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Money velocity, digital currency, and inflation dynamics*

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Abstract

This paper empirically investigates the impact of transaction cost-induced variations in the velocity of money on inflation dynamics, based on a structural New Keynesian Phillips curve (NKPC) with an explicit money velocity term. The money velocity effect arises from the role of money, both in physical and digital forms, in reducing the aggregate transaction costs and facilitating purchases of goods and services. We find a non-trivial aggregate impact in the context of the Indonesian economy: our benchmark estimates suggest that a 10% decrease in money velocity, which might be facilitated by a new digital currency (e.g. CBDC) issuance, would reduce the inflation rate by 0.6-1.7%, all else equal. Using the estimates and within a small-scale New Keynesian DSGE model, we analyze the potential implications of a CBDC issuance on aggregate fluctuations. A CBDC issuance that conservatively lowers the velocity of money by 5% is predicted to permanently raise the GDP level by 0.8% and lower the inflation rate by 0.8%. Both nominal and real interest rates are also permanently lower. Our findings imply that central banks could potentially use CBDCs as an additional stabilization policy tool by influencing the velocity.

JEL Classification: E31; E32; E41; E42; E51; E50; E58

Keywords: inflation dynamics; transaction cost; velocity of money; digital money; digital currency; central bank digital currency (CBDC); aggregate fluctuations;

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1 Introduction

Modern, mainstream macroeconomic theory has for some time regarded variations in the velocity of money as unimportant for inflation dynamics. Under the dominant cashless models (Goodfriend and King (1997), Rotemberg and Woodford (1997), Woodford (2003)), money only serves as a unit of account and the velocity of money is assumed to be constant.¹ The velocity, however, should vary over time if money also serves to facilitate purchases of goods and services by lowering the transaction costs (Tobin (1956), Cooley and Hansen (1989), Dotsey and Ireland (1996)). That is, for a given volume of transactions, an increase in the use of money to reduce the transaction costs would decrease the velocity of money. These variations in the velocity may in turn non-trivially affect the fluctuations of inflation and real aggregate variables.

The advent of digital money or currency—cryptocurrencies, stablecoins, and central bank digital currencies (CBDCs)—suggests that we should not ignore the potential impact of money velocity variations on aggregate fluctuations. While it remains uncertain whether private digital money would become a widely-acceptable means of payment, CBDCs are likely to be a dominant, universally-accepted legal tender in the near future. As of March 2023, 4 central banks, representing 11 countries, have fully launched a CBDC (Atlantic Council (2023)).² A further 104 countries were in various stages of CBDCs’ development, including China and India, each has launched a CBDC in a pilot phase.³ Similar to traditional paper money (cash), the issuance and use of digital money facilitates transactions, as there should be negligible transaction costs of using digital money such as CBDCs as a means of payment (Bank for International Settlements (2021)). The increased use of digital money is also therefore associated with a reduction in the aggregate transaction costs and induces variations in the velocity of money. As shown by Ireland (2001) and Kim and Subramanian (2006), these movements in the velocity may amplify inflation fluctuations through a Phillips-curve relationship, affecting the transmission of monetary policy. Whether this

¹The typical assumption is that the amount of real money holding M_t/P_t is equal to the volume of transaction Y_t , implying that the velocity of money is unity, and hence, constant.

²These four are the Central Bank of The Bahamas, the Eastern Caribbean Central Bank, the Central Bank of Nigeria, and the Bank of Jamaica.

³See also Kosse and Mattei (2022) for a recent survey on the exploration of CBDCs by central banks around the world.

effect is economically significant is an important empirical question, especially for central banks.

In this paper, we empirically investigate the impact of variations in the velocity of money on inflation dynamics, and given the estimates, analyze the potential implications of an issuance of a digital currency, e.g. a CBDC, on aggregate fluctuations. To this end, we proceed in two steps. First, we derive a structural New Keynesian Phillips curve (NKPC) with an explicit money velocity term, following the transaction-cost approach in Kim and Subramanian (2009), and estimate it using Indonesian data. The Indonesian economy offers a fitting environment for our analysis for two reasons: (i) the majority of transactions are still conducted using cash, indicating that the money-holding friction is non-trivial and money remains relevant as a means to facilitate transactions in Indonesia,⁴ and (ii) the central bank of Indonesia—Bank Indonesia (BI)—has a definitive plan to issue a CBDC (digital Rupiah) in the near future (Bank Indonesia (2022)). In the second step, to facilitate our analysis on the implications of a digital currency issuance through its impact on money velocity, we embed the estimated NKPC into a standard, small-scale New Keynesian DSGE model along the lines of Woodford (2003) and Galí (2015). While we only parsimoniously model the aggregate effect of digital money in this second step, the underlying mechanism is similar to those assumed in more-rigorous models (e.g. Barrdear and Kumhof (2022) and Minesso, Mehl and Stracca (2022) in the case of a CBDC issuance). That is, an issuance of a digital currency such as a CBDC acts as a technological innovation that reduces the aggregate transaction costs in purchasing goods and services.

Our benchmark, structural estimations show that a 10% increase in money velocity would raise the inflation rate by 0.6-1.7%, all else equal. Such an impact is non-trivial, even though it remains smaller compared to the effect of output gap variations, i.e. the traditional driving process of inflation fluctuations in a Phillips-curve relationship. We further show that, using the standard New Keynesian DSGE model, the non-trivial impact translates to other variables as well: even under a low-variance case, shocks to the velocity of money are

⁴In 2022, cash was still used in 51% of point-of-sale payments in Indonesia (FIS Global (2022)). Despite this, there has been a tremendous growth in the use of digital payments in Indonesia in the past decade, e.g. the volumes of electronic money and digital banking transactions grew by 42% and 35% year-on-year, respectively, in the first quarter of 2022 alone (source: Bank Indonesia).

responsible for 23% of output gap variations and 14% of nominal interest rate variations, vis-à-vis other shocks in the model (technology, cost-push, and monetary-policy shocks). Ignoring the variations in the velocity of money in the Philips curve relationship would therefore lead to an incorrect assessment of the relationship among aggregate variables, which in turn might lead to an inaccurate policy prescription if the central bank uses the model to guide its monetary policy conduct. On the potential impact of a CBDC issuance, which we treat as a near-permanent technological innovation in the model, our simulation shows that an issuance that conservatively lowers the velocity of money by 5% would permanently raise the GDP level by 0.8% and lower the inflation rate by 0.8%. Both nominal and real interest rates are also predicted to be permanently lower. These effects are broadly consistent with those found in the literature. For example, using a DSGE model with a rigorous CBDC modelling, Barrdear and Kumhof (2022) find that a CBDC issuance is associated with a higher GDP level and lower real interest rates, distortionary taxes, and aggregate transaction costs.

This paper is related to three strands of literature. The first strand involves the large literature on NKPC estimations using limited information methods (Galí and Gertler (1999), Rudd and Whelan (2005), Sbordone (2005), Barnes et al. (2011), Zhang and Murasawa (2011), Lie and Yadav (2017), Chen and Xia (2020), among others). Virtually all the studies in the literature, however, ignore the variations in the velocity of money. In terms of the form of the NKPC (with money velocity) and the estimation method (the generalized method of moments (GMM)), our paper is closely related to Kim and Subramanian (2009), who also find a non-trivial, positive impact of money velocity on inflation dynamics in the US data. We extend their finding by showing that the impact also translates to the Indonesian economy and in an open-economy setting, where the NKPC is an additional function of the real exchange rate, terms of trade, and foreign output gap. Further to this, we make a connection between the variations in money velocity and an issuance of digital currency such as a CBDC and analyze its potential impact on aggregate fluctuations.

Related to the first strand of literature, our paper adds to a small but growing literature on the estimation of the NKPC relationship using Indonesian data (Yanuarti (2007), Insukindro and Sahadewo (2010), Wimanda, Turner and Hall (2011), Wimanda, Turner and

Hall (2013)). Such an NKPC has also been used in various structural, business-cycle models for the Indonesian economy for policy analyses and simulations (Harmanta, Purwanto and Oktiyanto (2014), Idham, Wimanda and Winarto (2014), Lie (2019), Zams (2021), Juhro, Lie and Sasongko (2022)). These studies, however, typically assume a cashless environment, and hence, a constant velocity of money, which, as discussed above, is inconsistent with the prevalence of cash transactions in Indonesia. Variations in the velocity therefore play no role in inflation dynamics.⁵ Our estimates and subsequent analyses show that ignoring the money velocity term in such an economic environment may cause the monetary policymaker to miss a significant source of inflation variations in their projections and overlook an important monetary policy transmission channel.

We also contribute to the growing literature on the effect of digital money or currency, CBDC included, on aggregate fluctuations (Berentsen (1998), Humphrey, Kim and Vale (2001), Davoodalhosseini (2022), Barrdear and Kumhof (2022), Minesso, Mehl and Stracca (2022), Williamson (2022)).⁶ Our relative contribution is in making an explicit connection between a digital currency issuance and the transaction cost-induced variations in the velocity of money — as long as the digital currency is widely used as a means of payments and reduces the transaction costs, the aggregate effect would transpire through a reduction in the velocity of money. The current paper does this parsimoniously within a small-scale DSGE model. Future research, however, should model the connection more rigorously within a larger-scale model. Notwithstanding this limitation, our finding has an important policy implication: central banks could potentially use CBDCs as an additional stabilization policy tool by influencing the velocity.

The paper’s organization is as follows. Section 2 derives a benchmark New Keynesian Phillips curve (NKPC) with an explicit money velocity term. Section 3 presents and discusses the estimates, both under the reduced-form and structural estimations. We also perform several robustness exercises in this section, including the estimation of an open-economy NKPC. Given the estimates in Section 3, Section 4 investigates the impact of a digital currency issuance on aggregate nominal and real variables within a small-scale New

⁵For the determinants of the velocity of money in Indonesia, see Sharma and Syarifuddin (2019).

⁶See also Harahap et al. (2017), Syarifuddin and Bakhtiar (2021), and DKEM (2021) for related studies on the potential implications of a CBDC (digital Rupiah) issuance on the Indonesian economy.

Keynesian DSGE model. Section 5 concludes.

2 The Phillips-curve relationship: Inflation dynamics with transaction costs and money velocity

To model the aggregate effect of a digital currency issuance on inflation dynamics, we follow the tradition and basic idea behind the well-established transaction cost literature (Baumol (1952), Tobin (1956), Prescott (1987), Dotsey and Ireland (1996), Ireland (2001), Kim and Subramanian (2006)). Similar to traditional paper money or cash, digital money or currency could be used to facilitate purchases of goods and services by lowering the transaction costs, thus creating a demand for (digital) money. We do not take a stand on the exact form of the transaction costs — these costs could be in the form of communication and record-keeping costs in facilitating credit transactions (Dotsey and Ireland (1996)), or the credit time costs involved (Khan, King and Wolman (2003)), or something else entirely. Rather, following the setup in Kim and Subramanian (2006, 2009), we define the aggregate transaction cost τ_t as

$$\tau_t = c_t k_0 \left(\frac{M_t}{P_t c_t} \right)^{1-k_1} \exp(\varepsilon_t), \quad (1)$$

with $k_0 \geq 0$ and $k_1 > 1$. In this setup, an increase in the volume of transaction (consumption purchase), c_t , would raise the aggregate transaction cost, all else equal. As a means of payment, however, money can be used to facilitate transactions and reduce the transaction cost τ_t . For a given volume of transactions, a higher use of (or the issuance of) digital money would increase real money balances M_t/P_t , lowering the costs. ε_t can be generally treated as any other factors that affect the aggregate transaction cost, e.g. an exogenous shock to money creation, or, relevant for our paper, a new CBDC issuance by the central bank.

Since the velocity of money is defined as $v_t = P_t c_t / M_t$, equation (1) can be alternatively written as

$$\tau_t = c_t k_0 v_t^{k_1-1} \exp(\varepsilon_t). \quad (2)$$

Hence, there is a positive relationship between the velocity of money and the aggregate

transaction cost. A *lower* money velocity, which could arise due to a *higher* use of money M_t or a lower aggregate nominal transaction value $P_t c_t$, would decrease the cost τ_t , potentially influencing inflation dynamics and real fluctuations. The extent of this relationship is also dependent on the scale parameter $k_0 \geq 0$ and the curvature parameter $k_1 > 1$, which could be estimated from actual data.

The Phillips curve relationship As shown in the appendix, assuming the existence of transaction cost (2) and incorporating it into a standard New Keynesian model (Woodford (2003), Galí (2015)), we obtain the following structural, (log-linearized) *closed-economy* New Keynesian Phillips curve (NKPC):

$$\begin{aligned} \hat{\pi}_t = & \frac{\varrho}{1 + \beta\varrho} \hat{\pi}_{t-1} + \frac{\beta}{1 + \beta\varrho} E_t \hat{\pi}_{t+1} \\ & + \left[\frac{\kappa}{1 + \beta\varrho} (\sigma + \eta) \right] (\hat{y}_t - \hat{y}_t^f) \\ & + \left[\frac{\kappa}{(1 + \beta\varrho)} (k_1 - 1) v_0 \right] (\hat{v}_t - \hat{v}_t^f). \end{aligned} \quad (3)$$

Here, $\hat{\pi}_t$, $\hat{y}_t - \hat{y}_t^f$, $\hat{v}_t - \hat{v}_t^f$ denote inflation, the output gap, and the money velocity gap, respectively. We define the two "gap" variables as their log deviation from the level under the flexible-price equilibrium, i.e. the natural level. β is the subjective discount factor, ϱ is the degree of past price indexation, σ is the inverse elasticity of intertemporal substitution, and η is the inverse Frisch labor supply elasticity. The reduced-form parameter $\kappa \equiv (1-\theta)(1-\theta\beta)/\theta$ can be thought as the slope of the Phillips curve and is a compound function of β and the (Calvo (1983)) probability of non-optimal price adjustment θ . The parameter $v_0 \geq 0$ is related to the aggregate transaction cost function and is a function of the long-run level of money velocity. When there is no past indexation ($\varrho = 0$), (3) is purely forward-looking as in Kim and Subramanian (2009).

Provided that $v_0 \neq 0$, it is apparent from (3) that variations in the money velocity gap $\hat{v}_t - \hat{v}_t^f$ additionally affect inflation dynamics in our model. When no transaction cost is present, as in standard models, $v_0 = 0$ and the output gap becomes the only driving process. Whether $v_0 > 0$ and is economically and statistically significant is an empirical question,

which we investigate in the next section.

3 Estimates of the Phillips curve with money velocity

We estimate the velocity-enhanced NKPC relationship using a generalized method of moments (GMM) approach, which is a standard approach in the literature when estimating an NKPC relationship using a limited information method (see e.g. the seminal paper of Galí and Gertler (1999)).⁷ We first describe the data series used and the calibration of several parameters, prior to describing the estimation procedure and results.

3.1 Data and calibration

The following Indonesian quarterly data series are used in our study: the nominal short-term interest rate (Bank Indonesia (BI) 7-day reverse repo rate), headline CPI inflation rate, real GDP (output) gap, money (M1 and M2) velocity gap, and commodity price inflation. The interest rate, inflation, and money velocity data are sourced from Bank Indonesia's Indonesian Economic and Financial Statistics (SEKI). GDP data are sourced from OECD Main Economic Indicators. Commodity price inflation data, which are used as part of the instrument sets in the GMM estimation, are taken from the IMF's primary commodity price index. To obtain GDP and money velocity gaps, we extract the gap components from the respective raw data using the Hodrick-Prescott (HP) filter. Our benchmark estimation period is from 2005.Q3 to 2022.Q1, where the starting period coincides with the start of the full-fledged implementation of the inflation targeting framework (ITF) by Bank Indonesia.

Figure 1 plots the constructed M1-based and M2-based money velocity data, along with the headline CPI inflation rate. As shown in the top panel, both measures of money velocity vary over the sample, with M1-based velocity being the more volatile one. The bottom panel plots the M1-based velocity and the headline CPI inflation rate. Although not readily visible, the two series are positively correlated, as suggested by the theoretical model. While the correlation coefficient is moderate (0.37), it is the case that inflation dynamics are also

⁷An alternative partial-information method is to use a minimum distance estimator (Sbordone (2005), Cogley and Sbordone (2008), Barnes et al. (2011)).

driven by other factors such as inflation expectations and output gap variations. To assess and isolate the significance of the variations in velocity gap on inflation dynamics, one need to conduct a full econometric exercise.

3.1.1 Calibration of elasticity parameters

The parameters σ , η , and k_1 in (3) are not identified, and hence, need to be calibrated. We set $\sigma = 1$ and $\eta = 1$, as is standard in the literature (used e.g. in the estimated DSGE model for Indonesia in Juhro, Lie and Sasongko (2022)). The interest rate elasticity parameter is set to $k_1 = 12.3$, following our Newey-West OLS estimate of the model-implied money demand function. The appendix provides more details on the money demand function and the estimate of the parameter k_1 .

3.2 Reduced-form estimations

We first present the GMM estimates of the reduced-form coefficients γ_b , γ_f , λ_y and λ_v , based on the NKPC

$$\hat{\pi}_t = \gamma_b \hat{\pi}_{t-1} + \gamma_f E_t \hat{\pi}_{t+1} + \lambda_y (\hat{y}_t - \hat{y}_t^f) + \lambda_v (\hat{v}_t - \hat{v}_t^f). \quad (4)$$

The orthogonality (moment) condition under the GMM approach is given by

$$0 = E_t \left[\left\{ \hat{\pi}_t - \gamma_b \hat{\pi}_{t-1} - \gamma_f E_t \hat{\pi}_{t+1} - \lambda_y (\hat{y}_t - \hat{y}_t^f) - \lambda_v (\hat{v}_t - \hat{v}_t^f) \right\} \cdot Z_t \right], \quad (5)$$

where Z_t is a vector of variables dated t or earlier, i.e. the set of instruments. An orthogonality condition such as (5) forms the basis of the GMM approach: time- t (or later) expectation error should be orthogonal to the information set at time t . All variables in the instrument set contain relevant information for inflation forecasts. We use two sets of instruments in our estimations, both in the reduced-form estimations and in the structural estimations below, as part of our robustness check. The first set (henceforth, IS 1) comprises of four lags of inflation, four lags of real GDP gap, two lags of BI 7-day repo rate, and two lags of M1 velocity. The second, larger set (henceforth, IS 2) includes four lags of inflation, four lags

Figure 1: Velocity of money and inflation in Indonesia

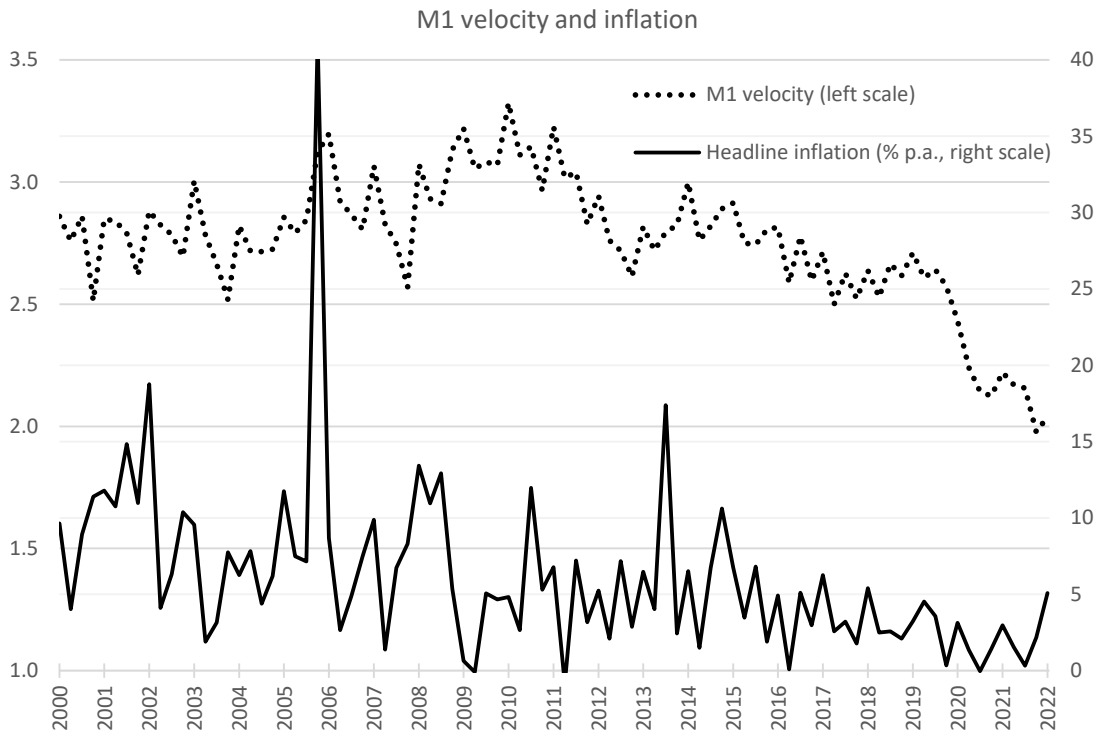
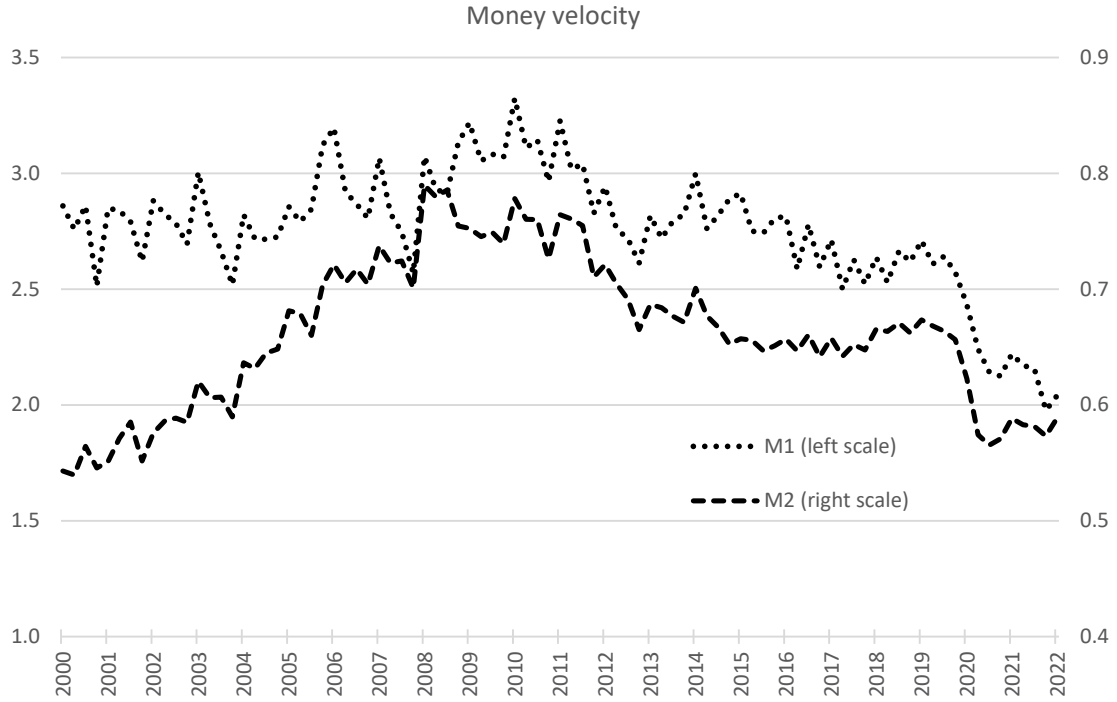


Table 1: NKPC with money velocity – Reduced-form estimation

	γ_b	γ_f	λ_y	λ_v	$J\text{-stat}$
No velocity					
Instrument set 1 (IS 1)	0.484 (0.098)	0.464 (0.085)	0.085 (0.026)	–	11.59 [0.17]
Instrument set 2 (IS 2)	0.458 (0.070)	0.349 (0.063)	0.096 (0.022)	–	14.54 [0.80]
With velocity					
IS 1	0.029 (0.065)	0.815 (0.081)	-0.045 (0.044)	0.054 (0.022)	11.83 [0.22]
IS 2	0.147 (0.024)	0.642 (0.034)	0.022 (0.030)	0.031 (0.012)	14.45 [0.76]
With velocity, purely forward-looking, no output gap					
IS 1	–	0.779 (0.043)	–	0.045 (0.012)	12.51 [0.33]
IS 2	–	0.738 (0.027)	–	0.048 (0.008)	14.42 [0.85]

Notes: (1) This table reports the GMM estimates of the reduced-form coefficients of the NKPC in Eq. (4), based on the orthogonality condition in Eq. (6); (2) the instrument set IS 1 includes four lags of inflation, four lags of real GDP gap, two lags of BI rate (7-day repo rate), and two lags of M1 velocity gap; (3) the instrument set IS 2 includes all the variables in IS 1 plus two additional lags of BI rate, two additional lags of M1 velocity gap, and four lags of commodity price inflation; (4) numbers in parantheses are standard errors, except for the J-statistic (probability value is reported instead); (5) sample period: 2005.Q3-2022.Q1.

of GDP gap, six lags of BI 7-day repo rate, six lags of M1 velocity gap, and four lags of commodity price index.

Table 1 reports the reduced-form estimates for several NKPC specifications. In the standard NKPC with no money velocity term (λ_v set to 0), all three coefficients are statistically significant, i.e. they have small standard errors. Moreover, the coefficients have the right signs. Lagged inflation is slightly more important than one-period ahead inflation expectation ($\gamma_b > \gamma_f$) in determining current inflation, irrespective of the instruments used in the GMM estimation. The point estimates of λ_y , 0.09 based on IS 1 and 0.10 based on IS 2, show that output gap is a relevant determinant of inflation fluctuations in Indonesia. These estimates are in line with other estimates of the NKPC slope using Indonesian

data (see e.g. Insukindro and Sahadewo (2010), Wimanda, Turner and Hall (2013), and Zams (2021)).⁸ The last column in Table 1 reports the J-statistic from Sargan-Hansen test (J-test), which show that we cannot reject the null of over-identifying restrictions against model misspecification.

The estimates for the NKPC with money velocity are reported in the middle panel. We find that once the velocity term is present, the forward-looking inflation term is much more important than the backward-looking term. Based on the first instrument set for example, $\gamma_f = 0.82$ with a low standard error, while $\gamma_b = 0.03$ and is not statistically different than zero. This finding that a purely forward-looking NKPC fits the Indonesian data better (rather than a hybrid one) is consistent with the findings from the full-information (Bayesian) DSGE-model estimates in Lie (2019), Zams (2021), and Juhro, Lie and Sasongko (2022). Importantly, we find that the estimates of λ_v are positive and both economically and statistically significant. $\lambda_v = 0.054$ (from IS 1 estimate) means that a 10% increase in the velocity of money would increase the (quarterly) inflation rate by 0.54%. We also find that, somewhat surprisingly, once we account for money velocity in the NKPC, the output gap is no longer a relevant determinant of inflation variation: irrespective of the instrument set used, the estimates of λ_y are not statistically significant from zero at 5% level.

The irrelevance of lagged inflation (based on IS 1) and the output gap prompts us to estimate an NKPC specification with only forward-looking inflation and money velocity terms (bottom panel of Table 1). This specification mirrors the standard purely forward-looking NKPC (see e.g. Yun (1996), Rotemberg and Woodford (1997), Woodford (2003), and Galí (2015)), albeit with money velocity as the driving process, instead of the output gap or real marginal cost. We find strong evidence of the importance of the forward-looking inflation and money velocity terms in driving inflation fluctuations. Here in particular, the estimates of λ_v have the right sign and both economically and statistically significant, in line with the previous estimates under the hybrid NKPC and with the output gap term included.

⁸In their linear reduced-form Phillips curve specification, for example, Wimanda, Turner and Hall (2013) find that the point estimate of the output gap coefficient is 0.12.

Table 2: NKPC with money velocity – Structural estimation

	β	ϱ	θ	v_0	Implied λ_y	Implied λ_v
Unrestricted						
Instrument set 1 (IS 1)	1.057 (0.056)	-0.255 (0.098)	0.629 (0.092)	0.055 (0.040)	0.540	0.168
Instrument set 2 (IS 2)	1.019 (0.054)	-0.187 (0.087)	0.614 (0.067)	0.027 (0.015)	0.580	0.089
Restricted β						
IS 1	0.9942	-0.247 (0.098)	0.646 (0.083)	0.050 (0.033)	0.521	0.148
IS 2	0.9942	-0.184 (0.087)	0.614 (0.053)	0.025 (0.011)	0.600	0.086
Restricted β , purely forward-looking						
IS 1	0.9942	0	0.592 (0.063)	0.029 (0.013)	0.567	0.094
IS 2	0.9942	0	0.578 (0.038)	0.017 (0.006)	0.620	0.060

Notes: (1) This table reports the GMM estimates of the structural parameters of the NKPC in Eq. (3), based on the orthogonality condition in Eq. (6); (2) numbers in parantheses are standard errors; (3) $\lambda_y = \frac{\kappa(\sigma+\eta)}{1+\beta\varrho}$, $\lambda_v = \frac{\kappa v_0(k_1-1)}{(1+\beta\varrho)}$, $\kappa \equiv (1-\theta)(1-\theta\beta)/\theta$, and in all cases, $\sigma = \eta = 1$, $k_1 = 12.3$; (4) the instrument set IS 1 includes four lags of inflation, four lags of real GDP gap, two lags of BI rate (7-day repo rate), and two lags of M1 velocity gap; (5) the instrument set IS 2 includes all the variables in IS 1 plus two additional lags of BI rate, two additional lags of M1 velocity gap, and four lags of commodity price inflation; (6) sample period: 2005.Q3-2022.Q1.

3.2.1 Structural estimations

We now estimate directly the structural parameters β , ϱ , θ , and v_0 , based on the orthogonality condition

$$0 = E_t \left[\left[\begin{array}{c} \theta(1+\beta\varrho)\hat{\pi}_t - \theta\varrho\hat{\pi}_{t-1} - \theta\beta\hat{\pi}_{t+1} \\ -(1-\theta)(1-\theta\beta)\{(\sigma+\eta)(\hat{y}_t - \hat{y}_t^f) \\ + v_0(k_1-1)(\hat{v}_t - \hat{v}_t^f)\} \end{array} \right] \cdot Z_t \right]. \quad (6)$$

Table 2 presents the structural estimation results. As in the reduced-form estimations, we report the estimates for two different instrument sets, IS 1 and IS 2. In addition to the

unrestricted, benchmark specification—where we estimate the parameters β , ρ , θ , and v_0 —we also report the estimates under restricted β (as in Gali and Gertler (1999)), both under the hybrid NKPC (unrestricted ρ) and purely forward-looking NKPC (restricted $\rho = 0$). The purely forward-looking case is motivated by the previous reduced-form estimation results. The last two columns in the table report the implied coefficients of the output gap (λ_y) and the velocity of money (λ_v), given the calibration and the point estimates of the structural parameters.⁹

Our estimates in the unrestricted case show that the impact of the velocity of money on inflation dynamics, represented by the parameter v_0 , is positive (as predicted by the theoretical model) and have low standard errors. This is true irrespective of the instrument set used, although the point estimate is somewhat lower under IS 2 (0.03 vs. 0.06 under IS 1). The point estimates of the Calvo parameter θ across the two instrument sets range from 0.61 to 0.63, which are in line with the estimates reported in various studies using Indonesian data.¹⁰ The estimates of β and ρ , however, are incongruent with the underlying theory. β are estimated to be above unity (although still close to 1). The estimates for ρ are negative and have low standard errors. Notwithstanding these estimates, we find that the implied coefficient on the money velocity term is significant: $\lambda_v = \{0.17, 0.09\}$. While the impact of money velocity is not as large as the output gap impact (λ_y), it is not insignificant and is larger than that based on the reduced-form estimation.

We next restrict β to be consistent with the underlying theory, i.e. we set $\beta = 0.9942$, per the estimate in Juhro, Lie and Sasongko (2022). Overall, the estimates are consistent with those in the unrestricted case. In particular, the estimated values of v_0 and the implied coefficients on the velocity term are positive and significant. In the bottom panel of Table 2, we also report the estimates when we restrict $\rho = 0$, rendering the Phillips curve purely forward-looking. The estimated values of θ and v_0 and the implied λ_v coefficients are now slightly smaller compared to their counterparts in the previous two cases in the table. Despite this, we still find strong evidence of a positive and significant contribution of money velocity

⁹The J-test results (not reported in the table) indicate that for all cases, we cannot reject the overidentifying restrictions.

¹⁰See e.g. Hermawan and Munro (2008), Harmanta, Purwanto and Oktiyanto (2014), Dutu (2016) Lie (2019), and Juhro, Lie and Sasongko (2022).

on inflation dynamics.

3.3 Robustness

This section presents the results from several robustness exercises. First, we extend the sample period to include the formative period of the implementation of inflation targeting framework (ITF) in Indonesia, starting from the 1st quarter of 2001. Second, instead of M1, we measure the velocity gap using broad money (M2). The third robustness exercise involves the use of an alternative moment condition in the structural estimation:

$$0 = E_t \left[\left\{ \begin{array}{l} \hat{\pi}_t - \frac{\rho}{1+\beta\rho} \hat{\pi}_{t-1} - \frac{\beta}{1+\beta\rho} \hat{\pi}_{t+1} \\ - \frac{(1-\theta)(1-\theta\beta)}{\theta(1+\beta\rho)} \{ (\sigma + \eta) (\hat{y}_t - \hat{y}_t^f) \\ + \frac{V_0(k_1-1)}{(1+V_0)} (\hat{v}_t - \hat{v}_t^f) \} \end{array} \right\} \cdot Z_t \right]. \quad (7)$$

The results from these robustness exercises are reported in Table 3.

Under the long sample (2001.Q1-2022.Q1), we still find the impact of money velocity to be positive in all considered specifications. The estimates of the velocity coefficient λ_v range from 0.023 to 0.096. Except for the reduced-form estimate under IS 1, the estimates (v_0 in the structural estimation case) are statistically significant. When we use M2 instead of M1 to construct the velocity gap, the estimates of v_0 in the structural estimation case turns out to be negative, which is inconsistent with underlying theory. It remains the case, however, that the reduced-form estimates show positive and significant λ_v values. In fact, the impact of the velocity term appears to be larger, e.g. $\lambda_v = 0.42$ in the IS 1 case. Our structural estimation based on the alternative moment condition produces an implausibly-large value of v_0 irrespective of the instrument set used (the estimates, however, have very large standard errors). This large value appears to be caused by a large estimate of the Calvo parameter ($\theta \approx 1$) — hence, it appears that the moment condition (7) fails to identify this parameter. Interestingly, it is still the case that the implied λ_v values are still positive and in line with the values previously reported in Table 2.

Open-economy dimension Our last robustness exercise concerns an open-economy extension of the structural NKPC in (3). As shown in the appendix, allowing for import-goods

Table 3: Sensitivity analysis (Robustness)

Long sample (2001.Q1 – 2022.Q1)							
	Reduced-form			Structural			
	γ_f	λ_y	λ_v	θ	v_0	Implied λ_y	Implied λ_v
IS 1	0.935 (0.040)	0.085 (0.084)	0.023 (0.027)	0.630 (0.055)	0.018 (0.015)	0.440	0.044
IS 2	1.019 (0.033)	-0.169 (0.043)	0.096 (0.012)	0.609 (0.051)	0.026 (0.009)	0.507	0.076
Broad money (M2)							
	Reduced-form			Structural			
	γ_f	λ_y	λ_v	θ	v_0	Implied λ_y	Implied λ_v
IS 1	0.816 (0.078)	-0.740 (0.152)	0.424 (0.072)	0.404 (0.064)	-0.058 (0.013)	1.767	-0.581
IS 2	0.796 (0.026)	-0.246 (0.064)	0.141 (0.025)	0.494 (0.027)	-0.030 (0.007)	1.042	-0.180
Alternative moment condition							
	Reduced-form			Structural			
	γ_f	λ_y	λ_v	θ	v_0	Implied λ_y	Implied λ_v
IS 1	–	–	–	0.999 (3.323)	776.8 (4x10 ⁶)	8x10 ⁻⁶	0.037
IS 2	–	–	–	0.997 (2.901)	352.6 (4x10 ⁵)	5x10 ⁻⁵	0.107

Notes: (1) Numbers in parantheses are standard errors; (2) In the structural estimation, $\lambda_y = \frac{\kappa(\sigma+\eta)}{1+\beta\varrho}$, $\lambda_v = \frac{\kappa v_0(k_1-1)}{(1+\beta\varrho)}$, $\kappa \equiv (1-\theta)(1-\theta\beta)/\theta$, and in all cases, we set $\sigma = \eta = 1$, $k_1 = 12.3$, $\beta = 0.9942$, $\varrho = 0$; (3) the instrument set IS 1 includes four lags of inflation, four lags of real GDP gap, two lags of BI rate (7-day repo rate), and two lags of M1 velocity gap; (4) the instrument set IS 2 includes all the variables in IS 1 plus two additional lags of BI rate, two additional lags of M1 velocity gap, and four lags of commodity price inflation; (5) the alternative moment (orthogonality) condition is given in Eq. (7); (6) for M2 and alternative moment condition estimates, the sample period is 2005.Q3-2022.Q1.

Table 4: Open-economy NKPC with money velocity – Structural estimation

	θ	α	τ	v_0	Implied λ_v
IS 1	0.547 (0.037)	0.070 (0.034)	0.285 (0.371)	0.014 (0.006)	0.056
IS 2	0.513 (0.027)	0.135 (0.023)	0.291 (0.128)	0.009 (0.004)	0.042

Notes: (1) This table reports the GMM estimates of the structural parameters of the open-economy NKPC in Eq. (8); (2) numbers in parantheses are standard errors; (3) $\lambda_v = \frac{\kappa(1-\alpha)(k_1-1)v_0}{(1+\beta\rho)}$, $\kappa \equiv (1-\theta)(1-\theta\beta)/\theta$, and in all cases, we set $\beta = 0.9942$, $\rho = 0$, $\sigma = \eta = 1$, $k_1 = 12.3$; (4) the instrument set IS 1 includes four lags of inflation, four lags of real GDP gap, two lags of BI rate (7-day repo rate), and two lags of M1 velocity gap; (5) the instrument set IS 2 includes all the variables in IS 1 plus two additional lags of BI rate, two additional lags of M1 velocity gap, and four lags of commodity price inflation; (6) sample period: 2005.Q3-2022.Q1.

price inflation (besides the domestic-goods price inflation) to additionally affect aggregate inflation dynamics yields the following open-economy NKPC:

$$\begin{aligned}
 \hat{\pi}_t = & \frac{\rho}{1+\beta\rho}\hat{\pi}_{t-1} + \frac{\beta}{1+\beta\rho}E_t\hat{\pi}_{t+1} \\
 & + \frac{\kappa}{1+\beta\rho}[\sigma + (1-\alpha)\eta] \left(\hat{y}_t - \hat{y}_t^f \right) \\
 & + \frac{\kappa}{1+\beta\rho} \left[(1-\alpha)(k_1-1)\frac{V_0}{1+V_0} \right] \left(\hat{v}_t - \hat{v}_t^f \right) \\
 & + \frac{\kappa}{1+\beta\rho}\alpha \left[(1-\sigma\tau) \left(\hat{q}_t - \hat{q}_t^f \right) - \sigma\tau \left(\hat{S}_t - \hat{S}_t^f \right) - \sigma \left(\hat{y}_t^* - \hat{y}_t^{*,f} \right) \right].
 \end{aligned} \tag{8}$$

Aggregate inflation fluctuations in the domestic economy are therefore also driven by variations in the real exchange rate $\hat{q}_t - \hat{q}_t^f$, the terms of trade gap $\hat{S}_t - \hat{S}_t^f$, and the foreign output gap $\hat{y}_t^* - \hat{y}_t^{*,f}$. Here, the additional parameters $\alpha \in [0, 1)$ and $\tau \geq 0$ denote the share of imported-goods in the aggregate consumption basket (i.e. the degree of openness) and the elasticity of substitution between home- and foreign-produced goods, respectively. When $\alpha = 0$, the economy is a closed one and (8) becomes the closed-economy NKPC in (3).

Table 4 presents the structural parameter estimates, based on (8).¹¹ We focus on estimating the purely forward-looking version of (8) and set $\varrho = 0$. We also restrict $\beta = 0.9942$ and as in the closed-economy NKPC estimations, calibrate $\sigma = \eta = 1$. In terms of additional data series, we construct terms of trade data using the import price and export price index data from Bank Indonesia’s Indonesian Economic and Financial Statistics (SEKI) database. Foreign output gap data are proxied using the US economy’s CBO output gap series, sourced from the Federal Reserve Economic Data (FRED) database. For real exchange rate, we construct the (model-consistent) data series using the nominal exchange rate data from the IMF’s International Financial Statistics database, Indonesia’s CPI series from SEKI, and the US CPI series from FRED.¹²

As shown in the table, the estimates of v_0 are still statistically and economically significant, irrespective of the instrument sets (IS 1 and IS 2). Compared to the closed-economy estimations in Table (2), however, the influence of the velocity of money on inflation dynamics is somewhat smaller. Here, based on the IS 2 estimates, the implied reduced-form coefficient on money velocity is $\lambda_v = 0.042$, smaller than 0.06 in the corresponding closed-economy case in Table (2). The presence of imports-goods inflation— α is estimated to be non-zero—thus weakens, but does not eliminate the impact of variations in the money velocity on inflation dynamics.

4 Implications of a digital currency issuance on inflation dynamics and real fluctuations

Having established the econometric evidence that money velocity variations have statistically and economically significant effect on inflation dynamics, we now assess the likely implications on real aggregate fluctuations. To this end, we utilize a structural, small-scale New Keynesian model, along the line of the textbook model in Woodford (2003) or Galí (2015).

¹¹The reduced-form estimates point to a similar conclusion to the structural parameter estimates, i.e. the impact of money velocity on inflation is positive and non-trivial. Results are available upon request.

¹²The model-consistent real exchange rate is defined as $q_t = E_t P_t^* / P_t$, where E_t is the nominal exchange rate (foreign currency in terms of units of domestic currency), P_t^* is the foreign price level, and P_t is the domestic price level.

This standard model consists of three (log-linearized) equations, which jointly determine the fluctuations in the output gap $\hat{x}_t \equiv \hat{y}_t - \hat{y}_t^f$, inflation $\hat{\pi}_t$, and the nominal interest rate \hat{R}_t :

$$\begin{aligned}\hat{x}_t &= E_t \hat{x}_{t+1} - \frac{1}{\sigma} \left(\hat{R}_t - E_t \hat{\pi}_{t+1} \right) + \hat{\varepsilon}_{x,t}, \\ \hat{\pi}_t &= \beta E_t \hat{\pi}_{t+1} + \frac{(1-\theta)(1-\theta\beta)}{\theta} \left[(\sigma + \eta) \hat{x}_t + v_0(k_1 - 1) \left(\hat{v}_t - \hat{v}_t^f \right) \right] + \hat{\varepsilon}_{m,t}, \\ \hat{R}_t &= \rho_R \hat{R}_{t-1} + (1 - \rho_R) [\phi_\pi \hat{\pi}_t + \phi_x \hat{x}_t] + \hat{\varepsilon}_{r,t}.\end{aligned}$$

The first equation is a standard forward-looking IS curve, the second equation is a (forward-looking) Phillips curve, and the third equation is a standard, Taylor-type monetary policy rule. $\hat{\varepsilon}_{x,t}$ is a supply shock, which can be thought as a function of productivity or technology shock. $\hat{\varepsilon}_{m,t}$ is a cost-push or mark-up shock. $\hat{\varepsilon}_{r,t}$ is an exogenous, unsystematic monetary-policy shock. These exogenous shocks follow

$$\begin{aligned}\hat{\varepsilon}_{x,t} &= \rho_x \hat{\varepsilon}_{x,t-1} + \eta_{x,t}, \\ \hat{\varepsilon}_{m,t} &= \rho_m \hat{\varepsilon}_{m,t-1} + \eta_{m,t}, \\ \hat{\varepsilon}_{r,t} &= \rho_r \hat{\varepsilon}_{r,t-1} + \eta_{r,t},\end{aligned}$$

where $\eta_{x,t} \sim i.i.d.N(0, \sigma_x^2)$, $\eta_{m,t} \sim i.i.d.N(0, \sigma_m^2)$, $\eta_{r,t} \sim i.i.d.N(0, \sigma_r^2)$.

For the purpose of our simulation below, the money velocity gap $\left(\hat{v}_t - \hat{v}_t^f \right)$ is simply assumed to negatively correlate with the aggregate transaction cost $\hat{\varepsilon}_{v,t}$ in the following fashion:

$$\left(\hat{v}_t - \hat{v}_t^f \right) = -\psi \hat{\varepsilon}_{v,t}. \quad (9)$$

ψ is a scale parameter that governs the extent of the relationship between the transaction cost and the velocity of money. The transaction cost is assumed to be exogenous in our simulation and is assumed to followed an AR(1) process,

$$\hat{\varepsilon}_{v,t} = \rho_v \hat{\varepsilon}_{v,t-1} + \eta_{v,t}, \quad (10)$$

$\eta_{v,t} \sim i.i.d.N(0, \sigma_v^2)$.

We note that the simple, reduced-form relationship (9) is consistent with the underlying theory presented in Section 2 and elaborated in the appendix. That is, money, physical or digital, could be used to reduce the transaction cost involved in purchasing goods and services. The presence of a digital currency that is widely acceptable as a means of payments decreases the aggregate transaction cost, which in turn should decrease the aggregate velocity of money, all else equal.¹³ Based on this hypothesis, a positive $\eta_{v,t}$ shock in (10) can thus be thought as representing an issuance of a new digital currency e.g. a CBDC issuance by the central bank.¹⁴

Calibration of parameters We calibrate $\sigma = \eta = 1$ and $k_1 = 12.3$ as in our estimation exercise. For the Phillips curve parameters, we set $\beta = 0.9942$, $\theta = 0.592$, and $v_0 = 0.029$, based on the structural estimation in the previous section under restricted β with instrument set 1 (see Table 2). The Taylor-rule parameters are set to $\rho_R = 0.75$, $\phi_\pi = 1.5$, and $\phi_x = 0.5/4$. For the standard deviations of exogenous shocks, we set $\sigma_x = 0.25$, $\sigma_m = 1$, $\sigma_r = 0.25$. For the money velocity-related parameters, we set $\rho_z = 0.995$ so that we can treat a CBDC issuance as a (near) permanent technology shock, following the literature on trend inflation (see e.g. Cogley and Sargent (2005), Justiniano and Primiceri (2008), and Barnes et al. (2011)). These calibrations are ad-hoc, as matching features of the data is not our primary aim in the current exercise. Our purpose in this section is to show that the presence of digital currency could have non-trivial implications for aggregate nominal and real variables, through its influence on the aggregate money velocity.

Variance decompositions We first assess the importance of money-velocity shocks on the model variables by computing the unconditional variance decomposition. Table 5 reports the decompositions for output, output gap, inflation, and nominal interest rate, for various values of the standard deviation of money-velocity shock, σ_v . When $\sigma_v = 0$, the velocity of money

¹³An alternative hypothesis is that the presence of a digital currency *increases* the velocity of money (see e.g. Berentsen (1998)). This hypothesis, however, relies on the assumptions that the digital currency replaces central bank currency and reduces the monetary base, which need not be the case.

¹⁴While it is possible to endogenously model the effect of a CBDC issuance (Barrdear and Kumhof (2022), Davoodalhosseini (2022), Minesso, Mehl and Stracca (2022)), we leave such a modelling for future research. Here, we simply treat the CBDC as affecting (reducing) the aggregate transaction costs, which is consistent with the implications of the endogenous models in the aforementioned studies.

Table 5: Variance decompositions for different variances of money velocity shock

	Productivity	Cost- push	Monetary policy	Money velocity
No money-velocity shock ($\sigma_v = 0$)				
Output	72.95	20.01	7.04	0
Output gap	0.04	73.95	26.01	0
Inflation	0.47	73.53	26.00	0
Nominal int. rate	6.05	69.34	24.61	0
Low variance ($\sigma_v = 0.25$)				
Output	67.62	18.55	6.52	7.31
Output gap	0.03	57.27	20.14	22.56
Inflation	0.46	71.76	25.38	2.41
Nominal int. rate	5.20	59.57	21.14	14.09
Medium variance ($\sigma_v = 0.50$)				
Output	55.47	15.21	5.35	23.97
Output gap	0.02	34.15	12.01	53.81
Inflation	0.43	66.92	23.67	8.99
Nominal int. rate	3.66	41.87	14.86	39.61
High variance ($\sigma_v = 1$)				
Output	32.27	8.85	3.11	55.77
Output gap	0.01	13.06	4.59	82.33
Inflation	0.34	52.71	18.64	28.31
Nominal int. rate	1.67	19.14	6.79	72.40

Notes: (1) Entries above are unconditional variance decompositions (in %), based on the small-scale structural New Keynesian model; (2) the standard deviations of the others shocks (productivity, cost-push, and monetary-policy) are kept at $\sigma_x = 0.25$, $\sigma_m = 1$, and $\sigma_r = 0.25$, respectively.

is constant and the shock does not contribute to the variations of all the variables. Output fluctuations are largely driven by productivity and cost-push shocks. Monetary-policy and cost-push shocks are the dominant drivers of the fluctuations in inflation, the output gap, and the nominal interest rate. Despite our ad-hoc calibration, these decompositions are not inconsistent with those produced by a larger-scale, estimated structural model for Indonesia such as in Lie (2019) and Juhro, Lie and Sasongko (2022).¹⁵

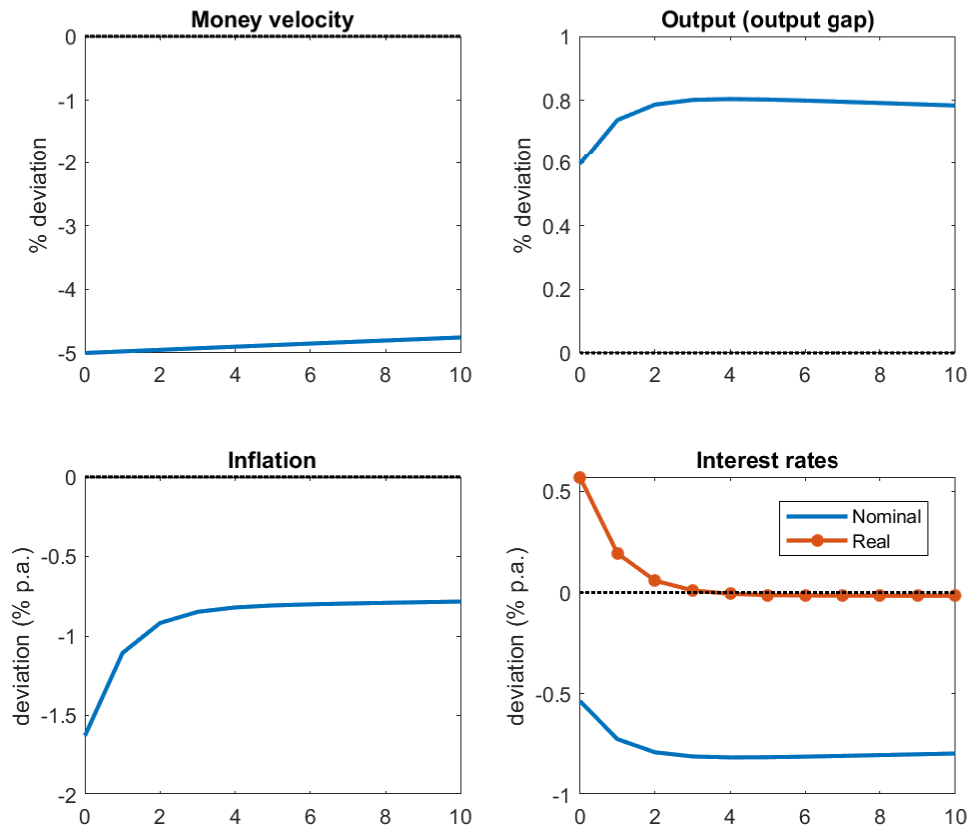
When $\sigma_v > 0$, we find that money-velocity shocks could be an important driver for aggregate nominal and real fluctuations. Here, when $\sigma_v = 0.25$ (the "low variance" case) and is equal to σ_x and σ_r , these shocks contribute to 2.4% of inflation variations and 7.3% of output variations. The impact on output gap and nominal interest rate variations are even larger, at 22.6% and 14.1%, respectively. As expected, the contribution of money-velocity shocks are even larger when σ_v is higher. In the "high variance" case ($\sigma_v = 1$), these shocks are now responsible for 28.3% of inflation variations and 82.3% of output gap variations. The contributions of the other three shocks decline as σ_v gets higher. Our finding in Table 5 has various policy implications. If, for example, the central bank uses the above (or similar) model to forecast inflation and the output gap, ignoring variations in the money-velocity gap would affect the accuracy of the forecasts. The degree of the inaccuracy might be non-trivial, even in the conservative low-variance case.

Impact of a CBDC issuance Next, we use our estimates and the model to assess the impact of a CBDC issuance by the central bank, which we modelled as a 5% near-permanent decrease in the velocity of money.¹⁶ The impulse responses are plotted in Figure 2. Qualitatively, this CBDC shock, which affects the aggregate fluctuations by permanently lowering the transaction costs and the velocity of money, has a similar implication to that arising from a permanent increase in technological progress or innovation. Quantitatively, our calibration implies that that such a CBDC issuance would decrease the long-run inflation rate by around

¹⁵Lie (2019), for example, finds that technology and monetary-policy shocks are the main drivers for output and inflation fluctuation in Indonesia.

¹⁶As CBDC is a relatively new subject (see Boar, Holden and Wadsworth (2020) for a recent survey on CBDCs issuance by central banks around the world), we have no prior empirical material regarding the impact of a CBDC issuance on money velocity to draw upon. The 5% aggregate money velocity decrease is simply a conservative estimate of the impact.

Figure 2: Impulse responses to a 5% decrease in money velocity (CBDC shock)



Notes: (1) The figure plots the impulse response to a CBDC issuance shock, modeled as a 5% near-permanent decrease in the velocity of money; (2) responses are based on the small-scale structural New Keynesian model.

0.8% per annum. The level of output would have permanently increased by 0.8%. Lower long-run inflation means that the nominal interest rate would be lower (by 0.8% as well). The model also predicts a permanently lower real interest rate, though the effect is minimal.

We note that despite our ad-hoc modelling of the impact of a CBDC issuance, the aggregate impact in Figure 2 is consistent with that arising from more-rigorous models (Barrdear and Kumhof (2022), Minesso, Mehl and Stracca (2022)). Barrdear and Kumhof (2022) for example find that a CBDC issuance is associated with a permanent decrease in transaction costs, which in turn would permanently raise GDP and lower the real interest rates.¹⁷

5 Conclusion

Do variations in the velocity of money have a non-trivial impact on inflation fluctuations? We answer this question empirically within a structural New Keynesian Phillips curve (NKPC) with an explicit money velocity term. This money velocity effect arises from the role of money, both in physical and digital forms, in reducing the aggregate transaction costs and facilitating purchases of goods and services. We find a significant aggregate impact in the context of the Indonesian economy: our benchmark estimates suggest that a 10% decrease in money velocity, which might be facilitated by a new digital currency (e.g. CBDC) issuance, would reduce the inflation rate by 0.6-1.7%, all else equal. Using the estimates and within a small-scale New Keynesian DSGE model, we analyze the potential implications of a CBDC issuance on aggregate nominal and real fluctuations. We show that a CBDC issuance would permanently increase output and lower inflation and the nominal and real interest rates. Shocks to the velocity of money are an important driver of aggregate fluctuations.

Our finding of a significant effect of money velocity on inflation dynamics implies that when a structural model is used in central banks' policy projections and forecasts, the Phillips curve relationship should include a money velocity term or its proxy. This is especially true in an economic environment where cash transactions are still commonplace such as in Indonesia. Ignoring the variations in money velocity might result in inaccurate policy prescriptions

¹⁷Specifically, Barrdear and Kumhof (2022) find that a CBDC issuance equivalent in value to 30% of GDP would permanently raise the level of GDP by 3%.

and forecasts. Our finding regarding the potential impact of a CBDC issuance should be taken with a degree of caution, given the use of a small-scale model and the parsimonious CBDC (digital currency) modelling. Despite this, as long as the digital currency is widely used as a means of payment in the marketplace (and hence, leads to a reduction in the aggregate transaction costs), our predictions as to the overall, qualitative impact are likely to hold. Under this scenario, a central bank with a CBDC could potentially use it as an additional stabilization policy tool by influencing the velocity. Future research should utilize a larger-scale model to more accurately quantify the impact of a CBDC issuance on aggregate fluctuations (through changes in the velocity of money) and on the conduct of monetary policy.

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Appendix

In this appendix we derive the New Keynesian Phillips curve (NKPC) with money velocity used in our estimation. Our approach closely follows the transaction cost assumption in Kim and Subramanian (2006) and Kim and Subramanian (2009). Here, households face a transaction cost when purchasing goods. Money, however, can be used to facilitate transactions and reduce the transaction costs. Under this scenario, an increase in the use of money would decrease the velocity of money, *ceteris paribus*. Hence, the aggregate transaction cost is a positive function of the money velocity. While money can be used to facilitate transactions and reduce the transaction cost, there is an opportunity cost of holding and using money, however, in the form of foregone interest rate from bondholding.

Households' problem

In each period the representative household chooses the amounts of consumption c_t , labor N_t , nominal one-period bond holding B_t , and nominal money holding M_t to maximize the lifetime utility function

$$U_0 = E_0 \sum_{t=0}^{\infty} \beta^t \left[\frac{c_t^{1-\sigma}}{1-\sigma} - \chi \frac{N_t^{1+\eta}}{1+\eta} \right],$$

subject to the nominal budget constraint

$$P_t c_t + M_t + B_t + P_t \tau_t \leq W_t N_t + M_{t-1} + (1 + i_{t-1}) B_{t-1} + \Pi_t^n + TR_t^n. \quad (\text{A.1})$$

Here, W_t is the nominal wage, i_t is the (net) nominal interest rate on the bond, Π_t^n is the dividend received by households from their ownerships of firms, TR_t^n is the nominal tax or transfer from the government, and P_t is the aggregate price level. β is the subjective discount factor, $\sigma > 0$ is the inverse of elasticity of intertemporal substitution, $\eta > 0$ is the inverse Frisch labor supply elasticity, and $\chi > 0$ is a labor scale parameter. τ_t is the associated (real) transaction cost incurred by households whenever they use credit (non-cash) when purchasing goods. This cost τ_t is defined as

$$\tau_t = e^{\varepsilon_t} c_t k_0 v_t^{k_1 - 1},$$

where $v_t \equiv \frac{P_t c_t}{M_t}$ is the velocity of money, ε_t is an exogenous money demand shock, $k_0 \geq 0$ is a scale parameter, and $k_1 > 1$ is a parameter that affects the curvature of the implied money demand function (together with k_0).

The nominal budget constraint (A.1) can be converted into a real one by dividing both sides by P_t :

$$c_t + m_t + b_t + e^{\varepsilon_t} c_t k_0 v_t^{k_1 - 1} \leq w_t N_t + \frac{m_{t-1}}{\pi_t} + (1 + i_{t-1}) \frac{b_{t-1}}{\pi_t} + \Pi_t + TR_t, \quad (\text{A.2})$$

where $m_t \equiv M_t/P_t$, $b_t \equiv B_t/P_t$, $w_t \equiv W_t/P_t$, $\Pi_t \equiv \Pi_t^n/P_t$, $TR_t \equiv TR_t^n/P_t$, and $\pi_t \equiv P_t/P_{t-1}$ denote real money holding, real bond holding, real wage, real dividend, real transfer, and gross price inflation, respectively. Denoting λ_t as the Lagrange multiplier, the first-order conditions (FOCs) of households' problem with respect to c_t , N_t , b_t , and m_t are given by

$$0 = c_t^{-\sigma} - \lambda_t (1 + e^{\varepsilon_t} k_0 k_1 v_t^{k_1 - 1}), \quad (\text{A.3})$$

$$0 = -\chi N_t^\eta + \lambda_t w_t, \quad (\text{A.4})$$

$$0 = \beta E_t \lambda_{t+1} \frac{(1 + i_t)}{\pi_{t+1}} - \lambda_t, \quad (\text{A.5})$$

$$0 = \beta E_t \lambda_{t+1} \frac{1}{\pi_{t+1}} - \lambda_t + \lambda_t e^{\varepsilon_t} k_0 (k_1 - 1) v_t^{k_1}. \quad (\text{A.6})$$

Money demand function

The money demand function is implicit in equation (A.6). Combining (A.5) and (A.6) yields

$$e^{\varepsilon_t} k_0 (k_1 - 1) v_t^{k_1} = 1 - R_t^{-1}, \quad (\text{A.7})$$

where $R_t = 1 + i_t$ is the gross nominal interest rate. Rearranging the above, we obtain

$$q_t^{-k_1} = \frac{1}{e^{\varepsilon_t} k_0 (k_1 - 1)} \left(\frac{i_t}{1 + i_t} \right), \quad (\text{A.8})$$

where

$$q_t \equiv v_t^{-1} \equiv \frac{M_t}{P_t c_t}$$

is the inverse money velocity. (A.8) is the associated money demand function, which could also be written in a log form,

$$\log(q_t) = \gamma_0 + \gamma_1 \log\left(\frac{i_t}{1+i_t}\right) + \eta_t, \quad (\text{A.9})$$

where

$$\begin{aligned} \gamma_0 &\equiv \frac{1}{k_1} \log(k_0(k_1 - 1)), \\ \gamma_1 &\equiv -\frac{1}{k_1}, \\ \eta_t &\equiv \frac{1}{k_1} \varepsilon_t. \end{aligned}$$

Given q_t ($\frac{M_t}{P_t c_t}$) and i_t data (see the main text), we estimate (A.9) and obtain the interest elasticity of money demand $\frac{1}{k_1}$. We use the Newey-West OLS method to estimate the relationship (A.9). For our benchmark estimation using the data from 2005.Q3-2022.Q1, we obtain the point estimate of $k_1 = 12.3$.

The log-linearized IS curve

We next linearize the first-order conditions (A.3)-(A.6) and derive the IS curve. Combining the resulting linearized equations yields the IS curve equation,

$$\begin{aligned} \sigma(E_t \hat{c}_{t+1} - \hat{c}_t) &= \hat{R}_t - E_t \hat{\pi}_{t+1} \\ &+ \frac{V_0(1-k_1)}{1+V_0} (E_t \hat{v}_{t+1} - \hat{v}_t) \\ &- \frac{V_0}{1+V_0} (E_t \hat{\varepsilon}_{t+1} - \hat{\varepsilon}_t), \end{aligned} \quad (\text{A.10})$$

where $V_0 \equiv e^\varepsilon k_0 k_1 v^{k_1-1} = k_0 k_1 v^{k_1-1}$, with v denotes the steady-state velocity of money. If there is no transaction cost, $V_0 = 0$ and all goods are purchased using credit — here, we have the standard, cashless-model IS curve equation (see Woodford (2003)), i.e. the last two terms in the RHS of (A.10) are absent. All the hatted variables are in terms of log deviations from the steady state values, except for $\hat{\varepsilon}_t \equiv \varepsilon_t - \varepsilon$, which is in level deviation.

Based on the FOCs, we also have the following log-linearized equations for real wage and

money velocity:

$$\begin{aligned}\hat{w}_t &= \sigma \hat{c}_t + \eta \hat{N}_t \\ &\quad + \frac{V_0(k_1 - 1)}{1 + V_0} \hat{v}_t + \frac{V_0}{1 + V_0} \hat{\varepsilon}_t,\end{aligned}\tag{A.11}$$

$$\hat{v}_t = \left[\frac{1}{k_1(R - 1)} \right] \hat{R}_t - \left[\frac{1}{k_1} \right] \hat{\varepsilon}_t.\tag{A.12}$$

Firms' problem and the NKPC with money velocity

Next, we derive the New Keynesian Phillips curve (NKPC), based on firms' optimal pricing problem. Monopolistically-competitive goods-producing firms (or retailers) problem is standard. We use a Calvo price assumption with a backward-looking indexation following the setup in Christiano, Eichenbaum and Evans (2005). Under this standard setup, we have the following hybrid NKPC (see e.g. Christiano, Eichenbaum and Evans (2005) or Barnes et al. (2011) for the derivation):

$$\hat{\pi}_t = \frac{\varrho}{1 + \beta\varrho} \hat{\pi}_{t-1} + \frac{\beta}{1 + \beta\varrho} E_t \hat{\pi}_{t+1} + \frac{\kappa}{1 + \beta\varrho} \widehat{mc}_t,\tag{A.13}$$

where \widehat{mc}_t is the real marginal cost and $\kappa \equiv (1 - \theta)(1 - \theta\beta)/\theta$. θ is the probability of *non-optimal* price adjustment. The parameter $\varrho \in [0, 1]$ denotes the degree of price indexation to past inflation. Given production function $y_t = A_t N_t$ (A_t is the level of technology), the real marginal cost is given by $mc_t = w_t/A_t$, or in log-linearized form, $\widehat{mc}_t = \hat{w}_t - \hat{A}_t$. Combining this with (A.11), $\hat{N}_t = \hat{y}_t - \hat{A}_t$ (from the production function), and $\hat{c}_t = \hat{y}_t$, we obtain

$$\widehat{mc}_t = (\sigma + \eta) \hat{y}_t - (1 + \eta) \hat{A}_t + \frac{V_0(k_1 - 1)}{1 + V_0} \hat{v}_t + \frac{V_0}{1 + V_0} \hat{\varepsilon}_t.\tag{A.14}$$

At the natural (flexible-price) equilibrium, the real marginal cost is constant, and hence, the natural output level \hat{y}_t^f is implicit in

$$0 = (\sigma + \eta) \hat{y}_t^f - (1 + \eta) \hat{A}_t + \frac{V_0(k_1 - 1)}{1 + V_0} \hat{v}_t^f + \frac{V_0}{1 + V_0} \hat{\varepsilon}_t,\tag{A.15}$$

obtained from applying the expression (A.14) under the flexible-price equilibrium. Here, \hat{v}_t^f is the natural velocity of money, i.e. the velocity under the flexible-price equilibrium.

Finally, combining equations (A.13), (A.14) and (A.15), we obtain the NKPC expression in terms of output gap $\hat{y}_t - \hat{y}_t^f$ and money velocity gap $\hat{v}_t - \hat{v}_t^f$,

$$\begin{aligned} \hat{\pi}_t &= \frac{\varrho}{1 + \beta\varrho} \hat{\pi}_{t-1} + \frac{\beta}{1 + \beta\varrho} E_t \hat{\pi}_{t+1} \\ &+ \left[\frac{\kappa(\sigma + \eta)}{1 + \beta\varrho} \right] (\hat{y}_t - \hat{y}_t^f) \\ &+ \left[\frac{\kappa V_0(k_1 - 1)}{(1 + \beta\varrho)(1 + V_0)} \right] (\hat{v}_t - \hat{v}_t^f). \end{aligned} \quad (\text{A.16})$$

Equation (A.16) is our benchmark, closed-economy NKPC to be estimated.¹⁸

Extension with an open-economy dimension

In this extension, we include an open-economy dimension to the NKPC (A.16). For this purpose, we adopt the standard New-Keynesian small open-economy model of Galí (2015), used e.g. in Lubik and Schorfheide (2007), Justiniano and Preston (2010), Jääskelä and Nimark (2011), and Lie (2019). We further adopt the simplifying approach in Kuttner and Robinson (2010) and Lie and Yadav (2017) by assuming that the domestic goods and the imported goods sectors have the same Calvo price-stickiness and indexation parameters. This assumption results in a single-equation Phillips curve (instead of two equations, each involving domestic inflation and import-good inflation). Following the same derivation procedure as in the closed-economy model above, the resulting NKPC as a function of domestic-goods real marginal cost \widehat{mc}_t^d and the import-goods real marginal cost \widehat{mc}_t^m is

$$\hat{\pi}_t = \frac{\varrho}{1 + \beta\varrho} \hat{\pi}_{t-1} + \frac{\beta}{1 + \beta\varrho} E_t \hat{\pi}_{t+1} + \frac{\kappa}{1 + \beta\varrho} \left((1 - \alpha) \widehat{mc}_t^d + \alpha \widehat{mc}_t^m \right). \quad (\text{A.17})$$

Here, $\alpha \in [0, 1)$ is the share of imported-goods in the aggregate consumption basket. When $\alpha = 0$, (A.17) becomes (A.13) and the economy is a closed economy.

¹⁸Instead of a function $\hat{v}_t - \hat{v}_t^f$, one could represent the NKPC as a function of the nominal interest \hat{R}_t , a byproduct of the money demand equation in (A.9). This positive relationship between the nominal interest rate and money velocity is highlighted e.g. in Hromcová (2003).

As in the closed-economy version we can write NKPC (A.17) in terms of the output gap and the money velocity gap. The equivalent expression to (A.15) for \widehat{mc}_t^d in an open-economy setting is

$$\begin{aligned} \widehat{mc}_t^d = & \left(\frac{\sigma}{1-\alpha} + \eta \right) \hat{y}_t - (1+\eta)\hat{A}_t + \frac{V_0(k_1-1)}{1+V_0}\hat{v}_t + \frac{V_0}{1+V_0}\hat{\varepsilon}_t \\ & + \alpha\hat{S}_t - \frac{\sigma\alpha}{1-\alpha} \left[\tau(2-\alpha)\hat{S}_t + \tau\hat{\Psi}_{F,t} + \hat{y}_t^* \right], \end{aligned} \quad (\text{A.18})$$

where \hat{S}_t , $\hat{\Psi}_{F,t}$, and \hat{y}_t^* denote the terms of trade (ratio of import to export prices), the law-of-one price (LOP) gap, and foreign output, respectively. The parameter τ is the elasticity of substitution between home- and foreign-produced goods. The first term in the second row of (A.18) relates to the expenditure switching effect: as the relative price of import to export (\hat{S}_t) increases there is an expenditure switching towards domestically-produced goods, leading to an increase in their marginal cost. The second term, $\tau(2-\alpha)\hat{S}_t + \tau\hat{\Psi}_{F,t} + \hat{y}_t^*$, relates to the net export demand, which is a positive function of the terms of trade, the law-of-one price gap, and foreign output. The presence of this term in (A.18) should be viewed as a net export adjustment on domestic consumption, i.e. as net export goes up, domestic consumption should decrease, which in turn *lowers* the domestic-goods real marginal cost, *ceteris paribus*. It is still the case, however, that a higher export demand would overall increase the domestic-goods marginal cost — this positive relationship is captured through the positive effect of \hat{y}_t on \widehat{mc}_t^d in (A.18).

Combining (A.17) and (A.18) and its flexible-price equilibrium expression yields an open-economy NKPC,

$$\begin{aligned} \hat{\pi}_t = & \frac{\varrho}{1+\beta\varrho}\hat{\pi}_{t-1} + \frac{\beta}{1+\beta\varrho}E_t\hat{\pi}_{t+1} \\ & + \frac{\kappa}{1+\beta\varrho} [\sigma + (1-\alpha)\eta] (\hat{y}_t - \hat{y}_t^f) \\ & + \frac{\kappa}{1+\beta\varrho} \left[(1-\alpha)(k_1-1)\frac{V_0}{1+V_0} \right] (\hat{v}_t - \hat{v}_t^f) \\ & + \frac{\kappa}{1+\beta\varrho}\alpha \left[(1-\sigma\tau) (\hat{q}_t - \hat{q}_t^f) - \sigma\tau (\hat{S}_t - \hat{S}_t^f) - \sigma (\hat{y}_t^* - \hat{y}_t^{*,f}) \right], \end{aligned} \quad (\text{A.19})$$

where $(\hat{q}_t - \hat{q}_t^f)$, $(\hat{S}_t - \hat{S}_t^f)$, and $(\hat{y}_t^* - \hat{y}_t^{*,f})$ denote the real exchange rate, terms of trade

gap, and the foreign output gap, respectively. To arrive at (A.19), we utilize the facts that $\hat{q}_t = \hat{\Psi}_{F,t} + (1 - \alpha)\hat{S}_t$ and $\widehat{mc}_t^m = \hat{\Psi}_{F,t}$. The first fact holds directly from the log-linearized aggregate price level equation under a CES aggregation, while the second one holds since the LOP gap is essentially the imported-goods aggregate real marginal cost in the model.¹⁹ When $\alpha = 0$, (A.19) becomes the closed-economy NKPC in (A.16).

¹⁹See e.g. the model appendix (Appendix A) of Lie (2019).