Two's not company: mis-aggregation and "supply-induced" unemployment increases

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Mis-aggregation and "Supply-Induced" Unemployment Increases

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Abstract

The seminal Blanchard and Quah (1989) study finds that a (positive) "supply" shock increases the unemployment rate. The ensuing debate between leading schools of macroeconomic thought has been lively. But should we use this model to study macroeconomic fluctuations? Faust and Leeper (1997) warn that it is not reliable for structural inference, nor is the related Bayoumi and Eichengreen (1993) model. In this paper, I revisit the disputed unemployment rate response from the viewpoint of IRF's bias. Using a tri-variate model encompassing these models and novel methodology to parse IRF's bias, I find that "supply-induced" unemployment rate increases can be explained by mis-aggregation of technology and labor-supply shocks.

JEL Classification Numbers: C32, C52, E32, E5

Keywords: Vector autoregression, Permanent and temporary shocks to output, Missing-variables, Fundamentalness, Mis-aggregated shocks, Co-mingled shocks, Impulse response function bias, Moving-average representation, Aggregate demand and supply shocks, Long-run neutrality, Labor-supply shocks, Contractionary technology shocks

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I. INTRODUCTION

The seminal Blanchard and Quah (1989) model (henceforth, BQ) engenders a vast literature.[2] But almost from the outset, controversy surrounded one of its findings: the unemployment rate, ur_t, increases following a "supply" shock. BQ explain that "…*in response to a positive supply shock, say an increase in productivity, aggregate demand does not initially increase enough to match the increase in output needed to maintain constant unemployment; real wage rigidities can explain why increases in productivity can lead to a decline"* over time (BQ, page 663).[3] "Supply-induced" ur_t increases are also reported for a number of G7 countries in Keating and Nye (1999) and using more recent U.S. data in Burnichon (2010).[4]

Two separate branches of literature stem from "supply-induced" ur_t increases. One branch, following Bayoumi and Eichengreen (1993) (henceforth, BE), "corrects" the BQ model by pairing output growth, Δy_t, with inflation, Δp_t, instead of ur_t.[5] Keating and Nye (1998) claim

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[2] This includes tri-variate models exploring differences in macroeconomic fluctuations under the Gold Standard (see Bordo et al, 2010) and larger models for the U.S. economy (Gali, 1992 and 1999). King et al. (1991) and Pagan and Pesaran (2008) discuss the implications of cointegration in identifying temporary and permanent structural shocks. In addition, the long-run recursive identification strategy was extended to small-open economies by Lastrapes (1992) and Hoffmaister and Roldós (2001). For a comprehensive review the literature and a discussion of the limitations of long-run restrictions in identifying structural shocks see Kilian and Lutkepohl (2017, pp. 278–96).

[3] Blanchard (1989) also reports "supply-induced" *ur* increases even when long-run restrictions are eschewed.

[4] While that study is closely related to BQ, it differs as it uses an HP-filter to extract the "cyclical" component of both series and, following Shapiro and Watson (1988), uses IV estimation to recover technology and aggregate demand shocks in a model estimated with four (not eight) lags. The shorter lag length could explain that, while Burnichon (2010) replicates the supply-induced ur_t increases in the short-run, it does not show subsequent declines in ur_t (*ibid*, page 1017, Figure 2). Indeed, those IRF's align with those of the BQ variable pairing when the model is estimated with (shorter) optimal lags (see IRF's depicted in green, Figure A1, Appendix II.C).

[5] BE considers whether European countries (and U.S. regions) constitute an optimal currency area (OCA). For cross-country comparisons, that study stresses the need to avoid the implicit assumption of a common size (normalization) in the identified structural shocks and use the reduced-from residuals' correlation matrix to recover structural shocks. For a recent discussion of the implications of such approach see Binet and Pentecote (2015). In addition, the BE model is used to explore whether: (1) macroeconomic fluctuations in G7 countries changed after the Bretton

the purported superiority of this pairing arguing that: "*…the use of unemployment rate makes it difficult to find impulse responses to permanent output shocks that are ~~in~~consistent with the effects of aggregate supply shocks.*" (strike-out added, ibid, page 234).[6] That study stresses that, in principle, technology and labor-supply shocks can have opposing effects on ur_t —the former decreasing ur_t over time and the latter increasing ur_t on impact—thus biasing impulse response functions (IRF's). Further, Keating and Nye (1998) argue that the BE model sidesteps this problem as both supply and labor-supply shocks reduce p_t .[7, 8]

The other branch, in contrast, embraces the BQ variable pairing and, in doing so, begets its own controversy. Specifically, a growing literature dating back to Gali (1999) notes that ur_t can increase following technological advances because, from a new-Keynesian view, technology shocks may decrease employment and hours worked. That study pairs Δy_t (per capita) with the *change* in hours worked and finds that hours decline following a positive supply shock.[9]

Woods System collapsed (Bayoumi and Eichegreen, 1994); (2) pre-World War I and post-World War II fluctuations differ in G7 countries (plus Denmark, Norway, and Sweden) (Keating and Nye, 1998); and, more recently, whether (3) earlier OCA evidence has changed (Campos and Machiarello, 2016, and Bayoumi and Eichengreen, 2020).

[6] The sentence comes at the end of a paragraph that makes clear that the authors meant to say "*…that are **consistent** with the effects of aggregate supply shocks*."

[7] This reasoning focuses on the impact effect and does not recognize the potential that, in the presence of wage rigidities, supply shocks can have opposing effects on prices over time. It also fails to account for the role of labor market information in the DGP for the U.S. post-World War II period. These issues are evident in the missing-variable tests and the augmented Keynesian framework discussed below.

[8] Keating (2013) notes two additional strengths of the BE variable pairing. First, it has a natural advantage because demand and supply models are formulated in terms of output and prices. But FL (page 350) note that the BE variable pairing needs to contend with potentially large price movements (such as those associated with the Korea War in 1950's), while the BQ variable pairing does not. The second strength is that it is ideal for "historical" studies given of the lack of robust unemployment data for earlier time periods. "*The unemployment data, even when collected, did not take on special significance until the Great Depression and the ascendance of the Keynesian theory of economic policy*." (Keating and Nye, 1999, page 266).

[9] Gali (1999) finds these results are robust to the inclusion of four (unidentified) temporary shocks.

Christiano, Eichenbaun, and Vigfusson (2003) argue however that hours worked do not contain a unit root. That study finds an "expansionary-employment" effect when Δy_t is paired with (log-levels of) hours and highlights that this result is consistent with an alternative view of the economy: the real-business cycle model of Kydland and Prescott (1982).

A lively scholarly debate ensues from these divergent views, fueled by contradictory empirical evidence often reflecting the time-series properties of labor market variables (see Kilian and Lutkepohl, 2017, pp. 590–608). The debate has also considered alternative empirical strategies and massive data efforts, notably Basu, Fernald, and Kimball (2006) (henceforth BFK). That study, using a modified growth-accounting framework to refine the measurement of the Solow residual, finds that directly measured "pure" technology shocks, Δbfk_t , are contractionary (on inputs).[10] But Christiano, et al. (2004) provide evidence against two key assumptions made in that study, namely the difference stationarity of hours and the exogeneity of technology shocks. Relaxing these assumptions reverses the effect of technology on hours.

But should we trust the BQ and BE bivariate models to study macroeconomic fluctuations in the first place? Faust and Leaper (1997) (henceforth, FL) warn that stringent conditions are needed for multiple permanent and temporary shocks to be correctly sorted in these models.[11] Indeed, FL uncover a disturbing fact about these bivariate models: permanent shocks from one and temporary shocks from the other are more correlated (0.36 and 0.34) than permanent shocks across models (0.30) (Table 1)! FL interpret this "cross-type" correlation as evidence of mis-

[10] In stripping out cyclical effects from the Solow residual, BFK account for time-varying utilization of capital and labor, imperfect competition and returns to scale, and sectoral aggregation effects.

[11] In fairness, BQ acknowledge the potential for "co-mingling" of temporary and permanent shocks and discuss the conditions for multiple shocks to be sorted correctly.

aggregated shocks: "…*we see no strong a priori or empirical grounds for selecting between the two models and conclude that neither provides a reliable basis for structural inference*" (FL, page 351).

Table 1. Correlation of Structural Shocks

	Blanchard-Quah	
	Permanent	Temporary
Bayoumi-Eichengreen:		
Permanent	0.30	0.34
Temporary	0.36	0.63

Note. These calculations replicate Table 2 in Faust and Leeper (1997). Structural shocks are identified using the long-run restrictions in Blanchard and Quah (1989) and Bayoumi and Eichengreen (1993). VAR models are estimated with eight lags using quarterly data from 1950:Q2 to 1987:Q4.

In this paper, I re-visit the reliability of the BQ and BE models and examine "supply-induced" ur_t increases from the point of view of mis-aggregation of structural shocks. Specifically, by taking advantage of advances in the understanding of, and testing for, missing-variables and fundamentalness, I trace the puzzling ur_t response back to FL's criticism and find evidence of a missing third variable and non-fundamentalness in these bivariate models.[12] I form a tri-variate model encompassing the BQ and BE models and, using both long-run and short-run restrictions, identify labor-supply shocks as well as technology and aggregate demand shocks;[13] the corresponding IRF's conform to their structural interpretation.

[12] These results are robust to alternative deterministic specifications, optimally selected lag length, and extending the sample period to include the 1990's low-inflation period and the 2007–08 financial crisis.

[13] Blanchard (1989) argues that the variation in a composite "supply shock" is mostly due to technology not labor-supply and dealing "*separately with labor-supply and productivity disturbances…is beyond the scope of this paper*." (ibid., page 1149). More recently, Brinca et al. (2021) suggests labor-supply shocks explain a large share of the variation in hours, particularly during the COVID-19 outbreak; Fernald and Li (2021) argue otherwise.

Using the tri-variate model as my benchmark, I document that BQ model's IRF's biases are small and the propagation of its shocks mimics those of the tri-variate model, except for the disputed "supply-induced" ur_t increases. In this case, the magnitude of the bias is such that it reverses the sign of the short-run impact of technology. For the BE model, I find that the IRF's biases are large for supply shocks but small for demand shocks.

I also find that the correlation between the tri-variate and the bivariate models' structural shocks are suggestive of mis-aggregation (Table 2). Specifically, permanent shocks from the BQ model are correlated with both the tri-variate model's technology (0.48) and labor-supply (0.83) shocks. For the BE model, I find that its permanent and temporary shocks are highly correlated with their tri-variate counterparts but nonetheless exhibit cross-type correlation.[14]

Table 2. Correlation of Structural Shocks including Labor Supply Shocks

	Tri-variate model		
	Permanent	Labor supply	Temporary
Blanchard-Quah:			
Permanent	0.48	0.83	0.20
Temporary	0.63	-0.50	0.46
Bayoumi-Eichengreen:			
Permanent	0.81	0.00	-0.34
Temporary	0.33	0.00	0.92

Note. For the tri-variate model, structural shocks are identified using the long- and short-run restrictions discussed in the main text; structural shocks for the Blanchard-Quah and Bayoumi-Eichengreen models are identified as in Table 1. All VAR models are estimated using eight lags using quarterly data from 1950:Q2 to 1987:Q4.

To understand these IRF's biases and patterns of structural-shock correlation, I develop a novel methodology to parse the sources of IRF's bias. With it, I find distinct afflictions in the BQ and

[14] Note that the BE structural shocks are "empirically orthogonal" to labor-supply shocks (zero up to six decimals). This reflects the fact that BE shocks are highly correlated with the tri-variate model's permanent and temporary shocks that are, by construction, orthogonal to labor-supply shocks.

the BE variable pairings: the latter endures missing-variable bias and the former exhibits mis-aggregated (technology and labor-supply) shocks. Moreover, mis-aggregation can explain "supply-induced" ur_t increases (see Figure 1). Combining the tri-variate model's IRF's for technology and labor-supply shocks (top panel), I can obtain a joint "technology-cum-labor-supply" shock that reproduces BQ's "puzzling" supply-induced ur_t response (bottom panel).[15] This result holds regardless of whether ur_t has a unit root or not.

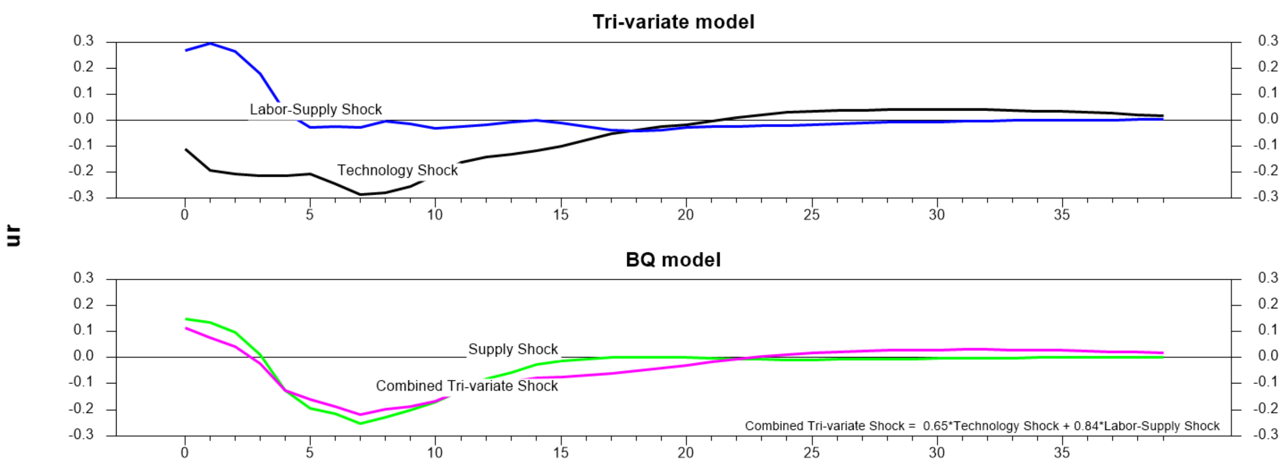

Figure 1. IRF's for the Tri-variate and BQ models

Given the new-Keynesian versus real business cycle debate noted, this paper also examines the relation of these VAR-based structural shocks and Δbfk_t. Two stylized facts emerge. First, demand shocks are not correlated to Δbfk_t suggesting an agreement between the methodologies in separating out shocks with permanent effects on output, as reported by Gali and

[15] The combined shock is the sum of 0.65 times technology and 0.84 times labor-supply shocks. These "weights" are the OLS estimates from a regression of the BQ model's IRF's for ur_t following a "supply" shock on the tri-variate model's IRF's for ur_t following technology and labor-supply shocks. The coefficient of determination of the regression is 0.88.

Rabanal (2004). And second, Δbfk_t is most correlated to the tri-variate model's labor-supply shocks, which is consistent with Christiano, et al. (2004).

The main contributions of this paper are twofold. First, I propose a novel method to parse the IRF's bias into mis-aggregated shocks and MA-representation distortion. These distortions are related to non-fundamentalness and missing variables discussed in Lutkepohl (1991), Giannone and Reichlin (2006), Forni and Gambetti (2014), and Canova and Hamidi Sahneh (2018). This parsing methodology can gauge the effects of these pathologies on the propagation of shocks at different time horizons and thereby complement the measurement of the "severity" of non-fundamentalness in Beaudry, Feve, Guay, and Portier (2019).[16]

Second by decomposing the IRF's bias in the BQ and BE models, I provide new insights into "supply-induced" ur_t increases (independent of a stochastic trend in ur_t). For the BQ model, the overall bias reflects the effect of comingled *supply* shocks (with opposing effects on the pairing variable); this bias follows directly from that predicted by FL's *Proposition 2 part 2* (FL, page 349). As noted, a combined technology and labor-supply shock can reproduce BQ's "supply-induced" ur_t increases. Further supportive evidence follows from the variance decomposition of ur_t pointing to the importance of labor-supply shocks in the short-run and supply shocks at longer horizons. For the BE model, the bias reflects the exclusion of relevant labor market information, and the corresponding missing-variable bias. An ancillary result emerges for Δbfk . Not only is Δbfk correlated with labor-supply shocks as noted, but its

[16] That study shows that the R^2 of the regression of the structural shocks on the missing variable(s) provides a straightforward measure of the severity of the non-fundamentalness problem in models "testing positive" for non-fundamentalness. (The higher the R^2 the more information is missing from the model, that is, the further from being fundamental.) In an illustrative example, Beaudry et al. (2019) show that an R^2 as small as 2 percent can trigger a rejection of the null hypothesis of fundamentalness.

propagation aligns well with those shocks. This is consistent with the finding that hours Granger-cause Δbfk in Christiano et al. (2004).

Granted, the empirical evidence in this paper is based solely on the U.S. post-World War II data and will need to be confirmed for earlier time periods and across countries. By the same token, further research will be needed to verify the usefulness of tri-variate model in studying macroeconomic fluctuations, including examining its: fundamentalness; explanation for the disputed "supply-induced" ur_t increases; and relation to Δbfk.

Methodologically, this study builds on the missing-variables IRF's bias literature, namely Braun and Mittnik (1993) (henceforth, BM), and follows Canova (2008) in recasting "co-mingling" in terms of that literature. The latter study stresses that missing-variables and mis-aggregation can provide an informative view of VAR representation problems.[17] In this connection, as noted, I derive a formula to parse the sources of IRF's bias—MA-representation distortion and mis-aggregation of shocks—using standard time-series notation. I show the formula's relation to the IRF's bias expression in BM, which is derived using (compact) lower-triangular Toeplitz matrix notation to represent simultaneously the IRF's for all shocks and up to a specific horizon of interest.[18] BM's non-recursive notation may have discouraged the exploration of the sources of IRF's bias as, to my knowledge, this paper is the first study to do so.

[17] Canova (2008) also highlights a deeper question for "small-scale" VAR models associated with a growing literature going back to Hansen and Sargent (1980, 1991). Can structural shocks be recovered from the information contained in linear combinations of current and past values of the variables included in the model, that is, are these models "fundamental?" Fernandez-Villaverde et al. (2007) derive formal conditions for VAR modeling techniques to recover structural shocks, and Canova and Hamidi (2018) discuss empirically testing "fundamentalness."

[18] Appendix I details the correspondence between BM's *Proposition 2* (page 328) and the time-series notation formula in this paper; the latter can be implemented directly with available econometric software packages.

This paper also relates to the empirical literature studying the effects of labor-supply shocks. Specifically, it is associated to earlier work by Shapiro and Watson (1988) (henceforth SW) exploring an SVAR model that relies exclusively on long-run restrictions to identify technology and labor-supply shocks.[19] As I do in this paper, SW assume that the long-run labor-supply (log-levels of hours) follows a random walk driven by its own shocks. But in contrast to this paper, SW introduces a stochastic trend in hours to identify labor-supply shocks; this is done to impose the long-run restriction that technology does not affect hours: the long-run labor supply is perfectly inelastic.[20] Not surprisingly in light of the academic debate noted above, SW find that the response of hours to technology are consistent with BQ's "supply-induced" ur_t increases: hours decrease initially and increase thereafter (ibid. page 126, Figure 2).

At first, this may seem to run counter to my claim that mis-aggregation underlies the puzzling ur_t response (SW, after all, distinguishes between technology and labor-supply shocks). But that study's results are reversed when the long-run labor-supply exogeneity restriction is relaxed. Specifically, by examining the "extensive" margin of labor-supply and focusing on ur_t allows

[19] The SW model contains two (not separately identified) "demand" shocks that "*…allow output and labor to move independently of the labor and productivity shocks in the short-run…*" (ibid. page 115) as well as an ad-hoc exogenous oil shock that on average is found to play a minor role in explaining macroeconomic fluctuations (ibid. page 144). These shocks are not material to the discussion in the main text.

[20] The core empirical SW model comprises $\Delta hours_t$, Δy_t , and $\Delta^2 p_t$, and the identification relies on three long-run-restrictions: (1) aggregate demand shocks not affect output (as in this paper) nor (2) do they affect hours, and (3) technology shocks not affect hours. The last two restrictions explicitly impose the assumption of an exogenous long-run (level of) labor-supply where real wages play no role. SW note that relaxing assumption (3) would not affect "*...the decomposition between supply and demand...*" and would "*...**only** affect the decomposition…into labor-supply and technology…*" (emphasis added, ibid page 114). Indeed, the variance decomposition in this paper (Table 4) confirms that the supply-demand divide for y_t and p_t is unaffected (and almost identical to Table 2, ibid, page 128). But the imposition of (3) downplays the role technology (versus labor-supply) in explaining hours. That study's conclusion that "*Increases in technology have little effect on hours*." (ibid. page 127) follows from the resulting IRF's depicted in Figure 2 and the (miniscule) fraction the variance of hours associated with technology in Table 2 (ibid. pages 126–8). The surprisingly low role played by technology was called out in Hall's comments to SW: "*What is surprising…is that shifts in labor supply are an important determinant of output in business cycle frequencies*." (SW, page 148).

this paper to identify labor-supply shocks using a weaker restriction: ur_t not respond on *impact* to technology shocks. Doing so, resolves the apparent contradiction: ur_t declines following a technology shock. Of note, ur_t declines whether I introduce a stochastic trend in ur_t or not. I thus argue that the "supply-induced" ur_t increases in SW reflect the restriction on the long-run effect technology on hours.

In addition, this paper relates to more recent literature associated with Brinca, Duarte, and Faria-e-Castro (2021). That study finds that labor-supply underlies recent movements in hours worked (using a bivariate VAR model for changes in both real wages and hours worked). Besides using sectoral (panel) data, that study employs a Bayesian methodology to identify structural shocks using (static) sign restrictions.[21] This agnostic approach is broadly consistent with Keating (2013) that stresses that many well-regarded macro models exhibit non-neutrality of aggregated demand shocks.[22] But by using sign restrictions, Brinca et al. (2021) implicitly assume that wages react contemporaneously to shocks, otherwise wage rigidities can result in time-smearing.[23] In addition, the identification of labor-supply versus labor-demand shocks can run into trouble with technology shocks unless technology and hours are positively correlated

[21] Brinca, et al. (2021) follow the general Baumeister and Hamilton (2015) methodology applicable to any market with price and quantity data. Brinca et al (2021) assume that labor-demand and labor-supply shocks have standard (sign) effects on real wages and hours. Their informative prior borrows available slope estimates to calibrate a truncated Student's *t* distributions describing slope parameters. Using Bayesian techniques, the posterior distribution combines the informative prior and the data.

[22] That study stresses the non-equivalency of temporary shocks and demand shocks and notes that avoiding long-run restriction on output (neutrality of demand shocks) can deepen our understanding of the effects and importance of permanent and temporary shocks.

[23] Consider a stylized wage-setting mechanism where wages are set one period in advance as in BQ, and consistent with Calvo-type wage setting as well as with "equilibrium" wage setting in Hall (2005). In this context, (static) sign restrictions will reflect the current shock in hours but will reflect the shock from the previous period in wages. This timing difference can distort the relative effect of shocks on hours and wages unless successive shocks have the same sign and size.

(the RBC view).[24] Whether agnostic sign restrictions or the combination of long-run and short-run restrictions (favored in this study) are better able to identify labor-supply shocks will depend on the validity of the restrictions. I provide evidence of the VAR model's ability to recover structural shocks independent of the tri-variate model's estimation.

The paper is organized as follows. Section II discusses the evidence of missing-variables and fundamentalness for the BQ and BE models. Section III considers the tri-variate model, its identification and IRF's and contrasts these to the bivariate models' IRF's; the effect of a stochastic trend in ur_t is also discussed. Section IV derives the novel expression for the sources of IRF's bias and applies the methodology to the bivariate models. Section V contrasts SVAR-based structural shocks with the directly measured Δbfk_t. Section IV briefly concludes.

II. Missing Third Variable and Fundamentalness

This section focuses on the BQ and BE models' ability to characterize the data generating process (DGP) by testing for a missing (third) variable and for fundamentalness. The evidence in this section requires taking a stand only on the specification and identification of these bivariate models and is robust to alternative specifications of the deterministic components, optimally selected lags, and in an extended sample period.[25]

A. Data and VAR Preliminaries

Test are performed (and the underlying bivariate models are estimated) using BQ's original quarterly data and sample period to maintain consistency with that literature. The data comprise

[24] When technology and hours are negatively correlated (the new-Keynesian view), static sign restrictions can be hard pressed to distinguish between labor-demand shocks associated with technology shocks and labor-supply shocks. The sign identification breaks down because technology (underlying the labor-demand shock) and labor-supply shocks both reduce hours and increase wages.

[25] Appendix II contains a robustness exercise for the main results in this paper.

the U.S. post-World War II data from 1950:Q2 to 1987:Q4 for seasonally adjusted GNP and the unemployment rate for males, ages 20 and over from "*BLS, February issue, 1982, Table A-39*" (BQ, page 661, footnote 5); the seasonally adjusted GDP deflator is used to deflate GNP.[26] Note that the lag length for all VAR models is set to eight following BQ and FL and, throughout this study, the base-case results are computed when the means of all variables are extracted before and after the 1973–74 oil shock.[27] While the treatment of the deterministic component for ur_t deviates from BQ and FL, it avoids the pathologies documented for the detrended ur_t case.[28]

B. Missing-variable and non-fundamentalness tests[29]

To fix notation, consider a *k*-variable VAR model for Y_t (of order *p*) partitioned as:

$$\begin{bmatrix} I - C^{(1,1)}(L) & -C^{(1,2)}(L) \\ -C^{(2,1)}(L) & I - C^{(2,2)}(L) \end{bmatrix} \cdot \begin{bmatrix} Y_{1,t} \\ Y_{2,t} \end{bmatrix} = \begin{bmatrix} U_{1,t} \\ U_{2,t} \end{bmatrix}$$

[26] The data are from the RATS replication code: https://www.estima.com/procs_perl/700/blanchardquahaer1989.zip. The extended sample for the robustness exercise runs through 2009:Q4; these data are from the FRED database (October 2021 vintage): GNP, UNRATE (number of people 16 and over actively searching for a job as a percentage of total labor force), and GDPDF. The quarterly unemployment series averages the monthly series.

[27] The original BE study uses annual data and sets the lag length equal to two lags.

[28] These problems are discussed in Appendix II. It is important to note nonetheless that base-case results in BQ are robust to alternative deterministic specifications, including, as in this paper, the removal of means before and the after the 1973–74 oil shock (BQ, page 661).

[29] These tests are performed with the information set spun by the BQ and BE models. Thus, the implementation of fundamentalness tests here differs from their typical application based on information comprising dozens, if not hundreds, of time series. Those series are used to extract principal components and test for fundamentalness by examining their statistical significance. Still, a rejection of fundamentalness with a single time series (as is the case in this paper) is telling as it means that the series is useful to forecast and/or predict the structural (or reduced-form) shocks from the k_1 -variable VAR model in question. Granted, a non-rejection is less trustworthy because it is conditional on the information in a single series and subject to being overturned in a richer information set.

where $Y_{1,t}$ and $Y_{2,t}$ denote respectively k_1 variables included in the sub-model under consideration and k_2 potentially missing variables, with $k = k_1 + k_2$ and $E[U \cdot U'] = \Omega$.

Missing-variable tests.[30] $Y_{2,t}$ is missing from the k_1 -variable sub-model for $Y_{1,t}$ when $Y_{2,t}$ Granger-causes (GC) $Y_{1,t}$. This can be tested directly as $H_0 : C^{(1,2)}(L) = 0$ or as a Sims test. Sims (1972) proves that in the regression:

$$Y_{2,t} = \{I - D_1(L)\} \cdot Y_{1,t} + B_1(F) \cdot Y_{1,t} + \xi_t$$

where $B_1(F)$ denotes (not a lag polynomial but) a "forward" polynomial, $Y_{2,t}$ GC $Y_{1,t}$ if and only if $B_1(F) \neq 0$. Thus, the null hypothesis for the Sims test is $H_0 : B_1(F) = 0$, that is, future values of $Y_{1,t}$ (conditioned on past and present values of $Y_{1,t}$) do not help predict $Y_{2,t}$.

Non-fundamentalness tests. Forni and Gambetti (2014) and Canova and Hamidi Sahneh (2018) propose to test for fundamentalness—defined heuristically as the structural shocks for the k_1 -variable model can be recovered from the information spun by $Y_{1,t}$—respectively with GC-type and Sims-type tests where $Y_{1,t}$ is suitably redefined.[31] Specifically, the former study proposes an orthogonality (OR) test based on a GC-type test where $Y_{1,t}$ denotes *structural* shocks from the k_1 -variable model. In this case, $H_0 : C^{(1,2)}(L) = 0$ implies that the information in $Y_{2,t}$ is not useful

[30] For simplicity, I focus the discussion on "system-wide" as opposed to "equation-by-equation" or "shock-by-shock" tests, as the latter expressions stem directly from those in the main text. See Appendix III for details.

[31] In the typical implementation of these tests, $Y_{2,t}$ would also be redefined as one or more principal components obtained from large set of relevant time-series.

in forecasting the model's *structural* shocks and thus the k_1 -variable VAR model is fundamental. The latter study proposes the Canova-Hamidi (CH) test based on a Sims-type test where $Y_{1,t}$ denotes *reduced-form* shocks from the k_1 -variable VAR model. In this case $H_0 : B_1(F) = 0$ implies that the future values of the reduced-form shocks from the k_1 -variable VAR model are not useful in predicting $Y_{2,t}$ so that the k_1 -variable VAR model is fundamental.

Empirical results. I use all four tests, thus taking an agnostic view on Canova and Hamidi Sahneh (2018).[32] That study notes the use of GC-type test in the (empirical) fundamentalness literature (Lutkepohl, 1991, and Giannone and Reichlin, 2006). But, while conceding the usefulness of these tests to examine missing variables, CH argue that GC-type test are not reliable tests of fundamentalness. This is particularly troublesome when a model includes cross-sectionally aggregated or proxy variables; this issue also afflicts the (GC-type) OR test for the unpredictability of structural shocks.[33] Forni, Gambetti, and Sala (2018) argue however that a truly structural "*aggregate*" model does not exist.

Model-wide and equation-by-equation test results for the BQ and BE models are reported in Table 3 (where $k_1 = 2$ and $k_2 = 1$). Consider the BQ variable pairing (Panel A), $Y_{1,t} = \left[\Delta y_t ,\quad ur_t\right]'$ and $Y_{2,t} = \left[\Delta p_t\right]$. At the 5 percent significance level, model-wide missing-variable test results are mixed: the lags of Δp_t are significant in the GC test but the leads of the

[32] To account for potential serial correlation in the original Sims test formulation, I use the Geweke, Meese, and Dent (1983) version of the Sims test (adding lagged dependent variables); I also use this specification for the CH test as in Forni, Gambetti, and Sala (2018). Either way, the tests results are (qualitatively) unaltered.

[33] Consider a VAR model with a cross-sectionally aggregated variable. Each variable forming the aggregate will Granger-cause the VAR model, regardless of whether the model is fundamental or not (see CH, page 1070).

BQ variables in the Sims test are not. Regarding fundamentalness tests, $Y_{1,t}=\left[\varepsilon_t^{BQ,S},\quad \varepsilon_t^{BQ,D}\right]'$, the OR test rejects the null: the lags of Δp_t in the corresponding bivariate VAR model are not zero. But the CH test, $Y_{1,t}=\left[\mu_t^{BQ,\Delta y},\quad \mu_t^{BQ,ur}\right]'$, does not reject the null of fundamentalness: the leads of the BQ model's reduced-form residuals in the Sims-regression are not different from zero. The equation-by-equation test results point consistently to missing-variables in the Δy_t equation and "non-fundamentalness" in the ur_t equation.[34]

Consider next the BE variable pairing tests (Panel B), $Y_{1,t}=\left[\Delta y_t,\quad \Delta p_t\right]'$ and $Y_{2,t}=\left[ur_t\right]$. Model-wide tests do not favor the BE model: both missing variable tests reject the null hypothesis that the third variable (ur_t) is not part of the model, and both fundamentalness tests $\left(Y_{1,t}=\left[\varepsilon_t^{BE,S},\quad \varepsilon_t^{BE,D}\right]' \text{ and } Y_{1,t}=\left[\mu_t^{BE,\Delta y},\quad \mu_t^{BE,\Delta p}\right]'\right)$ reject fundamentalness. These problems also appear in the equation-by-equation tests for Δy_t but not for Δp_t.

I conclude that ur_t cannot be excluded from the DGP without biasing the bivariate model's VAR coefficient estimates and, within the information set spun by these bivariate models, Δp_t should not be excluded as this can hinder the recovery of structural shocks. I thus explore a tri-variate model combining these models.[35]

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[34] Here I use the fundamentalness concept in the spirit of Forni and Gambetti (2014). That study argues that it is possible that a model-wide test reject fundamentalness but the information in the VAR could be "sufficient" to recover a single or sub-set structural shocks.

[35] I also find evidence in favor of combining the BQ and BE models from unreported non-nested hypothesis tests based the *C*-test and the *J*-test from the artificial-regression methodology in Mackinnon (1983). The tests reject the DGP for the common dependent variable, Δy_t, with the *J*-test rejections are "in the direction" of the competing model, thus favoring a combined model. See Pesaran and Weeks (2001) for a discussion of non-nested tests.

Table 3. Missing Third Variable and Fundamentalness Tests for BQ and BE models.

A. BQ model (Third variable: Δp_t)

Model-wide			Equation	Δy_t		ur_t	
1-Granger-Causality test for:							
F(16,101)	21.39		F(8,127)	2.45		1.79	
M-Signif	0.00	*	M-Signif	0.02	*	0.08	
2-Sims-exogeneity test of:							
F(16,100)	1.46		F(8,117)	2.22		1.42	
M-Signif	0.13		M-Signif	0.03	*	0.19	
3-OR (Forni-Gambetti) test for:							
	$\varepsilon_t^{BQ,S} + \varepsilon_t^{BQ,D}$			$\varepsilon_t^{BQ,S}$		$\varepsilon_t^{BQ,D}$	
F(16,93)	2.61		F(8,126)	1.57		2.27	
M-Signif	0.00	*	M-Signif	0.14		0.03	*
4-CH (Canova-Hamidi) test for:							
	$\mu_t^{BQ,\Delta y} + \mu_t^{BQ,ur}$			$\mu_t^{BQ,\Delta y}$		$\mu_t^{BQ,ur}$	
F(16,92)	1.65		F(8,109)	0.63		2.39	
M-Signif	0.07		M-Signif	0.75		0.02	*

B. BE model (Third variable: ur_t)

Model-wide			Equation	Δy_t		Δp_t	
1-Granger-Causality test for:							
F(16,101)	16.27		F(8,127)	3.20		1.85	
M-Signif	0.00	*	M-Signif	0.00	*	0.07	
2-Sims-exogeneity test of:							
F(16,100)	2.09		F(8,117)	3.88		1.13	
M-Signif	0.01	*	M-Signif	0.00	*	0.35	
3-OR (Forni-Gambetti) test for:							
	$\varepsilon_t^{BE,S} + \varepsilon_t^{BE,D}$			$\varepsilon_t^{BE,S}$		$\varepsilon_t^{BE,D}$	
F(16,93)	2.88		F(8,126)	3.86		0.88	
M-Signif	0.01	*	M-Signif	0.00	*	0.53	
4-CH (Canova-Hamidi) test for:							
	$\mu_t^{BE,\Delta y} + \mu_t^{BE,\Delta p}$			$\mu_t^{BE,\Delta y}$		$\mu_t^{BE,\Delta p}$	
F(16,92)	1.76		F(8,109)	2.88		0.90	
M-Signif	0.05	*	M-Signif	0.01	*	0.52	

Note. The Granger-causality tests the lags of the corresponding missing third variable in an augmented bivariate VAR model, which is estimated with eight lags. The Sims-exogeneity tests the leads of the BQ or BE variables in the regression where the third variable is regressed on its lags plus the leads and lags of the corresponding bivariate model. The OR test is a GC-type test for the third variable in a VAR model composed of the structural shocks in the corresponding bivariate model. The CH test is a Sims-exogeneity-type test that tests the leads of the BQ or BE variables in a the regression where the the third variable is regressed on its lags plus the leads and lags of the reduced-form residuals from the corresponding bivariate model. For all tests, the reduced-form residuals and structural shocks are obtained from the estimated BQ and BE models using eight lags. An "asterik" indicates rejection of the null hypothesis at the 5 percent significance level.

III. IRF'S: TRI-VARIATE MODEL VERSUS BQ AND BE MODELS

I propose a straightforward extension of BQ's illustrative Keynesian framework to identify the structural shocks of the tri-variate model encompassing the BQ and BE models. The corresponding IRF's are computed and compared to those from the (replicated) BQ and BE models. The effect of a unit root in ur_t is discussed. Note that the empirical results in this section (and Section IV) are conditional on the identification of the tri-variate and bivariate models but are robust to optimally selected lags and an extended sample (see Appendix II).

A. An Illustrative Three-Shock Keynesian Model[36]

To identify the tri-variate model's shocks and maintain close comparability to the BQ and BE models, I make two straightforward modifications to BQ's illustrative Keynesian framework. I introduce labor-supply shocks by replacing the constant full employment level with the stylized assumption that the labor force follows a random walk driven by labor-supply shocks.[37] And I include money directly in the price-setting equation to relax the (implicit) assumption that aggregate demand shocks not have an impact effect on prices.[38] The modified framework

[36] Compared to the prototypical three-equation new-Keynesian model in Gali (2018), this framework does not model monetary policy shocks, nor does it include an interest-rate rule. Instead, the framework implicitly assumes a simplified money market where shocks to nominal money are, in essence, shocks to (the inverse of) the velocity of money. These shocks may have short-run macroeconomic effects but are taken as neutral in the long-run. The "three-shock" model in this paper is thus not equipped to consider monetary policy and its macroeconomic effects. Specifically, the question of whether expansionary monetary policy can offset the contractionary effects of technology cannot be meaningfully addressed nor inferred from the results.

[37] This assumption is consistent with the labor-supply shock in SW. That study models the long-run labor supply (hours) as a random walk whose forcing variable (the labor-supply shock) is characterized as a general (stationary) AR process (ibid, page 114, equation 2.1).

[38] Note that, while grounded in the velocity theory of money, adding money to the price-setting equation is essential for technical reasons. Otherwise, the solution for Δp_t imposes the restriction that demand shocks not have an impact effect on p_t. This would imply a non-sensical restriction that the variance of demand shocks is zero! This can be verified by setting the elasticity of p_t with respect to nominal balances, λ, equal to zero in the solution for

borrows BQ's long-run neutrality assumption and wage-setting mechanism; the latter assumes that wages are set to be consistent with (expected) full employment one period in advance.[39]

Tri-variate model solution. The augmented illustrative model can be solved as the tri-variate system:

$$\Delta y_t = e_{s,t} + \theta \cdot \Delta e_{s,t} + e_{\tilde{n},t-1} + (1-\lambda) \cdot \Delta e_{d,t}$$

$$ur_t = -\theta \cdot e_{s,t} + e_{\tilde{n},t} - (1-\lambda) \cdot e_{d,t}$$

$$\Delta p_t = -e_{s,t} + \theta \cdot e_{s,t-1} - e_{\tilde{n},t-1} + \lambda \cdot e_{d,t} + (1-\lambda) \cdot e_{d,t-1}$$

where all variables are stationary and $e_{s,t}$ and $e_{d,t}$ denote "supply" (or technology) and "demand" (temporary) shocks as discussed in BQ and $e_{\tilde{n},t}$ denotes a labor-supply shock.[40] The latter can be interpreted as a shock emerging from labor-participation decisions in an involuntary unemployment model as in Christiano, Trabandt, and Walentin (2021).[41] Viewed in this light, a (positive) $e_{\tilde{n},t}$ shock reflects a low "aversion-to-work" realization prompting increases in the

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p_t below. This issue does not emerge in the BQ variable pairing because Δp_t is not part of the model. Note further that I implicitly assume that λ is between zero and one.

[39] This stylized wage-setting mechanism is broadly consistent with a Calvo-type wage setting and recent "equilibrium" wage stickiness in Hall (2005). The latter study shows that in model a with equilibrium wage stickiness, technology advances reduces ur_t ; Hall (2005) documents this effect using a univariate model for ur_t . Of note, the propagation of technology advances in that study is consistent with the results in this paper.

[40] Appendix IV details the modified model and its solution (including for wages, its "*fourth*" variable). Note that the tri-variate model's solution reproduces the BQ model solution when $\lambda = 0$ and $e_{\tilde{n},t} = 0$. And, as in BQ, all variables are stationary implying a cointegrating vector [1, -1] for labor supply, *n~*, and *n*. Note that viewed from the "extensive" margin of labor-supply, equation (2.6) in SW would imply cointegration as its lag-polynomial is assumed summable.

[41] In that study, unemployment reflects the lower utility of the unemployed (versus the employed) inducing workers to embark on costly job-search activities whenever their aversion-to-work does not exceed a threshold. Each period, a worker privately observes a random draw of their "aversion-to-work" and workers whose realization does not exceed their threshold pursue costly job search activities (becoming part of the labor force) and with a specific probability find a job. In expected terms the unemployment rate equals 1 minus this probability. Those whose "adversion-to-work" realization is high exit the labor force (so-called discouraged workers).

labor force that, given the prevailing wage, increases (involuntary) ur_t . The model's formulation thus focuses on the "extensive" margin of labor supply that underlies the vast majority of changes in U.S. labor supply as discussed in Blundel et al. (2011).

Note that the modified framework here is not rich enough to discuss business cycle effects on labor participation (and ur_t) discussed in Christiano et al. (2021). Here, labor participation decisions are associated exclusively with $e_{\tilde{n},t}$, which is orthogonal to $e_{d,t}$. The latter would however capture the intensification of job search of those already in the labor force.[42, 43]

It is important to note that (just as in BQ) the modified framework provides solutions for four variables. In this paper, the fourth variable is wage growth whose solution can be expressed as:

$$\Delta w_t = \theta \cdot e_{s,t-1} - e_{\tilde{n},t-1} + \left(1-\lambda\right) \cdot e_{d,t-1} .$$

It is useful to keep this expression in mind for the discussion of the identification restrictions below. Note that I also use this equation to verify the consistency of the tri-variate model's

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[42] The orthogonality does not capture wealth effects on labor participation from $e_{s,t}$ and $e_{d,t}$ in Gali (2011) and Gali et al. (2011). Christiano et al. (2021) note however that the resulting labor-supply declines are counterfactual.

[43] Alternatively, viewed from the prism of Hall (2008), $e_{\tilde{n},t}$ combines shocks to the employment supply and labor-force supply curves. The illustrative framework solution suggests that $e_{\tilde{n},t}$ increases ur_t , implying that reductions in ur_t from an employment shock are more than offset by increases from a labor-force supply shock. Also note that Hall's labor-demand shock combines $e_{d,t}$ and $e_{s,t}$ that both reduce ur_t (and increase wages as discussed below).

structural shocks, independently of the model's estimation and identification.[44] This is possible because Δw_t is not part of either and thus constitutes a "duck-test" for structural shocks.[45]

Implied identifying restrictions. Using Hamilton (1984) notation,[46] the identifying restrictions from the tri-variate model's solution can be represented as:

$$A_0 = \begin{bmatrix} * & 0 & * \\ * & * & * \\ * & 0 & * \end{bmatrix} \qquad A(1) = C(1) \cdot A_0 = \begin{bmatrix} * & * & 0 \\ * & * & * \\ * & * & * \end{bmatrix}$$

where A_0 and $A(1)$ denote respectively arrays grouping the contemporaneous impact and the long-run effects of structural shocks (with their typical elements denoted by $a(0)(i,j)$ and $a(1)(i,j)$). Here, asterisks represent unrestricted elements and zeros indicate the identifying restrictions that, together with the orthogonality and normalization of the structural shocks, exactly identify the model.[47]

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[44] Note that since $\Delta w_t = -ur_{t-1}$, an alternative tri-variate model could be defined using Δw_{t+1} or Δw_t as the labor market variable. Neither is appealing: the former introduces endogeneity in the corresponding VAR equation; the latter introduces three overidentifying restrictions. More importantly, neither provides direct comparison to BQ.

[45] The duck test relies on "abductive reasoning" yielding a plausible conclusion but not definitive proof: "If it looks like a duck, walks like a duck, and quacks like a duck, it's *probably* a duck."

[46] Using Hamilton's equation 11.4.22 (page 323) I express the underlying structural tri-variate and reduced-form models respectively as: $Y_t^{3V} = A(L) \cdot E_t^{3V}$ and $Y_t^{3V} = C(L) \cdot U_t^{3V}$ where $Y_t^{3V} = \left[\Delta y_t , \quad ur_t , \quad \Delta p_t \right]'$, $C(L) = \left[I - C_1 \cdot L - C_2 \cdot L^2 - \cdots - C_p \cdot L^p \right]^{-1}$, and $E_t^{3V} = \left[e_{s,t}, \quad e_{\tilde{n},t}, \quad e_{d,t} \right]'$. C_s and E_t^{3V} contain respectively the VAR model's coefficients for lag *s* and the orthogonal structural shocks obtained by post-multiplying their reduced-form counterparts by A_0; the latter satisfies $A_0 \cdot A_0' = \Omega^{3V}$ where Ω^{3V} denotes the VAR model's reduced-form covariance matrix.

[47] The tri-variate model conforms to the necessary and sufficient rank conditions to just-identify VAR models in Rubio-Ramirez, et al. (2010). Specifically, for models with both short- and long-run restrictions, these amount to a

For Δy_t , the identifying restrictions imply that $e_{d,t}$ not have long-run effects on y_t (long-run neutrality). These restrictions also impose that $e_{\tilde{n},t}$ not have an *impact* effect on y_t reflecting the fact that it increases the labor force (and ur_t) on impact but not employment. Subsequently, y_t and employment increase (ur_t decreases) as w_t are revised down. For Δp_t , these restrictions impose that $e_{\tilde{n},t}$ not have an impact effect on p_t . This is because $e_{\tilde{n},t}$ affects prices indirectly via their effect on w_t , which takes one period to adjust. Note that the wage-setting assumption also implies that declines in p_t following $e_{s,t}$ are (partially) offset by the delayed increases in w_t resulting in "under-shooting" p_t . Note further that the delayed effects of $e_{s,t}$ and $e_{\tilde{n},t}$ are of opposite signs.[48]

Before turning to the tri-variate model's IRF's, it is important to note that, as in the BQ model, the ur_t solution does not provide identifying restrictions but does imply that both $e_{s,t}$ and $e_{d,t}$ reduce ur_t .[49] This is because both shocks shift the labor-demand curve that, for a given w_t ,

"restriction-counting" exercise: the combined sum of the number of restrictions on the structural shocks in the columns of A_0 and $A(1)$ must be (in any order) exactly 0, 1, and 2.

[48] What does the tri-variate model's solution imply for the BQ and BE models? In both, $e_{s,t}$ shocks (scaled by θ) are combined (mis-aggregated) with $e_{\tilde{n},t}$ shocks, which have opposing effects on the pairing variable. As noted, FL would argue that because $e_{s,t}$ and $e_{\tilde{n},t}$ have opposing effects on ur_t in the model, the bivariate BQ solution does not meet the conditions in Proposition 2, part 2 (FL, page 349). An analogous problem emerges for the BE solution, though the opposing effects arise in the dynamics.

[49] Note that here *ur* solution can be interpreted as SW's equation (2.6) if one focuses on the "extensive" margin of labor supply and would imply that *ur* is stationary in here and in SW.

increases employment. But $e_{\tilde{n},t}$ increases ur_t because it increases the labor force (shifts out the labor-supply curve) while the quantity of labor demanded remains unchanged for a given w_t .

B. Impulse Response Functions

Impulse response functions are defined and computed in the standard way,

$$\frac{\partial Y_{t+s}^{3V}}{\partial e_{j,t}} = \Psi_s^{3V} \cdot p_j^{3V}, \text{ for } s = 0, 1, 2, 3, \ldots$$

where Y_{t+s}^{3V} and $e_{j,t}$ denote respectively the variables in the tri-variate model (Δy_t , ur_t , and Δp_t), and the j^{th} structural shock; Ψ_s^{3V} and p_j^{3V} denote respectively the (3×3) matrix grouping the lag-*s* coefficients of the VAR model's MA-representation and the j^{th} column of A_0 that reflects the tri-variate model's identifying restrictions.[50]

Foreshadowing the discussion of the sources of IRF's bias, I express the IRF's with the following function:

$$IRF_{j,s}\left(\Psi_s^{3V}, p_j^{3V}\right) = \Psi_s^{3V} \cdot p_j^{3V}, \text{ for } s = 0, 1, 2, 3, \ldots$$

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[50] I obtain A_0 as the solution to the following set of (nine) non-linear equations:

$$\left[\text{vech}\left(A_0 \cdot A_0 ' - \Omega^{3V}\right), \quad a(0)(1,2), \quad a(0)(3,2), \quad a(1)(1,3)\right]' = \left[0, \quad 0, \quad 0, \quad \cdots, 0\right]' ,$$

where $\text{vech}\left(A_0 \cdot A_0 ' - \Omega^{3V}\right)$, $a(0)(. \,, \,.)$, and $a(1)(. \,, \,.)$ denote respectively the "vectorization" of the six conditions associated with orthogonality of structural shocks (and their normalized variances) and the tri-variate model's restrictions. To solve these equations, I use the Newton-Raphson method (see Kilian and Lutkephol, 2017, pp. 310–16) starting the algorithm with an initial $\hat{A}_0$ equal to the Cholesky decomposition of Ω^{3V} and an initial $\hat{A}(1)$ equal to the Cholesky decomposition of $C(1) \cdot \Omega^{3V} \cdot C(1)'$ (the factor for a long-run recursive tri-variate model). The solution algorithm iterates until the squared norm of the nine non-linear equations is less than 0.000001.

Figure 2 depicts the IRF's for the tri-variate's structural shocks (labeled *3V*); the model is estimated with eight lags over the BQ sample period noted above. Using the tri-variate model as the DGP, bootstrapping methods are used to generate 2,000 quasi-data samples and repeatedly estimate the tri-variate model, compute IRF's, and generate confidence bands at the 68 and 95 percent levels. To facilitate comparisons, Figure 2 also depicts the IRF's (point estimates) for the replicated BQ and BE models (labeled *BQ* and *BE*).

The IRF's are consistent with the structural shocks' interpretation from the modified Keynesian framework.[51] A supply shock increases y_t persistently and lowers ur_t in the short run. p_t decline on impact and further decline slightly thereafter but tend to increase over the long term. A labor-supply shock temporarily increases ur_t and does not affect y_t nor p_t on impact (by construction). Subsequently, a labor-supply shock results in lasting increases in y_t and reductions in p_t. A demand shock results in the familiar hump-shaped response in y_t (vanishing by construction) and a virtual mirror-image response for ur_t . p_t increase through the medium term and falls back slightly in the long run.

[51] SW find technological advances decrease hours in the short-run. FG report a consistent (though more persistent) result for ur_t when the information content of a BQ-related bivariate model is augmented (with at least one principal component) (FG, Figure 2, page 134). In both studies, the propagation is reminiscent of that of the BQ model and consistent with Gali (1999) where, as noted, hours include a stochastic trend.

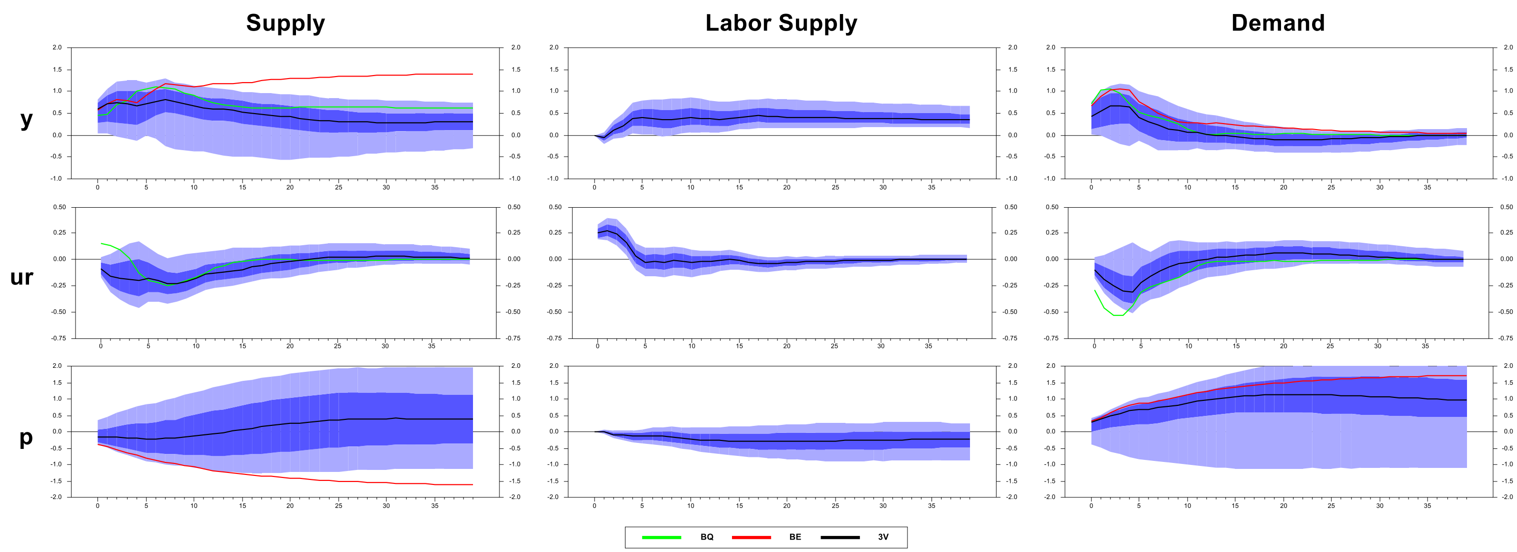

Figure 2. IRF's for the Tri-variate model (shaded 68% and 95% bands), BQ, and BE models

Contrasting these IRF's to the bivariate models' IRF's, I note that for:

- *Supply shocks*. The BQ model's y_t responses are qualitatively the same to the tri-variate model, though understating short-run and overstating long-run effects. As noted, "supply-induced" ur_t increases appear to combine the opposing forces in its "comingled" permanent shock. The BE's model's y_t and p_t responses differ markedly resulting in large deviations over time.
- *Demand shocks*. The BQ and BE model's y_t responses exhibit standard hump-shaped responses that vanish over time. Regarding the pairing variables, both models' responses are qualitatively the same to those of the tri-variate model. But, but the BQ model's ur_t response overstates declines and the BE model's p_t response overstates p_t increases in the long-run.

Propagation of structural shocks for wages and the labor force. Further evidence of the validity of the interpretation of the tri-variate model's structural shocks stems from the responses of variables not included in the estimated model. For this I use BFK's near-VAR framework to trace the dynamic responses for a single variable of interest, x_t , to exogenously determined shocks, ξ_t . Specifically, consider the near-VAR model:

$$\begin{bmatrix} \Delta S_t \\ x_t \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ -c_{2,1}^{s} & 1-c_{22}(L) \end{bmatrix}^{-1} \cdot \begin{bmatrix} \xi_t \\ \mu_t \end{bmatrix}$$

where ΔS_t denotes a vector of random walks associated with tri-variate model's structural shocks, $\xi_t = \left[e_{s,t}, e_{\tilde{n},t}, e_{d,t}\right]'$, and μ_t is the reduced-form shock of x_t. Note that the near-VAR model is not used to identify structural shocks: ξ_t is identified by the tri-variate model.[52]

Figure 3 depicts the propagation of the tri-variate model's structural shocks for two variables of particular interest: $x_t = w_t$ and the labor force (for reference, the figure also shows the propagation for ur_t). The near-VAR model broadly captures (qualitatively) the IRF's of ur_t from the tri-variate model: ur_t decreases following $e_{s,t}$ or $e_{d,t}$ and increases following $e_{\tilde{n},t}$. For w_t, the propagation aligns with the augmented illustrative Keynesian framework's predictions: w_t increase following $e_{s,t}$ and $e_{d,t}$, and decrease following $e_{\tilde{n},t}$ (and, consistent with the illustrative framework, these are of the opposite sign of the ur_t responses). For the labor force, it too responds mostly as expected, though $e_{d,t}$ results in puzzling declines. Regardless, these responses are imprecisely measured.[53]

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[52] Also note that for notational simplicity, I omit constants included near-VAR models; the latter contain eight lags of x_t and are estimated with quarterly data over the sample period noted in the main text.

[53] For wages, the ratio of the estimated $-c_{2,1}^{s}$ to its standard error is low 1.6, 1.5, and 2.3 respectively for $e_{s,t}$, $e_{\tilde{n},t}$, and $e_{d,t}$. The labor force coefficients are estimated with less precision (these ratios are less than 1). But the estimates are precise for ur_t : these ratios exceed 6 for all three shocks.

Figure 3. Labor market response to tri-variate shocks (near-VAR)

Stochastic trend in ur_t. The key results from the IRF's are robust to a stochastic trend in ur_t (Figure 4). I introduce the stochastic trend by replacing ur_t with Δur_t in the tri-variate model; for illustrative purposes, I also do so in the BQ model.[54] The corresponding IRF's for all three shocks remain consistent with their structural interpretation and reproduce qualitatively those without a unit root. The most important difference is that $e_{s,t}$ now results in a persistent decrease in ur_t : the 95% confidence bands do not include zero (at all horizons).[55] While these confidence bands are not a formal statistical test, restricting the long-run effect to zero (as in SW) would not

[54] The IRF's for ur_t are the accumulated sums of the Δur_t responses for the tri-variate (and BQ) model, where the model is estimated and identified as in the main text but the model now includes Δur_t instead of ur_t . As noted, the tri-variate model's ur_t solution does not imply short-run or long-run restrictions so the unit-root does not alter the identification but can alter the estimation: if ur_t is stationary (as assumed in BQ), the resulting IRF's would reflect "over-differencing."

[55] Also note that the BQ model's "supply-induced" ur_t increases vanish when ur_t is contains a stochastic trend.

seem consistent with the data. Imposing this restriction will require that long-run decreases in ur_t be offset by "supply-induced" ur_t increases.

In contrast with received wisdom, I thus argue that the time-series properties of the labor market variable do not underlie a negative or positive relation between technology and employment. By disentangling technology and labor-supply shocks—while eschewing long-run restrictions on the labor-market variable—the tri-variate model paves the way for an alternative understanding of "supply-induced" ur_t increases, one independent of the labor market variable's time series properties: mis-aggregation of technology and labor-supply shocks.

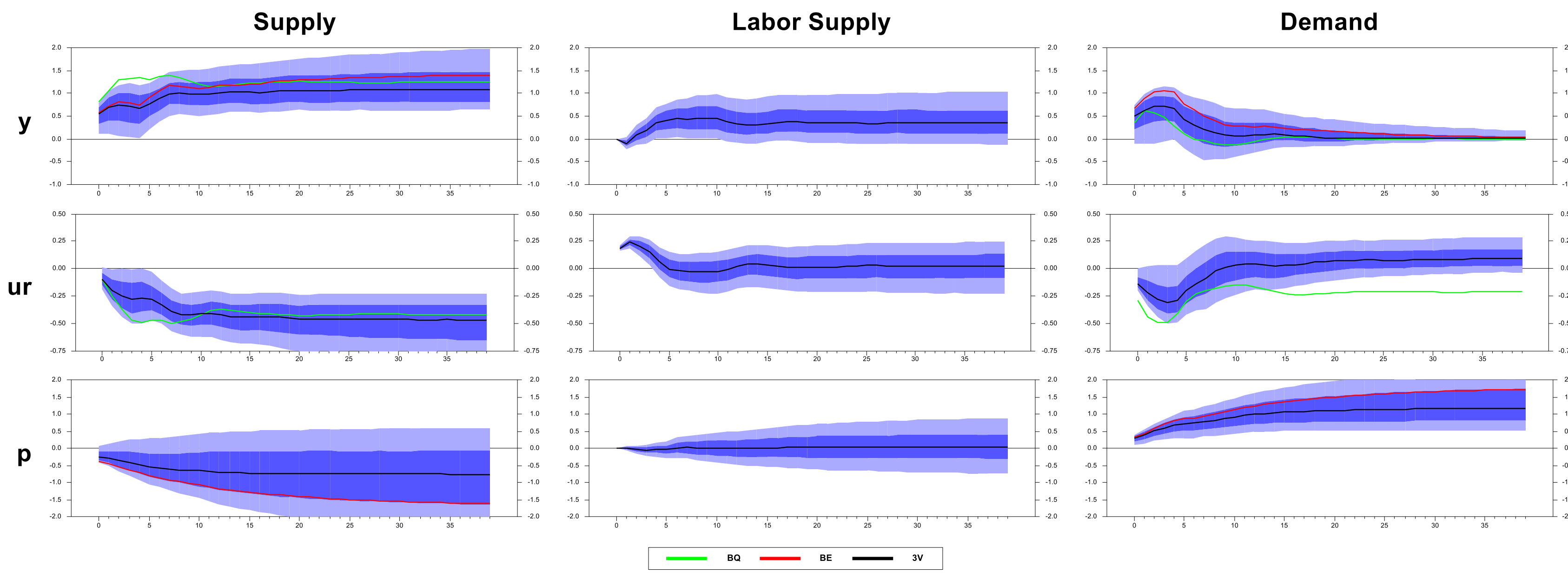

Figure 4. IRF's for the Tri-variate model (shaded 68% and 95% bands), BQ, and BE models (ur stochstic trend)

III. SOURCES OF IMPULSE RESPONSE FUNCTION MISSING-VARIABLE BIAS.

To shed light on the bivariate models' IRF's bias, I derive an expression to parse the sources of bias and gauge the importance of mis-aggregation and missing-variable bias. As noted, this expression is consistent with *Proposition 2* in BM but obtained using time-series notation (see Appendix I).

To begin, I partition the IRF's for the tri-variate model into a bivariate pairing and a third variable. Without loss of generality, I define $Y_{1,t}$ to contain the BQ variables with $Y_{2,t}$ denoting the (potentially) missing third variable, $Y_{1,t} = \left[\Delta y_t \;\; ur_t\right]'$ and $Y_{2,t} = \left[\Delta p_t\right]$. Next, I partition the tri-variate model's IRF's as follows:

$$IRF_{j,s}^{3V}\left(\Psi_s^{3V}, p_j^{3V}\right) = \begin{bmatrix} IRF_{j,s}^{3V,BQ}\left(\Psi_s^{3V,BQ}, p_j^{3V}\right) \\ IRF_{j,s}^{3V,\Delta p}\left(\Psi_s^{3V,\Delta p}, p_j^{3V}\right) \end{bmatrix} = \begin{bmatrix} \Psi_s^{3V,BQ} \\ \Psi_s^{3V,\Delta p} \end{bmatrix} \cdot p_j^{3V}$$

where the tri-variate model's MA representation Ψ_s^{3V} (3×3) is divided into $\Psi_s^{3V,BQ}$ (2×3) and $\Psi_s^{3V,\Delta p}$ (1×3). Note that since the object $IRF_{j,s}^{3V,BQ}$ is identical to IRF's for $Y_{1,t}$ in $IRF_{j,s}^{3V}$, I focus on $IRF_{j,s}^{3V,BQ}$ when comparing these responses to the BQ model's IRF's; the latter expressed as:

$$IRF_{j,s}^{BQ,BQ}\left(\hat{\Psi}_s^{BQ}, \hat{p}_j^{BQ}\right) \equiv \frac{\partial Y_{t+s}^{BQ}}{\partial e_{j,t}^{BQ}} = \hat{\Psi}_s^{BQ} \cdot \hat{p}_j^{BQ}, \quad \text{for } s = 0, 1, 2, 3, \ldots$$

where $\hat{\Psi}_s^{BQ}$ and $\hat{p}_j^{BQ}$ are respectively of dimensions (2×2) and (2×1).

Following BM (Section 4, pp. 330–8), I define the BQ model's IRF's missing-variable bias as the difference between the IRF's of the bivariate model and (benchmark) tri-variate model:

$$\Delta_{IFR_{j,s}} = IRF_{j,s}^{BQ,BQ}\left(\hat{\Psi}_s^{BQ}, \hat{p}_j^{BQ}\right) - IRF_{j,s}^{3V,BQ}\left(\Psi_s^{3V,BQ}, p_j^{3V}\right).$$

Note that the bias corresponds to the vertical distance between the IRF's from the BQ and the tri-variate models (at horizon *s*).

Next, I define $\hat{\Psi}_s^{BQ} = \Psi_s^{3V,BQ} + \Delta_\Psi$ and $\hat{p}_j^{BQ} = p_j^{3V} + \Delta_{p_j}$ where Δ_Ψ and Δ_{p_j} denote respectively the change in the MA-representation and the j^{th} column of A_0 when $Y_{2,t}$ is excluded from the model.[56] That is, Δ_Ψ and Δ_{p_j} reflect the bivariate model's MA-representation distortion and mis-aggregation (compared to benchmark tri-variate model).

The IRF bias can now be re-expressed as:

$$\Delta_{IFR_{j,s}} = IRF_{j,s}\left(\Psi_s^{3V,BQ} + \Delta_\Psi,\ p_j^{3V} + \Delta_{p_j}\right) - IRF_{j,s}\left(\Psi_s^{3V,BQ},\ p_j^{3V}\right),$$

and the sources of bias follows from the implied multiplication:[57]

$$\Delta_{IFR_{j,s}} = IRF_{j,s}\left(\Psi_s^{3V,BQ}, \Delta_{p_j}\right) + IRF_{j,s}\left(\Delta_{\Psi_s}, p_j^{3V}\right) + IRF_{j,s}\left(\Delta_{\Psi_s}, \Delta_{p_j}\right).$$

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[56] To avoid addition notation, I implicitly "re-dimension" $\hat{\Psi}_s^{BQ}$ and $\hat{p}_j^{BQ}$ to conform to the dimensions of the tri-variate model. Specifically, these arrays are expanded respectively from (2×2) to (2×3) and from (2×1) to (3×1) by adding a column of zeros to $\hat{\Psi}_s^{BQ}$ and a zero element to $\hat{p}_j^{BQ}$. The IRF's from these expanded arrays replicate those from the bivariate BQ model, but also contain zeros for the responses for a notional "third variable" and to a notional labor-supply shock.

[57] In essence, this is the product-rule for discrete changes. Intuitively, it emerges because omitting variables in a VAR model can simultaneously bias the two arrays underlying the IRFs calculation: $C(L)$ and Ω. In deriving the bias expression, I use the distributive property of matrix multiplication that $IRF_{j,s}(\cdot,\cdot)$ inherits from the IRF's calculation. Namely, for conformable matrices A, B, C, and D:
$IRF_{j,s}(A+B, C+D) = IRF_{j,s}(A,C) + IRF_{j,s}(A,D) + IRF_{j,s}(B,C) + IRF_{j,s}(B,D)$.

The first term in this expression captures the bias from mis-aggregated shocks as it computes the IRF's when the correct MA-representation, $\Psi_s^{3V,BQ}$, is post-multiplied by the bias in the identification factor, Δ_{p_j}. The second term reflects the bias from the MA-representation distortion as it computes the IRF's with the correct identification factor, p_j^{3V}, but pre-multiplied by the bias in the MA-representation, Δ_{Ψ_s}. And the third term captures the interaction of these biases as each is computed holding the other constant.[58]

With this expression, I am now equipped to examine the sources of IRF's biases and assess the role of mis-aggregation bias in "supply-induced" ur_t increases. Figures 5 and 6 depict respectively the BQ and BE models' overall bias in the first column and the sources of bias in subsequent columns (supply and demand shocks are in the top-two and bottom-two rows). Confidence bands for the 68 and 95 percent levels are computed using bootstrapping methods taking the tri-variate model as the DGP to obtain 2,000 quasi-data samples. These samples are used to repeatedly estimate the BQ, BE, and tri-variate models, and compute their respective IRF's as well as the bivariate models' IRF's bias and sources of bias.

[58] Note that while Δ_{IRF} is not a closed-form expression for the IRF's bias—Δ_{Ψ_s} and Δ_{p_j} are not derived analytically—it shows how to compute the sources of IRF bias from the empirical counterparts of those objects. Specifically, Δ_{Ψ_s} and Δ_{p_j} can be obtained respectively as the difference between the bivariate and tri-variate models' (estimated) MA-representations and (estimated) j^{th} columns of the identification factors. For the closed-form expression of Δ_{Ψ_s} see BM's equations 25 and 26 (BM, page 328).

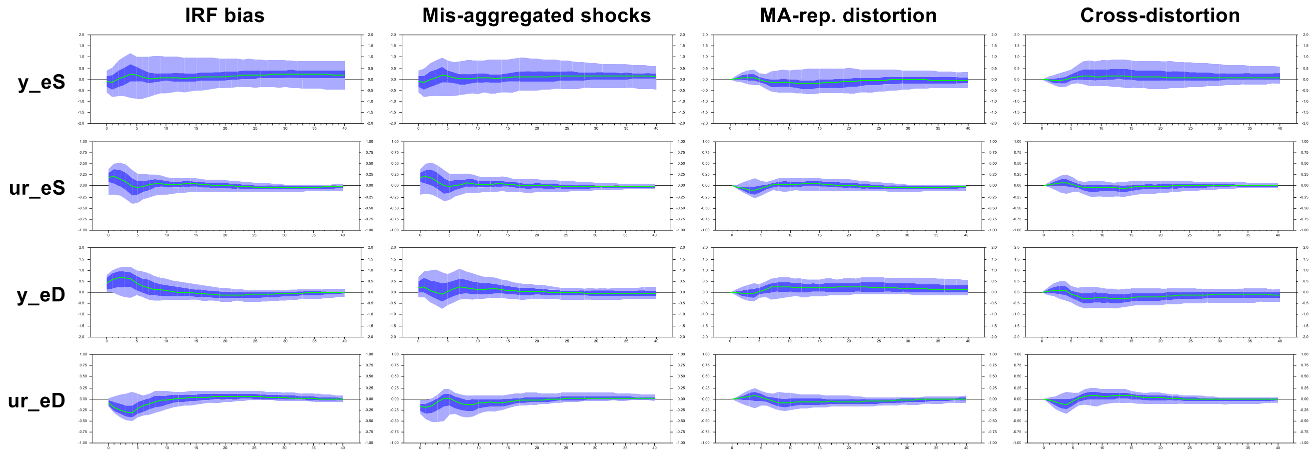

Figure 5. BQ model's IRFs bias decomposition (shaded 68% and 95% bands)

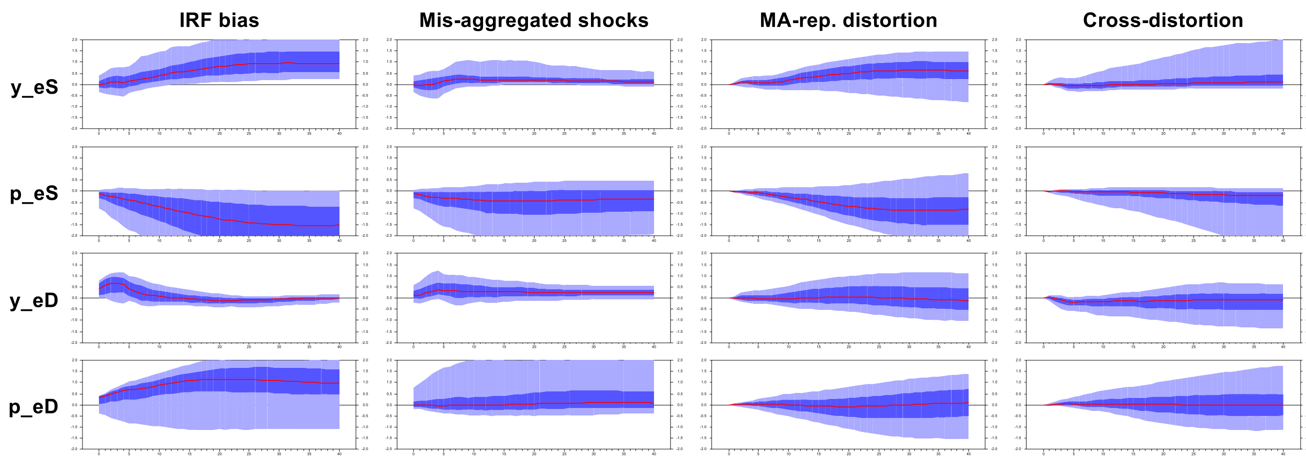

Figure 6. BE model's IRFs bias decomposition (shaded 68% and 95% bands)

The bias decompositions provide a clear picture of the overall IRF's bias and the principal pathology afflicting each bivariate model. Specifically,

- the BQ model's IRF's bias reflects primarily mis-aggregation of shocks with a lesser bias from MA-representation distortions (Figure 5, columns 2 and 3). Of particularly interest, mis-aggregation clearly underlies the "positive" IRF's bias of ur_t following a supply shock in the BQ model (row 2, columns 1 and 2). The magnitude of the mis-aggregation bias is such that it reverses the sign of the ur_t response from the tri-variate model, particularly in the first four quarters. Note also that the prevalence of mis-aggregation bias aligns with the evidence from cross-type correlations in Table 2 and, to a lesser extent, with the test results in Table 3. (Consistent with the IRF's discussion above, the IRF's bias in demand shocks is small and does not change qualitatively the responses.)

- the BE model's IRF's bias—overstating y_t and understating p_t—reflect mis-aggregation in the short run, but MA-representation distortions dominate at longer horizons (Figure 6, columns 2 versus 3). For demand shocks, the overall bias is smaller (column 1, rows 3 and 4) reflecting partially offsetting effects of mis-aggregation and MA-representation distortions (columns 2 and 3, rows 3 and 4). The MA-representation distortion mirrors the missing ur_t variable test results (Table 3).

Of note, the magnitudes of the IRF's biases align well with the measurement of the "severity" of non-fundamentalness discussed in Beaudry et al. (2019). Consider the IRF's bias in each model: larger biases in the BQ and the BE model follow demand shocks in the former and supply shocks in the latter. This suggests more "severe" non-fundamentalness for demand shocks in the BQ model and supply shocks in the BE model. Indeed, for the BQ model, the R^2 of a regression of demand shocks on the "missing" third variable (0.11) is almost three times larger than for supply shocks (0.04). And for the BE model, the R^2 of a regression of supply shocks on the "missing" third variable (0.16) is eight times that of demand shocks (0.02).

IV. ADDITIONAL QUESTIONS

To what extent do mis-aggregated shocks and MA-representation distortion alter assessments of the relative importance of supply versus demand factors in macroeconomic fluctuations from these bivariate models? Should we rely on VAR-based structural shocks to infer the effect of technology shocks on ur_t (or employment) or favor instead directly measured "pure" Δbfk_t shocks? To address these questions, I turn to variance decompositions, FL cross-type correlations, and BFK's near-VAR propagation methodology noted above.

Variance decompositions (Table 4). Consider the tri-variate model. Up to 4 quarters, the dominant source of y_t and ur_t fluctuations are supply shocks ($e_{s,t}$ and $e_{\tilde{n},t}$) while p_t fluctuations are dominated by demand shocks ($e_{d,t}$). In contrast, both the BQ and BE overstates (understates) the importance of $e_{d,t}$ ($e_{s,t}$) shocks in explaining y_t fluctuations through forecasting horizons up to 24 quarters. Regarding their respectively pairing variables, the BQ model and BE model respectively *overstates* the importance of $e_{d,t}$ shocks in explaining ur_t fluctuations and *understates* the importance of $e_{d,t}$ shocks in explaining p_t fluctuations. At face value and compared to the tri-variate model, ignoring the missing-variable and non-fundamentalness a researcher would infer greater roles than justified for aggregate demand and demand management policies in explaining macroeconomic fluctuations, except for p_t where lesser roles would be inferred.

Table 4. Variance Decompositions (percent due to structural shocks)

	Permanent					***Demand***		
	Trivariate			BQ	BE	Trivariate	BQ	BE
	Supply	Labor	Sum					
				y_t				
Horizon								
0	71.6	0.0	71.6	27.0	42.0	28.4	73.0	58.0
1	68.2	0.4	68.6	21.2	40.6	31.4	78.8	59.4
2	63.5	0.8	64.2	25.5	39.3	35.8	74.5	60.7
3	59.9	1.8	61.7	30.2	38.1	38.3	69.8	61.9
4	55.6	5.6	61.2	38.7	37.2	38.8	61.3	62.8
8	62.6	13.9	76.5	59.6	55.4	23.5	40.4	44.6
12	63.2	19.2	82.4	67.3	66.7	17.6	32.7	33.3
24	57.7	28.8	86.5	75.3	82.5	13.5	24.7	17.5
40	51.7	37.2	88.8	81.3	90.2	11.2	18.7	9.8
				ur_t				
Horizon								
0	13.0	76.2	89.2	21.2	...	10.8	78.8	...
1	16.2	63.3	79.5	12.2	...	20.5	87.8	...
2	16.8	53.5	70.3	8.0	...	29.7	92.1	...
3	17.3	43.9	61.2	5.5	...	38.8	94.5	...
4	18.5	35.4	53.9	5.9		46.1	94.1	
8	33.8	24.4	58.2	17.0	...	41.9	83.0	...
12	41.6	21.9	63.5	21.0	...	36.5	79.0	...
24	41.7	20.6	62.4	21.2	...	37.6	78.8	...
40	42.0	20.0	62.0	21.2	...	38.0	78.8	...
				p_t				
Horizon								
0	15.8	0.0	15.8	...	54.8	84.2	...	45.2
1	12.7	0.1	12.8	...	51.3	87.2	...	48.7
2	10.3	0.5	10.8	...	48.1	89.2	...	51.9
3	9.4	1.0	10.3	...	46.3	89.7	...	53.7
4	8.6	1.6	10.2		45.0	89.8		55.0
8	6.7	1.7	8.5	...	46.4	91.5	...	53.6
12	4.0	2.4	6.3	...	46.5	93.7	...	53.5
24	2.9	2.9	5.7	...	46.3	94.3	...	53.7
40	6.3	2.5	8.9	...	46.3	91.1	...	53.7

Note. All VAR models are estimated using eight lags over the sample period 1950:02 to 1987:04. Before estimation I remove the sample means for the three variables before and after the first OPEC oil shock (1973:04). This treatment of the data series is consistent with the BE sample split but differs from BQ's base case. Nonetheless, differences are quantitative in nature. Structural shocks are identified using short- and long-run restrictions as discussed in the main text.

Beyond the "demand-supply" divide, the relative importance of technology versus labor-supply shocks is informative. For y_t fluctuations, $e_{\tilde{n},t}$ shocks come into play in the medium-term, explaining roughly a third of the variance. For ur_t fluctuations, the reverse is true: $e_{\tilde{n},t}$ shocks play a dominant role in the shorter term and $e_{s,t}$ shocks become more important at longer horizons. This pattern aligns with the literature's conjecture about "supply-induced" ur_t increases in BQ model: the initial increase is associated with the importance of labor-supply shocks in short-run variations and subsequently decreases as supply shocks take hold. For price fluctuations, supply plays a minor role in the first four quarters and subsequently an even a smaller role; $e_{\tilde{n},t}$ does not play a role.

Of note, the demand-supply divide for y_t and p_t in Table 4 closely reproduces that reported by SW (Table 2, page 128). But notable differences arise when apportioning fluctuations between technology and labor-supply. SW finds that labor-supply shocks explain about half of output fluctuations in the first year and 40 percent at eight quarters; in Hall's comments, he calls out the surprising result that: "*...shifts in labor-supply are an important determinant of output in business cycle frequencies*" (ibid, page 148). In contrast, the variance decompositions in this paper point to a negligible role for labor-supply shocks in explaining output fluctuations at four quarters and only about 15 percent at eight quarters. The marked difference with SW on the role of labor supply at business cycle frequencies reflects, as noted above and acknowledged in SW (page 114), the long-run identifying restriction that technology shocks do not affect hours.

Δbfk_t versus VAR-model based structural shocks

FL cross-type correlations (Table 5). To compute the correlations between Δbfk_t and VAR-based structural shocks discussed above, I use the quarterly version of Δbfk_t from Fernald (2014). That study does not correct for deviations from constant returns to scale nor from perfect competition due to data availability constraints, but the quarterly series "*…goes a long way towards cleansing the Solow residual of non-technological cyclicality*." (ibid, page 15).

Table 5. Correlation of BFK shocks and SVAR-based shocks

	Permanent	Labor supply	Temporary
Blanchard-Quah	0.18	…	-0.06
Bayoumi-Eichengreen	0.09	…	-0.05
Tri-variate model	0.11	0.15	-0.07

Note. Quarterly BFK shocks are taken from Fernald (2014). VAR-based structural shocks are identified using the long- and short-run restrictions discussed in the main text for the tri-variate model and the long-run restrictions in Blanchard-Quah and Bayoumi-Eichengreen. All VAR models are estimated using eight lags using quarterly data from 1950:Q2 to 1987:Q4

The correlations are low, not surprisingly, as Δbfk_t is obtained from a different empirical methodology.[59] Nonetheless as noted, two stylized facts emerge. First, VAR-model based demand shocks are not correlated to Δbfk_t (these are slightly negatively correlated). And second, Δbfk_t is most correlated with BQ's supply (0.18) and the tri-variate model's labor-supply shocks (0.15); and to a lesser extent with supply shocks from the BE and tri-variate models.

[59] Gali and Rabanal (2004) report higher correlations, which may reflect that study's use of annual data (and the averaging of quarterly VAR-model shocks).

These stylized facts point to a general agreement between the BFK's direct measurement and SVAR approaches in separating demand (cyclical) from supply (long-run) factors; this is also noted in Gali and Rabanal (2004). Moreover, that study's conclusion: "…*VAR-based permanent shocks may indeed be capturing exogenous variations in technology, in a way consistent with the interpretation in Gali (1999)*…" (ibid, page 245) is not fully at odds with the correlations in Table 5. Still, Δbfk_t is more correlated to labor-supply shocks than to technological shocks.

BFK's near-VAR propagation. I again turn to BFK's near-VAR methodology, this time as a common framework to compare the propagation of VAR-based supply shocks to those of Δbfk_t . To do so, I define ξ_t to be a scalar denoting one of five *supply* shocks: those recovered from BQ and BE models, the technology and labor-supply shocks from the tri-variate model, and the BFK's technology shock, Δbfk_t . The near-VAR models are estimated using quarterly data with eight lags over the sample period used above; following BFK, I estimate the near-VAR model with SUR techniques.

Figure 7 depicts the dynamic responses for the labor force and wages for the five supply shocks noted.[60] The labor force increases persistently following all shocks. At face value, this is inconsistent with the modified Keynesian framework as it does not accommodate labor force movements beyond labor-supply shocks.[61] But in the real world, it is not unreasonable for labor

[60] I use changes in the log of the "civilian labor force, 16 years-old and over" (CLF16OV) (thousands of persons, seasonally adjusted) and in the log of the "average hourly earnings of production and nonsupervisory employees, total private" (AHETPI) (dollars per hour seasonally adjusted) from FRED (June 2022 vintage).

[61] Note also, that, to the extent that hours worked and the labor force are correlated, this propagation clashes with an assumption of the long-run exogeneity of hours.

participation decisions to also reflect other factors, notably future wage information implicit in technology shocks; this information may underlie the depicted labor force propagation.[62]

More intriguing, however, are the wage responses to supply shocks. Save one of the five supply shocks, wages persistently decrease in response to "supply" shocks. This suggests that the "supply" shocks recovered from the BQ and BE models and Δbfk_t all suffer from a degree of mis-aggregation as these propagate to wages as if these were labor-supply shocks.[63] Note that, to the extent that hours reflect labor-supply, wage responses to the Δbfk_t shock are consistent with the finding in Christiano, et al. (2004) that hours Granger-cause Δbfk_t . Note further that I find that wages increase persistently only to a technology shock from the tri-variate model: the tri-variate model's $e_{s,t}$ "quacks" like technology!

[62] I find supportive evidence of this conjecture using a simple extension of the near-VAR used for the labor force responses. Specifically, I include both technology and labor-supply shocks in the near-VAR model and focus on the estimates of the impact effects of these shocks, respectively $c_{2,1}^{s}$ and $c_{2,1}^{\tilde{n}}$, in the near-VAR model. In this case, the estimate for $c_{2,1}^{s}$ is cut in half compared to its estimate when only the technology shock is included. This suggests that, in the near-VAR model, the technology shock indeed contains information about the labor force. The reverse however is not true: estimates for $c_{2,1}^{\tilde{n}}$ remains virtually unchanged whether the technology shock is included in the near-VAR model or not.

[63] I also explore MA-representation distortions as an alternative explanation for the propagation of Δbfk_t shocks depicted in Figure 7. In unreported results, I take Δbfk_t shocks to be correctly "identified" and extend BFK's near-VAR approach to encompass the tri-variate model. I do this by adding to tri-variate VAR model an equation for an exogenously determined Δbfk_t shock. (I also use an analogous approach to form near-VAR models for the BQ and BE bivariate models; for completeness I also used univariate near-VAR to trace out the propagation.) The dynamic responses show that, regardless of the MA-representation employed, ur_t increases following Δbfk_t shocks. This suggests that ur_t increases are intrinsic to the BFK shock and not associated with MA-representation distortions.

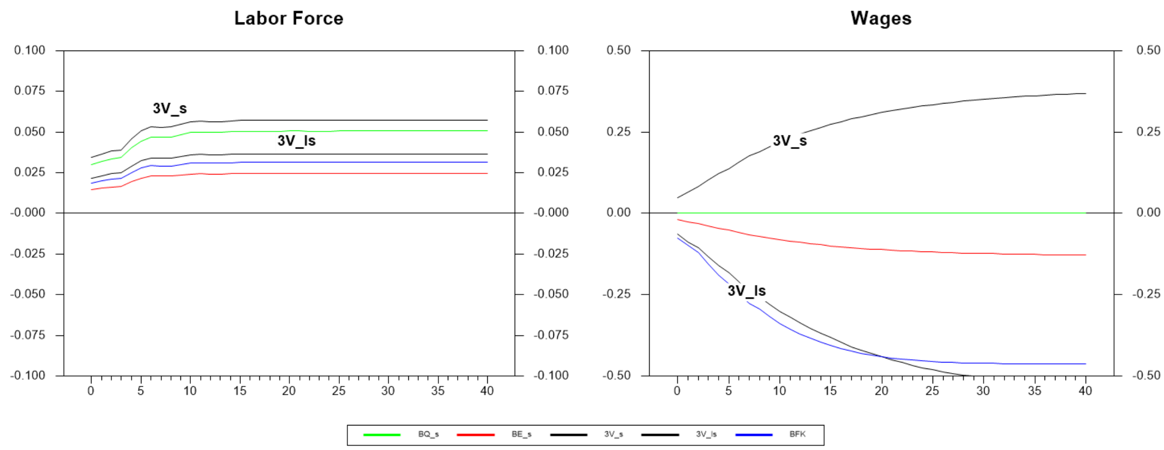

Figure 7. Labor Force and Wage responses to supply shocks (univariate near-VAR)

V. CONCLUDING REMARKS

Using a tri-variate model, I identify labor-supply, technology, and aggregate demand shocks without restricting the long-run response of the labor-market variable. I find that the IRF's biases in the bivariate BQ and BE models are primarily due to mis-aggregation of structural shocks in the former and MA-representation distortions in the latter. These assessments are supported by FL cross-type correlations, missing-variables and non-fundamentalness tests, and variance decompositions, as well as by a novel calculation of the sources of IRF's bias. This parsing methodology provides a full account of the IRF's biases with an added benefit of measuring the sources of bias at different time horizons.

For the U.S. post-World War II data, I come to two broad conclusions. First, labor market and price information should be included to respectively assess macroeconomic fluctuations and account for the demand-side factors. In other words, at least a tri-variate model is essential in this regard. This conclusion echoes FL's stern warning about the reliability and usefulness of the bivariate BQ and BE models to study macroeconomic fluctuations. And second, "supply-induced" ur_t increases reflect mis-aggregated technology and labor-supply shocks, regardless of whether the labor-market variable contains a stochastic trend or not. I also find that state-of-the-art direct measurements of technology are more correlated to, and their propagation more aligned with, labor-supply shocks.

Granted, further evidence is needed for earlier time periods and across countries to judge more broadly the weaknesses of bivariate models, evaluate the tri-variate model's ability to assess macroeconomic fluctuations, and confirm the role of labor-supply shocks in ur_t responses. I conclude that for the BQ and BE variable pairings: "two's *not* company." Whether "*three's* company" remains an open question.

Appendix I. Δ_{IRF} and BM's *Proposition 2*

To fix notation, I briefly review IRF's non-recursive notation and its relation to textbook time-series notation.

Non-recursive notation. BM express IRF's in non-recursive form (Mittnik, 1987) using lower-triangular block Toeplitz matrix notation. Specifically, for a *k*-dimensional VARMA model and up to horizon *h*, IRF's are expressed compactly in BM equation (24) (page 328) as:

$$\Psi(\Theta) = M^{-1}(\Theta)\cdot B(\Theta)\cdot \Sigma^{1/2}(\Theta)$$

where $\Psi(\cdot)$ $(k\cdot(h+1)\times k)$ contains the IRF's for all *k* variables to all *k* structural shocks up to horizon *h*.[64]

In BM, IRF's are computed as the product of three arrays:

(1) the $(k\cdot(h+1)\times k\cdot(h+1))$ Toeplitz matrix containing the MA-representation of the VAR part of the VARMA model,

$$M^{-1}(\cdot) = \begin{bmatrix} I & 0 & 0 & 0 & \cdots & 0 & 0 \\ C_1 & I & 0 & 0 & \cdots & 0 & 0 \\ C_2 & C_1 & I & 0 & \cdots & 0 & 0 \\ C_3 & C_2 & C_1 & I & \cdots & 0 & 0 \\ \vdots & \vdots & \vdots & \vdots & \ddots & \vdots & \vdots \\ C_{h-1} & C_{h-2} & C_{h-3} & C_{h-4} & \cdots & I & 0 \\ C_h & C_{h-1} & C_{h-2} & C_{h-3} & \cdots & C_1 & I \end{bmatrix}$$

(2) the $(k\cdot(h+1)\times k)$ MA part of the VARMA model, $B(\cdot)$,

[64] A reader familiar with panel data might find useful to think about $\Psi(\cdot)$ in terms of *time-specific* formulation where each variable is a "panel unit" and its columns stack the responses for the *k* "units" for each time period (horizon).

$$B(\cdot)' = \begin{bmatrix} B_0, & B_1, & B_2, & B_3, & \cdots, & B_{h-1}, & B_h \end{bmatrix}$$ and

(3) the ($k \times k$) “square-root” of the model’s covariance matrix, $\Sigma^{1/2}(\cdot)$.[65]

Each of these arrays are evaluated at Θ, the vector collecting the VARMA model’s coefficients. Note that the block-elements in $\Psi(\cdot)$, $M^{-1}(\cdot)$, and $B(\cdot)$ are of dimension ($k \times k$).

To focus on VAR models, I assume $B^I(\Theta)' = \begin{bmatrix} I & I & I & I & \cdots & I & I \end{bmatrix}$ and express $\Psi(\cdot)$ explicitly as:

$$\begin{aligned} \Psi(\cdot) &= M^{-1}(\cdot) \cdot B^I(\cdot) \cdot \Sigma^{1/2}(\cdot) \\ &= \left[\Sigma^{1/2}, \quad \left[I + C_1 \right] \cdot \Sigma^{1/2}, \quad \left[I + C_1 + C_2 \right] \cdot \Sigma^{1/2}, \quad \left[I + C_1 + C_2 + C_3 \right] \cdot \Sigma^{1/2}, \quad \cdots \quad , \quad \left[I + \sum_{s=1}^{h} C_s \right] \cdot \Sigma^{1/2} \right]' \end{aligned}$$

where each block-element in $\Psi(\cdot)$ contains the IRF’s for all k shocks for a specific horizon.

Relation to time-series notation. The block-elements in $\Psi(\cdot)$ correspond to $\Sigma^{1/2}$ pre-multiplied by arrays $\tilde{C}_i$ defined implicitly in:

$$\begin{aligned} C(L) &= \left[I - C_1 \cdot L - C_2 \cdot L^2 - \cdots - C_p \cdot L^p \right]^{-1} \\ &= I + \tilde{C}_1 \cdot L + \tilde{C}_2 \cdot L^2 + \tilde{C}_3 \cdot L^3 + \cdots \end{aligned}$$

Thus, $\Psi(\cdot)$ can be expressed as:

$$\Psi(\cdot) = \left[I \cdot \Sigma^{1/2}, \ \tilde{C}_1 \cdot \Sigma^{1/2}, \ \tilde{C}_2 \cdot \Sigma^{1/2}, \ \ldots, \ \tilde{C}_h \cdot \Sigma^{1/2} \right]',$$

which contains the IRF’s for a VAR model for all shocks up to horizon h. Note that

[65] BM’s IRF’s are the product of three elements because these are for VARMA models.

$$\tilde{C}_0 = I \text{ and } \tilde{C}_i = I + \sum_{ii=1}^{h} C_{ii},\ h \geq 1.$$

Standard time-series IRF's notation however focuses on a single shock and time horizon. To pick out to the j^{th} shock (the j^{th} column in $\Psi(\cdot)$) define the $(k \times 1)$ unit vector q_j (with "1" in the j^{th} position). Using q_j, the IRF's to the j^{th} shock can be expressed as:

$$\begin{aligned}\Psi(\cdot)_j &= \Psi(\cdot)\ \cdot q_j \\ &= \left[p_j,\ \ \tilde{C}_1 \cdot p_j,\ \ \tilde{C}_2 \cdot p_j,\ \ \tilde{C}_3 \cdot p_j,\ \ \cdots\ \ \tilde{C}_{h-1} \cdot p_j,\ \ \tilde{C}_h \cdot p_j \right]'\end{aligned}$$

where $\Psi(\cdot)_j$ is of dimension $(k \cdot (h+1) \times 1))$ as it stacks the responses to the j^{th} shock from s= 0, 1, 2, 3, …, h. To pick out the responses for period s, define the $(k \times k \cdot (h+1))$ matrix $R_s = \left[0,\ \ 0,\ \ 0,\ \ \cdots,\ \ I,\ \ \cdots,\ \ 0\right]$ that contains "h+1" block elements of dimension $(k \times k)$, all of zero except for a single $(k \times k)$ identity matrix located in rows $(k \times s + 1)$ to $(k \times (s+1))$.

Using R_s, the IRF's for shock j and period s can be expressed as:

$$\Psi(\cdot)_{j,s} = R_s \cdot \Psi(\cdot)_j = \tilde{C}_s \cdot p_j \text{ for } s = 0, 1, 2, 3, ...$$

where $\Psi(\cdot)_{j,s}$ of dimension $(k \times 1)$ is the standard IRF's for the k variables to the j^{th} shock for horizon s.

BM's *Proposition 2*.[66] To start, I first reproduce equation (26) in *Proposition 2* (BM, page 328) that details the closed-form analytical solution for IRF's bias:

$$\delta_{IRF}^{BM} = \Delta_c \cdot \Sigma^{1/2}(\Theta) + M^{-1}\left(\text{plim } \hat{\Theta}_T\right) \cdot B\left(\text{plim } \hat{\Theta}_T\right) \cdot \tilde{\Sigma}^{1/2}\ ,$$

[66] *Proposition 2* covers a broader range of VAR model (coefficient) misspecification than omitted variables, including "omitted lags" and "omitted MA-components."

where Δ_c, $\tilde{\Sigma}^{1/2}$, and $\Sigma^{1/2}(\Theta)$ denote respectively, for all shocks and up to horizon *h*, the inconsistencies of the MA-representation $(k \cdot (h+1) \times k)$, the bias of the "square-root" of covariance matrix $(k \times k)$, and the unbiased "square-root" of the covariance matrix $(k \times k)$.[67] $M^{-1}(\cdot)$ and $B(\cdot)$ are defined above. Note that this expression for the IRF's bias corresponds to a VARMA model. To focus on VAR models, I assume no missing MA components, $B\left(\text{plim}\,\hat{\Theta}_T\right) = B^I$.

To obtain the one-to-one mapping of δ_{IRF}^{BM} and $\Delta_{IFR_{j,s}}$, I take BM's equation (26) above and express it as the product rule, and then extract the *j*th shock and the corresponding response for the *s*-period.[68] I do this by using BM's general plim expression and write $M^{-1}\left(\text{plim}\,\hat{\Theta}_T\right) \cdot B^I$ as the equivalent expression $M^{-1}(\Theta) \cdot B^I + \Delta_c$ and substituting into BM's equation (26) above: [69]

$$\begin{aligned}\delta_{IRF}^{BM} &= \Delta_c \cdot \Sigma^{1/2}(\Theta) + \left\{M^{-1}(\Theta) \cdot B^I + \Delta_c\right\} \cdot \tilde{\Sigma}^{1/2} \\ &= \Delta_c \cdot \Sigma^{1/2}(\Theta) + M^{-1}(\Theta) \cdot B^I \cdot \tilde{\Sigma}^{1/2} + \Delta_c \cdot \tilde{\Sigma}^{1/2}\end{aligned}$$

where the three terms are matrices $(k \cdot (h+1) \times k)$ comprising the product-rule (for all shocks and up to period *h*). Note that these terms capture respectively the MA-representation distortion, mis-aggregation, and the cross-product distortion.

[67] The equivalency between *Proposition 2* and the expressing in main text can be obtained without reference to the explicit analytical expressions for Δ_c and $\tilde{\Sigma}^{1/2}$ detailed in BM equation (25) and Corollary 2 (BM, page 329). As noted in the main text, I use the empirical counterparts to compute the sources of IRF's biases in this paper.

[68] Of note, BM do not express equation (26) in terms of the product rule. I do so here to show the relation to the sources of IRF's bias in the main text.

[69] Specifically, BM (page 328) define inconsistencies as "*…the quantities derived from the plim* $\hat{\Theta}_T$. *For example,* $\text{plim}\Psi(\hat{\Theta}_T) = \Psi(\text{plim}\hat{\Theta}_T) = \Psi(\Theta + \tilde{\Theta}) = \Psi(\Theta) + \Delta_\Psi$, …". This definition allows me to express the term $M^{-1}\left(\text{plim}\,\hat{\Theta}_T\right) \cdot B^I$ as follows: $M^{-1}(\Theta) \cdot B^I + \Delta_c$, where $\Delta_c \equiv \Delta_\Psi \cdot B^I$.

Next, I post-multiply δ_{IRF}^{BM} by the unit vector q_j to extract the expression for the bias corresponding to the j^{th} shock (up to period h) and pre-multiply by the $(k \times k \cdot (h+1))$ R_s matrix to pick out the responses for the *s*-period. Specifically,

$$\begin{aligned}\delta_{IRFj,s}^{BM} &= R_s \cdot \left[\Delta_c \cdot \Sigma^{1/2}\left(\Theta\right) + \left\{ M^{-1}\left(\Theta\ \right) \cdot B' + \Delta_c \right\} \cdot \tilde{\Sigma}^{1/2} \right] \cdot q_j \\ &= \Delta_{c,s} \cdot p_j(\Theta) + M^{-1}\left(\Theta\ \right)_s \cdot \Delta_{\tilde{p}_j} + \Delta_{c,s} \cdot \Delta_{\tilde{p}_j}\end{aligned},$$

where $\Delta_{c,s}$, $p_j(\Theta)$, $\Delta_{\tilde{p}_j}$, and $M^{-1}\left(\Theta\right)_s$ denote respectively the *s*-period responses contained in Δ_c, the j^{th} columns of $\Sigma^{1/2}\left(\Theta\right)$ and $\tilde{\Sigma}^{1/2}$, and the s^{th} block from $M^{-1}\left(\Theta\right)$.

And using the vector IRF's function notation from the main text:

$$\delta_{IRFj,s}^{BM} = \boldsymbol{IRF}_{j,s}\left(\Delta_{c,s}, p_j(\Theta)\right) + \boldsymbol{IRF}_{j,s}\left(M^{-1}(\Theta)_s, \Delta_{\tilde{p}_j}\right) + \boldsymbol{IRF}_{j,s}\left(\Delta_{c,s}, \Delta_{\tilde{p}_j}\right),$$

where the terms on the right-hand side reflect respectively the MA-representation distortion, mis-aggregation bias, and the cross-product distortion. Save the order of the terms and obvious differences in the nomenclature, this expression provides the one-to-one correspondence of BM's *Proposition 2* and IRF's bias decomposition in the main text.

APPENDIX II. ROBUSTNESS

Here I discuss the robustness of the results of cross-type FL correlations, the missing-variable and fundamentalness tests, the IRF's of the bivariate and tri-variate models, duck tests, variance decompositions, and comparisons to Δbfk_t. I recompute these for three cases. First, all VAR models contain two lags in line with the Hannan-Quinn (HQ) Criterion (Table A1). Ivanov and Kilian (2005) find that the HQ criterion is best able to select the lag length that minimizes the distortions in the resulting IRF's when the estimation is based on at least 120 quarterly observations; for shorter quarterly samples the Schwarz Information Criterion (SIC) is preferred. Second, the relevant VAR models include deterministically detrended ur_t consistent with the base-case results in BQ and FL. Note that in this case, even though the estimates for the BE model do not change (as it omits ur_t), the BE model's correlations and statistical tests can because these reference ur_t. And third, the models are estimated using an extended sample period through 2009:04 using data downloaded from FRED (October 2021 vintage): GNP, UNRATE (number of people 16 and over actively searching for a job as a percentage of total labor force), and GDPDF. The quarterly unemployment series averages the monthly series.

Table A1. Lag Order Selection Analysis for BQ, BE, and tri-variate VAR models.

	Akaike Information Criterion			Hannan-Quinn Criterion			Schwarz Information		
	(AIC)			(HQ)			Criterion (SIC)		
	BQ	BE	Tri-variate	BQ	BE	Tri-variate	BQ	BE	Tri-variate
Lag length									
1	-2.461	-1.290	-3.697	-2.381	-1.210	**-3.517** *	-2.429	-1.257	-3.624
2	-2.544	-1.375	-3.841	**-2.384** *	**-1.215** *	-3.482	**-2.479** *	**-1.310** *	**-3.695** *
3	**-2.557** *	-1.374	-3.842	-2.317	-1.134	-3.303	-2.459	-1.277	-3.623
4	-2.523	-1.336	-3.772	-2.203	-1.016	-3.053	-2.393	-1.206	-3.480
5	-2.521	**-1.402** *	**-3.869** *	-2.122	-1.003	-2.970	-2.359	-1.240	-3.504
6	-2.512	-1.356	-3.852	-2.033	-0.876	-2.773	-2.317	-1.161	-3.414
7	-2.482	-1.335	-3.815	-1.923	-0.775	-2.556	-2.255	-1.107	-3.304
8	-2.467	-1.312	-3.732	-1.828	-0.673	-2.293	-2.207	-1.052	-3.147

Note. The criterium are calculated for a fixed sample of 150 quarterly observations (1950:Q2 to 1987:Q4) using formulas detailed in Ivanov and Kilian (2005). I rely on HQ because Monte Carlo experiments suggest it results in the most accurate IRF's for a quarterly data sample exceeding 120 observations (see Ivanov and Kilian, 2005) . An asterisk symbol denotes the selected lag length for individual criteria and VAR models.

A. FL cross correlation of structural shock.

(*Case 1*) Setting the number of lags to two, I find structural shock correlations that are very similar to the base-case results (Table A2, panel A). As above, the correlations for the BQ variable pairing point to high correlation of its supply shock with the tri-variate model's "technology" and labor-supply, with limited correlation to temporary shocks. Also, its demand shock is correlated with all three of the tri-variate models structural shocks. Likewise, the correlations for the BE variable pairing point to high correlation of the model's supply and demand shocks with their tri-variate model's counterparts and limited cross-type correlation; the BE model's shocks remain "empirical" orthogonal to labor-supply shocks. (*Case 2*) Using detrended (instead of means-removed) ur_t in the relevant VAR models results in several revisions in the structural shock correlations (Table A2, panel B). For the BQ model, its supply shocks are less with the tri-variate model's technology should but more correlated to the labor-supply shock. For the BE model, both supply and demand shocks exhibit far higher cross-type correlation, with the latter's correlation pattern changes sign! The missing-variable and fundamentalness tests results discussed below go a long way in explaining these changes. (*Case*

3) For the extended sample period, the bivariate models virtually replicate the base-case results in the main text.

Table A2. Correlation of Structural Shocks, Robustness

	Tri-variate model		
	Permanent	Labor Supply	Temporary
A. VAR models with two lags			
Blanchard-Quah:			
Permanent	0.40	0.90	0.14
Temporary	0.74	-0.40	0.54
Bayoumi-Eichengreen:			
Permanent	0.89	0.00	0.27
Temporary	-0.27	0.00	0.96
B. Detrended unemployment rate			
Blanchard-Quah:			
Permanent	0.10	0.94	0.26
Temporary	0.78	-0.17	0.42
Bayoumi-Eichengreen:			
Permanent	0.09	0.00	0.88
Temporary	0.94	0.00	-0.15
C. Extended sample			
Blanchard-Quah:			
Permanent	0.53	0.79	0.27
Temporary	0.62	-0.59	0.45
Bayoumi-Eichengreen:			
Permanent	0.93	0.00	0.08
Temporary	-0.11	0.00	0.97

Note. Structural shocks are identified using the long- and short-run restrictions discussed in the main text for the tri-variate model and long-run restrictions in Blanchard-Quah and Bayoumi-Eichengreen. The VAR models used for Panels A and B are estimated using the sample period in Blanchard-Quah (1950:02 to 1987:04). The VAR models used in Panel C are estimated using the sample period 1950:02 to 2009:04.

B. Missing-variable and fundamentalness tests (Table A3)

(*Case 1*) Decreasing the number of lags to two, results in very similar missing-variable and fundamentalness test results as those in the main text. But now the BQ model only fails the GC test; the BE model continues to fail all four tests. (*Case 2*) Linear detrending of ur_t wreaks havoc on the test results: both the BQ and the BE models reject three out of four tests! (The common exception is the non-rejection of the CH test.) These results can explain the dramatic changes in the correlation of structural shocks above and serve as a flag not to place too much credence on the results based on a linearly detrended ur_t discussed below. It is precisely because of this that I take as my base-case not to be BQ's base-case. (*Case 3*) Extending the sample through 2009:04 the BQ model now fails three of the four test (it passes the Sims test) and the BE model continues to fail all four tests.

Table A3. Missing Third Variable and Fundamentalness Tests for BQ and BE models, Robustness

A. BQ model (Third variable: Δp)						*B. BE model (Third variable: ur_t)*					
Two lags		Detrended ur_t		Extended Sample		Two lags		Detrended ur_t		Extended Sample	
1-Granger causality tests for:											
F(4,143)	4.792	F(16,101)	36.257	F(16,181)	27.129	F(4,101)	24.617	F(16,101)	15.514	F(16,181)	63.345
M-Signif	0.00 *	M-Signif	0.00 *	M-Signif	0.00 *	M-Signif	0.00 *	M-Signif	0.00 *	M-Signif	0.00 *
2-Sims-exogeneity test of:											
F(4,143)	1.114	F(16,101)	2.2794	F(16,181)	1.045	F(4,101)	5.506	F(16,101)	1.974	F(16,181)	3.064
M-Signif	0.35	M-Signif	0.01 *	M-Signif	0.41	M-Signif	0.00 *	M-Signif	0.02 *	M-Signif	0.00 *
3-Orthogonality (Forni-Gambetti) test for:											
$\varepsilon_t^{BQ,S} + \varepsilon_t^{BQ,D}$						$\varepsilon_t^{BE,S} + \varepsilon_t^{BE,D}$					
F(4,141)	0.454	F(16,93)	3.65	F(16,181)	2.020	F(4,141)	7.345	F(16,93)	3.154	F(16,181)	5.445
M-Signif	0.77	M-Signif	0.00 *	M-Signif	0.01 *	M-Signif	0.00 *	M-Signif	0.00 *	M-Signif	0.00 *
4-Canova-Hamidi test for:											
$\mu_t^{BQ,\Delta y} + \mu_t^{BQ,ur}$						$\mu_t^{BE,\Delta y} + \mu_t^{BE,\Delta p}$					
F(4,141)	1.104	F(16,93)	1.535	F(16,181)	1.896	F(4,141)	4.989	F(16,93)	1.1712	F(16,181)	3.084
M-Signif	0.36	M-Signif	0.10	M-Signif	0.02 *	M-Signif	0.00 *	M-Signif	0.31	M-Signif	0.00 *

Note. The Granger-causality tests the lags of the corresponding missing third variable in an augmented bivariate VAR model. The Sims-exogeneity tests the leads of the BQ or BE variables in the regression where the third variable is regressed on its lags and the leads and lags of the corresponding bivariate model. The Forni-Gambetti test is a type of Granger-causality test of the third variable in VAR model whose variables are the corresponding structural shocks. The Canova-Hamidi test is a type of Sims-exogeneity test that tests the leads of the BQ or BE variables in a the regression where the the third variable is regressed on its lags and the leads and lags of the reduced-form residuals from the corresponding bivariate model. The reduced-form residuals and the structural shocks are obtained by estimating the BQ and BE models with eight lags or two lags and detrended ur (both over the sample 1950:02 to 1987:04); the extended sample covers data from 1950:02 to 2009:04. An "asterik" indicates

C. IRF's relative to the tri-variate model (Figures A1 to A3)

(Case 1) Decreasing lags to two in all VAR models, three out of four of the BQ model's IRF's compared to the tri-variate model remain broadly unchanged (Figure A1). The exception is the y_t response to supply shocks, which now lies outside of the tri-variate model's confidence bands. For the BE model, the IRF's are comparable to the base-case, though the p_t response to supply shocks no longer shows the persistent deviation from the tri-variate model's IRF's. (*Case 2*) For completeness, I provide the IRF's using (in the relevant VAR models) detrended ur_t (Figure A2). But, as noted, these results are best taken with a grain of salt as both bivariate models exhibit missing variables and fail fundamentalness tests. Note that since the BE does not include ur_t, its IRF's are unchanged. Note further, that the tri-variate model's IRF's continue to show that ur_t declines (increases) following supply (labor-supply) shocks but p_t fail to decline as expected. (*Case 3*) Extending the sample through 2009:04 does not qualitatively change the conclusions for the BQ model as its IRF's closely resemble those in the base-case results. Though a bit more deviation is observed in the IRF's following demand shocks (Figure A3). For the BE model, its p_t response to supply shocks are more aligned to the tri-variate model.

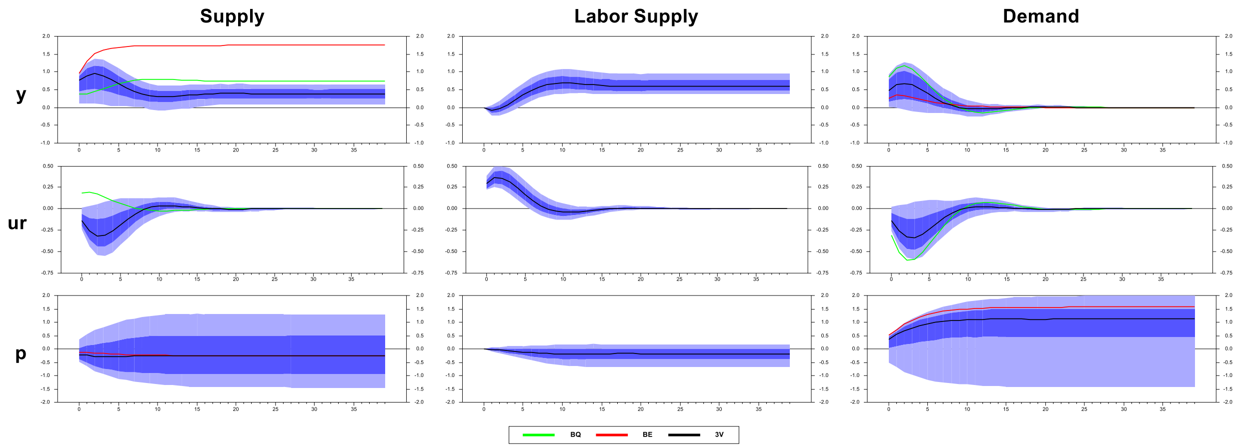

Figure A1. IRF's for the Tri-variate model (with shaded 68% and 95% bands), BQ, and BE models (two lags)

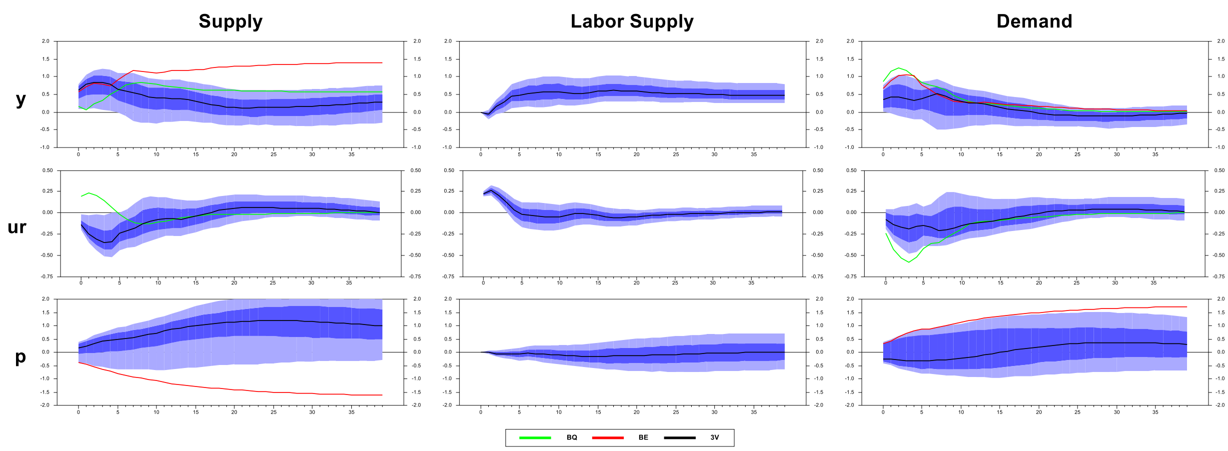

Figure A2. IRF's for the Tri-variate model (with shaded 68% and 95% bands), BQ, and BE models (ur detrend)

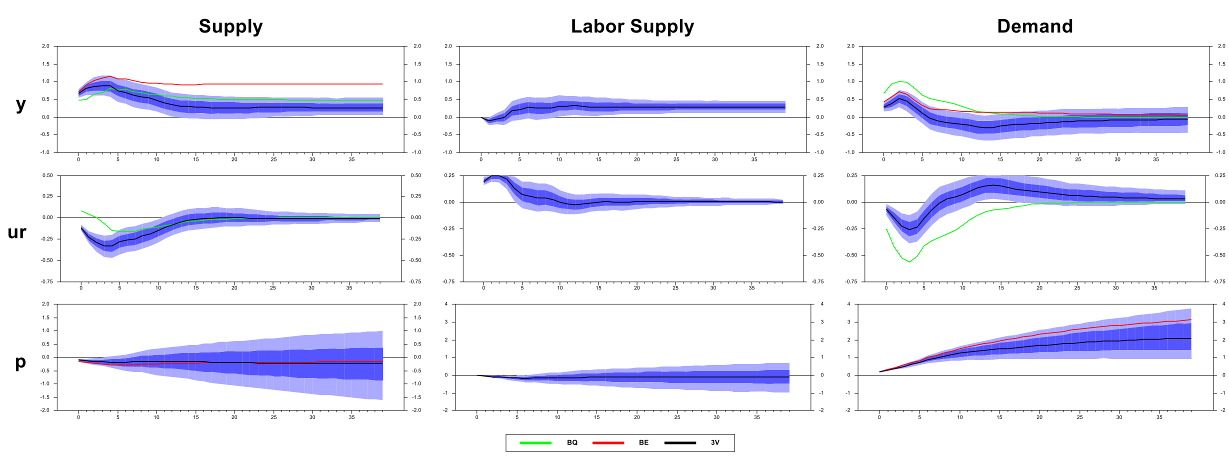

Figure A3. IRF's for the Tri-variate model (with shaded 68% and 95% bands), BQ, and BE models (extended sample)

D. "Duck-tests"

The dynamic responses of wages and the labor force (Figures A4 to A6): (*Case 1*) Decreasing lags to two in the univariate near-VAR models, does not qualitatively change the results for wages but it now the labor force responses of demand and supply are the opposite (see Figure 3). (*Case 2*) The propagation for wages is unchanged qualitatively but, as in *Case 1*, the labor force responses are the opposite as the base-case. (*Case 3*) The responses of wages and labor force are qualitatively unchanged.

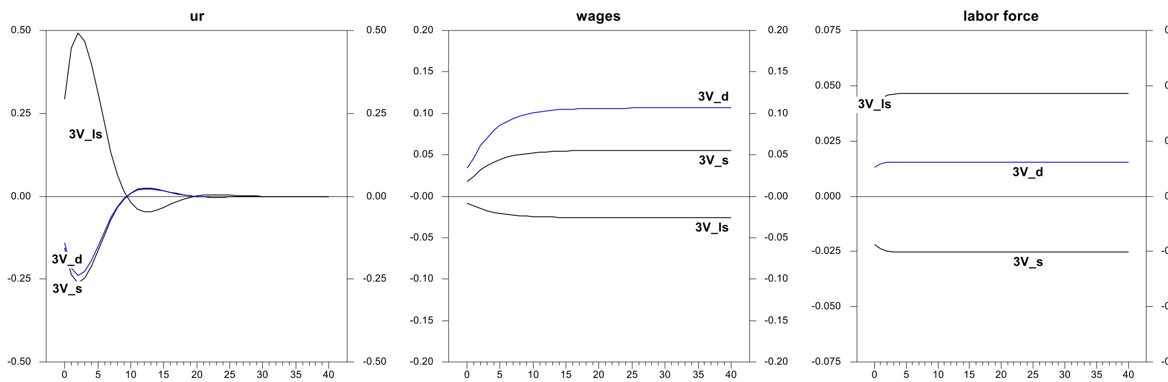

Figure A4. Labor market response to tri-variate shocks (near-VAR, two lags)

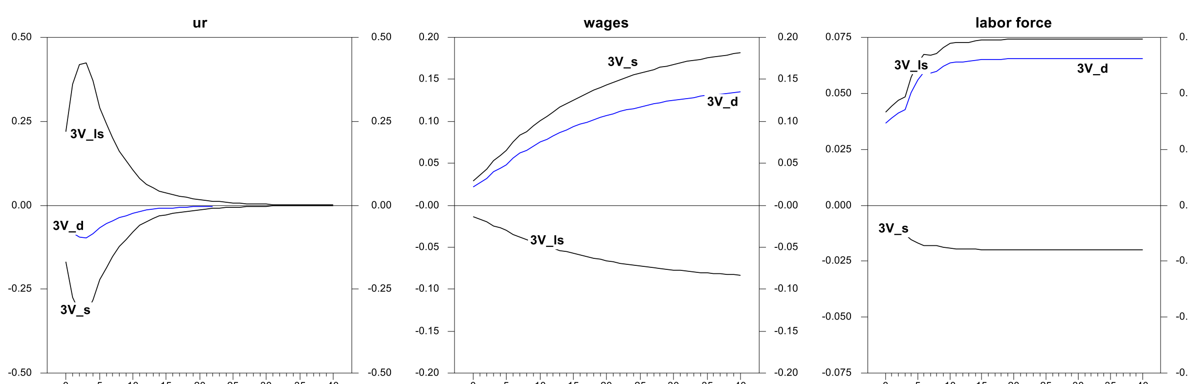

Figure A5. Labor market response to tri-variate shocks (near-VAR, detrended ur)

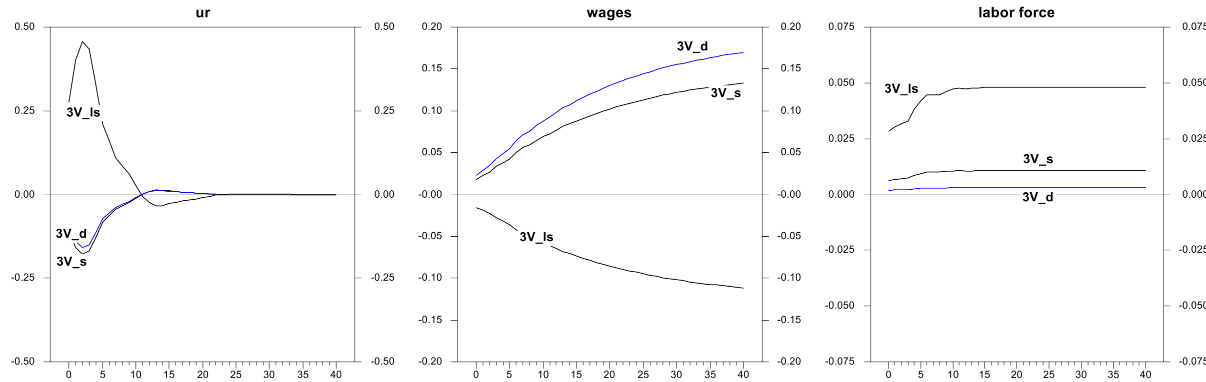

Figure A6. Labor market response to tri-variate shocks (near-VAR, extended sample)

E. Variance decompositions (Table A4)

(*Case 1*) Decreasing the number of lags to two, does not qualitatively change the stylized fact that the BQ model overstates the importance of demand shocks (Table A4, panel A). In contrast to the base-case, the BE model now understates the importance of demand for output and overstates their importance for p_t fluctuations. As in the base-case, the variance decomposition for the tri-variate model continues to point to the importance of labor-supply shocks in the short-run and supply shocks as the forecast horizon is extended in explaining movements in ur_t . The supply-demand split continues to be consistent with SW and, as in the base case, the labor-supply shocks play a minor role in output and price fluctuations. (*Case 2*) For completeness, I provide the variance decomposition when detrended ur_t is used in the relevant VAR models results (Table A4, panel B). Qualitatively, these show similar results as those form the base-case, that is, BQ and BE overstate the importance of demand shocks. But now the BE model tends to understate the importance of demand shocks for p_t fluctuations in the short-run, while overstating their importance in the long-run. However, as noted, these results should be taken with a grain of salt as both bivariate models fail missing variables and fundamentalness tests.

Regarding the tri-variate model, movements in ur_t are associated with labor-supply shocks in the short-run with the importance of supply shocks increase at longer horizons. (*Case 3*). Extending the sample through 2009:04 does not qualitatively change the conclusions regarding the importance of demand shocks being overstated in the BQ model. But for the BE model it does change output. And for the tri-variate model, fluctuations in ur_t continue to be dominated by labor-supply and supply shocks respectively in the short- and longer-term.

Table A4 Variance Decompositions (percent due to structural shocks)

A. Two lags

	Permanent					Demand		
	Trivariate			BQ	BE	Trivariate	BQ	BE
	Supply	Labor	Sum					
				y_t				
Horizon								
0	70.17	0.00	70.2	16.5	93.1	29.8	83.45	6.91
1	66.53	0.34	66.9	13.2	93.0	33.1	86.81	7.03
2	65.17	0.25	65.4	13.2	94.2	34.6	86.79	5.84
3	64.30	0.26	64.6	14.9	95.1	35.4	85.08	4.90
4	63.58	0.87	64.5	17.9	95.9	35.5	82.14	4.10
8	56.98	12.75	69.7	35.3	97.7	30.3	64.66	2.28
12	47.96	28.07	76.0	48.9	98.5	24.0	51.13	1.51
24	38.03	46.78	84.8	67.3	99.3	15.2	32.72	0.74
40	32.94	56.85	89.8	77.9	99.6	10.2	22.12	0.44
				ur_t				
Horizon								
0	18.43	66.22	84.6	26.2	...	15.4	73.76	...
1	23.64	53.29	76.9	16.7	...	23.1	83.30	...
2	26.41	45.99	72.4	12.3	...	27.6	87.68	...
3	27.51	41.65	69.2	10.0	...	30.8	89.96	...
4	27.87	38.89	66.8	8.8		33.2	91.23	
8	27.41	35.34	62.8	7.5	...	37.3	92.49	...
12	27.45	35.36	62.8	7.6	...	37.2	92.42	...
24	27.51	35.33	62.8	7.5	...	37.2	92.46	...
40	27.51	35.33	62.8	7.5	...	37.2	92.46	...
				p_t				
Horizon								
0	19.96	0.00	20.0	...	4.0	80.0	...	95.95
1	15.70	0.00	15.7	...	2.7	84.3	...	97.35
2	14.61	0.02	14.6	...	2.4	85.4	...	97.56
3	13.44	0.06	13.5	...	2.3	86.5	...	97.71
4	12.42	0.15	12.6		2.2	87.4		97.75
8	8.95	0.73	9.7	...	2.2	90.3	...	97.80
12	6.97	1.19	8.2	...	2.2	91.8	...	97.80
24	5.32	1.54	6.9	...	2.2	93.1	...	97.80
40	4.84	1.63	6.5	...	2.2	93.5	...	97.79

Note. All VAR models are estimated using eight lags over the sample period 1950:02 to 1987:04. Before estimation I remove the sample means for the three variables before and after the first OPEC oil shock (1973:04). This treatment of the data series is consistent with the BE sample split but differs from BQ's base case. Nonetheless, differences are quantitative in nature. Structural shocks are identified using short- and long-run restrictions as discussed in the main text.

Table A4 Variance Decompositions (percent due to structural shocks) (continued)

B. Detrended ur

	Permanent					Demand		
	Trivariate			BQ	BE	Trivariate	BQ	BE
	Supply	Labor	Sum					
				y_t				
Horizon								
0	74.3	0.0	74.3	3.2		25.7	96.84	58.02
1	76.3	0.8	77.1	1.3		22.9	98.71	59.39
2	79.2	0.9	80.1	2.3		19.9	97.72	60.66
3	80.5	2.1	82.6	3.7		17.4	96.30	61.91
4	79.1	5.9	85.1	6.9		14.9	93.07	62.77
8	57.8	18.9	76.7	25.7		23.3	74.33	44.62
12	48.0	26.6	74.6	37.1		25.4	62.87	33.34
24	37.0	41.7	78.7	52.2		21.3	47.84	17.47
40	32.2	49.8	81.9	62.6		18.1	37.36	9.82
				ur_t				
Horizon								
0	35.5	61.4	96.8	38.7	…	3.2	61.29	…
1	42.9	50.8	93.7	28.1	…	6.3	71.85	…
2	52.4	40.1	92.5	20.7	…	7.5	79.31	…
3	60.4	31.2	91.6	15.6	…	8.4	84.45	…
4	66.1	25.2	91.3	12.6		8.7	87.44	
8	58.4	16.8	75.3	10.8	…	24.7	89.22	…
12	49.4	14.8	64.1	12.1	…	35.9	87.89	…
24	45.8	13.3	59.1	12.2	…	40.9	87.75	…
40	47.0	12.3	59.3	12.3	…	40.7	87.74	…
				p_t				
Horizon								
0	36.0	0.0	36.0	…	54.8	64.0	…	45.23
1	40.8	0.1	40.9	…	51.3	59.1	…	48.74
2	44.8	0.2	45.0	…	48.1	55.0	…	51.93
3	47.9	0.3	48.2	…	46.3	51.8	…	53.68
4	50.6	0.3	50.9		45.0	49.1		55.03
8	58.3	0.2	58.5	…	46.4	41.5	…	53.61
12	68.1	0.2	68.2	…	46.5	31.8	…	53.53
24	88.1	0.1	88.2	…	46.3	11.8	…	53.67
40	93.0	0.1	93.1	…	46.3	6.9	…	53.74

Note. All VAR models are estimated using eight lags over the sample period 1950:02 to 1987:04. Before estimation I remove the sample means for the three variables before and after the first OPEC oil shock (1973:04). This treatment of the data series is consistent with the BE sample split but differs from BQ's base case. Nonetheless, differences are quantitative in nature. Structural shocks are identified using short- and long-run restrictions as discussed in the main text.

Table A4 Variance Decompositions (percent due to structural shocks) (concluded)

C. Extended sample

y_t

	Permanent					***Demand***		
	Trivariate			BQ	BE	Trivariate	BQ	BE
	Supply	Labor	Sum					
Horizon								
0	70.32	0.00	70.3	35.4	79.9	29.68	64.59	20.13
1	66.45	0.88	67.3	27.9	77.3	32.66	72.09	22.72
2	62.11	0.55	62.7	28.6	74.2	37.34	71.36	25.79
3	61.73	0.39	62.1	30.1	74.2	37.89	69.91	25.78
4	62.26	1.08	63.3	34.7	75.2	36.66	65.28	24.84
8	68.59	5.19	73.8	46.3	82.6	26.22	53.72	17.37
12	68.50	9.72	78.2	51.8	86.2	21.78	48.17	13.82
24	62.54	19.00	81.5	60.9	91.3	18.46	39.11	8.74
40	57.46	27.27	84.7	68.3	94.4	15.28	31.72	5.57

ur_t

Horizon	Supply	Labor	Sum	BQ	BE	Trivariate	BQ	BE
0	19.29	67.91	87.2	9.3	...	12.81	90.69	...
1	25.80	52.51	78.3	3.4	...	21.69	96.64	...
2	29.02	41.88	70.9	1.7	...	29.10	98.35	...
3	33.19	34.17	67.4	2.1	...	32.64	97.91	...
4	36.99	28.15	65.1	4.2		34.86	95.77	
8	48.82	20.56	69.4	9.5	...	30.62	90.48	...
12	53.60	18.11	71.7	11.0	...	28.29	89.02	...
24	50.45	16.03	66.5	11.1	...	33.51	88.86	...
40	48.95	15.47	64.4	11.1	...	35.59	88.86	...

p_t

Horizon	Supply	Labor	Sum	BQ	BE	Trivariate	BQ	BE
0	36.34	0.00	36.3	...	28.5	63.66	...	71.53
1	34.71	0.04	34.8	...	26.0	65.25	...	74.02
2	30.30	0.64	30.9	...	22.0	69.07	...	77.98
3	27.90	1.05	28.9	...	19.5	71.06	...	80.52
4	25.34	1.41	26.8		16.9	73.25		83.07
8	22.65	1.40	24.0	...	13.8	75.96	...	86.23
12	22.73	0.93	23.7	...	13.4	76.34	...	86.61
24	22.54	0.68	23.2	...	13.1	76.79	...	86.95
40	21.11	0.66	21.8	...	12.9	78.23	...	87.08

Note. All VAR models are estimated using eight lags over the sample period 1950:02 to 1987:04. Before estimation I remove the sample means for the three variables before and after the first OPEC oil shock (1973:04). This treatment of the data series is consistent with the BE sample split but differs from BQ's base case. Nonetheless, differences are quantitative in nature. Structural shocks are identified using short- and long-run restrictions as discussed in the main text.

F. Δbfk_t and VAR-based Structural Shocks

FL "cross-type" correlations (Table A5). (*Case 1*) The pattern of the FL structural shock correlations with two lags remains unchanged: the highest correlation of Δbfk_t is with the BQ supply shock, and Δbfk_t remains more correlated to the tri-variate model's labor-supply shocks than to technological shocks. Also, it continues to exhibit limited correlation with the temporary/demand shocks. (*Case 2*) Though the pattern remains when using detrended ur_t . As noted above, these results should be taken with a grain of salt. (*Case 3*) The highest correlation remains to the BQ supply shock but now the correlation with the supply shocks from the BE and tri-variate models have increased. And because of these increases, these correlations now exceed those with the labor-supply shock, which remains as strong as before.

Table A5. Correlation of BFK shocks with VAR-based shocks, Robustness

	Permanent	Labor supply	Temporary
A. Two lags			
Blanchard-Quah	0.24	...	0.00
Bayoumi-Eichengreen	0.09	...	-0.07
Tri-variate model	0.14	0.19	-0.04
B. Detrended ur			
Blanchard-Quah	0.19	...	0.03
Bayoumi-Eichengreen	0.09	...	-0.05
Tri-variate model	-0.01	0.18	0.15
C. Extended sample			
Blanchard-Quah	0.28	...	0.00
Bayoumi-Eichengreen	0.23	...	-0.09
Tri-variate model	0.23	0.19	-0.10

Note. Quarterly BFK shocks are taken from Fernald (2014). VAR-based structural shocks are identified using the long- and short-run restrictions discussed in the main text for the tri-variate model and the long-run restrictions in Blanchard-Quah and Bayoumi-Eichengreen. VAR models are estimated using eight lags using quarterly data from 1950:Q2 to 1987:Q4

The dynamic responses of the labor force and wages. (Figures A7 to A9): (*Case 1*) Reducing the lags reverses the puzzling wage responses to the supply shocks identified by the BQ and BE models: wages now increase. The results for Δbfk_t (and the tri-variate model shocks) are unchanged. (*Cases 2 and 3*) These remain unchanged qualitatively, except for the supply shocks from the BQ model that now results in wage declines.

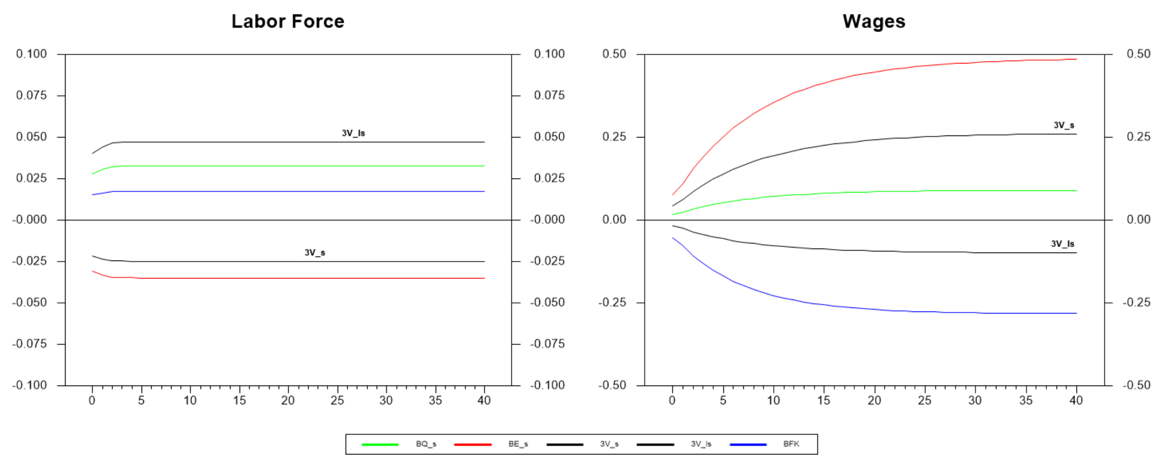

Figure A7. Labor Force and Wage responses to supply shocks (univariate near-VAR) (two lags)

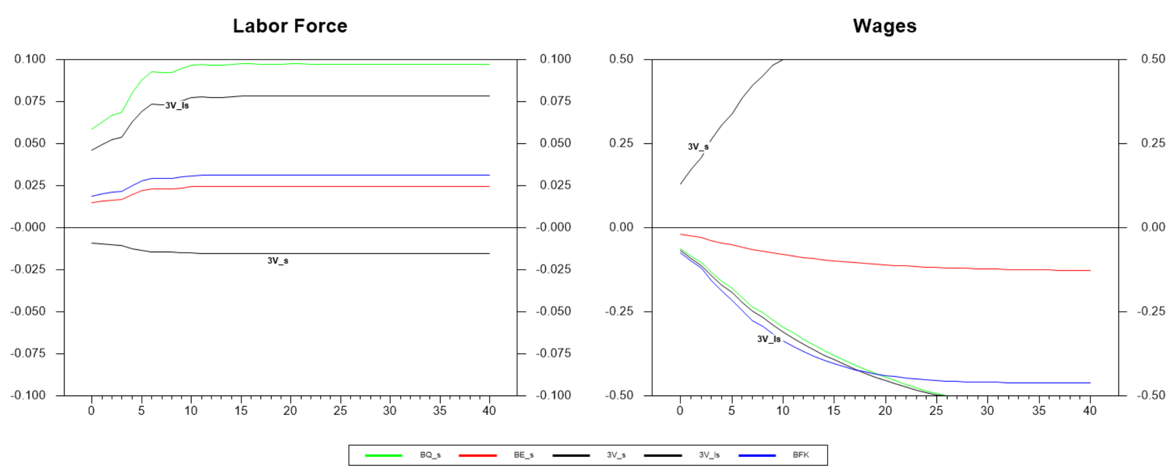

Figure A8. Labor Force and Wage responses to supply shocks (univariate near-VAR) (extended sample)

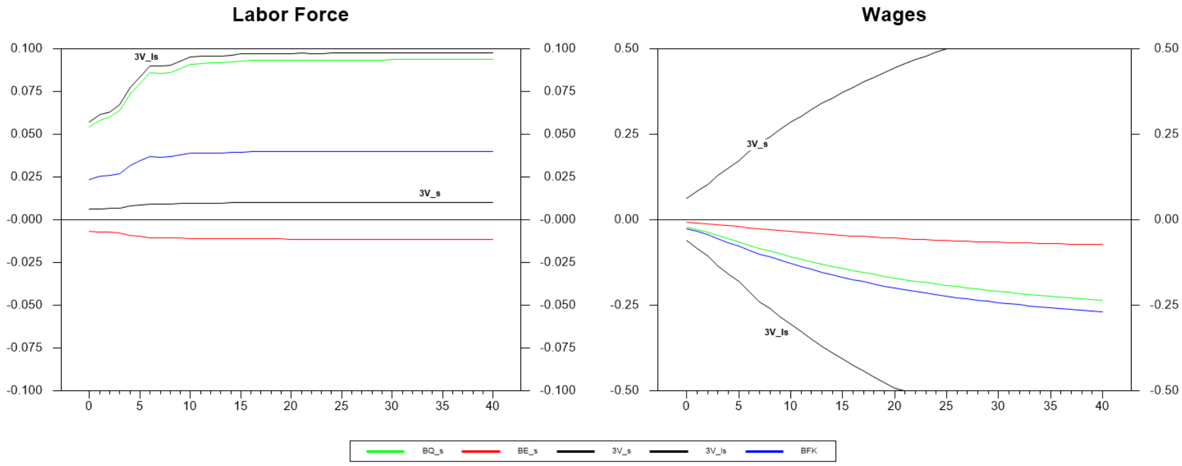

Figure A9. Labor Force and Wage responses to supply shocks (univariate near-VAR) (extended sample)

Appendix III. Missing-Variable and Fundamentalness Tests

Here I detail the formulas for the four tests reported in Tables 3 and A3. Namely, the three tests discussed in CH to check for missing variables and assess fundamentalness, plus the Sims test as modified by Geweke, Meese, and Dent (1982) (henceforth GMD).

Consider the partitioned tri-variate model in the main text:

$$\left[I - C(L)\right] \cdot \begin{bmatrix} Y_{1,t} \\ Y_{2,t} \end{bmatrix} = U_t^{3V} ,$$

where $Y_{1,t}$ and $Y_{2,t}$ contain respectively the BQ (or BE) variable pairing and the corresponding "third" variable. As noted, the information set is limited to that spun by $Y_{1,t}$ and $Y_{2,t}$.

Granger-causality test*:*

System-wide:

$$Y_{1,t} = C^{(1,1)}(L) \cdot Y_{1,t-1} + C^{(1,2)}(L) \cdot Y_{2,t-1}$$

F-test for $C^{(1,2)}(L) = 0$

Equation-by-equation:

$$Y_{1,t}(i) = C_i^{(1,1)}(L) \cdot Y_{1,t-1}(i) + C_i^{(1,2)}(L) \cdot Y_{2,t-1} \text{ for } i\text{=1, 2}$$

F-test for $C_i^{(1,2)}(L) = 0$

Sims-exogeneity test (GMD version):[70]

System-wide:

$$Y_{2,t} = C^{(2,2)}(L)\cdot Y_{2,t-1} + C^{(2,1)}(L)\cdot Y_{1,t} + D^{(2,1)}(F)\cdot Y_{1,t+1}$$

F-test for $D^{(2,1)}(F) = 0$

Equation-by-equation:

$$Y_{2,t} = C^{(2,2)}(L)\cdot Y_{2,t-1} + C_i^{(2,1)}(L)\cdot Y_{1,t}(i) + D_i^{(2,1)}(F)\cdot Y_{1,t+1}(i) \text{ for } i\text{=1, 2}$$

F-test for $D_i^{(2,1)}(F) = 0$

Orthogonality (Forni-Gambetti) test:

System-wide:

$$E_{1,t} = H(L)\cdot E_{1,t-1} + G(L)\cdot Y_{2,t-1}$$

F-test for $G(L) = 0$

Equation-by-equation:

$$e_{i,t} = H_i\,(L)\cdot e_{i,t-1} + G_i\,(L)\cdot Y_{2,t-1} \text{ for } i\text{= s, d}$$

F-test for $G_i\,(L) = 0$

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[70] Note that the lead elements in $C_i^{(1,2)}$ and $C^{(1,2)}$ are respectively 1 and *I*.

Canova-Hamidi test (GMD version):

System-wide:

$$Y_{2,t} = C^{(2,2)}(L) \cdot Y_{2,t-1} + G(L) \cdot U_{1,t} + \tilde{G}(F) \cdot U_{1,t+1}$$

F-test for $\tilde{G}(F) = 0$

Equation-by-equation:

$$Y_{2,t} = C^{(2,2)}(L) \cdot Y_{2,t-1} + G_i(L) \cdot \mu_{i,t} + \tilde{G}_i(F) \cdot \mu_{i,t+1} \text{ for } i=1,2$$

F-test for $\tilde{G}_i(F) = 0$

APPENDIX IV. TRI-VARIATE MODEL SOLUTION

The augmented illustrative Keynesian model can be expressed as:

$$y_t = m_t - p_t + \theta \cdot a_t \qquad \text{(aggregate demand equation)}$$

$$y_t = n_t + a_t \qquad \text{(production function equation)}$$

$$p_t = w_t - a_t + \lambda \cdot m_t \qquad \text{(price-setting equation)}$$

$$w_t = w| \; \left\{ E_{t-1}\left[\left(\tilde{n}_t - n_t \right) \right] = 0 \right\} \qquad \text{(wage-setting equation)}$$

$$ur_t = \tilde{n}_t - n_t \qquad \text{(unemployment definition)}$$

where y_t, m_t, p_t, a_t, and n_t denote the logs of output, money, prices, productivity, and employment, with (the log of) full employment denoted by $\tilde{n}_t$. As in the original BQ model, aggregate demand is a function of real balances and productivity, while production reflects employment and productivity. But prices, in addition to reflecting wages and productivity, now also reflect m_t. Wages are set one-period in advance, as in the BQ model, such that full employment is achieved (in expected terms). I define $\Delta a_t = e_{s,t}$ and $\Delta m_t = e_{d,t}$ as the technology and demand shocks consistent with BQ, and add a labor-supply shock by replacing the BQ's original model (implicit assumption) $\Delta \tilde{n}_t = 0$ with the assumption that it follows a random walk, $\Delta \tilde{n}_t = e_{\tilde{n},t}$, where $e_{\tilde{n},t}$ denotes labor-supply shocks. Thus, labor-supply shocks are modeled as in SW equation (2.1).

Note that, in terms of the involuntary unemployment model in Christiano et al. (2021) a positive $e_{\tilde{n},t}$ can be interpretated as a low "adversion-to-work" realization prompting increases in labor participation and the labor force that, given the prevailing wage, increases (involuntary) ur_t .

Also, in terms of the model's discussed in Hall (2008), $e_{\tilde{n},t}$ can be interpretated as a "labor-*force* supply" shock. This requires re-interpreting unemployment, for a given wage rate, as the difference between the labor-force supply (n^{LF}) and the equilibrium employment (n_t^* ,) from the labor market (the equality of labor demand and employment supply). In this case, "equilibrium" wages would be set to be consistent with a labor market equilibrium that (in expected terms) results in a constant "natural" rate of unemployment:

$$w_t = w| \ \left\{E_{t-1}\left[\left(n_t^{NF} - n_t^*\right)\right] = ur^{(natural)}\right\}$$

Regardless, the augmented illustrative Keynesian model can be expressed in tri-variate form as:

$$\begin{bmatrix} 1 & 0 & 1 \\ 1 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} \cdot \begin{bmatrix} y_t \\ ur_t \\ p_t \end{bmatrix} = \begin{bmatrix} m_t + \theta \cdot a_t \\ \tilde{n}_t + a_t \\ w_t - a_t + \lambda \cdot m_t \end{bmatrix}$$

or:

$$\begin{bmatrix} y_t \\ ur_t \\ p_t \end{bmatrix} = \begin{bmatrix} 1 & 0 & -1 \\ -1 & 1 & 1 \\ 0 & 0 & 1 \end{bmatrix} \cdot \begin{bmatrix} m_t + \theta \cdot a_t \\ \tilde{n}_t + a_t \\ w_t - a_t + \lambda \cdot m_t \end{bmatrix}$$

so that the tri-variate model's equations can be expressed as:

$$y_t = (1-\lambda) \cdot m_t + (1+\theta) \cdot a_t - w_t$$

$$ur_t = -m_t - \theta \cdot a_t + \tilde{n}_t + w_t + \lambda \cdot m_t$$

$$p_t = w_t - a_t + \lambda \cdot m_t$$

The full solution follows from replacing w_t with model-consistent wage expectations, w_t^*. The latter obtained by taking the expectation of w_t (at time *t-1*) using the ur_t equation (solved for wages),

$$w_t = ur_t + \left(m_t + \theta \cdot a_t - \tilde{n}_t - \lambda \cdot m_t\right),$$

that assuming full employment (in expected terms) can be written as:

$$w_t^* = E_{t-1}\left[ur_t + m_t - \tilde{n}_t + \theta \cdot a_t - \lambda \cdot m_t \mid E_{t-1}\left[\tilde{n}_t - n_t\right] = 0\right]$$

Taking the expectation and using the definitions $a_t = a_{t-1} + e_{s,t}$, $\tilde{n}_t = \tilde{n}_{t-1} + e_{\tilde{n},t}$, and $m_t = m_{t-1} + e_{d,t}$ where $e_{\cdot,t}$ denote a zero-mean structural shocks:

$$w_t^* = \left(1-\lambda\right) \cdot m_{t-1} - \tilde{n}_{t-1} + \theta \cdot a_{t-1}$$ [71]

Setting $w_t = w_t^*$, the solution of the illustrative tri-variate structural model can be expressed as follows:

(1) Δy_t:

$$\begin{aligned} y_t &= \left(1-\lambda\right) \cdot m_t + \left(1+\theta\right) \cdot a_t - \left\{\left(1-\lambda\right) \cdot m_{t-1} - \tilde{n}_{t-1} + \theta \cdot a_{t-1}\right\} \\ &= \left(1-\lambda\right) \cdot m_t - \left(1-\lambda\right) \cdot m_{t-1} + \left(1+\theta\right) \cdot a_t + \tilde{n}_{t-1} - \theta \cdot a_{t-1} \end{aligned}$$

---

[71] Note that the expression for w_t^* (and the corresponding solution for ur_t) are augmented by an additive constant, $ur^{(natural)}$ when $e_{\tilde{n},t}$ is interpreted as a labor *force* supply shock. This constant cancels out in the solutions of other variables, including for its fourth variable, Δw_t .

and taking the first difference:

$$\begin{aligned}\Delta y_t &= (1-\lambda)\cdot\Delta e_{d,t} + (1+\theta)\cdot e_{s,t} - \theta\cdot e_{s,t-1} + \varepsilon_{\tilde{n},t-1} \\ &= e_{s,t} + \theta\cdot\Delta e_{s,t} + \varepsilon_{\tilde{n},t-1} + (1-\lambda)\cdot\Delta e_{d,t}\end{aligned}$$

(2) ur_t:[72]

$$\begin{aligned}ur_t &= -m_t - \theta\cdot a_t + \tilde{n}_t + \{(1-\lambda)\cdot m_{t-1} - \tilde{n}_{t-1} + \theta\cdot a_{t-1}\} + \lambda\cdot m_{t-1} \\ &= -(1-\lambda)\cdot\Delta m_t - \theta\cdot\Delta a_t + \Delta\tilde{n}_t \\ &= -\theta\cdot e_{s,t} + e_{\tilde{n},t} - (1-\lambda)\cdot e_{d,t}\end{aligned}$$

and (3) Δp_t:

$$\begin{aligned}p_t &= \{(1-\lambda)\cdot m_{t-1} - \tilde{n}_{t-1} + \theta\cdot a_{t-1}\} - a_t + \lambda\cdot m_t \\ &= \lambda\cdot m_t + (1-\lambda)\cdot m_{t-1} - \tilde{n}_{t-1} - a_t + \theta\cdot a_{t-1}\end{aligned}$$

and taking the first difference:

$$\begin{aligned}\Delta p_t &= \lambda\cdot\Delta m_{t-1} + (1-\lambda)\cdot\Delta m_{t-1} - \Delta\tilde{n}_{t-1} - \Delta a_t + \theta\cdot\Delta a_{t-1} \\ &= -e_{s,t} + \theta\cdot e_{s,t-1} - e_{\tilde{n},t-1} + \lambda\cdot e_{d,t} + (1-\lambda)\cdot e_{d,t-1}\end{aligned}$$

Note that the implicit solution for the tri-variate model's "fourth" can be obtained from the solution for $w_t\ (= w_t^*)$ and taking the first difference:

$$\Delta w_t = \Delta w_t^* = \theta\cdot e_{s,t-1} - e_{\tilde{n},t-1} + (1-\lambda)\cdot e_{d,t-1}.$$

Note further that $\Delta w_t = -ur_{t-1}$.

[72] By focusing on the "extensive-margin" of labor-supply and expressing the model in terms of a stationary *ur*, the tri-variate model, in essence, assumes a cointegrating vector [1, -1] for full employment (long-run) labor supply and the labor input.

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