Intertemporal equilibrium with physical capital and financial asset: role of dividend taxation

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Intertemporal equilibrium with physical capital and financial asset: role of dividend taxation*

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Abstract

The paper introduces dividend taxation and productive government spending in an infinite-horizon general equilibrium model with heterogeneous agents and financial market imperfections. We point out that imposing a dividend tax and using the revenue from this tax to finance productive government spending may prevent economic recession and promote economic growth. We also investigate the issue of optimal dividend taxation and the role of dividend taxation on the asset price bubble.

Keywords: Intertemporal equilibrium, recession, economic growth, productive government spending, dividend taxation, asset price bubbles.
JEL Classifications: C62, D31, D91, G10, E44.

1 Introduction

The interplay between the financial market and the production sector is an important issue in economics. On the one hand, the financial market having financial frictions may amplify the macroeconomic impacts of exogenous changes (Kiyotaki and Moore, 1997), and, in some cases, it is considered as a source of economic recession. On the other hand, Le Van and Pham (2016) point out that the key factor to prevent recessions is the productivity of the production sector and that financial assets may be beneficial to the productive sector by providing financial support for the purchase of physical capital.

Note that the productivity of firms in the above papers is exogenous and they did not investigate the role of taxation. While there is an extensive literature on capital and labor income taxation,[1] few papers focus on the impacts of dividend taxation on

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[1]See Atkinson and Sandmo (1980), Chamley (1986), Judd (1985), Kocherlakota (2010), Straub and Werning (2020).

economic growth and asset price bubble. Motivated by these observations, we propose a policy to enhance the firms' productivity: the government sets a tax on financial asset dividends and then uses the tax revenue to finance productive government spending which in turn improves the productivity of firms.[2] We then investigate the impacts of this policy on the economic growth and asset price bubble as well as the issue of optimal dividend taxation.[3]

To address these questions, we construct an infinite-horizon general equilibrium model with heterogeneous consumers, a firm, and a government. In this economy, there are a long-lived asset and a good (which can be consumed or/and used to produce). If consumers buy the long-lived asset, they may resell it after receiving exogenous dividends (in term of consumption good). This asset is similar to the Lucas tree (Lucas, 1978) or security (Santos and Woodford, 1997) or stock (Kocherlakota, 1992). We will call it the *financial asset*. The government taxes the dividends on the financial asset and then spends the tax revenue on financing productive projects (for example, public infrastructure) that improve the firm's productivity. This kind of endogenous growth is in the spirit of Barro (1990). The representative firm maximizes its profit by choosing its capital demand. Consumers maximize their intertemporal utility by choosing their allocation of consumption, capital stock and financial asset holding. They can also borrow by selling financial asset but there is a borrowing constraint: the repayment cannot exceed a fraction of their (physical) capital income.

Our contribution is three-fold. First, we explore the role of productive government spending (which is financed by dividend taxation) on economic recessions and growth. We say that an economic recession appears if the aggregate capital stock used for production falls below some critical level. We prove that recessions occur at infinitely many dates if the firm's productivity is too low. However, when the government employs the above policy, the productivity of firms is improved. By consequence, economic recessions can be prevented and we may have economic growth in the long run. This happens if the governance quality is good and the size of dividend is high. By contrast, when these conditions are violated (for example, when the tax revenue were used for wasteful government spending), the economy cannot escape from recession.

These findings contribute to the endogenous growth theory. The added-value is that our results are obtained in a general equilibrium model with heterogeneous agents and borrowing constraints, which raises technical difficulties that methods in the standard optimal growth theory (Le Van and Dana, 2003; Acemoglu, 2009) are no longer applicable. It should be noticed that our results hold for any equilibrium, including recursive ones. Although some authors (Acemoglu and Jensen, 2015; Datta et al., 2018) study comparative statics of recursive equilibria, intertemporal equilibria in our paper maybe not recursive, and therefore their methods cannot be directly applied in our framework.

Our second contribution concerns the optimal dividend taxation. This question

[2] Gourio and Miao (2010, 2011) study the effects of dividend taxation in a dynamic general equilibrium model which consists of a continuum of corporate firms, a representative household, and a government. However, unlike our paper, they abstract away from government spending.

[3] See Alstadsaeter et al. (2017) and references therein for the role of dividend taxes on corporate investment. However, this is not the aim of our paper.

matters because when the government increases the tax rate (τ) on dividends, the net dividends decrease but the production level may increase. Hence, the total amount of good may decrease or increase. So, it would be important to study the optimal dividend taxation to grasp this trade-off. In this respect, we assume that the government maximizes the total consumption of households at the steady state by choosing the tax rate. If the productivity of firms or the effect of the productive government spending are high, the government should choose the highest feasible tax rate on dividends. By contrast, if these factors are low, the government should apply the lowest tax rate. In the intermediate case for productivity, the size of dividend and the effect of the productive government spending, the optimal level of dividend taxation can be explicitly computed as a function of these three factors. We show that the optimal dividend tax rate is increasing in the governance quality and the firm's productivity, but decreasing in the size of dividend.

This result is closely related to Barro and Sala-i-Martin (1995) (Section 4.4.1) where they also study the interplay between productive government services and economic growth. They assume the production function of firms depend on the productive government services (which are equal to a proportion of the output). While this spending is financed by the dividend taxation in our paper, it is financed by a lump-sum tax in Barro and Sala-i-Martin. Another difference is that Barro and Sala-i-Martin focus on the case where the government maximizes the rate of growth of the balanced growth path. With specific setups,[4] they find that the optimal rate of tax equals the output elasticity of the productive government services. By contrast, the optimal dividend tax in our paper depends on several factors and we can provide comparative statics as mentioned above.[5]

Our third contribution is about the impact of dividend taxation on asset prices and bubbles. Following Santos and Woodford (1997), we say that an asset price bubble arises if, at equilibrium the fundamental value (i.e., the sum of discounted values) of asset dividends (after-tax) exceeds the asset's equilibrium price. Although there is a large literature on the non-existence of rational bubble in general equilibrium models,[6] few examples of bubbles of assets having positive dividends have been provided. We present a model where there may be a continuum of equilibria with bubble. Asset price bubbles may exist if endowments of agents fluctuate over time. Indeed, with such a fluctuation, at any date there is at least one agent who needs to buy asset (even when the asset price exceeds the fundamental value) because this agent has to transfer her wealth from this date to the next date (this is the only way she can smooth

[4]Cobb-Douglas production function and CRRA utility function

[5]Recall that Chamley (1986), Judd (1985) study the capital and labor income taxation and show that if an equilibrium has an asymptotic steady state, then the optimal policy is eventually to set the tax rate on capital to zero. Recall also that the limiting value of optimal capital tax can be different from zero in some situations (see (Aiyagari, 1995), Straub and Werning (2020) or Ljungqvist and Sargent (2018)'s Chapter 16). Observe that in Chamley (1986), Judd (1985), the government spending does not affect the productivity of firms as in Barro and Sala-i-Martin (1995) (Section 4.4.1) or in our paper.

[6]See Tirole (1982), Santos and Woodford (1997) or more recently Bosi et al. (2022). Moreover, Brunnermeier and Oehmke (2013) and Martin and Ventura (2018) provide more complete surveys on bubbles.

consumption because she is prevented from borrowing).[7] Our paper is different from Le Van and Pham (2016) because the asset's fundamental value in our model is not monotonic in dividends while, in Le Van and Pham (2016), it is monotonic. The reason is that we introduce a dividend taxation which makes the real returns and discount factors in our example depend on dividends through productive government investment. Interestingly, we show that asset bubbles are more likely to arise when dividend taxes increase. The intuition is that if such taxes increase, then the after-tax dividends decrease, making the asset's fundamental value decrease and lower than the asset price.

The paper is organized as follows. Section 2 presents the model and provides some basic equilibrium properties. Section 3 investigates the role of dividend taxation on recessions and economic growth. Section 4 studies the optimal dividend taxation. Section 5 considers the role of dividend taxation on asset bubbles. Section 6 concludes. Formal proofs are gathered in Appendix A.

2 Framework

Our model is based on Santos and Woodford (1997), Le Van and Pham (2016). We consider a deterministic infinite-horizon general equilibrium model à la Ramsey with three types of agents: a representative firm without market power, m heterogeneous households and a government. With respect to Le Van and Pham (2016), we introduce a government who imposes a dividend tax and uses it to finance productive government spending which in turn improves the productivity of firms.

Households

Each household invests in physical or financial asset, and consumes.

Consumption good: there is a single good which can be consumed or used to produce. p_t is its price at period t and $c_{i,t}$ the amount of good consumed by agent i.

Physical capital: $\delta \in (0, 1)$ denotes the capital depreciation rate, while r_t the return of capital. If agent i buys $k_{i,t} \geq 0$ units of physical capital at date $t-1$, then she will receive in the following period $(1-\delta)k_{i,t}$ units of physical capital (after depreciation) and returns $r_t k_{i,t}$.

Financial asset: if agent i buys $a_{i,t}$ units of financial asset at a price q_t at date t, she will receive in the following period ξ_{t+1} units of consumption good as dividend. Then, at the next date, she will be able to resell $a_{i,t}$ units of financial asset at a price q_{t+1}.[8] The government sets a tax on asset dividends: for each unit of dividend, consumers must pay τ units of consumption good.

[7] This mechanism is related to several examples in the literature as Bewley (1980) (Section 13), Townsend (1980), Kocherlakota (1992) (Example 1), Santos and Woodford (1997) (Example 4.1), Ljungqvist and Sargent (2018) (Chapter 27). The difference is that we consider a Ramsey model with physical capital and dividend taxation while these papers focus on exchange economies.

[8] This long-lived asset takes on different meanings: land or Lucas' tree (this is the case where $f_i = 0$ for any i) or security as in Santos and Woodford (1997) or stock as in Kocherlakota (1992)

Household i takes the sequence of prices $(p, q, r) := (p_t, q_t, r_t)_{t=0}^{\infty}$ as given, and solves the following program:

$$
\begin{aligned}
&(P_i(p,q,r)) : \max_{(c_{i,t},k_{i,t+1},a_{i,t})_{t=0}^{\infty}} \left[\sum_{t=0}^{\infty} \beta_i^t u_i(c_{i,t})\right] && \text{(1a)}\\
&\text{subject to: } k_{i,t+1} \geq 0, && \text{(1b)}\\
&\qquad p_t(c_{i,t} + k_{i,t+1} - (1-\delta)k_{i,t}) + q_t a_{i,t} \\
&\qquad \leq r_t k_{i,t} + q_t a_{i,t-1} + p_t \xi_t (1-\tau) a_{i,t-1} + \theta_t^i \pi_t, && \text{(1c)}\\
&\qquad (q_{t+1} + (1-\tau)p_{t+1}\xi_{t+1}) a_{i,t} \geq -f_i\Big[p_{t+1}(1-\delta) + r_{t+1}\Big] k_{i,t+1}, && \text{(1d)}
\end{aligned}
$$

where $k_{i,0} \geq 0$ and $a_{i,-1} \geq$ are exogenously given. At date t, π_t is the firm's profit, $(\theta_t^i)_{i=1}^m$ is the exogenous share of profit with $\theta_t^i \geq 0$ for any i and t, and $\sum_{i=1}^m \theta_t^i = 1$ for any t.

In our model, consumers can borrow by using the financial asset but they face the borrowing constraint (1d). Precisely, (1d) means that agent i can borrow an amount but the repayment of this amount does not exceed a fraction, say f_i, of the market value of her physical capital income (including returns and depreciation). Here, the physical capital stock plays the role of collateral. The fraction f_i is less than 1 to ensure that the market value of collateral of each agent is greater than her debt. The exogenous parameter $f_i \in [0, 1]$ represents the borrowing limit of agent i and can be viewed as an index of financial development. At equilibrium, as we will see (after Lemma 1), the borrowing constraint (1d) becomes equivalent to $q_t a_{i,t} \geq -f_i p_t k_{i,t+1}$.

The government

In our model, the government levies a constant proportional tax (τ) on dividends and uses it to finance productive government spending (for example, public infrastructure). The aggregate tax revenue (in terms of consumption good) is denoted by T_t. By construction, we have

$$T_t = \sum_{i=1}^{m} \tau \xi_t a_{i,t-1}.$$

Let us denote by G_t the productive government spending at date t. In the spirit of Barro (1990) (see all Barro and Sala-i-Martin (1995), Section 4.4), we assume that the productive government spending will improve the productivity of all firms at the next date. More precisely, the production function at date t is given by

$$F_g(G_{t-1}, K) = f(G_{t-1})F(K)$$

where f is an increasing function and $f(0) = 1$. When there is no productive government spending, we have $F_g(G_{t-1}, K) = f(0)F(K) = F(K)$.

The value $f(G)$ represents the effect of the productive government spending on the productivity of firms. This effect depends not only on the spending G but also on the governance quality.

Firm

At date t, the representative firm takes prices (p_t, r_t) and government spending G_{t-1} as given and maximizes its profit by choosing the physical capital amount K_t.

$$(P(p_t, r_t, G_{t-1})): \quad \pi_t := \max_{K_t \geq 0} \Big[p_t F_g(G_{t-1}, K_t) - r_t K_t\Big]. \tag{2}$$

2.1 Equilibrium

We denote an infinite-horizon sequence of prices and quantities by

$$(p, q, r, (c_i, k_i, a_i)_{i=1}^m, K, G, T)$$

with $(x) := (x_t)_{t\geq 0}$ for $x \in \{p, q, r, c_i, a_i, K, G, T\}$ and $(k_i) := (k_{i,t+1})_{t\geq 0}$ for any i.

The economy is denoted by $\mathcal{E}$ and it is characterized by a list of fundamentals

$$\mathcal{E} := \Big((u_i, \beta_i, k_{i,0}, a_{i,-1}, f_i, \theta^i)_{i=1}^m, F, f, (\xi_t)_{t=0}^\infty, \delta, \tau\Big).$$

Definition 1. *A list* $\Big(\bar{p}_t, \bar{q}_t, \bar{r}_t, (\bar{c}_{i,t}, \bar{k}_{i,t+1}, \bar{a}_{i,t})_{i=1}^m, \bar{K}_t, \bar{G}_t, \bar{T}_t\Big)_{t=0}^\infty$ *is an equilibrium of the economy* $\mathcal{E}$ *if the following conditions are met.*

(i) Price positivity: $\bar{p}_t, \bar{q}_t, \bar{r}_t > 0$ *for* $t \geq 0$.

(ii) Market clearing conditions: for any $t \geq 0$,

$$\begin{aligned} \textit{good: } & \sum_{i=1}^m (\bar{c}_{i,t} + \bar{k}_{i,t+1} - (1-\delta)\bar{k}_{i,t}) = f(\bar{G}_{t-1})F(\bar{K}_t) + (1-\tau)\xi_t, \\ \textit{capital: } & \bar{K}_t = \sum_{i=1}^m \bar{k}_{i,t}, \\ \textit{financial asset: } & \sum_{i=1}^m \bar{a}_{i,t} = 1, \end{aligned}$$

(iii) Optimal consumption plans: for any i, $(\bar{c}_{i,t}, \bar{k}_{i,t+1}, \bar{a}_{i,t})_{t=0}^\infty$ *is a solution of the problem* $(P_i(\bar{p}, \bar{q}, \bar{r}))$.

(iv) Optimal production plan: for any $t \geq 0$, $\bar{K}_t$ *is a solution of the problem* $(P(\bar{p}_t, \bar{r}_t, \bar{G}_{t-1}))$.

(v) Government: $\bar{G}_t = \bar{T}_t$ *where* $\bar{T}_t = \sum_{i=1}^m \tau\xi_t \bar{a}_{i,t-1}$.

At equilibrium, we have $G_t = T_t = \tau\xi_t$. Therefore, the consumption market clearing condition writes

$$C_t + K_{t+1} + G_t = f(G_{t-1})F(K_t) + (1-\delta)K_t + \xi_t, \tag{3}$$

where $C_t := \sum_{i=1}^m c_{i,t}$, $K_t := \sum_{i=1}^m k_{i,t}$. The output of the economy is $f(G_{t-1})F(K_t) + (1-\delta)K_t + \xi_t$ and decomposes into three parts: private consumption C_t, private investment K_{t+1} and public expenditure G_t.

In the rest of this paper, when we do not explicitly mention, the following assumptions are required.

Assumption 1. *(i) u_i is continuously differentiable, strictly increasing and concave with $u_i(0) = 0$ and $u_i'(0) = \infty$.*
(ii) The function $F(\cdot)$ is continuously differentiable, strictly increasing, concave with $F(0) \geq 0$, $F(\infty) = \infty$. The function $f(\cdot)$ is continuous, increasing, $f(0) = 1$, $f(\infty) = \infty$.
(iii) $0 < \xi_t < \infty, \forall t$. $f_i \in [0, 1), \forall i$.
(iv) $k_{i,0}, a_{i,-1} \geq 0$, and $(k_{i,0}, a_{i,-1}) \neq (0, 0)$, $\forall i$. Moreover, $\sum_{i=1}^{m} a_{i,-1} = 1$ and $K_0 := \sum_{i=1}^{m} k_{i,0} > 0$.
(v) $\sum_{t=0}^{\infty} \beta_i^t u_i(D_t) < \infty$ where $D_0 := F_g(\xi_0, K_0) + (1-\delta)K_0 + \xi_0$, $D_t := F_g(\xi_{t-1}, D_{t-1}) + (1-\delta)D_{t-1} + \xi_t, \forall t \geq 0$.

While conditions (i)-(iv) are standard, condition (v) ensures that the intertemporal utility of consumers is finite because D_t is an upper bound of the aggregate consumption.

Before presenting the equilibrium analysis, we prove the existence of equilibrium.

Proposition 1. *Under assumptions (H1, H2, H3, H4, H5), there exists an equilibrium.*

Proof. See Appendix A.1. □

The detailed proof of Proposition 1 is presented in Appendix A.

Price normalization: Since $p_t > 0$ $\forall t$ at equilibrium, in the rest of the paper, we will normalize by setting $p_t = 1$ $\forall t$. In this case, we also call $\Big(q_t, r_t, (c_{i,t}, k_{i,t+1}, a_{i,t})_{i=1}^m, K_t, G_t, T_t\Big)_{t\geq 0}$ an equilibrium.

2.2 Basic properties of equilibrium

Let $\Big(q_t, r_t, (c_{i,t}, k_{i,t+1}, a_{i,t})_{i=1}^m, K_t, G_t, T_t\Big)_{t\geq 0}$ be an equilibrium. Denote by $\mu_{i,t}$ and $\nu_{i,t+1}$ the multipliers associated to the budget and the borrowing constraint of the agent i at date t. Denote $\lambda_{i,t+1}$ the multiplier associated with constraint $k_{i,t+1} \geq 0$. We have

$$\beta_i^t u_i'(c_{i,t}) = \mu_{i,t} \tag{4a}$$
$$\mu_{i,t} = (r_{t+1} + 1 - \delta)(\mu_{i,t+1} + f_i \nu_{i,t+1}) + \lambda_{i,t+1} \tag{4b}$$
$$q_t \mu_{i,t} = (q_{t+1} + (1-\tau)\xi_{t+1})(\mu_{i,t+1} + \nu_{i,t+1}). \tag{4c}$$

Notice that $k_{i,t+1}\lambda_{i,t+1} = 0$ and $\nu_{i,t+1}\Big[\Big(q_{t+1} + (1-\tau)\xi_{t+1}\Big)a_{i,t} + f_i\Big(1 - \delta + r_{t+1}\Big)k_{i,t+1}\Big] = 0$.

The following lemma sums up the FOCs.

Lemma 1. *The following non-arbitrage condition is obtained*

$$r_{t+1} + 1 - \delta \leq \frac{q_{t+1} + (1-\tau)\xi_{t+1}}{q_t} = \frac{1}{\max_i \Big\{ \frac{\beta_i u_i'(c_{i,t+1})}{u_i'(c_{i,t})} \Big\}} \tag{5}$$

for any t. Moreover, the inequality holds with equality if $K_{t+1} > 0$.

It should be noticed that in the presence of borrowing constraints, we only have the following Euler inequality, instead of Euler equation as in the representative consumer model without financial frictions,

$$1 \geq (r_{t+1} + 1 - \delta) \max_i \left\{ \frac{\beta_i u_i'(c_{i,t+1})}{u_i'(c_{i,t})} \right\}. \tag{6}$$

According to Lemma 1, we have that

$$f_i(1 - \delta + r_{t+1})k_{i,t+1} = f_i \frac{q_{t+1} + (1-\tau)\xi_{t+1}}{q_t} k_{i,t+1}. \tag{7}$$

Therefore, borrowing constraint (1d) is equivalent to $-q_t a_{i,t} \leq f_i k_{i,t+1}$. This means that, if agent i borrows at date t by selling $-a_{i,t}$ units of financial asset, the borrowing amount does not exceed a fraction f_i of the value of physical capital stock.

3 Dividend taxation and aggregate production

This section will investigate two questions: (1) Why do economic recessions appear? (2) How to avoid them and eventually achieve economic growth? First of all, we introduce the concept of recession.

Definition 2. *Let $\bar{k} \geq 0$. We say that there is a $\bar{k}$-recession in the productive sector at date t if the aggregate capital is less than $\bar{k}$, i.e., $K_t \leq \bar{k}$.*

We say that there is an extreme recession at date t if $K_t = 0$.[9]

The following result provides conditions under which recessions occur.

Proposition 2. *Assume that $\bar{\xi} := \sup_t \xi_t < \infty$, $\underline{\xi} := \inf_t \xi_t > 0$.*

Let $\bar{k} \geq 0$. If $f(\tau\bar{\xi})F'(\bar{k}) \leq \delta$, then, there exists an infinite sequence of times $(t_n)_{n=0}^{\infty}$ such that $K_{t_n} \leq \bar{k}$ for every $n \geq 0$.

Proof. See Appendix A.3 □

According to this result, recessions frequently occur if $f(\tau\bar{\xi})F'(\bar{k}) \leq \delta$ which can happen if (1) the productivity $F'(\bar{k})$ is low, and (2) the effect of the productive government spending $f(\tau\bar{\xi})$ is small. The idea is the following. Consumers diversify their portfolio by investing in capital and the financial asset. During the periods t and $t+1$, the real return on physical capital is $f(\tau\xi_t)F'(k_{t+1}) + 1 - \delta$ while the real return on the financial asset is $\frac{q_{t+1}+(1-\tau)\xi_{t+1}}{q_t}$. Households compare these two returns when making their investment decision (invest in capital or in financial asset). If the production sector has a low productivity and hence a low return, households will prefer to buy financial asset. This phenomenon generates a recession in the production sector (i.e., $K_t \leq \bar{k}$).

[9]The notion of extreme recession has been investigated in Le Van and Pham (2016). Notice that Le Van and Pham (2016) do not introduce dividend taxation.

It is clear that the main cause of economic recessions is not the financial market, but rather the low productivity and a weak effect of the productive government spending (for example, when the tax revenue is used for wasteful government spending).

We now examine whether recessions can be avoided. Formally, we will find conditions under which the aggregate physical capital exceeds the threshold $\bar{k}$. Since we are focusing on intertemporal equilibrium in economies with financial constraint, there is no easy way to investigate the characteristics of the aggregate capital path (K_t) (recall that we do not have Euler equations as in standard models à la Ramsey). To overcome this difficulty and get intuitive insights, we introduce an additional assumption on utility functions.

Assumption 2. *There exists the function $y_i(\cdot)$: $\mathbb{R}^+ \to \mathbb{R}^+$ such that*

(1) $y_i(x) > 0$ and $y_i'(x) > 0$ for any $x > 0$. Moreover, $\lim_{x\to\infty} y_i(x) = \infty$.

(2) For each $x > 0$ and i, we have $\frac{(u_i')^{-1}\left(\frac{u_i'(a)}{x}\right)}{a} \geq y_i(x)$ for any $a > 0$, where $(u_i')^{-1}$ is the inverse function of u_i'.

Notice that this assumption is satisfied with standard utility functions (for example, if $u_i(c) = c^{1-\sigma_i}/(1-\sigma_i)$ with $\sigma_i \in (0,1)$, then the function $y_i(x) = x^{\frac{1}{\sigma_i}}$ satisfies Assumption 2). Condition 2 in Assumption 2 leads to an important property: if $xu_i'(b) \leq u_i'(a)$, then $b \geq y_i(x)a$.[10] This property allows us to obtain the following result showing that a $\bar{k}-$recession can be avoided.

Proposition 3. *Assume that $\xi_t = \xi > 0$ for any t. Consider an equilibrium.*

1. *If*
$$f(\tau\xi)F'(0) > \delta + \max_{i=1,\dots,m} \left\{\frac{1}{\beta_i} - 1\right\} \tag{8}$$
then there is no extreme recession, i.e., $K_t > 0$ for any t.

2. *Let Assumption 2 be satisfied and assume that $\lim_{a\to\infty} \frac{a}{\min_i\{y_i(\beta_i a)\}} < \infty$.*[11] *Given $\bar{k} > 0$, there exists $\xi_{\bar{k}}$ such that $K_t > \bar{k}$ for any $\xi > \xi_{\bar{k}}$ and for any $t \geq 1$.*

Proof. See Appendix A.4. □

If we write $\beta_i = \frac{1}{1+r_i}$, then $r_i = \frac{1}{\beta_i} - 1$ may be interpreted as the exogenous subjective interest rate of agent i. So, condition (8) means that the marginal productivity at the point zero is higher than the total cost of investment. This happens if the effect of the productive government spending $f(\tau\xi)$ and the original productivity $F'(0)$ are high enough. In this case, there is at least one household who invests in physical capital, and hence we can avoid an extreme recession (i.e., the situation where the aggregate capital is zero).

[10] Indeed, since u_i' is decreasing, we have $b \leq (u_i')^{-1}\left(\frac{u_i'(a)}{x}\right)$. According to Assumption 2, we have $(u_i')^{-1}\left(\frac{u_i'(a)}{x}\right) \geq y_i(x)a$ which implies that $b \geq y_i(x)a$.

[11] Note that condition $\lim_{a\to\infty} \frac{a}{\min_i\{y_i(\beta_i a)\}} < \infty$ holds for standard utility functions, for example, when $u_i(c) = c^{1-\sigma_i}/(1-\sigma_i)$ with $\sigma_i < 1$.

Point 2 of Proposition 3 complements point 1 by focusing on the size of dividend and indicating that when dividends are high enough, any $\bar{k}$-recession can be avoided. To prove this point, we need Assumption 2 and $\lim_{a\to\infty} \frac{a}{\min_i\{y_i(\beta_i a)\}} < \infty$. Let us explain key steps of our proof. We have the Euler inequality $u_i'(c_{i,t-1}) \geq \Big(f(\tau\xi)F'(K_t) + 1 - \delta\Big)\beta_i u_i'(c_{i,t})\ \forall i$. By combining Assumption 2 and the Euler inequality, we can prove that

$$c_{i,t} \geq y_i\Big[\Big(f(\tau\xi)F'(K_t) + 1 - \delta\Big)\beta_i\Big]c_{i,t-1}, \quad \forall i.$$

Since the total consumption of households depends on the aggregate capital and dividend (according to the market clearing condition: $\sum_{i=1}^{m} c_{i,t} = F(K_t) + (1-\delta)K_t + (1-\tau)\xi$), we can find a lower bound of the aggregate capital K_t. Then, we use condition 1 in Assumption 2 and $\lim_{a\to\infty} \frac{a}{\min_i\{y_i(\beta_i a)\}} < \infty$, $\forall i$, to prove that this bound is higher than $\bar{k}$ when the asset dividend ξ is high enough.

We may wonder whether the dividend taxation can lead to unbounded growth. Observe that, when $F'(\infty) = 0$ and $\sup_t \xi_t < \infty$, we can prove that the aggregate output is uniformly bounded from above. So, if we assume that $\sup_t \xi_t < \infty$, which is a natural assumption, then an unbounded growth requires that $F'(\infty) > 0$. The following result provides conditions under which $\lim_{t\to\infty} K_t = \infty$.

Proposition 4. *Let Assumption 2 be satisfied and $\bar{\xi} := \sup_t \xi_t < \infty$, $\underline{\xi} := \inf_t \xi_t > 0$. Assume also that there exists A so that $F'(K) \geq A > 0$ for any K.*

Denote $x := \min_i \Big\{ y_i\Big(\beta_i\Big(Af(\tau\underline{\xi}) + 1 - \delta\Big)\Big)\Big\}$, where the function $y_i(\cdot)$ is defined in Assumption 2. Then, we have $\lim_{t\to\infty} K_t = \infty$ at equilibrium if

$$x \geq \frac{\xi_t}{\xi_{t-1}}, \forall t \tag{9a}$$

$$\textit{and} \quad x\Big(Af(\tau\underline{\xi}) - \delta\Big) > 1 - \delta + A. \tag{9b}$$

Proof. See Appendix A.5. □

Conditions (9a) and (9b) ensure that the rate of growth of the economy is always higher than 1, which in turns implies that $\lim_{t\to\infty} K_t = \infty$. By definition of x and the function y_i, x increases in $f(\tau\underline{\xi})$ and A. Hence, conditions (9a, 9b) are more likely satisfied if $\tau\underline{\xi}$, b, A are high.[12] By the way, our result shows that the size of dividends (ξ_t) and the effect of the productive government spending $f(\tau\underline{\xi})$ play a key role on economic growth. Our theoretical results on the role of the effect of the government spending are supported by empirical evidences such as Furceri and Li (2017).[13]

Proposition 4 leads to an interesting implication. To see better the insight, let us consider a particular case with linear technology $F(K) = AK$ and the productivity is low in the sense that $A < \delta$.

[12] In a particular case where $u_i(c) = \frac{c^{1-\sigma_i}}{1-\sigma_i}$ with $\sigma_i < 1$, we find $y_i(a) = a^{\frac{1}{1-\sigma_i}}$ and $x = \min_i \beta_i^{\frac{1}{1-\sigma_i}} \left(Af(\tau\underline{\xi}) + 1 - \delta\right)^{\frac{1}{1-\sigma_i}}$.

[13] For the quality of government, see La Porta et al. (1999).

1. If there is no dividend ($\xi_t = 0$ for any t), then, according to (3), we have $K_{t+1} \leq (A + 1 - \delta)K_t$ for any t, which implies that $\lim_{t \to \infty} K_t = 0$: the economy collapses.

2. In the case with constant positive dividend ($\xi_t = \xi > 0$ for any t), Proposition 4 suggests that, if the government levies taxes on asset dividends and invests in productive projects which improve the productivity of firms (in the sense of condition (9a), (9b)), the economy may have unbounded growth.

Our result is related to the literature on optimal growth with increasing returns (Jones and Manuelli, 1990; Kamihigashi and Roy, 2007; Bruno et al., 2009). Our added-value is twofold. First, we point out the role of dividend taxation which can finance productive government spending, and thanks to this, the host country may grow. Second, we consider a decentralized economy while these authors study centralized economies. Working in a heterogeneous agent model is, in general, more difficult than in a representative agent model. The reason is that, in general equilibrium models, there may not exist a representative agent who chooses the level of aggregate capital K_t to maximize her intertemporal utility. So, it is not easy to obtain some nice properties like monotonicity and convergence of capital stock (K_t) as in the standard optimal growth theory (see Le Van and Dana (2003); Acemoglu (2009) among others).

Acemoglu and Jensen (2015), Datta et al. (2018) study comparative statics of recursive equilibria. However, intertemporal equilibria in our paper may not be recursive and therefore their methods cannot be directly applied here. It should also be noticed that equilibrium indeterminacy may arise in our model (see Proposition 6 in Section 5).

4 Optimal dividend taxation

When the government raises the tax rate τ, the net dividend $(1 - \tau)\xi_t$ drops but the output increases. By consequence, the consumption of households may increase or decrease. It is worthy to deepen this trade-off by studying the optimal taxation on dividends. To this purpose, we assume that the government chooses $\tau \in [\underline{\tau}, \bar{\tau}] \subset [0, 1]$, where $\underline{\tau}$ and $\bar{\tau}$ are exogenous parameters,[14] in order to maximize the total consumption of households at the steady state.[15]

Let us now define the steady state formally.

[14]The exogenous parameters $\underline{\tau}$ and $\bar{\tau}$ represent political or institutional constraints that we do not microfound in our paper.

[15]Some authors (see Chapter 16 in Ljungqvist and Sargent (2018) for instance) assume that the government's objective function is a positively weighted average of households' intertemporal utilities. In our framework, addressing the issue of optimal dividend taxation with this objective function rises some concerns. The main issue is that, given a tax rate τ, the intertemporal equilibrium may fail to be unique (see Proposition 6 for instance; see also Bosi et al. (2022) and references therein for real indeterminacy in infinite-horizon general equilibrium models with financial constraints). Moreover, finding intertemporal equilibrium is far from trivial because there are heterogeneous agents and borrowing constraints. For these reasons, we look at the steady state because it is unique in general setups. Since the private consumption is an important macroeconomic indicator (Stiglitz et al., 2009), we focus on the total consumption of households.

Definition 3. *Assume that* $\xi_t = \xi > 0, \forall t$.
A steady state is an equilibrium $\Big(q_t, r_t, (c_{i,t}, k_{i,t+1}, a_{i,t})_{i=1}^m, K_t, G_t, T_t\Big)_t$ *such that* $q_t = q, r_t = r$, $c_{i,t} = c_i, k_{i,t} = k_i$ *and* $a_{i,t} = a_i$ *for any* i *and* t, *and* $K_t = K$, $G_t = G$, $T_t = T$ *for any* t.

We provide now sufficient conditions for steady state uniqueness.

Lemma 2. *Let* $\beta_1 > \beta_i$ *for any* $i \geq 2$. *Assume also that* $\xi_t = \xi, \forall t$ *and that* F *is strictly concave with* $F'(0) = \infty$. *Then, there is a unique steady state and it is determined by*

$$1 = \beta_1\Big(f(\tau\xi)F'(K) + 1 - \delta\Big) \tag{10a}$$

$$r = f(\tau\xi)F'(K) \text{ and } q = \frac{(1-\tau)\xi\beta_1}{1-\beta_1} \tag{10b}$$

$$k_1 = K,\ a_1 = 1 \text{ and } c_1 = (r-\delta)K + \theta_1\pi + (1-\tau)\xi \tag{10c}$$

$$a_i = k_i = 0 \text{ and } c_i = \theta_i\pi \text{ for } i = 2, \dots, m \tag{10d}$$

where the profit is given by $\pi = f(\tau\xi)\Big(F(K) - F'(K)K\Big)$.

Proof. See Appendix A.2. □

Since $\beta_1 > \beta_i$ for any $i = 2, \dots, m$, the borrowing constraints of any consumer $i = 2, \dots, m$ are binding. Moreover, condition $f_i < 1, \forall i$, implies that no agent $i = 2, \dots, m$ will invest in physical capital. Hence, the income of any agent $i = 2, \dots, m$ equals their profit share.[16]

Since the aggregate capital level K is determined by (10a) and F is strictly concave, we see that K is uniquely determined. Moreover, we also see that K is increasing in β_1, τ and ξ, and decreasing in δ.

The total consumption of households is $C = (1-\tau)\xi + f(\tau\xi)F(K) - \delta K$. For the sake of simplicity, we consider a Cobb-Douglas production function $F(K) = AK^\alpha$ with $\alpha \in (0,1)$. In this case, we have

$$K = \left(\frac{\alpha A f(\tau\xi)}{\frac{1}{\beta} + \delta - 1}\right)^{\frac{1}{1-\alpha}} \tag{11a}$$

$$C = f(\tau\xi)AK^\alpha - \delta K + (1-\tau)\xi = B_1\Big(Af(\tau\xi)\Big)^{\frac{1}{1-\alpha}} + (1-\tau)\xi \tag{11b}$$

where $B_1 := \alpha^{\frac{\alpha}{1-\alpha}} \dfrac{\frac{1}{\beta_1} - 1 + \delta(1-\alpha)}{\left(\frac{1}{\beta_1} - 1 + \delta\right)^{\frac{1}{1-\alpha}}}$.

For the sake of tractability, we consider $f(G) = (1+bG)^{\alpha_1}$ with $\alpha_1 \in (0,1)$. Here, the parameter b represents the governance quality regarding the productive government spending. In this case, the government's problem writes

$$\max_{\tau \in [\underline{\tau}, \bar{\tau}]} \Big[B_1 A^{\frac{1}{1-\alpha}}(1 + b\xi\tau)^\sigma - \xi\tau\Big] \tag{12}$$

[16] Notice that, when there are at least 2 agents, say 1 and 2, whose rates of time preference are $\beta_1 = \beta_2 > \beta_i$ for any $i = 3, \dots, m$, the aggregate capital stock K remains unique and still determined by (10a) but their income distribution depends on their initial distribution of capital.

where $\sigma := \frac{\alpha_1}{1-\alpha}$. If $\alpha_1 < 1-\alpha$, then, $\sigma < 1$, which implies in turn that the objective function in (12) is strictly concave.[17] By consequence, we obtain the following result.

Proposition 5. *Let $\beta_1 > \beta_i, \forall i \geq 2$. Assume also that $\xi_t = \xi, \forall t$. Let $F(K) = AK^\alpha$ and $f(x) = (1+bx)^{\alpha_1}$ with $\alpha + \alpha_1 < 1$. Then, there are three possibilities.*

1. *If $\sigma b B_1 A^{\frac{1}{1-\alpha}} \geq (1+b\bar{\tau}\xi)^{1-\sigma}$, then $\tau^* = \bar{\tau}$.*
2. *If $\sigma b B_1 A^{\frac{1}{1-\alpha}} \leq (1+b\underline{\tau}\xi)^{1-\sigma}$, then $\tau^* = \underline{\tau}$.*
3. *If $(1+b\underline{\tau}\xi)^{1-\sigma} < \sigma b B_1 A^{\frac{1}{1-\alpha}} < (1+b\bar{\tau}\xi)^{1-\sigma}$, then τ^* is the solution of the following equation $\sigma b B_1 A^{\frac{1}{1-\alpha}} = (1+b\tau\xi)^{1-\sigma}$, i.e.,*

$$\tau^* = \frac{\left(\sigma b B_1 A^{\frac{1}{1-\alpha}}\right)^{\frac{1}{1-\sigma}} - 1}{b\xi}.$$

Comparative statics

Consider the role of parameters b and A that represent the governance quality and the original TFP respectively. Proposition 5 shows that when the governance quality b and TFP A are very high (in the sense of the first point in Proposition 5), the optimal tax rate equals $\bar{\tau}$, the highest affordable tax rate. But, when b and A are low enough, the optimal tax rate equals $\underline{\tau}$ and the government implements the lowest taxation.

The most interesting case corresponds to point 3 in Proposition 5. From the formula of τ^*, we get that:

Corollary 1. *In the third case of Proposition 5, the optimal dividend tax rate τ^* is increasing in β_1, A and b, but decreasing in ξ.*

Our result is closely related to Barro and Sala-i-Martin (1995), Section 4.4.1. There are important differences. First, Barro and Sala-i-Martin assume that the production function of each firm i takes the Cobb-Douglas form: $Y_i = AL_i^{1-\alpha}K_i^\alpha G^{1-\alpha}$, where the government spending equals a fraction of the aggregate output: $G = \tau Y$ (this in turn generates an endogenous growth). By contrast, in Proposition 5, the productive government spending is financed by the dividend tax and the production function has the form: $Y = AK^\alpha(1+bG)^{\alpha_1}$. Second, Barro and Sala-i-Martin investigate the optimal level of τ by maximizing the rate of growth of the balance growth path (i.e., when the growth rates of private consumption, capital and output all equal the same constant) while we maximize the steady-state private consumption. Recall that, in Barro and Sala-i-Martin (1995)'s Section 4.4.1, they find the optimal value $\tau^* = 1-\alpha$.

In our model, we can use (3) to prove that: if $\frac{C_{t+1}}{C_t} = \frac{K_{t+1}}{K_t} = \frac{Y_{t+1}}{Y_t} = \gamma > 0, \forall t$, then we have $\frac{\xi_{t+1}}{\xi_t} = \gamma, \forall t$. It means that the rate of growth of the balanced growth path (if it exists) must equal the rate of growth of dividend. By consequence, in our model with exogenous dividend, it is not relevant to study the optimal dividend taxation along the balanced growth path.

[17] If $\alpha_1 \geq 1-\alpha$, the objective function is convex and the solution becomes either $\underline{\tau}$ or $\bar{\tau}$.

5 Dividend taxation and asset price bubbles

This section investigates the impact of the dividend tax on asset price and bubbles. We allow for non-stationary tax (τ_t). Before starting, a definition of asset price bubble is needed. Since Lemma 1 still holds with non-stationary tax rates τ_t, we have the following asset-pricing equation:

$$q_t = \gamma_{t+1}(q_{t+1} + (1 - \tau_{t+1})\xi_{t+1})$$

where $\gamma_{t+1} := \max_i \dfrac{\beta_i u_i(c_{i,t+1})}{u_i(c_{i,t})}$ is the discount factor of the economy from date t to date $t+1$. Then, by using (5), we can decompose the asset price q_0 (in term of consumption good at the initial date) into two parts:

$$q_0 = \sum_{t=1}^{\infty} Q_t(1 - \tau_t)\xi_t + \lim_{T\to\infty} Q_T q_T$$

where $Q_t := \prod_{s=1}^{t} \gamma_t$ is the discount factor of the economy from the initial date to date t. Following Kocherlakota (1992), Santos and Woodford (1997), we define the fundamental value and bubble of asset.

Definition 4. *$\sum_{t=1}^{\infty} Q_t(1 - \tau_t)\xi_t$ is the asset fundamental value. Bubbles exist at equilibrium if the asset price exceeds the fundamental value: $q_0 > \sum_{t=1}^{\infty} Q_t(1 - \tau_t)\xi_t$.*

Apply the same argument by Montrucchio (2004) and Bosi et al. (2022), we can prove that bubbles exist (i.e., $\lim_{t\to\infty} Q_t q_t > 0$) if and only if $\sum_{t\geq 1} \frac{(1-\tau_t)\xi_t}{q_t} < \infty$. This implies that there is no bubble at the steady state equilibrium studied in Section 4. By consequence, in order to investigate the role of dividend taxation on bubbles, we should not focus on the steady state equilibrium. As recognized by Santos and Woodford (1997), Kocherlakota (2008), and Bosi et al. (2022), it is not easy to find a general equilibrium model with bubbles. We present here a tractable model, inspired by Section 6.1 in Le Van and Pham (2016), where bubbles may arise and look at the role of dividend taxation. We will work under the following assumption.

Assumption 3. *Assume that there are 2 consumers H and F. Let $u_i(c) = ln(c)$, $\beta_i = \beta \in (0, 1)$ and $f_i = 0$ for $i = \{H, F\}$ (so that households cannot borrow). Agents' initial endowments are given by $k_{H,0} = 0$, $a_{H,-1} = 0$, $k_{F,0} > 0$ and $a_{F,-1} = 1$, while their profit shares by:*

$$\left(\theta^H_{2t}, \theta^H_{2t+1}\right) = (1, 0), \quad \left(\theta^F_{2t}, \theta^F_{2t+1}\right) = (0, 1) \quad \forall t \geq 0.$$

We assume that $F(K) = AK + B$, with $A, B > 0$, and $\beta(1 - \delta + f(\bar{\xi})A) \leq 1$ where $\bar{\xi} := \sup_t \xi_t$.[18]

[18] Condition $\beta(1 - \delta + f(\bar{\xi})A) \leq 1$ ensures that FOCs are satisfied. This and condition (8) are not mutually exclusive since (8) implies $K_t > 0$.

Notice that $F_g(G_{t-1}, K_t) = f(\tau_{t-1}\xi_{t-1})(AK_t + B)$ and the firm's profit equals $\pi_t = f(\tau_{t-1}\xi_{t-1})B$ for any t.

Let us now construct an equilibrium. The allocation of consumer H is given by

$$k_{H,2t} = 0, a_{H,2t-1} = 0 \tag{13a}$$
$$c_{H,2t-1} = (1-\delta+r_{2t-1})K_{2t-1} + q_{2t-1} + (1-\tau_{2t-1})\xi_{2t-1} \tag{13b}$$
$$k_{H,2t+1} = K_{2t+1}, a_{H,2t} = 1 \tag{13c}$$
$$c_{H,2t} = \pi_{2t} - K_{2t+1} - q_{2t} \tag{13d}$$

while the allocation of consumer F by

$$k_{F,2t} = K_{2t}, a_{F,2t} = 1 \tag{14a}$$
$$c_{F,2t-1} = \pi_{2t-1} - K_{2t} - q_{2t-1} \tag{14b}$$
$$k_{F,2t+1} = 0, a_{F,2t} = 0 \tag{14c}$$
$$c_{F,2t} = (1-\delta+r_{2t})K_{2t} + q_{2t} + (1-\tau_{2t})\xi_{2t}. \tag{14d}$$

Prices and the aggregate capital solve the following system: for any t,

$$K_{t+1} + q_t = \frac{\beta}{1+\beta}(F_t(K_t) - r_tK_t) = B_t \tag{15a}$$
$$q_{t+1} + (1-\tau_{t+1})\xi_{t+1} = q_t(r_{t+1} + 1 - \delta) \tag{15b}$$
$$q_t > 0, \quad K_t > 0 \tag{15c}$$
$$p_t = 1 \text{ and } r_t = f(\tau_{t-1}\xi_{t-1})A, \text{ where } B_t := \frac{\beta f(\tau_{t-1}\xi_{t-1})B}{1+\beta} \tag{15d}$$

By using Lemma 3 in Appendix A.6, we can prove that any sequence of allocations and prices satisfying the above conditions is an equilibrium.

The asset's fundamental value equals $FV := \sum_{s=1}^{\infty}(1-\tau_s)\xi_sQ_s$ where

$$Q_s := \frac{1}{(1-\delta+f(\tau_0\xi_0)A)\cdots(1-\delta+f(\tau_{s-1}\xi_{s-1})A)}$$

is the discount factor of the economy.

We see that the fundamental value FV is decreasing in τ_t for any t. Moreover, FV is not monotonic in dividend ξ_t. Note that, in Le Van and Pham (2016) (Section 6.1), the fundamental value of the asset is increasing in dividends. This difference is from the fact that the interest rates and discount factors in our economy depend on dividends through the productive government spending[19] while, in Le Van and Pham

[19]Indeed, given ξ_t, we write

$$FV = \Big(\sum_{s=1}^{t-1}(1-\tau_s)\xi_sQ_s\Big) + (1-\tau_t)\xi_tQ_t + \frac{\sum_{s=t+1}^{\infty}(1-\tau_s)\xi_s\frac{Q_s}{Q_{t+1}}}{(1-\delta+f(\tau_0\xi_0)A)\cdots(1-\delta+f(\tau_t\xi_t)A)}.$$

The first term does not depend on ξ_t. The second term increases in ξ_t but the last term decreases in ξ_t.

(2016), these variables do not depend on dividends. Notice that if $\tau_t = 0\ \forall t$, we recover Le Van and Pham (2016).

To find an equilibrium, we have to find a sequence $(K_{t+1}, q_t)_{t\geq 0}$ satisfying the system (15a, 15b, 15c). To do so, we choose $q_0 \geq FV$ and $(q_t)_{t\geq 1}$ such that

$$q_0 = \sum_{s=1}^{t}(1-\tau_s)\xi_s Q_s + q_t Q_t \tag{16a}$$

$$q_t < \frac{\beta f(\tau_{t-1}\xi_{t-1})B}{1+\beta}. \tag{16b}$$

Condition (16b) ensures that $K_{t+1} > 0$ while condition $q_0 \geq FV$ implies that $q_t > 0$ for any t. Then, such a sequence $(q_t)_{t\geq 0}$ is a sequence of equilibrium prices because it satisfies the system (15a, 15b, 15c). In this case, a bubble exists when $q_0 > FV$. Summing up, we obtain the following result.

Proposition 6. *Let Assumption 3 be satisfied. Any sequence (q_t) with $q_0 \in [FV, B_0)$ and $(q_t)_{t\geq 1}$ satisfying (16a, 16b) is a sequence of equilibrium asset price. Moreover, there are two cases:*
(1) If $q_0 = FV$, then the equilibrium is bubbleless.
(2) If $q_0 > FV$, then the equilibrium is bubbly.

Our result is also related to the seminal paper Tirole (1985) where he shows that there may be a continuum of bubbly equilibria but the difference is that he works in an overlapping generations model while we consider an infinite-horizon general equilibrium model.

Let us provide some implications of Proposition 6.

- **Asset bubble and dividend taxes**. Since $q_0 \geq FV$, Proposition 6 indicates that FV is the minimum level above which q_0 is an equilibrium price with bubbles. Since FV is decreasing in each τ_t, we can say that bubbles are more likely to appear when sequence of tax τ_t increases. The intuition is that, when the tax rates τ_t increases, the after-tax dividend $(1-\tau_t)\xi_t$ decreases and the financial asset fundamental value may turn out to be lower than its price. In this case, an asset bubble arises.

- **Asset price and dividend taxes**. In Proposition 6, let $q_0 = FV + \bar{d}$ with $\bar{d} \in [0, B_0 - FV)$, and then there is an asset price bubble. According to (16a), we can compute the asset price at date t as follows

$$\begin{aligned} q_t = &\Big((1-\delta+f(\tau_0\xi_0)A\Big)\cdots\Big(1-\delta+f(\tau_{t-1}\xi_{t-1})A\Big)\bar{d} \\ &+ \sum_{s=t+1}^{\infty}\frac{(1-\tau_s)\xi_s}{(1-\delta+f(\tau_t\xi_t)A)\cdots(1-\delta+f(\tau_{s-1}\xi_{t-1})A)} \end{aligned} \tag{17}$$

  Observe that q_t is increasing τ_s for any $s \leq t-1$ but decreasing in τ_s for any $s \geq t$.

6 Conclusion

We have shown that low productivity entails recessions at infinitely many dates. However, when the government taxes asset dividends and spends this fiscal revenue to finance productive government spending, the productivity of firms is enhanced, and hence, recessions may be avoided. This happens if: (1) the productive government spending is productive enough and (2) dividends are high. We have also investigated the optimal dividend taxation. When the government's objective function is the total consumption of households at the steady state, the optimal level of dividend taxation increases in the firm's productivity and the governance quality but decreases in the size of dividends. We have presented a simple model where there is room for equilibrium indeterminacy. In this model, asset price bubbles are more likely to exist if dividend taxes increase.

We conclude our paper by outlining some avenues for future research. Firstly, in our current model, R&D and human capital are not treated separately. Therefore, it would be interesting is to endogenize human capital, as suggested by Lucas (1988), and incorporate it into our model. Then, we use this multi-sector growth model to investigate (1) the interaction between productive government spending (partially financed by dividend taxation), physical and human capital accumulation, and (2) how this interaction affects the economic development.[20]

Secondly, we have considered exogenous dividends. It would be valuable to extend our analysis for the case of endogenous dividends.[21] In such a case, imposing a dividend tax may affect corporate investment (Alstadsaeter et al., 2017). However, productive government spending (financed by dividend tax revenues) may improve the firms' productivity. So, it is important to study the optimal dividend taxation in this context.

A Appendix: Formal proofs

A.1 Proof of Proposition 1: The existence of equilibrium.

We consider the intermediate economy $\tilde{\mathcal{E}}$ as the economy $\mathcal{E}$ but the asset dividend is $\tilde{\xi}_t := (1-\tau)\xi_t$, the production function $\tilde{F}_t$ is by $\tilde{F}_t(K) := F_g(\tau\xi_{t-1}, K)$, and the government is not taken into account. According to Le Van and Pham (2016), there exists an equilibrium $\left(\tilde{p}_t, \tilde{q}_t, \tilde{r}_t, (\tilde{c}_{i,t}, \tilde{k}_{i,t+1}, \tilde{a}_{i,t})_{i=1}^m, \tilde{K}_t\right)_{t=0}^{\infty}$ of the economy $\tilde{\mathcal{E}}$, i.e., the following conditions hold:

1. $\tilde{p}_t, \tilde{q}_t, \tilde{r}_t > 0$ for $t \geq 0$.

2. For any $t \geq 0$, we have $\sum_{i=1}^{m}(\tilde{c}_{i,t} + \tilde{k}_{i,t+1} - (1-\delta)\tilde{k}_{i,t}) = \tilde{F}_t(\tilde{K}_t) + (1-\tau)\xi_t$,

[20] It would be relevant to introduce ideas in this multi-sector growth model by using the approach in Section 4 of Jones (2005).

[21] We can follow Kamihigashi (2008), Gourio and Miao (2010), Gourio and Miao (2011) to model endogenous dividends

$$\tilde{K}_t = \sum_{i=1}^{m} \tilde{k}_{i,t}, \quad \text{and } \sum_{i=1}^{m} \bar{a}_{i,t} = 1.$$

3. Optimal consumption plans: for any i, $(\tilde{c}_{i,t}, \tilde{k}_{i,t+1}, \tilde{a}_{i,t})_{t=0}^{\infty}$ is a solution of the problem $(P_i(\tilde{p}, \tilde{q}, \tilde{r}))$.

4. Optimal production plan: for any $t \geq 0$, $\bar{K}_t$ is a solution of the following problem

$$\max_{K_t \geq 0} \Big[\tilde{p}_t \tilde{F}_t(\tilde{K}_t) - \tilde{r}_t \tilde{K}_t\Big]. \tag{A.1}$$

It is easy to see that $\Big(\tilde{p}_t, \tilde{q}_t, \tilde{r}_t, (\tilde{c}_{i,t}, \tilde{k}_{i,t+1}, \tilde{a}_{i,t})_{i=1}^{m}, \tilde{K}_t, G_t, T_t\Big)_{t=0}^{\infty}$, where $G_t = \tau\xi_t, T_t = \tau\xi_t$, is an equilibrium the economy $\mathcal{E}$.

A.2 Proof of Lemma 2

Let $\Big(q, r, (c_i, k_i, a_i)_{i=1}^{m}, K, G, T\Big)$ be a steady state equilibrium. By FOCs, there exists multipliers $m_i \geq 0$, and $n_i \geq 0$ such that

$$1 = (r + 1 - \delta)(\beta_i + f_i m_i) + n_i \tag{A.2a}$$

$$q = (q + (1 - \tau)\xi)(\beta_i + m_i) \tag{A.2b}$$

$$k_i n_i = 0, \quad m_i\Big((q + (1 - \tau)\xi)a_i + f_i(1 - \delta + r)k_i\Big) = 0. \tag{A.2c}$$

According to (A.2b) and $\beta_1 > \beta_i$ for any $i \geq 2$, we have $m_1 = 0$ and $m_i > 0$ for any $i \geq 2$ which implies that $(q + (1 - \tau)\xi)a_i + f_i(1 - \delta + r)k_i = 0$ for any $i \geq 2$.

Since $F'(0) = \infty$, we have $r + 1 - \delta = \frac{q+(1-\tau\xi)}{q} = \frac{1}{\beta_i + m_i}$. According to (A.2a), we obtain that, for any i,

$$1 = \frac{\beta_i + f_i m_i}{\beta_i + m_i} + y_i \tag{A.3}$$

For each $i \geq 2$, since $m_i > 0$, and $f_i < 1$, we obtain that $n_i > 0$. Therefore, we get that $k_i = 0$, and hence $a_i = 0$ for each $i \geq 2$. So, we can compute $c_i = \theta_i \pi$ for each $i \geq 2$.

Since $F'(0) = \infty$, we have $K > 0$ which implies that $k_1 = K > 0$. According to (A.2a), we see that K is determined by

$$1 = \Big(f(\xi\tau)F'(K) + 1 - \delta\Big)\beta_1. \tag{A.4}$$

It is now easy to obtain that $a_i = 1$ and $c_1 = (r - \delta)K + \theta_1 \pi + (1 - \tau)\xi$.

A.3 Proof of Proposition 2

We firstly claim that there exists an infinite increasing sequence $(t_n)_{n=0}^{\infty}$ such that $q_{t_n} + (1 - \tau)\xi_{t_n} > q_{t_n - 1}$ for every $n \geq 0$. Indeed, if not, there exists t_0 such that

$q_{t+1} + (1-\tau)\xi_{t+1} \leq q_t$ for every $t \geq t_0$. Combining with $\xi_t \geq \underline{\xi}, \forall t \geq 0$ and by using induction argument, we can easily prove that

$$q_{t_0} \geq q_{t+t_0} + t(1-\tau)\underline{\xi}$$

for every $t \geq 0$. Let $t \to \infty$, we have $q_{t_0} = \infty$, then a contradiction is reached.[22]

Therefore, there exists a sequence (t_n) such that for every $n \geq 0$, $q_{t_n} + (1-\tau)\xi_{t_n} > q_{t_n-1}$. By consequence, we have

$$\frac{q_{t_n} + (1-\tau)\xi_{t_n}}{q_{t_n-1}} > 1 \geq f(\tau\bar{\xi})F'(\bar{k}) + 1 - \delta.$$

where the last inequality is from assumption: $f(\tau\bar{\xi})F'(\bar{k}) \leq \delta$

We now claim that $K_{t_n} \leq \bar{k}$ for any n. Indeed, if $K_{t_n} > \bar{k}$, then $K_{t_n} > 0$. According to Lemma 1, we see that

$$\frac{q_{t_n} + (1-\tau)\xi_{t_n}}{q_{t_n-1}} = f(\tau\xi_{t_n})F'(K_{t_n}) + 1 - \delta \leq f(\tau\bar{\xi})F'(\bar{k}) + 1 - \delta.$$

This is a contradiction. Therefore, $K_{t_n} \leq \bar{k}$ for any n.

A.4 Proof of Proposition 3

Point 1. If $K_{t+1} = 0$, we have

$$\begin{aligned}
\sum_{i=1}^{m} c_{i,t} &= F_g(G_{t-1}, K_t) + (1-\delta)K_t + (1-\tau)\xi, \\
\sum_{i=1}^{m} c_{i,t+1} + K_{t+2} &= (1-\tau)\xi.
\end{aligned}$$

Therefore, we have

$$\sum_{i=1}^{m} c_{i,t} \geq F_g(G_{t-1}, K_t) + (1-\tau)\xi \geq (1-\tau)\xi \geq \sum_{i=1}^{m} c_{i,t+1}. \tag{A.5}$$

Consequently, there exists $i \in \{1, \dots, m\}$ such that $c_{i,t} \geq c_{i,t+1}$. This implies that $u_i'(c_{i,t+1}) \geq u_i'(c_{i,t})$, and, hence,

$$\frac{1}{f(\tau\xi)F'(0) + 1 - \delta} \geq \max_j \frac{\beta_j u_j'(c_{j,t+1})}{u_j'(c_{j,t})} \geq \frac{\beta_i u_i'(c_{i,t+1})}{u_i'(c_{i,t})} \geq \beta_i$$

Thus, $1 \geq \Big(f(\tau\xi)F'(0) + 1 - \delta\Big)\beta_i$, which is a contradiction. By consequence, we have $K_{t+1} > 0$.

[22] Our result is still valid if the condition "$\xi_t \geq \xi > 0$ for every $t \geq 0$" is replaced by a weaker one "$\sum_{t=0}^{\infty} \xi_t = \infty$".

Point 2. We see that

$$\frac{\beta_i u_i'(c_{i,t})}{u_i'(c_{i,t-1})} \leq \max_j \frac{\beta_j u_j'(c_{j,t})}{u_j'(c_{j,t-1})} \leq \frac{1}{f(\tau\xi)F'(K_t) + 1 - \delta}.$$

Let us denote $B_i(\tau\xi, K_t) := \Big(f(\tau\xi)F'(K_t) + 1 - \delta\Big)\beta_i$. The above inequality implies that $c_{i,t} \geq y_i\Big(B_i(\tau\xi, K_t)\Big)c_{i,t-1}$ for any i, where the function $y_i(\cdot)$ is defined in Assumption 2.

Denote

$$x(\tau\xi, K_t) := \min_i \Big\{y_i\Big(B_i(\tau\xi, K_t)\Big)\Big\}.$$

Notice that $x(\tau\xi, K_t)$ is increasing in $f(\tau\xi)$ and $\tau\xi$ but decreasing in K_t.

According to Assumption 2, there exists ξ^* such that $x(\tau\xi, 0) > 1$ for any $\xi \geq \xi^*$. In the following we focus on the case where $\xi \geq \xi^*$.

Since $c_{i,t} \geq y_i(B_i(\tau\xi, K_t))c_{i,t-1} \geq x(\tau\xi, K_t)c_{i,t-1}$ for any i. Thus, we have $C_t \geq x(\tau\xi, K_t)C_{t-1}$. By market clearing conditions, we have

$$C_{t-1} + K_t = f(G_{t-2})F(K_{t-1}) + (1-\delta)K_{t-1} + (1-\tau)\xi \tag{A.6a}$$

$$C_t + K_{t+1} = f(G_{t-1})F(K_t) + (1-\delta)K_t + (1-\tau)\xi. \tag{A.6b}$$

By consequence, condition $C_t \geq x(\tau\xi, K_t)C_{t-1}$ implies that

$$\begin{aligned}&f(\tau\xi)F(K_t) + (1-\delta)K_t + (1-\tau)\xi \\ &\geq C_t \geq x(\tau\xi, K_t)C_{t-1} = x(\tau\xi, K_t)\Big(f(\tau\xi)F(K_{t-1}) + (1-\delta)K_{t-1} + (1-\tau)\xi - K_t\Big).\end{aligned}$$

From this, we have

$$\begin{aligned}\frac{f(\tau\xi)F(K_t) + (1-\delta)K_t + (1-\tau)\xi}{x(\tau\xi, K_t)} + K_t &\geq f(\tau\xi)F(K_{t-1}) + (1-\delta)K_{t-1} + (1-\tau)\xi \\ &\geq (1-\tau)\xi.\end{aligned}$$

Thus, we get

$$\frac{f(\tau\xi)F(K_t)}{x(\tau\xi, K_t)} + \frac{(1-\delta)K_t}{x(\tau\xi, K_t)} + K_t - (1-\tau)\xi\frac{x(\tau\xi, K_t) - 1}{x(\tau\xi, K_t)} \geq 0. \tag{A.7}$$

By definition of the function x, we see that $x(\tau\xi, K_t)$ decreases in K_t, and so does $\dfrac{x(\tau\xi, K_t) - 1}{x(\tau\xi, K_t)}$. Hence, the left hand side of (A.7) is an increasing function of K_t. By consequence, we have that: for each $\xi \geq \xi^*$, there exists a unique $K(\xi)$ such that

$$\frac{f(\tau\xi)F(K(\xi))}{x(\tau\xi, K(\xi))} + \frac{(1-\delta)K(\xi)}{x(\tau\xi, K(\xi))} + K(\xi) = (1-\tau)\xi\frac{x(\tau\xi, K(\xi)) - 1}{x(\tau\xi, K(\xi))}, \tag{A.8}$$

and, hence, condition (A.7) implies that $K_t \geq K(\xi)$.

Let ξ tend to infinity. We prove that $\lim\limits_{\xi\to\infty} K(\xi) = \infty$. Suppose $M = \sup_\xi K(\xi) < \infty$. By definition of the functions x and y_i, we have that $\lim_{\xi\to\infty} x(\tau\xi, K(\xi)) = \infty$. Thus,

$$\lim_{\xi\to\infty}(1-\tau)\xi\frac{x(\tau\xi, K(\xi))-1}{x(\tau\xi, K(\xi))} = \infty \tag{A.9}$$

$$\lim_{\xi\to\infty}\frac{(1-\delta)K(\xi)}{x(\tau\xi, K(\xi))} + K(\xi) < \infty \tag{A.10}$$

Using the assumption $\lim_{a\to\infty}\frac{a}{\min_i\{y_i(\beta_i a)\}} < \infty$, we have

$$\begin{aligned}
&\lim_{\xi\to\infty}\frac{f(\tau\xi)}{x(\tau\xi, K(\xi))} \leq \lim_{\xi\to\infty}\frac{f(\tau\xi)}{x(\tau\xi, M)} = \lim_{\xi\to\infty}\frac{f(\tau\xi)}{\min\limits_i\left\{y_i\Big(\big(f(\tau\xi)F'(M)+1-\delta\big)\beta_i\Big)\right\}}\\
&= \lim_{\xi\to\infty}\frac{f(\tau\xi)}{f(\tau\xi)F'(M)+1-\delta}\frac{f(\tau\xi)F'(M)+1-\delta}{\min\limits_i\left\{y_i\Big(\big(f(\tau\xi)F'(M)+1-\delta\big)\beta_i\Big)\right\}}\\
&= \frac{1}{F'(M)}\lim_{a\to\infty}\frac{a}{\min\limits_i\{y_i\big(a\beta_i\big)\}} < \infty
\end{aligned} \tag{A.11}$$

Combining (A.9-A.11), we see that the left hand side of (A.8) is bounded while the right hand side tends to infinity when ξ tends to infinity, which is a contradiction. By consequence, we get that $\lim\limits_{\xi\to\infty} K(\xi) = \infty$. So, there exists $\xi_{\bar{k}} > 0$ such that $K(\xi_{\bar{k}}) > \bar{k}$ for any t. Therefore, $K_t > \bar{k}$ for any t.

A.5 Proof of Proposition 4

We see that

$$\frac{\beta_i u_i'(c_{i,t})}{u_i'(c_{i,t-1})} \leq \max_j \frac{\beta_j u_j'(c_{j,t})}{u_j'(c_{j,t-1})} \leq \frac{1}{f(\tau\xi_{t-1})F'(K_t)+1-\delta} \leq \frac{1}{f(\tau\underline{\xi})A+1-\delta}$$

where the last inequality comes from the fact that $F'(K) \geq A$ for any K.

Let us denote $B_i := \big(f(\tau\underline{\xi})A + 1 - \delta\big)\beta_i$. The above inequality implies that $B_i u_i'(c_{i,t}) \leq u_i'(c_{i,t-1})$. According to Assumption 2, we get that $c_{i,t} \geq x_i c_{i,t-1}$ for any i, where $x_i := y_i(B_i)$. Denote $x := \min\limits_i y_i(B_i)$. Using the same argument in the proof of Proposition 3, we have

$$\frac{f(\tau\xi_{t-1})F(K_t) + (1-\delta)K_t + (1-\tau)\xi_t}{x} + K_t \geq f(\tau\xi_{t-2})F(K_{t-1}) + (1-\delta)K_{t-1} + (1-\tau)\xi_{t-1}$$

which implies that

$$\begin{aligned}
f(\tau\xi_{t-1})F(K_t) + (1-\delta)K_t + xK_t &\geq xf(\tau\xi_{t-2})F(K_{t-1}) + x(1-\delta)K_{t-1} + (1-\tau)(x\xi_{t-1}-\xi_t)\\
&\geq x\Big(f(\tau\xi_{t-2})F(K_{t-1}) + (1-\delta)K_{t-1}\Big)
\end{aligned} \tag{A.12}$$

where the last inequality is based on condition $x\xi_{t-1} - \xi_t \geq 0$.

Denote $\lambda_t := \frac{1-\delta+x+A}{1-\delta+Af(\tau\xi_{t-1})}$. We can verify that $\frac{1}{A} = \frac{\lambda_t f(\tau\xi_{t-1})-1}{1-\delta+x-\lambda_t(1-\delta)}$. According to our assumption $x(Af(\tau\underline{\xi}) - \delta) > 1 - \delta + A$, we have $\lambda f(\tau\underline{\xi}) - 1 > 0$ which implies that $\lambda f(\tau\xi_{t-1}) - 1 > 0$ and $1 - \delta + x - \lambda(1-\delta) > 0$.

Since $F(K) \geq F'(K)K \geq AK$, we have $K_t \leq \frac{F(K_t)}{A}$. By consequence, we can check that

$$f(\tau\xi_{t-1})F(K_t) + (1-\delta)K_t + xK_t < \lambda_t\Big(f(\tau\xi_{t-1})F(K_t) + (1-\delta)K_t\Big). \tag{A.13}$$

Combining (A.12) and (A.13), we get that

$$\frac{f(\tau\xi_{t-1})F(K_t) + (1-\delta)K_t}{f(\tau\xi_{t-2})F(K_{t-1}) + (1-\delta)K_{t-1}} \geq \frac{x\Big(1-\delta+Af(\tau\xi_{t-1})\Big)}{1-\delta+x+A} \geq \frac{x\Big(1-\delta+Af(\tau\underline{\xi})\Big)}{1-\delta+x+A}.$$

Since $x(Af(\tau\underline{\xi}) - \delta) > 1 - \delta + A$, we have $\frac{x\Big(1-\delta+Af(\tau\underline{\xi})\Big)}{1-\delta+x+A} > 1$. By consequence, we get $\lim_{t\to\infty} f(\tau\xi_{t-1})F(K_t) + (1-\delta)K_t = \infty$. Since $\sup_t \xi_t < \infty$, we obtain that $\lim_{t\to\infty} K_t = \infty$.

A.6 A sufficient condition for the equilibrium

Let us denote $I := \{1, 2, \ldots, m\}$. We give sufficient conditions for a sequence

$$\Big(p_t, q_t, r_t, (c_{i,t}, k_{i,t+1}, a_{i,t})_{i\in I}, K_t, G_t, T_t\Big)_t$$

to be an equilibrium. This result is used in our model with bubble in Section 5. Notice that the utility may satisfy $u_i(0) = -\infty$.

Lemma 3. *Let $f_i = 0$ for any i (i.e., consumers cannot borrow).*

A sequence $\Big(p_t, q_t, r_t, (c_{i,t}, k_{i,t+1}, a_{i,t})_{i\in I}, K_t, G_t, T_t\Big)_t$ is an equilibrium, if the sequence $\Big(p_t, q_t, r_t, (c_{i,t}, k_{i,t+1}, a_{i,t}, \zeta_{i,t}, \epsilon_{i,t})_{i\in I}, K_t, G_t, T_t\Big)_t$ satisfies the following conditions.

(i) For any i and t, $c_{i,t} > 0$, $k_{i,t+1} \geqslant 0$, $a_{i,t} \geqslant 0$, $\zeta_{i,t} \geqslant 0$ and $\epsilon_{i,t} \geqslant 0$.

For any t, $p_t = 1$, $q_t > 0$ and $r_t > 0$.

(ii) The first-order conditions:

$$\frac{1}{r_{t+1} + 1 - \delta} = \frac{\beta_i u_i'(c_{i,t+1})}{u_i'(c_{i,t})} + \zeta_{i,t}, \quad \frac{q_t}{q_{t+1} + (1-\tau_{t+1})\xi_{t+1}} = \frac{\beta_i u_i'(c_{i,t+1})}{u_i'(c_{i,t})} + \epsilon_{i,t}$$

with $\zeta_{i,t}k_{i,t+1} = 0$ and $\epsilon_{i,t}a_{i,t} = 0$.

(iii) The transversality conditions: $\lim_{t\to\infty} \beta_i^t u_i'(c_{i,t})k_{i,t+1} = \lim_{t\to\infty} \beta_i^t u_i'(c_{i,t})q_t a_{i,t} = 0$.

(iv) For any t, $F_g(G_{t-1}, K_t) - r_t K_t = \max_{K:K\geq 0}\{F_g(G_{t-1}, K) - r_t K\}$.

(v) For any t, $c_{i,t} + k_{i,t+1} - (1-\delta)k_{i,t} + q_t a_{i,t} = r_t k_{i,t} + (q_t + (1-\tau_t)\xi_t)a_{i,t-1} + \theta_t^i \pi_t$ where $\pi_t = F_g(G_{t-1}, K_t) - r_t K_t$.

(vi) *For any* t, $K_t = \sum_{i \in I} k_{i,t}$, $\sum_{i \in I} a_{i,t} = 1$.

(vii) *For any* t, $G_t = T_t = (1 - \tau_t)\xi_t$.

Proof. It suffices to prove the optimality of households. This can be done by using the standard argument in dynamic programming. $\square$

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