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Sensitivity Analysis of Inputs of an Organization: A Profit Maximization Exploration

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Abstract

This article tries to discuss sensitivity analysis of various inputs of an organization during profit maximization investigations. In this study Cobb-Douglas production function is analyzed with a detail mathematical analysis. The method of Lagrange multiplier is a very useful and powerful technique in multivariate calculus, and it is applied here to obtain higher dimensional unconstrained problem from the lower dimensional constrained one. In the article determinant of the 6×6 bordered Hessian matrix and 6×6 Jacobian are operated for consulting sensitivity analysis efficiently.

Keywords: Lagrange multiplier, profit maximization, sensitivity analysis

1. Introduction

At the present globalized world, mathematical modeling becomes a part and parcel in economics (Samuelson, 1947). In the 21st century, it is widely used to investigate optimization strategy (Carter, 2001). It plays an important role in modern economics for the development of global financial structure (Ferdous & Mohajan, 2022). In the society economics sees its benefits and also sees welfare of the global humankind. Profit maximization policy provides sustainable environment of the organization, and in parallel it increases welfare of the society (Eaton & Lipsey, 1975; Mohajan et al., 2013). To obtain maximum profit, an organization must be very sincere in every step of its operation, such as in production, financial balance, demand and supply strategy, transportation, total management system, etc. (Mohajan, 2015; Mohajan & Mohajan, 2022a).

Cobb-Douglas production function is one of the most widely used production function in mathematical economics that helps the organization to take rational decision on the quantity of each factor inputs to employ so as to minimize the production cost for its profit maximization (Cobb & Douglas, 1928). In this paper we have used the determinant of 6×6 bordered Hessian matrix, 6×6 Jacobian and four input variables. For convenient we have applied Implicit Function Theorem of multivariable calculus. In this article we have tried to show mathematical calculations in some detail.

2. Literature Review

In any research work, the literature review is an elementary section where a researcher shows the works of previous researchers in the same field within the existing knowledge (Polit & Hungler, 2013). It deals with a secondary research source and does not report a new or a coming research work (Gibbs, 2008). In 1928, two American dedicated persons, mathematician Charles W. Cobb (1875-1949) and economist Paul H. Douglas (1892-1976) have derived the functional distribution of income between capital and labor (Cobb & Douglas, 1928). After a few years in 1984, another two American professors: mathematician John V. Baxley and economist John C.

Moorhouse have provided a mathematical formulation for nontrivial constrained optimization problem with special reference to application in economics (Baxley & Moorhouse, 1984).

Steven D. Levitt shows that profit maximizing behavior by firms is one of the most fundamental and widely applied policies in all of economics (Levitt, 2006). Well-established Bangladeshi mathematician Jamal Nazrul Islam and his coauthors have given reasonable interpretation of the Lagrange multipliers. In some research papers they have tried to examine the behavior of optimization problems (Islam et al., 2009a,b, 2010a,b). Cambodian professor Pahlaj Moolio and his coauthors have stressed on optimization of output in an organization (Moolio et al., 2009). Devajit Mohajan and Haradhan Kumar Mohajan have discussed utility maximization and profit maximization policies. They have studied the Cobb-Douglas production function with detailed mathematical analysis (Mohajan et al., 2012, 2013; Mohajan & Mohajan, 2022a-j, 2023a-s).

On the other hand, Haradhan Kumar Mohajan has considered three inputs in his optimization investigation (Mohajan, 2021a). Earlier, Lia Roy and her coauthors have shown that cost minimization is essential for the sustainable development of an industry (Roy et al., 2021). Jannatul Ferdous and Haradhan Kumar Mohajan have developed a profit maximization problem that is a splendid work in mathematical economics (Mohajan, 2012; Ferdous & Mohajan, 2022).

3. Research Methodology of the Study

Research is a vital and significant device to the academicians for the leading in academic empire (Pandey & Pandey, 2015). On the other hand, methodology is a guideline to perform a good research that follows scientific methods efficiently (Kothari, 2008). Therefore, research methodology is the collection of a set of principles for organizing, planning, designing and conducting a good research (Legesse, 2014). In this study we have tried to discuss sensitivity analysis for the welfare of the organization and humankind. First we have considered a Cobb-Douglas production function, and then we have used Lagrange multiplier to make the paper interesting to the readers. Moreover, we have used the 6×6 bordered Hessian matrix and 6×6 Jacobian to make the profit function easier.

To prepare this paper, we have followed both qualitative and quantitative research approaches (Mohajan, 2018a, 2020). Throughout the study we have tried our best to maintain the reliability and validity properly (Das & Mohajan, 2014a,b,c; Mohajan, 2017b). In this paper, we have depended on the secondary data sources of optimization, such as journal articles, books of famous authors, conference papers, internet, websites, etc. (Islam et al., 2009a,b, 2010a,b, 2011a,b,c, 2012a,b,c, Mohajan, 2011a-d, 2012a-h, 2013a-j, 2014a-g, 2015a-e, 2016a,b,c, 2017a-g, 2018a-e, 2020a-e, 2021a-e, 2022a-d, Rahman & Mohajan, 2019).

4. Objective of the Study

The main objective of this study is to discuss sensitivity analysis of various inputs if interest rate of the organization increases during profit maximization investigation. Other minor but related objectives are as follows:

- to show the calculation properly,
- to highlight matrix representation, and
- to provide predictions properly.

5. Economic Model

Let us consider that an organization produces and distributes its products to achieve maximum profit within a year using a_1 quantity of capital, a_2 quantity of labor, a_3 quantity of principal raw materials, and a_4 quantity of other inputs. The profit function is represented by the Cobb-Douglas production function (Islam et al., 2011; Mohajan, 2017a),

$$P = f\left(a_1, a_2, a_3, a_4\right) = Aa_1^x a_2^y a_3^z a_4^w \qquad (1)$$

where *A* is the technical process of economic system that indicates total factor productivity. Here *x*, *y*, *z*, and *w* are parameters; *x* indicates the output of elasticity of capital measures the percentage change in *P* for 1% change in a_1, while a_2, a_3, and a_4 are held constants. Similar properties carry parameters *y*, *z*, and *w*. The values of *x*, *y*, *z*, and *w* are determined by the available technologies, and must satisfy the following four inequalities (Mohajan, 2021a; Roy et al., 2021):

$$0 < x < 1,\ 0 < y < 1,\ 0 < z < 1, \text{ and } 0 < w < 1. \quad (2)$$

A strict Cobb-Douglas production function, in which $\Pi = x + y + z + w = 1$ indicates constant returns to scale, $\Pi < 1$ indicates decreasing returns to scale, and $\Pi > 1$ indicates increasing returns to scale (Moolio et al., 2009; Mohajan, 2021b).

Now we consider that the budget constraint,

$$B(a_1, a_2, a_3, a_4) = ka_1 + la_2 + ma_3 + na_4, \quad (3)$$

where *k* is rate of interest or services of capital per unit of capital a_1; *l* is the wage rate per unit of labor a_2; *m* is the cost per unit of principal raw material a_3; and *n* is the cost per unit of other inputs a_4.

Now we introduce a single Lagrange multiplier λ, as a device; and by using equations (1) and (3) we can represent the Lagrangian function $L(a_1, a_2, a_3, a_4, \lambda)$, in a 5-dimensional unconstrained problem as follows (Mohajan et al., 2013; Mohajan, 2022):

$$L(a_1, a_2, a_3, a_4, \lambda) = Aa_1^x a_2^y a_3^z \alpha_4^w + \lambda(B - ka_1 - la_2 - ma_3 - na_4). \quad (4)$$

where $\frac{\partial B}{\partial a_1} = B_1$, $\frac{\partial B}{\partial a_2} = B_2$, $\frac{\partial L}{\partial a_1} = L_1$, $\frac{\partial^2 L}{\partial a_1 \partial a_3} = L_{31}$, $\frac{\partial^2 L}{\partial a_2^2} = L_{22}$, etc. are partial derivatives.

Let us consider the determinant of the 5×5 bordered Hessian matrix as,

$$H = \begin{vmatrix} 0 & -B_1 & -B_2 & -B_3 & -B_4 \\ -B_1 & L_{11} & L_{12} & L_{13} & L_{14} \\ -B_2 & L_{21} & L_{22} & L_{23} & L_{24} \\ -B_3 & L_{31} & L_{32} & L_{33} & L_{34} \\ -B_4 & L_{41} & L_{42} & L_{43} & L_{44} \end{vmatrix}. \quad (5)$$

Taking first-order partial differentiations of (3) we get,

$$B_1 = k,\ B_2 = l,\ B_3 = m, \text{ and } B_4 = n. \quad (6)$$

Taking second-order and cross partial derivatives of (4) we get,

$$L_{11} = x(x-1)Aa_1^{x-2} a_2^y a_3^z a_4^w,$$

$$L_{22} = y(y-1)Aa_1^x a_2^{y-2} a_3^z a_4^w,$$

$$L_{33} = z(z-1)Aa_1^x a_2^y a_3^{z-2} a_4^w,$$

$$L_{44} = w(w-1)Aa_1^x a_2^y a_3^z a_4^{w-2},$$

$$L_{12} = L_{21} = xyAa_1^{x-1} a_2^{y-1} a_3^z a_4^w,$$

$$L_{13} = L_{31} = xzAa_1^{x-1} a_2^y a_3^{z-1} a_4^w,$$

$$L_{14} = L_{41} = xwAa_1^{x-1} a_2^y a_3^z a_4^{w-1}, \tag{7}$$

$$L_{23} = L_{32} = yzAa_1^x a_2^{y-1} a_3^{z-1} a_4^w,$$

$$L_{24} = L_{42} = ywAa_1^x a_2^{y-1} a_3^z a_4^{w-1},$$

$$L_{34} = L_{43} = zwAa_1^x a_2^y a_3^{z-1} a_4^{w-1}.$$

Now we expand the bordered Hessian (5) as,

$$|H| = \frac{A^3 B^2 xyzw a_1^{3x} a_2^{3y} a_3^{3z} a_4^{3w}}{a_1^2 a_2^2 a_3^2 a_4^2 \Pi} > 0 \tag{8}$$

where $A > 0$, $x, y, z, w > 0$, and budget, $B > 0$, therefore, $|H| > 0$. Hence, the profit is maximized (Mohajan & Mohajan, 2022b; Moolio et al., 2009).

6. Highlights on Matrix Operations

We have observed that the second order condition is satisfied, so that the determinant of (5) survives at the optimum, i.e., $|J| = |H|$; hence, we can apply the implicit function theorem. Let $\mathbf{G}$ be the vector-valued function of ten variables $\lambda^*, a_1^*, a_2^*, a_3^*, a_4^*, k, l, m, n$, and B, and we define the function $\mathbf{G}$ for the point $(\lambda^*, a_1^*, a_2^*, a_3^*, a_4^*, k, l, m, n, B) \in R^{10}$, and take the values in R^5. By the implicit function theorem of multivariable calculus, the equation,

$$F(\lambda^*, a_1^*, a_2^*, a_3^*, a_4^*, k, l, m, n, B) = 0, \tag{9}$$

may be solved in the form of

$$\begin{bmatrix} \lambda \\ a_1 \\ a_2 \\ a_3 \\ a_4 \end{bmatrix} = \mathbf{G}(k, l, m, n, B). \tag{10}$$

Now the 5×5 Jacobian matrix for $\mathbf{G}$; regarded as $J_G = \dfrac{\partial(\lambda, a_1, a_2, a_3, a_4)}{\partial(k, l, m, n, B)}$, and is presented by;

$$J_G = \begin{bmatrix} \frac{\partial \lambda}{\partial k} & \frac{\partial \lambda}{\partial l} & \frac{\partial \lambda}{\partial m} & \frac{\partial \lambda}{\partial n} & \frac{\partial \lambda}{\partial B} \\ \frac{\partial a_1}{\partial k} & \frac{\partial a_1}{\partial l} & \frac{\partial a_1}{\partial m} & \frac{\partial a_1}{\partial n} & \frac{\partial a_1}{\partial B} \\ \frac{\partial a_2}{\partial k} & \frac{\partial a_2}{\partial l} & \frac{\partial a_2}{\partial m} & \frac{\partial a_2}{\partial n} & \frac{\partial a_2}{\partial B} \\ \frac{\partial a_3}{\partial k} & \frac{\partial a_3}{\partial l} & \frac{\partial a_3}{\partial m} & \frac{\partial a_3}{\partial n} & \frac{\partial a_3}{\partial B} \\ \frac{\partial a_4}{\partial k} & \frac{\partial a_4}{\partial l} & \frac{\partial a_4}{\partial m} & \frac{\partial a_4}{\partial n} & \frac{\partial a_4}{\partial B} \end{bmatrix}. \tag{11}$$

$$= -J^{-1} \begin{bmatrix} -a_1 & -a_2 & -a_3 & -a_4 & 1 \\ -\lambda & 0 & 0 & 0 & 0 \\ 0 & -\lambda & 0 & 0 & 0 \\ 0 & 0 & -\lambda & 0 & 0 \\ 0 & 0 & 0 & -\lambda & 0 \end{bmatrix}.$$

The inverse of Jacobian matrix is, $J^{-1} = \dfrac{1}{|J|} C^T$, where $C = (C_{ij})$, the matrix of cofactors of J, where T indicates transpose, then (11) becomes (Mohajan, 2017a; Mohajan & Mohajan, 2022f),

$$= -\frac{1}{|J|} \begin{bmatrix} C_{11} & C_{21} & C_{31} & C_{41} & C_{51} \\ C_{12} & C_{22} & C_{32} & C_{42} & C_{52} \\ C_{13} & C_{23} & C_{33} & C_{43} & C_{53} \\ C_{14} & C_{24} & C_{34} & C_{44} & C_{54} \\ C_{15} & C_{25} & C_{35} & C_{45} & C_{55} \end{bmatrix} \begin{bmatrix} -a_1 & -a_2 & -a_3 & -a_4 & 1 \\ -\lambda & 0 & 0 & 0 & 0 \\ 0 & -\lambda & 0 & 0 & 0 \\ 0 & 0 & -\lambda & 0 & 0 \\ 0 & 0 & 0 & -\lambda & 0 \end{bmatrix}$$

$$J_G = -\frac{1}{|J|}\begin{bmatrix} -a_1C_{11}-\lambda C_{21} & -a_2C_{11}-\lambda C_{31} & -a_3C_{11}-\lambda C_{41} & -a_4C_{11}-\lambda C_{51} & C_{11} \\ -a_1C_{12}-\lambda C_{22} & -a_2C_{12}-\lambda C_{32} & -a_3C_{12}-\lambda C_{42} & -a_4C_{12}-\lambda C_{52} & C_{12} \\ -a_1C_{13}-\lambda C_{23} & -a_2C_{13}-\lambda C_{33} & -a_3C_{13}-\lambda C_{43} & -a_4C_{13}-\lambda C_{53} & C_{13} \\ -a_1C_{14}-\lambda C_{24} & -a_2C_{14}-\lambda C_{34} & -a_3C_{14}-\lambda C_{44} & -a_4C_{14}-\lambda C_{54} & C_{14} \\ -a_1C_{15}-\lambda C_{25} & -a_2C_{15}-\lambda C_{35} & -a_3C_{15}-\lambda C_{45} & -a_4C_{15}-\lambda C_{55} & C_{15} \end{bmatrix}. \quad (12)$$

In (11) total 25 comparative statics are available, and for sensitivity analysis we will try some of them to predict the economic analysis for the profit maximization (Baxley & Moorhouse, 1984; Wiese, 2021).

7. Sensitivity Analysis

Now we observe the effect on capital a_1 when its interest rate, *k* increases. Taking T_{21} (i.e., term of 2nd row and 1st column) from both sides of (12) we get (Islam et al., 2010b; Mohajan, 2021a, b),

$$\frac{\partial a_1}{\partial k} = \frac{a_1}{|J|}[C_{12}] + \frac{\lambda}{|J|}[C_{22}]$$

$$= \frac{a_1}{|J|}\text{Cofactor of } C_{12} + \frac{\lambda}{|J|}\text{Cofactor of C}_{22}$$

$$= -\frac{a_1}{|J|}\begin{vmatrix} -B_1 & L_{12} & L_{13} & L_{14} \\ -B_2 & L_{22} & L_{23} & L_{24} \\ -B_3 & L_{32} & L_{33} & L_{34} \\ -B_4 & L_{42} & L_{43} & L_{44} \end{vmatrix} + \frac{\lambda}{|J|}\begin{vmatrix} 0 & -B_2 & -B_3 & -B_4 \\ -B_2 & L_{22} & L_{23} & L_{24} \\ -B_3 & L_{32} & L_{33} & L_{34} \\ -B_4 & L_{42} & L_{43} & L_{44} \end{vmatrix}$$

$$= -\frac{a_1}{|J|}\left\{ -B_1\begin{vmatrix} L_{22} & L_{23} & L_{24} \\ L_{32} & L_{33} & L_{34} \\ L_{42} & L_{43} & L_{44} \end{vmatrix} - L_{12}\begin{vmatrix} -B_2 & L_{23} & L_{24} \\ -B_3 & L_{33} & L_{34} \\ -B_4 & L_{43} & L_{44} \end{vmatrix} + L_{13}\begin{vmatrix} -B_2 & L_{22} & L_{24} \\ -B_3 & L_{32} & L_{34} \\ -B_4 & L_{42} & L_{44} \end{vmatrix} - L_{14}\begin{vmatrix} -B_2 & L_{22} & L_{23} \\ -B_3 & L_{32} & L_{33} \\ -B_4 & L_{42} & L_{43} \end{vmatrix} \right\}$$

$$+ \frac{\lambda}{|J|}\left\{ B_2\begin{vmatrix} -B_2 & L_{23} & L_{24} \\ -B_3 & L_{33} & L_{34} \\ -B_4 & L_{43} & L_{44} \end{vmatrix} - B_3\begin{vmatrix} -B_2 & L_{22} & L_{24} \\ -B_3 & L_{32} & L_{34} \\ -B_4 & L_{42} & L_{44} \end{vmatrix} + B_4\begin{vmatrix} -B_2 & L_{22} & L_{23} \\ -B_3 & L_{32} & L_{33} \\ -B_4 & L_{42} & L_{43} \end{vmatrix} \right\}$$

$$= -\frac{a_1}{|J|}\left[-B_1\left\{ L_{22}(L_{33}L_{44} - L_{43}L_{34}) + L_{23}(L_{42}L_{34} - L_{32}L_{44}) + L_{24}(L_{32}L_{43} - L_{42}L_{33}) \right\} \right.$$

$$-L_{12}\{-B_2(L_{33}L_{44}-L_{43}L_{34})+L_{23}(-B_4L_{34}+B_3L_{44})+L_{24}(-B_3L_{43}+B_4L_{33})\}$$

$$+L_{13}\{-B_2(L_{32}L_{44}-L_{42}L_{34})+L_{22}(-B_4L_{34}+B_3L_{44})+L_{24}(-B_3L_{42}+B_4L_{32})\}$$

$$-L_{14}\{-B_2(L_{32}L_{43}-L_{42}L_{33})+L_{22}(-B_4L_{33}+B_3L_{43})+L_{23}(-B_3L_{42}+B_{4d}L_{32})\}]$$

$$+\frac{\lambda}{|J|}[B_2\{-B_2(L_{33}L_{44}-L_{43}L_{34})+L_{23}(-B_4L_{34}+B_3L_{44})+L_{24}(-B_3L_{43}+B_4L_{33})\}$$

$$-B_3\{-B_2(L_{32}L_{44}-L_{42}L_{34})+L_{22}(-B_4L_{34}+B_3L_{44})+L_{24}(-B_3L_{42}+B_4L_{32})\}$$

$$+B_4\{-B_2(L_{32}L_{43}-L_{42}L_{33})+L_{22}(-B_4L_{33}+B_3L_{43})+L_{23}(-B_3L_{42}+B_4L_{32})\}]$$

$$=-\frac{a_1}{|J|}\{-B_1L_{22}L_{33}L_{44}+B_1L_{22}L_{43}L_{34}-B_1L_{23}L_{42}L_{34}+B_1L_{23}L_{32}L_{44}-B_1L_{24}L_{32}L_{43}+B_1L_{24}L_{42}L_{33}$$

$$+B_2L_{12}L_{33}L_{44}-B_2L_{12}L_{43}L_{34}+B_4L_{12}L_{23}L_{34}-B_3L_{12}L_{23}L_{44}+B_3L_{12}L_{24}L_{43}-B_4L_{12}L_{24}L_{33}$$

$$-B_2L_{13}L_{32}L_{44}+B_2L_{13}L_{42}L_{34}-B_4L_{13}L_{22}L_{34}+B_3L_{13}L_{22}L_{44}-B_3L_{13}L_{24}L_{42}+B_4L_{13}L_{24}L_{32}$$

$$+B_2L_{14}L_{32}L_{43}-B_2L_{14}L_{42}L_{33}+B_4L_{14}L_{22}L_{33}-B_3L_{14}L_{22}L_{43}+B_3L_{14}L_{23}L_{42}-B_4L_{14}L_{23}L_{32}\}$$

$$+\frac{\lambda}{|J|}\{-B_2^2L_{33}L_{44}+B_2^2L_{43}L_{34}-B_2B_4L_{23}L_{34}+B_2B_3L_{23}L_{44}-B_2BL_{24}L_{43}+B_2B_4L_{24}L_{33}$$

$$+B_2B_3L_{32}L_{44}-B_2B_3L_{42}L_{34}+B_3B_4L_{22}L_{34}-B_3^2L_{22}L_{44}+B_3^2L_{24}L_{42}-B_3B_4L_{24}L_{32}$$

$$-B_2B_4L_{32}L_{43}+B_2B_4L_{42}L_{33}-B_4^2L_{22}L_{33}+B_3B_4L_{22}L_{43}-B_3B_4L_{23}L_{42}+B_4^2L_{23}L_{32}\}$$

$$=-\frac{a_1}{|J|}\{-B_1L_{22}L_{33}L_{44}+B_1L_{22}L_{34}^2-B_1L_{23}L_{42}L_{34}+B_1L_{23}^2L_{44}-B_1L_{24}L_{32}L_{43}+B_1L_{24}^2L_{33}$$

$$+B_2L_{12}L_{33}L_{44}-B_2L_{12}L_{34}^2+B_4L_{12}L_{23}L_{34}-B_3L_{12}L_{23}L_{44}+B_3L_{12}L_{24}L_{43}-B_4L_{12}L_{24}L_{33}$$

$$-B_2L_{13}L_{32}L_{44}+B_2L_{13}L_{42}L_{34}-B_4L_{13}L_{22}L_{34}+B_3L_{13}L_{22}L_{44}-B_3L_{13}L_{24}^2+B_4L_{13}L_{24}L_{32}$$

$$+B_2L_{14}L_{32}L_{43}-B_2L_{14}L_{42}L_{33}+B_4L_{14}L_{22}L_{33}-B_3L_{14}L_{22}L_{43}+B_3L_{14}L_{23}L_{42}-B_4L_{14}L_{23}^2\}$$

$$+\frac{\lambda}{|J|}\{-B_2^2L_{33}L_{44}+B_2^2L_{34}^2-B_2B_4L_{23}L_{34}+B_2B_3L_{23}L_{44}-B_2B_3L_{24}L_{43}+B_2B_4L_{24}L_{33}$$

$$+B_2B_3L_{32}L_{44}-B_2B_3L_{42}L_{34}+B_3B_4L_{22}L_{34}-B_3^2L_{22}L_{44}+B_3^2L_{24}^2-B_3B_4L_{24}L_{32}$$

$$-B_2B_4L_{32}L_{43}+B_2B_4L_{42}L_{33}-B_4^2L_{22}L_{33}+B_3B_4L_{22}L_{43}-B_3B_4L_{23}L_{42}+B_4^2L_{23}^2\}$$

$$= -\frac{1}{|J|}\frac{A^3 a_1^{3x} a_2^{3y} a_3^{3z} a_4^{3w}}{a_2^2 a_3^2 a_4^2}\ \{-ka_1 y(y-1)z(z-1)w(w-1) + ka_1 y(y-1)z^2 w^2 - ka_1 y^2 z^2 w^2$$
$$+ ka_1 y^2 z^2 w(w-1) - ka_1 y^2 z^2 w^2 + ka_1 y^2 z(z-1)w^2 + la_2 xyz(z-1)w(w-1) - la_2 xyz^2 w^2 + na_4 xy^2 z^2 w$$
$$- ma_3 xy^2 zw(w-1) + ma_3 xy^2 zw^2 - na_4 xy^2 z(z-1)w - la_2 xyz^2 w(w-1) + la_2 xyz^2 w^2$$
$$- na_4 xy(y-1)z^2 w + ma_3 xy(y-1)zw(w-1) - ma_3 xy^2 zw^2 + na_4 xy^2 z^2 w + la_2 xyz^2 w^2$$
$$- la_2 xyz(z-1)w^2 + na_4 xy(y-1)z(z-1)w - ma_3 xy(y-1)zw^2 + ma_3 xy^2 zw^2 - na_4 xy^2 z^2 w\}$$

$$+ \frac{\lambda}{|J|}\frac{A^2 a_1^{2x} a_2^{2y} a_3^{2z} a_4^{2w}}{a_2^2 a_3^2 a_4^2}\ \{-l^2 a_2^2 z(z-1)w(w-1) + l^2 a_2^2 z^2 w^2 - nla_2 a_4 yz^2 w + lma_2 a_3 yzw(w-1)$$
$$- lma_2 a_3 yzw^2 + nla_2 a_4 yz(z-1)w + lma_2 a_3 yzw(w-1) - lma_2 a_3 yzw^2 + nla_3 a_4 y(y-1)zw$$
$$- m^2 a_3^2 y(y-1)w(w-1) + m^2 a_3^2 y^2 w^2 - nla_3 a_4 y^2 zw - nla_2 a_4 yz^2 w + nla_2 a_4 yz(z-1)w$$
$$- n^2 a_4^2 y(y-1)z(z-1) + mna_3 a_4 y(y-1)zw - mna_3 a_4 y^2 zw + n^2 a_4^2 y^2 z^2\}$$

$$= -\frac{1}{|J|}\frac{A^3 xyzw a_1^{3x} a_2^{3y} a_3^{3z} a_4^{3w}}{a_2^2 a_3^2 a_4^2}\ \{-ka_1 x^{-1}(y-1)(z-1)(w-1) + ka_1 x^{-1}(y-1)zw - ka_1 x^{-1} yzw$$
$$+ ka_1 x^{-1} yz(w-1) - ka_1 x^{-1} yzw + ka_1 x^{-1} y(z-1)w + la_2(z-1)(w-1) - la_2 zw + na_4 yz - ma_3 y(w-1)$$
$$+ ma_3 yw - na_4 y(z-1) - la_2 z(w-1) + la_2 zw - na_4(y-1)z + ma_3(y-1)(w-1) - ma_3 yw + na_4 yz$$
$$+ la_2 zw - la_2(z-1)w + na_4(y-1)(z-1) - ma_3(y-1)w + ma_3 yw - na_4 yz\}$$

$$+ \frac{\lambda}{|J|}\frac{A^2 yzw a_1^{2x} a_2^{2y} a_3^{2z} a_4^{2w}}{a_2^2 a_3^2 a_4^2}\ \{-l^2 a_2^2 y^{-1}(z-1)(w-1) + y^{-1} l^2 a_2^2 zw - z^{-1} m^2 a_3^2 (y-1)(w-1)$$
$$+ z^{-1} m^2 a_3^2 yw - w^{-1} n^2 a_4^2 (y-1)(z-1) + w^{-1} n^2 a_4^2 yz\}$$

$$= -\frac{1}{|J|}\frac{A^3 xyzw a_1^{3x} a_2^{3y} a_3^{3z} a_4^{3w} B}{a_2^2 a_3^2 a_4^2 \Pi} + \frac{1}{|J|}\frac{A^2 yzw a_1^{2x} a_2^{2y} a_3^{2z} a_4^{2w}}{a_2^2 a_3^2 a_4^2}\ \frac{A a_1^x a_2^y a_3^z a_4^w \Pi}{B}\ \frac{B^2}{\Pi^2}(y+z+w)$$

$$\frac{\partial a_1}{\partial k} = \frac{1}{|J|}\frac{A^3 xyzw a_1^{3x} a_2^{3y} a_3^{3z} a_4^{3w} B}{a_2^2 a_3^2 a_4^2 \Pi}(\Pi - 2x). \tag{13}$$

If $(\Pi - 2x) > 0$ in equation (13), we get,

$$\frac{\partial a_1}{\partial k} < 0\,. \tag{14}$$

The relation (14) specifies that if the interest rate of the capital a_1 increases, the organization may decrease the level of input capital a_1 for the sustainability of its production and also for profit maximization. In this situation we observe that the organization may face decreasing returns to scale (Moolio et al., 2009; Mohajan & Mohajan, 2022a).

Again if $(\Pi - 2x) < 0$ in equation (13), we get,

$$\frac{\partial a_1}{\partial k} > 0. \qquad (15)$$

The inequality (15) indicates that when the interest rate of the capital a_1 increases, the organization also increases its capital structure. It seems that for the increasing demand of the products, and for the maintenance of the total cost; the organization also needs to increase its capital structure for profit maximization. From this analysis we see that the organization may face increasing returns to scale (Islam et al., 2011; Mohajan & Mohajan, 2022c).

Now we observe the effect on wage α_2 when its interest rate, k of the capital α_1 increases. Taking T_{31} (i.e., term of 3rd row and 1st column) from both sides of (12) we get (Moolio et al., 2009; Mohajan, 2021c),

$$\frac{\partial a_2}{\partial k} = \frac{a_1}{|J|}[C_{13}] + \frac{\mu}{|J|}[C_{23}]$$

$$= \frac{a_1}{|J|}\text{Cofactor of } C_{13} + \frac{\lambda}{|J|}\text{Cofactor of } \mathrm{C}_{23}$$

$$= \frac{a_1}{|J|}\begin{vmatrix} -B_1 & L_{11} & L_{13} & L_{14} \\ -B_2 & L_{21} & L_{23} & L_{24} \\ -B_3 & L_{31} & L_{33} & L_{34} \\ -B_4 & L_{41} & L_{43} & L_{44} \end{vmatrix} - \frac{\lambda}{|J|}\begin{vmatrix} 0 & -B_1 & -B_3 & -B_4 \\ -B_2 & L_{21} & L_{23} & L_{24} \\ -B_3 & L_{31} & L_{33} & L_{34} \\ -B_4 & L_{41} & L_{43} & L_{44} \end{vmatrix}$$

$$= \frac{a_1}{|J|}\left\{ -B_1\begin{vmatrix} L_{21} & L_{23} & L_{24} \\ L_{31} & L_{33} & L_{34} \\ L_{41} & L_{43} & L_{44} \end{vmatrix} - L_{11}\begin{vmatrix} -B_2 & L_{23} & L_{24} \\ -B_3 & L_{33} & L_{34} \\ -B_4 & L_{43} & L_{44} \end{vmatrix} + L_{13}\begin{vmatrix} -B_2 & L_{21} & L_{24} \\ -B_3 & L_{31} & L_{34} \\ -B_4 & L_{41} & L_{44} \end{vmatrix} - L_{14}\begin{vmatrix} -B_2 & L_{21} & L_{23} \\ -B_3 & L_{31} & L_{33} \\ -B_4 & L_{41} & L_{43} \end{vmatrix} \right\}$$

$$- \frac{\lambda}{|J|}\left\{ B_1\begin{vmatrix} -B_2 & L_{23} & L_{24} \\ -B_3 & L_{33} & L_{34} \\ -B_4 & L_{43} & L_{44} \end{vmatrix} - B_3\begin{vmatrix} -B_2 & L_{21} & L_{24} \\ -B_3 & L_{31} & L_{34} \\ -B_4 & L_{41} & L_{44} \end{vmatrix} + B_4\begin{vmatrix} -B_2 & L_{21} & L_{23} \\ -B_3 & L_{31} & L_{33} \\ -B_4 & L_{41} & L_{43} \end{vmatrix} \right\}$$

$$= -\frac{a_1}{|J|}[-B_1\{L_{21}(L_{33}L_{44} - L_{43}L_{34}) + L_{23}(L_{41}L_{34} - L_{31}L_{44}) + L_{24}(L_{31}L_{43} - L_{41}L_{33})\}$$

$$- L_{11}\{-B_2(L_{33}L_{44} - L_{43}L_{34}) + L_{23}(-B_4L_{34} + B_3L_{44}) + L_{24}(-B_3L_{43} + B_4L_{33})\}$$

$$+ L_{13}\{-B_2(L_{31}L_{44} - L_{41}L_{34}) + L_{21}(-B_4L_{34} + B_3L_{44}) + L_{24}(-B_3L_{41} + B_4L_{31})\}$$

$$-L_{14}\{-B_2(L_{31}L_{43}-L_{41}L_{33})+L_{21}(-B_4L_{33}+B_3L_{43})+L_{23}(-B_3L_{41}+B_4L_{31})\}]$$

$$-\frac{\lambda}{|J|}[B_1\{-B_2(L_{33}L_{44}-L_{43}L_{34})+L_{23}(-B_4L_{34}+B_3L_{44})+L_{24}(-B_3L_{43}+B_4L_{33})\}$$
$$-B_3\{-B_2(L_{31}L_{44}-L_{41}L_{34})+L_{21}(-B_4L_{34}+B_3L_{44})+L_{24}(-B_3L_{41}+B_4L_{31})\}$$
$$+B_4\{-B_2(L_{31}L_{43}-L_{41}L_{33})+L_{21}(-B_4L_{33}+B_3L_{43})+L_{23}(-B_3L_{41}+B_4L_{31})\}]$$

$$=-\frac{a_1}{|J|}\{B_1L_{21}L_{33}L_{44}\ -B_1L_{21}L_{43}L_{34}+B_1L_{23}L_{41}L_{24}-B_1L_{23}L_{31}L_{44}+B_1L_{24}L_{31}L_{43}-B_1L_{24}L_{41}L_{33}$$
$$-B_2L_{11}L_{33}L_{44}+B_2L_{11}L_{43}L_{34}-B_4L_{11}L_{23}L_{34}+B_3L_{11}L_{23}L_{44}-B_3L_{11}L_{24}L_{43}+B_4L_{11}L_{24}L_{33}+B_2L_{13}L_{31}L_{44}$$
$$-B_2L_{13}L_{41}L_{34}+B_4L_{13}L_{21}L_{34}-B_3L_{13}L_{21}L_{44}+B_3L_{13}L_{24}L_{41}-B_4L_{13}L_{24}L_{31}-B_2L_{14}L_{31}L_{43}+B_2L_{14}L_{41}L_{33}$$
$$-B_4L_{14}L_{21}L_{33}+B_3L_{14}L_{21}L_{43}-B_3L_{14}L_{23}L_{41}+B_4L_{14}L_{23}L_{31}\}+\frac{\lambda}{|J|}\{B_1B_2L_{33}L_{44}\ -B_1B_2L_{34}^2+B_1B_4L_{23}L_{34}$$
$$-B_4B_3L_{23}L_{44}+B_1B_3L_{24}L_{43}-B_1B_4L_{24}L_{33}+B_2B_3L_{31}L_{44}-B_2B_3L_{41}L_{34}+B_3B_4L_{21}L_{34}-B_3^2L_{21}L_{44}$$
$$+B_3^2L_{24}L_{41}-B_3B_4L_{24}L_{31}-B_2B_4L_{31}L_{43}+B_2B_4L_{41}L_{33}-B_4^2L_{21}L_{33}+B_3B_4L_{21}L_{43}-B_3B_4L_{23}L_{41}$$
$$+B_4^2L_{23}L_{31}\}$$

$$=-\frac{a_1}{|J|}\{B_1L_{21}L_{33}L_{44}\ -B_1L_{21}L_{34}^2+B_1L_{23}L_{41}L_{34}-B_1L_{23}L_{31}L_{44}+B_1L_{24}L_{31}L_{43}-B_1L_{24}L_{41}L_{33}$$
$$-B_2L_{11}L_{33}L_{44}+B_2L_{11}L_{34}^2-B_4L_{11}L_{23}L_{34}+B_3L_{11}L_{23}L_{44}-B_3L_{11}L_{24}L_{43}+B_4L_{11}L_{24}L_{33}+B_2L_{13}^2L_{44}$$
$$-B_2L_{13}L_{41}L_{34}+B_4L_{13}L_{21}L_{34}-B_3L_{13}L_{21}L_{44}+B_3L_{13}L_{24}L_{41}-B_4L_{13}^2L_{24}-B_2L_{14}L_{31}L_{43}+B_2L_{14}^2L_{33}$$
$$-B_4L_{14}L_{21}L_{33}+B_3L_{14}L_{21}L_{43}-B_3L_{14}^2L_{23}\ +B_4L_{14}L_{23}L_{31}\}+\frac{\lambda}{|J|}\{B_1B_2L_{33}L_{44}-B_1B_2L_{34}^2+B_1B_4L_{23}L_{34}$$
$$-B_1B_3L_{23}L_{44}+B_1B_3L_{24}L_{43}-B_1B_4L_{24}L_{33}+B_2B_3L_{31}L_{44}-B_2B_3L_{41}L_{34}+B_3B_4L_{24}L_{34}-B_3^2L_{21}L_{44}$$
$$+B_3^2L_{24}L_{41}-B_3B_4L_{24}L_{31}-B_2B_4L_{31}L_{43}+B_2B_4L_{41}L_{33}-B_4^2L_{21}L_{33}+B_3B_4L_{21}L_{43}-B_3B_4L_{23}L_{41}$$
$$+B_4^2L_{23}L_{31}\}$$

$$=-\frac{a_1}{|J|}\frac{A^3a_1^{3y}a_2^{3x}a_3^{3z}a_4^{3w}}{a_1^2a_2^2a_3^2a_4^2}\ \{ka_1a_2xyz(z-1)w(w-1)-ka_1a_2xyz^2w^2+ka_1a_2xyz^2w^2-ka_1a_2xyz^2w(w-1)$$
$$+ka_1a_2xyz^2w^2-ka_1a_2xyz(z-1)w^2\ -la_2^2x(x-1)z(z-1)w(w-1)+la_2^2x(x-1)z^2w^2$$
$$-na_2a_4x(x-1)yz^2w+ma_2a_3x(x-1)yzw(w-1)-ma_2a_3x(x-1)yzw^2+na_2a_4x(x-1)yz(z-1)w$$
$$+la_2^2x^2z^2w(w-1)-la_2^2x^2z^2w^2\ +na_2a_4x^2yz^2w-ma_2a_3x^2yzw(w-1)+ma_2a_3x^2yzw^2$$
$$-na_2a_4x^2yz^2w-la_2^2x^2z^2w^2\ +la_2^2x^2z(z-1)w^2\ -na_2a_4x^2yz(z-1)w+ma_2a_3x^2yzw^2$$
$$-ma_2a_3x^2yzw^2\ +na_2a_4x^2yz^2w\}+\frac{\lambda}{|J|}\frac{A^2a_1^{2y}a_2^{2x}a_3^{2z}a_4^{2w}}{a_1^2a_2^2a_3^2a_4^2}\ \{kla_1^2a_2^2z(z-1)w(w-1)-kla_1^2a_2^2z^2w^2$$
$$+kna_1^2a_2a_4yz^2w-kma_1^2a_2a_3yzw(w-1)+kma_1^2a_2a_3yzw^2-kna_1^2a_2a_4yz(z-1)w$$

$$+ lma_1a_2^2a_3xzw(w-1) - lma_1a_2^2a_3xzw^2 + mna_1a_2a_3a_4xyzw - m^2a_1a_2a_3^2xyw(w-1) + m^2a_3^2xyw^2$$
$$- mna_1a_2a_3a_4xyzw - nla_1a_2^2a_4xz^2w + nla_1a_2^2a_4xz(z-1)w - n^2a_1a_2a_4^2xyz(z-1) + mna_1a_2a_3a_4xyzw$$
$$- mna_1a_2a_3a_4xyzw + n^2a_1a_2a_4^2xyz^2 \}$$

$$= -\frac{a_1}{|J|}\frac{A^3xyzwa_1^{3y}a_2^{3x}a_3^{3z}a_4^{3w}}{a_1^2a_2a_3^2a_4^2}\left\{-ka_1(z-1)w + la_2(1-w-z)y^{-1} + ma_3 + na_4\right\}$$
$$+\frac{\lambda}{|J|}\frac{A^2zwa_1^{2x}a_2^{2y}a_3^{2z}a_4^{2w}}{a_1a_2a_3^2a_4^2}\{kla_1a_2(1-z-w) + kna_1a_4y + kma_1a_3y - lma_2a_3x + z^{-1}m^2a_3^2xy$$
$$- nla_2a_4x + w^{-1}n^2a_4^2xyz\}$$

$$= -\frac{1}{|J|}\frac{A^3xyzwa_1^{3y}a_2^{3x}a_3^{3z}a_4^{3w}}{a_1a_2a_3^2a_4^2}\left\{\frac{B}{\Pi}(1-w-z) + \frac{zB}{\Pi} + \frac{wB}{\Pi}\right\} + \frac{1}{|J|}\frac{A^2xyzwa_1^{2x}a_2^{2y}a_3^{2z}a_4^{2w}}{a_1a_2a_3^2a_4^2}\ \frac{Aa_1^xa_2^ya_3^za_4^w\Pi}{B}$$
$$\times\left\{\frac{B^2}{\Pi^2}(1-z-w) + \frac{zB^2}{\Pi^2} + \frac{wB^2}{\Pi^2}\right\}$$

$$\frac{\partial a_2}{\partial k} = -\frac{1}{|J|}\frac{A^3xyzwa_1^{3y}a_2^{3x}a_3^{3z}a_4^{3w}}{a_1a_2a_3^2a_4^2}\left(\frac{B}{\Pi} - \frac{B}{\Pi}\right) = 0. \qquad (16)$$

Equation (16) notices that there will be no effect on the level of labor a_2, if the interest rate of capital a_1 increases. Hence, there is no relation between labor a_2 and capital a_1 in the production procedure of the organization. In this situation the organization may increase or decrease its total labor. It seems that the organization may face constant returns to scale (Islam et al., 2010a,b; Mohajan & Mohajan, 2022f).

Now we observe the effect on principal raw material a_3 when interest rate of capital, *k* increases. Taking T_{41} (i.e., term of 4th row and 1st column) from both sides of (12) we get (Mohajan, 2021a),

$$\frac{\partial a_3}{\partial k} = \frac{a_1}{|J|}[C_{14}] + \frac{\lambda}{|J|}[C_{24}]$$

$$= \frac{a_1}{|J|}\text{Cofactor of } C_{14} + \frac{\lambda}{|J|}\text{Cofactor of } \mathrm{C}_{24}$$

$$= -\frac{a_4}{|J|}\begin{vmatrix} -B_1 & L_{11} & L_{12} & L_{14} \\ -B_2 & L_{21} & L_{22} & L_{24} \\ -B_3 & L_{31} & L_{32} & L_{34} \\ -B_4 & L_{41} & L_{42} & L_{44} \end{vmatrix} + \frac{\lambda}{|J|}\begin{vmatrix} 0 & -B_1 & -B_2 & -B_4 \\ -B_2 & L_{21} & L_{22} & L_{24} \\ -B_3 & L_{31} & L_{32} & L_{34} \\ -B_4 & L_{41} & L_{42} & L_{44} \end{vmatrix}$$

$$= -\frac{a_4}{|J|}\left\{ -B_1\begin{vmatrix} L_{21} & L_{22} & L_{24} \\ L_{31} & L_{32} & L_{34} \\ L_{41} & L_{42} & L_{44} \end{vmatrix} - L_{11}\begin{vmatrix} -B_2 & L_{22} & L_{24} \\ -B_3 & L_{32} & L_{34} \\ -B_4 & L_{42} & L_{44} \end{vmatrix} + L_{12}\begin{vmatrix} -B_2 & L_{21} & L_{24} \\ -B_3 & L_{31} & L_{34} \\ -B_4 & L_{41} & L_{44} \end{vmatrix} - L_{14}\begin{vmatrix} -B_2 & L_{21} & L_{22} \\ -B_3 & L_{31} & L_{32} \\ -B_4 & L_{41} & L_{42} \end{vmatrix} \right\}$$

$$+ \frac{\lambda}{|J|}\left\{ B_1\begin{vmatrix} -B_2 & L_{22} & L_{24} \\ -B_3 & L_{32} & L_{34} \\ -B_4 & L_{42} & L_{44} \end{vmatrix} - B_2\begin{vmatrix} -B_2 & L_{21} & L_{24} \\ -B_3 & L_{31} & L_{34} \\ -B_4 & L_{41} & L_{44} \end{vmatrix} + B_4\begin{vmatrix} -B_2 & L_{21} & L_{22} \\ -B_3 & L_{31} & L_{32} \\ -B_4 & L_{41} & L_{42} \end{vmatrix} \right\}$$

$$= -\frac{a_4}{|J|}\left[-B_1\{L_{21}(L_{32}L_{44} - L_{42}L_{34}) + L_{22}(L_{41}L_{34} - L_{31}L_{44}) + L_{24}(L_{31}L_{42} - L_{41}L_{32})\} \right.$$

$$- L_{11}\{-B_2(L_{32}L_{44} - L_{42}L_{34}) + L_{22}(-B_4L_{34} + B_3L_{44}) + L_{24}(-B_3L_{42} + B_4L_{32})\}$$

$$+ L_{12}\{-B_2(L_{31}L_{44} - L_{41}L_{34}) + L_{21}(-B_4L_{34} + B_3L_{44}) + L_{24}(-B_3L_{41} + B_4L_{31})\}$$

$$\left. - L_{14}\{-B_2(L_{31}L_{42} - L_{41}L_{32}) + L_{21}(-B_4L_{32} + B_3L_{42}) + L_{22}(-B_3L_{41} + B_4L_{31})\} \right]$$

$$+ \frac{\lambda}{|J|}\left[B_1\{-B_2(L_{32}L_{44} - L_{42}L_{34}) + L_{22}(-B_4L_{34} + B_3L_{44}) + L_{24}(-B_3L_{42} + B_4L_{32})\} \right.$$

$$- B_2\{-B_2(L_{31}L_{44} - L_{41}L_{34}) + L_{21}(-B_4L_{34} + B_3L_{44}) + L_{24}(-B_3L_{41} + B_4L_{31})\}$$

$$\left. + B_4\{-B_2(L_{31}L_{42} - L_{41}L_{32}) + L_{21}(-B_4L_{32} + B_3L_{42}) + L_{22}(-B_3L_{41} + B_4L_{31})\} \right]$$

$$= -\frac{a_4}{|J|}\{-B_1L_{21}L_{32}L_{44} + B_1L_{21}L_{42}L_{34} - B_1L_{22}L_{41}L_{34} + B_1L_{22}L_{31}L_{44} - B_1L_{24}L_{31}L_{42} + B_1L_{24}L_{41}L_{32}$$

$$+ B_2L_{11}L_{32}L_{44} - B_2L_{11}L_{42}L_{34} + B_4L_{11}L_{22}L_{34} - B_3L_{11}L_{22}L_{44} + B_3L_{11}L_{24}L_{42} - B_4L_{11}L_{24}L_{32}$$

$$- B_2L_{12}L_{31}L_{44} + B_2L_{12}L_{41}L_{34} - B_4L_{12}L_{21}L_{34} + B_3L_{12}L_{21}L_{44} - B_3L_{12}L_{24}L_{41} + B_4L_{12}L_{24}L_{31}$$

$$+ B_2L_{14}L_{31}L_{42} - B_2L_{14}L_{41}L_{32} + B_4L_{14}L_{21}L_{32} - B_3L_{14}L_{21}L_{42} + B_3L_{14}L_{22}L_{41} - B_4L_{14}L_{22}L_{31}\}$$

$$+ \frac{\lambda}{|J|}\{-B_1B_2L_{32}L_{44} + B_1B_2L_{42}L_{34} - B_1B_4L_{22}L_{34} + B_1B_3L_{22}L_{44} - B_1B_3L_{24}L_{42} + B_1B_4L_{24}L_{32}$$

$$+ B_2^2L_{31}L_{44} - B_2^2L_{41}L_{34} + B_2B_4L_{21}L_{34} - B_2B_3L_{21}L_{44} + B_2B_3L_{24}L_{41} - B_2B_4L_{24}L_{31}$$

$$- B_2B_4L_{31}L_{42} + B_2B_4L_{41}L_{32} - B_4^2L_{21}L_{32} + B_3B_4L_{21}L_{42} - B_3B_4L_{22}L_{41} + B_4^2L_{22}L_{31}\}$$

$$
\begin{aligned}
&= -\frac{1}{|J|}\frac{A^3 a_1^{3x} a_2^{3y} a_3^{3z} a_4^{3w}}{a_1^2 a_2^2 a_3^2 a_4} \{-ka_1a_3xy^2zw(w-1) + ka_1a_3xy^2zw^2 - ka_1a_3xy(y-1)zw^2 \\
&+ ka_1a_3xy(y-1)zw(w-1) - ka_1a_3xy^2zw^2 + ka_1a_3xy^2zw^2 + la_2a_3x(x-1)yzw(w-1) \\
&- la_2a_3x(x-1)yzw^2 + na_3a_4x(x-1)y(y-1)zw - ma_3^2x(x-1)y(y-1)w(w-1) + ma_3^2x(x-1)y^2w^2 \\
&- na_3a_4x(x-1)y^2zw - la_2a_3x^2yzw(w-1) + la_2a_3x^2yzw^2 - na_3a_4x^2y^2zw + ma_3^2x^2y^2w(w-1) \\
&- ma_3^2x^2y^2w^2 + na_3a_4x^2y^2zw + la_2a_3x^2yzw^2 - la_2a_3x^2yzw^2 + na_3a_4x^2y^2zw - ma_3^2x^2y^2w^2 \\
&+ ma_3^2x^2y(y-1)w^2 - na_3a_4x^2y(y-1)zw\} + \frac{1}{|J|}\frac{A^3 a_1^{3x} a_2^{3y} a_3^{3z} a_4^{3w}\Lambda}{a_1^2 a_2^2 a_3^2 a_4^2 B}\{-kla_1^2a_2a_3yzw(w-1) \\
&+ kla_1^2a_2a_3yzw^2 - kna_1^2a_3a_4y(y-1)zw + kma_1^2a_3^2y(y-1)w(w-1) - kma_1^2a_3^2y^2w^2 + kna_1^2a_3a_4y^2zw \\
&+ l^2a_1a_2^2a_3xzw(w-1) - l^2a_1a_2^2a_3xzw^2 + nla_1a_2a_3a_4xyzw - lma_1a_2a_3^2xyw(w-1) + lma_1a_2a_3^2xyw^2 \\
&- nla_1a_2a_3a_4xyzw - nla_1a_2a_3a_4xyzw + nla_1a_2a_3a_4xyzw - n^2a_1a_3a_4^2xy^2z + mna_1a_3^2a_4xy^2w \\
&- mna_1a_3^2a_4xy(y-1)w + n^2a_1a_3a_4^2xy(y-1)z\}
\end{aligned}
$$

$$
\begin{aligned}
&= -\frac{1}{|J|}\frac{A^3 xyzw a_1^{3x} a_2^{3y} a_3^{3z} a_4^{3w}}{a_1^2 a_2^2 a_3 a_4} \{-ka_1y(w-1) - ka_1(y-1)w + ka_1(y-1)(w-1) + ka_1yw \\
&+ la_2(x-1)(w-1) - la_2x(w-1) + la_2xw - la_2(x-1)w - 2ma_3xyz^{-1}w + ma_3x(y-1)z^{-1}w \\
&- ma_3(x-1)(y-1)z^{-1}(w-1) + ma_3xyz^{-1}(w-1) + ma_3(x-1)yz^{-1}w + na_4(x-1)(y-1) - na_4(x-1)y \\
&+ na_4xy - na_4x(y-1)\} + \frac{1}{|J|}\frac{A^3 a_1^{3x} a_2^{3y} a_3^{3z} a_4^{3w}\Pi}{a_1 a_2^2 a_3 a_4^2 B}\{-kla_1a_2yzw(w-1) + kla_1a_2yzw^2 \\
&+ kma_1a_3y(y-1)w(w-1) - kma_1a_3y^2w^2 + kna_1a_4y^2zw - kna_1a_4y(y-1)zw + l^2a_2^2xzw(w-1) \\
&- l^2a_2^2xzw^2 - lma_2a_3xyw(w-1) + lma_2a_3xyw^2 + mna_3a_4xy^2w - mna_3a_4xy(y-1)w - n^2a_4^2xy^2z \\
&+ n^2a_4^2xy(y-1)z\}
\end{aligned}
$$

$$
\frac{\partial a_3}{\partial k} = -\frac{1}{|J|}\frac{A^3 xyzw a_1^{3x} a_2^{3y} a_3^{3z} a_4^{3w} B}{a_1^2 a_2^2 a_3 a_4 \Pi} < 0 \qquad (17)
$$

From (17) we have realized that if the interest rate of the capital a_1 increases, the organization may decrease purchasing of principal raw material a_3. Consequently, in this situation the production rate of the organization may decrease. It seems that the organization may face decreasing returns to scale (Moolio et al., 2009; Mohajan, 2022).

Now we observe the effect on other inputs a_4 when its interest rate, *k* increases. Taking T_{21} (i.e., term of 2nd row and 1st column) from both sides of (12) we get (Mohajan, 2021a; Roy et al., 2021),

$$\frac{\partial a_4}{\partial k} = \frac{a_1}{|J|}[C_{15}] + \frac{\lambda}{|J|}[C_{25}]$$

$$= \frac{a_1}{|J|}\text{Cofactor of } C_{15} + \frac{\lambda}{|J|}\text{Cofactor of } \mathrm{C}_{25}$$

$$= \frac{a_1}{|J|}\begin{vmatrix} -B_1 & L_{11} & L_{12} & L_{13} \\ -B_2 & L_{21} & L_{22} & L_{23} \\ -B_3 & L_{31} & L_{32} & L_{33} \\ -B_4 & L_{41} & L_{42} & L_{43} \end{vmatrix} - \frac{\lambda}{|J|}\begin{vmatrix} 0 & -B_1 & -B_2 & -B_3 \\ -B_2 & L_{21} & L_{22} & L_{23} \\ -B_3 & L_{31} & L_{32} & L_{33} \\ -B_4 & L_{41} & L_{42} & L_{43} \end{vmatrix}$$

$$= \frac{a_1}{|J|}\left\{ -B_1\begin{vmatrix} L_{21} & L_{22} & L_{23} \\ L_{31} & L_{32} & L_{33} \\ L_{41} & L_{42} & L_{43} \end{vmatrix} - L_{11}\begin{vmatrix} -B_2 & L_{22} & L_{23} \\ -B_3 & L_{32} & L_{33} \\ -B_4 & L_{42} & L_{43} \end{vmatrix} + L_{12}\begin{vmatrix} -B_2 & L_{21} & L_{23} \\ -B_3 & L_{31} & L_{33} \\ -B_4 & L_{41} & L_{43} \end{vmatrix} - L_{13}\begin{vmatrix} -B_2 & L_{21} & L_{22} \\ -B_3 & L_{31} & L_{32} \\ -B_4 & L_{41} & L_{42} \end{vmatrix} \right\}$$

$$- \frac{\lambda}{|J|}\left\{ B_1\begin{vmatrix} -B_2 & L_{22} & L_{23} \\ -B_3 & L_{32} & L_{33} \\ -B_4 & L_{42} & L_{43} \end{vmatrix} - B_2\begin{vmatrix} -B_2 & L_{21} & L_{23} \\ -B_3 & L_{31} & L_{33} \\ -B_4 & L_{41} & L_{43} \end{vmatrix} + B_3\begin{vmatrix} -B_2 & L_{21} & L_{22} \\ -B_3 & L_{31} & L_{32} \\ -B_4 & L_{41} & L_{42} \end{vmatrix} \right\}$$

$$= \frac{a_1}{|J|}\left[-B_1\{L_{21}(L_{32}L_{43} - L_{42}L_{33}) + L_{22}(L_{41}L_{33} - L_{31}L_{43}) + L_{23}(L_{31}L_{42} - L_{41}L_{32})\}\right.$$

$$- L_{11}\{-B_2(L_{32}L_{43} - L_{42}L_{33}) + L_{22}(-B_4L_{33} + B_3L_{43}) + L_{23}(-B_3L_{42} + B_4L_{32})\}$$

$$+ L_{12}\{-B_2(L_{31}L_{43} - L_{41}L_{33}) + L_{21}(-B_4L_{33} + B_3L_{43}) + L_{23}(-B_3L_{41} + B_4L_{31})\}$$

$$\left. - L_{13}\{-B_2(L_{31}L_{42} - L_{41}L_{32}) + L_{21}(-B_4L_{32} + B_3L_{42}) + L_{22}(-B_3L_{41} + B_4L_{31})\}\right]$$

$$- \frac{\lambda}{|J|}\left[B_1\{-B_2(L_{32}L_{43} - L_{42}L_{33}) + L_{22}(-B_4L_{33} + B_3L_{43}) + L_{23}(-B_3L_{42} + B_4L_{32})\}\right.$$

$$- B_2\{-B_2(L_{31}L_{43} - L_{41}L_{33}) + L_{21}(-B_4L_{33} + B_3L_{43}) + L_{23}(-B_3L_{41} + B_4L_{31})\}$$

$$\left. + B_3\{-B_2(L_{31}L_{42} - L_{41}L_{32}) + L_{21}(-B_4L_{32} + B_3L_{42}) + L_{22}(-B_3L_{41} + B_4L_{31})\}\right]$$

$$= \frac{a_1}{|J|}\{-B_1L_{21}L_{32}L_{43} + B_1L_{21}L_{42}L_{33} - B_1L_{22}L_{41}L_{33} + B_1L_{22}L_{31}L_{43} - B_1L_{23}L_{31}L_{42} + B_1L_{23}L_{41}L_{32}$$

$$+ B_2L_{11}L_{32}L_{43} - B_2L_{11}L_{42}L_{33} + B_4L_{11}L_{22}L_{33} - B_3L_{11}L_{22}L_{43} + B_3L_{11}L_{23}L_{42} - B_4L_{11}L_{23}L_{32}$$

$$- B_2L_{12}L_{31}L_{43} + B_2L_{12}L_{41}L_{33} - B_4L_{12}L_{21}L_{33} + B_3L_{12}L_{21}L_{43} - B_3L_{12}L_{23}L_{41} + B_4L_{12}L_{23}L_{31}$$

$$+B_2L_{13}L_{31}L_{42}-B_2L_{13}L_{41}L_{32}+B_4L_{13}L_{21}L_{32}-B_3L_{13}L_{21}L_{42}+B_3L_{13}L_{22}L_{41}-B_4L_{13}L_{22}L_{31}\}$$

$$-\frac{\lambda}{|J|}\{-B_1B_2L_{32}L_{43}+B_1B_2L_{42}L_{33}-B_1B_4L_{22}L_{33}+B_1B_3L_{22}L_{43}-B_1B_3L_{23}L_{42}+B_1B_4L_{23}L_{32}$$

$$+B_2^2L_{31}L_{43}-B_2^2L_{41}L_{33}+B_2B_4L_{21}L_{33}-B_2B_3L_{21}L_{43}+B_2B_3L_{23}L_{41}-B_2B_4L_{23}L_{31}$$

$$-B_2B_3L_{31}L_{42}+B_2B_3L_{41}L_{32}-B_3B_4L_{21}L_{32}+B_3^2L_{21}L_{42}-B_3^2L_{22}L_{41}+B_3B_4L_{22}L_{31}\}$$

$$=\frac{1}{|J|}\frac{A^3a_1^{3x}a_2^{3y}a_3^{3z}a_4^{3w}}{a_1a_2^2a_3^2a_4^2}\{-ka_1a_4xy^2z^2w+ka_1a_4xy^2z(z-1)w-ka_1a_4xy(y-1)z(z-1)w$$
$$+ka_1a_4xy(y-1)z(z-1)w-ka_1a_4xy^2z^2w+ka_1a_4xy^2z^2w+la_2a_4x(x-1)yz^2w$$
$$-la_2a_4x(x-1)yz(z-1)w+na_4^2x(x-1)y(y-1)z(z-1)-ma_3a_4x(x-1)y(y-1)zw$$
$$+ma_3a_4x(x-1)y^2zw-na_4^2x(x-1)y^2z^2-la_2a_4x^2yz^2w+la_2a_4x^2yz(z-1)w-na_4^2x^2y^2z(z-1)$$
$$+ma_3a_4x^2y^2zw-ma_3a_4x^2y^2zw+na_4^2x^2y^2z^2+la_2a_4x^2yz^2w-la_2a_4x^2yz^2w+na_4^2x^2y^2z^2$$
$$-ma_3a_4x^2y^2zw+ma_3a_4x^2y(y-1)zw-na_4^2x^2y(y-1)z^2\}-\frac{\lambda}{|J|}\frac{A^2a_1^{2x}a_2^{2y}a_3^{2z}a_4^{2w}}{a_1^2a_2^2a_3^2a_4^2}\{-kla_1^2a_2a_4yz^2w$$
$$+kla_1^2a_2a_4yz(z-1)w-kna_1^2a_4^2y(y-1)z(z-1)+kma_1^2a_3a_4y(y-1)zw-kma_1^2a_3a_4y^2zw$$
$$+kna_1^2a_4^2y^2z^2+l^2a_1a_1^2a_4xz^2w-l^2a_1a_1^2a_4xz(z-1)w+nla_1a_2a_4^2xyz(z-1)-lma_1a_2a_3a_4xyzw$$
$$+lma_1a_2a_3a_4xyzw-nla_1a_2a_4^2xyz^2-lma_1a_2a_3a_4xyzw+lma_1a_2a_3a_4xyzw-mna_1a_3a_4^2xy^2z$$
$$+m^2a_1a_3^2a_4xy(y-1)w-m^2a_1a_3^2a_4xy^2w+mna_1a_3a_4^2xy(y-1)z\}$$

$$=\frac{1}{|J|}\frac{A^3xyzwa_1^{3x}a_2^{3y}a_3^{3z}a_4^{3w}}{a_1a_2^2a_3^2a_4}\{-ka_1yz+ka_1y(z-1)+la_2(x-1)z-la_2(x-1)(z-1)+la_2x(z-1)$$
$$-la_2xz-ma_3(x-1)(y-1)+ma_3(x-1)y-ma_3xy+ma_3x(y-1)+na_4(x-1)(y-1)(z-1)w^{-1}$$
$$-na_4(x-1)yzw^{-1}-na_4xy(z-1)w^{-1}+2na_4xyzw^{-1}-na_4x(y-1)zw^{-1}\}$$
$$-\frac{1}{|J|}\frac{Aa_1^xa_2^ya_3^za_4^w\Lambda}{B}\frac{A^2a_1^{2x}a_2^{2y}a_3^{2z}a_4^{2w}}{a_1a_2^2a_3^2a_4}\{-kla_1a_2yz^2w+kla_1a_2yz(z-1)w-kna_1a_4y(y-1)z(z-1)$$
$$+kna_1a_4y^2z^2+kma_1a_3y(y-1)zw-kma_1a_3y^2zw+l^2a_2^2xz^2w-l^2a_2^2xz(z-1)w+nla_2a_4xyz(z-1)$$
$$-nla_2a_4xyz^2+m^2a_3^2xy(y-1)w-m^2a_3^2xy^2w-mna_3a_4xy^2z+mna_3a_4xy(y-1)z\}$$

$$=-\frac{1}{|J|}\frac{A^3xyzwa_1^{3x}a_2^{3y}a_3^{3z}a_4^{3w}B}{a_1a_2^2a_3^2a_4\Pi}(xy+1)+\frac{1}{|J|}\frac{A^3xyzwa_1^{3x}a_2^{3y}a_3^{3z}a_4^{3w}B}{a_1a_2^2a_3^2a_4\Pi}(2z+1)$$

$$=\frac{1}{|J|}\frac{A^3xyzwa_1^{3x}a_2^{3y}a_3^{3z}a_4^{3w}B}{a_1a_2^2a_3^2a_4\Pi}(2z-xy). \tag{18}$$

From (2) we see that $xy << 1$ and also $z < 1$, so that we can consider $2z - xy > 0$; then from (18) we see that

$$\frac{\partial a_4}{\partial k} > 0. \tag{19}$$

From (19) we have realized that if the interest rate of the capital a_1 increases, the use of other inputs also increases. This seems that other inputs for this organization are necessary and these have no substitutes. It seems that the organization may face increasing returns to scale (Moolio et al., 2009; Mohajan, 2021b).

Also from (2) we see that $xy < 1$ and also $z < 1$, so that we can consider $2z - xy < 0$; then from (18) we see that

$$\frac{\partial a_4}{\partial k} < 0. \tag{20}$$

From (20) we have realized that if the interest rate of the capital a_1 increases; the use of other inputs decreases. Therefore, in this situation it seems that other inputs are not necessary for this organization or the organization has many substitutes of these. It seems that the organization may face decreasing returns to scale (Islam et al., 2011; Mohajan, 2022).

8. Conclusions

In this study we have considered the sensitivity analysis during profit maximization. We have used Lagrange multiplier method to make the constrained problem as unconstrained. For mathematical techniques we have analyzed the Cobb-Douglas production function with the subject to budget constraint. We have tried to predict on future production for maximum profit of the organization. Throughout the paper we have tried to show mathematical calculations in some details.

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