Numerical Simulation of Reaching a Steady State: Effects of Economic Rents on Development of Economic Inequality

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Abstract

It is not easy to numerically simulate the path to a steady state because there is no closed form solution in dynamic economic growth models in which households behave generating rational expectations. In contrast, it is easy if households are supposed to behave under the MDC (maximum degree of comfortability)-based procedure to reach a steady state where MDC indicates the state at which a household feels most comfortable with its combination of incomes and assets. In this paper, I simulate the development of economic inequality when households obtain economic rents heterogeneously and behave under the MDC-based procedure. The results of simulations indicate that if a government does not intervene, the level of economic inequality continues to increase, but if it intervenes appropriately, households can reach a stabilized (steady) state that is not approximately but "purely" optimal in the sense that they can feel they are at MDC.

JEL Classification: C60, D63, E10, H30, I30

Keywords: Balanced growth path; Economic rent; Economic inequality; Government transfer; Heterogeneity; Simulation; Steady state

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1 INTRODUCTION

It is not easy to numerically simulate the path to a steady state in dynamic economic growth models in which households behave generating rational expectations because there is no closed form solution in these models. This difficulty raises an important question about the assumption of behaviors generating rational expectations because to generate rational expectations, ordinary households generally have to do something equivalent to computing complex non-linear dynamic macro-econometric models in their everyday lives.

Harashima (2018a[1]) showed an alternative procedure for households to reach a steady state. With this procedure, households maintain their capital-wage ratio (CWR) at the maximum degree of comfortability (MDC) where MDC indicates the state at which a household feels most comfortable with its combination of incomes and assets. He showed that the behavior of households based on rational expectations (i.e., the behavior under the RTP [rate of time preference]-based procedure) is equivalent to that under the MDC-based procedure (Harashima 2018a, 2021a, 2022a[2]). Furthermore, Harashima (2018a) showed that if preferences of households are heterogeneous under the MDC-based procedure, there is no guarantee of a steady state as with the case of the RTP-based procedure (Becker 1980; Harashima 2010, 2012). However, Harashima (2010[3], 2012[4], 2014) also showed that there is a state in which all optimality conditions of all heterogeneous households are satisfied (sustainable heterogeneity, SH) even if the abovementioned heterogeneities exist. Although SH cannot necessarily be naturally achieved, it can be achieved if a government intervenes appropriately.

Unlike the case of the RTP-based procedure, the path to a steady state will easily be simulated if we suppose that households behave under the MDC-based procedure because households are not required to do something equivalent to computing complex models. Indeed, Harashima (2022c) numerically simulated the path to a steady state under the MDC-based procedure and showed that households can reach a stabilized (steady) state without generating any rational expectations as predicted theoretically (Harashima, 2010, 2012, 2014). Furthermore, Harashima (2022c) numerically showed that a government can achieve a stabilized (steady) state by appropriately intervening even if households behave unilaterally in the sense that they do not consider the optimality of other households, although heterogeneous households cannot necessarily reach their intrinsic CWRs at MDC (this state is called the "approximate SH").

[1] Harashima (2018a) is also available in Japanese as Harashima (2019).
[2] Harashima (2022a) is also available in Japanese as Harashima (2022b).
[3] Harashima (2010) is also available in Japanese as Harashima (2017b).
[4] Harashima (2012) is also available in Japanese as Harashima (2020a).

However, in numerical simulations undertaken in Harashima (2022c), economic rents are not considered. Harashima (2016[5], 2017a, 2018c[6], 2018d, 2020b, 2021a) theoretically showed that economic rents derived from ranking preference and values in addition to those from exploitative contracting (Harashima, 2020b) play an important role in the development of economic inequality. That is, if economic rents are obtained heterogeneously among households and if the government does not intervene, the level of economic inequality will continue to increase as in the case of heterogeneous preferences across households, as shown theoretically in Becker (1980) and Harashima (2010, 2012, 2014) and numerically in simulations in Harashima (2022c). In addition, Harashima (2021c) showed that the heterogeneous success rates of investment play a similar role as economic rents in the sense that they make households' capital accumulations heterogeneous and cause continuous increases in economic inequality.

The purpose of this paper is to numerically simulate the development of economic inequality when households obtain economic rents heterogeneously and behave under the MDC-based procedure. In the simulations, a household was set to increase or decrease its consumption according to simple formulae that are supposed to well capture and represent a household's behaviors under the MDC-based procedure. The results of simulations indicate that if only some households obtain economic rents and a government does not intervene, no stabilized (steady) state can be achieved as predicted theoretically (Harashima, 2020b, 2021a), and the level of economic inequality continues to increase. This nature does not change if economic rents are given every period or intermittently or if they are given deterministically or randomly. However, if a government intervenes appropriately (i.e., it appropriately transfers money from households that obtain economic rents to those that do not), a stabilized (steady) state can be achieved also as predicted theoretically (Harashima, 2020b, 2021a).

An important difference between the cases of heterogeneous preferences (CWRs at MDC) and economic rents is that only approximate SHs can exist in the former case, but a "pure" SH can exist in the latter case in the sense that all heterogeneous households can satisfy their intrinsic CWRs at MDC. Nevertheless, approximate SHs also exist in a population with heterogeneous economic rents. Because there is no mechanism for a government to be forced to select only a "pure" SH, it can select from a wide range of approximate SHs.

2 ECONOMIC RENTS

[5] Harashima (2016) is also available in Japanese as Harashima (2018b).
[6] Harashima (2018c) is also available in Japanese as Harashima (2021b).

2.1 Economic rents derived from ranking preference and mistakes in business dealings

Harashima (2016, 2018c) showed the existence of a type of economic rent that had not been discussed previously: monopoly profits (rents) derived from people's ranking preferences. These rents enable some individuals to be superstars in the worlds of sport, art, or music (Harashima, 2016, 2018c), and enable some corporate executives to earn extremely high compensations (Harashima, 2018d). Ranking preference is an important element in product differentiation that allows companies to accrue large amounts of monopoly rent (Harashima, 2017a). As a result, product differentiation is one of the most important strategies a company uses to prosper (Porter, 1980, 1985), and monopoly rents derived from product differentiation owing to ranking preference are highly likely to be found in economies.

Furthermore, Harashima (2020b) discussed the importance of another kind of economic rent, which arises from heterogeneity in mistakes made in business. Here, a "mistake" means, for example, that a household purchases a product at a price that is higher than the cost to produce it plus a normal margin, or that a worker accepts a wage that is lower than their marginal productivity would indicate is appropriate. Harashima (2020b) showed that because heterogeneity certainly exists in people's ability to make fewer mistakes in business dealings, economic rents derived from mistakes probably exist ubiquitously and on a large scale across an economy.

2.2 Family lines

Family lines consist of households that are descended from common ancestors and therefore share similar traits. In addition, in accordance with local customs and various other reasons, many groups of people mostly marry within the same or similar groups. Therefore, it is highly likely that abilities (e.g., those related to obtaining economic rents) are exogenously and unevenly given (Harashima, 2020c, 2020d). Therefore, the average abilities of people in a given group (or family line) will remain different from those in other groups (Harashima, 2020c, 2020d). This means that there are groups (or family lines) that indefinitely obtain persistent economic rents. At the same time, there are groups (or family lines) that are indefinitely exploited because of these persistent economic rents. As a result, many economic rents will be enjoyed persistently by only a small number of households and family lines; that is, the persistent economic rents will be distributed very unevenly.

3 SIMULATION METHOD

Simulations in this paper are undertaken on the basis of the SH concepts presenting in Harashima (2010, 2012, 2014) and the MDC-based procedure developed in Harashima (2018a, 2021a, 2022a). These concepts are briefly summarized in Appendixes 1 and 2. The method of simulations is basically the same as that used in Harashima (2022c), which is explained in Appendix 3, but it was slightly modified to include the effect of economic rents.

3.1 Basic simulation assumptions

No technological progress and capital depreciation are assumed, and all values are expressed in real and per capita terms. It is assumed that there are H economies in a country, the number of households in each of economy is identical, and households within each economy are identical. The production function of Economy i ($1 \leq i \leq H$) is

$$y_{i,t} = \omega_i A_t^{\alpha} k_{i,t}^{1-\alpha} \ , \tag{1}$$

where $y_{i,t}$ and $k_{i,t}$ are the production and capital of a household in Economy i in period t, respectively; ω_i is the productivity of a household in Economy i; A_t is technology in period t; and α ($0 < \alpha < 1$) is a constant and indicates the labor share. All variables are expressed in per capita terms. In simulations, I set $\alpha = 0.65$, $A_t = 1$, and $\omega_i = 1$ for any t and i. The initial capital a household owns is set at 1 for any household.

By equation (1), the production of a household in Economy i in period t ($y_{i,t}$) is calculated, for any i, by

$$y_{i,t} = k_{i,t}^{1-\alpha} \ .$$

The amount of capital used (not owned) by each household (i.e., $k_{i,t}$) is kept identical among households although the amount of capital owned (not used) by each household can be heterogeneous. For any i,

$$k_{i,t} = \frac{\sum_{i=1}^{H} \check{k}_{i,t}}{H} \ ,$$

where $\check{k}_{i,t}$ is the amount of capital a household in Economy i owns (not uses).

The capital income of a household in Economy i in period t ($x_{K,t}$) is calculated by

$$x_{K,i,t} = r_t \check{k}_{i,t} \ ,$$

where r_t is the real interest rate in period t and

$$r_t = \frac{\partial k_{i,t}}{\partial y_{i,t}} \ .$$

The labor income of a household in Economy i in period t $(x_{L,i,t})$ is calculated by extracting its capital income from its production such that

$$x_{L,i,t} = y_{i,t} - r_t k_{i,t} = y_{i,t} - r_t \frac{\sum_{i=1}^{H} \breve{k}_{i,t}}{H} \ .$$

Household savings in Economy i in period t ($s_{i,t}$) are calculated by

$$s_{i,t} = x_{L,i,t} + x_{K,i,t} - c_{i,t} \ \ ,$$

where $c_{i,t}$ is the consumption of a household in Economy i in period t. In period t + 1, these savings $(s_{i,t})$ are added to the capital the household owns, and therefore,

$$\breve{k}_{i,t+1} = \breve{k}_{i,t} + s_{i,t} \ .$$

The following simple consumption formula is used.

Consumption formula 1: The consumption of a household in Economy i in period t is

$$c_{i,t} = \left(x_{L,i,t} + x_{K,i,t}\right)\left(\frac{\Gamma(\tilde{s}_i)}{\Gamma_{i,t}}\right)^{\gamma} \ \ ,$$

and equivalently

$$c_{i,t} = \left(x_{L,i,t} + x_{K,i,t}\right)\left(\frac{\theta_i}{\Gamma_{i,t}^{\frac{1-\alpha}{\alpha}}}\right)^{\gamma},$$

where $\Gamma_{i,t}$ is the capital-wage ratio (CWR) of a household in Economy i in period t, $\Gamma(\tilde{s}_i)$ is $\Gamma_{i,t}$ of a household in Economy i in period t when the household is at its MDC, and γ is a parameter. In this paper, I set the value of γ to be 0.5. It is assumed that the intrinsic $\Gamma(\tilde{s}_i)$ (i.e., CWR at MDC) of a household is identical across households and economies, and I set this common $\Gamma(\tilde{s}_i)$ to be $0.04 \times 0.65/(1 - 0.65) = 0.0743$, which corresponds to an RTP of 0.04.

In a heterogeneous population, Consumption formula 1 should be modified to Consumption formula 2. Let $\Gamma_{R,i,t}$ be the adjusted value of $\Gamma_{i,t}$ of a household in Economy i in period t in a heterogeneous population, and $\Gamma(S_t)$ be the CWR of the country (i.e., the aggregate CWR).

Consumption formula 2: In a heterogeneous population, the consumption of a household in Economy i in period t is

$$c_{i,t} = \left(x_{L,i,t} + x_{K,i,t}\right)\left(\frac{\Gamma(\tilde{s}_i)}{\Gamma_{R,i,t}}\right)^{\gamma}$$
$$= \left(x_{L,i,t} + x_{K,i,t}\right)\left(\frac{\Gamma(\tilde{s}_i)}{r_t \dfrac{\alpha}{1-\alpha}}\right)^{\gamma} = \left(x_{L,i,t} + x_{K,i,t}\right)\left(\frac{\Gamma(\tilde{s}_i)\dfrac{1-\alpha}{\alpha}}{r_t}\right)^{\gamma},$$

and equivalently,

$$c_{i,t} = \left(x_{L,i,t} + x_{K,i,t}\right)\left(\frac{\theta_i}{r_t}\right)^{\gamma} .$$

In simulations with government transfers, it is assumed for simplicity that there are two economies (Economies 1 and 2) in a country. Let κ be the $\check{k}_{1,t}$ that a government aims for to force a household in Economy 1 to own capital at a stabilized (steady) state (i.e., κ is the target value set by the government). Under these conditions, the bang-bang control of government transfers is set as follows.

Transfer rule: The amount of government transfers from a household in Economy 1 to a household in Economy 2 in period t is T_{low} if $\check{k}_{1,t}$ is lower than κ, and T_{high} if $\check{k}_{1,t}$ is higher than κ, where T_{low} and T_{high} are constant amounts of capital predetermined by the government.

In the simulations, I set T_{low} to be -0.1 and T_{high} to be 0.5. The value of κ is varied in each simulation depending on what stabilized (steady) state the government is aiming to achieve.

3.2 Modification to examine the effect of economic rents

To examine the effect of economic rents, it is assumed for simplicity that a country has two economies (Economies 1 and 2) that are identical except for the economic rents that households obtain in each economy. A household in Economy 1 is set to obtain economic rents, and these rents are set to be added to the capital it owns. Consequently, the capital that a household in Economy 2 owns is set to decrease by the same amount as the economic rents a household in Economy 1 obtains. The amount of economic rents each household in Economy 1 obtains is identical, and the amount of capital decrease in each household in Economy 2 due to the economic rents in Economy 1 is also identical. Economic rents may be obtained every period or intermittently, and they may be obtained deterministically or stochastically.

Let ρ_t be the amount of economic rents a household in Economy 1 obtains in period t. Hence,

$$\check{k}_{1,t} = \check{k}_{1,t-1} + s_{1,t-1} + \rho_t$$

and

$$\check{k}_{2,t} = \check{k}_{2,t-1} + s_{2,t-1} - \rho_t \ .$$

4 RESULTS OF SIMULATIONS

Because population is heterogeneous in economic rents, each household behaves according to Consumption formula 2. I simulate the following three cases: (1) a household in Economy 1 obtains 0.2 as economic rents in every period, (2) a household obtains a total of 2 in economic rents in every 10 periods, and (3) a household stochastically obtains economic rents from 0 to 0.4 in each period every 10 periods and the sum of the rents during the 10 periods averages 2.

4.1 The case of unilateral behavior without government intervention

First, I simulate the situation where households behave unilaterally and the government of the country does not intervene. The results of simulations of cases (1), (2), and (3) are shown in Figures 1, 2, and 3, respectively. Note that the results shown in the figures are 10-period moving averages. Clearly, neither economy can reach a stabilized (steady) state in all three cases, which means that the level of economic inequality continues to increase to the limit. Even if the amounts or intervals of economic rents differ, the nature of the (lack of) stabilized (steady) state does not change.

A household in Economy 1 that can obtain rents continues to accumulate capital, and a household in Economy 2 that cannot obtain rents continues to lose capital and eventually owes debts to households in Economy 1. This result completely matches the theoretical predictions of Harashima (2016, 2017a, 2018c, 2018d, 2020b, 2021a).

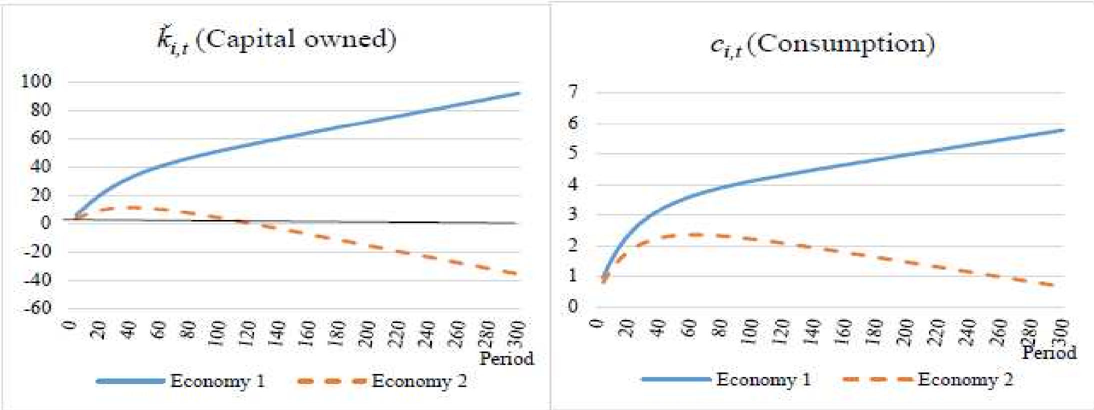

Figure 1. Simulation of capital owned by each household ($\breve{k}_{i,t}$) and consumption ($c_{i,t}$) in case (1) with unilateral behavior and without government intervention

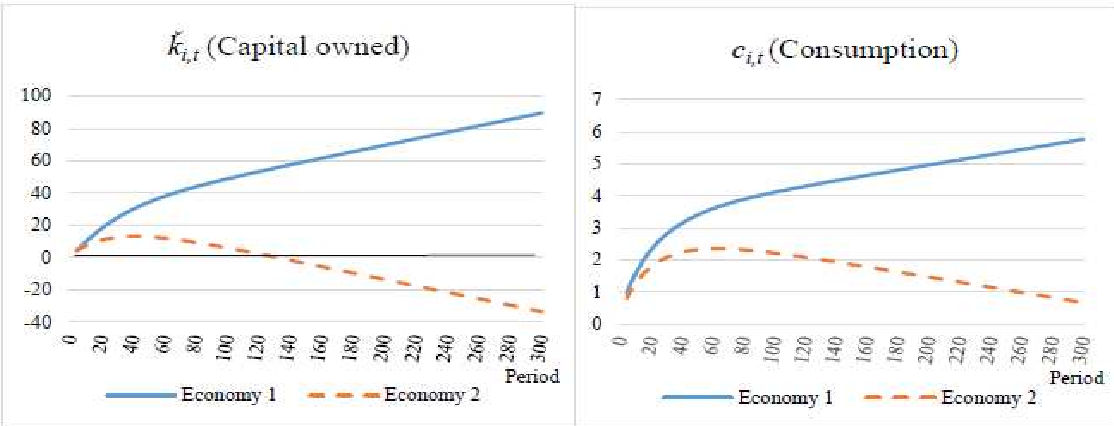

Figure 2. Simulation of capital owned by each household ($\breve{k}_{i,t}$) and consumption ($c_{i,t}$) in case (2) with unilateral behavior and without government intervention

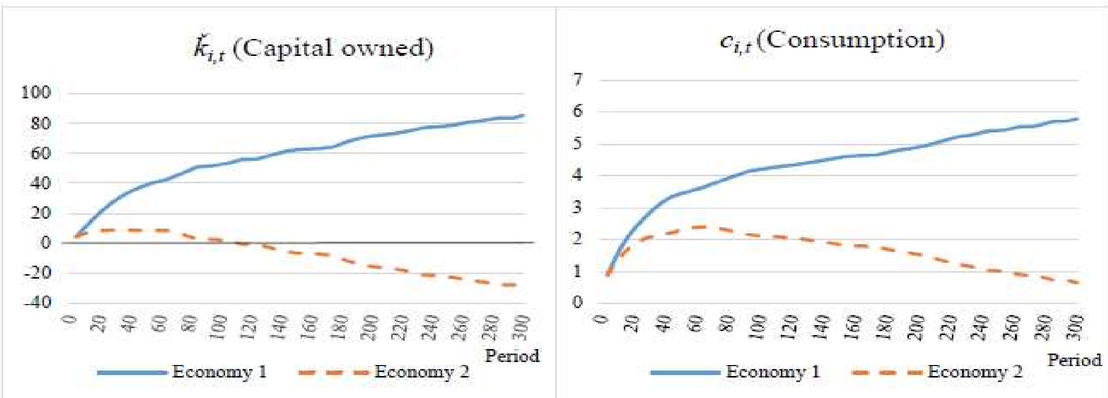

Figure 3. Simulation of capital owned by each household ($\breve{k}_{i,t}$) and consumption ($c_{i,t}$) in case (3) with unilateral behavior and without government intervention

4.2 The case of "pure" SH with government intervention

4.2.1 SH by government intervention

Next, I simulate the situation where households behave unilaterally, but the government of the country intervenes according to the Transfer rule shown in Section 3.1. Figures 4, 5, and 6 show the results of cases (1), (2), and (3), respectively.

In all three cases, capital, consumption, and the real interest rate are all eventually stabilized. That is, even though economic rents exist, a stabilized (steady) state can be achieved if the government appropriately intervenes, which means that the level of economic inequality does not increase. Note that because the results are presented as 10-period moving averages, the amount of government transfers in period t (T_t) in the figures indicates the weighted sum of T_{low} and T_{high} during the 10 periods surrounding period t.

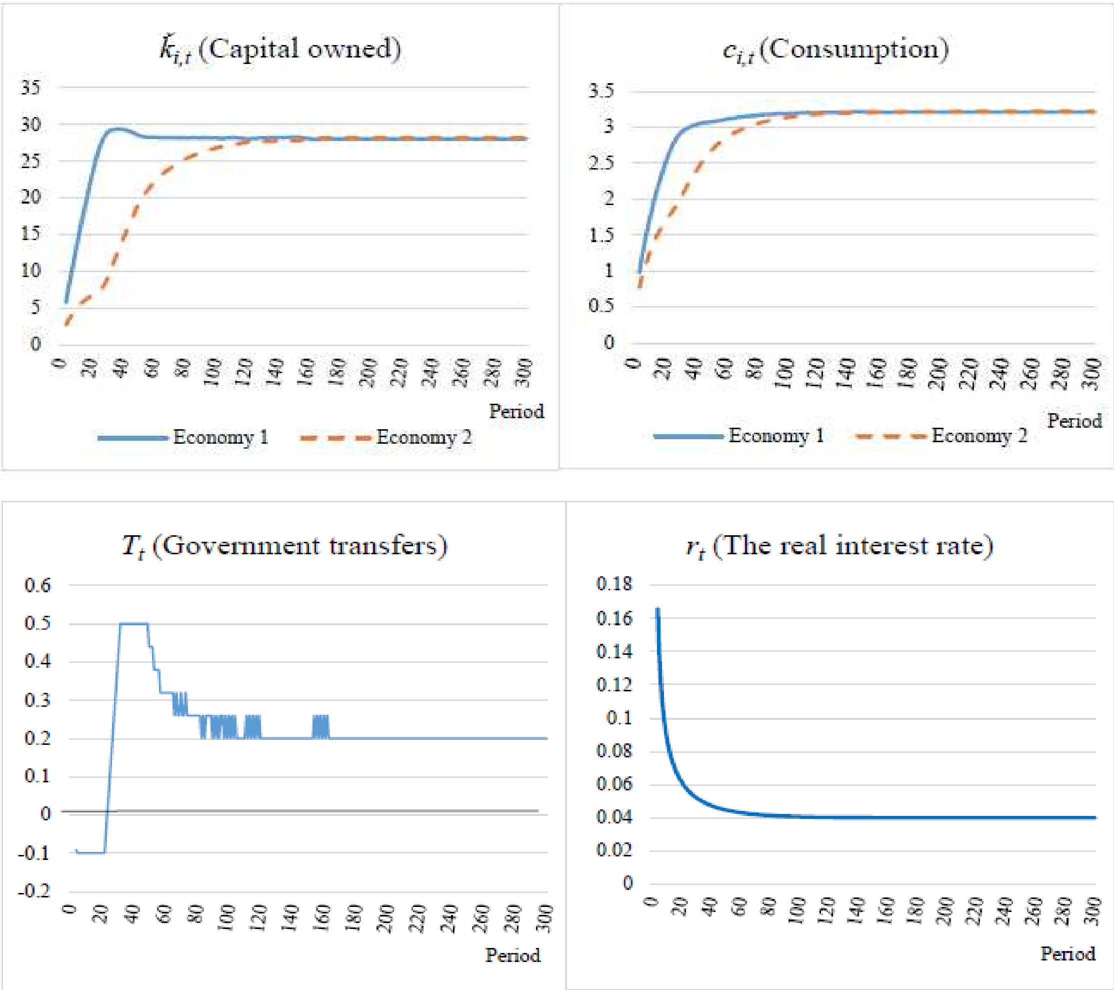

Figure 4. Simulation of capital owned by each household ($\check{k}_{i,t}$), consumption ($c_{i,t}$), government transfers (T_t), and real interest rate (r_t) in case (1) with unilateral behavior and government intervention to achieve a "pure" SH

After stabilization, the CWRs of households in the two economies are commonly almost identical to their common intrinsic CWR at MDC (i.e., $0.04 \times 0.65/(1 - 0.65) = 0.0743$, which corresponds to an average RTP of 0.04). This is not coincidental; rather, it is a result that the value of κ, which is intentionally set to make them almost identical. In actuality, there can be many stabilized (steady) states at which the CWRs of households are different from their CWRs at MDC. These stabilized (steady) states differ depending on how the government intervenes (i.e., the value it sets for κ).

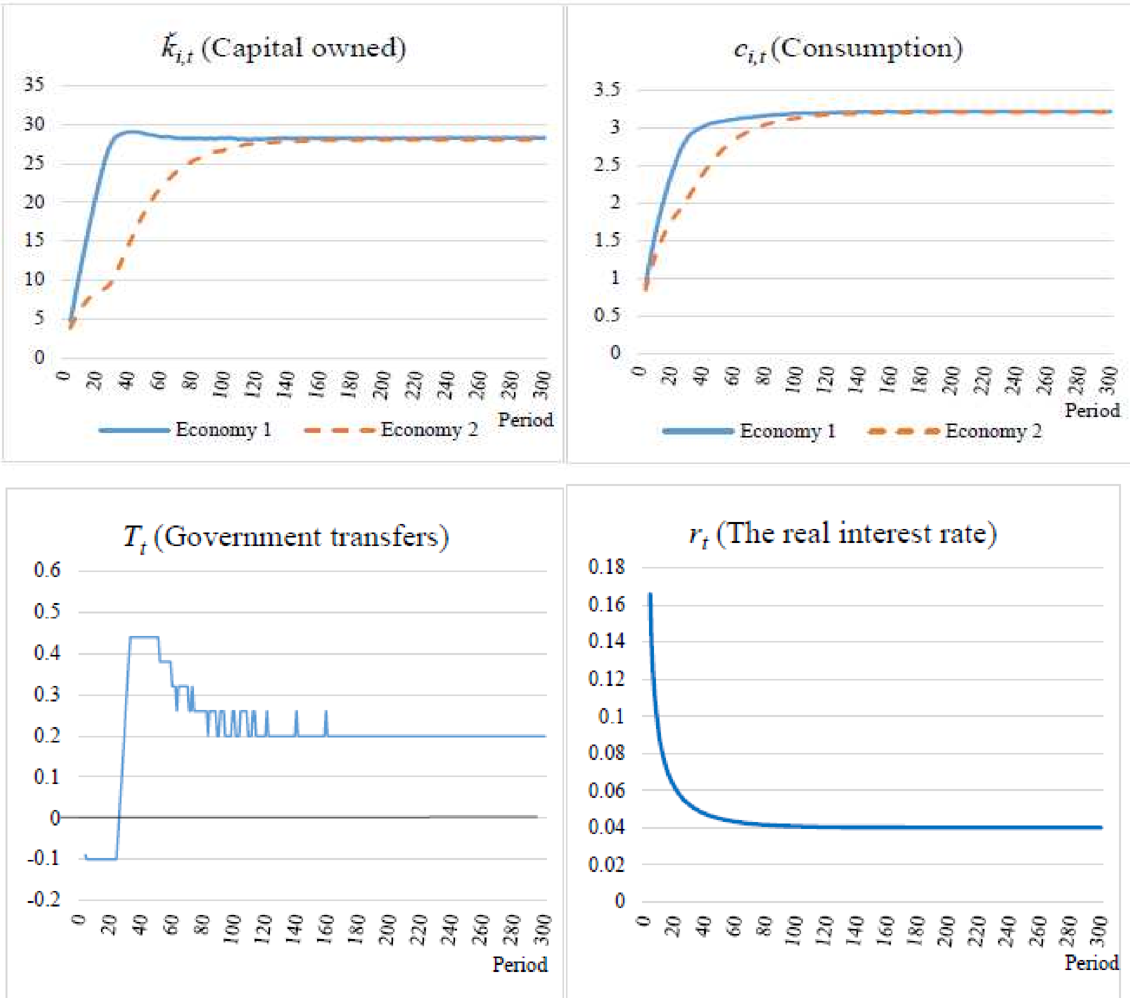

Figure 5. Simulation of capital owned by each household ($\check{k}_{i,t}$), consumption ($c_{i,t}$), government transfers (T_t), and real interest rate (r_t) in case (2) with unilateral behavior and government intervention to achieve a "pure" SH

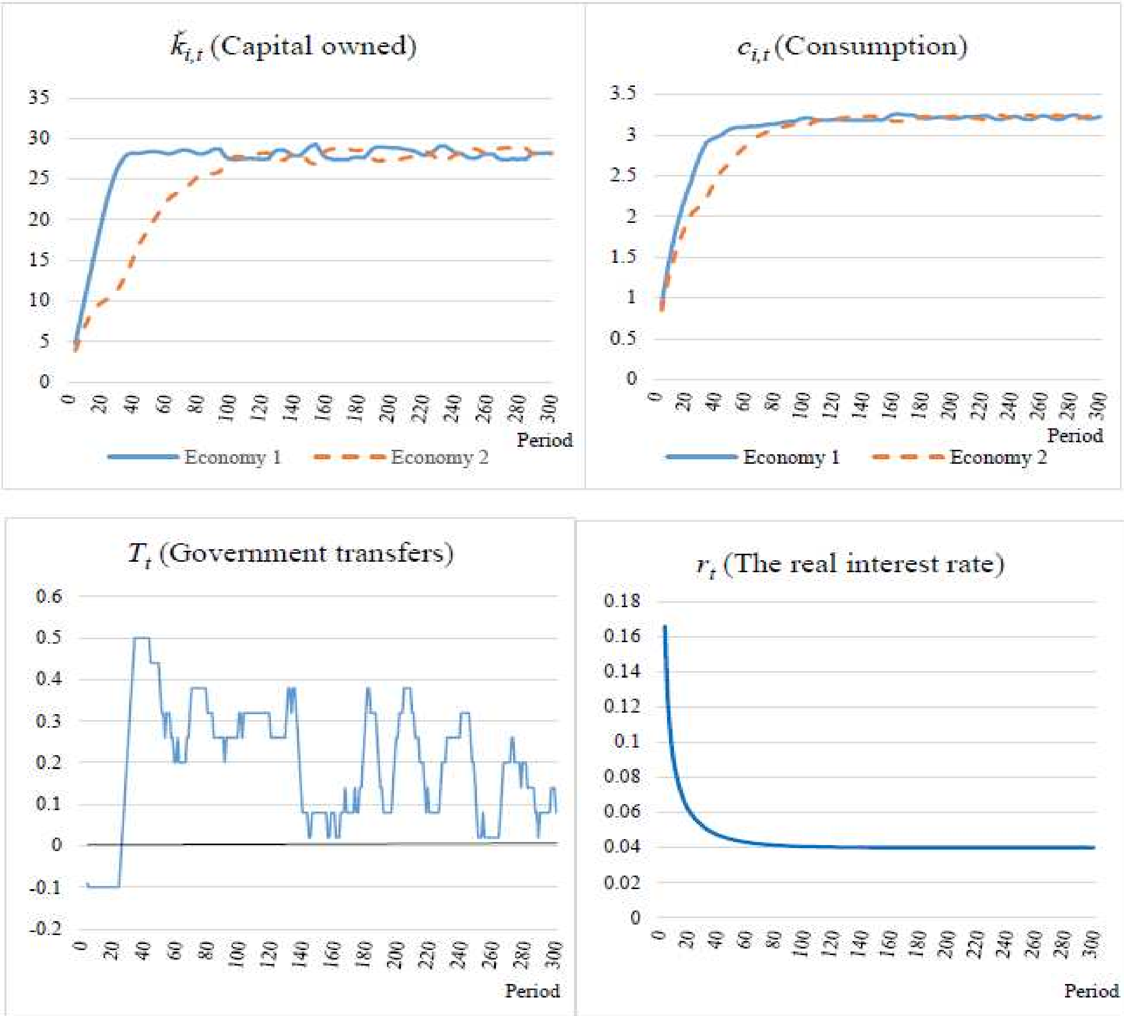

Figure 6. Simulation of capital owned by each household ($\check{k}_{i,t}$), consumption ($c_{i,t}$), government transfers (T_t), and real interest rate (r_t) in case (3) with unilateral behavior and government intervention to achieve a "pure" SH

4.2.2 Multilateral steady state

As shown in Harashima (2010, 2012, 2014), even though a government does not intervene, a SH can be achieved if households that are heterogeneous in RTP behave multilaterally in the sense that they behave intentionally to satisfy all optimal conditions of all households. Harashima (2016, 2017a, 2018c, 2018d, 2020b, 2021a) theoretically showed that this is also true in the case of heterogeneous economic rents and called these SHs multilateral steady states. Only one multilateral steady state exists for each heterogeneous population, and in two-economy models where Economy 1 has the advantage over Economy 2, $\check{k}_{1,t} < \check{k}_{2,t}$ holds at the multilateral steady state. Harashima (2010, 2012, 2014) showed that a multilateral steady state is identical to the SH achieved by government interventions when government transfers after stabilization are controlled to maintain $\breve{k}_{1,t} = \breve{k}_{2,t}$.

Harashima (2022c) showed that a state that corresponds to a multilateral steady state can be replicated in simulations in the case of heterogeneous CWRs at MDC. This will be also true in the case of heterogeneous economic rents. Indeed, at the stabilized (steady) states shown in Figures 4, 5, and 6, $\breve{k}_{1,t} = \breve{k}_{2,t}$ is almost held, and therefore these states can be interpreted to correspond to multilateral steady states. Note that, as mentioned in Section 4.2.1, there can be many stabilized (steady) states that do not correspond to the multilateral steady state.

4.3 The case of an approximate SH

An important nature of the stabilized (steady) state indicated in Figures 4, 5, and 6 is that, unlike the case of heterogeneous preferences (CWRs at MDC) shown in Harashima (2022c), households in both economies feel comfortable at these stabilized (steady) states. That is, they feel that they are at MDC because their adjusted CWRs that are felt to be equal to the real interest rate are equal to their intrinsic CWRs at MDC. In this sense, these stabilized (steady) states (Figures 4, 5, and 6) are "pure" SHs. In the case of heterogeneous preferences, only an approximate SH at which households do not necessarily feel comfortable can be achieved even if the approximate SH corresponds to a multilateral steady state (Harashima, 2018a, 2022c).

In contrast, both "pure" and approximate SHs can exist in the case of heterogeneous economic rents. Figures 7 and 8 show the result of a simulation of case (1) in which the government sets κ to be 10% higher and lower, respectively, than the amount of $\breve{k}_{1,t}$ that is needed for a "pure" SH to be achieved. In both simulations, stabilized (steady) states are achieved, but $\breve{k}_{1,t} \neq \breve{k}_{2,t}$. Specifically, $\breve{k}_{1,t} < \breve{k}_{2,t}$ in Figure 7 and $\breve{k}_{1,t} > \breve{k}_{2,t}$ in Figure 8 at each stabilized (steady) state, which means that these stabilized (steady) states are approximate SHs.

Figures 9 and 10 show the result of a simulation of case (3) in which the government sets κ to be 10% higher and lower, respectively, than the amount of $\breve{k}_{1,t}$ that is needed for a "pure" SH to be achieved. The results also indicate that stabilized (steady) states are achieved, but $\breve{k}_{1,t} \neq \breve{k}_{2,t}$. Specifically, $\breve{k}_{1,t} < \breve{k}_{2,t}$ in Figure 9 and $\breve{k}_{1,t} > \breve{k}_{2,t}$ in Figure 10 at each stabilized (steady) state.

Even if a household cannot feel most comfortable at an approximate SH, it can at least reach a stabilized (steady) state by simply sticking to their own intrinsically given heterogeneous preferences throughout all periods. In this sense, even an approximate SH can be seen to be "optimal."

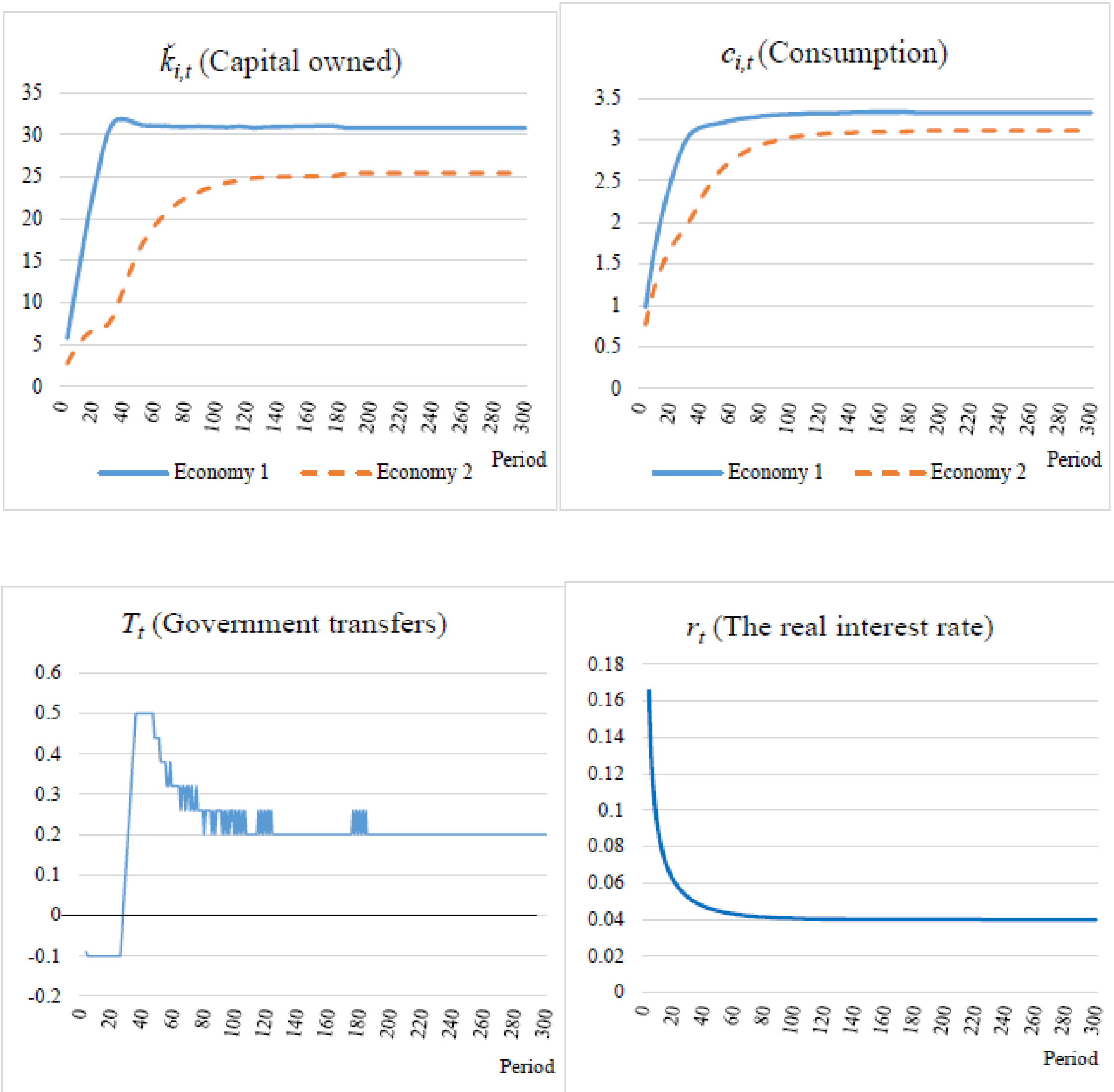

Figure 7. Simulation of capital owned by each household ($\breve{k}_{i,t}$), consumption ($c_{i,t}$), government transfers (T_t), and real interest rate (r_t) in case (1) for a κ that is 10% higher than the amount of $\breve{k}_{1,t}$ that is needed for a "pure" SH to be achieved in the case of an approximate SH with government intervention to achieve an approximate SH

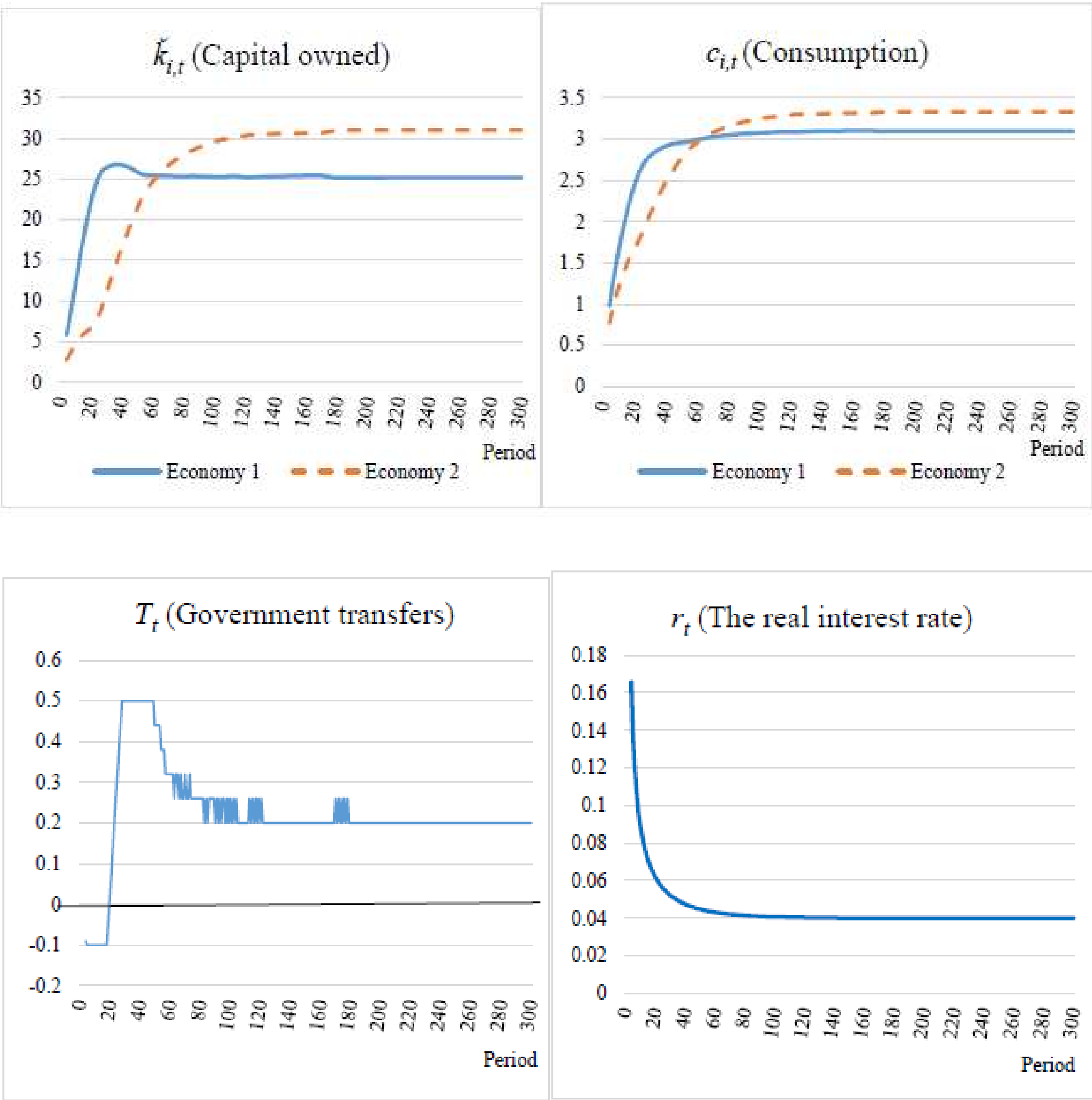

Figure 8. Simulation of capital owned by each household ($\check{k}_{i,t}$), consumption ($c_{i,t}$), government transfers (T_t), and real interest rate (r_t) in case (1) for a κ that is 10% lower than the amount of $\check{k}_{1,t}$ that is needed for a "pure" SH to be achieved in the case of an approximate SH with government intervention to achieve an approximate SH

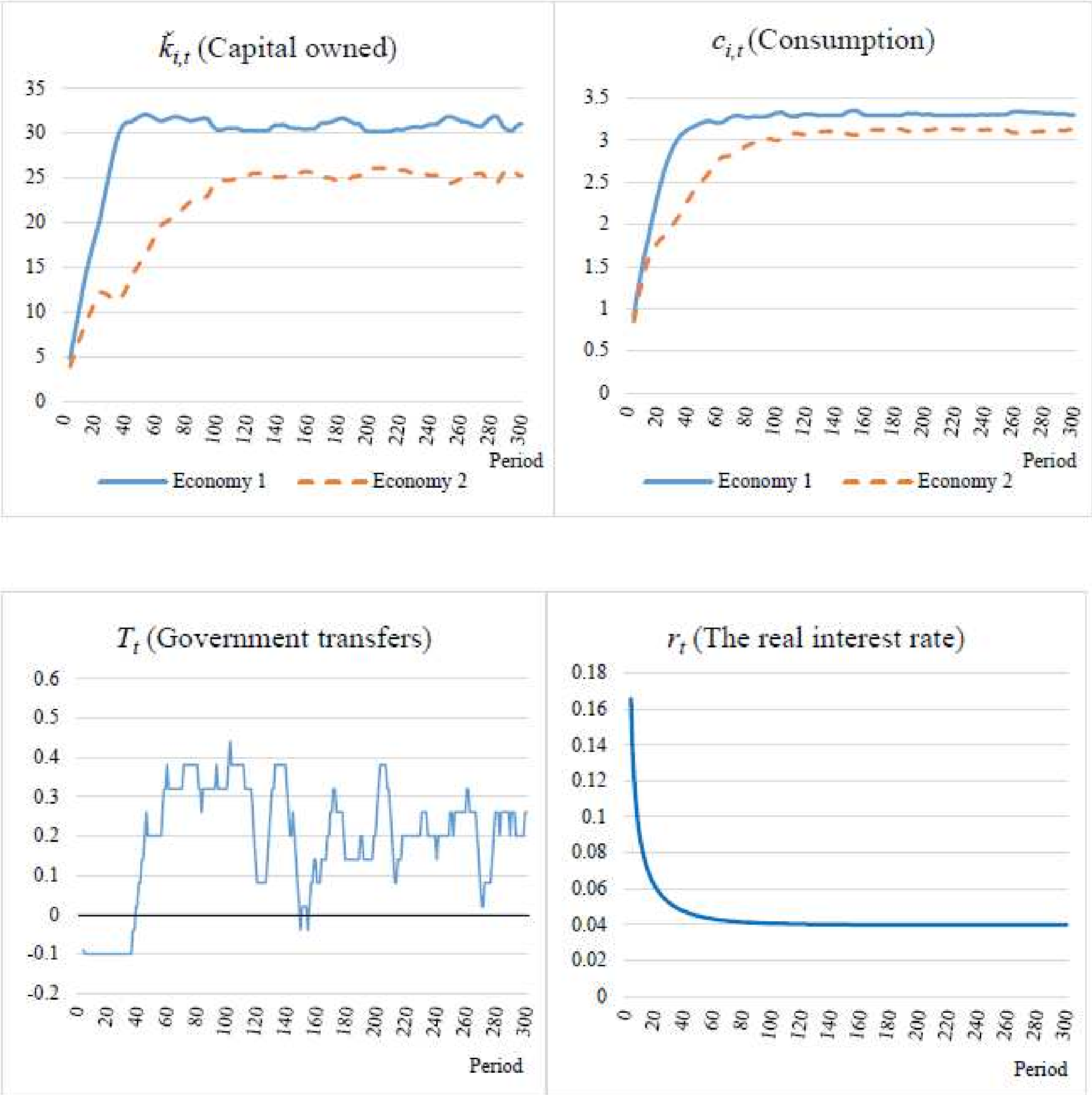

Figure 9. Simulation of capital owned by each household ($\breve{k}_{i,t}$), consumption ($c_{i,t}$), government transfers (T_t), and real interest rate (r_t) in case (3) for a κ that is 10% higher than the amount of $\breve{k}_{1,t}$ that is needed for a "pure" SH to be achieved in the case of an approximate SH with government intervention

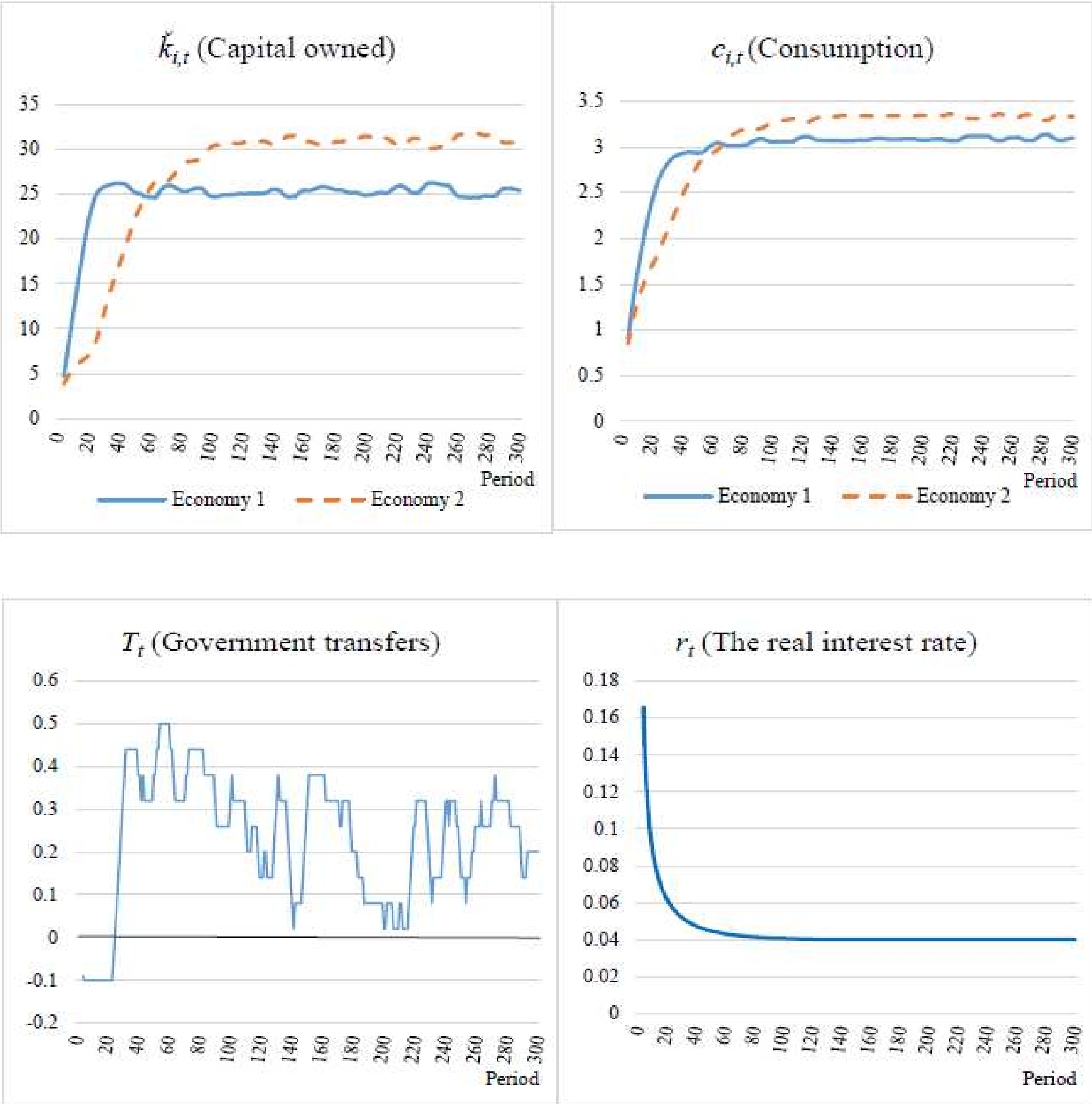

Figure 10. Simulation of capital owned by each household ($\check{k}_{i,t}$), consumption ($c_{i,t}$), government transfers (T_t), and real interest rate (r_t) in case (3) for a κ that is 10% lower than the amount of $\check{k}_{1,t}$ that is needed for a "pure" SH to be achieved in the case of an approximate SH with government intervention to achieve an approximate SH

The results shown in Figures 7–10 mean that a government can achieve almost any kind of approximate SH, i.e., both $\check{k}_{1,t} < \check{k}_{2,t}$ and $\check{k}_{1,t} > \check{k}_{2,t}$ can be achieved and these values could be notably higher or lower. As shown in Harashima (2018a), under the MDC-based procedure, a government only concerns itself about winning a majority of votes in elections and therefore behaves so as to make the number of votes cast in response to increases in the level of economic inequality equivalent to that in response to decreases. This behavior does not necessarily guarantee that the government sets the value of κ so as to achieve a "pure" SH. Indeed, there is no mechanism through which a government can be forced to achieve only “pure” SHs.

5 CONCLUDING REMARKS

It is not easy to numerically simulate the path to a steady state in dynamic economic growth models in which households behave generating rational expectations because there is no closed form solution in these models. However, if we suppose that households behave under the MDC-based procedure, the path to a steady state can easily be simulated. Harashima (2022c) numerically simulated the path to a steady state under the MDC-based procedure and showed that households can reach a stabilized (steady) state without generating any rational expectations, and the results were consistent with theoretical predictions (Harashima, 2010, 2012, 2014). However, Harashima (2022c) did not consider economic rents. In this paper, I numerically simulate the development of economic inequality when households obtain economic rents heterogeneously and behave under the MDC-based procedure.

In the simulations, households were set to increase or decrease consumption according to simple formulae that are supposed to well capture and represent household behaviors under the MDC-based procedure. The results of simulations indicate that if only some households obtain economic rents and a government does not intervene, no stabilized (steady) state can be achieved as predicted theoretically (Harashima, 2020b, 2021a), and the level of economic inequality continues to increase. This nature does not change whether economic rents are given in every period or intermittently or if they are given deterministically or randomly. However, if a government appropriately intervenes (i.e., it appropriately transfers money from households that obtain economic rents to those that do not), a stabilized (steady) state can be achieved as predicted theoretically (Harashima, 2020b, 2021a).

An important difference between the cases of heterogeneous preferences (CWRs at MDC) and economic rents is that only an approximate SH can exist in the former case, but a “pure” SH can exist in the latter case. Nevertheless, both “pure” and approximate SHs exist in a population with heterogeneous economic rents. Furthermore, because there is no mechanism for a government to be forced to select only a “pure” SH, a government can select from a range of approximate SHs in place of a “pure” SH.

APPENDIX 1: Sustainable heterogeneity

A1.1 SH

Here, three heterogeneities—RTP, degree of risk aversion (DRA), and productivity—are considered. Suppose that there are two economies (Economy 1 and Economy 2) that are identical except for RTP, DRA, and productivity. Each economy is interpreted as representing a group of identical households, and the population in each economy is constant and sufficiently large. The economies are fully open to each other, and goods, services, and capital are freely transacted between them, but labor is immobilized in each economy. Households also provide laborers whose abilities are one of the factors that determine the productivity of each economy. Each economy can be interpreted as representing either a country or a group of identical households in a country. Usually, the concept of the balance of payments is used only for international transactions, but in this paper, this concept and the associated terminology are used even if each economy represents a group of identical households in a country.

The production function of Economy i (= 1, 2) is

$$y_{i,t} = A_t^{\alpha} k_{i,t}^{1-\alpha} \ ,$$

where $y_{i,t}$ and $k_{i,t}$ are the production and capital of Economy i in period t, respectively; A_t is technology in period t; and α ($0 < \alpha < 1$) is a constant and indicates the labor share. All variables are expressed in per capita terms. The current account balance in Economy 1 is τ_t and that in Economy 2 is $-\tau_t$. The accumulated current account balance

$$\int_0^t \tau_s ds$$

mirrors capital flows between the two economies. The economy with current account surpluses invests them in the other economy. Since $\frac{\partial y_{1,t}}{\partial k_{1,t}} \left(= \frac{\partial y_{2,t}}{\partial k_{2,t}} \right)$ is returns on investments,

$$\frac{\partial y_{1,t}}{\partial k_{1,t}} \int_0^t \tau_s ds \quad \text{and} \quad \frac{\partial y_{2,t}}{\partial k_{2,t}} \int_0^t \tau_s ds$$

represent income receipts or payments on the assets that an economy owns in the other economy. Hence,

$$\tau_t - \frac{\partial y_{2,t}}{\partial k_{2,t}} \int_0^t \tau_s ds$$

is the balance on goods and services of Economy 1, and

$$\frac{\partial y_{1,t}}{\partial k_{1,t}} \int_0^t \tau_s ds - \tau_t$$

is that of Economy 2. Because the current account balance mirrors capital flows between the economies, the balance is a function of capital in both economies such that

$$\tau_t = \kappa(k_{1,t}, k_{2,t}) \ .$$

This two-economy model can be easily extended to a multi-economy model. Suppose that a country consists of H economies that are identical except for RTP, DRA, and productivity (Economy 1, Economy 2, … , Economy H). Households within each economy are identical. $c_{i,t}$, $k_{i,t}$, and $y_{i,t}$ are the per capita consumption, capital, and output of Economy i in period t, respectively; and θ_i, $\varepsilon_q = -\frac{c_{1,t} u_i''}{u_i'}$, ω_i, and u_i are the RTP, DRA, productivity, and utility function of a household in Economy i, respectively (i = 1, 2, …, H). The production function of Economy i is

$$y_{i,t} = \omega_i A_t^{\alpha} k_{i,t}^{1-\alpha} \ .$$

In addition, $\tau_{i,j,t}$ is the current account balance of Economy i with Economy j, where i, j = 1, 2, … , H and $i \neq j$.

Harashima (2010) showed that if, and only if,

$$\lim_{t\to\infty} \frac{\dot{c}_{i,t}}{c_{i,t}} = \left(\frac{\sum_{q=1}^{H} \varepsilon_q \omega_q}{\sum_{q=1}^{H} \omega_q} \right)^{-1} \left\{ \left[\frac{\varpi \alpha \sum_{q=1}^{H} \omega_q}{Hmv(1-\alpha)} \right]^{\alpha} - \frac{\sum_{q=1}^{H} \theta_q \omega_q}{\sum_{q=1}^{H} \omega_q} \right\} \qquad \text{(A1.1)}$$

for any i (= 1, 2, … , H), all the optimality conditions of all heterogeneous economies are satisfied, where m, v, and ϖ are positive constants. Furthermore, if, and only if, equation (A1.1) holds,

$$\lim_{t\to\infty}\frac{\dot{c}_{i,t}}{c_{i,t}}=\lim_{t\to\infty}\frac{\dot{k}_{i,t}}{k_{i,t}}=\lim_{t\to\infty}\frac{\dot{y}_{i,t}}{y_{i,t}}=\lim_{t\to\infty}\frac{\dot{A}_{t}}{A_{t}}=\lim_{t\to\infty}\frac{\dot{\tau}_{i,j,t}}{\tau_{i,j,t}}=\lim_{t\to\infty}\frac{\frac{d\int_0^t \tau_{i,j,s}ds}{dt}}{\int_0^t \tau_{i,j,s}ds}$$

is satisfied for any i and j ($i \neq j$). Because all the optimality conditions of all heterogeneous economies are satisfied, the state at which equation (A1.1) holds is SH by definition.

A1.2 SH with government intervention

As shown above, SH is not necessarily naturally achieved, but if the government properly transfers money or other types of economic resources from some economies to other economies, SH is achieved.

Let Economy 1+2+…+ (H – 1) be the combined economy consisting of Economies 1, 2, …, and (H – 1). The population of Economy 1+2+… + (H – 1) is therefore (H – 1) times that of Economy i (= 1, 2, 3, …, H). $k_{1+2+\cdots+(H-1),t}$ indicates the capital of a household in Economy 1+2+…+ (H – 1) in period t. Let g_t be the amount of government transfers from a household in Economy 1+2+…+ (H – 1) to households in Economy H, and $\overline{g}_t$ be the ratio of g_t to $k_{1+2+\cdots+(H-1),t}$ in period t to achieve SH. That is,

$$g_t=\overline{g}_t k_{1+2+\cdots,+(H-1),t} \ .$$

$\overline{g}_t$ is solely determined by the government and therefore is an exogenous variable for households.

Harashima (2010) showed that if

$$\lim_{t\to\infty}\overline{g}_t=\left(\frac{\sum_{q=1}^{H}\varepsilon_q\omega_q}{\omega_H}\right)^{-1}\left\{\frac{\varepsilon_H\sum_{q=1}^{H}\omega_q-\sum_{q=1}^{H}\varepsilon_q\omega_q}{\sum_{q=1}^{H-1}\omega_q}\left[\frac{\varpi\alpha\sum_{q=1}^{H}\omega_q}{Hmv(1-\alpha)}\right]^{\alpha}-\frac{\varepsilon_H\sum_{q=1}^{H}\theta_q\omega_q-\theta_H\sum_{q=1}^{H}\varepsilon_q\omega_q}{\sum_{q=1}^{H-1}\omega_q}\right\}$$

is satisfied for any i (= 1, 2, …, H) in the case that Economy H is replaced with Economy i, then equation (A1.1) is satisfied (i.e., SH is achieved by government interventions even if households behave unilaterally). Because SH indicates a steady state, $\lim_{t\to\infty}\overline{g}_t$ = constant.

Note that the amount of government transfers from households in Economy 1+2+ … + (H – 1) to a household in Economy H at SH is

$$(H-1)g_t=(H-1)\ k_{1+2+\cdots+(H-1),t}\lim_{t\to\infty}\overline{g}_t \ .$$

Note also that a negative value of g_t indicates that a positive amount of money or other type of economic resource is transferred from Economy H to Economy $1+2+\cdots+(H-1)$ and vice versa.

APPENDIX 2: The MDC-based procedure

A2.1 "Comfortability" of CWR

Let k_t and w_t be per capita capital and wage (labor income), respectively, in period t. Under the MDC-based procedure, a household should first subjectively evaluate the value of $\frac{\widetilde{w}_t}{\tilde{k}_t}$ where $\tilde{k}_t$ and $\widetilde{w}_t$ are household k_t and w_t, respectively. Let Γ be the subjective valuation of $\frac{\widetilde{w}_t}{\tilde{k}_t}$ by a household and Γ_i be the value of $\frac{\widetilde{w}_t}{\tilde{k}_t}$ of household i ($i = 1, 2, 3, \ldots, M$). Each household assesses whether it feels comfortable with its current Γ (i.e., its combination of income and capital expressed by CWR). "Comfortable" in this context means "at ease," "not anxious," and other similar feelings.

Let the "degree of comfortability" (DOC) represent how comfortable a household feels with its Γ. The higher the value of DOC, the more a household feels comfortable with its Γ. For each household, there will be a most comfortable CWR value because the household will feel less comfortable if CWR is either too high or too low. That is, for each household, a maximum DOC exists. Let $\tilde{s}$ be a household's state at which its DOC is the maximum (MDC). MDC therefore indicates the state at which the combination of revenues and assets is felt most comfortable. Let $\Gamma(\tilde{s})$ be a household's Γ when it is at $\tilde{s}$. $\Gamma(\tilde{s})$ indicates the Γ that gives a household its MDC, and $\Gamma(\tilde{s}_i)$ is household i's Γ_i when it is at $\tilde{s}_i$.

A2.2 Homogeneous population

I first examine the behavior of households in a homogeneous population (i.e., all households are assumed to be identical).

A2.2.1 Rules

Household i should act according to the following rules:

Rule 1-1: If household i feels that the current Γ_i is equal to $\Gamma(\tilde{s}_i)$, it maintains the same level of consumption for any i.

Rule 1-2: If household i feels that the current Γ_i is not equal to $\Gamma(\tilde{s}_i)$, it adjusts its level of consumption until it feels that Γ_i is equal to $\Gamma(\tilde{s}_i)$ for any i.

A2.2.2 Steady state

Households can reach a steady state even if they behave only according to Rules 1-1 and 1-2. Let S_t be the state of the entire economy in period t and $\Gamma(S_t)$ be the value of $\frac{w_t}{k_t}$ of the entire economy at S_t (i.e., the economy's average CWR). In addition, let $\tilde{S}_{MDC}$ be the steady state at which MDC is achieved and kept constant by all households, and $\Gamma(\tilde{S}_{MDC})$ be $\Gamma(S_t)$ for $S_t = \tilde{S}_{MDC}$. Let also $\tilde{S}_{RTP}$ be the steady state under the RTP-based procedure; that is, it is the steady state in a Ramsey-type growth model in which households behave based on rational expectations generated by discounting utilities by θ, where θ (> 0) is the RTP of a household. In addition, let $\Gamma(\tilde{S}_{RTP})$ be $\Gamma(S_t)$ for $S_t = \tilde{S}_{RTP}$.

Proposition 1: If households behave according to Rules 1-1 and 1-2, and if the value of θ that is calculated from the values of variables at $\tilde{S}_{MDC}$ is used as the value of θ under the RTP-based procedure in an economy where θ is identical for all households, then $\Gamma(\tilde{S}_{MDC}) = \Gamma(\tilde{S}_{RTP})$.
Proof: See Harashima (2018a).

Proposition 1 indicates that we can interpret $\tilde{S}_{MDC}$ to be equivalent to $\tilde{S}_{RTP}$. This means that both the MDC-based and RTP-based procedures can function equivalently and that CWR at MDC can be substituted for RTP as a guide for household behavior.

A2.3 Heterogeneous population

In actuality, however, households are not identical—they are heterogeneous—and if heterogeneous households behave unilaterally, there is no guarantee that a steady state other than corner solutions exists (Becker 1980; Harashima 2010, 2012). However, Harashima (2010, 2012) has shown that SH exists under the RTP-based procedure. In addition, Harashima (2018a) has shown that SH also exists under the MDC-based procedure, although Rules 1-1 and 1-2 have to be revised, and a rule for the government should be added in a heterogeneous population.

Suppose that households are identical except for their MDCs (i.e., their values of $\Gamma(\tilde{s})$). Let $\tilde{S}_{MDC,SH}$ be the steady state at which MDC is achieved and kept constant by any household (i.e., SH in a heterogeneous population under the MDC-based procedure), and let $\Gamma(\tilde{S}_{MDC,SH})$ be $\Gamma(S_t)$ for $S_t = \tilde{S}_{MDC,SH}$. In addition, let Γ_R be a household's numerically adjusted value of Γ for SH based on its estimated value of

$\Gamma(\tilde{S}_{MDC,SH})$ and several other related values. Specifically, let $\Gamma_{R,i}$ be Γ_R of household i, T be the net transfer that a household receives from the government with regard to SH, and T_i be the net transfer that household i receives (i = 1,2,3, … , M).

A2.3.1 Revised and additional rules

Household i should act according to the following rules in a heterogeneous population:

Rule 2-1: If household i feels that the current $\Gamma_{R,i}$ is equal to $\Gamma(\tilde{s}_i)$, it maintains the same level of consumption as before for any i.
Rule 2-2: If household i feels that the current $\Gamma_{R,i}$ is not equal to $\Gamma(\tilde{s}_i)$, it adjusts its level of consumption or revises its estimated value of $\Gamma(\tilde{S}_{MDC,SH})$ so that it perceives that $\Gamma_{R,i}$ is equal to $\Gamma(\tilde{s}_i)$ for any i.

At the same time, the government should act according to the following rule:

Rule 3: The government adjusts T_i for some i if necessary so as to make the number of votes cast in elections in response to increases in the level of economic inequality equivalent to the number cast in response to decreases.

A2.3.2 Steady state

Even if households and the government behave according to Rules 2-1, 2-2, and 3, there is no guarantee that the economy can reach $\tilde{S}_{MDC,SH}$. However, thanks to the government's intervention, SH can be approximately achieved. Let $\tilde{S}_{MDC,SH,ap}$ be the state at which $\tilde{S}_{MDC,SH}$ is approximately achieved (an approximate SH), and $\Gamma(\tilde{S}_{MDC,SH,ap})$ be $\Gamma(S_t)$ at $\tilde{S}_{MDC,SH,ap}$ on average. Here, let $\tilde{S}_{RTP,SH}$ be the steady state that satisfies SH under the RTP-based procedure, that is, in a Ramsey-type growth model in which households that are identical except for their θs behave generating rational expectations by discounting utilities by their θs. Furthermore, let $\Gamma(\tilde{S}_{RTP,SH})$ be $\Gamma(S_t)$ for $S_t = \tilde{S}_{RTP,SH}$.

Proposition 2: If households are identical except for their values of $\Gamma(\tilde{s})$ and behave unilaterally according to Rules 2-1 and 2-2, if the government behaves according to Rule 3, and if the value of θ_i that is calculated back from the values of variables at $\tilde{S}_{MDC,SH,ap}$ is used as the value of θ_i for any i under the RTP-based procedure in an economy where households are identical except for their θs, then $\Gamma(\tilde{S}_{MDC,SH,ap}) = \Gamma(\tilde{S}_{RTP,SH})$.
Proof: See Harashima (2018a).

Proposition 2 indicates that we can interpret $\tilde{S}_{MDC,SH,ap}$ as being equivalent to $\tilde{S}_{RTP,SH}$. No matter what values of T, Γ_R, and $\Gamma(\tilde{S}_{MDC,SH})$ are estimated by households, any $\tilde{S}_{MDC,SH,ap}$ can be interpreted as the objectively correct and true steady state. In addition, a government need not necessarily provide the objectively correct T_i for $\tilde{S}_{MDC,SH,ap}$ even though the $\tilde{S}_{MDC,SH,ap}$ is interpreted as objectively correct and true.

APPENDIX 3: Simulation method

A3.1 Simulation assumptions

A3.1.1 Environment

No technological progress and capital depreciation are assumed, and all values are expressed in real and per capita terms. It is assumed that there are H economies in a country, the number of households in each of economy is identical, and households within each economy are identical.

A3.1.2 Production

The production function of Economy i ($1 \leq i \leq H$) is

$$y_{i,t} = \omega_i A_t^{\alpha} k_{i,t}^{1-\alpha} \quad , \tag{A3.1}$$

where ω_i is the productivity of a household in Economy i. Because α indicates the labor share, I set $\alpha = 0.65$. In addition, I set $A_t = 1$ and $\omega_i = 1$ for any t and i. The initial capital a household owns is set at 1 for any household.

With $A_t = 1$ and $\omega_i = 1$, by equation (A3.1), the production of a household in Economy i in period t ($y_{i,t}$) is calculated, for any i, by

$$y_{i,t} = k_{i,t}^{1-\alpha} \; . \tag{A3.2}$$

A3.1.3 Capitals

Because the marginal productivity is kept equal across economies within the country through arbitrage in markets, the amount of capital used (not owned) by each household (i.e., $k_{i,t}$) is kept identical among households in all economies in any period; that is, $k_{i,t}$ is identical for any i although the amount of capital each household owns (not uses) can be heterogeneous. Hence, by equation (A3.2), the amount of production ($y_{i,t}$) is always identical across households and economies regardless of how much capital a household in Economy i owns, when $\omega_i = 1$. In addition, for any i,

$$k_{i,t} = \frac{\sum_{i=1}^{H} \breve{k}_{i,t}}{H} \ ,$$

where $\breve{k}_{i,t}$ is the amount of capital a household in Economy *i* owns (not uses). As shown above, I set the initial capital of a household owns to be 1 (i.e., $\breve{k}_{i,0} = 1$ for any *i*) throughout simulations in this paper.

A3.1.4 Incomes

The capital income of a household in Economy *i* in period *t* ($x_{K,t}$) is calculated by

$$x_{K,i,t} = r_t \breve{k}_{i,t} \ ,$$

where r_t is the real interest rate in period *t* and

$$r_t = \frac{\partial k_{i,t}}{\partial y_{i,t}} \ . \tag{A3.3}$$

Hence, by equations (A3.1) and (A3.3), the real interest rate r_t is calculated by

$$r_t = (1-\alpha) k_{i,t}^{-\alpha} = (1-\alpha) \left(\frac{\sum_{i=1}^{H} \breve{k}_{i,t}}{H} \right)^{-\alpha} \ .$$

The labor income of a household in Economy *i* in period *t* ($x_{L,i,t}$) is calculated by extracting its capital income from its production such that

$$x_{L,i,t} = y_{i,t} - r_t k_{i,t} = y_{i,t} - r_t \frac{\sum_{i=1}^{H} \breve{k}_{i,t}}{H} \ .$$

Because the amount of capital used and the amount of labor inputted by a household is identical for any household in any economy when $\omega_i = 1$, household labor income is identical across economies. Note that if productivity ($\omega_{i.t}$) is heterogeneous among economies, production and labor income differ in proportion to their productivities. Note also that in a homogeneous population, the labor income becomes equal to $\alpha y_{i,t}$ for any household.

A3.1.5 Savings

Household savings in Economy i in period t ($s_{i,t}$) are calculated by

$$s_{i,t} = x_{L,i,t} + \ x_{K,i,t} - c_{i,t} \ .$$

In period t + 1, these savings ($s_{i,t}$) are added to the capital the household owns, and therefore,

$$\breve{k}_{i,t+1} = \breve{k}_{i,t} + s_{i,t} \ .$$

A3.2 Cconsumption formula

A3.2.1 Consumption formula in a homogeneous population

For a simulation to be implemented, the consumption formula that describes how a household adjusts its consumptions needs to be set beforehand. However, under the MDC-based procedure, there is no strict consumption formula for households. A household just has to behave roughly feeling and guessing (i.e., not exactly calculating) its CWR and CWR at MDC in each period. It increases its consumption somewhat if it feels that $\Gamma(\tilde{s}_i)$ is larger than $\Gamma_{i,t}$ and decreases its consumption somewhat if it feels the opposite way. The amount of the increase/decrease will differ by period. In this sense, the actual formula of consumption under the MDC-based procedure is lax and vague; therefore, it is difficult to set a strict consumption formula with a mathematical functional form.

Nevertheless, if we consider the average consumption over some periods (i.e., moving averages), it will be possible to describe a mathematical form of the consumption formula because households will behave in a similar manner on average. Considering this nature, I introduce the following simple consumption formula because it seems to simply but correctly capture the behavior of households under the MDC-based procedure on average. Please note that that this consumption formula is not the only possible choice. Other, possibly more complex and subtle, functional forms could be chosen.

Consumption formula 1: The consumption of a household in Economy i in period t is

$$c_{i,t} = \left(x_{L,i,t} + \ x_{K,i,t}\right)\left(\frac{\Gamma(\tilde{s}_i)}{\Gamma_{i,t}}\right)^{\gamma} \ , \qquad \text{(A3.4)}$$

where $\Gamma_{i,t}$ is the CWR of household in Economy i in period t and γ is a parameter.

Because

$$\theta_i = \left(\frac{1-\alpha}{\alpha}\right)\Gamma(\tilde{s}_i) \ , \tag{A3.5}$$

as shown in Harashima (2018a, 2021a, 2022a), by equation (A3.5), equation (A3.4) is equal to

$$c_{i,t} = \left(x_{L,i,t} + \ x_{K,i,t}\right)\left(\frac{\theta_i}{\Gamma_{i,t}\frac{1-\alpha}{\alpha}}\right)^{\gamma} \ .$$

Athough a household is set to precisely follow equation (A3.4) in the simulations, in reality, they do not behave by calculating equation (A3.4). Furthermore, they are not even aware of Consumption formula 1 itself and cannot know the exact numerical value of each $\Gamma(\tilde{s}_i) = \theta_i\alpha/(1-\alpha)$. Instead, households feel and guess whether they should increase or decrease consumption considering their income and wealth.

That is, Consumption formula 1 is set only for the convenience of calculation in the simulation. It seems to well capture the essence of household behavior in that it increases or decreases consumption depending on a household's feelings with regard to $\Gamma_{i,t}$ and $\Gamma(\tilde{s}_i)$. In this context, the value of parameter γ represents the average adjustment velocity of increase or decrease in consumption.

Consumption formula 1 means that a household's consumption is roughly equal to the sum of its incomes ($x_{L,i,t} + \ x_{K,i,t}$). The reason for this equality is that there is no technological progress and capital depreciation, so savings stay around zero at the stabilized (steady) state. As mentioned above, the adjustment velocity of consumption in each period is determined by the value of γ in equation (A3.4). As the value of γ is larger, a stabilized (steady) state can be achieved more quickly (if it can be achieved). In this paper, I set the value of γ to be 0.5.

A3.2.2 Consumption formula in a heterogeneous population

As shown in Harashima (2018a, 2021a, 2022a), in a heterogeneous population, a household behaving under the MDC-based procedure does not use its CWR ($\Gamma_{i,t}$) to make decisions about its consumption. Instead, it uses an adjusted value of CWR considering the behaviors of other heterogeneous households and the government because the entire economic state of the country depends on these heterogeneous behaviors in a heterogeneous population. Accordingly, in a heterogeneous population, Consumption formula 1 has to be modified to accommodate the adjusted CWR. Let $\Gamma_{R,i,t}$ be the adjusted value of $\Gamma_{i,t}$ of a household in Economy i in period t and $\Gamma(S_t)$ be the CWR of the country (i.e., the aggregate capital-wage ratio).

A3.2.2.1 Consumption formula 2

Unilateral behavior implies that a household behaves supposing that other households must behave in the same manner as it does. In other words, it assumes that other households' preferences are almost identical to its preferences, or at least, its preferences are not exceptional but roughly the same as the preferences of the average household (Harashima, 2018a). If all households behaved in the same manner as a household in Economy *i* did, the real interest rate (r_t) would be equal to the household's $\Gamma_{R,i,t}(1-\alpha)/\alpha$ and eventually converge at its $\Gamma(\tilde{s}_i)(1-\alpha)/\alpha$. Hence, if a household in Economy i behaves unilaterally in a heterogeneous population, it feels and guesses that its $\Gamma_{R,i,t}$ $(1-\alpha)/\alpha$ is roughly identical to the real interest rate (r_t). That is, the real interest rate will be used as $\Gamma_{R,i,t}(1-\alpha)/\alpha$, and $r_t\alpha/(1-\alpha)$ will be used as its adjusted CWR ($\Gamma_{R,i,t}$).

Therefore, even if a unilaterally behaving household's raw (unadjusted) CWR is accidentally equal to its CWR at MDC, the household does not feel that it is at its MDC unless at the same time r_t is accidentally equal to its $\Gamma(\tilde{s}_i)(1-\alpha)/\alpha$. The household will instead feel that the value of r_t will soon change, and accordingly, its raw (unadjusted) CWR will also change soon. That is, it feels and guesses that the entire economic state of the country is not yet stabilized because r_t is not equal to its $\Gamma(\tilde{s}_i)(1-\alpha)/\alpha$. As a result, the household will still continue to change its consumption to accumulate or diminish capital (see Lemma 2 in Harashima, 2018a).

Considering the above-shown nature of the adjusted CWR, Consumption formula 1 can be modified to Consumption formula 2 to use in simulations with a heterogeneous population.

Consumption formula 2: In a heterogeneous population, the consumption of a household in Economy *i* in period *t* is

$$c_{i,t} = \left(x_{L,i,t} + x_{K,i,t}\right)\left(\frac{\Gamma(\tilde{s}_i)}{\Gamma_{R,i,t}}\right)^{\gamma}$$
$$= \left(x_{L,i,t} + x_{K,i,t}\right)\left(\frac{\Gamma(\tilde{s}_i)}{r_t\dfrac{\alpha}{1-\alpha}}\right)^{\gamma} = \left(x_{L,i,t} + x_{K,i,t}\right)\left(\frac{\Gamma(\tilde{s}_i)\dfrac{1-\alpha}{\alpha}}{r_t}\right)^{\gamma} \quad \text{(A3.6)}$$

and equivalently, by equations (A3.5) and (A3.6),

$$c_{i,t} = \left(x_{L,i,t} + x_{K,i,t}\right)\left(\frac{\theta_i}{r_t}\right)^{\gamma} .$$

As with $\Gamma_{i,t}$ in Consumption formula 1, the use of r_t in equation (A3.6) does not mean that households always actually behave by paying attention to r_t. What Consumption formula 2 means is that, on average, unilaterally behaving households will feel and guess that r_t represents their adjusted CWRs.

Under the RTP-based procedure, a household changes its consumption according to

$$\frac{\dot{c}_{i,t}}{c_{i,t}}=\varepsilon^{-1}(r_t-\theta_i)\ ,$$

where ε is the degree of relative risk aversion. That is, a household changes its consumption by comparing r_t and its $\theta_i=\Gamma(\tilde{s}_i)(1-\alpha)/\alpha$. The household changes consumption as r_t increasingly differs from $\theta_i=\Gamma(\tilde{s}_i)(1-\alpha)/\alpha$. This household's behavior under the RTP-based procedure is very similar to that according to Consumption formula 2, which means that the formula is basically consistent with a household's behavior under the RTP-based procedure.

In addition, in a homogeneous population, r_t is always equal to a homogenous household's $\Gamma_{i,t}(1-\alpha)/\alpha$ because all households behave in the same manner. Hence, equation (A3.4) is practically identical to equation (A3.6) (i.e., Consumption formula 1 is practically identical to Consumption formula 2) because $\Gamma_{i,t}$ in equation (A3.4) can be replaced with $r_t\frac{\alpha}{1-\alpha}$.

A3.2.2.2 Consumption formula 2-a

In Consumption formula 2, a household is supposed to feel that its preferences are not exceptional and almost the same as the preferences of the average household, but it may not actually feel that way. It may instead feel that its preferences are different from those of the average household. In this case, the household will not only feel its preferences are different, but it will also have to guess how far its preferences are from the average (i.e., by how much its adjusted CWR is different from the real interest rate).

For example, a household in Economy i may feel and guess that its adjusted CWR is

$$\Gamma_{R,i,t}=\frac{\alpha}{1-\alpha}\ (r_t+\chi_i) \qquad \text{(A3.7)}$$

instead of $\Gamma_{R,i,t}=r_t\frac{\alpha}{1-\alpha}$ in Consumption formula 2, where χ_i is a constant and $\chi_i\neq\chi_j$ for any i and j. χ_i represents the magnitude of how much a household in Economy i feels it is different from the average household. I refer to a modified version of Consumption formula 2 in which $r_t \frac{\alpha}{1-\alpha}$ is replaced with $\frac{\alpha}{1-\alpha}\ (r_t + \chi_i)$ shown in equation (A3.7) as Consumption formula 2-a. In this case, a household in Economy *i* behaves feeling that

$$\Gamma_{R,i,t} = \frac{\alpha}{1-\alpha}\ (r_t + \chi_i) = \Gamma_{i,t} \tag{A3.8}$$

holds at a stabilized (steady) state that will be realized at some point in the future.

A3.2.2.3 Consumption formula 2-b

In both Consumption formulae 2 and 2-a, the raw (unadjusted) CWR is not included and therefore plays no role. Nevertheless, a household may utilize a piece of information derived from its raw (unadjusted) CWR because past behaviors may contain some useful information for guiding future behavior. As indicated in Section A3.2.2.2, χ_i is a parameter that indicates how far a household is from the average household. In general, the value of the parameter should be adjusted if households obtain any new and additional pieces of information. This implies that a piece of information derived from the raw (unadjusted) CWR may be used to adjust the value of parameter χ_i.

For example, a household in Economy *i* may use its raw (unadjusted) CWR ($\Gamma_{i,t}$) to adjust the value of χ_i such that

$$\chi_{i,t} = \chi_{i,t-1} + \zeta_i \left(\Gamma_{i,t} \frac{1-\alpha}{\alpha} - r_{t-1} - \chi_{i,t-1} \right) , \tag{A3.9}$$

where $\chi_{i,t}$ is χ_i in period *t*, and ζ_i is a positive constant and its value is close to zero. Equation (A3.9) means that a household in Economy *i* increases the value of $\chi_{i,t}$ a little if its raw (unadjusted) CWR is higher than its adjusted CWR ($r_{t-1} + \chi_{i,t-1}$) in the previous period and vice versa. It fine-tunes $\chi_{i,t}$ in this manner because it feels that equation (A3.8) will eventually hold at some point in the future, as shown in Section A3.2.2.2. The value of ζ_i is close to zero because $\Gamma_{i,t}$ is highly likely to be almost equal to $\Gamma_{i,t-1}$, and therefore, the guess of $\chi_{i,t}$ in period *t* will not change largely from that of $\chi_{i,t-1}$ in period $t-1$. I refer to the modified version of Consumption formula 2-a in which χ_i is replaced with $\chi_{i,t}$ shown in equation (A3.9) as Consumption formula 2-b.

A3.3 Rule of government transfer

Although governments implement transfers among households in complex and subtle manners, a simple bang-bang control is adopted in simulations in this paper as the rule of government transfer for simplicity. In addition, government transfers in each period are assumed to be added to or extracted from the capital of each relevant household in the next period.

In simulations with government transfers, it is assumed for simplicity that there are two economies (Economies 1 and 2) in a country, the economies are identical except for each $\Gamma(\tilde{s}_i)(1-\alpha)/\alpha = \theta_i$, and all households in each economy are identical. Let κ be the $\breve{k}_{1,t}$ that a government aims for to force a household in Economy 1 to own capital at a stabilized (steady) state (i.e., κ is the target value set by the government). Under these conditions, the bang-bang control of government transfers is set as follows.

Transfer rule: The amount of government transfers from a household in Economy 1 to a household in Economy 2 in period t is T_{low} if $\breve{k}_{1,t}$ is lower than κ and T_{high} if $\breve{k}_{1,t}$ is higher than κ, where T_{low} and T_{high} are constant amounts of capital predetermined by the government.

In the simulations, I set T_{low} to be -0.1 and T_{high} to be 0.5. The value of κ is varied in each simulation depending on what stabilized (steady) state the government is aiming to achieve. Note that because of the discontinuous control signal in bang-bang control, flow variables may show discontinuous zigzag paths but stock variables can move relatively smoothly. These zigzag paths may look unnatural, but they are generated only because of the bang-bang control method that is adopted for simplicity.

Even if a household knows about the existence of government transfers, it still behaves based on Consumption formula 2 (or 2-a and 2-b) with no government transfer. That is, a household uses $x_{L,i,t} + x_{K,i,t}$, not $x_{L,i,t} + x_{K,i,t}$ + government transfers (T_{low} or T_{high}), as the "base" consumption in determining whether it should increase or decrease its consumption. This behavior superficially may mean that a household does not consider government transfers in the process of adjusting its CWR. However, it is implicitly assumed that a household knows that government transfers exist and that they are an exogenous factor. Therefore, the household feels that the transfers should be removed from the elements that it can change or control freely. Furthermore, it is implicitly assumed that a household correctly knows the exact amount of government transfers.

However, these assumptions may be oversimplifications, and they can be relaxed to allow for incorrect guesses on the amount of government transfers. This relaxation enables a household to use $x_{L,i,t} + x_{K,i,t}$ + government transfers (T_{low} or T_{high}) instead of $x_{L,i,t} + x_{K,i,t}$ in determining its consumption.

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