A simulation model of public debt sustainability

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Abstract

There is a long period since the problem of public debt sustainability captures the attention of economists. However, there is no unanimity concerning an adequate unique sustainability indicator or function generally accepted. Just in this line of elaborating new models and improving methodologies in order to quantify the impact of various factors on public debt sustainability is our paper. Moreover, last years, during its pre- and post-accession into EU period, Romanian economy is facing to numerous problems. Among these, the public debt sustainability plays a central role, its implications practically expanding on all fields connected to the economic dynamics.

Key words: public debt, sustainability function, contour plot, primary deficit, interest rate
JEL Codes: C15, C51, H63, H68

1. Public debt equations

To quantify the dynamics of public sector debt, often it starts from the well-known definition relation of the government's budget constraint. So, the change in public sector debt, D, between two periods (years) t and t-1, is given by the following relation:

$$D_t - D_{t-1} = i_t D_{t-1} + \Pi_t + a_t D_{t-1} - \Delta B_t \quad (1)$$

where i is the average nominal interest rate on public sector debt, Π is the primary deficit (net of interest payments), a is the revaluation effect on existing debt (in Romania this was supposed to be integrally due to the depreciation of ROL) and ΔB is the direct financing of budget from the Central Bank. In order to estimate parameters i and a, we also used the following relations: $i_t = Db_t / D_{t-1}$, where Db is the effective interest paid on public debt, and respectively $a_t = (D_t / D_{t-1})\ [1 - (CS_{t-1} / CS_t)]$, CS being the exchange rate (ROL/USD or ROL/EUR) at the end of year (for details see Albu, 2002).

Dividing both sides of equation (1) by nominal GDP, Y_t, and manipulating we obtain:

$$d_t - d_{t-1} = (i_t + a_t - g_t)\ [d_{t-1}/(1 + g_t)] + \pi_t - b_t \quad (2)$$

where d_t and d_{t-1} are the public sector debt to GDP ratio in two consecutive years, t and t –1, π is the primary public sector deficit as a percent of GDP, g is the nominal GDP growth rate between years t and t-1 and b is ΔB/Y. Alternatively we can approximate the nominal growth rate g as the sum of the change in GDP deflator p and the real GDP growth rate q and rewrite equation (2) as follows:

$$d_t - d_{t-1} = (is_t - q_t)\,[d_{t-1}/(1 + g_t)] + \pi_t - b_t \qquad (3)$$

where is could be defined as the real effective average interest rate on public sector debt (it is equal to the average real interest rate, i-p, plus the revaluation effect, a).
To see what the dynamics of debt accumulation involves, we can solve equation (3) recursively to obtain

$$d_T = d_0\, v^T + \Sigma\,(\pi^m - b_m)\, v^{T-m} \qquad (m = 1, 2, \dots, T) \qquad (4)$$

where v = (1 + is + p) / (1 + q + p), while it has been assumed, in order to simplify calculations, that the real effective interest rate, is, the real growth rate, q, and the change in the GDP deflator, p, are constant: $is_t = is$, $q_t = q$, $p_t = p$. Using equation (4) we can predict the debt ratio to GDP ratio for some future moments T, making assumptions about the relevant parameters. A high real growth rate relative to the effective real interest rate tends to reduce the debt to GDP ratio, d, while persistent primary deficits net of (real) Central Bank financing tend to increase it.

Actions in Romania to diminish inflation in order to stabilise economy and to achieve the conditions to be accepted in future period within European Monetary Union, restricts its ability to increase the direct financing of budget deficits by Central Bank, while it also implies that (real) interest rates will have to tend to European levels. A further safe, and quite helpful – regarding calculations – assumption to make is that the growth rate q will be equal to the average effective real interest rate, is, on public debt. It has also a theoretical reason: it corresponds to the "golden rule of accumulation" of optimum growth theory. In fact, it is speaking about the approach of accumulating debt problem by using equations with finite differences in conditions of indeterminacy in the case g = i, while the method that we shall use in next section of paper, starting from equation (3) and solving it recursively to obtain equation (4), avoids it (OECD, 1989). Under the assumption q = is, equation (4) could be written as follows:

$$d_T = d_0 + \Sigma\,(\pi^m - b_m) \qquad (5)$$

In fact, d_T will always tend to infinity for a very large T, unless the "average" future primary deficit is zero. An interesting, and empirically appealing, case arises, when the primary deficit is positive but declining. It can be shown (using the so-called d'Alambert's theorem on the convergence of infinite series) that d_T will converge to a finite limit for a very large T, if the primary deficit, π-b, is declining at a constant rate. If q>is, it can be shown from equation (4) that d_T will always be bounded, provided that primary deficits remain bounded. In a special case, in which the primary deficit, π-b, is constant, d_T will converge to (π-b) / (1-v) for a very large T. It should be noted, however, that this limit would be a very large one (and may not be practically sustained). Finally, if $q<is$, the debt to GDP ratio increases without limit (this is the so-called Domar's law).

2. Dynamics of debt in long term and sustainability function

One of the most important results of our investigation is the so-called sustainability function, $f(\pi, b, is, q, p, d)$, which must tend to zero in dynamics (or at least to a very small constant value), as a fundamental condition for sustainability:

$$f1\,(\pi, b, is, q, p, d) = [\,(\pi - b)/d\,] + (is - q)/(1 + p + q) \tag{6}$$

or

$$f2\,(\pi, b, is, q, p, d) = [\,(\pi - b)/d\,] + (is - q)/(1 + p + q + pq) \tag{7}$$

The computed values of function f during last fifteen years demonstrate that the sustainability function entered an oscillating regime, its values being situated around equilibrium. However, some risks of new jumps in the future continue to exist. That is why we shall concentrate on the behaviour of terms in sustainability function and implicitly on virtuous or vicious circle that could occur in the economic dynamics.

First term of sustainability function express the impact of the direct governmental policies (budgetary policies) and respectively those of central monetary authorities (monetary policies). Second term, expressed by the ratio $(is-q)/(1+p+q)$, or more precisely by $(is-q)/(1+p+q+pq)$, describes the behaviour of the real economy.

To study the behaviour of real economy, we used two partial models in order to simulate the following correlations: investment rate – growth rate and respectively investment rate – investment efficiency. The main hypotheses on which are based the models are referring to the existence of a direct positive correlation either between investment rate (α) and GDP growth rate (q), on the one hand, and between investment rate and its efficiency (η), on the other hand.

Other hypothesis is that, at limit, in case of an investment efficiency equal to the interest rate (noted by i or is), the investment process is stopped, i.e. $\alpha = 0$ (in this limit-case, the economic agents will be stimulated to place their savings in bank, economic investment as an alternative ensuring no supplementary money return).

In order to illustrate the mentioned hypotheses, in Figure 1 there is presented the output of simulation in case of the two partial theoretic models, their parameters being estimated on data covering last fifteen years of (1993-2007) and conforming to the following equations of regression:

$$qest_t = a\,\alpha_{t-1} + b \tag{8}$$

$$\eta est_t = c\,\alpha_{t-1} + is_{t-1} \tag{9}$$

where a, b, and c are estimated coefficients. In case of estimated efficiency (ηest) we considered the definition relation of efficiency in real terms, $\eta_t = \Delta Yd / I_{t-1} = (Yd_t - Yd_{t-1}) / I_{t-1}$ (where Yd is the disposable income in private sector and households after the extraction of all taxes, Tx, i.e. Yd=Y-Tx, and I is investment) and the dynamics of prices as well as. On the theoretical graphs of the GDP growth rate and investment efficiency (qT and respectively ηT), corresponding to a hypothetic variation of the investment rate (αM) within 0-0.35, we noted also some significant values such as: the minimum level of investment rate under which the GDP growth rate becomes negative (αcr); the average investment rate for the considered period (αM) and respectively the average saving rate (αEM); the theoretic efficiency corresponding to the average saving rate (ηTEM); and the average rate of interest on public debt computed implicitly on the "is" base (isM).

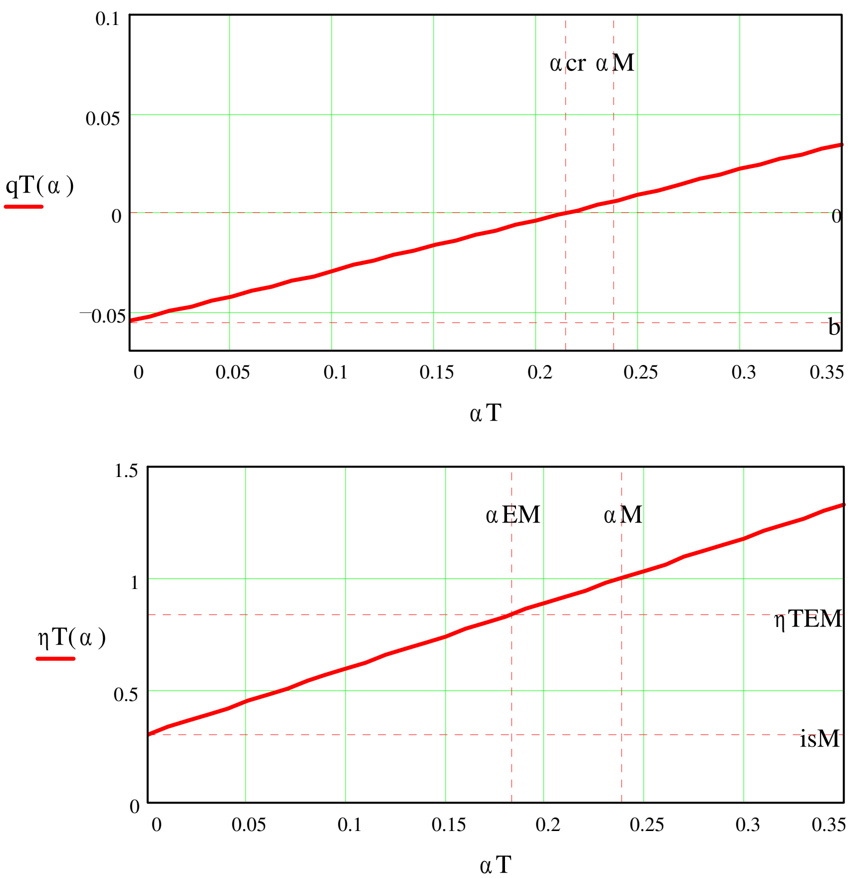


Figure 1.

In order to study the sustainability behaviour on the real side of economy, we combined the two partial models. After some algebraic operations and using the so-called technique backward perfect foresight, we can write explicitly the interest rate function, R, as follows:

$$R\ (q,\ tx,\ \Delta tx) = [qa^2\ (1\text{-}tx+\Delta tx) + \Delta txa^2]\ /\ [\text{-}Kq^2 + K(a+2b)q - ab - Kb^2] \quad (10)$$

where

$$K = [(kE - 1\ )\ a]\ /\ (qE - b) \quad (11)$$

qE is GDP growth rate corresponding to the saving rate (used to replace the investment rate), according to the first partial model; kE - the ratio between the efficiency corresponding to the level of brute savings (used to replace the volume of brute investments) and the interest rate, according to the second partial model; tx=Tx/Y. Now, considering, by simplification reasons, Δtx=0, qE=q and the following relation of kE:

$$kE = 1 + [\ (e\ \alpha\ c)\ /\ is\ ] \quad (12)$$

where e is the report between savings and investments (or in equilibrium case e=1)

$$ke\ 1 + [\ (\alpha\ c)\ /\ is\ ] \quad (13)$$

we obtained the following expressions for the function of interest rate:

$$R(q, kE, tx) := \frac{q\cdot a^2\cdot(1-tx)}{-\frac{(kE-1)\cdot a}{q-b}\cdot q^2 + \frac{(kE-1)\cdot a}{q-b}\cdot(a+2\cdot b)\cdot q - a\cdot b - \frac{(kE-1)\cdot a}{q-b}\cdot b^2} \quad (14)$$

$$Re(q, ke, tx) := \frac{q\cdot a^2\cdot(1-tx)}{-\frac{(ke-1)\cdot a}{q-b}\cdot q^2 + \frac{(ke-1)\cdot a}{q-b}\cdot(a+2\cdot b)\cdot q - a\cdot b - \frac{(ke-1)\cdot a}{q-b}\cdot b^2} \quad (15)$$

However, in line with the sustainability function, we are interested in the difference is-q, noted this time as G and having the following two forms (the second one is in case of fulfilling the equilibrium condition between saving and investment):

$$G(q, kE, tx) := \frac{q\cdot a^2\cdot(1-tx)}{-\frac{(kE-1)\cdot a}{q-b}\cdot q^2 + \frac{(kE-1)\cdot a}{q-b}\cdot(a+2\cdot b)\cdot q - a\cdot b - \frac{(kE-1)\cdot a}{q-b}\cdot b^2} - q \quad (16)$$

$$Ge(q, ke, tx) := \frac{q\cdot a^2\cdot(1-tx)}{-\frac{(ke-1)\cdot a}{q-b}\cdot q^2 + \frac{(ke-1)\cdot a}{q-b}\cdot(a+2\cdot b)\cdot q - a\cdot b - \frac{(ke-1)\cdot a}{q-b}\cdot b^2} - q \quad (17)$$

Graphical representation of simulations is shown in Figure 2. Some conclusions could be extracted from the simulation model, as follows:

- The optimum level for the sustainability function, G, is obtained for a growth rate, q, of 3.6%.
- In case of growth rates larger than 7% or less than 1.5-2% the sustainability is dramatically compromised.
- In case of interest function, the optimum level (minimum) is obtained for a growth rate, q, of 2.4%.
- In case of a growth rate of 7% the corresponding interest rate continues to be below 15%.

Focussing more on the origin neighbourhood zone permitted a better specification of the structure of local map, local behaviour, and some of characteristic mechanisms that govern the dynamics of system, being this time plausible from a normal economic viewpoint. The conclusion is that the dynamics of the sustainability function, despite of the imposed simplifying hypotheses, demonstrates a vast complexity. The change of values for certain fundamental parameters till in the neighbourhood of some zones of turbulence, as those near the asymptotes, could attract the system to enter regimes of chaotic behaviour, i.e. non-predictable ones. From the viewpoint of the policymakers, these could be roads toward errors and uncontrolled measures, returns and abrupt changes, either in legislation or in practice, and could create a negative impact on long-run economic evolution.

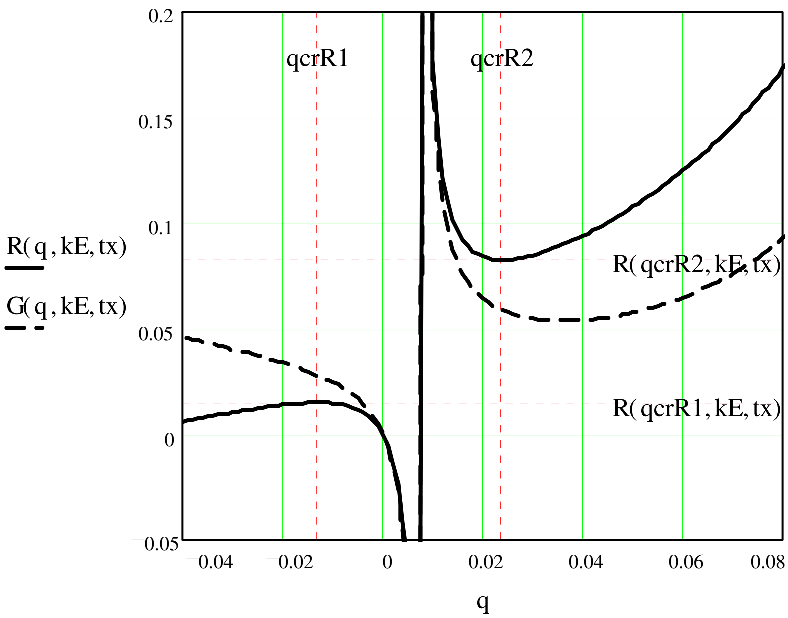
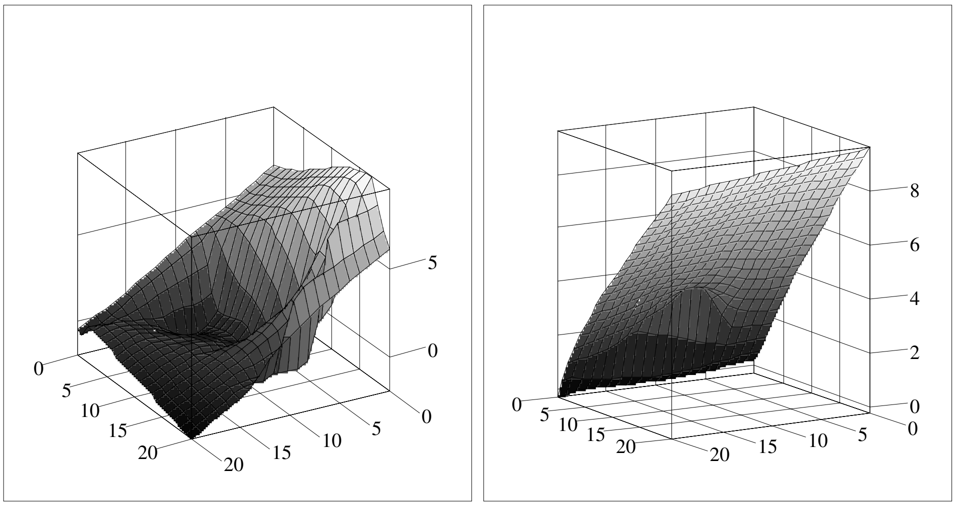


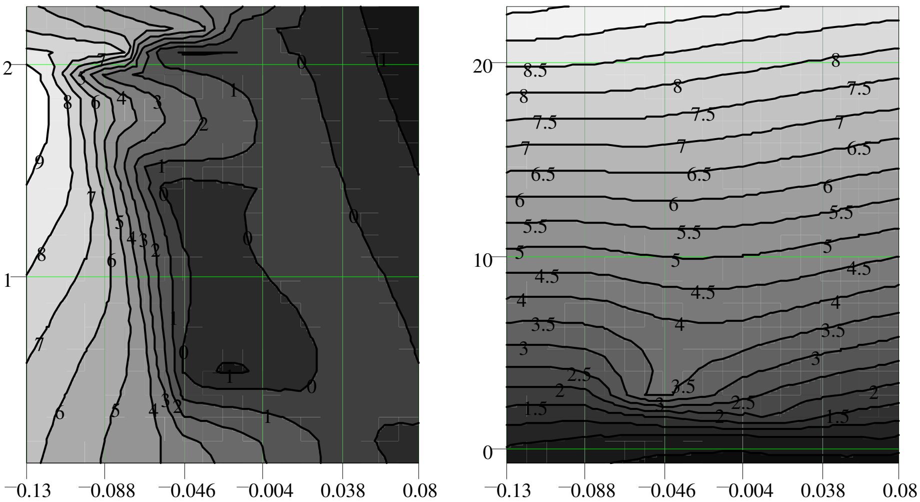
Figure 2.

Relating to the question "how is the imagine of sustainability function?", two 3D graphical representations and respectively two contour plot maps in case of f1 are shown in Figures 3 and 4.



q, p, f1 q, is, f1

Figure 3.



q, p, f1 q, is, f1

Figure 4.

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