that households restrict their consumption and labor supply so that the activity of the economy is restricted.

Where there is no demand-supply gap, **equilibrium** or **Walrasian equilibrium (WE)** is labeled.

Bénassy (1975b) extended Barro-Grossman's interpretation of Patinkin-Clower's disequilibrium models, with the descriptions of the microeconomic view of transactions; the monetary exchange, and the rationing schemes.[2] He presented systematic tools to describe the dual-decision hypothesis and demonstrated the existence of a Keynesian equilibrium (K-equilibrium), which is defined by *effective* demands.

One characteristic of Bénassy's K-equilibrium is the specification of the perception of quantity constraint. We start with a certain set of expressed excess demand, where the goods are indexed with i, $\{\tilde{z}_{ih}\}_{i,h}$, where h is the individual index. The perceived quantity constraint is a set of correspondences $\{G_{ih}\}_{i,h}$. For simplicity, he assumes that the rationing scheme is described as a function: $\bar{\bar{z}}_{ih} = G_{ih}(\{\tilde{z}_i\}_i)$ is the quantity constraint on the

[2]His paper on monetary exchange was published in the same year (Bénassy, 1975a). He was interested in Barro-Grossman's general disequilibrium model and Clower's monetary theory (Clower, 1967); see Section 3.3 for money in disequilibrium.

trade of goods i of individual h. The announced excess demand, $\{\tilde{z}_{ih}\}_{i,h}$ affects individuals' perceptions of trade possibilities. Returning to the excess demand expressions, the effective demand of individual h is defined as the set $\{\tilde{z}_{ih}\}_i$ which is optimum under the budget constraint and is affected by the perceived quantity constraints $\{\bar{\bar{z}}_{ih}\}_i$. To emphasize this dual decision, Bénassy's effective demand concept focuses only on the spillover effect. That is, the individual repeatedly solves the optimization problem of every good, in which the budget constraint for good h is not affected by $\bar{\bar{z}}_{ih}$. [3] Finally, realized transactions are described as a set $\{\bar{z}_{ih}\}_{i,h}$, which is determined by the rationing scheme $\bar{z}_{ih} = F_{ih}(\{\tilde{z}_i\}_i)$, where F_{ih} is the rationing function.

Bénassy defined K equilibrium as a set of three types of excess demand vectors $\{\tilde{z}_{ih}, \bar{z}_{ih}, \bar{\bar{z}}_{ih}\}_{ih}$, which satisfy (1) $\bar{\bar{z}}_{ih} = G_{ih}(\{\tilde{z}_i\}_i)$; (2) $\tilde{z}_{ih}$ is optimal under the quantity constraint $\{\bar{\bar{z}}_{ih}\}_i$; and (3) $\bar{z}_{ih} = F_{ih}(\{\tilde{z}_i\}_i)$ for each i. It is worth noting that the first two conditions say that K-equilibrium is an equilibrium of perception of quantity rationing and dual-decision under constraint; we should interpret the transaction as the equilibrium point of "quantity-tâtonnement." The realized quantity itself seems independent of the tâtonnement process, although the perceived quantity constraints are often equated with realized rationing ($F_{ih} = G_{ih}$) in sequential work.

Barro-Grossman's and Bénassy's canonical disequilibrium models stimulated macroeconomics. For macroeconomists in those days, it was important to know how the disequilibrium regime (in particular, KU) originated and how persistent it was. They use comparative statistics to formulate the relationships between price variables, stock variables, and regimes. Korliras (1975) extended Patinkin's employment model and explored how the two price variables (real wage and interest rate) affect involuntary unemployment in the short term. Barro and Grossman (1976) modified their model in 1971 by introducing dynamic optimization. Malinvaud (1977, 1980) developed simple models and emphasized the persistence of the KU regime. Muellbauer and Portes (1978) presented a graphical interpretation of the four disequilibrium regimes using offer curves which are derived from optimization problems of households and firms.[4]

[3] The Bénassy's effective excess demand for good i of household h is described as follows:

$$\max_{z_{ih}} U_h(\omega_h + z_h, M_h) \text{ subject to } p \cdot z_h + M_h \leq M_{h0},$$
$$\omega_h + z_h \geq 0, M_h \geq 0,$$
$$|z_{i'h}| \leq |\bar{\bar{z}}_{i'h}|, \text{ where } i' \neq i,$$

where U_h is h's utility function, M_h is the money held, M_{h0} is the initial money held, and ω_h is the initial allocation vector.

[4] Stoneman (1979) also provides a graphical interpretation of a disequilibrium model.

2.3 Toward a general disequilibrium model of microeconomics

Although Keynes's macroeconomics have influenced disequilibrium economics, some researchers have explored the microeconomic features of disequilibrium as an extension of the general equilibrium model.

In 1971, a draft was written about the fixed-price equilibrium model Drèze (1975) (Backhouse and Boianovsky, 2012, p.111). He explored the world with rigid prices and quantity constraints and found the existence of trade, or the Drèze equilibrium.[5] His work was extended by Grandmont and Laroque (1976) as work on Keynesian equilibrium; however, Drèze himself initially treated his work as one in general equilibrium economics (Backhouse and Boianovsky, 2012, p.111).

Although the Drèze equilibrium is well "micro-founded" in that the descriptions about preferences and choices are consistent with the usual general equilibrium model, it lacked an important aspect of disequilibrium economics: the spill-over effect with quantity signals. Since the quantity constraints are fully perceived before the agents decide their demands and supplies, no one can notice the demand-supply gap in the realized trade. In other words, no one was surprised by the rationing. However, Bénassy's concept of K-equilibrium lacks consistency. Agents repeatedly solve the optimization problems for each good, and the expressed demands and supplies can violate their budget constraints.[6]

Some researchers have introduced a stochastic rationing scheme for Keynesian macroeconomics to overcome Bénassy's inconsistency and Drèze's intractability(see Gale (1979), Green (1980), and Svensson (1980)). Although this concept overcomes the microeconomic consistency problem of rationing models, stochastic rationing is rarely used in macroeconomic analyses, with the exceptions of Ioannides (1983), Weinrich (1984), and Honkapohja and Ito (1985).

[5]Drèze (1975) used an inequality constraint on prices, such as downward rigidity and quantity constraint for individual i, as the set of ranges: $x_i \in \prod_k [l_{ki}, L_{ki}]$, where k is a good index. Although Drèze allowed a slight change in prices, Younès (1975) reinterpreted this as a p-equilibrium, which is a fixed-price equilibrium.

[6]See Svensson (1980) for an example of this violation. For comparative studies of equilibria with quantity rationings, see Grandmont (1977, 1982) and Silvestre (1982, 1983). Böhm and Lévine (1979) suggested a game-theoretic interpretation of temporary equilibrium in which the actual net trade could be same as those in Bénassy's and Drèze's equilibrium.

3 Prosperity of disequilibrium economics: its characteristics

Based on disequilibrium theories in the early days, the disequilibrium model became a tool for economic analysis (Backhouse and Boianovsky, 2012, p.79) in the mid-1970s; for example, Dixit (1978) and Cuddington et al. (1984) used it for international trade. This section discusses several characteristic issues in disequilibrium economics.

3.1 Dynamical characteristic of disequilibrium

As previously stated, the persistence of disequilibrium, particularly in the Keynesian unemployment regime, is one of the most important issues in disequilibrium macroeconomics. Although the early models such as Solow and Stiglitz (1968), Barro and Grossman (1976) and Muellbauer and Portes (1978) examined dynamic properties, it was Ito (1979) who presented the use of "discontinuous-righthand-side" differential equation technique that marked a turning point.[7][8]

The most characteristic dynamic property of the disequilibrium model is the regime switching. The usual disequilibrium macroeconomic model has three or four regimes, depending on the demand-supply gaps in the goods and labor markets. The realized transaction is described using different equations, and discontinuity emerges in the dynamic equations. For example, the dynamic equations in Malinvaud (1977), for which Honkapohja and Ito (1983, Section 4) provided an additional mathematical explanation.

There are three disequilibrium regimes (KU, CU, and RI), and WE are located at their center. The locations are determined by a combination of price and (nominal) wage (P, W). The price and wage adjustments are determined by excess demand in the goods and labor markets, respectively. Therefore, differential equations vary when the regime changes. Let R_x denote the region of the disequilibrium regime denoted by x. The dynamics

[7] For the basic concept of solution for this problem, see Filippov (1988).

[8] Of course, many works had analyzed the disequilibrium dynamics. Varian (1977) proved the existence of stable non-Walrasian and unstable Walrasian equilibrium, however, his analysis is limited because the regime-switching is not included. The complex price-adjustment process was shown in Böhm (1978), although the study was limited to within-regime dynamics and graphical explanations. Löfgren (1979) investigated the Barro-Grossman model's dynamics and concluded that there was not sufficient analysis on regime-switching. Malinvaud (1980) showed the stable KU steady state, but his analysis also lacked the regime-switching. Blad and Zeeman (1982) introduced the lagged adjustment to avoid the on-the-regime-boundary analysis.

are described as follows.

$$(\dot{P}, \dot{W}) = \begin{cases} (f^1(P,W), g^1(P,W)) & \text{if } (P,W) \in \text{int}R_{KU}, \\ (f^2(P,W), g^2(P,W)) & \text{if } (P,W) \in \text{int}R_{CU}, \\ (f^3(P,W), g^3(P,W)) & \text{if } (P,W) \in \text{int}R_{RI}, \end{cases} \tag{3}$$

where $(f^i(P^*, W^*), g^i(P^*, W^*)) = (0, 0), i = 1, 2, 3$. Figure 1 illustrates the dynamics. It is worth noting that the dynamics on the RI-KU border are intractable using the usual techniques because the two systems meet at this boundary. Honkapohja and Ito (1983) used the Filippov solution, in which the dynamics are determined by the combination of vectors. See x_0 in the figure. We know that $(f^1(x_0), g^1(x_0))$ and $(f^3(x_0), g^3(x_0))$, and then we should calculate and combine them. The newly produced vector goes along the line tangent to the boundary, so that the economy would go on a "sliding trajectory" and would converge to the WE. However, the economy can diverge along a sliding trajectory, an example of which is shown as x_1. Whether the sliding path converges or diverges depends on the shape of the boundary and velocities of P and W (see Honkapohja and Ito (1983, Theorem 4.1)). If the vectors are balanced, the dynamics stop, even though all f^i and g^i are not zero. This is called the *quasi-equilibrium* in Filippov (1988) and is shown as the QE in the figure.

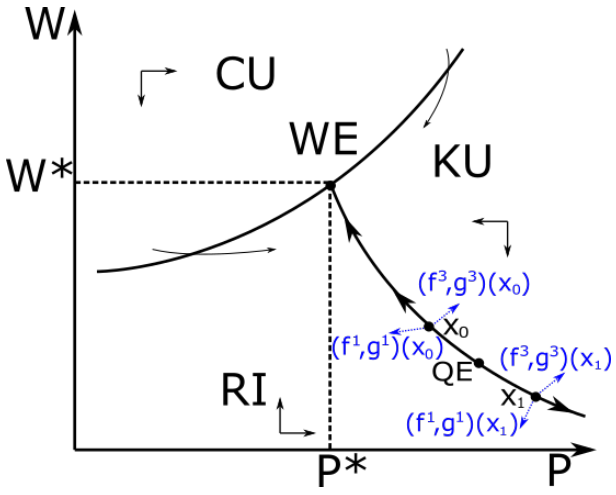

Figure 1: The phase diagram of Malinvaud's dynamics

Michaillat and Saez (2015) pointed out that this discontinuity makes disequilibrium research intractable, but it can explicitly express the intrinsic idea of disequilibrium: the dynamics under disequilibrium is qualitatively different from that under equilibrium. Therefore, they have been applied to

address several economic issues. The shape of the Phillips curve depends on the regime, as demonstrated by Picard (1983) and Chiarella et al. (2000, Chapter 5). Colombo and Weinrich (2003) also showed that the Phillips curve emerges as an attractor of chaotic dynamics of a simple disequilibrium model. The dynamics of inventory in disequilibrium are notable. As Blinder (1980) argued, the existence of inventory moderates the dynamics because the spillover effect is weakened unless a stockout occurs. For instance, Eckalbar (1985) discovered that the number of regimes drops to two.[9]

3.2 Disequilibrium growth

As dynamics are explored, growth theory is often analyzed in disequilibrium economics. The seminal work is Ito's neoclassical growth model (Ito, 1978, 1980a). He introduced sluggish wage adjustments in Solow (1956). Although the regime changes during transitional dynamics, the economy converges into a neoclassical steady state, in which the markets are in notional equilibria. This story seems to overlap Solow's view on long-run dynamics in Solow (1988, p.312); it seems "neoclassical synthesis" in disequilibrium economics since the balanced growth path or the steady-state in the long run is the same as that in equilibrium economics, but the short- and medium-run dynamics emphasize disequilibrium regimes such as Keynesian unemployment. This synthesized view is sometimes utilized: Ginsburgh et al. (1985) explored a Ramsey model, Sgro (1984) extended Ito (1980a) with monetary growth, and van Marrewijk and Verbeek (1994) used a two-sector approach. Ogawa (2022) shows that the Walrasian equilibrium is locally stable but that the convergence takes long time in KU regime. These disequilibrium dynamic models often focus on transitional dynamics rather than the steady state. This is because equilibrium is not regarded as a standard state in disequilibrium economics.

Many growth models do not depend on neoclassical syntheses. Nikaido (1980) and Hénin and Michel (1982) explored Harrodian instability in quantity constrained models. Böhm and Puhakka (1988) and Weddepohl and Yildirim (1993) built overlapping generation models although their main issues are not the growth path but optimality and expectation, respectively. Keynesian macroeconomists, who are not New Keynesian, focus on the KU regime and have developed numerous dynamic models; see Flaschel (1999), Chiarella and Flaschel (2000), Chiarella et al. (2000), Chiarella et al. (2005), Asada et al. (2011), and Murakami (2014).

[9]For disequilibrium dynamics with inventories, see Honkapohja and Ito (1980), Green and Laffont (1981), Simonovits (1982), Duménil and Lévy (1987), and Hsu (1992, 1994).

3.3 The money on disequilibrium

As Clower treated this as the main issue in his 1967 paper (Clower, 1967), money is sometimes considered in disequilibrium economics (see Gale (1983)). Particularly, money is thought to play an important role in transactions and as the medium of exchange. The buyer handles the money and the seller receives it in pairwise trades (see Ostroy and Starr (1974)). Indeed, money promotes trade if sufficient information is shared but hinders trade in effective demand failures.[10] In the Keynesian coordination failure situation (KU regime), the employee and the employer may improve their realized trade if they bartered the workforce and commodity. However, as Bénassy (1975b) demonstrated, each trade requires an exchange of money. This payment constraint prevents resolution of Keynesian unemployment.[11] Since money is always needed for payment, it is natural to address the cash-in-advance (CIA) constraint problem.[12] However, we should note that Clower himself denied that his idea was inherited in CIA analyses (see Plassard (2017)). Furthermore, the dual-decision hypothesis seems to be a stricter version of the CIA constraint, since the expressions of demand and supply are restricted by the quantity constraint of the realized transaction, which adds flow constraints.

However, monetary constraints have not been treated *completely*, even in disequilibrium economics. As Rogers (1989, Chapter 3) argued, Clower and his followers cannot describe monetary constraints on the transaction process in detail; the constraint works explicitly in *ex-post* transactions. This might be an issue in all economic theories rather than one particular to disequilibrium economics.

3.4 Econometrics for disequilibrium

Economists have also conducted empirical studies to reinforce the progress of theoretical analyses in disequilibrium economics. For the basic assumption, Kawasaki et al. (1982, 1983) empirically showed that quantity adjustment is preferred to price changes in the German industry, using microdata. They also presented a simplified dynamic model with inventory dynamics in Kawasaki et al. (1983) and demonstrated that firms tend to choose quantity

[10] See Younès (1975) and Bénassy (1975a).

[11] His textbooks of disequilibrium (Bénassy, 1982, 1986) start with comparing barter economy and monetary economy using the diagrams in Clower (1967).

[12] Grandmont and Younès (1972) and Grandmont (1985) tried to build a general model for Clower's exchange with money payment, with Hicksian temporary equilibrium. In particular, Grandmont (1985) point out that the real balance effect with price adjustment cannot resolve the persistent excess supply situation (which could be interpreted as the KU regime).

adjustment when the demand shock is transitory (or when they expect demand to recover soon). Ashenfelter (1980) suggested the way how to find the dual-decision effect using the Slutsky equations with or without quantity constraints. Although these empirical works succeeded in justifying the critical assumption that the individual prefers quantity adjustment, theoretical analyses of the assumption within the disequilibrium framework have not been successful.

Several econometric analyses of disequilibrium economics have focused on regimes, whether the economy is demand-or supply constrained. For the equilibrium framework, the observed economic activity, such as the prevailing price and traded quantity of goods, is supposed to be located on both demand and supply curves. However, for the disequilibrium framework without friction, the realized and observed transactions can differ from demand or supply. Distinguishing between demanded and supplied quantities using the observed quantity is a central issue.

Fair and Jaffee (1972), Maddala and Nelson (1974), and Rosen and Quandt (1978) presented some methods to estimate whether markets are demand or supply-constrained using the observed price dynamics; see Ito (1980b) and Quandt (1988) for the summary. Although some studies reject the hypothesis of market equilibrium, their estimations depend heavily on the price adjustment formulation. This implies that the models usually assume that Walrasian price adjustment dominates. Therefore, the estimation remains a difficult and unresolved problem.

Recently, Juselius (2021) suggested a cointegrated-VAR (CVAR) approach supported by Colander et al. (2008) and inspired by Guzman and Stiglitz (2020). Guzman and Stiglitz's theoretical work on disequilibrium dynamics emphasizes trade inconsistency, although the critical difference from fluctuations in orthodox equilibrium models is not explicitly described in mathematical theory. Juselius argues that her CVAR model, which does not require consistency or stationarity in the base model, is consistent with the non-stationary time series in a disequilibrium economy.

3.5 Keynesian disequilibrium: Iwai model

Iwai (1974, 1981, 2018) also studied disequilibrium dynamics from a Keynesian and Wicksellian perspective, but the perspective is different from disequilibrium economics treated in this study.[13] He used quantity rationing for transactions but denied quantity adjustment in one period; instead, he

[13]Iwai says that he did not accept the Keynesian view by Clower and Leijonhufvud in his biography (Iwai, 2015).

described the streams of production and market transactions. Because production takes time, the firm should plan the quantity of goods supply and labor demand before opening the commodity market. Furthermore, the firm should also determine the goods' prices and wages before going to the market. Iwai supplied the microeconomic theory for these uncertainties and showed that a flexible price change would result in instability.

From the perspective of disequilibrium theory, one of the prominent characteristics of Iwai's model is that it distinguishes the planning time of quantity and the time when trades occur. This theory makes it difficult to reproduce the spillover effect of quantity adjustment (the dual-decision hypothesis) but sounds natural and realistic. Future disequilibrium research on quantity adjustments should integrate the advantages of this model.[14]

4 Impasse of analysis and fraud of disequilibrium models

When we look at the standard textbooks of macroeconomics today, we would rarely see descriptions of the disequilibrium models. As argued, disequilibrium economics is almost forgotten. When we learn macroeconomics, this usually means that we learn (dynamic) general *equilibrium* economics.

After a prominent book (Barro and Grossman, 1976) was published, they immediately moved away from disequilibrium. Barro is now known as an economist of equilibrium rather than disequilibrium. As "New Classical" economics appeared, the concern about disequilibrium declined during the 1980s, particularly in the US.[15] Economists have pointed out that disequilibrium economics has certain shortcomings.

4.1 Shortcomings of disequilibrium theory

First, the reason the price is "sticky" is not explained in the disequilibrium theory, as Drazen (1980) pointed out. The most explicit difference between equilibrium and disequilibrium economics is the existence of an auctioneer, as argued by Leijonhufvud (1968). From the equilibrium view, it is important to explain how price stickiness occurs and why agents quickly adjust their quantities in "micro-foundations," which usually means that the agents behave

[14]For instance, Smolny (1993, 1998) consider the models with lagged sales in disequilibrium framework.

[15]Ito (1985) says that disequilibrium economics seemed accepted calmly in Europe and Japan and the equilibrium economics is preferred in the US. Mankiw (1990, p.1655) also mentioned this difference.

with some consistent choice. Although several researchers have explored this, their answers do not seem to be well-accepted.[16] New Keynesian economics solved this problem after disequilibrium economics almost disappeared by using frictions and imperfections about information and competition. It seems natural that New Keynesian economics would be a modern version of disequilibrium economics. However, caution should be exercised while accepting this idea. Most micro-founded price stickiness may not be suitable for the possible inconsistency in disequilibrium (see the definition of disequilibrium economics below). For instance, the Calvo pricing named after Calvo (1983) in standard macrodynamic models seems to assume that sales expectations are consistent; thus, it assumes that there is no disequilibrium in the economy. Equilibrium with stickiness does not substitute disequilibrium economics.

Second, transactions and rationing are not fully explained (Howitt, 1979, p.60). Although Bénassy (1975b) presented a disequilibrium model in which the rationing scheme can take various forms, it is difficult to interpret each rationing as in decentralized markets. At this point, the micro-foundation of disequilibrium has not yet been completed.

Third, the equilibrium point of quantity adjustment is ambiguous. Referring to Leijonhufvud (1968), almost all disequilibrium models describe the transaction as the equilibrium point of the quantity adjustment; see Hayashi (1977) for stability analysis. This is the counterpart of the tâtonnement process in the general equilibrium model. However, this scheme is not a perfect answer to Leijonhufvud's claim since we need an auctioneer for quantity. Few models show how individuals reach an equilibrium point of quantity adjustment without an auctioneer's help.[17]

When we emphasize the spillover effect that arises from the quantity constraint, this is a little inconvenient; the agents make dual decisions because the transactions are different from the planned ones, but these unexpected transactions continue to occur, and the agents never change their policies. Although many studies are related to this problem, there seems to be no complete or intuitive answer.[18] This problem may arise from the static

[16]For instance, Azariadis (1975) and Azariadis and Stiglitz (1983) presented the implicit contrast theory. In Keynesian economics, the perceived kinked demand curve may be a good tool for quantity-constrained micro-foundations (see Bénassy (1976), Dreze (1979), and Negishi (1979). Their monopolistic price setting is based on Arrow (1959).

[17]One possible example is the ins and outs of the labor market. Shimer (2012) shows that the U.S. labor market reaches an equilibrium point of the ins and outs within a quarter. Job seekers and vacant posts are quantity variables, so the equilibrium of the job search model can be interpreted as that of quantity adjustment. Ogawa (2021) argues that we should then decompose the frictional unemployment and unemployment due to goods market's disequilibrium.

[18]For instance, the non-tâtonnement process such as Hahn and Negishi (1962) and

framework. In static models, researchers usually treat the economy as an equilibrium point of adjustment. However, disequilibrium economics focuses on the disequilibrium (or inconsistency) in trade.

4.2 Confusing terms

In disequilibrium economics, there is a serious problem that is not crucial for theoretical consistency, but one that confuses researchers: various names and confusing technical terms.

Disequilibrium economics is called *quantity-constrained* (Negishi, 1979; Latham, 1980), *quantitative rationing* (Muellbauer and Portes, 1978; Sneessens, 1984; Fourgeaud et al., 1981; Movshovich, 1994), *fixed (ed) price equilibrium* (Eckalbar, 1981; Silvestre, 1982, 1983; Weddepohl, 1983; Michaillat and Saez, 2015), *non-Walrasian equilibrium* (Drazen, 1980), and *non-clearing market (NCM)* (Bénassy, 1993). This variation appears to originate from the difficult concept of disequilibrium.

The most serious problem is the word "disequilibrium." Many studies include the word "disequilibrium" when researching disequilibrium economics on the web. However, the aims and frameworks of these studies vary. It is constructive to discuss disequilibrium here.[19] Disequilibrium is an antonym of equilibrium; therefore, we compare the disequilibrium economics presented by Bénassy (1975b) with the general "equilibrium" framework.

Dixon (1990) supplied three properties of equilibrium economics. (P1) The behavior of agents is consistent, (P2) no agent has an incentive to change their behavior, and (P3) the equilibrium is the outcome of some dynamic processes. Table 1 summarizes the properties of the general equilibrium and Bénassy's equilibrium. The properties of the general equilibrium model follow those in Dixon's original description. In this study, we consider the properties of K-equilibrium as disequilibrium economics. First, consistency (P1) is ambiguous in the K-equilibrium. As expressed demand and supply are determined by the optimization problem with quantity signals perceived, agents seem consistent with their choices. However, actual trade could easily betray their expressions (trade plans) since the realization and perception

Uzawa (1962) explored the dynamics of price adjustment in which the transaction by quantity rationing also occurs. However, agents are consistent in that they are never surprised by the realized quantity and make no dual decisions. Stochastic rationing, as mentioned previously, can overcome this problem. In equilibrium with rational expectation, however, we again return to the problem of consistency; the agents are unaffected by quantity rationing, so we do not need a disequilibrium framework.

[19]The discussion follows the concept in non-Walrasian and neo-Keynesian disequilibrium economics.

of constraints are independent of each other. The consistency property is regarded as violated when we focus on realized transactions. Second, P2 is ensured through monetary payments. In K-equilibrium, the agents do not change the expressions of demand and supply because they react optimally to quantity and price signals. If they change the transaction, they should go through the monetary payment process (they cannot trade goods and labor directly). They cannot improve their transactions even if they are ineffective (see Bénassy (1975b, Section 5)). Third, K-equilibrium is an equilibrium point of the quantity-perception tâtonnement process (see Bénassy (1975b, p. 509)).

From this comparison, we find that the most prominent property of disequilibrium economics is inconsistency, which we also emphasize in Guzman and Stiglitz (2020). In other words, what the word disequilibrium means is that realized trade incorporates inconsistency. The main issue for disequilibrium economists is how to address this inconsistency in disequilibrium economics. In the next section, we present how we treat inconsistency (the gap between desired or planned trade and realized trade) and the importance of the spillover effect.

Table 1: Equilibrium properties of equilibrium and disequilibrium models

	Equilibrium	K-equilibrium (disequilibrium)
P1	Demand equals supply	Unsatisfied?
P2	Actual trade equals desired trade	Payment constant
P3	Tâtonnement	Adjustment of perception and expression

5 Redefine non-Walrasian disequilibrium

Although the concern about disequilibrium had once declined, some researchers refer to disequilibrium models such as Barro and Grossman (1971) when they explore the unemployment and "Keynesian situation" in *secular stagnation* these days, as mentioned in the introduction. Sometimes, the terms of disequilibrium economics are abused in analyses of involuntary unemployment and several important issues are overlooked.[20] In this section, we examine and define the doctrinal and mathematical features of disequilibrium economics. We focus on two features of disequilibrium trading:

1. The realized quantity can differ from the planned quantity for each individual. This condition implies a gap between expressed demand and

[20]It does NOT mean that the analyses fail; our purpose is to redefine the disequilibrium economics and to find an unused analytical tool which is peculiar to it.

expressed supply. In general equilibrium economics, demand and supply must be the same in transactions because the tâtonnement process finishes before the actual transaction occurs. However, in disequilibrium economics, individuals do not know whether their planned (desired) transactions have been realized. This corresponds to rationing in many disequilibrium models, but it has a wider range. We allowed for instantaneous price adjustments.[21] This condition states that an economic individual never knows the realized transaction before making a decision.

For a model analysis, this definition is difficult to interpret. Notably, it simply states that $X \neq X^d$ or $X \neq X^s$ holds in the market of X. However, we must clarify the meanings of X^d and X^s. For instance, households somehow know how much rationing of their labor supply occurs; $L = (1-u)L^s$ always holds, as $u \in (0,1)$. They optimize L^s, which means that the realized employment is also optimized, because u is known to them (they are overbidding). In this example, they know that their labor supply is not realized but that the intended employment is realized; this is the same as an equilibrium model. To address this problem for the disequilibrium model, we set the realized value of u as unknown when households make decisions. It should be noted that they do not know whether their planned quantity is realized or whether they perceive a possible quantity constraint. This unexpected constraint condition induces inefficient trade owing to insufficient information. As Bénassy (1975b) showed, agents fear the constraint and, as a result, reduce demand, which causes persistent inefficient trades.

2. The demand (and supply) quantities under the perception of quantity constraints and without are different. When an individual perceives a possible quantity constraint on their transaction in at least one market, this means that their budget constraint, including the realized quantities, is different from the planned budget constraint without the perception of quantity constraint. This condition corresponds to Clower's *dual decision* hypothesis. This condition requires the individual to decide by considering the (expected) realized quantity.

Demand and supply should be affected by quantity signals so that the expected sales Y^e or employment L^e are included in the labor demand and goods demand functions; $L^d = L^d(Y^e)$ and $Y^d = Y^d(L^e)$ hold. In addition, the expectation (quantity signal) is affected by the realization or the realization itself at the equilibrium point of quantity adjustment; that is, $Y^e = Y^e(Y)$ and $L^e = L^e(L)$ hold. Finally, each demand (and supply) is

[21] On this point, our view is closer to the Keynesian economics in Negishi (1979) than that in Leijonhufvud (1968). However, our economics are not just Keynesian; they only emphasize the possibility of disequilibrium.

affected by transactions realized in other markets. This is the spillover effect emphasized by Patinkin (1956) and Clower (1965).

In summary, individuals do not know the realizations of every market, but the realizations in the markets are (indirectly) connected. This relationship should be maintained when moving from microstructure to macroeconomic analysis. In fact, K-equilibrium satisfies the two features: effective demand in each market is determined without knowing how much rationing occurs on it, and each effective demand is determined through the dual-decision hypothesis.

5.1 Rigidity models: rigid wage and ZLB

Here we conduct a brief comparison between a "non-Walrasian disequilibrium model" defined above and an equilibrium model which has a disequilibrium flavor. First, we consider a model with downward wage rigidity.

The real wage is pegged at $\bar{w}$ such that *nominal* labor demand can be lower than labor supply:

$$L = L^d(\bar{w}) < L^s = \bar{L} = \text{const.} \tag{4}$$

This is the classical unemployment regime (see figure 2). On this point, disequilibrium economics still seems alive; in fact, Dupor et al. (2019) refer to Barro and Grossman (1971) as a sticky wage model.

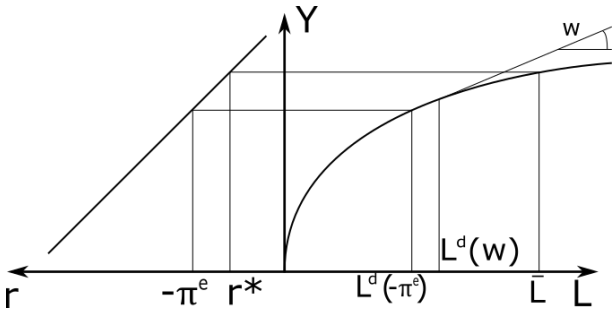

Figure 2: A simple example of ZLB model and rigid wage model

Second, we consider a simple example of zero-lower-bound (ZLB) economic model. Production Y is determined by the goods market equilibrium and depends on the real interest rate r and real wage w. Goods demand is derived by intertemporal optimization under the given price variables r and w. We define natural interest rate r^* as the rate at which full employment $\bar{L}$ is ensured. Employment is determined by the production $L = L(Y)$.

Therefore, the system consists of the following equations:

$$Y = \min[Y^d(r, w), Y^s(w)], \tag{5}$$
$$L = L(Y), \tag{6}$$
$$w = \bar{w}, \tag{7}$$
$$r = \max[r^*, -\pi^e], \tag{8}$$

where π^e is the expected inflation rate. When the ZLB constraint holds ($r > r^*$), involuntary unemployment ($L < \bar{L}$) continues (see figure. 2). Although unemployment in ZLB economics sometimes occurs through the goods demand term, it is difficult to regard it as Keynesian unemployment because the rigidity of price in the funds market (real interest rate) is a source of unemployment. Although these models are often complicated, the core structure appears to be the stickiness of the real (or nominal) interest rate.

Because employment is quantity adjusted, the employment conditions in the above models can be rearranged as follows.

$$L = \min[L^d(r, w), \bar{L} = L^s]. \tag{9}$$

This rationing equation is the same as that in disequilibrium models; therefore, the ZLB works and secular stagnation analyses sometimes refer to disequilibrium economics.[22] However, the core of non-Walrasian disequilibrium economics is usually overlooked, that is, the spillover effect in disequilibrium, or the dual-decision hypothesis.

Although involuntary and unexpected unemployment occurs, in the example above, a household's behavior is not affected by existing unemployment. In other words, neither labor supply nor the goods demand function differs from that of full employment. This indicates that the disequilibrium analysis was incomplete. If the spillover effect is incorporated, the goods demand function depends on the realized employment.

$$Y^d = Y^d(r, w, L) \tag{10}$$

If we proceed with the quantity adjustment process, the transaction is described as the system $Y = Y(L)$ and $L = L(Y)$ under the given (r, w). We should notice that even if the price variables are equilibrium values (r^*, w^*), on which the full employment is ensured in the standard ZLB model, the

[22]For instance, see Eggertsson et al. (2019, footnote 19). The simplified ZLB model in Palley (2019) does not specify the employment function in detail but seems to adopt this rationing scheme.

full employment is not ensured in the quantity adjustment model.[23] This is because each demand (supply) function is quantitatively different from that in the equilibrium.

5.2 Simple non-Walrasian disequilibrium model

We now present a simple example of the disequilibrium model on the ZLB. The system determines the quantity of goods produced Y and employment L:

$$Y = \min\{Y^d(r, w, x, L), Y^{s*}(w), \bar{Y}^s(\bar{L})\}, \tag{11}$$

$$L = \min\{\tilde{L}^d(Y^d, x), L^{d*}(w), \bar{L}\}, \tag{12}$$

$$w = \bar{w}, \tag{13}$$

$$r = \max\{r^*, -\pi^e\}, \tag{14}$$

where x is the sales expectation. The notional supply Y^{s*} and notional demand L^{d*} are derived from usual optimization problems without quantity constraints. The labor supply is inelastic $L^s = \bar{L}$, such that there is a physical constraint on production capacity $\bar{Y}^s$. If we suppose that the monetary authority has complete information about the real economy and that there is no demand saturation, such that some r^* satisfies $\tilde{L}^d(Y^d(r^*), x) = \bar{L}$, then the resource of unemployment is price rigidity, and if r or w is too high, unemployment occurs. We should note that the unemployment mechanism itself does not change from the standard ZLB or rigid wage model but that the quantity adjustment process is added to this model. The shortage of goods demand is enhanced through the spillover effect in labor demand $Y = Y^d(L(Y))$ when goods demand determines production (see figure 3). In a preliminary textbook Blanchard (2016, Chapter 6), Blanchard utilized the IS-LM framework to show ZLB constraint. His model partially depicts this spillover effect, since the IS curve is derived through the ordinal Keynesian Cross (goods market equilibrium under quantity adjustment).

However, there is a possibility that another problem may cause involuntary unemployment. Suppose that sales expectation x is affected by realized production Y. If the firm becomes risk-avoiding and underestimates x during a recession, there might not be a natural interest rate: $\tilde{L}^d(Y^d(r), x(Y^d(r))) < \bar{L}$ holds for any r. This implies that switching in dynamics occurs and the demand function changes quantitatively. The pessimistic expectation of sales is maintained for the current shortage of demand for goods, and the ZLB is

[23] Barro and Grossman (1971) showed the first example of unemployment under the equilibrium prices.

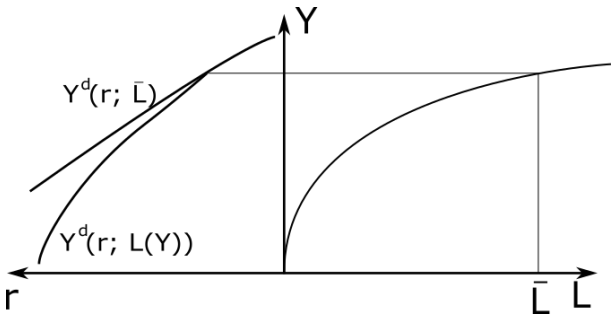

Figure 3: ZLB model with spill-over (dual-decision)

no longer the main source of unemployment.[24] Figure 4 presents an example. The second quadrant depicts the two good demand curves. The higher one has a sufficient and fixed sales expectation $\bar{x}$ and the lower one has an underestimated $x(Y)$ function.

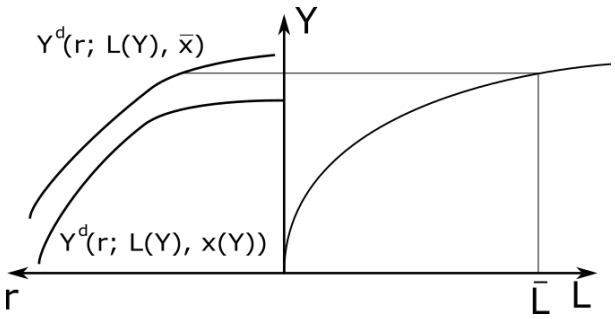

Figure 4: The case in which there is no natural interest rate

5.3 Problems on expectations

Subsequently, our interest moves to the formulation of sales expectations: Why do firms underestimate it? If we set the perfect foresight model as the starting point, we must forcibly add inconsistency or irrationality to the canonical model. This implies that the equilibrium model, in which the consistency of individuals and the model itself is embedded, cannot explain the persistent downturn in their natural assumptions, as Guzman and Stiglitz (2020) emphasized. Therefore, we cannot utilize tools for mathematical formulations of expectations that have grown in equilibrium without consider-

[24]Varian (1977) showed the persistent underestimated sales expectation and shortage of goods demand.

ing them. Instead, it is natural to construct formulations of micro-founded expectations within a disequilibrium framework.

Neary and Stiglitz (1983) found that the self-fulfilling prophecy works in a disequilibrium economy; the optimistic expectation for sales constraint in the next period promotes the current economic activities so that the current sales would be stimulated. Building overlapping generations model with investment problem was also referred to by Weddepohl and Yildirim (1993).

These works imply that the steady state, which is usually accompanied with market equilibrium, could not be the starting point of expectation theory of disequilibrium. If we use it, the results may mislead us to under evaluate the persistent disequilibrium. One possible way is to extend the learning dynamics models summarized in Evans and Honkapohja (1999), emphasizing the inconsistency due to disequilibrium transactions. We should consider some spillover effects among markets invoked by the demand-supply gap, since the canonical models in the learning dynamics are based on equilibrium trades. This is a difficult but interesting future issue for the disequilibrium theory.

6 Concluding remarks and future issues

In this study, we present a brief survey of the disequilibrium economics. Disequilibrium economics is greatly affected by changes in the macroeconomic analysis tools, and is now almost forgotten. However, it has the potential to research various economic issues such as secular stagnation.

The most important characteristic of disequilibrium research is the dual-decision hypothesis, in which quantity signals have a spillover effect, and this issue has often been overlooked. A spillover effect exists on the relationship between realized transactions and non-desired transactions. The dual-decision hypothesis comes from the most important feature of inconsistency. We should include this term in disequilibrium models and estimate them in the future.

Although disequilibrium economics has developed its tools and derived many interesting conclusions, it remains a minor part of economics. How can we promote discussions of disequilibrium? There seem to be two types of strategy. One is to extend the existing fixed-price models to a static framework à la equilibrium economics. This implies to analyze the equilibrium point of the quantity adjustment. The theory has two characteristic tools: rationing and dual decisions. Although New Keynesian (equilibrium) economics has succeeded in modern macroeconomics, the perspective of disequilibrium provides several good tools through the theory of spillover effects

among markets. This strategy is shown in the former section. The other challenge is to decompose the time series and directly describe the disequilibrium dynamics. This implies denying the adjustment process before trade to abandon property P3 in 4.2. The model of this type was presented by Iwai (1981) as the stream of trade during price adjustment. The key is to describe the intertemporal spillover effect: how does the demand-supply gap in one market and one period spread to the economy in the future? This would strongly emphasize inconsistency since the agents keep revising their trade plans.

In either case, we must consider the unsolved problems in 4.1. Particularly, we require a sound micro-foundation for the dominance of quantity adjustment *in disequilibrium*. Although Drazen (1980) expected progress in endogenous price setting, this task has not been completed. He referred to the conjectural equilibrium concept by Hahn (1977, 1978), which is related to the kinked demand theory for Keynesians in Negishi (1979) and Palley (1997). These days, for instance, Ilut et al. (2020) tackle this issue from Knightian uncertainty theory. Endogenous price stickiness due to possible disequilibrium has not been synthesized with the disequilibrium macro, and remains the most important issue.

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