Controlling Environmental Pollution, Sectoral Composition and Factor Prices: A H-O and SFM Hybrid Approach

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Controlling Environmental Pollution, Sectoral Composition and Factor Prices: A H-O and SFM Hybrid Approach

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Abstract: The consternation regarding environment is manifold. One of them is environmental quality which has both short-run and long-run implications including sustainable development goals. In view of such apprehension, this paper develops a Heckscher-Ohlin nugget kind of competitive general equilibrium model with four sectors and four factors of production to analyse the effect of tax policy to curb environmental pollution. Surprisingly we find that environmental tax on the polluting sector eventually raises the production of polluting output and widens the wage inequality between skilled and unskilled labour. On the other hand, taxing the non-polluting sector yields the desired outcome in both production and factor income. The possibility of vanishing sector strengthens the counterintuitive results we get in case of taxing the non-polluting sector. Such an intriguing outcome is driven by the recursive nature of structure of the H-O nugget model. We empirically test the efficacy of environmental taxes using panel data of 10 OECD countries for 1997-2020 and find that environmental taxes have a deleterious effect on pollution.

Keywords: General Equilibrium, Environmental Tax, H-O Nugget, Wage Inequality

JEL Classification: F180, F110, F620

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1. Introduction

The debate over the effects of liberalised trade on the environment has been carried out over decades now. Environmentalists assert that international trade and economic growth result in increased economic activity and have unambiguously detrimental effects on the environment. For a while, economists defended globalisation with the argument that higher economic activity in terms of higher national income leads to a structural transformation where environmental protection is granted greater importance. The crux of this debate lies in the complex interconnectedness of the economy and the environment. This is further exacerbated by the varying degree of importance that is attached to the environment, by policymakers across nations.

The literature on effects of trade on the environment came in two waves – initially the literature took a normative approach analysing the gains from trade and optimal trade, but the modern literature looks more into the positive aspects by assessing the impact of trade and environmental policies (Copeland and Taylor, 2004). It should be specified that the interlinkage of trade and environmental policies is not unidirectional. While it is obvious that trade policies have environmental implications, the causality runs in the other direction as well. It is observed that stricter environmental policies and renunciation of non-green goods can have unfavourable effects on trade by reducing competitiveness and restricting entry in foreign markets (Barrett, 1994). Exporters of pollution-intensive goods favour free trade while importers prefer coherence of environmental policy with trade agreements (Copeland, 2000). Thus, less stringent pollution control rules in low-income countries allow them to develop a comparative advantage in pollution-intensive industries, and affect trade patterns.

The central argument offered by economists that higher income leads to greater consideration for the environment was formalised by Grossman and Krueger (1993) as the *Environmental Kuznets Curve* (EKC) hypothesis. As trade is liberalised, there is economic growth, which has an evident impact on the environmental quality. The EKC (Grossman and Krueger, 1995; Shafik and Bandyopadhyay,1992; Selden and Song, 1994) hypothesises an inverted U-shaped relation between per capita income and environmental quality: as income increases in a country, pollution increases but gradually decreases as the economy turns into a post-industrial service economy.

The effects of trade on environment can be decomposed into the “technique”, “composition” and “scale” effects. The technique and composition effects refer to the respective shift of production and consumption preferences toward cleaner processes and goods. But the scale effect conveys the increase in pollution levels as a result of increased economic activity. Thus, the EKC hypothesis is contingent upon the technique and composition effects outweighing the scale effect beyond the turning-point national income (Esty, 2001).

Conventional trade theory has also been employed to show the ill-effects of trade on the environment, especially in smaller nations. Less stringent environmental policy in poorer nations can shift the polluting industries away from richer countries to poorer countries. This is the *pollution haven hypothesis* which lacked any empirical evidence earlier but recent literature has shown the pollution haven effect to influence trade flows (Levinson and Taylor, 2004).

However, following the Earth Summit, 1992 and subsequent UN summits, major emphasis has been laid on the *Sustainable Development Goals* (SDGs). The 2030 Agenda for Sustainable Development adopted in 2015 has provided 17 SDGs which deal with the primary goals of ending poverty and boosting economic growth while simultaneously improving health and education, reducing inequality and tackling climate change. The SDGs of 'Good health and well-being', 'Clean Water and Sanitation' and 'Climate Action' especially prioritise the conservation of the environment. Therefore, the question has shifted from "Growth or Environment?" to "How much of growth and how much of environment?".

International trade and environmental protection might not be at loggerheads but a certain trade-off is associated when implementing policies. The 2030 Agenda calls for policy coherence, which is indicative of the synergy that is required between environmental and trade policies to ensure sustainability. This requires trade to be exercised within the purview of environmentally sound norms and standards. The protection of the environment in the presence of free trade and economic growth requires controlling pollution. While the chances of a reversal to pre-industrial environmental conditions is infinitesimal, an immediate check on pollution can ensure the existence of a green future.

In this context, the rudimentary method of controlling pollution is levying a Pigouvian tax on the pollution generating good. A Pigouvian tax eliminates the negative externalities by capturing the social cost of pollution which is usually neglected under free market operations. The immediate criticism faced by Pigouvian taxation was its anti-economic effect and how it causes a divergence from the free market outcome. But this was countered with the double-dividend hypothesis[1] which highlights the revenue raising property of environmental taxes in addition to its externality correcting capacity. The hypothesis suggests that shifting from distortionary taxes to an environmental tax might propel economic efficiency by reducing the burden on workers and driving down unemployment. This added benefit in excess of environmental protection is considered the 'double' dividend but it is conditional upon the revenue generating capacity and associated costs of the environmental tax structure.

Environmental taxes are not the only means of controlling pollution. Other means include quantity-based instruments and Command-and-Control (CAC). Quantity based instruments may include the issuance of tradable permits which can be traded following an Ambient Permit System (APS), Environmental Permit System (EPS) or a hybrid Pollution Offset System (PO). On the other hand, the government might implement a CAC approach where it not only determines the optimum level of pollution but also sets guidelines and regulations for each stage of the production process, the defiance of which results in fines and penalties.

It is undeniable that pollution levels need to be reduced and the customary practise is to impose environmental taxes. Thus, some insights into the definition and characteristics of an environmental tax are required. Alternatively referred to as an ecotax or green tax, the Eurostat defines an environmental tax as "A tax whose tax base is a physical unit (or a proxy of a physical unit) of something that has a proven, specific negative impact on the environment, and which is identified in ESA as a tax." While this definition encompasses a broad spectrum of taxes, given lack of consensus as to what comprises an environmental tax, the OECD provides multiple definitions: "Pigouvian taxes; indirect taxes on production inputs or consumer goods which hamper the environment (e.g., excise taxes on gasoline); environment-

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[1] Fullerton and Metcalf (1997); Carraro, et al. (1996); Bento and Jacobsen (2007)

related provisions in other taxes; and accelerated depreciation provisions and lower tax rates for equipment and production methods that save energy and reduce pollution."

Again, there are contrasting views on the implications of environmental taxes. For instance, it is observed that environmental taxes are not able to meet their goal of large revenue generation but in some cases like those of Denmark and Sweden, green taxes have served their double dividend purpose by allowing relaxation of taxes which adversely affect employment. More importantly, environmental tax reforms cut down on international competitiveness in the absence of compensating measures. This trade theoretic argument has led to exemptions on green taxes which have further hampered their double-edged capability.

We often view environmental taxes as regressive even though no major regressive impacts are observed if the taxes are considered in reference to the environmental benefits they generate. There is also a conflict between fiscal and environmental goals since ecotaxes aim to reduce their own tax base. But it is found that in many nations the taxbase is stable or even growing in the presence of these taxes.

This raises a question regarding the efficacy of environmental taxes in controlling pollution. Most studies follow the partial equilibrium approach to assess the impact of such taxes on emissions. But partial equilibrium fails to capture the essence of the structural form of the economy when estimating the impact. We aim to develop a general equilibrium framework for a small open economy to verify the conventional wisdom on the efficacy of environmental taxes. We consider an amalgamation of Heckscher-Ohlin setting with a Ricardo-Viner-Jones setup, often referred to as a Heckscher-Ohlin "nugget" following Marjit (1990) and Jones and Marjit (1992).

The paper is organised as follows: section 2 reviews the existing literature, section 3 introduces the basic model and propositions, section 4 extends the model and discusses the vanishing sector possibility and section 5 concludes the paper.

2. Literature Review

The ideal starting point is the seminal work of Copeland and Taylor (1994,1995) where they develop a static general equilibrium model to provide the theoretical basis for the pollution haven hypothesis where the differences in environmental policy (which are income-induced) determine patterns of trade. Environmental quality is a normal good and higher income leads to demand for stricter environmental policies. Thus, in the absence of other differences among countries, richer countries develop a comparative advantage in clean goods resulting in pollution havens in the poorer countries. They further posit that trade leads to increase in global pollution and growth in richer (poorer) countries leads to an increase (decrease) in world pollution.

Subsequently, Copeland and Taylor (1997) develop a simple 2-sector model to show that if income differences are small, patterns of trade can be determined by abundance of factors. This implies that the rich, capital abundant country will export the pollution-intensive good, following the Heckscher-Ohlin theorem. Interestingly, when income differences are larger relative to factor differences, income patterns determine trade i.e., the capital abundant country will import polluting good as a result of more stringent environmental policies. This can be a possible explanation to the Leontief Paradox where the H-O theorem does not hold since pattern of trade is determined by income-induced environmental policies. The most striking result is that when factor differences dominate income difference, trade shifts production of polluting good to richer country with stricter regulations and results in reduction of world pollution.

Copeland and Taylor (2004) provide a conclusive review of all theoretical and empirical literature till 2004. They develop a static model to deal with production generated pollution. Treating pollution as an endogenously supplied factor of production, the optimum level of pollution and environmental tax is determined as result of the interaction of demand and supply of pollution. Under an exogenous tax policy, the government determines the optimal tax by implementing the standard Samuelson rule where the tax rate is equated to the sum of marginal damage across all representative individuals. It is shown that welfare implications of trade liberalisation depend on the trade patterns, policy instrument and level of policy stringency. If the emission intensities are fixed, the pollution haven hypothesis is backed where countries with comparative advantage in dirty (clean) goods face an increase (decrease) in pollution. On the other hand, under an ideal endogenous policy, trade is always welfare augmenting although there might be an increase in pollution in countries with comparative advantage in dirty industries. Hence, a trade-off is implied between optimal environmental quality and consumption.

Literature on the general equilibrium analysis of the interlinkages between trade and environment is limited and especially the effects of taxing polluting sectors have not been explored in much depth. However, Fullerton and Heutel (2007) study the distributional effects of a pollution tax in general equilibrium. They treat pollution as an input along with labour and capital, and allow for varying degree of substitutability between the inputs. It is found that the intuition that, taxing the income of the factor used relatively intensively in the polluting sector will place disproportionate burdens on that factor, does not hold. The effect of a pollution tax depends on the substitutability of pollution with other factors and the relative factor intensities of the sectors. Consequently, Fullerton and Heutel (2010) show that regulations that restrict emissions have the same effect as imposing taxes. In stead of considering taxes, they look at nonrevenue-raising measures which also affect relative factor prices, output and commodity prices. These might include environmental mandates in the form of emission quantity limits (whether tradable or not), performance standards (emissions per unit output) and technology mandates (emissions per unit of input).

Williams (2016) studies the effect of environmental policy reforms and reviews empirical literature to estimate the effects of such reforms on income inequality, tax revenue and emissions. They find that environmental taxes usually have a negative impact on the level of greenhouse emissions. An overall assessment of the impact of ecotaxes showed that “getting a reduction roughly three times as large requires a tax rate more than three times as high.”

Scrimgeour, et al (2005) analyse the effectiveness of different forms of environmental taxes on New Zealand. Their study suggests that a carbon tax is effective in reducing emissions but an energy tax can also serve as an effective instrument. They also support the claim that there exists a trade-off between growth and taxes as the international competitiveness of home industries is hampered.

The conclusions following most literature shows that environmental taxes do have a significant negative effect on the level of emissions. It is also obvious that GDP and its components are usually adversely affected following the imposition of an ecotax. Oueslati, et al. (2016) offers the same conclusion but they go on to show the effect of ecotaxes on income inequality. It is found that the effect on income inequality is dependent on the structure of the redistribution scheme of tax revenue. If a robust revenue redistribution mechanism is present, environmental taxes have a negative effect on income inequality. The absence of such mechanisms leads to a positive effect.

It is difficult to believe that in the presence of such complex structures and mechanisms in an economy, the effect of taxes on pollution is unequivocal. Especially when an economy participates in trade, such partial equilibrium results cease to apply. Structures which result in paradoxes are prevalent in the trade literature. One such anomaly was shown by Marjit (1990), where he used the framework provided by Gruen and Corden (1970) in a simple production structure to derive counterintuitive results. He uses the mix of a Heckscher-Ohlin and Jones (1971) settings, referred to as the Heckscher-Ohlin nugget. This nugget shows that “an increase in the relative price for a particular good might lead to a contraction in the output of that sector, and this might be accompanied by a decline in the absolute return of a factor which is exclusively used in the sector affected favourably by the relative price movement.” In the environmental context, this insinuates the inefficacy that pollution taxes might result in by raising the effective price of polluting goods.

A generalisation of this structure was utilised by Jones and Marjit (1992) where they integrated the Jones (1971) n-factor, ‘n+1’-commodity setup with a generalization of the Gruen and Corden (1970) model, where each industry is tied to a production “nugget” by the use of a common factor. They analyse the infamous Dutch Disease phenomenon where certain traded sectors boom at the expense of other relatively dormant or non-traded sectors. As a result of such Dutch disease type phenomenon, traded industries can be wiped out by non-traded industries which are shielded from global competition.

This vanishing sector case is of particular interest owing to its policy connotations. Marjit and Kar (2009) state that while a vanishing sector is not feasible in a specific factor model since the specific factor has to be employed, the case of a vanishing sector is possible in a Heckscher-Ohlin model under complete specialization. But, if one allows unemployment of any factor in the economy, SFM can also explain the case of the vanishing sector. Again, the H-O nugget and vanishing sector can also be examined simultaneously in a situation where one sector from the H-O setup may vanish.

Mandal and Marjit (2012) mention that a sector might vanish as result of disparity between unit cost and unit price. If the unit price of a commodity falls below the unit cost, free mobility of resources and perfect competition make production of that commodity unviable. Similarly, when unit price exceeds unit cost, the factor price of a specific factor must rise and draw the mobile factor from other sectors leading to the vanishing of some other sector.

Mandal (2022) shows another such application of finite change in international trade. He considers a simple Ricardian model with constant opportunity costs and incorporates corruption into the model. The most interesting result which follows is that an in the case of asymmetric transaction costs of corruption, an increase in the transaction cost causes the corrupted sector to vanish.

These unconventional results which follow from a Heckscher-Ohlin nugget and finite changes in international trade, inspires us to develop such a production structure to theoretically analyse the effects of environmental taxes. We also aim to assess the effects of such taxes on the wage inequality.

3. The Basic Model and Policy Ineffectiveness

To tackle the problem of pollution, governments use multiple policy instruments: Command and Control, Quantity Instruments and Price Instruments. As discussed earlier, the basic and most commonly used instrument is a Pigouvian tax. This has an almost certain externality eliminating effect and hence reduces pollution, when considered in a partial equilibrium approach. But if the price effect is used to analyse the changes in factor incomes, the outcomes are not so straightforward and requires a general equilibrium approach. Therefore, we develop such a general equilibrium model which uses a H-O nugget kind of setup to delve into the effects of such taxes on polluting sectors.

We consider a small open economy with perfectly competitive markets. Production exhibits constant returns to scale, diminishing marginal productivity and resources are fully employed. There are four sectors- the service sector (I), the government sector (X), the manufacturing sector (Y) and the agricultural sector (Z). The four factors of production are skilled labour (S), unskilled labour (L), Capital (K) and Land (T).

I is treated as an ITeS (Information Technology enabled Services) sector. X is the government sector which is unionized and *non-IteS*. It is *non-polluting*. It may include teaching, tourism, transportation, healthcare etc. Y is the manufacturing sector which is conventionally polluting. Z is the agricultural sector with sector-specific factor, land.

The *Price System*[2] of the model is described by:

$$w_S a_{SI} + r a_{KI} = P_I \qquad (1)$$

$$\bar{w}_S a_{SX} + \bar{w} a_{LX} + r a_{KX} = P_x \qquad (2)$$

$$w a_{LY} + r a_{KY} = P_Y(1-\tau) \qquad (3)$$

$$w a_{LZ} + R a_{TZ} = P_Z \qquad (4)$$

A tax is levied on the production of the traditionally polluting sector, Y at the rate, τ.

The *Quantity System* ensuring full employment of resources is given by:

$$a_{SI} I + a_{SX} X = \bar{S} \qquad (5)$$

$$a_{LX} X + a_{LY} Y + a_{LZ} Z = \bar{L} \qquad (6)$$

$$a_{KI} I + a_{KX} X + a_{KY} Y = \bar{K} \qquad (7)$$

$$a_{TZ} Z = \bar{T} \qquad (8)$$

This model has 8 unknown variables - w_S, w, r, R, X, Y, Z and I. We need to solve for these unknown variables from 8 equations (1)-(8). Thus, the system is solvable. Since prices are exogenous to the system, the price system follows a recursive structure which allows to solve for the factor prices in a step-wise method independently of the factor endowments. This helps us to separate the price and quantity systems.

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[2] Symbols have conventional meanings. $\bar{w}$ and $\bar{w}_S$ are the unionized returns to unskilled and skilled labour.

Since prices are internationally determined and wages are fixed in the government sector, r is determined from (2). Once the value of r is obtained, w_s and w can be obtained from (1) and (3) respectively. We can substitute this value of w in (4) to obtain R.

At this stage we assume, I is skill-intensive, Y is most capital-intensive, X is more capital-intensive than I and more labour-intensive than Y.

Consider an increase in the environmental tax imposed on the polluting sector, Y.

It should be emphasised that prices are determined exogenously in a small open economy. Since, r is determined from (2), given P_x, r is left unchanged. Now, in (3), an increase in τ, lowers $P_Y(1-\tau)$ which depresses w, since r is unchanged. There is no effect in the market for I, and w_s is unaffected in (1). But, the fall in w causes R to rise in equation (4), since P_Z is exogenous.

Differentiating equations (2), (3), (1) and (4) and using the envelope conditions, we get:

$$\hat{r} = 0 \qquad (9)$$

$$\hat{w} = -\frac{\hat{\tau}\tau}{\theta_{LY}} < 0 \qquad (10)$$

$$\widehat{w_S} = 0 \qquad (11)$$

$$\hat{R} = \frac{\hat{\tau}\tau\theta_{LZ}}{\theta_{LY}\theta_{TZ}} > 0 \qquad (12)$$

We can directly observe deterioration of the wage inequality (w_s-w).

To arrive at the changes in output, we first determine the change in output of Z, attainable by differentiating (8):

$$\hat{Z} = -\widehat{a_{TZ}} \qquad (13)$$

We then use the concept of elasticity of substitution and envelope conditions (Jones, 1965) to obtain:

$$\widehat{a_{TZ}} = -\sigma_Z\hat{\tau}\tau\frac{\theta_{LZ}}{\theta_{LY}\theta_{TZ}} < 0 \qquad (14)$$

(For the complete derivation, see Appendix)

The expansion of Z depends upon its elasticity of substitution between labour and land. As rent on land rises and the wages fall, Z will tend to employ more of cheaper labour. Thus, higher the elasticity of substitution, more labour will be substituted and greater will be the expansion of output.

Thus, we obtain

$$\hat{Z} = \sigma_Z\hat{\tau}\tau\frac{\theta_{LZ}}{\theta_{LY}\theta_{TZ}} > 0 \qquad (15)$$

Differentiating the Quantity Equations (5)-(7), we get

$$\lambda_{SI}\hat{I} + \lambda_{SX}\hat{X} = 0 \qquad (16)$$

$$\lambda_{LX}\hat{X} + \lambda_{LY}\hat{Y} = -\sigma_Z\hat{\tau}\tau\frac{\theta_{LZ}\lambda_{LZ}}{\theta_{LY}\theta_{TZ}} \qquad (17)$$

$$\lambda_{KI}\hat{I} + \lambda_{KX}\hat{X} + \lambda_{KY}\hat{Y} = 0 \qquad (1814)$$

We get 3 equations in 3 unknowns. Thus, the system is solvable if the coefficient matrix is non-singular. We find that the determinant is non-zero and takes a positive value given the factor intensity assumptions.

Solving for the rate of change of output in each sector, we get

$$\hat{I} = \frac{1}{A}\lambda_{SX}\lambda_{KY}\sigma_z\hat{\tau}\tau\frac{\theta_{LZ}\lambda_{LZ}}{\theta_{LY}\theta_{TZ}} > 0 \; if \; A > 0 \qquad (19)$$

$$\hat{X} = -\frac{1}{A}\lambda_{SI}\lambda_{KY}\sigma_z\hat{\tau}\tau\frac{\theta_{LZ}\lambda_{LZ}}{\theta_{LY}\theta_{TZ}} < 0 \; if \; A > 0 \qquad (20)$$

$$\hat{Y} = \frac{1}{A}(\lambda_{SI}\lambda_{KX} - \lambda_{KI}\lambda_{SX})\hat{\tau}\tau\frac{\theta_{LZ}\lambda_{LZ}}{\theta_{LY}\theta_{TZ}} > 0 \; if \; A > 0 \qquad (21)$$

where, $A = \lambda_{SI}\lambda_{LX}\lambda_{KY} - \lambda_{SI}\lambda_{KX}\lambda_{LY} + \lambda_{KI}\lambda_{SX}\lambda_{LY}$

$A > 0 \; if \; X \; is \; Labor - intensive \; and \; Y \; is \; capital - intensive$

i.e., if $\lambda_{LX}\lambda_{KY} - \lambda_{KX}\lambda_{LY} > 0$

Following an increase in the tax rate on production of the polluting sector, Y, there is an increase in the rent on land, R. An increase in R makes land expensive and a_{TZ} falls. The full employment of land implies that this will result in an increase in the production of Z. As Z production increases, it draws unskilled labour from sectors X and Y. This causes contraction of the labour-intensive government sector X, which releases both skilled labour and capital. The skilled labour and capital released by X is absorbed into the IT sector but factor intensity assumptions guarantee that more capital was released than absorbed by I. The excess capital moves into the capital-intensive manufacturing sector, Y. Therefore, the output of the polluting sector expands.

Thus, the intuitive policy of taxing the polluting sector is rendered ineffective in our small open economy structure as we obtain counterproductive results of increased pollution and worsened wage inequality.

PROPOSITION-I: Depending on certain factor intensity assumptions, an increase in the tax on the polluting sector causes

(i) an expansion of the polluting sector, and

(ii) worsening of the wage inequality between skilled and unskilled labour i.e., (ws – w) increases

Proof: See Discussion Above

4A. Extended Model and Policy Effectiveness

Since we have a counterintuitive outcome in the basic model that an imposition of τ on the environment polluting sector is leading to expansion of that sector, we may check an alternative policy formulation to see what happens to the polluting sector.

Since restriction on polluting sector causes expansion of Y and contraction of X which is surely not a desirable outcome, we attempt to extend the model by imposing the tax β on X instead of τ on Y. Here, β is not a pollution/environmental tax, but it is an ad valorem tax on the output of the government sector X.

$$w_S a_{SI} + r a_{KI} = P_I \qquad (1A)$$

$$\bar{w}_S a_{SX} + \bar{w} a_{LX} + r a_{KX} = P_X(1-\beta) \qquad (2A)$$

$$w a_{LY} + r a_{KY} = P_Y \qquad (3A)$$

$$w a_{LZ} + R a_{TZ} = P_Z \qquad (4A)$$

Full employment conditions remain the same.

Prices are exogenously given. Here, the starting point is the change in r. As β increases, r falls since $\bar{w}$ and $\bar{w}_S$ are fixed in (2A). From (3A), as r falls, w increases. It follows that w_S increases and R falls from equations (1A) and (4A) respectively.

Differentiating equations (2A), (1A), (3A) and (4A) and using the envelope conditions, we get,

$$\hat{r} = -\frac{\hat{\beta}\beta}{\theta_{KX}} < 0 \qquad (9A)$$

$$\widehat{w_S} = \frac{\hat{\beta}\beta\theta_{KI}}{\theta_{SI}\theta_{KX}} > 0 \qquad (10A)$$

$$\hat{w} = \frac{\hat{\beta}\beta\theta_{KY}}{\theta_{KX}\theta_{LY}} > 0 \qquad (11A)$$

$$\hat{R} = -\frac{\hat{\beta}\beta\theta_{KY}\theta_{LZ}}{\theta_{KX}\theta_{LY}\theta_{TZ}} < 0 \qquad (12A)$$

Following the technique of the basic model, using *elasticity of substitution* and *envelope conditions*

$$\widehat{a_{TZ}} = \frac{\sigma_Z\hat{\beta}\beta\theta_{KY}\theta_{LZ}}{\theta_{KX}\theta_{LY}\theta_{TZ}} > 0 \qquad (14A)$$

From full employment condition of land,

$$\hat{Z} = -\frac{\sigma_Z\hat{\beta}\beta\theta_{KY}\theta_{LZ}}{\theta_{KX}\theta_{LY}\theta_{TZ}} < 0 \text{ (Unlike the basic model)} \qquad (15A)$$

Again, contraction of Z is dependent upon the elasticity of substitution between land and labour in Z. But here, w has increased and R has fallen. If Z uses more land per unit of output, Z will contract. Thus, lower the elasticity of substitution, lesser will be the contraction of agricultural output.

Here, differentiating the quantity system (5)-(7), we get

$$\lambda_{SI}\hat{I} + \lambda_{SX}\hat{X} = 0 \tag{16A}$$

$$\lambda_{LX}\hat{X} + \lambda_{LY}\hat{Y} = \lambda_{LZ}\frac{\sigma_Z\hat{\beta}\beta\theta_{KY}\theta_{LZ}}{\theta_{KX}\theta_{LY}\theta_{TZ}} \tag{17A}$$

$$\lambda_{KI}\hat{I} + \lambda_{KX}\hat{X} + \lambda_{KY}\hat{Y} = 0 \tag{18A}$$

As in the basic model, we get 3 equations in 3 unknowns. Thus, the system is solvable if the coefficient matrix is non-singular. We find that the determinant is non-zero and takes a positive value given the factor intensity assumptions.

Solving for the rate of change in output of the sectors from (16A)-(18A),

$$\hat{I} = -\frac{1}{A}\lambda_{\mathrm{LZ}}\lambda_{SX}\lambda_{KY}\frac{\sigma_Z\hat{\beta}\beta\theta_{KY}\theta_{LZ}}{\theta_{KX}\theta_{LY}\theta_{TZ}} < 0 \tag{19A}$$

$$\hat{X} = \frac{1}{A}\lambda_{\mathrm{SI}}\lambda_{KY}\lambda_{LZ}\frac{\sigma_Z\hat{\beta}\beta\theta_{KY}\theta_{LZ}}{\theta_{KX}\theta_{LY}\theta_{TZ}} > 0 \tag{20A}$$

$$\hat{Y} = -\frac{1}{A}\lambda_{\mathrm{LZ}}(\lambda_{SI}\lambda_{KX} - \lambda_{KI}\lambda_{SX})\frac{\sigma_Z\hat{\beta}\beta\theta_{KY}\theta_{LZ}}{\theta_{KX}\theta_{LY}\theta_{TZ}} < 0 \tag{21A}$$

$$\lambda_{SI}\lambda_{LX}\lambda_{KY} - \lambda_{SI}\lambda_{KX}\lambda_{LY} + \lambda_{KI}\lambda_{SX}\lambda_{LY} = A$$

$$A > 0\ if\ X\ is\ Labor - intensive\ and\ Y\ is\ capital - intensive$$

i.e., if $\lambda_{LX}\lambda_{KY} - \lambda_{KX}\lambda_{LY} > 0$

On the wage inequality front,

$$\widehat{w_S} - \hat{w} = \frac{\hat{\beta}\beta}{\theta_{KX}}\left(\frac{\theta_{KI}\theta_{LY} - \theta_{SI}\theta_{KY}}{\theta_{SI}\theta_{KX}\theta_{LY}}\right) \tag{22}$$

$$\therefore (\widehat{w_S} - \hat{w}) \gtreqless 0 if (\theta_{KI}\theta_{LY} - \theta_{SI}\theta_{KY}) \gtreqless 0$$

Since I is S-intensive and Y is most K-intensive, $(\theta_{KI}\theta_{LY} - \theta_{SI}\theta_{KY})<0$. Hence, wage inequality must ameliorate due to an increase in tax on X.

The driving force in this model is the change in r. Here, r decreases, eventually causing R to fall. This increases the per unit use of land in agriculture. T being specific to Z, causes contraction of agricultural output. This releases unskilled labour to absorbed by sectors X and Y. Rybczynski argument suggests expansion of labour-intensive X and contraction of capital-intensive Y. Expansion of X also draws capital and skilled labour from Y and I. Thus, the IT sector also contracts. But the interesting outcome here is the contraction of the manufacturing sector, Y and hence a reduction in the level of pollution.

Therefore, in our structure, the unorthodox policy of taxing the non-polluting government sector leads to favourable outcomes both in the form of control on pollution and reduction in the wage gap.

PROPOSITION-II: Under the same factor intensity assumptions as in the basic model, an increase in the tax on the non-polluting government sector, β causes

(i) polluting sector to contract, and

(ii) wage inequality between skilled and unskilled labour to improve i.e., ($w_S - w$) decreases.

Proof: See Discussion Above

4B. The Case of Vanishing Sector

We try to decipher another possibility where finite change in international trade theory may be applied. When land is not willing to accept a fall in R, Z production stops. Z vanishes from the system. Land becomes completely unemployed. So, in a sense the model no longer remains a full employment model. This sort of analysis is in fact an extreme application of complete specialisation.

Mathematically, equation (6) in the full employment condition becomes

$$a_{LX}X + a_{LY}Y = \bar{L} - \frac{a_{LZ}}{a_{TZ}}\bar{T} \qquad (6A)$$

Thus, differentiating (6A), we get

$$\lambda_{LX}\hat{X} + \lambda_{LY}\hat{Y} = \sigma_Z\hat{\beta}\beta\frac{\theta_{KY}}{\theta_{KX}\theta_{LY}\theta_{TZ}}$$

We now use the differentiated quantity equations to solve for the changes in output,

$$\hat{I} = -\frac{1}{A}\sigma_Z\hat{\beta}\beta\frac{\theta_{KY}}{\theta_{KX}\theta_{LY}\theta_{TZ}}\lambda_{SX}\lambda_{KY} < 0 \qquad (19B)$$

$$\hat{X} = \frac{1}{A}\sigma_Z\hat{\beta}\beta\frac{\theta_{KY}}{\theta_{KX}\theta_{LY}\theta_{TZ}}\lambda_{SI}\lambda_{KY} > 0 \qquad (20B)$$

$$\hat{Y} = -\frac{1}{A}\sigma_Z\hat{\beta}\beta\frac{\theta_{KY}}{\theta_{KX}\theta_{LY}\theta_{TZ}}(\lambda_{SI}\lambda_{KX} - \lambda_{KI}\lambda_{SX}) < 0 \quad (21B)$$

$$\lambda_{SI}\lambda_{LX}\lambda_{KY} - \lambda_{SI}\lambda_{KX}\lambda_{LY} + \lambda_{KI}\lambda_{SX}\lambda_{LY} = A$$

$$A > 0 \; if \; X \; is \; Labor - intensive \; and \; Y \; is \; capital - intensive$$

i.e., if $\lambda_{LX}\lambda_{KY} - \lambda_{KX}\lambda_{LY} > 0$

When Z shuts down it releases L which must be employed in other sectors like X and Y. This guarantees a round of Rybczynski effect.

Released L will seek employment in either X or Y since I does not use L. Out of X and Y, X is labour-intensive and Y is capital-intensive. Conventional Rybczynski argument suggests expansion of labour-intensive X sector and contraction of capital-intensive Y sector.

Moreover, the expansion of X requires some S and K. The simultaneous shrinkage of Y frees up K and the factor intensities of X and Y ensure that more capital is released than absorbed by X.

However, it is also not to forget that increase in X calls for an employment of some more S which has to be released from I. If S is released from I, I should ideally decrease. But that may not happen since factor substitution is possible in I. I may use more of K and less of S. This is also corroborated by an increase in w_S and decline in r in the extended model. In such a situation excess capital left after absorption in X moves to I. Hence, first round contraction of I is going to be dampened slightly due to Rybczynski effect, whereas for X and Y the output effects are reinforced.

Thus, the taxed sector expands while the polluting sector, Y contracts. The same results follow from the extended model even when sector Z vanishes. Although land is completely unemployed, the policy is effective to curb the polluting industry and reduce wage inequality.

This opens up the possibility of a new or emerging sector where the unemployed factor can be employed. This requires government intervention. The agricultural land can be utilised for real-estate, fisheries, dairies, etc. In case a new sector emerges, we can also introduce a new factor or vocational training of unskilled labour for specific activities.

PROPOSITION-III: Under the same factor intensity assumptions as in the basic model, an increase in the tax on the non-polluting government sector, β causes

(i) the agricultural sector to vanish,
(ii) contraction of polluting sector, and
(iii) wage inequality between skilled and unskilled labour to shrink i.e., $(w_S - w)$ decreases.

Proof: See Discussion Above

Thus, our results of the extended model are further strengthened in the presence of a vanishing sector.

5. Conclusion

We have developed a Heckscher-Ohlin nugget kind of general equilibrium framework with four sectors and four factors of production. We impose certain conditions on the relative factor-intensities of the four sectors. The counterintuitive result, that imposing a tax on the polluting sector, expands the polluting output follows from the recursive structure of the setup. Furthermore, an environmental tax widens the wage gap between skilled and unskilled labour. We have shown in the extension to the model that taxing the non-polluting sector achieves the goal of curbing the polluting sector while also reducing the wage gap. The validity of these results in the case of a vanishing sector reinvigorates the model.

It is necessary to emphasise on the driving force behind the result. The change in the rent on capital affects the factor incomes which in turn determine the output changes. The factor intensities of the sectors and the structure of the model play a crucial role in the final outcomes. Hence, an in-depth analysis of the structure of the economy is required before making policy decisions. As is evident, environmental policy might not always be the ideal measure to attain environmental goals.

Appendix

A. Deriving the change in a_{TZ} using the concepts of elasticity of substitution and envelope conditions:

$$\hat{Z} = -\widehat{a_{TZ}} \qquad (13)$$

Using the concept of *elasticity of substitution*:

$$\sigma_Z = -\frac{\widehat{a_{LZ}} - \widehat{a_{TZ}}}{\widehat{w} - \hat{r}}$$

$$\widehat{a_{LZ}} = \widehat{a_{TZ}} - \sigma_Z(\widehat{w} - \hat{r})$$

$$\widehat{a_{TZ}} = \widehat{a_{LZ}} + \sigma_Z(\widehat{w} - \hat{r})$$

From the *envelope conditions*

$$\widehat{a_{LZ}} = -\frac{\widehat{a_{TZ}}\theta_{TZ}}{\theta_{LZ}}$$

$$\widehat{a_{TZ}} = -\frac{\widehat{a_{LZ}}\theta_{LZ}}{\theta_{TZ}}$$

$$\widehat{a_{TZ}} = -\sigma_z\hat{\tau}\tau\frac{\theta_{LZ}}{\theta_{LY}\theta_{TZ}} < 0 \qquad (14)$$

B. Following the technique of the basic model, to compute the change in a_{TZ}

$$\widehat{a_{TZ}} = \sigma_Z\left(\widehat{w} - \widehat{R}\right)\theta_{LZ}$$

$$\widehat{a_{TZ}} = \sigma_Z\left[\frac{\widehat{\beta}\beta\theta_{KY}}{\theta_{KX}\theta_{LY}} + \frac{\widehat{\beta}\beta\theta_{KY}\theta_{LZ}}{\theta_{KX}\theta_{LY}\theta_{TZ}}\right]\theta_{LZ}$$

$$\widehat{a_{TZ}} = \frac{\sigma_Z\widehat{\beta}\beta\theta_{KY}\theta_{LZ}}{\theta_{KX}\theta_{LY}\theta_{TZ}} > 0 \qquad (14A)$$

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