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Abstract

Local governments have recently adopted place-based policies in order to revitalize decayed shopping areas in downtown areas. Developing a multipurpose shopping model, we evaluate the welfare impacts of place-based policies for downtown retail agglomeration. In the model, retail stores are under monopolistic competition, and consumers are free to choose where to reside. Results show that, whether or not place-based policies are efficient depends on the recipients of government subsidies, even if the policies promote retail agglomeration in downtown areas. Specifically, subsidizing consumers residing near downtown areas is inevitably harmful from the viewpoint of welfare, whereas subsidizing retail stores can be efficient.

Keywords: Agglomeration; Monopolistic competition; Multipurpose shopping; Place-based policy.

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1. Introduction

Shopping is an indispensable daily activity in our lives. The decline of retail stores operating in downtown areas has been regarded as an urban problem over the past several decades. Local governments have recently implemented place-based policies in order to make retail stores agglomerate in downtown areas. A feature of place-based policies is that stores and/or consumers in a targeted area are subsidized. For example, the city of Albuquerque in the U.S.A. subsidizes retail stores operating in the downtown area. Toyama in Japan subsidizes consumers who migrate from outside to an area around the downtown area.

Impacts of place-based policies on retail stores have been empirically investigated (e.g., Givord et al., 2013; Neumark and Simpson, 2015; Iwata and Kondo, 2021). For example, Givord et al. (2013) empirically show that, in France, the government has promoted the agglomeration of retail stores by a place-based policy, which indicates that place-based policies can revitalize downtown areas. However, the place-based policy does not ensure that social welfare increases because it can produce deadweight losses in the policy-implemented market, and can cause a decline in the number of retail stores in other areas. We theoretically clarify which place-based policies increase social welfare, and which decrease social welfare.

Since agglomeration of retail stores generally involves market failures, a place-based policy may increase social welfare. Examples of such market failures are the shopping externality generated by multipurpose shopping (O'Sullivan, 1993), which is purchasing goods from stores on a single trip, and price distortions caused by imperfect competition among stores. Arentze et al. (2005) empirically show that agglomeration of retail stores relates to multipurpose shopping.

General equilibrium models in which consumers engage in multipurpose shopping with imperfect competition in a marketplace (e.g., shopping streets and shopping malls) have been developed (Henkel et al., 2000; Arakawa, 2006; Tabuchi, 2009; Ushchev et al., 2015). Most multipurpose shopping models have two features. One feature is that retail stores operating in marketplaces are under monopolistic competition. The other feature is that the spatial distribution of consumers is exogenous.

In order to evaluate place-based policies, we need to consider the endogenous spatial distribution of consumers rather than the exogenous spatial distribution. Some studies develop spatial competition models in which consumers and firms compete in the land market (e.g., Fujita and Thisse, 1986; Fujita, 1988; Liu and Fujita, 1991). However, these studies do not answer how place-based policies affect social welfare. One of the place-based policies is to subsidize consumers to reside around downtown areas. This policy intends to agglomerate retail stores in downtown areas by encouraging more consumers to reside close to the downtown areas and visit the downtown areas for shopping. This policy can be adopted in a sprawled city to revitalize the center of the city. Another place-based policy is direct subsidies for stores to agglomerate.

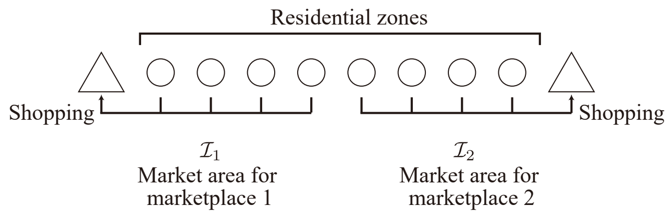
We evaluate the welfare impacts of place-based policies for retail agglomeration by developing a multipurpose shopping model. In the model, retail stores are under monopolistic competition, and consumers are free to choose where to reside. We focus on two place-based policies which have been adopted by local governments. One is location subsidies to consumers, and the other is location subsidies to stores. We evaluate the welfare impacts of these policies in terms of social surplus.

The welfare impacts of place-based policies can be decomposed into three terms, according to Harberger's welfare change measurement formula (Harberger, 1971). The first term is the total change in deadweight losses caused by the price distortions of the varieties supplied in marketplaces. The second term is the net social benefit generated by variety distortion (e.g., Kanemoto, 2013a; Behrens et al., 2015). These two terms are generated by monopolistic competition among retail stores. On the other hand, the third term is the migration fiscal externality generated by income transfer inefficiency by a place-based policy (Boadway and Flatters, 1982; Kono et al., 2007). Place-based policies change social welfare through these three channels, changeing the scales of the deadweight losses and fiscal externalities.

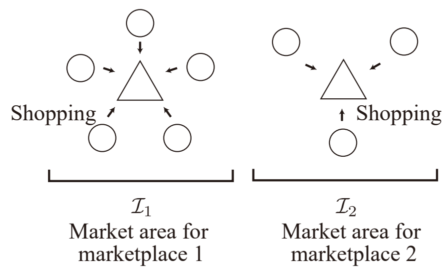
Using the derived Harberger's welfare change measurement formula, we show that whether or not place-based policies are socially efficient depends on the recipients of the subsidies, even if the policies promote downtown retail agglomeration. Specifically, with the constant elasticity of substitution (CES) utility function of varieties, location subsidies to consumers is harmful from the viewpoint of welfare, whereas location subsidies to stores is desirable. We show that location subsidies to stores can increase social welfare because the policy generate positive large benefits with respect to the variety distortion, which exceed deadweight losses caused by price distortions. In contrast, location subsidies to residents do not change the scales of the price distortion and the variety distortion, but increases the fiscal externality because the policy increase asymmetric income transfers across residents.

In order to validate the theoretical results, we numerically evaluate the welfare impacts with the CES utility function. We show that the numerical results are the same as the theoretical results. Moreover, we conduct numerical analyses with a variable elasticity of substitution (VES) utility function in order to examine whether or not relaxing the assumption regarding the elasticity affects the welfare impacts. We show that the welfare impacts are qualitatively the same as the results of the CES function. In the numerical analyses as well as the theoretical derivation, we decompose the welfare change into net benefits generated by the price distortion, the variety distortion, and the fiscal externality by using Harberger's welfare measurement formula. With the location subsidies to residents, all the net benefits are negative. With subsidizing retail stores, the net benefit generated by the price distortion and the variety distortion are negative and positive, respectively, and the latter exceeds the former.

Our paper is organized as follows. Basic assumptions are introduced in Section



(a) A line segment city with two symmetric marketplaces.



(b) A city with a large marketplace and a small marketplace.

Figure 1: Two examples of the model city with two marketplaces given market areas. Circles: residential zones in the city; Triangles: marketplaces in the city.

2. The formula to evaluate the welfare impact of place-based policies is introduced in Section 3. Welfare analysis is conducted in Section 4. Section 5 concludes our paper.

2. Model

2.1. Basic assumptions

The model city is a closed city where $\overline{N}$ homogeneous consumers reside. This city consists of residential zones and marketplaces. Let $\mathcal{I} \equiv \{1, \ldots, I\}$ and $\mathcal{J} \equiv \{1, \ldots, J\}$ denote the sets of the residential zones and marketplaces, respectively ($I, J > 2$). Consumers reside in a residential zone and visit a marketplace for shopping. They can choose where to reside.

In order to simply conduct the welfare analysis of place-based policies, we focus on an equilibrium such that all consumers in the same residential zone visit the same marketplace. In order to express such an equilibrium, we introduce market area. Let

$\mathcal{I}_j$ ($\subset \mathcal{I}$) denote the residential zones where consumers visit the jth marketplace for shopping.[1] $\mathcal{I}_j$ is the market area represented by residential zones.[2] Figure 1(a) and (b) represent examples of geographical patterns of residential zones and marketplaces with market areas. If each residential zone is small and the zones densely line up, shown in Figure 1, then we can interpret the geographical setting in our model as a discrete version of a continuous geographical space employed by most multipurpose shopping models (Tabuchi, 2009; Ushchev et al., 2015).

Consumers pay travel cost to visit marketplaces for shopping. Let $j(i)$ ($\in \mathcal{J}$) denote the marketplace that consumers residing in residential zone i visit for shopping.[3] Consumers residing in zone i pay travel cost t_i in order to visit the $j(i)$th marketplace. Following assumption employed in the multipurpose shopping models, we assume that travel cost between a marketplace and a residential zone is determined with the straight-line distance between the marketplace and the residential zone. Travel costs determine the geographical pattern of the residential zones and the marketplaces, as shown in Figure 1.

2.2. Consumers

We explain the utility and the budget constraint of consumers residing in residential zone i ($\in \mathcal{I}$) and visiting marketplace $j(i)$ ($\in \mathcal{J}$) for shopping. Consumers in the city

[1]We introduce market equilibrium conditions given market area $\{\mathcal{I}_j\}_{j\in\mathcal{J}}$ in Section 2.5. One may consider that the market area should be endogenous. Note that our aim is to investigate how place-based policies affect social welfare at market equilibrium. Hence, we can accomplish our aim by conducting welfare analysis for any given market area. We will conduct the theoretical analysis in Sections 3 and 4.

[2]We have $\mathcal{I}_j \neq \emptyset$ ($\forall j \in \mathcal{J}$), $\mathcal{I}_{j_1} \cap \mathcal{I}_{j_2} = \emptyset$ ($j_1 \neq j_2$), and $\mathcal{I} = \cup_{j=1}^{J} \mathcal{I}_j$.

[3]The formal definition of $j(i)$ is as follows. We define mappings $J_1 : i \mapsto \mathcal{I}_j$, where $i \in \mathcal{I}_j$, and $J_2 : \mathcal{I}_j \mapsto j$. Note that J_1 is well-defined since the definition of the market area determines unique $\mathcal{I}_j$ for each i ($\in \mathcal{I}$). $j()$ is the composite mapping of J_1 and J_2: $j(i) \equiv (J_2 \circ J_1)(i)$.

derive utility from differentiated goods, housing measured in floor area, and a composite good. The utility of consumers residing in residential zone i is given by $U_i(M_i, h_i, a_i)$, where M_i is the composite index of the consumption of differentiated goods, h_i is the consumption of housing measured by floor area, and a_i is the consumption of the composite good which is the numéraire. M_i is assumed to be an additively separable function over the varieties (i.e., the differentiated goods) supplied in a marketplace:

$$M_i = \int_0^{m_{j(i)}} u(q_i(k))\mathrm{d}k, \tag{1}$$

where $q_i(k)$ is the consumption of the kth variety and $m_{j(i)}$ is the mass of varieties supplied in the $j(i)$th marketplace.

We assume public ownership of land and firms for simplicity. Consumers' net income y_i is composed of common income y, travel cost to the marketplace t_i, equal share of profits and rents Π, and subsidy (or tax) $s_i(s)$:

$$y_i = y - t_i + \Pi + s_i(s). \tag{2}$$

Each place-based policy determines $s_i(s)$ and s (≥ 0) expresses tax level to finance place-based policies. We call s the policy instrument.

Our paper focus on two place-based policies: location subsidies to stores, and location subsidies to consumers.[4] Consumers (retail stores) in the same zone can receive the same amount of subsidy with these policies. Let n_i and $s_j^M(s)$ denote the total number of consumers residing in residential zone i and the total subsidy provided to retail stores in marketplace j, respectively. The formal definitions for the place-based policies are as follows.

Definition 1. *Let $N_{\widehat{\mathcal{I}}}$ denote the total population in target area $\widehat{\mathcal{I}}$ ($\subset \mathcal{I}$) (i.e., $N_{\widehat{\mathcal{I}}} = \sum_{a \in \widehat{\mathcal{I}}} n_a$). Location subsidies to consumers in target area $\widehat{\mathcal{I}}$ is the place-based*

[4]Similar policies to both policies are adopted by local governments in the real world (e.g., Albuquerque in the U.S.A. and Toyama in Japan).

policy such that the following equations hold.

$$s_i(s) = \begin{cases} \left(\overline{N} - N_{\widehat{\mathcal{I}}}\right) s/N_{\widehat{\mathcal{I}}} & (i \in \widehat{\mathcal{I}}), \\ -s & (i \notin \widehat{\mathcal{I}}), \end{cases} \qquad s_j^M(s) = 0 \quad \forall j \in \mathcal{J}. \tag{3}$$

Definition 2. *Location subsidies to stores is the place-based policy such that the following equations hold.*

$$s_i(s) = -s \quad \forall i \in \mathcal{I}, \quad s_j^M(s) = \begin{cases} s\overline{N} & (j = 1), \\ 0 & (j \neq 1). \end{cases} \tag{4}$$

Location subsidies to consumers implies that consumers residing in a target area are subsidized. Location subsidies to stores implies that retail stores operating in a marketplace are subsidized. The subsidies with the policies are paid by consumers:

$$\sum_{i \in \mathcal{I}} n_i s_i(s) + \sum_{j \in \mathcal{J}} s_j^M(s) = 0. \tag{5}$$

In order to evaluate the welfare impacts of the place-based policies with Harberger's welfare change measurement formula (Harberger, 1971), we focus on the following expenditure minimization problem:

$$\min_{\{q_i(k)\}_k, h_i, a_i} \int_0^{m_{j(i)}} p_{j(i)}^M(k) q_i(k) \mathrm{d}k + p_i^H h_i + a_i \quad \text{s.t.} \quad \text{Eq. (1) and } U_i = \overline{U}, \tag{6}$$

where $p_{j(i)}^M(k)$ is the price of kth variety supplied in the $j(i)$th marketplace, p_i^H is the price per square foot of housing in residential zone i, and $\overline{U}$ is the target utility. We decompose the expenditure minimization problem into two problems regarding two-stage budgeting. The conditional demands are functions of k, $\{p_{j(i)}^M(k)\}_k$, m_j, and M_i:

$$q_i^*(k) = \widetilde{q}_i^*(\{p_{j(i)}^M(k)\}_k, m_{j(i)}, M_i) \quad \forall k \in [0, m_{j(i)}], \tag{7}$$

where superscript " $*$ " and tilde " $\widetilde{\bullet}$ " denote the optimal solution and a function that maps arguments onto " $\bullet$ ", respectively. We assume that all the consumers consume all the varieties (i.e., $q_i^*(k) > 0$ ($\forall k \in [0, m_j]$)). The demand functions are functions of $\{p_{j(i)}^M(k)\}_k$, $m_{j(i)}$, p_i^H, and $\overline{U}$:

$$M_i^* = \widetilde{M_i^*}\left(\{p_{j(i)}^M(k)\}_k, m_{j(i)}, p_i^H, \overline{U}\right),$$
$$h_i^* = \widetilde{h_i^*}\left(\{p_{j(i)}^M(k)\}_k, m_{j(i)}, p_i^H, \overline{U}\right),$$
$$a_i^* = \widetilde{a_i^*}\left(\{p_{j(i)}^M(k)\}_k, m_{j(i)}, p_i^H, \overline{U}\right).$$

Substituting M_i^* into $q_i^*(k)$ yields

$$q_i^*(k) = \widetilde{q_i^*}\left(\{p_{j(i)}^M(k)\}_k, m_{j(i)}, \widetilde{M_i^*}\left(\{p_{j(i)}^M(k)\}_k, m_{j(i)}, p_i^H, \overline{U}\right)\right). \tag{8}$$

See Appendix A.1 for detailed derivation of the demand functions.

2.3. Retail stores

Retail stores supply differentiated goods in marketplaces. Each retail store supplies a variety in a marketplace. They are under monopolistic competition. Hence, the total mass of retail stores in each marketplace is endogenously determined by free entry. They rent units of land in marketplaces.

All the retail stores incur the same marginal production cost c to supply varieties. The retail store that supplies the kth variety incurs $k + r_j(k)$ for the fixed cost, where k also represents the fixed cost that depends on varieties, and $r_j(k)$ is land rent of a constant unit of land for a store. Some retail stores can receive subsidies, as shown in Definition 2.

Let $Q_j(k)$ and $\pi_j^M(k)$ denote the supply of the kth variety and the profit of the retail store supplying the kth variety in marketplace j, respectively. $\pi_j^M(k)$ is given by

$$\pi_j^M(k) = (p_j^M(k) - c)Q_j(k) - k + \frac{s_j^M(s)}{m_j} - r_j(k) \quad \forall k \in [0, m_j]. \tag{9}$$

We assume that each store pays the bid rent. Using profit (9) yields the maximum land rent that each store can pay:

$$r_j(k) = \max_{p_j^M(k)} \left((p_j^M(k) - c)Q_j(k) - k + \frac{s_j^M(s)}{m_j} \right). \tag{10}$$

Eq. (10) implies that the more demand for a variety in a marketplace, the larger the bid rent. Hence, if the prices of a variety supplied in some marketplaces are the same, then a retail store operating in a larger marketplace can propose a higher bid rent.

The total supply (or demand) is given by

$$Q_j(k) = \sum_{a \in \mathcal{I}_j} n_a q_a^*(k). \tag{11}$$

The first order condition for maximization problem (10) is given by

$$\frac{Q_j(k)}{p_j^M(k)} (p_j^M(k) + (p_j^M(k) - c)\eta_j^M(k)) = 0, \tag{12}$$

where $\eta_j^M(k)$ is the price elasticity of the total demand: $\eta_j^M(k) = \partial \ln Q_j(k) / \partial \ln p_j^M(k)$. Eq. (12) implies that retail stores with heterogeneous technologies in terms of fixed cost determine profit-maximizing prices based on their common marginal cost and demands of consumers. For simplicity, we focus on a symmetric price equilibrium[5] such that the demands satisfy the following symmetric elasticity condition:

$$\eta_j^M(k) = \eta_j^M(k') \quad \forall k, k' \in [0, m_j]. \tag{13}$$

We can obtain the above equation with subutility function M_i employed in Section 4 (e.g., CES function). Using first order condition (12) and Eq. (13) yields the prices of varieties supplied in marketplace j:

$$p_j^M(k) = \widetilde{p_j^M}(\{n_i\}_{i \in \mathcal{I}_j}, m_j, \{p_i^H\}_{i \in \mathcal{I}_j}, \overline{U}) \quad \forall k \in [0, m_j]. \tag{14}$$

Since the prices do not depend on k, we express $p_j^M(k)$ as p_j^M. Furthermore, under the symmetric price equilibrium, the total demand for varieties supplied in the same marketplace are the same: $Q_j(k) = Q_j(k')$ ($\forall\ k, k' \in [0, m_j]$). Hence, we express $Q_j(k)$ as Q_j.

[5] DellaVigna and Gentzkow (2019) empirically show that retail stores in the U.S.A. charge nearly uniform prices across stores. Based on this result, we focus on a symmetric price equilibrium.

2.4. Developers

Developers are assumed to be perfectly competitive and homogeneous. They supply residential buildings in residential zones.

Following Brueckner (2007) and Domon et al. (2022), we specify developers as follows. Buildings are produced by combining one unit of land and housing capital (or building materials). The area of land in each residential zone is assumed to be one unit. The building output per unit of land is expressed as $g(b)$, where g is the housing production function and b is the capital-to-land ratio. Let π_i^H and H_i denote the developers' net profit in residential zone i and the housing output, respectively. π_i^H is given by

$$\pi_i^H = p_i^H H_i - g^{-1}(H_i) - R_i^H, \tag{15}$$

where g^{-1} is the inverse function of g and R_i^H $(i \in \mathcal{I})$ is the total land rent in residential zone i.

We assume that developers pay the bid land rent. Using profit (15) yields the maximum land rent that developers can pay:

$$R_i^H = \max_{H_i} \left(p_i^H H_i - g^{-1}(H_i) \right). \tag{16}$$

The first order condition for maximization problem (16) is

$$p_i^H - \frac{\partial g^{-1}(H_i)}{\partial H_i} = 0 \quad \forall i \in \mathcal{I}. \tag{17}$$

Using this condition, we can obtain $H_i^* = \widetilde{H_i^*}(p_i^H)$. Hence, the bid rent is expressed as

$$R_i^H = p_i^H H_i^* - g^{-1}(H_i^*) \quad \forall i \in \mathcal{I}. \tag{18}$$

2.5. Market equilibrium condition

We introduce market equilibrium condition. In the equilibrium, given the spatial distribution of consumers (i.e., $(n_i)_{i \in \mathcal{I}}$), the market clearing condition of housing holds and the mass of retail stores is determined.

The market clearing condition for housing is given by

$$\widetilde{H_i^*}(p_i^H) = n_i \widetilde{h_i^*}(p_{j(i)}^M, m_{j(i)}, p_i^H, \overline{U}) \quad \forall i \in \mathcal{I}, \tag{19}$$

Next, we focus on the mass of retail stores (i.e., m_j). Since p_j^M and Q_j do not depend on k, $(p_j^M - c)Q_j + s_j^M(s)/m_j$ also does not depend on k. Land rent $r_j(k)$, shown by Eq. (10), monotonously decreases with an increase in k. Using this monotonicity, $r_j(k) \geq 0$ ($\forall k \in [0, m_j]$), and Eq. (10), we obtain the following condition for the mass of stores m_j:

$$r_j(m_j) = (p_j^M - c)Q_j - m_j + \frac{s_j^M(s)}{m_j} = 0 \quad \forall j \in \mathcal{J}. \tag{20}$$

Eq. (20) implies that sales are equals to the cost for the store supplying variety m_j.

Let $\boldsymbol{n} \equiv (n_i)_{i \in \mathcal{I}}$ denote the spatial distribution of the consumers in the city. The total number of equations, which are Eqs. (19) and (20), is equal to that of endogenous variables, which are m_j and p_i^H. Using these equations, we can obtain these variables as functions of spatial distribution $\boldsymbol{n}$, target utility $\overline{U}$, and policy instrument s:

$$m_j = \widetilde{m_j}(\boldsymbol{n}, \overline{U}, s), \quad p_i^H = \widetilde{p_i^H}(\boldsymbol{n}, \overline{U}, s).$$

Substituting these functions into $\widetilde{p_j^M}$ (i.e., Eq. (14)), we obtain $\widetilde{p_j^M}$ as a function of $\boldsymbol{n}$, $\overline{U}$, and s. Since the prices and the mass are functions of $\boldsymbol{n}$, $\overline{U}$, and s, the demand functions are also functions of $\boldsymbol{n}$, $\overline{U}$, and s in the equilibrium.

3. Marginal welfare impacts of place-based policies

3.1. Allais surplus

We investigate the welfare impact of place-based policies. In this paper, we measure the welfare impact in terms of the Allais surplus (Allais, 1977). The Allais surplus is defined as a surplus of goods that can be taken up with a policy to keep the utility levels constant. There are two advantages of employing the Allais surplus when we evaluate welfare impact. First, we can evaluate the welfare impact in terms of the compensation criterion.[6] Second, we can interpret the welfare impact in terms of distortions generated by market failure; our paper focuses on this advantage.

The Allais surplus is the weighted sum of income minus the expenditure function of consumers with the weights being the number of consumers. As Wheaton (1977) and Kono and Kishi (2018) show, we cannot uniquely obtain the Allais surplus in our model because of population migration. Following them, we impose the equality of surpluses across residential zones to define the Allais surplus.[7]

Using net income shown in Eq. (2), we obtain the substructed income of consumers residing in zone i with the expenditure: $y_i - e_i = y - t_i + \Pi + s_i(s) - e_i$, where e_i is the expenditure function of consumers residing zone i. Using the assumption of the public ownership, we obtain equal share of profits and rents Π:

$$\Pi = \overline{N}^{-1} \left(\sum_{i \in \mathcal{I}} \left(\pi_i^H + R_i^H \right) + \sum_{j \in \mathcal{J}} \left(\int_0^{m_j} \pi_j^M(k) \mathrm{d}k + \int_0^{m_j} r_j(k) \mathrm{d}k \right) \right). \tag{21}$$

Substituting Eqs. (9) and (18) into the above Π yields

$$\Pi = \overline{N}^{-1} \left(\sum_{i \in \mathcal{I}} (p_i^H H_i^* - g^{-1}(H_i^*)) + \sum_{j \in \mathcal{J}} \left((p_j^M - c) Q_j m_j - \frac{m_j^2}{2} + s_j^M(s) \right) \right). \tag{22}$$

The condition for the equal surpluses among the residential zones is given by

$$y - t_i + \Pi + s_i(s) - e_i = \overline{E} \quad \forall i \in \mathcal{I}, \tag{23}$$

where $\overline{E}$ ($\in \mathbb{R}$) is the surplus level in each residential zone. Moreover, population constraint condition holds in the closed city:

$$\sum_{i \in \mathcal{I}} n_i = \overline{N}. \tag{24}$$

[6]We can conduct welfare analysis with the compensation criterion for spatial economic models with population migration as well as multipurpose shopping models. See Charlot et al. (2006) for an example of welfare analysis with the compensation criterion for a New Economic Geography model.

[7]Our definition of the Allais surplus is called the Equalized-β measure as shown in Kono and Kishi (2018).

Using Eqs. (23) and (24), we can obtain spatial distribution $\boldsymbol{n}$ as a function of policy instrument s: $\boldsymbol{n} = \boldsymbol{n}(s)$.

Let AS denote the Allais surplus. We can obtain AS as a function of s, keeping the utility level at $\overline{U}$:

$$AS(\boldsymbol{n}(s), s, \overline{U}) = \sum_{i \in \mathcal{I}} n_i(y - t_i + \Pi + s_i(s) - e_i) = \overline{N} \times \overline{E}. \tag{25}$$

3.2. Marginal change in Allais surplus AS with a change in policy instrument s

Investigating marginal change in AS with an increase in policy instrument s, we evaluate the welfare impact of adopting a place-based policy. In order to evaluate the welfare impact, we need to determine target utility level $\overline{U}$. We set the target utility level such that all the consumers maximize their utility under the equilibrium prices with no policy. That is, we focus on target utility level $\overline{U^*}$ and spatial distribution $\boldsymbol{n}^*$ such that following equation holds:

$$y - t_i + \Pi + s_i(0) - e_i = 0 \quad \forall i \in \mathcal{I}, \tag{26}$$

which is condition (23) at $s = 0$ and $\overline{E} = 0$. This condition implies that under the equilibrium prices and $\overline{U^*}$, all the consumers maximize their utility because expenditure e_i equals net income y_i (See Eq. (2)).

We focus on marginal change in AS from $(\boldsymbol{n}^*, \overline{U^*})$ with an increase in s from zero along the equilibrium path. We can obtain $\mathrm{d}AS/\mathrm{d}s$ with the market distortions generated by monopolistic competition and place-based policies, which is consistent with Harberger's welfare change measurement formula (Harberger, 1971).

Lemma 1. *For any given market area $\{\mathcal{I}_j\}_{j\in\mathcal{J}}$, the following holds for $s \geq 0$:*

$$\frac{\mathrm{d}AS}{\mathrm{d}s} = PDH + PD + VD + FD, \tag{27}$$

where

$$PDH = \sum_{i\in\mathcal{I}} \left[\left(\underbrace{p_i^H - \frac{\partial g^{-1}}{\partial H_i}}_{=0} \right) \frac{\mathrm{d}H_i^*}{\mathrm{d}s} \right], \quad PD \equiv \sum_{j\in\mathcal{J}} \underbrace{(p_j^M - c)m_j}_{\geq 0} \frac{\mathrm{d}Q_j}{\mathrm{d}s},$$

$$VD \equiv \sum_{j\in\mathcal{J}} \left(\underbrace{\sum_{a\in\mathcal{I}_j} \left(\frac{n_a p_j^M u(q_a)}{u'(q_a)} \right) - cQ_j - m_j}_{\geq 0} \right) \frac{\mathrm{d}m_j}{\mathrm{d}s}, \quad FD = \sum_{i\in\mathcal{I}} (-s_i) \frac{\mathrm{d}n_i}{\mathrm{d}s}.$$

Proof. See Appendix A.2. □

Eq. (27) shows that $\mathrm{d}AS/\mathrm{d}s$ is decomposed into four parts.[8] *PDH* and *PD* express the total change in deadweight losses in the housing markets and the differentiated goods markets, respectively. *VD* is caused by variety distortion (Kanemoto, 2013a,b; Behrens et al., 2015). *FD* is caused by asymmetric income transfer among residential zones with a place-based policy. We can interpret *FD* as the migration fiscal externality generated by income transfer inefficiency by a place-based policy (Boadway and Flatters, 1982; Kono et al., 2007). *FD* indicates that place-based policies distort market allocation and decrease surplus. For example, if population in residential zone i where consumers can receive subsidy increases by a place-based policy, then the city loses $s_i \times \mathrm{d}n_i/\mathrm{d}s$ of surplus.

We can decompose *PD* and *VD* into two effects, which are employed in Section 4. One is the effect generated by population migration, whereas the other is the effect generated by only subsidy. Let *EP* and *ES* denote the former effect and the latter effect, respectively. Using *PD* and *VD*, we can express *EP* and *ES* as

$$EP = PD_P + VD_P, \quad ES = PD_S + VD_S,$$

where

$$PD_P = \sum_{j\in\mathcal{J}} \left[(p_j^M - c) m_j \sum_{a\in\mathcal{I}} \frac{\partial Q_j}{\partial n_a} \frac{\mathrm{d}n_a}{\mathrm{d}s} \right], \tag{28}$$

[8]If the geographical space of the city is continuous, then the welfare impact generated by change in a market boundary is added to the welfare measurement formula. In our model, the welfare impact is composed of a difference between travel costs from the market boundary to marketplaces. Hence, if the difference is small, then the welfare impact is almost the same as that of the discrete model.

$$PD_S = \sum_{j \in \mathcal{J}} \left[(p_j^M - c) m_j \frac{\partial Q_j}{\partial s} \right], \tag{29}$$

$$VD_P = \sum_{j \in \mathcal{J}} \left[\left(\sum_{a \in \mathcal{I}_j} \left(\frac{n_a p_j^M u(q_a)}{u'(q_a)} \right) - cQ_j - m_j \right) \left(\sum_{a \in \mathcal{I}} \frac{\partial m_j}{\partial n_a} \frac{\mathrm{d}n_a}{\mathrm{d}s} \right) \right], \tag{30}$$

$$VD_S = \sum_{j \in \mathcal{J}} \left(\sum_{a \in \mathcal{I}_j} \left(\frac{n_a p_j^M u(q_a)}{u'(q_a)} \right) - cQ_j - m_j \right) \frac{\partial m_j}{\partial s}. \tag{31}$$

We can interpret VD as follows. Using the first-order condition for expenditure minimization (6), we can interpret $u'(q_a)/p_j^M$ $(a \in \mathcal{I}_j)$ as the marginal utility of shopping expenditure. This interpretation implies that $p_j^M u(q_a)/u'(q_a)$ is the benefit that a consumer can obtain by consuming an additional variety of goods supplied in marketplace j. Hence, the first term of VD is the total benefit that the consumers in the city can obtain. Furthermore, since $cQ_j + m_j$ is the cost that new retail stores entering in marketplace j must incur, the second term is the total cost caused by a place-based policy. That is, we can interpret VD as the total benefit subtracted by the total cost.

Since developers are under perfect competition, the price of housing and the marginal cost are the same. This implies $PDH = 0$; $\mathrm{d}AS/\mathrm{d}s$ is composed of PD, VD, and FD. This equation is similar to welfare change measurement formulae with monopolistic competition (Kanemoto, 2013a,b; Behrens et al., 2015). In contrast to these studies, we focus on income transfer among consumers and retail stores by a place-based policy. FD, which does not appear in Kanemoto (2013a,b) and Behrens et al. (2015), is added to the welfare change measurement formula because our model takes account of place-based policies generating migration fiscal distortions.

To explore welfare analyses of policies, we explain the signs of coefficients in PD and VD. Since each retail store operating in a marketplace supplies a good at a price larger than marginal cost c, we have $(p_j^M - c)m_j \geq 0$ $(\forall j \in \mathcal{J})$.

We can obtain the sign of the coefficient of $\mathrm{d}m_j/\mathrm{d}s$ as follows. Using the love of variety condition (e.g., Behrens and Murata, 2007; Behrens et al., 2015) yields

$u(q_i)/u'(q_i) \geq q_i$ ($\forall i \in \mathcal{I}$). Using this inequality, the definition of total demand, and Eq. (20), we can obtain the following inequality for $s = 0$:

$$\sum_{a \in \mathcal{I}_j} \left(\frac{n_a p_j^M u(q_a)}{u'(q_a)} \right) - cQ_j - m_j \geq p_j^M Q_j - cQ_j - m_j = 0. \tag{32}$$

The signs of the coefficients of $\mathrm{d}Q_j/\mathrm{d}s$ and $\mathrm{d}m_j/\mathrm{d}s$ are non-negative, whereas those of $\mathrm{d}Q_j/\mathrm{d}s$ and $\mathrm{d}m_j/\mathrm{d}s$ depend on place-based policies. We can intuitively predict that policies promoting marketplace j_1 generate $\mathrm{d}Q_{j_1}/\mathrm{d}s, \mathrm{d}m_{j_1}/\mathrm{d}s > 0$ and $\mathrm{d}Q_j/\mathrm{d}s, \mathrm{d}m_j/\mathrm{d}s \leq 0$ ($j \neq j_1$). We, however, cannot determine $\mathrm{d}AS/\mathrm{d}s > 0$ for such a policy since all the coefficients are non-negative. Hence, the welfare impact of a place-based policy depends on how we specify the utility function and the place-based policy. Specifying the utility function in Section 4, we investigate the welfare impact of place-based policies.

4. Welfare analysis of place-based policies with the constant elasticity of substitution and the variable elasticity of substitution cases

We evaluate the welfare impact of adopting place-based policies. We focus on two place-based policies shown in Definitions 1 and 2 (i.e., location subsidies to consumers and location subsidies to stores).

4.1. Model specification with the constant elasticity of substitution

Specifying the utility function and the housing production function, we demonstrate how place-based policies improve social welfare with the Allais surplus defined in Section 3.1.

Most multipurpose shopping models in which retail stores are under monopolistic competition represent consumers' love of variety with constant elasticity of substitution (CES) function (e.g., Henkel et al., 2000; Tabuchi, 2009; Ushchev et al., 2015). We evaluate the welfare impact with the following utility function:

$$U_i = \frac{\sigma\mu}{\sigma - 1} \ln M_i + (1-\mu) \ln h_i + a_i, \quad 0 < \mu < 1, \tag{33}$$

where $M_i = \int_0^{m_j} q_j(k)^{(\sigma-1)/\sigma} \mathrm{d}k$. σ and μ are the elasticities of substitution between any two varieties and the shopping expenditure, respectively.[9] In addition to the above specification for consumers' preference, we specify the housing production function employed by urban economics models (e.g., Brueckner, 2007; Kono et al., 2019; Domon et al., 2022):

$$g(b) = \theta b^{\beta} \quad (0 < \theta,\ 0 < \beta < 1). \tag{34}$$

4.1.1. Properties of $\mathrm{d}AS/\mathrm{d}s$

We show properties of $\mathrm{d}AS/\mathrm{d}s$ with the specification in order to discuss the welfare impacts of the place-based policies. We can obtain the variables to express EP and ES with the market equilibrium conditions (see Appendix B.1 for the derivation):

$$m_j = \left(\frac{\mu}{\sigma} \sum_{a \in \mathcal{I}_j} n_a + s_j^M(s) \right)^{1/2} \quad \forall j \in \mathcal{J}, \tag{35}$$

$$q_i^* = \mu (p^M m_{j(i)})^{-1} \quad \forall i \in \mathcal{I}, \tag{36}$$

$$Q_j = \sum_{a \in \mathcal{I}_j} n_a q_a^* \quad \forall j \in \mathcal{J}, \tag{37}$$

where $p^M = c\sigma/(\sigma - 1)$ is the equilibrium price of varieties. The following lemma holds with the above model specification.

Lemma 2. *If the utility function and the production function are expressed by (33) and (34) respectively, then $PD_P = VD_P = 0$ holds at the market equilibrium (i.e., $(\boldsymbol{n}, s, \overline{U}) = (\boldsymbol{n}^*, 0, \overline{U^*})$).*

Proof. See Appendix B.3. □

[9] In addition, $1 - \mu$ implies the housing expenditure share.

Lemma 2 shows that $PD = PD_S$, $VD = VD_S$, and $\mathrm{d}AS/\mathrm{d}s = ES$ hold. This result would be obtained because all the retail stores in the city supply varieties at the same price. Such pricing is caused when we assume the CES preference because the CES preference causes the price elasticity of the total demand to be constant (i.e., σ).

4.1.2. Location subsidies to consumers

We focus on location subsidies to consumers. If a place-based policy does not subsidize retail stores, then mass of variety m_j is not affected by policy instrument s (see Eq. (35)). Hence, $\mathrm{d}AS/\mathrm{d}s = ES = 0$ holds for any location subsidies to consumers. This result indicate AS at $(\boldsymbol{n}, s, \overline{U}) = (\boldsymbol{n}^*, 0, \overline{U^*})$ is locally maximized.

Formulating a maximization problem for AS, we examine whether or not AS is locally maximized for place-based policies that do not generate ES. The maximization problem of AS is defined as follows:

$$\max_{\boldsymbol{n}} AS \quad (38)$$
$$\text{s.t.} \quad \gamma_i(\boldsymbol{n}) \equiv -n_i \leq 0 \quad (i \in \mathcal{I}), \quad \Gamma(\boldsymbol{n}) \equiv \overline{N} - \sum_{i \in \mathcal{I}} n_i = 0.$$

We analyze this maximization problem with the Karush-Kuhn-Tucker (KKT) condition. The results regarding the first-order necessary conditions and the second-order sufficient conditions are as follows.

Lemma 3. *At the market equilibrium (i.e., $(\boldsymbol{n}, s, \overline{U}) = (\boldsymbol{n}^*, 0, \overline{U^*})$), $\boldsymbol{n}^*$ satisfies the KKT conditions of maximization problem (38).*

Proof. See Appendix B.3. □

Lemma 4. *At the market equilibrium (i.e., $(\boldsymbol{n}, s, \overline{U}) = (\boldsymbol{n}^*, 0, \overline{U^*})$), $\boldsymbol{n}^*$ satisfies the second-order sufficient conditions of maximization problem (38) if $\mu/(1-\mu) < 2(\sigma - 1)(1 - \beta)$.*

Proof. See Appendix B.3. □

Even though there is a market failure generated by monopolistic competition (i.e., imperfect competition), Lemma 4 implies that any inner market equilibrium is locally maximized if the expenditure share of differentiated goods and housing is lower than $2(\sigma - 1)(1 - \beta)$. Low σ implies that consumers love variety, whereas high β implies that developers are more productive. Lemma 4 implies that any policy that generates population migration (i.e., change in $\boldsymbol{n}$), as well as location subsidies to consumers, decrease the Allais surplus. For example, adopting land-use regulation decreases the Allais surplus. We restate Lemma 4 in the following proposition:

Proposition 1. *The market equilibria are locally efficient regarding the spatial distribution of consumers, even though there are price distortions and the variety distortions generated by monopolistic competition.*

Proposition 1 is similar to one of the results shown by Dhingra and Morrow (2019). Dhingra and Morrow (2019) compare the allocation at market equilibrium with that at the socially optimal state in an economy that consists of workers and firms under monopolistic competition. In particular, they show that if workers' demands for varieties are expressed by the CES preference, then the allocation at the market equilibrium is socially optimal in a non-space economy with no migration. We show that, in an economy with population migration and monopolistic competition, allocations determined by $\boldsymbol{n}$ at the equilibrium are locally efficient, in contrast to the results of Dhingra and Morrow.

Regarding Proposition 1, we should note that at the equilibrium, mass of variety m_j is determined by $\boldsymbol{n}$ (see Eq. (35)). Since policymakers can choose the level of the mass, Proposition 1 does not ensure that the equilibrium is first-best, which is a difference between Proposition 1 and the results of Dhingra and Morrow (2019). For example, place-based policies that generate positive direct benefit (i.e., $MS > 0$) increase the Allais surplus.

4.1.3. Location subsidies to stores

We explore location subsidies to stores. This place-based policy is an example in which $ES \neq 0$ holds. Using Eq. (4) in Definition 2 and Eq. (35) yields $\partial m_1/\partial s \neq 0$. Furthermore, we obtain the following result.

Lemma 5. *At the market equilibrium (i.e., $(\boldsymbol{n}, s, \overline{U}) = (\boldsymbol{n}^*, 0, \overline{U^*})$), the following holds.*

$$\frac{\mathrm{d}AS}{\mathrm{d}s} = \underbrace{PD}_{<0} + \underbrace{VD}_{>0} + \underbrace{FD}_{=0} > 0.$$

Proof. See Appendix B.3. □

Lemma 5 shows that adopting location subsidies to stores marginally increases the Allais surplus. While the deadweight loss generated by the price distortion decreases AS, the total net benefit generated by the variety distortion exceeds the loss.

In this section, we have shown that subsidizing retail stores operating in a marketplace (e.g., the downtown area in a city) is desirable from the viewpoint of welfare, whereas subsidizing consumers residing near the marketplace is harmful. Hence, whether or not place-based policies are socially efficient depends on the recipients of the subsidies due to the market distortions, even if the policies promote retail agglomeration in the downtown area.

4.2. Numerical examples

Conducting numerical analysis of the equilibrium and the Allais surplus on the equilibrium for $s \geq 0$, we demonstrate how the surplus changes on the equilibrium. We consider the model city shown in Figure 1(b). That is, this city consists of the downtown area and the suburb. The downtown area and the suburb have one marketplace (i.e., $J = 2$). There are more residential zones in the downtown area than the zones in the suburb. We represent the assumption as $\mathcal{I} = \{1, 2, \ldots, 8\}$, $\mathcal{I}_1 = \{1, 2, \ldots, 5\}$, and $\mathcal{I}_2 = \{6, 7, 8\}$. The travel costs to the marketplaces are the same: $t_i = 10$ ($\forall i \in \mathcal{I}$). We set common income of consumers y at 1000. Hence, 1% of the common income is the travel cost to the marketplace. $\overline{N}$ is set at 1; n_i is interpreted as the ratio to the total population in the city.

4.2.1. How the place-based policies change the Allais surplus

We conduct numerical analysis with utility function (33) and production function (34). There are five exogenous parameters: θ, β, μ, σ, and c. Referring to the empirical results shown by Domon et al. (2022), we set θ and β at 0.0028 and 0.75, respectively. We set μ at 0.4, which means that the ratio of the shopping expenditure to the housing expenditure is about 66%. σ and c are set at 6.0 and 1.0, respectively.

We numerically evaluate the two place-based policies defined in Section 2. One is location subsidies to consumers residing in the downtown area:

$$s_i(s) = \begin{cases} ((\sum_{a \in \mathcal{I}_1} n_a)^{-1} - 1)s & (i \in \mathcal{I}_1), \\ -s & (i \in \mathcal{I}_2), \end{cases} \qquad s_j^M(s) = 0 \quad \forall j \in \mathcal{J}, \tag{39}$$

which is the case for $\widehat{\mathcal{I}} = \mathcal{I}_1$ in Definition 1. The other is location subsidies to stores operating in the downtown area:

$$s_i(s) = -s, \quad s_j^M(s) = \begin{cases} s & (j = 1), \\ 0 & (j = 2), \end{cases} \tag{40}$$

which is the case for $J = 2$ in Definition 2.

We investigate the equilibrium and the Allais surplus for $0 \leq s \leq 10$. Figure 2 shows the population in residential zone 1 (i.e., n_1) and the Allais surplus (i.e., AS), which changes as policy instrument s changes. The Allais surplus increases with the place-based policy expressed by Eq. (40) and decreases with the place-based policy expressed by Eq. (39) from $s = 0$. Both results are consistent with the theoretical results shown in Section 4.1 (i.e., Lemmas 4 and 5). We also check that the Allais surplus monotonously decreases for $0.04 \leq s \leq 10$. In order to clearly show that the

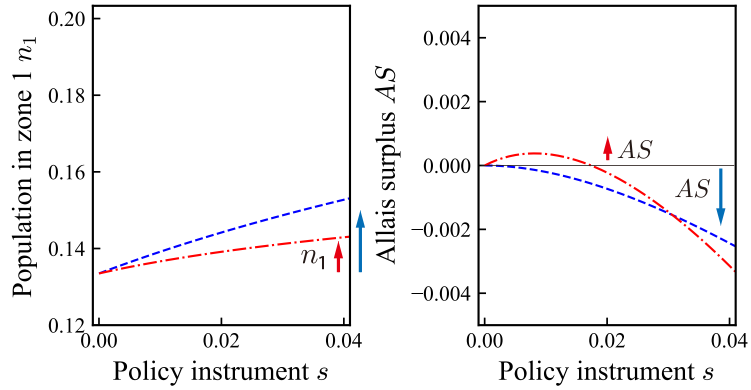


Figure 2: Population and the Allais surplus on the equilibria with utility function (33) and production function (34) . Left: population in residential zone 1. Right: the Allais surplus. Red dashed-dotted line: the result obtained for policy function (40); blue dashed line: the result obtained for policy function (39).

Allais surplus increases for location subsidies to the retail stores, the results for the range are not shown.

We investigate how market distortions affect the Allais surplus shown in Figure 2. Integrating Eq. (27) with respect to policy instrument s, we can decompose Allais surplus AS into three factors:

$$AS(\boldsymbol{n}(s), s, \overline{U^*}) = \int_0^s PD \, \mathrm{d}s + \int_0^s VD \, \mathrm{d}s + \int_0^s FD \, \mathrm{d}s. \tag{41}$$

The first term of Eq. (41) is the factor related to the price distortion, the second term is that to the variety distortion, and the third term is that to the fiscal externality. Figure 3 shows the Allais surplus and the three factors generated by applying the place-based policies for $s \geq 0$. As Figure 3 (a) shows, the location subsidies to consumers does not affect the price distortion and the variety distortions. The fiscal externality alone decreases the Allais surplus. This result is caused by asymmetric income transfer among consumers with the place-based policy. In order to efficiently increase the surplus, policy makers need to apply place-based policies with symmetric income transfer.

Figure 3 (b) shows the Allais surplus and the three factors generated by location

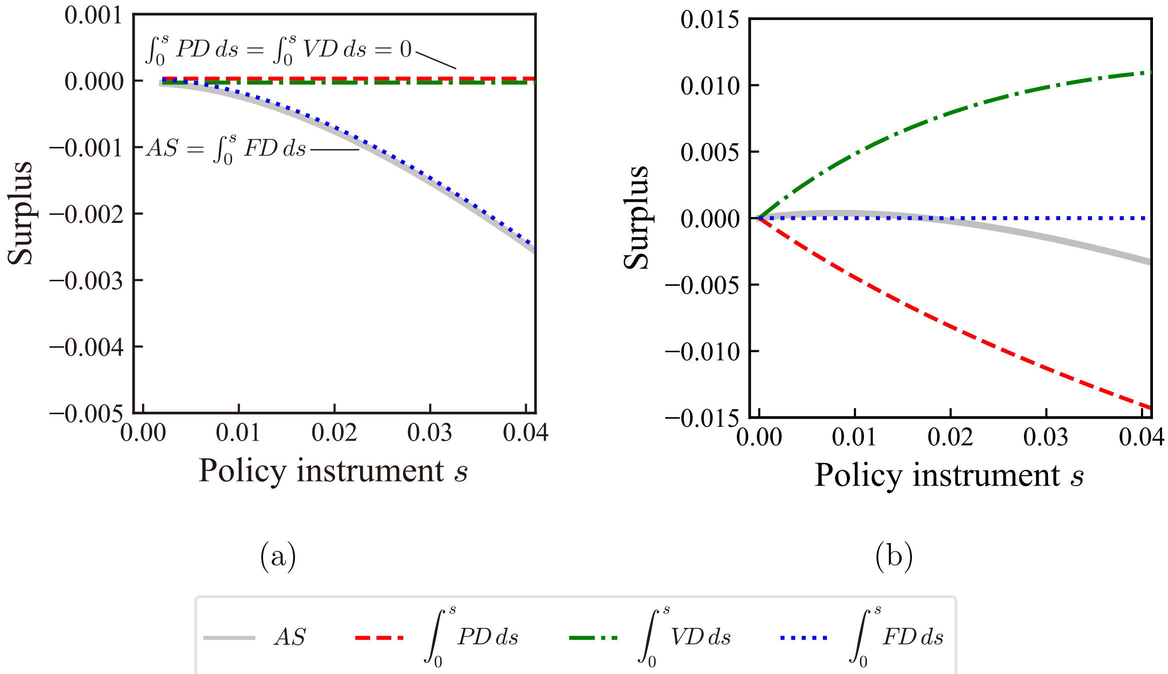


Figure 3: Decomposition of AS into three factors related to the three market distortions: the price distortion, the variety distortion, and the fiscal externality. Gray solid line : Allais surplus AS; red dashed line: price distortion; green dashed-dotted line: variety distortion; blue dotted line: fiscal externality. (a) Location subsidies to consumers (b) Location subsidies to stores.

subsidies to stores. Since all the consumers pay the same amount of tax, income transfer with the policy is symmetric. Hence, the fiscal externality dose not arise unlike location subsidies to consumers. The variety distortion and the price distortion are positive and negative, respectively. Moreover, the variety distortion exceeds the price distortion for $s \leq 0.016$. Location subsidies to stores with asymmetric income transfer will decrease the Allais surplus.

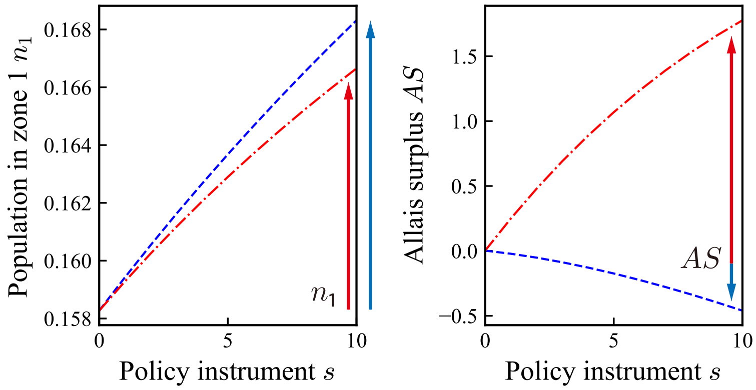


Figure 4: Population and the Allais surplus on the equilibria with utility function (42) and production function (34). Left: population in residential zone 1. Right: the Allais surplus. Red dashed-dotted line: the result obtained for policy function (40); blue dashed line: the result obtained for policy function (39).

4.2.2. Relaxing the assumption regarding the elasticity of substitution between varieties

We show the theoretical results under the constant elasticity of substitution between varieties in Section 4.1. In this section, relaxing this assumption, we explore how the welfare impacts of adopting the place-based policies change. We employ the Constant Absolute Risk Aversion (CARA) utility function as a utility function that represents the variable elasticity of substitution between varieties. Behrens and Murata (2007) show that a pro-competitive effect emerges when we employ this function. In our model, the pro-competitive effect implies that the price of varieties in a marketplace p_j^M decreases with an increase in the mass of the varieties m_j.

We investigate the welfare impacts of adopting the place-based policies with the following utility function:

$$U_i = \mu_1 \ln M_i + \mu_2 \ln h_i + \mu_3 \ln a_i, \tag{42}$$

Table 1: The composition of $\mathrm{d}AS/\mathrm{d}s$.

	PD			VD			Total
	PD_P	PD_S	Total	VD_P	VD_S	Total	
Location subsidies to stores	-0.00570	-0.0450	-0.0507	-0.00536	0.321	0.316	0.27
Location subsidies to consumers	-0.00645	0	-0.00645	-0.00607	0	-0.00607	-0.013

Notes: PD_P, PD_S, VD_P, and VD_S are given by Eqs. (28)–(31). *For the location subsidies to* consumers, policy instrument s does not affect the Hicks demands and the equilibrium conditions; this value affects only population migration. Hence, we obtain $PD_S = VD_S = 0$ for the policy.

where $\mu_1, \mu_2, \mu_3 > 0$, $\mu_1 + \mu_2 + \mu_3 = 1$, and

$$M_i = \int_0^{m_j} 1 - \alpha \exp(-\omega q_i(k))\mathrm{d}k, \quad \alpha, \omega > 0. \tag{43}$$

We employ the same production function (i.e., Eq. (34)). Because the specification is so intractable that we cannot obtain even the closed forms of the expenditure function and the indirect utility, we resort to conducting only numerical analysis for the welfare impacts. The set of parameters $\alpha = 1$, $\omega = 1$, $\mu_1 = 0.1$, $\mu_2 = 0.3$, and $\mu_3 = 0.6$ is employed to investigate the welfare impacts.

Figure 4 shows population in residential zone 1 and the Allais surplus on the equilibria with the utility function and the production function for $0 \leq s \leq 10$. The Allais surplus increases with the place-based policy expressed by Eq. (40) and decreases with the place-based policy expressed by Eq. (39) from $s = 0$. Hence, these results are qualitatively the same as the CES case.

Table 1 shows the composition of $\mathrm{d}AS/\mathrm{d}s$ at the market equilibrium for $s = 0$. Table 1 shows that PD_P and VD_P are negative for both policies unlike the CES case (Lemma 2). With location subsidies to consumers, both welfare changes generated by the price and the variety distortions are negative. With location subsidies to stores, the former

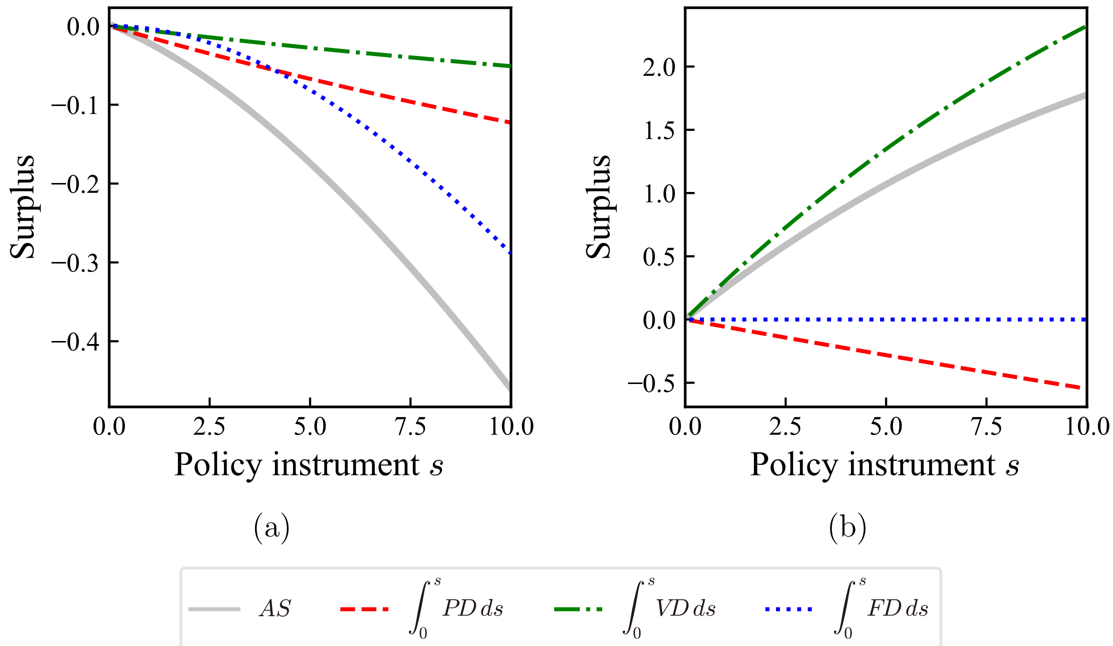


Figure 5: Decomposition of AS into three factors related to the three market distortions: the price distortion, the variety distortion, and the fiscal externality. Gray solid line : Allais surplus AS; red dashed line: price distortion; green dashed-dotted line: variety distortion; blue dotted line: fiscal externality. (a) Location subsidies to consumers (b) Location subsidies to stores.

and the latter are negative and positive, respectively, and the latter exceeds the former. The results shown in Table 1 indicate that policy makers should directly subsidize retail stores in order to increase social welfare. Figure 5 shows the Allais surplus and the three factors generated by the place-based policies. Even if utility function is the VES utility function, the welfare impacts generated by the variety distortion and the fiscal externality are qualitatively the same as the results shown in Table 1.

5. Conclusion

We have evaluated how place-based policies affect social welfare. Conducting theoretical analyses with constant and variable elasticity of substitution between varieties supplied in marketplaces, we obtain two main findings: (1) subsidizing retail stores operating in downtown areas is desirable from the viewpoint of welfare, and (2) subsidizing consumers residing near the downtown areas is harmful. The main reason for the difference is the level of variety distortion generated by a place-based policy. Since directly subsidizing retail stores generates a positive net benefit with the variety distortion, we obtain these results. Furthermore, we have shown that adopting policies that change the spatial distribution of consumers (e.g., land-use regulation) is harmful with the constant elasticity of substitution, even though there are market distortions generated by monopolistic competition.

The numerical simulation in the current paper focuses on demonstrating to validate our theoretical results, and does not calibrate the parameters using real data. We do such numerical analyses in Aizawa and Kono (2023), using data in the city of Sendai in Japan. One extension is to develop a structural model that expresses the agglomeration of retail stores in marketplaces. By doing so, we can evaluate the benefit of place-based policies with empirically valid parameters. For example, combining the so-called Quantitative Spatial Economics model (e.g., Redding and Rossi-Hansberg, 2017; Behrens et al., 2020) and our multipurpose shopping model will enable us to elaborately evaluate the benefit.

Appendix

A. Theoretical details of Section 3

We show theoretical details given market area $\{\mathcal{I}_j\}_{j\in\mathcal{J}}$ in this Appendix. Using the market area, we focus on equilibrium such that consumers in residential zone i ($\in \mathcal{I}$) visit marketplace $j(i)$ ($\in \mathcal{J}$) for shopping.

A.1. First order conditions for the expenditure minimization problem

We solve the following expenditure minimization problem:

$$\min_{\{q_i(k)\}_k} \int_0^{m_{j(i)}} p^M_{j(i)}(k) q_i(k)\mathrm{d}k, \quad \text{s.t. } M_i = \int_0^{m_{j(i)}} u(q_i(k))\mathrm{d}k. \tag{A1}$$

The first order condition for the optimization of problem (A1) is given by

$$p^M_{j(i)}(k) = \rho_1 u'(q_i(k)) \quad \forall k, \tag{A2}$$

$$M_i = \int_0^{m_{j(i)}} u(q_i(k))\mathrm{d}k, \tag{A3}$$

where ρ_1 is the Lagrange multiplier. Solving this problem with the above first order condition, we can obtain conditional demand (7), as shown in Section 2:

$$q_i^*(k) = \widetilde{q_i^*}(\{p^M_{j(i)}(k)\}_k, m_{j(i)}, M_i).$$

Let e_i^M be expenditure function regarding the above conditional demands. This is given by

$$e_i^M(\{p^M_{j(i)}(k)\}_k, m_{j(i)}, M_i) = \int_0^{m_{j(i)}} p^M_{j(i)}(k) q_i^*(k)\mathrm{d}k.$$

Next, we solve the following expenditure minimization problem:

$$\min_{M_i, h_i, a_i} \; p_i^H h_i + e_i^M(\{p^M_{j(i)}(k)\}_k, m_{j(i)}, M_i) + a_i, \quad \text{s.t. } U_i = \overline{U}. \tag{A4}$$

The first order condition for the above optimization problem is given by

$$p_i^H = \rho_2 \frac{\partial U_i}{\partial h_i}, \tag{A5}$$

$$\frac{\partial e_i^M}{\partial M_i} = \rho_2 \frac{\partial U_i}{\partial M_i}, \tag{A6}$$

$$1 = \rho_2 \frac{\partial U_i}{\partial a_i}, \tag{A7}$$

$$U_i = \overline{U}, \tag{A8}$$

where ρ_2 is the Lagrange multiplier. Solving this problem with the above first order condition, we can obtain the Hicksian demand functions:

$$M_i^* = \widetilde{M_i^*}(\{p_{j(i)}^M(k)\}_k, m_{j(i)}, p_i^H, \overline{U}),$$

$$h_i^* = \widetilde{h_i^*}(\{p_{j(i)}^M(k)\}_k, m_{j(i)}, p_i^H, \overline{U}),$$

$$a_i^* = \widetilde{a_i^*}(\{p_{j(i)}^M(k)\}_k, m_{j(i)}, p_i^H, \overline{U}).$$

Substituting M_i^* into conditional demand $q_i^*(k)$ yields

$$q_i^*(k) = \widetilde{q_i^*}\left(\{p_{j(i)}^M(k)\}_k, m_{j(i)}, \widetilde{M_i^*}(\{p_{j(i)}^M(k)\}_k, m_{j(i)}, p_i^H, \overline{U})\right).$$

Using the Hicksian demands, we obtain expenditure function for consumers residing in zone i:

$$e_i = p_i^H h_i^* + e_i^M(\{p_{j(i)}^M(k)\}_k, m_{j(i)}, M_i^*) + a_i^*. \tag{A9}$$

A.2. Proof of Lemma 1

We focus on a marginal change in the Allais surplus at $(\boldsymbol{n}(s), \overline{U^*})$ with respect to s. We have

$$\frac{\mathrm{d}AS}{\mathrm{d}s} = \sum_{i \in \mathcal{I}} n_i \frac{\mathrm{d}}{\mathrm{d}s}\left(y - t_i + \Pi + s_i(s) - e_i\right) + \left(y - t_i + \Pi + s_i(s) - e_i\right)\frac{\mathrm{d}n_i}{\mathrm{d}s}. \tag{A10}$$

Since $y - t_i + \Pi + s_i(s) - e_i = \overline{E}$ and $\sum_{i\in\mathcal{I}} \mathrm{d}n_i/\mathrm{d}s = 0$ hold by conditions (23) and (24), the second term is zero. Furthermore, y and t_i are not functions of s. Hence, we have

$$\frac{\mathrm{d}AS}{\mathrm{d}s} = \sum_{i \in \mathcal{I}} n_i \frac{\mathrm{d}}{\mathrm{d}s}\left(s_i(s) + \Pi - e_i\right). \tag{A11}$$

We focus on the derivative of the expenditure function. Under the price equilibrium of varieties, the prices of the varieties supplied in a marketplace are the same as shown in (14). Hence, the derivative of expenditure function (A9) is given by

$$\frac{\mathrm{d}e_i}{\mathrm{d}s} = h_i^* \frac{\mathrm{d}p_i^H}{\mathrm{d}s} + p_i^H \frac{\mathrm{d}h_i^*}{\mathrm{d}s} + \frac{\partial e_i^M}{\partial p_{j(i)}^M} \frac{\mathrm{d}p_{j(i)}^M}{\mathrm{d}s} + \frac{\partial e_i^M}{\partial m_{j(i)}} \frac{\mathrm{d}m_{j(i)}}{\mathrm{d}s} + \frac{\partial e_i^M}{\partial M_i} \frac{\mathrm{d}M_i^*}{\mathrm{d}s} + \frac{\mathrm{d}a_i^*}{\mathrm{d}s}. \tag{A12}$$

Substituting the Hicksian demands into the utility function yields $U_i(M_i^*, h_i^*, a_i^*) = \overline{U^*}$. The derivative of the utility is given by

$$\frac{\mathrm{d}U_i}{\mathrm{d}s} = \frac{\partial U_i}{\partial M_i} \frac{\mathrm{d}M_i^*}{\mathrm{d}s} + \frac{\partial U_i}{\partial h_i} \frac{\mathrm{d}h_i^*}{\mathrm{d}s} + \frac{\partial U_i}{\partial a_i} \frac{\mathrm{d}a_i^*}{\mathrm{d}s} = 0. \tag{A13}$$

Using first order conditions (A5)–(A7) for expenditure minimization problem (A4) yields

$$\frac{\partial e_i^M}{\partial M_i} = \left(\frac{\partial U_i}{\partial M_i}\right) \left(\frac{\partial U_i}{\partial a_i}\right)^{-1}, \tag{A14}$$

$$p_i^H = \left(\frac{\partial U_i}{\partial h_i}\right) \left(\frac{\partial U_i}{\partial a_i}\right)^{-1}. \tag{A15}$$

Multiplying both sides of Eq. (A13) by $(\partial U_i / \partial a_i)^{-1}$ and substituting (A14) and (A15) into the equation yields

$$\frac{\partial e_i^M}{\partial M_i} \frac{\mathrm{d}M_i^*}{\mathrm{d}s} + p_i^H \frac{\mathrm{d}h_i^*}{\mathrm{d}s} + \frac{\mathrm{d}a_i^*}{\mathrm{d}s} = 0. \tag{A16}$$

In the equilibrium, using first order conditions (A2) and (A3) for expenditure minimization problem (A1) yields

$$\frac{\partial e_i^M}{\partial p_{j(i)}^M} = m_{j(i)} q_i^*, \tag{A17}$$

$$\frac{\partial e_i^M}{\partial m_{j(i)}} = -\frac{p_{j(i)}^M u(q_i^*)}{u'(q_i^*)} + p_{j(i)}^M q_i^*. \tag{A18}$$

Substituting Eqs. (A16)–(A18) into (A12) yields

$$\frac{\mathrm{d}e_i}{\mathrm{d}s} = h_i^* \frac{\mathrm{d}p_i^H}{\mathrm{d}s} + m_{j(i)} q_i^* \frac{\mathrm{d}p_{j(i)}^M}{\mathrm{d}s} + \left(-\frac{p_{j(i)}^M u(q_i^*)}{u'(q_i^*)} + p_{j(i)}^M q_i^*\right) \frac{\mathrm{d}m_{j(i)}}{\mathrm{d}s}. \tag{A19}$$

Substituting the derivative of total profit (22) and Eq. (A19) into (A11) and using Eq. (5) yields

$$\begin{aligned}\frac{\mathrm{d}AS}{\mathrm{d}s} =&\sum_{i\in\mathcal{I}}\left[(H_i^* - n_i h_i^*)\frac{\mathrm{d}p_i^H}{\mathrm{d}s} + \left(p_i^H - \frac{\partial g^{-1}}{\partial H_i}\right)\frac{\mathrm{d}H_i^*}{\mathrm{d}s}\right]\\ &+\sum_{j\in\mathcal{J}}\left[\left(Q_j m_j - \sum_{a\in\mathcal{I}_j} n_a m_j q_a^*\right)\frac{\mathrm{d}p_j^M}{\mathrm{d}s} + (p_j^M - c)m_j\frac{\mathrm{d}Q_j}{\mathrm{d}s}\right.\\ &\qquad\left. + \left((p_j^M - c)Q_j - m_j + \sum_{a\in\mathcal{I}_j} n_a\left(\frac{p_{j(a)}^M u(q_a^*)}{u'(q_a^*)} - p_j^M q_a^*\right)\right)\frac{\mathrm{d}m_j}{\mathrm{d}s}\right]\\ &+\left(\sum_{i\in\mathcal{I}} n_i\frac{\mathrm{d}s_i}{\mathrm{d}s}\right) + \left(\sum_{j\in\mathcal{J}}\frac{\mathrm{d}s_j^M}{\mathrm{d}s}\right). \end{aligned} \tag{A20}$$

Using equilibrium conditions (11) and (19), and Eq. (5) yields

$$\sum_{i\in\mathcal{I}}(H_i^* - n_i h_i^*)\frac{\mathrm{d}p_i^H}{\mathrm{d}s} = 0, \tag{A21}$$

$$\sum_{j\in\mathcal{J}}\left(Q_j m_j - \sum_{a\in\mathcal{I}_j} n_a m_j q_a^*\right)\frac{\mathrm{d}p_j^M}{\mathrm{d}s} = 0, \tag{A22}$$

$$\sum_{i\in\mathcal{I}}\left(n_i\frac{\mathrm{d}s_i}{\mathrm{d}s} + s_i\frac{\mathrm{d}n_i}{\mathrm{d}s}\right) + \sum_{j\in\mathcal{J}}\frac{\mathrm{d}s_j^M}{\mathrm{d}s} = 0. \tag{A23}$$

Substituting Eqs. (A21)–(A23) into Eq. (A20) and using equilibrium condition (11) yields Eq. (27).

B. Theoretical details of Section 4

B.1. Endogenous variables in the equilibrium with the specification in Section 4.1

Following the discussion in Section 2, we obtain endogenous variables in the equilibrium and the Allais surplus with the specification in Section 4.1.

Solving (A1) with $M_i = \int_0^{m_{j(i)}} q_{j(i)}(k)^{(\sigma-1)/\sigma}\mathrm{d}k$, we obtain the conditional demand:

$$q_{j(i)}^*(k) = p_{j(i)}^M(k)^{-\sigma} P_{j(i)}^{\sigma} M_i^{\sigma/(\sigma-1)}, \tag{B1}$$

where $P_j = \left(\int_0^{m_j} p_j^M(k)^{1-\sigma}\mathrm{d}k\right)^{1/(1-\sigma)}$ is the price index of differentiated goods supplied in marketplace j. We can obtain the expenditure function as a function of the price index and the composite index: $e_i^M = \int_0^{m_{j(i)}} p_{j(i)}^M(k) q_{j(i)}(k)\mathrm{d}k = P_{j(i)} M_{j(i)}^{\sigma/(\sigma-1)}$. Solving (A4) gives us the Hicksian demands:

$$M_i^* = \left(\mu P_{j(i)}^{-1}\right)^{(\sigma-1)/\sigma}, \tag{B2}$$

$$h_i^* = (1-\mu)/p_i^H, \tag{B3}$$

$$a_i^* = \overline{U} + \mu \ln\left(P_{j(i)}\right) + (1-\mu)\ln(p_i^H) - \mu\ln(\mu) - (1-\mu)\ln(1-\mu). \tag{B4}$$

Substituting Eq. (B2) into Eq. (B1) yields $q_i^*(k) = \mu p_{j(i)}^M(k)^{-\sigma} P_{j(i)}^{\sigma-1}$. The expenditure function is given by

$$e_i = \overline{U} + \mu \ln\left(P_{j(i)}\right) + (1-\mu)\ln(p_i^H) - \mu\ln(\mu) - (1-\mu)\ln(1-\mu) + 1. \tag{B5}$$

We focus on retail stores and developers. Using $q_i^*(k)$ gives us the total demand:

$$Q_j(k) = \mu p_j^M(k)^{-\sigma} P_j^{\sigma-1} \sum_{i\in\mathcal{I}_j} n_i \quad (j \in \mathcal{J}).$$

The price elasticity of the total demand is $\eta_j^M(k) = -\sigma$ $(j \in \mathcal{J})$. Using Eq. (12) gives us the equilibrium price: $p_j^M(k) = c\sigma/(\sigma-1)$ $(\forall j, k)$. We express $p_j^M(k)$ as p^M. Applying Eq. (17) to $g(b) = \theta b^\beta$ gives us the profit maximizing supply as a function of the price: $H_i^* = \theta^{1/(1-\beta)}(\beta p_i^H)^{\beta/(1-\beta)}$ $(\forall i \in \mathcal{I})$. Using this function gives us the bid rent in the residential zones:

$$R_i^H = \theta^{1/(1-\beta)}(\beta^{\beta/(1-\beta)} - \beta^{1/(1-\beta)})(p_i^H)^{1/(1-\beta)}.$$

We focus on the short-run equilibrium. Under the equilibrium price for the retail stores, we have

$$q_i^*(k) = \mu(p^M m_{j(i)})^{-1}, \tag{B6}$$

$$Q_j(k) = \mu(p^M m_j)^{-1} \sum_{i\in\mathcal{I}_j} n_i. \tag{B7}$$

Substituting $Q_j(k)$ into equilibrium condition (20) yields the equilibrium mass:

$$m_j = \left(\frac{\mu}{\sigma} \sum_{a \in \mathcal{I}_j} n_a + s_j^M(s) \right)^{1/2} \qquad \forall j \in \mathcal{J}. \tag{B8}$$

The market clearing condition regarding housing (19) gives us the equilibrium price: $p_i^H = (\theta\beta^\beta)^{-1} \left((1-\mu)n_i\right)^{1-\beta}$ $(i \in \mathcal{I})$.

Next, we will obtain the Allais surplus. Substituting p_i^H and H_i^* into the first term of (22) yields

$$\overline{N}^{-1} \sum_{i \in \mathcal{I}} (p_i^H H_i^* - g^{-1}(H_i^*)) = \overline{N}^{-1}(1-\beta)(1-\mu) \sum_{i \in \mathcal{I}} n_i = (1-\beta)(1-\mu). \tag{B9}$$

Substituting p^M, $Q_j(k)$, and m_j into the second term of (22) yields

$$\overline{N}^{-1} \sum_{j \in \mathcal{J}} \left((p_j^M - c) Q_j m_j - \frac{m_j^2}{2} + s_j^M(s) \right) = \frac{\mu}{2\sigma} + \overline{N}^{-1} \sum_{j \in \mathcal{J}} s_j^M(s). \tag{B10}$$

Substituting (B9) and (B10) into (22) yields

$$\widetilde{\Pi}(\boldsymbol{n}, s, \overline{U}) = (1-\beta)(1-\mu) + \mu/(2\sigma) + \overline{N}^{-1} \sum_{j \in \mathcal{J}} s_j^M(s). \tag{B11}$$

In addition, we can obtain expenditure function (B5) with equilibrium mass m_j, price of varieties p^M, and housing price p_i^H:

$$\widetilde{e}_i(\boldsymbol{n}, s, \overline{U}) = \overline{U} - \zeta_1 \ln \left(\frac{\mu}{\sigma} \sum_{a \in \mathcal{I}_{j(i)}} n_a + s_{j(i)}^M(s) \right) + \zeta_2 \ln n_i + \Psi, \tag{B12}$$

where ζ_1, ζ_2, and Ψ are constant values:

$$\zeta_1 = \frac{\mu}{2(\sigma-1)}, \quad \zeta_2 = (1-\mu)(1-\beta),$$
$$\Psi = \mu \ln p^M - \mu \ln \mu - \beta(1-\mu)\ln(1-\mu) - (1-\mu)(\ln\theta + \beta\ln\beta) + 1.$$

Substituting Eqs. (B11) and (B12) into Eq. (25) yields the Allais surplus.

B.2. Lemmas proving Lemmas 3 and 4

We introduce two lemmas employed to prove Lemmas 3 and 4 in Section 4. These lemmas are related to algebraic properties of the model.

We can express Allais surplus (25) with matrices:

$$AS = \boldsymbol{n}^\top \boldsymbol{Y}, \tag{B13}$$

where $\boldsymbol{Y} = \boldsymbol{s} + \boldsymbol{y} + \widetilde{\Pi} \cdot \mathbf{1}_I - \widetilde{\boldsymbol{e}}$, $\boldsymbol{s} = (s_i(s))_{i\in\mathcal{I}}$, $\boldsymbol{y} = (y - t_i)_{i\in\mathcal{I}}$, $\widetilde{\boldsymbol{e}} = (\widetilde{e}_i(\boldsymbol{n}, \overline{U}, s))_{i\in\mathcal{I}}$, and $\mathbf{1}_I$ is the I dimensional vector with each component equaling one. $\widetilde{\boldsymbol{e}}$ has a symmetric property expressed by the following lemma.

Lemma 6. *For $s = 0$, $\partial\widetilde{\boldsymbol{e}}/\partial\boldsymbol{n}$ is a symmetric matrix and the following holds:*

$$\frac{\partial \widetilde{\boldsymbol{e}}}{\partial \boldsymbol{n}} = -\zeta_1 E_1 + \zeta_2 E_2, \tag{B14}$$

$$\left(\frac{\partial \widetilde{\boldsymbol{e}}}{\partial \boldsymbol{n}}\right)^\top \boldsymbol{n} = -(\zeta_1 - \zeta_2)\mathbf{1}_I, \tag{B15}$$

where

$$E_1 = \begin{pmatrix} (\sum_{a\in\mathcal{I}_1} n_a)^{-1}\mathbf{1}_{I_1}\mathbf{1}_{I_1}^\top & & & \\ & (\sum_{a\in\mathcal{I}_2} n_a)^{-1}\mathbf{1}_{I_2}\mathbf{1}_{I_2}^\top & & \\ & & \ddots & \\ & & & (\sum_{a\in\mathcal{I}_J} n_a)^{-1}\mathbf{1}_{I_J}\mathbf{1}_{I_J}^\top \end{pmatrix},$$

$$E_2 = \mathrm{diag}(n_1^{-1}, n_2^{-1}, \ldots, n_I^{-1}).$$

Proof. See Supplement SA.1. □

The following lemma has an important role in proving Lemma 4.

Lemma 7. *For $n \geq 3$ and $a_1, a_2, \ldots, a_n, b_1, b_2, \ldots, b_n \in \mathbb{R}$, the following holds:*

$$\left(\prod_{i=1}^n a_i\right)\left(\sum_{i=1}^n b_i\right)^2 - \left(\sum_{i=1}^n a_i\right)\sum_{i=1}^n\left(b_i^2 \prod_{j\in\mathcal{N}\setminus\{i\}} a_j\right)$$

$$= -\sum_{i,j\in\mathcal{N},i\neq j}\left(\frac{1}{2}(a_ib_j - a_jb_i)^2\prod_{k\in\mathcal{N}\setminus\{i,j\}}a_k\right), \tag{B16}$$

where $\mathcal{N} = \{1, 2, \ldots, n\}$.

Proof. See Supplement SA.2. □

B.3. Proofs of main lemmas shown in Section 4

Proof of Lemma 2

We prove $PD_P = 0$. Using Eq. (11) yields

$$\sum_{a\in\mathcal{I}_j}\frac{\partial Q_j}{\partial n_a}\frac{\mathrm{d}n_a}{\mathrm{d}s} = \sum_{a\in\mathcal{I}_j}\left(q_a^* + \sum_{b\in\mathcal{I}_j}n_b\frac{\partial q_b^*}{\partial n_a}\right)\frac{\mathrm{d}n_a}{\mathrm{d}s}. \tag{B17}$$

Using Eqs. (B1), (B2), and (B8) yields the derivative of q_b^* and m_j for $b \in \mathcal{I}_j$ and $s = 0$:

$$\frac{\partial q_b^*}{\partial n_a} = -\frac{\mu}{p^M m_j^2}\frac{\partial m_j}{\partial n_a}, \quad \frac{\partial m_j}{\partial n_a} = \frac{\mu}{2\sigma m_j}.$$

Using these equations yields $(\partial q_b^*/\partial n_a) = -\mu^2(2\sigma p^M m_j^3)^{-1}$. Substituting (B6) and this equation into (B17) yields

$$\sum_{a\in\mathcal{I}_j}\frac{\partial Q_j}{\partial n_a}\frac{\mathrm{d}n_a}{\mathrm{d}s} = \frac{\mu}{2p^M m_j}\sum_{a\in\mathcal{I}_j}\frac{\mathrm{d}n_a}{\mathrm{d}s}. \tag{B18}$$

Because we have $\sum_{i\in\mathcal{I}}(\mathrm{d}n_i/\mathrm{d}s) = 0$ with Eq. (24), using this equation yields $PD_P = 0$.

The proof of $VD_P = 0$ is similar to the above proof:

$$\sum_{j\in\mathcal{J}}\left(\sum_{a\in\mathcal{I}_j}\left(\frac{n_a p_{j(a)}^M u(q_a)}{u'(q_a)}\right) - cQ_j - m_j\right)\sum_{a\in\mathcal{I}_j}\frac{\partial m_j}{\partial n_a}\frac{\mathrm{d}n_a}{\mathrm{d}s} = \frac{\mu}{2\sigma}\left(\frac{\sigma^2}{\sigma-1} - \frac{c\sigma}{p^M} - 1\right)\sum_{i\in\mathcal{I}}\frac{\mathrm{d}n_i}{\mathrm{d}s} = 0. \tag{B19}$$

Proof of Lemma 3

Let L denote the Lagrangian for maximization problem (38). L is given by

$$L = AS - \sum_{i\in\mathcal{I}}\nu_i\gamma_i - \lambda\Gamma, \tag{B20}$$

where ν_i and λ are the Lagrange multipliers. The KKT conditions for this problem are as follows:

$$\frac{\partial AS}{\partial \boldsymbol{n}} = \sum_{i\in\mathcal{I}} \nu_i \frac{\partial \gamma_i(\boldsymbol{n})}{\partial \boldsymbol{n}} + \lambda \frac{\partial \Gamma(\boldsymbol{n})}{\partial \boldsymbol{n}}, \tag{B21}$$

$$\gamma_i(\boldsymbol{n}) \leq 0, \quad \Gamma(\boldsymbol{n}) = 0, \tag{B22}$$

$$\nu_i \geq 0, \quad \nu_i \gamma_i = 0. \tag{B23}$$

Using (B13) gives us the derivative of the Allais surplus:

$$\frac{\partial AS}{\partial \boldsymbol{n}} = \boldsymbol{Y} + \boldsymbol{1}_I \left(\frac{\partial \Pi}{\partial \boldsymbol{n}}\right)^\top \boldsymbol{n} - \left(\frac{\partial \widetilde{\boldsymbol{e}}}{\partial \boldsymbol{n}}\right)^\top \boldsymbol{n}. \tag{B24}$$

We focus on the first term of (B24). Since consumers maximize their utility for $s = 0$, we have $V_i = \overline{U}$ and $\widetilde{e}_i(\boldsymbol{n}^*, \overline{U}, 0) = y - t_i + \widetilde{\Pi}(\boldsymbol{n}^*, \overline{U}, 0)$ $(\forall i \in \mathcal{I})$. Hence, $\boldsymbol{Y} = \boldsymbol{0}$ holds for $s = 0$. The second term is zero because $(\partial \Pi / \partial \boldsymbol{n}) = 0$ holds for $s = 0$ by Eq. (B11). In addition, we have $(\partial \widetilde{\boldsymbol{e}} / \partial \boldsymbol{n})^\top \boldsymbol{n} = -(\zeta_1 - \zeta_2)\boldsymbol{1}_I$ by Lemma 6. Hence we have

$$\frac{\partial AS}{\partial \boldsymbol{n}} = (\zeta_1 - \zeta_2)\boldsymbol{1}_I. \tag{B25}$$

Substituting (B25) into (B21) gives us $\boldsymbol{\nu} + (\zeta_1 - \zeta_2 + \lambda)\boldsymbol{1}_I = \boldsymbol{0}_I$, where $\boldsymbol{\nu} = (\nu_i)_{i\in\mathcal{I}}$. Since we focus on an inner equilibrium, $\gamma_i < 0$ and $\Gamma = 0$ hold. Furthermore, if we set $\boldsymbol{\nu} = \boldsymbol{0}$ and $\lambda = -\zeta_1 + \zeta_2$, then the other conditions are satisfied.

Proof of Lemma 4

The Hessian of Lagrangian (B20) is given by

$$\frac{\partial^2 L}{\partial \boldsymbol{n}^2} = \frac{\partial^2 \left(\boldsymbol{n}^\top \boldsymbol{Y}\right)}{\partial \boldsymbol{n}^2} = \frac{\partial \boldsymbol{Y}}{\partial \boldsymbol{n}} + \frac{\partial}{\partial \boldsymbol{n}} \left(\left(\frac{\partial \boldsymbol{Y}}{\partial \boldsymbol{n}}\right)^\top \boldsymbol{n}\right). \tag{B26}$$

We have $(\partial \boldsymbol{Y} / \partial \boldsymbol{n}) = -(\partial \widetilde{\boldsymbol{e}} / \partial \boldsymbol{n})$. Hence, Lemma 6 yields

$$\frac{\partial^2 L}{\partial \boldsymbol{n}^2} = \frac{\partial \boldsymbol{Y}}{\partial \boldsymbol{n}} = \zeta_1 E_1 - \zeta_2 E_2. \tag{B27}$$

We focus on $I \geq 3$. We define the following:

$$\mathcal{M}^+ \equiv \left\{ \boldsymbol{z} \in \mathbb{R}^I \;\middle|\; \left(\frac{\partial \gamma_i}{\partial \boldsymbol{n}}\right)^\top \boldsymbol{z} = 0 \right\} = \left\{ \boldsymbol{z} \in \mathbb{R}^I \;\middle|\; \sum_{i\in\mathcal{I}} z_i = 0 \right\}.$$

Since we have $\zeta_1 < \zeta_2$ by assumption of Lemma 4, this inequality and E_1 and E_2, shown in Lemma 6, gives us the following for any $\boldsymbol{z} \in \mathcal{M}^+ \backslash \{\boldsymbol{0}\}$:

$$\boldsymbol{z}^\top \frac{\partial \boldsymbol{Y}}{\partial \boldsymbol{n}} \boldsymbol{z} = \zeta_1 \boldsymbol{z}^\top E_1 \boldsymbol{z} - \zeta_2 \boldsymbol{z}^\top E_2 \boldsymbol{z} < \zeta_1 \sum_{j \in \mathcal{J}} Z_j, \tag{B28}$$

where $Z_j = (\sum_{a \in \mathcal{I}_j} n_a)^{-1} (\sum_{a \in \mathcal{I}_j} z_a)^2 - \sum_{a \in \mathcal{I}_j} (z_a^2 / n_a)$. Because $Z_j \leq 0$ ($\forall j \in \mathcal{J}$) holds by Lemma 7, we obtain $\boldsymbol{z}^\top (\partial \boldsymbol{Y} / \partial \boldsymbol{n})\, \boldsymbol{z} < 0$.

A similar argument to the above discussion shows that $\boldsymbol{z}^\top (\partial \boldsymbol{Y} / \partial \boldsymbol{n})\, \boldsymbol{z} < 0$ holds for $I = 2$.

Proof of Lemma 5

It is obvious that $FD = 0$ holds at $s = 0$. Using (B8) gives us the mass of stores for the given policy function:

$$m_j = \left(\frac{\mu}{\sigma} \left(\sum_{a \in \mathcal{I}_j} n_a \right) + \delta_j s \right)^{1/2} \qquad \forall j \in \mathcal{J}, \tag{B29}$$

where $\delta_1 = 1$ and $\delta_j = 0$ ($j \neq 1$).

Lemma 2 yields $\mathrm{d}AS/\mathrm{d}s = PD_S + VD_S$. Furthermore, the following hold:

$$\frac{\partial p_1^M}{\partial s} = \cdots = \frac{\partial p_J^M}{\partial s} = 0, \quad \frac{\partial Q_2}{\partial s} = \cdots = \frac{\partial Q_J}{\partial s} = 0, \quad \frac{\partial m_2}{\partial s} = \cdots = \frac{\partial m_J}{\partial s} = 0.$$

Hence, we have

$$PD_S = (p_1^M - c) m_1 \frac{\partial Q_1}{\partial s}, \quad VD_S = \left(\sum_{a \in \mathcal{I}_1} \left(\frac{n_a p_{j(a)}^M u(q_a)}{u'(q_a)} \right) - cQ_1 - m_1 \right) \frac{\partial m_1}{\partial s}.$$

Using Eqs. (B7) and (B29) yields for $s = 0$: $PD_S = -\overline{N}/2 < 0$, while using Eqs. (B1) and (B29) yields for $s = 0$: $VD_S = \overline{N}\sigma/(2(\sigma - 1)) > 0$. Substituting PD_S and VD_S into $\mathrm{d}AS/\mathrm{d}s$ yields $\mathrm{d}AS/\mathrm{d}s = \overline{N}(2(\sigma - 1))^{-1} > 0$.

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Supplement: This part is not printed but shown on the authors' website.

SA. Proofs of Lemmas shown in Appendix B

SA.1. Proof of Lemma 6

We express $\widetilde{\boldsymbol{e}}$ as a matrix: $\widetilde{\boldsymbol{e}} = -\zeta_1 (Lo \circ G_1)(\boldsymbol{n}) + \zeta_2\, Lo(\boldsymbol{n}) + (\overline{U} - \Psi)\mathbf{1}_I$, where

$$
\begin{aligned}
&Lo(z_1, z_2, \dots, z_I) = (\ln z_1, \ln z_2, \dots, \ln z_I)^\top,\\
&G_1(\boldsymbol{n}) = \left(\left(\frac{\mu}{\sigma} \sum_{a \in \mathcal{I}_1} n_a + s_1^M(s) \right) \mathbf{1}_{I_1}^\top, \dots, \left(\frac{\mu}{\sigma} \sum_{a \in \mathcal{I}_J} n_a + s_J^M(s) \right) \mathbf{1}_{I_J}^\top \right)^\top.
\end{aligned}
$$

$\partial \widetilde{\boldsymbol{e}} / \partial \boldsymbol{n}$ is given by

$$
\frac{\partial \widetilde{\boldsymbol{e}}}{\partial \boldsymbol{n}} = -\zeta_1 \frac{\partial ((Lo \circ G_1)(\boldsymbol{n}))}{\partial \boldsymbol{n}} + \zeta_2 \frac{\partial Lo(\boldsymbol{n})}{\partial \boldsymbol{n}}. \tag{SA1}
$$

Using the chain rule, we obtain the Jacobian matrix of $(Lo \circ G_1)(\boldsymbol{n})$ and $Lo(\boldsymbol{n})$ for $s = 0$:

$$
\frac{\partial ((Lo \circ G_1)(\boldsymbol{n}))}{\partial \boldsymbol{n}} = \begin{pmatrix} (\sum_{a \in \mathcal{I}_1} n_a)^{-1} \mathbf{1}_{I_1} \mathbf{1}_{I_1}^\top & & & \\ & (\sum_{a \in \mathcal{I}_2} n_a)^{-1} \mathbf{1}_{I_2} \mathbf{1}_{I_2}^\top & & \\ & & \ddots & \\ & & & (\sum_{a \in \mathcal{I}_J} n_a)^{-1} \mathbf{1}_{I_J} \mathbf{1}_{I_J}^\top \end{pmatrix}, \tag{SA2}
$$

$$
\frac{\partial Lo(\boldsymbol{n})}{\partial \boldsymbol{n}} = \mathrm{diag}(n_1^{-1}, n_2^{-1}, \dots, n_I^{-1}). \tag{SA3}
$$

Since the sum of symmetric matrices is also a symmetric matrix, $\partial \boldsymbol{V} / \partial \boldsymbol{n}$ is a symmetric matrix. Furthermore, substituting (SA2) and (SA3) into (SA1), we obtain (B14) and (B15).

SA.2. Proof of Lemma 7

Using the mathematical induction, we prove the lemma.

We obtain (B16) for $n = 3$ by a simple deformation:

$$a_1a_2a_3(b_1+b_2+b_3)^2 - (a_1+a_2+a_3)(a_2a_3b_1^2 + a_3a_1b_2^2 + a_1a_2b_3^2)$$
$$= -a_3(a_2b_1 - a_1b_2)^2 - a_2(a_3b_1 - a_1b_3)^2 - a_1(a_2b_3 - a_3b_2)^2.$$

Next, we assume that Eq. (B16) holds for n. We verify whether Eq. (B16) holds for $n+1$. That is, we will show the following holds:

$$\left(\prod_{i=1}^{n+1} a_i\right)\left(\sum_{i=1}^{n+1} b_i\right)^2 - \left(\sum_{i=1}^{n+1} a_i\right)\sum_{i=1}^{n+1}\left(b_i^2 \prod_{j\in\widehat{\mathcal{N}}\setminus\{i\}} a_j\right)$$
$$= -\sum_{i,j\in\widehat{\mathcal{N}},i\neq j}\left(\frac{1}{2}(a_ib_j - a_jb_i)^2 \prod_{k\in\widehat{\mathcal{N}}\setminus\{i,j\}} a_k\right), \qquad \text{(SA4)}$$

where $\widehat{\mathcal{N}} = \{1, 2, \ldots, n+1\}$.

We define $B_n = \sum_{i=1}^{n} b_i$. We focus on the LHS of (SA4):

$$\text{(LHS)} = \left(\prod_{i=1}^{n+1} a_i\right)\left(B_n^2 + 2B_nb_{n+1} + b_{n+1}^2\right)$$
$$- \left(\sum_{i=1}^{n+1} a_i\right)\sum_{i=1}^{n}\left(b_i^2 \prod_{j\in\widehat{\mathcal{N}}\setminus\{i\}} a_j + b_{n+1}^2 \prod_{j\in\widehat{\mathcal{N}}\setminus\{n+1\}} a_j\right)$$
$$= X + Y, \qquad \text{(SA5)}$$

where

$$X = B_n^2\left(\prod_{i=1}^{n+1} a_i\right) - \left(\sum_{i=1}^{n} a_i\right)\left(\sum_{i=1}^{n} b_i^2 \prod_{j\in\widehat{\mathcal{N}}\setminus\{i\}} a_j\right),$$

$$Y = -a_{n+1}\left(\sum_{i=1}^{n} b_i^2 \prod_{j\in\widehat{\mathcal{N}}\setminus\{i\}} a_j\right) + (2B_nb_{n+1} + b_{n+1}^2)\left(\prod_{i=1}^{n+1} a_i\right)$$
$$- b_{n+1}^2\left(\sum_{i=1}^{n+1} a_i\right)\left(\prod_{j\in\widehat{\mathcal{N}}\setminus\{n+1\}} a_j\right).$$

Using the assumption, we deform X as follows:

$$X = -\sum_{i,j\in\mathcal{N},i\neq j} \frac{1}{2}(a_i b_j - a_j b_i)^2 \prod_{k\in\widehat{\mathcal{N}}\setminus\{i,j\}} a_k.$$

We deform Y as follows:

$$Y = -a_{n+1}\sum_{i=1}^{n}\left(b_i^2 \prod_{j\in\widehat{\mathcal{N}}\setminus\{i\}} a_j\right) + 2B_n b_{n+1}\prod_{i=1}^{n+1} a_i + Y_1, \tag{SA6}$$

where

$$Y_1 = b_{n+1}^2\left(\prod_{i=1}^{n+1} a_i - \left(\sum_{i=1}^{n+1} a_i\right)\prod_{j\in\widehat{\mathcal{N}}\setminus\{n+1\}} a_j\right). \tag{SA7}$$

We deform Y_1 as follows:

$$Y_1 = -b_{n+1}^2\left(\sum_{i=1}^{n} a_i\right)\prod_{i\in\widehat{\mathcal{N}}\setminus\{n+1\}} a_i. \tag{SA8}$$

Using (SA8), we deform Y as follows:

$$\begin{aligned} Y = &-\sum_{i=1}^{n}\frac{1}{2}(a_{n+1}b_i - a_i b_{n+1})^2 \prod_{j\in\widehat{\mathcal{N}}\setminus\{i,n+1\}} a_j \\ &-\sum_{i=1}^{n}\frac{1}{2}(a_i b_{n+1} - a_{n+1}b_i)^2 \prod_{j\in\widehat{\mathcal{N}}\setminus\{i,n+1\}} a_j. \end{aligned} \tag{SA9}$$

Substituting X and Y into (SA5) implies that LHS of (SA4) equals the RHS of (SA4).

SB. The case of continuous space

In this section, we obtain the welfare impact of a place-based policy in a continuous space. We show that the difference between a discrete space and the continuous space is the marginal welfare change generated by a change in a market boundary. Assumptions in the continuous space model other than geographical space of the city are the same as the discrete space model. We assume that functions are so continuous that we can obtain derivatives.

SB.1. Model setting

The geographical space of the continuous space model is given by $[0, I]$ $(I > 0)$. Consumers can reside in $(0, I)$ and retail stores can operate at 0 or I $(0, I \in [0, I])$. The utility of consumers who reside in x $(\in (0, I))$ and visit marketplace j $(\in \{0, I\})$ for shopping is given by $U(M_j(x), h(x), a(x))$, where $M_j(x) = \int_0^{m_j} u(q(x, k))\mathrm{d}k$. The budget constraint is given by:

$$\int_0^{m_j} p_j^M(k)q(x, k)\mathrm{d}k + p^H(x)h(x) + a(x) = y_j(x), \tag{SB1}$$

where $y_j(x) = \widetilde{y}_j(x, s) \equiv y - t_j(x) + \Pi + s_j(x, s)$. The profit of retail store supplying the kth variety in marketplace j is given by

$$\pi_j^M(k) = (p_j^M(k) - c)Q_j(k) - k + \frac{s_j^M(s)}{m_j} - r_j(k) \quad \forall k \in [0, m_j], \tag{SB2}$$

where $Q_j(k) = \int_0^I q(x, k)\mathrm{d}x$. The developers' net profit at x $(\in [0, x])$ is given by

$$\pi^H(x) = p^H(x)H(x) - g^{-1}(H(x)) - R^H(x). \tag{SB3}$$

An equal division of the profits and rents is given by

$$\Pi = \overline{N}^{-1} \left(\int_0^I \pi^H(x) + R^H(x)\,\mathrm{d}x + \sum_{j \in \mathcal{J}} \left(\int_0^{m_j} \pi_j^M(k)\mathrm{d}k + \int_0^{m_j} r_j(k)\mathrm{d}k \right) \right),$$

where $\mathcal{J} = \{0, I\}$.

Equilibrium conditions are given by

$$H(x) = n(x, s)h(x) \quad \forall x \in (0, I), \tag{SB4}$$

$$(p_j^M - c)Q_j - m_j + \frac{s_j^M(s)}{m_j} = 0 \quad \forall j \in \mathcal{J} \equiv \{0, I\}, \tag{SB5}$$

$$y(x, s) = y - t_{j(x)}(x) + \Pi + s_{j(x)}(x, s) \quad \forall x \in [0, I], \tag{SB6}$$

where $j(x)$ $(\in \mathcal{J})$ is the marketplace that consumers residing in x $(\in (0, I))$ visit for shopping. Let $e(x, j, s)$ denote the expenditure function of consumers who reside in x and visit marketplace j. The equilibrium conditions for the Allais surplus are given by

$$y - t_{j(x)} + \Pi + s_{j(x)}(x, s) - e(x, j, s) = \overline{E} \quad \exists \overline{E} \in \mathbb{R}\ \forall x \in (0, I), \tag{SB7}$$

$$\int_0^I n(x,s)\,\mathrm{d}x = \overline{N}. \tag{SB8}$$

SB.2. Welfare impact of a place-based policy for the continuous space model

We can obtain the differentiation of the Allais surplus with the continuous space model. Let $b(s) \in (0, I)$ denote the market boundary given policy instrument s with equilibrium conditions (SB4)–(SB8). We focus on a utility level at which consumers residing in $x \in (0, b(s)]$ visit marketplace 0 and consumers residing in $x \in [b(s), I)$ visit marketplace I. At the utility level, we obtain the Allais surplus:

$$\begin{aligned} AS &= \int_0^I n(x,s)\overline{E}\,\mathrm{d}x \\ &= \int_0^I n(x,s)\left(y - t_{j(x)}(x) + \Pi + s_{j(x)}(x,s) - e(x, j(x), s)\right)\mathrm{d}x. \end{aligned} \tag{SB9}$$

We differentiate the Allais surplus with respect to s:

$$\begin{aligned} \frac{\mathrm{d}AS}{\mathrm{d}s} &= \int_0^I n(x,s)\frac{\mathrm{d}}{\mathrm{d}s}\left(y - t_{j(x)}(x) + \Pi + s_{j(x)}(x,s) - e(x, j(x), s)\right)\mathrm{d}x \\ &\quad + \int_0^I \frac{\mathrm{d}n(x,s)}{\mathrm{d}s}\left(y - t_{j(x)}(x) + \Pi + s_{j(x)}(x,s) - e(x, j(x), s)\right)\mathrm{d}x + BD, \end{aligned}$$

where

$$\begin{aligned} BD &= n(b(s), s)(Y_I - Y_0)\frac{\mathrm{d}b(s)}{\mathrm{d}s}, \\ Y_0 &= t_0(b(s)) + e(x, 0, s) - s_0(b(s), s), \quad Y_I = t_I(b(s)) + e(x, I, s) - s_I(b(s), s). \end{aligned}$$

When we evaluate the welfare impact with the continuous model, the welfare impact generated by the change in the market boundary BD is added to the welfare measurement formula. The same discussion for the derivation of the derivative of the Allais surplus, shown in Appendix A.2, gives us

$$\frac{\mathrm{d}AS}{\mathrm{d}s} = PD + VD + FD + BD, \tag{SB10}$$

where

$$PD \equiv \sum_{j\in\mathcal{J}} (p_j^M - c)m_j\frac{\mathrm{d}Q_j}{\mathrm{d}s},$$

$$
\begin{aligned}
VD \equiv & \left(\int_0^{b(s)} \frac{n(x,s) p_0^M u(q(x))}{u'(q(x))} \mathrm{d}x - cQ_0 - m_0 \right) \frac{\mathrm{d}m_0}{\mathrm{d}s} \\
& + \left(\int_{b(s)}^I \frac{n(x,s) p_I^M u(q(x))}{u'(q(x))} \mathrm{d}x - cQ_I - m_I \right) \frac{\mathrm{d}m_I}{\mathrm{d}s}, \\
FD = & \int_0^I -s_{j(x)}(x,s) \frac{\mathrm{d}n(x,s)}{\mathrm{d}s} \, \mathrm{d}x.
\end{aligned}
$$

Since $Y_I - Y_0 = t_I(b(0)) - t_0(b(0))$ holds for $s = 0$, we have $BD = (t_I(b(0)) - t_0(b(0)))\mathrm{d}b(s)/\mathrm{d}s$. If the difference in the travel costs is small, then BD is small. That is, BD hardly affects the welfare impact of adopting a place-based policy.