# On the Time Consistency of Universal Basic Income

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**Abstract**

I examine the effects of government commitment on the optimal provision of Universal Basic Income (UBI) in an incomplete-markets model by characterizing a dynamic game between individuals and a benevolent government according to its commitment technologies. I find that, throughout the transition, commitment determines how the government balances *income redistribution* through taxes and UBI with *pecuniary externalities* from changes in the factor income composition. In a calibrated economy, commitment leads to significant welfare improvements by counterbalancing these forces over the entire time horizon. Commitment enables a substantial long-run UBI provision by increasing taxes, which generates long-run welfare losses from stagnant income redistribution and unfavorable factor price changes for low-income individuals. However, this long-run UBI provision induces front-loaded welfare gains from factor price changes favoring low-income individuals and income redistribution facilitated by reduced precautionary savings. Without commitment, this time-lagged strategy is not credible because the government balances the two economic forces every period in a forward-looking manner, disregarding the long-run UBI impacts on the short-run economy. This time-consistent strategy results in smaller welfare improvements.

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# 1 Introduction

UBI has drawn growing social and political attention in many developed countries. A large body of recent literature responds by investigating various policy aspects of UBI: different welfare implications across demographic groups (Guner et al., 2021), its interactions with labor market frictions (Santos and Rauh, 2022), various financing plans and their welfare consequences (Luduvice, 2021; Conesa et al., 2023), and its intergenerational implications (Daruich and Fernández, 2023). Despite the extensive analyses regarding its intriguing policy implications, very few studies have examined UBI in terms of one of the timeless policy issues: *Time Inconsistency* (Kydland and Prescott, 1977; Calvo, 1978; Barro and Gordon, 1983; Lucas and Stokey, 1983; Chari and Kehoe, 1990; Athey et al., 2005; Domínguez, 2007a; Klein et al., 2008). This paper fills this gap by using a dynamic game between individuals and a benevolent government in the standard incomplete-markets model with uninsurable income risk (the Bewly-Hugget-Ayagari model). In this context, I examine how different the optimal tax-UBI schemes are along transition paths according to the availability of government commitment technologies.

Specifically, I compare two economies: one with commitment, where a Ramsey planner exists, and one without commitment. In the economy with commitment, the government is able to commit to future tax and UBI policies, choosing a sequence of income taxes and UBI that maximizes the utilitarian welfare function along the transition path. By contrast, in the case without commitment, the economy is in Markov-perfect equilibria (MPEs), as in Krusell et al. (1996); Klein et al. (2008). The government can only set a tax rate and UBI for the next period and cannot commit to them thereafter. As a result, the government sequentially chooses a tax and UBI policy maximizing the utilitarian welfare function, and this action continues perpetually, which implies a *time-consistent policy*. In both economies, balancing its budget in each period, the government levies a tax from labor and capital income, covers a predetermined level of government spending from the tax revenue, and redistributes the rest to households through lump-sum transfers—UBI.[1]

This paper makes the first contribution to the literature by providing a theoretical understanding of government policy decisions on UBI. To this end, using the generalized Euler equation (GEE) approach, as proposed by Klein et al. (2008), I characterize the equilibria by computing the first-order condition (FOC) of the government's policy decision. The analysis reveals that the government does not consider the direct impact of individual decisions on consumption, saving, and labor supply on welfare, as these terms are offset in the government's optimal condition through the envelope theorem. Instead, the government focuses on two types of economic forces while making policy decisions: (i) *income redistribution* through taxes and UBI and (ii) *pecuniary externalities* arising from changes in the factor composition of individual income.[2]

[1] The assumption of the tax base is relaxed later in the paper.

[2] Pecuniary externalities have been thoroughly investigated in the literature on constrained efficiency in incomplete-

The two types of economic forces are heterogeneous across individuals and have opposite impacts on individuals' welfare according to their income level. For instance, when UBI increases with a rise in income tax rate, it raises the after-tax income for low-income individuals but lowers it for high-income individuals. This provides welfare gains to low-income individuals but welfare losses to high-income individuals. At the same time, the increased UBI and income taxes reduce overall savings and aggregate capital, resulting in an increase in the equilibrium interest rate and a decrease in the equilibrium market wage due to general equilibrium effects—pecuniary externalities. These changes impact individual welfare differently, with low-income individuals, who have a higher proportion of labor income, being worse off (negative pecuniary externalities), while high-income individuals, who have a higher proportion of capital income, are better off (positive pecuniary externalities). The government must balance the opposite impacts of these forces on individual welfare at different income levels when making policy decisions.

Commitment technologies determine how to strike a balance between the two types of economic forces along the equilibrium path. With commitment, the Ramsey planner balances the two types of economic forces over the entire time horizon, resulting in time-consistent outcomes. Suppose that the Ramsey planner implements a substantial UBI with a high income tax rate in the long run. In this case, in the long run, the Ramsey planner must endure welfare losses from stagnant income redistribution and efficiency losses from the tax burden.

In the short run, however, this long-run provision of UBI leads to front-loaded welfare gains. The long-run UBI decreases the expected probability of hitting the borrowing constraints among individuals, thus reducing precautionary savings and the proportion of capital income out of the disposable income for low-income individuals. Because capital income has more severe inequality than labor income and UBI, the long-run provision of UBI reduces the overall income inequality early in the transition, resulting in front-loaded welfare gains. This policy, which is desired at time 0, is not optimal when evaluated in a forward-looking manner, leading to a time-inconsistent outcome. This finding is intuitive because with a discount factor of households less than one, it is more efficient to delay losses in the long run while achieving gains in the short run.

Without commitment, the government implements time-consistent policies by balancing income redistribution and pecuniary externalities in a forward-looking manner in each period. Therefore, the time-lagged balancing, which works in the case with commitment, is not possible. This finding implies that without commitment, the government disregards the effects of long-run UBI on the past economy. This time-consistent policy leads to sub-optimal outcomes in terms of the entire time horizon.

Another contribution of this paper is to provide a numerical solution algorithm that enables quantitative assessments of these theoretical findings. Specifically, I propose a computational

markets models, as demonstrated in studies such as Davila et al. (2012).

method for the case when the government lacks commitment technologies. Note that although I have used this method to quantify my theoretical findings, it is not limited to the examples presented in this paper but applicable to other general games with incomplete financial markets, including economies with general optimal policies or political procedures.

The general structure of this game, as Krusell et al. (1996); Krusell and Ríos-Rull (1999) demonstrate, is complex because it involves the interplay of three equilibrium objects: individual decisions, the aggregate law of motion for the household distribution, and the endogenous government policy function, all of which must be consistent with each other. This becomes more challenging with one-shot deviations required to solve sub-perfect Nash equilibria because the deviations produce a sequence of taxes/transfers that are off-the-equilibrium paths—alternative paths of the economy that are not selected in the equilibrium but still need to be defined to shape the equilibrium. To address these computational issues, the paper draws on the backward induction method of Reiter (2010) and makes modifications to suit the characteristics of the MPE, considering the existence of off-the-equilibrium paths, while still retaining its computational benefits.[3] For the case with commitment, I employ the numerical method from Dyrda and Pedroni (2022).[4]

For quantitative assessments of these theoretical findings, I calibrate the model to the U.S. economy as a starting point and compare the differences in taxes and transfers (UBI) based on government commitment technologies. I find that commitment makes substantial differences in the aggregate economy, inequality, and welfare. The Ramsey planner, with its time-inconsistent optimal policy, chooses more substantial income taxes than the government with the time-consistent optimal policy over the entire transition path. Compared to the calibrated initial economy, the Ramsey planner gradually increases income taxes by 16 percentage points. However, with a lack of commitment, the optimal income tax rate rapidly increases by 2 percentage points.

This gap in tax policies results in differences in the size of UBI. The ratio of UBI-to-initial GDP in the case with commitment increases by 9.2 percentage points, but that in the case without commitment increases by 4.7 percentage points. These differences in the tax-UBI scheme result in diverse dynamics in the economy and distributions. The economy managed by the Ramsey planner is less efficient but more equal due to the imposition of higher income taxes and larger UBI. Aggregate consumption, capital, and output are higher in the time-consistent optimal policy scenario, but their inequality is lower in the time-inconsistent scenario. The welfare gain, as measured by the consumption equivalent variation, is greater in the time-inconsistent optimal income tax-UBI scenario (+2.19%) than in the time-consistent scenario (+0.57%).

To shed light on the economic reasoning behind differences in the government UBI decisions,

[3] Section 5 explains the key ideas and Appendix A demonstrates each step of the algorithm in detail, including outcomes related to its efficiency and accuracy.

[4] Using a Ramsey planner in the standard incomplete-market model, Dyrda and Pedroni (2022) provides a quantitative analysis of optimal fiscal policies that cover a wide range of scenarios.

I apply my theoretical findings to the quantitative results. Through this analysis, I find that the Ramsey planner, with its commitment, achieves front-loaded welfare gains through both reduced income inequality (considerable income redistribution) and changes in the factor composition of income (positive pecuniary externalities). To understand this result, note that the government is inclined to represent the interests of low-income individuals more, as these forces are weighted by the marginal utility of consumption. On the one hand, the committed scenario, which results in a more substantial increase in taxes and UBI, rapidly reduces after-tax income inequality in the early stages of the transition, leading to welfare gains for low-income individuals. At the same time, this policy change induces an increase in the market wage and a decrease in the market interest rate in the early phase of the transition because the adjustment of aggregate capital is slower than that of aggregate labor—positive pecuniary externalities. Therefore, the two types of economic forces improve welfare in the early phase of the transition, leading the government to attain upfront welfare gains.

These front-loaded welfare gains are feasible because the government with commitment can provide substantial UBI in the long run by preserving initially increased taxes along the transition. This long-run provision of large UBI leads to rapid income redistribution early in the transition by reducing precautionary savings for low-income individuals. This reduction in precautionary savings means that low-income individuals increase the proportion of labor income and lump-sum transfers (UBI) out of their total income by reducing the portion of capital income. Because capital income has more inequality compared to labor income and UBI, this income composition change reduces overall income inequality for the low-income group, generating front-loaded welfare gains. Moreover, this change in income composition for the low-income group is expedited by the early decrease in the market interest rate during the transition. Middle- and high-income individuals do not experience a permanent reduction in the proportion of capital income. Although they temporarily decrease their savings early in the transition due to a fall in the market interest rate, they increase the proportion of capital income when the market interest rate increases again in the long run.

In exchange for these front-loaded welfare gains, the government with commitment must bear long-run welfare losses from stagnant income redistribution and negative pecuniary externalities caused by changes in the factor composition of income that are unfavorable to low-income individuals. The increased taxes and UBI lead to a higher market interest rate and a lower market wage in the long run due to its lower capital-to-labor ratio. These price changes act as negative externalities for low-income individuals whose income composition is biased toward labor. Additionally, since UBI is already substantial in the long run, welfare gains from reducing income inequality (i.e., income redistribution) become insignificant.

These findings imply that the Ramsey planner counterbalances these two types of economic forces over the entire time horizon, by achieving upfront welfare gains from reduced income inequality and factor price changes favoring low-income individuals while delaying welfare losses arising from unfavorable factor price changes for low-income individuals and stagnant income redistribution later on. The significant welfare gain (+2.19%) implies that this method of balancing throughout the entire time horizon is effective when commitment is available.

Without commitment, the previously discussed strategy is not credible because the government cannot provide substantial long-run insurance. If the government without commitment finds itself in the long-run equilibrium of the committed scenario, it will ignore the upfront welfare gains and balance the two types of economic forces in a forward-looking manner in each period. In other words, the government perceives this economy as one where negative pecuniary externalities from changes in the factor composition of income—a reduction in the market wage and an increase in the market interest rate—outweigh income redistribution through taxes and UBI that is mitigated in the long run. As a result, the government will consider a one-time reduction in taxes and UBI, which is all it can do due to its lack of commitment, in the next period to improve welfare.

Households will rationally anticipate this government incentive and understand that this government has no ability to provide as considerable long-run UBI as the government with commitment. This changed expectation on the policy increases precautionary motives, thereby increasing the proportion of capital income among low-income individuals, leading to a deviation from the long-run equilibrium in the committed scenario. The quantitative results imply that this way of making policy decisions by the government results in smaller welfare gains (+0.57%). I conduct the same quantitative exercises, but change the tax base, and find that the outcomes are consistent with those under proportional income taxes.

These results suggest that the degree of government commitment when delivering UBI programs is important for their effectiveness. Relying on legal rules can be more effective than fulfilling the administration's discretion.

**Related Literature:** This paper is closely related to the growing literature that examines various perspectives of UBI using macroeconomic general equilibrium models, such as (Guner et al., 2021; Luduvice, 2021; Santos and Rauh, 2022; Conesa et al., 2023; Daruich and Fernández, 2023). Among them, Conesa et al. (2023) has similarities with this paper in the sense that it examines on the welfare consequences of UBI along the transition. However, the policy aspect this paper focuses on differs. While Conesa et al. (2023) analyzes the effect of various financing plans on welfare outcomes, this paper focuses on the impacts of government commitment on the welfare consequences of UBI. Daruich and Fernández (2023) is also closely related to this study because it investigates the medium- and long-term implications of UBI through intergenerational channels. This paper is complementary to Daruich and Fernández (2023) by revealing another medium- and long-term mechanism of UBI: the feasibility of long-run government commitment is important for the effectiveness of UBI.

This paper is also linked to another strand of literature that examines to what extent unconditional transfers can be part of the optimal tax-transfer schemes (Heathcote and Tsujiyama, 2021; Boar and Midrigan, 2022; Ferriere et al., 2022). They focus on the trade-offs between the progressivity of taxes and the size of transfers in designing optimal policies. This paper is complementary to these studies by focusing on how a different policy aspect—time inconsistency—is involved in the optimal design of taxes and transfers through redistribution channels over time.

This paper also belongs to the stream of macroeconomic literature that examines the implications of time-inconsistent features for government policies, following the seminal study of Kydland and Prescott (1977). A branch of this literature does this task by investigating the features of time-consistent policies in MPEs that depend on the fundamental economic state variables (Cohen and Michel, 1988; Krusell et al., 1996; Krusell and Ríos-Rull, 1999; Klein and Ríos-Rull, 2003; Klein et al., 2008; Corbae et al., 2009; Azzimonti, 2011; Song et al., 2012; Bassetto et al., 2020; Laczó and Rossi, 2020).[5] This paper is closely aligned with this stream, relying on the concept of MPE for the case without commitment. Like this paper, these studies with Markov-perfect policies have found that a lack of commitment makes a significant difference in the optimal design of policies. However, this paper distinguishes itself by providing an understanding of how the interaction between government commitment and individual heterogeneity affects optimal policy design through the incomplete financial markets channel in a theoretical framework.

This paper is also related to macroeconomic studies on constrained efficiency in dynamic general equilibrium models with incomplete-markets that focus on pecuniary externalities through wages and interest rates. For example, Davila et al. (2012) analyzes constrained efficiency in models presented in Aiyagari's (1994) model; Park (2018) conducts a similar analysis with human capital; and Itskhoki and Moll (2019) characterizes the optimal distribution of development policies between workers and entrepreneurs in an economy with financial constraints. While pecuniary externalities play a crucial role in these studies based on centralized economies, the mechanism in the present paper, which is based on a decentralized economy, differs from them. In a centralized economy, constrained efficiency can be achieved through consideration of both dynamic allocations of consumption at the individual level and pecuniary externalities at the aggregate level. In this paper, the government endogenously responds to efficient individual decisions made in competitive equilibrium, considering only the externalities and ignoring the direct welfare impacts of individual decisions, which cancel out in the GEE through the envelope theorem.

[5] Another branch of this literature has focused on designing policies to overcome the time-inconsistent issues through the use of rules (Barro and Gordon, 1983; Rogoff, 1985; Athey et al., 2005), a richer range of policy instruments (Lucas and Stokey, 1983; Debortoli et al., 2017, 2021), and reputational equilibria (Chari and Kehoe, 1990; Domínguez, 2007a,b), using representative-agent models.

The solution method in this paper is a non-negligible, independent contribution to the literature. Broadly, two types of methods are often used to solve macroeconomic models with MPEs. The first is the Klein, Krusell and Ríos-Rull's (2008) approach, which is a solution method using the GEE. This method is accurate and efficient; however, it focuses on solving long-run equilibria and is not general enough to handle heterogeneous-agent models with incomplete markets. The method in this paper is a solution method applicable to heterogeneous-agent models. The other approach is the Krusell and Smith's (1998) method, which applicable to heterogeneous agent models. For example, Corbae et al. (2009) used this approach in their heterogeneous agent economy. However, this simulation-based method is computationally costly because economies without commitment would have more than one aggregate law of motion (e.g., the law of motion for the distributions and the endogenous tax policy function). This structure increases the computational burden in an exponential manner. Additionally, in some cases, the endogenous policy function can be highly non-linear, which poses a challenge for capturing it accurately using the parameterized law of motion in the Krusell and Smith's (1998) method. The approach used in this paper is more accurate and efficient by following the non-simulation-based solution approach that captures the non-linearity in a non-parametric way as in Reiter (2010).

The remainder of this paper proceeds as follows. Section 2 presents the model and defines the equilibria. Section 3 characterizes the equilibria by using the GEE. Section 4 explains the core ideas of the numerical solution algorithm. Section 5 describes the calibration strategy. Section 6 presents the results of the policy analysis. Section 7 concludes this paper. Finally, Appendix A demonstrates the full details of the numerical solution algorithm.

# 2 Model

The quantitative model here builds upon the canonical model of Aiyagari (1994), incorporating wealth effects of labor supply. In this model, given a tax policy rule, heterogeneous households make decisions on consumption, savings, and labor supply at the intensive margin, as in standard incomplete-markets models. A notable difference from standard models is the manner in which the tax/UBI policy is set. It is determined endogenously, according to the government's commitment technology. In an equilibrium state, the tax/UBI policy, individual decisions, and the evolution of the distribution are all consistent with one another, given the government's commitment constraint.

## 2.1 Environment

The model economy is populated by a continuum of infinitely lived households. Their preferences follow

$$E\left[\sum_{t=0}^{\infty}\beta^t u(c_t, 1-n_t)\right] \tag{1}$$

where $c_t$ is consumption, $n_t \in [0,1]$ is labor supply in period $t$ ($(1-n_t)$ refers to leisure), and $\beta$ is the discount factor. Preferences are represented by

$$u(c_t, 1-n_t) = \frac{{c_t}^{1-\sigma}}{1-\sigma} + B\,\frac{(1-n_t)^{1-1/\chi}}{1-1/\chi} \tag{2}$$

where $\sigma$ is the coefficient of relative risk aversion, $B$ is the utility of leisure, and $\chi$ is the Frisch elasticity of labor supply. Note that the preferences here capture the wealth effects of labor supply. Such wealth effects are crucial for a welfare analysis, closely related to efficiency loss. An increase in UBI, for example, decreases the overall labor supply, shrinking the size of the aggregate economy and playing a role in reducing welfare.

The representative firm produces output with a constant returns to scale. The firm's technology is represented by

$$Y_t = zF(K_t, N_t) = zK_t^{\theta}N_t^{1-\theta} \tag{3}$$

where $z$ is the total factor productivity (TFP), $K_t$ is the quantity of aggregate capital, $N_t$ is the quantity of aggregate labor, and $\theta$ is the capital income share. Capital depreciates at the rate of $\delta$ each period.

In each period, households confront an uninsurable, idiosyncratic shock $\epsilon_t$ to their wage that follows an AR-1 process:

$$\log(\epsilon_{t+1}) = \rho_\epsilon \log(\epsilon_t) + \eta_{t+1}^{\epsilon} \tag{4}$$

where $\eta_{t+1}^{\epsilon} \sim N(0, \sigma_\epsilon^2)$. Using the method in Rouwenhorst (1995), I approximate the AR-1 process as a finite-state Markov chain with transition probabilities $\pi_{\epsilon,\epsilon'}$ from state $\epsilon$ to state $\epsilon'$. Households earn $w_t\epsilon_t n_t$ as their labor income where $w_t$ is the market equilibrium wage. They can self insure through assets $a_t$. Such households have capital income of as much as $r_t a_t$ where $r_t$ is the equilibrium risk-free interest rate.

The government obtains its tax revenue by levying taxes on household capital and labor income at proportional flat tax rate, $\tau_t$.[6] Given its tax revenue, the government covers a constant government spending $G$, and the rest is used for UBI $T_t \geq 0$ (lump-sum transfers). The government runs a balanced budget each period:

$$G + T_t = \tau_t \left[ r_t K_t + w_t N_t \right] \text{ where } T_t \geq 0. \tag{5}$$

It is worth noting several points regarding the government budget presented above. First, in representative agent models, the tax-transfer scheme presented above always results in zero transfers, regardless of the degree of government commitment. Therefore, individual heterogeneity is the main driver for the provision of UBI, and this paper will explore how the availability of government commitment affects the extent of this provision. Second, the progressivity of transfers is not taken into account in the baseline economy, similar to Daruich and Fernández (2023), but for a different reason. This setting is chosen to conduct policy exercises with MPEs later, where the baseline economy must be in one of its equilibria. Since UBI is in the form of lump-sum transfers, a state with progressive transfers is incompatible with MPEs.

## 2.2 Competitive Equilibrium, Exogenous Policy

In this section, I define the competitive equilibrium, given an exogenous tax and transfer policy. I start with a setting to address time-inconsistent policies. To describe problems with commitment (the Ramsey problem), household dynamic problems need to be represented in a sequential manner. At the beginning of each period, households differ from one another in asset holdings $a$ and labor productivity $\epsilon$. $\mu_t(a, \epsilon)$ denotes the distribution of households in period $t$. Given a sequence of prices $\{r_t, w_t\}_{t=0}^{\infty}$, income taxes $\{\tau_t\}_{t=0}^{\infty}$, and UBI $\{T_t\}_{t=0}^{\infty}$, households in period $t$ solve

$$v_t(a, \epsilon) = \max_{c_t,\, a_{t+1},\, n_t} u(c_t(a, \epsilon), 1 - n_t(a, \epsilon)) + \beta \sum_{\epsilon_{t+1}} \pi_{\epsilon_t, \epsilon_{t+1}} v_{t+1}(a_{t+1}(a, \epsilon), \epsilon_{t+1}) \tag{6}$$

such that

$$c_t + a_{t+1} = (1 - \tau_t) w_t \epsilon_t n_t + (1 + r_t(1 - \tau_t))a + T_t.$$

**Definition 2.2.1.** ***Sequential Competitive Equilibrium (SCE), given a Sequence of Taxes***
Given $G$, an initial distribution $\mu_0(\cdot)$, and a sequence of income taxes and UBI $\{\tau_t, T_t\}_{t=0}^{\infty}$, an SCE is a sequence of prices $\{w_t, r_t\}_{t=0}^{\infty}$, a sequence of allocations $\{c_t, n_t, a_{t+1}, K_t, N_t\}_{t=0}^{\infty}$, a sequence of value functions $\{v_t(\cdot)\}_{t=0}^{\infty}$, and a sequence of distributions over the state space $\{\mu_t(\cdot)\}_{t=1}^{\infty}$ , such that for all $t$

[6] In a later section, I relax the assumption of the tax base.

(i) Given $\{\tau_t, T_t\}_{t=0}^{\infty}$ and $\{w_t, r_t\}_{t=0}^{\infty}$, the decision rules $a_{t+1}(a, \epsilon)$ and $n_t(a, \epsilon)$ solve the household problem in (6), and $v_t(a, \epsilon)$ is the associated value function.

(ii) The representative agent firm engages in competitive pricing:

$$w_t = (1 - \theta) z \left( \frac{K_t}{N_t} \right)^{\theta} \tag{7}$$

$$r_t = \theta z \left( \frac{K_t}{N_t} \right)^{\theta - 1} - \delta. \tag{8}$$

(iii) The factor markets clear:

$$K_t = \int a \, \mu_t(\mathbf{d}(a \times \epsilon)) \tag{9}$$

$$N_t = \int \epsilon \, n_t(a, \epsilon) \, \mu_t(\mathbf{d}(a \times \epsilon)) \tag{10}$$

(iv) The government budget constraint (5) is satisfied.

(v) Let $\mathcal{B}(A \times E)$ denote the Borel $\sigma$-algebra on $A \times E$. For any $B \in \mathcal{B}(A \times E)$, the sequence of distributions over individual $\{\mu_t(\cdot)\}_{t=1}^{\infty}$ satisfies

$$\mu_{t+1}(B) = \int_{\{(a,\epsilon) | (a_{t+1}(a,\epsilon), \epsilon_{t+1}) \in B\}} \sum_{\epsilon_{t+1}} \pi_{\epsilon, \epsilon_{t+1}} \mu_t(\mathbf{d}(a \times \epsilon)). \tag{11}$$

In contrast, to handle problems without commitment, it is convenient to present the household dynamic problems in a recursive manner. In addition to the individual state variables $a$ and $\epsilon$, there are two aggregate state variables, including the distribution of households $\mu(a, \epsilon)$ over $a$ and $\epsilon$ and income tax $\tau$.[7] A variable with a prime symbol denotes its value in the next period.

Let $v(a, \epsilon; \mu, \tau)$ denote the value of households associated with a state of $(a, \epsilon; \mu, \tau)$. They

[7]Note that either $\tau$ or $T$ is a state variable because once one of them chosen, the other is fixed in the balanced government budget, $G + T = \tau[rK + wN]$. I here choose $\tau$ as a state variable.

solve

$$v(a,\epsilon;\mu,\tau) = \max_{c>0,\ a'\geq \underline{a},\ 0\leq n\leq 1} \left[ \frac{c^{1-\sigma}}{1-\sigma} + B\,\frac{(1-n)^{1-1/\chi}}{1-1/\chi} + \beta \sum_{\epsilon'} \pi_{\epsilon,\epsilon'} v(a',\epsilon';\mu',\tau') \right] \tag{12}$$

such that

$$c + a' = (1-\tau)\,w(\mu)\,\epsilon\, n + (1 + r(\mu)(1-\tau))\,a + T$$
$$\tau' = \Psi(\mu,\tau)$$
$$\mu' = \Gamma(\mu,\tau,\tau') = \Gamma(\mu,\tau,\Psi(\mu,\tau))$$

where $\underline{a} \leq 0$ is a borrowing limit, $\tau' = \Psi(\mu,\tau)$ is the perceived law of motion of taxes, and $\mu' = \Gamma(\mu,\tau,\tau')$ is the law of motion for the distribution over households. Note that households here solve the above problem given an exogenous tax policy function $\tau' = \Psi(\mu,\tau)$.

**Definition 2.2.2.** ***Recursive Competitive Equilibrium (RCE), given a Law of Motion for Tax.*** Given $G$ and $\Psi(\mu,\tau)$, an RCE is a set of prices $\{w(\mu), r(\mu)\}$, a set of decision rules for households $g^a(a,\epsilon;\mu,\tau)$ and $g^n(a,\epsilon;\mu,\tau)$, a value function $v(a,\epsilon;\mu,\tau)$, a distribution of households $\mu(a,\epsilon)$ over the state space, and the law of motion for the distribution of households $\Gamma(\mu,\tau,\Psi(\mu,\tau))$ such that

(i) Given $\{w(\mu), r(\mu)\}$, the decision rules $a' = g^a(a,\epsilon;\mu,\tau)$ and $n = g^n(a,\epsilon;\mu,\tau)$ solve the household problem in (12), and $v(a,\epsilon;\mu,\tau)$ is the associated value function.

(ii) The representative agent firm engages in competitive pricing:

$$w(\mu) = (1-\theta) z \left(\frac{K}{N}\right)^{\theta} \tag{13}$$
$$r(\mu) = \theta z \left(\frac{K}{N}\right)^{\theta-1} - \delta. \tag{14}$$

(iii) The factor markets clear:

$$K = \int a\,\mu(\mathbf{d}(a\times\epsilon)) \tag{15}$$
$$N = \int \epsilon\, g^n(a,\epsilon;\mu,\tau)\,\mu(\mathbf{d}(a\times\epsilon)) \tag{16}$$

(iv) The government budget constraint (5) is satisfied.

(v) The law of motion for the distribution of households $\mu' = \Gamma(\mu,\tau,\Psi(\mu,\tau))$ is consistent with individual decision rules and the stochastic process of $\epsilon$.

## 2.3 Competitive Equilibrium, Endogenous Policy

In this section, I define competitive equilibria where the income tax is endogenously determined. I model the tax choice in two ways: the optimal income tax with commitment (Ramsey problem) and the optimal income tax without commitment (time-consistent case). I begin with the Ramsey problem.

**Definition 2.3.1.** ***The Ramsey Problem:***
***An SEC with the Optimal Income Tax and UBI with Commitment***
Given $\mu_0$, the government chooses $\{\tau_t\}_{t=0}^{\infty}$ such that

$$\{\tau_t\}_{t=0}^{\infty} = \underset{\{\tilde{\tau}_t\}_{t=0}^{\infty}}{\operatorname{argmax}} \int E_0 \sum_{\hat{t}=0}^{\infty} \beta^{\hat{t}} u\big(c^*_{\hat{t}}(a,\epsilon|\{\tilde{\tau}_{\hat{t}}\}_{t=0}^{\infty}), 1-n^*_{\hat{t}}(a,\epsilon|\{\tilde{\tau}_t\}_{t=0}^{\infty})\big)\mu_0(\mathbf{d}(a\times\epsilon))$$

where $\big(c^*_{\hat{t}}(a,\epsilon|\{\tau_t\}_{t=0}^{\infty}), n^*_{\hat{t}}(a,\epsilon|\{\tau_t\}_{t=0}^{\infty})\}_{\hat{t}=0}^{\infty}\big)$ is an allocation in Definition (2.2.1) in period $\hat{t}$, given $\{\tilde{\tau}_t\}_{t=0}^{\infty}$.

Note that the consumption and labor decisions at time $t$, $(c^*_t, n^*_t)$, are affected not only by the policy in this period but also by a sequence of income taxes.[8] Therefore, the current decisions are influenced by past and future taxes, which can lead to the time-inconsistency issue.

For the case without commitment, I have employed the definition in Krusell and Ríos-Rull (1999); Klein and Ríos-Rull (2003).

**Definition 2.3.2.** ***An RCE with the Optimal Income Tax and UBI without Commitment***

(i) A set of functions $\{w(\cdot), r(\cdot), g^a(\cdot), g^n(\cdot), v(\cdot), \Gamma(\cdot)\}$ satisfy Definition (2.2.2).

(ii) For each $(\mu,\tau)$, the government chooses $\tau^{WO}(\mu,\tau)$ such that

$$\tau^{WO}(\mu,\tau) = \underset{\tilde{\tau}'}{\operatorname{argmax}} \int \hat{V}(a,\epsilon;\mu,\tau,\tilde{\tau}')\mu(\mathbf{d}(a\times\epsilon)) \tag{17}$$

where

$$\hat{V}(a,\epsilon;\mu,\tau,\tilde{\tau}') = \max_{c>0,\, a'\geq \underline{a},\, 0\leq n\leq 1}\left[\frac{c^{1-\sigma}}{1-\sigma} + B\,\frac{(1-n)^{1-1/\chi}}{1-1/\chi} + \beta\sum_{\epsilon'}\pi_{\epsilon,\epsilon'}v(a',\epsilon';\mu',\tilde{\tau}')\right]$$

such that

$$c + a' = (1-\tau)\,w(\mu)\,\epsilon\, n + (1+r(\mu)(1-\tau))\,a + T$$

$$\tau' = \tilde{\tau}',\ \text{and thereafter } \tau'' = \Psi(\mu',\tau'=\tilde{\tau}') \tag{18}$$

$$\mu' = \Gamma(\mu,\tau,\tilde{\tau}),\ \text{and thereafter } \mu'' = \Gamma(\mu',\tilde{\tau},\tau''=\Psi(\mu',\tau'=\tilde{\tau}')) \tag{19}$$

[8] Note that taxes have a one-for-one response to UBI here.

(iii) $a' = \hat{g}_a(a, \epsilon; \mu, \tilde{\tau} : \tilde{\tau}')$ and $n = \hat{g}_n(a, \epsilon; \mu, \tilde{\tau} : \tilde{\tau}')$ solve (17) at prices that clear markets and satusfy the government budget constraint, and $\Gamma$ is consistent with individual decisions and the stochastic process of $\epsilon$.

(iv) For each $(\mu, \tau)$, the policy outcome function satisfies $\Psi(\mu, \tau) = \tau^{WO}(\mu, \tau)$.

Note that the government's solution to the problem is consistent with the sub-perfect Nash equilibrium obtained with the one-shot deviation principle. In the economy with the optimal income tax without commitment, the government implements a time-consistent optimal policy as in Klein and Ríos-Rull (2003); Corbae et al. (2009): a tax rate that is sequentially chosen only for the next period while maximizing its utilitarian welfare under this commitment constraint. The government cannot commit to the future tax rate from the period after the next period. Thus, if the chosen tax rate $\tilde{\tau}'$ deviates from the equilibrium tax policy function $\Psi(\cdot)$, future tax rates will follow the equilibrium tax policy function $\Psi(\cdot)$ because of the lack of commitment. (18) presents such dynamics. The law of motion for the distribution of households $\Gamma(\cdot)$ has to capture all the changes in the evolution of distributions caused by the deviation of the income tax from the equilibrium tax function, as shown in (19). In equilibrium, for each aggregate state $(\mu, \tau)$, the government's choice of the tax rate, $\tau^{WO}(\mu, \tau)$, should be equal to the equilibrium tax function $\psi(\mu, \tau)$, as presented in (iv).

# 3 Characterization of the Equilibria

## 3.1 The Case without Commitment

Despite the definitions that demonstrate the government's decision-making process for taxes and UBI, the underlying economic trade-offs that inform these decisions can be challenging to observe. This section employs the generalized-Euler equation (GEE) approach, introduced by Klein et al. (2008), to characterize the equilibria, thereby illuminating the economic trade-offs considered by the government in making decisions on tax and UBI policies.

The GEE approach provides insight into the economic forces driving the policymaker's decision through its FOC. This condition can be derived by utilizing the Benveniste-Scheinkman condition, also known as the envelope condition, to eliminate terms related to the partial derivative of the value function. In this section, I will first analyze the case without commitment and then proceed to the case with commitment. To determine the FOC of the government, I take the partial derivative of the government's value, represented by $\hat{V}$, with respect to the next period's income tax rate, represented by $\tilde{\tau}'$, in its vicinity of the equilibrium value $\tau'$:

$$\begin{aligned}
0 =& \frac{d}{d\tilde{\tau}'}\Big|_{\tilde{\tau}'=\tau'} \int \hat{V}(a,\epsilon;\mu,\tau,\tilde{\tau}')\mu(d(a\times\epsilon)) \\
=& \int \frac{d}{d\tilde{\tau}'}\Big|_{\tilde{\tau}'=\tau'} \\
& \Big[u((1-\tau)\, w(\mu)\epsilon\, \tilde{g}^n(a,\epsilon;\mu,\tau,\tilde{\tau}') + (1+r(\mu)(1-\tau))a + T - \tilde{g}^a(a,\epsilon;\mu,\tau,\tilde{\tau}'), 1-\tilde{g}^n(a,\epsilon;\mu,\tau,\tilde{\tau}')) \\
& + \beta\sum_{\epsilon'}\pi_{\epsilon,\epsilon'} v(\tilde{g}^a(a,\epsilon;\mu,\tau,\tilde{\tau}'),\epsilon';\mu'=\Gamma(\mu,\tau,\tilde{\tau}'),\tilde{\tau}')\Big]\mu(\mathbf{d}(a\times\epsilon))
\end{aligned} \tag{20}$$

Note that the tilde over $g^a$ and $g^n$ means that the deviation of $\tilde{\tau}'$ from its equilibrium value $\tau'$ makes the decision rules for assets and labor supply different from those in equilibrium.

An obscure part in computing the FOC (20) is the derivative of $v$ with respect to $\mu'$. Let $m_q$ denote the $q-th$ moment of $\mu$. I assume that $Q \in \mathbb{N}$ exists such that $\{m_q\}_{q=1}^Q$ is a sufficient statistic of $\mu$. This assumption allows me to replace $\mu$ with $\{m_q\}_{q=1}^Q$ in the value function. Then, the FOC (20) is given by:

$$\begin{aligned}
0 = \int \Big[& u_c(c,1-n)\cdot\Big((1-\tau)w(m_1)\epsilon\frac{\partial\tilde{g}^n(a,\epsilon;\{m_q\}_{q=1}^Q,\tau,\tau')}{\partial\tau'} - \frac{\partial\tilde{g}^a(a,\epsilon;\{m_q\}_{q=1}^Q,\tau,\tau')}{\partial\tau'}\Big) \\
& - u_n(c,1-n)\cdot\frac{\partial\tilde{g}^n(a,\epsilon;\{m_q\}_{q=1}^Q,\tau,\tau')}{\partial\tau'} \\
& + \beta\sum_{\epsilon'}\pi_{\epsilon,\epsilon'}\Big\{\frac{\partial v(a',\epsilon',\{m'_q\}_{q=1}^Q,\tau')}{\partial a'}\cdot\frac{\partial\tilde{g}^a(a,\epsilon;\{m_q\}_{q=1}^Q,\tau,\tau')}{\partial\tau'} \\
& + \sum_{q=1}^Q\Big(\frac{\partial v(a',\epsilon',\{m'_q\}_{q=1}^Q,\tau')}{\partial m'_q}\cdot\frac{dm'_q}{d\tau'}\Big) + \frac{\partial v(a',\epsilon',\{m'_q\}_{q=1}^Q,\tau')}{\partial\tau'}\Big\}\Big]\mu(\mathbf{d}(a\times\epsilon)).
\end{aligned} \tag{21}$$

where $u_c(c)$ is the derivative of $u$ in $c$ and $m_1$ is the first moment of $\mu$. Note that the first moment is sufficient to determine $w$ and $r$. I will eliminate the derivative terms of the value $\frac{\partial v}{\partial a'}$, $\frac{\partial v}{\partial m'_q}$, and $\frac{\partial v}{\partial\tau'}$ by using the Benveniste-Scheinkman condition and then interpret the economic logic behind the equations. First, $\frac{\partial v}{\partial a'}$ is given by:

$$\frac{\partial v(a,\epsilon;\{m_q\}_{q=1}^Q,\tau)}{\partial a} = u_c(c,1-n)(1+r(m_1)(1-\tau)). \tag{22}$$

Similarly, with $\frac{\partial T}{\partial \tau} = rK + wN$, $\frac{\partial v}{\partial \tau}$ is given by:

$$\begin{aligned}
\frac{\partial v(a,\epsilon;\{m_q\}_{q=1}^{Q},\tau)}{\partial \tau} =& u_c(c,1-n)\Big(w(m_1)(N-\epsilon\cdot g^n(a,\epsilon;\{m_q\}_{q=1}^{Q},\tau)+r(m_1)(K-a)\Big)\\
&+\omega(a,\epsilon,\{m_q\}_{q=1}^{Q},\tau)\cdot\frac{\partial g^a(a,\epsilon;\{m_q\}_{q=1}^{Q},\tau)}{\partial \tau}\\
&+\zeta(a,\epsilon;\{m_q\}_{q=1}^{Q},\tau)\cdot\frac{\partial g^n(a,\epsilon;\{m_q\}_{q=1}^{Q},\tau)}{\partial \tau}\\
&+\beta\sum_{\epsilon'}\pi_{\epsilon,\epsilon'}\Big\{\sum_{q=1}^{Q}\Big(\frac{\partial v(a',\epsilon';\tau',\{m'_q\}_{q=1}^{Q})}{\partial m'_q}\cdot\frac{dm'_q}{d\tau}\Big)\\
&+\frac{\partial v(a',\epsilon';\tau',\{m'_q\}_{q=1}^{Q})}{\partial \tau'}\cdot\frac{\partial \Psi(\tau,\mu)}{\partial \tau}\Big\}\\
\text{where}\;\; &\zeta(a,\epsilon;\{m_q\}_{q=1}^{Q},\tau) = -\big(u_c(c,1-n)\cdot(1-\tau)w(m_1)\epsilon+u_n(c,1-n)\big)\\
&\omega(a,\epsilon;\{m_q\}_{q=1}^{Q},\tau) = -u_c(c,1-n)+\beta(1+r(m'_1)(1-\tau'))\sum_{\epsilon'}\pi_{\epsilon,\epsilon'}u_c(c',1-n').
\end{aligned} \tag{23}$$

$\xi$ and $\omega$ represent wedges in the Euler equation for consumption and the FOC for optimal leisure choice, respectively.

Note that $u_c(c,1-n)(w(N-\epsilon\cdot g^n)+r(K-a))$ implies an individual welfare change driven by income redistribution via changes in taxes/UBI. This term is one of the key economic forces in characterizing the MPE. Let $\chi$ denote this term in the subsequent discussion:

$$\chi(a,\epsilon;\{m_q\}_{q=1}^{Q},\tau) = u_c(c,1-n)\Big(w(m_1)(N-\epsilon\cdot g^n(a,\epsilon;\{m_q\}_{q=1}^{Q},\tau)+r(m_1)(K-a)\Big). \tag{24}$$

$\chi$ represents *income redistribution* via taxes and UBI because they measure the difference between before- and after-tax income, the extent of which varies across individuals. Individuals with a lower pre-tax income will have more income after receiving UBI, while those with a higher pre-tax income will have less due to heaver taxes. Thus, if an individual's effective labor $\epsilon g^n$ and asset holdings $a$ are below the average values ($N$ and $K$), $\chi$ becomes positive for that individual. In contrast, if an individual's effective labor and assets are above average, $\chi$ becomes negative.

The next step is to eliminate $\frac{\partial v(a',\epsilon';\{m'_q\}_{q=1}^{Q},\tau')}{\partial m'_q}$. It is difficult to determine the required value of $Q$ to obtain sufficient statistics for $\mu$. For the purposes of this analysis, I assume that $Q = 1$, which means that $m_1 = K$ is sufficient for capturing the evolution of the distribution, as in Krusell and Smith (1998). An alternative interpretation of this assumption is that the government only considers changes in future prices and not higher moments of the future distribution when determining income taxes and UBI. This assumption enables a further characterization of the MPE.

With the Benveniste-Scheinkman condition, $\frac{\partial v}{\partial K}$ is given by:

$$\begin{aligned}\frac{\partial v(a,\epsilon;K,\tau)}{\partial K} =& u_c(c,1-n)\Big((1-\tau)(f_{NK}(K)\epsilon\cdot g^n(a,\epsilon;K,\tau)+f_{KK}(K)a)+\frac{\partial T}{\partial K}\Big)\\ &+\zeta(a,\epsilon;K,\tau)\cdot\frac{g^n(a,\epsilon;K,\tau)}{\partial K}+\omega(a,\epsilon;K,\tau)\cdot\frac{g^a(a,\epsilon;K,\tau)}{\partial K}\\ &+\beta\sum_{\epsilon'}\pi_{\epsilon,\epsilon'}\Big\{\frac{\partial v(a',\epsilon'_j;K',\tau')}{\partial K'}\cdot\frac{\partial\Gamma(K,\tau,\tau')}{\partial K}+\frac{\partial v(a',\epsilon';K',\tau')}{\partial\tau'}\cdot\frac{\partial\Psi(K,\tau)}{\partial K}\Big\}. \qquad (25)\end{aligned}$$

Note that $u_c(c,1-n)\big((1-\tau)(f_{NK}(K)\epsilon+f_{KK}(K)a)+\frac{\partial T}{\partial K}\big)$ implies an individual welfare change driven by variations in the factor composition between capital and labor income following an increase in the current tax $\tau$. As discussed in Davila et al. (2012), this effect differs across individuals and depends on the composition of their income. To clarify how this effect is linked to the factor composition of individual income, I proceed with further steps following Davila et al. (2012). Because $f$ is homogeneous of degree 1, $f_{KK}(K,N)K+fKN(K,N)N=0$. In addition, because $T=\tau(rK+wN)-G=\tau(f_KK+f_NN)-G$, $\frac{\partial T}{\partial K}=f_K(K)\tau$. Then, with these conditions, $\frac{\partial v}{\partial K}$ is given by:

$$\begin{aligned}\frac{\partial v(a,\epsilon;K,\tau)}{\partial K} =& u_c(c,1-n)\big((1-\tau)(-\frac{\epsilon\cdot g^n(a,\epsilon;K,\tau)}{N}+\frac{a}{K})f_{KK}(K)K+f_K(K)\tau\big)\\ &+\zeta(a,\epsilon;K,\tau)\cdot\frac{g^n(a,\epsilon;K,\tau)}{\partial K}+\omega(a,\epsilon;K,\tau)\cdot\frac{g^a(a,\epsilon;K,\tau)}{\partial K}\\ &+\beta\sum_{\epsilon'}\pi_{\epsilon,\epsilon'}\Big\{\frac{\partial v(a',\epsilon'_j;K',\tau')}{\partial K'}\cdot\frac{\partial\Gamma(K,\tau,\tau')}{\partial K}+\frac{\partial v(a',\epsilon';K',\tau')}{\partial\tau'}\cdot\frac{\partial\Psi(K,\tau)}{\partial K}\Big\} \qquad (26)\end{aligned}$$

The first term is another important economic force in characterizing the MPE. For further discussion, I refer to this term as $\phi$:

$$\phi(a,\epsilon;K,\tau)=u_c(c,1-n)\big((1-\tau)(-\frac{\epsilon\cdot g^n(a,\epsilon;K,\tau)}{N}+\frac{a}{K})f_{KK}(K)K+f_K(K)\tau\big). \qquad (27)$$

$\phi$ represents *pecuniary externalities* arising from changes in the factor composition of income. It measures the welfare change following a shift of the factor composition of income between $r$ and $w$, which is driven by general equilibrium effects after an increase in $K$. $\phi$ is considered a type of externality because individuals take a sequence of factor prices as given in competitive equilibrium and do not consider the possibility that these prices may vary due to endogenous government policies and their impacts on welfare. Another feature of $\phi$ is that the extent of pecuniary externalities differs across individuals based on their factor composition of income, as noted by Davila et al. (2012). Note that because $f_{KK}(K)K<0$, $f_K(K)\tau>0$, the sign of this term highly depends on the value of $(-\frac{\epsilon\cdot g^n}{N}+\frac{a}{K})$. For example, suppose that there is an increase in $K$. In this case, if $\frac{\epsilon\cdot g^n}{N}$ is substantially larger than $\frac{a}{K}$, indicating that the factor income is biased toward labor (the factor composition of income of the consumption-poor), then $\frac{\partial v}{\partial K}$ is positive because $w$ increases while $r$ is reduced in general equilibrium. In contrast, the consumption-rich, whose factor income is biased more toward capital, are more likely to experience a loss in welfare due to a decline in $r$ reducing their income.

Next, I substitute (22), (23), and (26) into the derivative of the government value in $\tilde{\tau}'$, $\frac{\partial \hat{v}}{\partial \tilde{\tau}'}$, in the FOC (21) and eliminate the partial derivatives of the future values by substituting them out. Additionally, I simplify the notation by employing sequential terms. I refer to $y_{i,t}$ as the variable of $y$ for individual $i$ in period $t$.

Then, $\frac{\partial \hat{V}_{i,t}}{\partial \tau_{t+1}}$ is given by:

$$\frac{\partial \hat{V}_{i,t}}{\partial \tau_{t+1}} = E_{i,t}\left[\sum_{s=1}^{\infty} \beta^s \cdot \left( \underbrace{\phi_{i,t+s}}_{\text{Pecuniary Externalities}} \cdot \underbrace{\frac{\Delta K_{t+s}}{\Delta \tau_{t+1}}}_{\text{Efficiency Effect}} + \underbrace{\chi_{i,t+s}}_{\text{Income Redist.}} \cdot \underbrace{\frac{\Delta \tau_{t+s}}{\Delta \tau_{t+1}}}_{\text{Policy Scale Effect}} \right)\right] \tag{28}$$

where $E_{i,t}$ is the conditional expectation for individual $i$ at time $t$; $\frac{\Delta K_{t+s}}{\Delta \tau_{t+1}}$ is the total variation of aggregate capital $K$ in period $t+s$ caused by a tax rate change at $t+1$; and $\frac{\Delta \tau_{t+s}}{\Delta \tau_{t+1}}$ is the total variation in the taxes in period $t+s$ caused by a tax rate change at $t+1$.[9] A notable feature is that the wedges related to individual decisions disappear above. More precisely, the product terms between these wedges and the derivative of individual decisions, such as $\omega \cdot \frac{\partial g^a}{\partial K}$ and $\zeta \cdot \frac{\partial g^n}{\partial K}$, are offset through the envelope theorem. This canceling out implies that the government does not consider the direct impact of distortions that are involved with individual decisions on welfare. The government recognizes that given a set of policies, individuals make optimal decisions on consumption, saving, and labor supply in competitive equilibrium; therefore, there is no room for varying individual welfare via this channel.

Instead, the government considers the two types of economic forces that are heterogeneous across individuals—pecuniary externalities and income redistribution —along the transition path. Pecuniary externalities work through changes in aggregate capital. For example, if the government increases $\tau_{t+1}$, efficiency is reduced because aggregate capital $K$ falls below the initial level for a period of time ($\frac{\Delta K_{t+s}}{\Delta \tau_{t+1}} < 0$). These changes in $K$ along the transition path induce variations in the factor composition of income between $r$ and $w$ over time. In this case, while $w$ decreases, $r$ increases over the transition path. Meanwhile, income redistribution works through changes in taxes and UBI. For instance, if the economy converges to the long-run equilibrium in a mean-reverting way, its tax rate $\tau$ remains above the initial level for a time once $\tau_{t+1}$ increases ($\frac{\Delta \tau_{t+s}}{\Delta \tau_{t+1}} > 0$). These increases in $\tau$ imply more transfers along the transition path, influencing income redistribution.

Finally, I substitute (28) into the FOC (21). Then, the government-optimal condition is given

[9] The precise definitions are presented in Appendix C.

by:

$$\underbrace{-\int E_{i,t}\left[\sum_{s=1}^{\infty}\beta^s\cdot\phi_{i,t+s}\cdot\frac{\Delta K_{t+s}}{\Delta\tau_{t+1}}\right]\mu_t(\mathbf{d}(a_{i,t}\times\epsilon_{i,t}))}_{\text{Aggregate Pecuniary Externalities}}=\underbrace{\int E_{i,t}\left[\sum_{s=1}^{\infty}\beta^s\cdot\chi_{i,t+s}\cdot\frac{\Delta\tau_{t+s}}{\Delta\tau_{t+1}}\right]\mu_t(\mathbf{d}(a_{i,t}\times\epsilon_{i,t}))}_{\text{Aggregate Income Redistribution}} \tag{29}$$

where

$$\phi_{i,\tilde{t}}=u_c(c_{i,\tilde{t}},1-n_{i,\tilde{t}})\big((1-\tau_{\tilde{t}})(-\frac{\epsilon\cdot g^n_{i,\tilde{t}}}{N_{\tilde{t}}}+\frac{a_{i,\tilde{t}}}{K_{\tilde{t}}})f_{KK}(K_{\tilde{t}})K_{\tilde{t}}+f_K(K_{\tilde{t}})\tau_{\tilde{t}}\big)$$

$$\chi_{i,\tilde{t}}=u_c(c_{\tilde{t}},1-n_{\tilde{t}})\bigg(w_{\tilde{t}}(N_{\tilde{t}}-\epsilon_{\tilde{t}}\cdot g^n_{i,\tilde{t}}+r_{\tilde{t}}(K_{\tilde{t}}-a_{i,\tilde{t}})\bigg).$$

The above government-optimal condition implies that for each period $t$, when deciding $\tau_{t+1}$, the government without commitment strikes a balance between the two types of economic forces at the aggregate level. For further discussion, I refer to the left-hand side as *aggregate pecuniary externalities* because the absolute value of this term indicates the sum of the present discounted value of pecuniary externalities caused by variations in the factor composition of income over all individuals. Analogously, I refer to the right-hand side as *aggregate income redistribution* because this term means the sum of the present discounted value of income redistribution caused by changes in taxes and UBI over all individuals.

The above optimal condition (29) reveals what the government takes into account in its policy decisions. First, the government places greater weight on the interests of the consumption-poor because the two types of economic forces $\phi$ and $\chi$ are weighted with the marginal utility of consumption. Additionally, the distribution of consumption is right-skewed, leading the government to further consider the interests of the consumption-poor. This government attention to the consumption-poor allows me to regard its policy decisions as a cost-benefit analysis of the consumption-poor for taxes and transfers. Second, the consumption-poor experience different welfare changes from the two types of economic forces, as mentioned previously with (24) and (27) . Suppose that the government decides to increase $\tau$. While income redistribution $\chi$ improve welfare for the consumption-poor by increasing their after-tax income via UBI, pecuniary externalities $\phi$ are negative for the consumption-poor—whose income is more biased toward labor—because the tax change reduces $w$ and increases $r$ in general equilibrium. Third, for the consumption-poor, pecuniary externalities $\phi$ are less than or equal to $f_k(K)\tau$ ($\phi\geq f_k(K)\tau$) and income redistribution is non negative ($\chi\geq 0$). Finally, as incomes for the consumption-poor become more equally distributed $(a,\epsilon\cdot g^n)\rightarrow(K,N)$, pecuniary externalities $\phi$ converge to $f_K(K)\tau$ from a negative value, and their income redistribution $\chi$ converges to 0 from a positive value. These findings leads to the following proposition.

**Proposition 1.** *For each period $t$, the government without commitment makes a policy decision as follows:*

1. *When aggregate pecuniary externalities are* ***greater*** *than aggregate income redistribution, the government* ***reduces*** $\tau_{t+1}$.
2. *When aggregate pecuniary externalities are* ***less*** *than aggregate income redistribution, the government* ***raises*** $\tau_{t+1}$.
3. *As the income of the consumption-poor approaches the average, the government* ***reduces*** $\tau_{t+1}$.

The first and second statements posit that pecuniary externalities can be seen as the consumption-poor's marginal cost of increasing $\tau_{t+1}$ and income redistribution can be regarded as their marginal benefit. Accordingly, the government decides to increase (decrease) its tax rate when the marginal cost (i.e., aggregate pecuniary externalities) is greater (less) than the marginal benefit (i.e., aggregate income redistribution ). The third statement is closely related to how $\chi$ and $\phi$ change as the income of the consumption-poor approaches the average. Specifically, the marginal benefit of increasing $\tau_{t+1}$ for the consumption-poor, $\phi$, converges to 0 from a positive value as their income approaches the average. Additionally, as their income approaches the average, in their marginal cost of raising $\tau_{t+1}$, $\chi$, the impact of $f_k(K)\tau$ becomes more pronounced. Therefore, the most effective way to minimize losses from $\chi$ is to reduce $\tau$ when the income of the consumption-poor approaches the average. This insight is distilled into the third statement.

## 3.2 Comparison with Constrained Efficiency

Another interesting investigation for the optimal conditions (29) is to compare this to the planner's optimal condition in Davila et al. (2012) because pecuniary externalities are thoroughly investigated in their study. The planner's optimal condition in Davila et al. (2012) is given by:

$$\omega_{i,t} + \beta \int E_{i,t}\big[\phi_{t+1}\big]\mu(\mathbf{d}(a \times \epsilon)) = 0 \tag{30}$$

where

$$\omega_{i,t} = -u'(c_{i,t}) + \beta(1 + r(K_{t+1})) \sum_{\epsilon'} \pi_{\epsilon,\epsilon'} u'(c_{t+1}) \tag{31}$$

$$\phi_{i,t} = u'(c_{i,t})\big((-\frac{\epsilon_{i,t}}{L_t} + \frac{a_{i,t}}{K_t})f_{KK}(K_t)K_t\big) \tag{32}$$

In contrast with the government's optimal conditions (29) in this paper, the consumption Euler equation part in Davila et al. (2012) does not need to be null. What matters for the social planner is to satisfy this optimal condition considering the pecuniary externality and distortions embedded in the consumption Euler equation. This distinction arises because of different assumptions between the two economies. The Davila, Hong, Krusell and Ríos-Rull's (2012) economy is centralized: the social planner can manipulate individual saving decisions while preserving the constraints caused by incomplete-markets and uninsurable idiosyncratic income risk. This centralized economy assumption makes the consumption Euler equation non-zero. However, the economy in my paper is decentralized. Although the government exists and endogenously determines taxes, individuals optimally choose consumption and saving; therefore, the government has no room for improvement regarding individual consumption dynamic allocations.

### 3.3 Comparison with the Case with Commitment

Regarding the case with commitment (the Ramsey problem), I assume that the government has already taken an optimal sequence of taxes/UBI and now considers a change in its tax rate at time $t+1$. Then, repeating the previous calculations leads me to obtain the government's optimal condition as follows:

$$\underbrace{-\int E_{i,0}\left[\sum_{s=1}^{\infty}\beta^s\cdot\phi_{i,s}\cdot\frac{\Delta K_s}{\Delta\tau_{t+1}}\right]\mu_0(\mathbf{d}(a_{i,0}\times\epsilon_{i,0}))}_{\text{Aggregate Pecuniary Externalities at } t=0} = \underbrace{\int E_{i,0}\left[\sum_{s=1}^{\infty}\beta^s\cdot\chi_{i,s}\cdot\frac{\Delta\tau_s}{\Delta\tau_{t+1}}\right]\mu_0(\mathbf{d}(a_{i,0}\times\epsilon_{i,0}))}_{\text{Aggregate Income Redistribution at } t=0}. \tag{33}$$

In comparison with (29), the government's optimal condition (33) highlights the role of commitment technologies. Although the government always balances the two types of economic forces in both cases, commitment makes a difference in how the balance is struck. With commitment, the government (the Ramsey planner) balances the externalities by taking conditional expectations at time $0$ over the entire time horizon. In contrast, as (29) shows, a lack of commitment causes the government to take a conditional expectation in each period. Because of this difference, commitment leads the government to consider the effect of taxes and transfers in a backward manner. The optimal condition (33) suggests that the government considers how a tax change in period $t+1$ affects not only the current and future economy but also the past. In contrast, as (29) shows, a lack of commitment causes the government to consider the effect of policies only on the current and future economy without taking into account their effect on the past. These findings suggest that the government with commitment makes policy decisions that are optimal at time $0$ but not necessarily desirable when evaluated in a forward-looking manner, thereby leading to time-inconsistent outcomes.

The comparison above clearly illustrates the qualitative differences in the government's policy decisions with and without commitment. However, it does not provide a quantitative assessment of the magnitude of these differences. Additionally, it does not explain how individuals' decisions are affected by the contrasting balancing methods that arise due to the presence or absence of commitment. In a later section, I will conduct a quantitative analysis to address these questions. Before doing so, in the following section, I propose a numerical solution method that allows me to conduct this quantitative analysis.

# 4 Numerical Solution Algorithm

Here, I focus on conveying the key ideas of the numerical solution algorithm. Appendix A demonstrates each step of the algorithm with details, including outcomes related to its efficiency and accuracy.

Although the characterization of the MPE in the previous section helps us better understand the government's decisions on policies, it is not very useful in numerically computing the equilibrium because of its sequential feature. Basically, solving the model entails a substantial computational burden. The law of motion for the distribution of households $\Gamma(\cdot)$ has to be consistent with individual decisions. Additionally, because the labor supply is endogenous with wealth effects, the two factor markets—K and N—must clear. Furthermore, perhaps the most challenging part is finding the equilibrium policy function $\Psi(\cdot)$ that must be consistent with individual decisions and the law of motion for the distribution of households. That is, three equilibrium objects—specifically, individual decisions, $g^n(\cdot)$ and $g^a(\cdot)$, the law of motion for the distribution, $\Gamma(\cdot)$, and the policy function, $\Psi(\cdot)$—interact and have to be consistent with one another in an MPE.

I address the above computational issues by taking ideas from the backward induction method of Reiter (2010). This study introduced a non-simulation-based solution method to solve an incomplete-markets economy with aggregate uncertainty. As in Krusell and Smith's (1998), the Reiter's (2010) approach also reduces the dimension of distributions in the law of motion $\Gamma(\cdot)$ to some finite moments of the distribution, and they are defined across the aggregate finite grid points. However, the way of finding $\Gamma(\cdot)$ differs substantially between the two methods. In Krusell and Smith (1998), their algorithm repeatedly simulates the model economy through the inner and outer loops. In the inner loops, the value is solved given a perceived law of motion for the distribution of households, and the law of motion is updated after a simulation in the outer loop. This procedure is repeated until the perceived law of motion is equal to the updated one.

By contrast, the backward induction method of Reiter (2010) does not simulate the economy to update the law of motion for the distribution of households $\Gamma(\cdot)$; rather, this is updated while solving for the value given a set of proxy distributions across the aggregate finite grid points. Given a proxy distribution, finding the law of motion for the distribution of households $\Gamma(\cdot)$ is feasible by using the moment-consistent conditions. For example, individual decision rules for assets allow me to obtain the information (e.g., the mean or variance) on the aggregate capital in the next period. A simulation step is followed not to update the law of motion for the distribution of households $\Gamma(\cdot)$ but to update a set of proxy distributions across the finite nodes in the aggregate state. Simulations are required much less in Reiter (2010) than in Krusell and Smith (1998), which improves the computational efficiency of the backward induction method. Additionally, with these proxy distributions, the backward induction method allows me to approximate not only the aggregate law of motion for the distribution $\Gamma(\cdot)$ but also the tax policy function $\Psi(\cdot)$. This is feasible because, with the value function, these endogenous tax functions can be directly obtained by solving (17).

However, I wish to clarify that I cannot directly apply the Reiter's (2010) method to the model in this paper because of the presence of off-equilibrium paths after one-shot deviations that are required to find the sub-perfect Nash equilibrium. In the incomplete-markets economy with aggregate uncertainty, for which the Reiter's (2010) method is originally designed, the distribution of aggregate shocks (TFP) $Z$ is ergodic. Thus, all the aggregate states $Z$ are not measure zero. With a positive probability, all the states in $Z$ are realized on the equilibrium path. However, cases in the MPE do not have this property. For example, in the economy without commitment, the government chooses a tax rate by comparing one-time deviation policies, as in (17). Some tax paths will not be reached on the equilibrium path but the corresponding value needs defining to solve the problem that the government confronts.

To cope with this issue, I make three variations to the original backward induction method of Reiter (2010). First, as mentioned above, I approximate not only the aggregate law of motion for the distribution of households but also the endogenous tax policy function. I find these mappings in a nonparametric way as in Reiter (2010). Second, I arrange distributions for all types of off the equilibrium paths by taking the initial distribution of the simulations as the previous proxy distribution for each finite grid point of the aggregate state. Figure 1 shows various transitions from off the equilibrium to the steady-state equilibrium in the case without commitment. Finally, I modify the way of constructing the reference distributions, which is required to update the proxy distributions in Reiter (2010), by reflecting the features of the MPE, in which how many times a tax rate off-the-equilibrium takes place is unknown before simulation. Appendix A demonstrates the full details of the solution method, in addition to its performances in efficiency and accuracy.

Because of these somewhat complex variations in the Reiter's (2010) method, one might consider simply using the Krusell and Smith's (1998) method to solve this model. However, their approach is less efficient in addressing this class of models in MPE. First, finding the two aggregate laws of motion–$\Gamma$ and $\Psi$—is computationally very costly when using this simulation-based solution method. When this method is employed to solve the economy in this paper, this process is the same as adding another outer loop to the outer loop in the Krusell and Smith's (1998) origi-

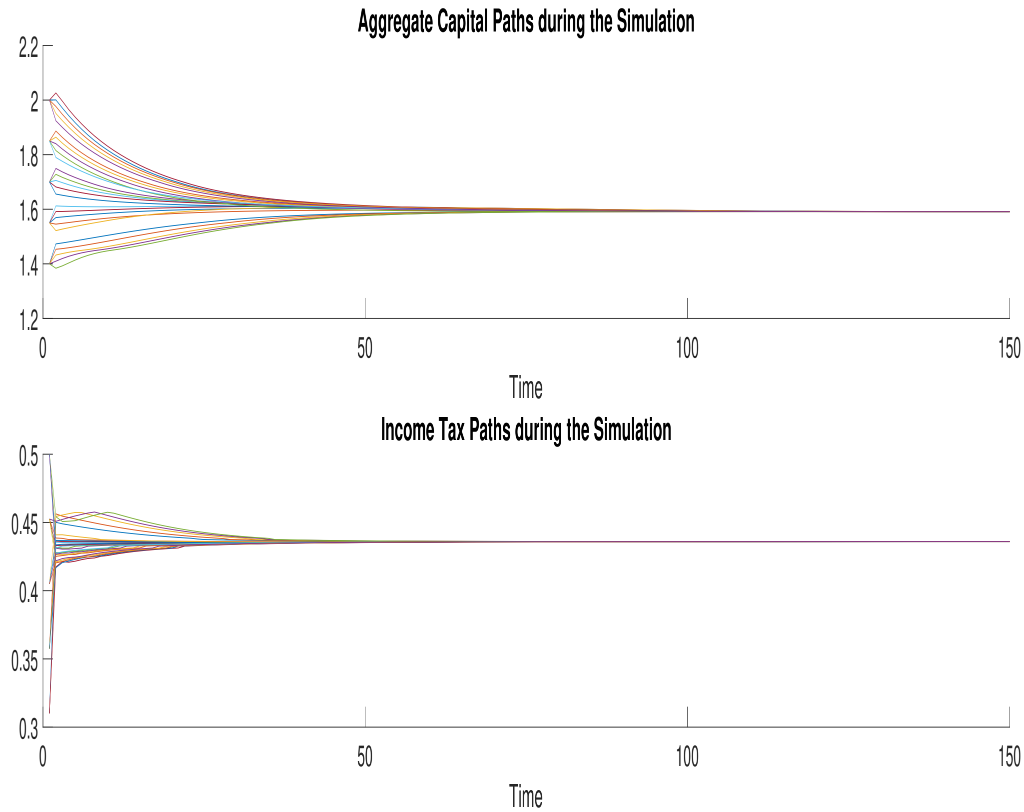


Figure 1: Transitions from off the Equilibrium to the Equilibrium

nal algorithm, thereby exponentially increasing the computational burden. Second, the parametric assumption of the Krusell and Smith's (1998) approach acts as a barrier because the equilibrium tax function $\Psi(\cdot)$ could be severely non-linear in the aggregate state. The parametric assumption works well when the law of motion for household distributions $\Gamma(\cdot)$ is close to linear. However, it is possible for $\Psi(\cdot)$ to be severely non-linear and the method in this paper can cope with this non-linearity.

Regarding the case with commitment (the Ramsey problem), I employ the approach in Dyrda and Pedroni (2022) by parameterizing the transition path of income taxes as follows:

$$\tau_t = \left(\sum_{i=0}^{m_{x0}} \alpha_i^x P_i(t)\right) \exp(\lambda t) + (1 - \exp(-\lambda t)) \left(\sum_{j=0}^{m_{xF}} \beta_j^x P_j(t)\right), \quad t \leq t_F \tag{34}$$

where $\{P_i(t)\}_{i=1}^{m_{x0}}$ and $\{P_j(t)\}_{j=0}^{m_{xF}}$ are families of the Chebyshev polynomial; $m_{x0}$ and $m_{xF}$ are the orders of the polynomial approximation for the short- and long run dynamics, respectively; $\{\alpha_i^x\}_{i=0}^{m_{x0}}$ and $\{\beta_j^x\}_{j=0}^{m_{xF}}$ are weights on the consecutive elements of the family; and $\lambda$ controls the convergence rate of the fiscal instrument. This setting assumes that the economy has the long-run steady state at the latest in period $t_F$. I first choose $m_{x0} = m_{xF} = 2$ and $t_F = 250$. Then, I seek $\{\alpha_0^x, \alpha_1^x, \alpha_2^x, \beta_0^x, \beta_1^x, \beta_2^x, \lambda\}$ that maximize the welfare function of the utilitarian government at time 0.[10]

# 5 Calibration

I calibrate the model to capture the features of the U.S. economy. I divide the parameters into two groups. The first set of parameters requires solving the stationary distribution of the model to match the moments generated by the model with their empirical counterparts. The other set of the parameters is determined outside the model. I take the values of these parameters from the macroeconomic literature and policies.

Table 1: Parameter Values of the Baseline Economy

| | Description (Target) | Value |
|---|---|---|
| $\beta$ | Discount factor ($K/Y = 3$) | 0.951 |
| $B$ | Utility of leisure (AVG Wrk Hrs = 1/3) | 3.803 |
| $\sigma$ | Relative risk aversion | 2 |
| $\chi$ | Frisch elasticity of labor supply | 0.75 |
| $\underline{a}$ | Borrowing constraint | 0 |
| $z$ | TFP | 1 |
| $\theta$ | Capital income share | 0.36 |
| $\delta$ | Depreciation rate | 0.08 |
| $\rho_\epsilon$ | Persistence of wage shocks | 0.955 |
| $\sigma_\epsilon$ | STD of wage shocks | 0.20 |
| $G$ | Government spending | $G/Y = 0.19$ |
| $\tau$ | AVG income tax | 0.31 |

Table 1 displays the parameters. I internally calibrate two parameters: the discount factor $\beta$ and the utility of leisure $B$. $\beta$ is selected to match a capital-to-output ratio of 3, and $B$ is chosen to reproduce an average of hours worked of 8 hours a day. The other parameters are determined outside the model. The coefficient of relative risk aversion  is set to 2. The Frisch elasticity of labor supply $\chi$ is taken to be $0.75$. I set the borrowing constraint as $\underline{a} = 0$. The TFP $z$ is set as 1, and the capital income share $\theta$ is chosen to reproduce the empirical finding that the share of capital income is 0.36. The annual depreciation rate $\delta$ is 8 percent. The persistence of wage shocks $\rho_\epsilon$ is set to be 0.955, and the standard deviation of wage shocks $\sigma_\epsilon$ is taken as 0.20. The values of $\rho_\epsilon$ and $\sigma_\epsilon$ lie in the range of those frequently used in the literature. Government spending $G$ is set so

[10]The inclusion of lump-sum transfers prevents the non-existence of a Ramsey steady state, which is examined in Straub and Werning (2020). Further details are provided in Dyrda and Pedroni (2022).

that the fraction of government spending out of GDP is equal to 0.19. The flat income tax rate is chosen as 0.31 in the baseline economy, implying the ratio of transfers to GDP to be 0.046, which is closer to its empirical counterpart of, 0.044.[11]

# 6 Results

In this section, I quantitatively explore how commitment technologies affect the aggregate economy, inequality, and welfare. For this, I compare the economy with the time-consistent optimal policy to the economy with the time-inconsistent optimal policy with the Ramsey planner. I assume that the initial economy begins at the calibrated steady-state through all the exercises and compare their equilibrium results along the transition path. I begin with the exercise based on proportion income taxes. Later, I will change the tax base.

## 6.1 Time-Consistent versus Time-Inconsistent Policy: Income Tax

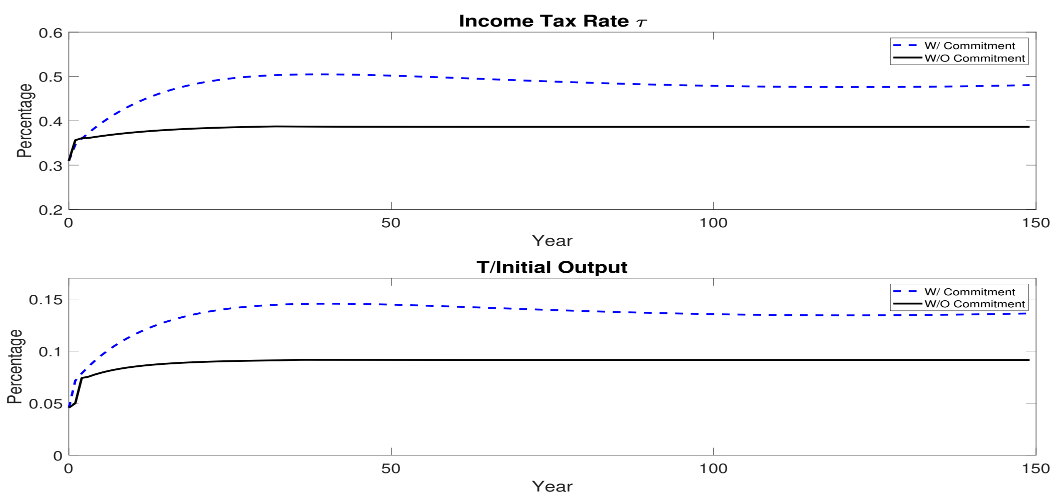


Figure 2: Time-Consistent and Time-Inconsistent Policies: Taxes/UBI Transition Paths

Figure 2 shows the time-consistent and time-inconsistent optimal taxes and the implied ratio of UBI to the initial output. The top panel of Figure 2 implies that the government with commitment chooses more substantial income taxes than the government with the time-consistent optimal policy over the entire transition path. The Ramsey planner initially increases income taxes by 16 percentage points and then reduces them thereafter. However, without commitment, the government with the optimal income tax raises taxes by only 2 percentage points. This gap in tax policies

[11] I take the value from Jang et al. (2021) that excludes Social Security and Medicare in their calculation to reflect the lack of a lifetime structure.

results in differences in the size of UBI. The time-inconsistent optimal income tax-UBI economy generates larger UBI than the time-consistent optimal income tax-UBI economy. The ratio of UBI to the initial GDP in the case with commitment gradually increases by 9.2 percentage points, but in the case without commitment, it only increases by 4.7 percentage points.

Table 2: Welfare Outcomes According to Commitment (Income Taxes)

| Welfare (CEV) | With Commitment | Without Commitment |
|---|---|---|
| OPT INC TAX | +2.19% | +0.57% |

This distinction in income taxes and UBI creates different welfare consequences. Table 2 shows that welfare, measured by the consumption equivalent variation (CEV) of the utilitarian welfare function, is significantly higher in the case with commitment. The time-inconsistent optimal policy improves welfare by 2.19 percent, while the time-consistent optimal policy improves welfare by 0.57 percent. To understand this disparity in welfare consequences, I examine how differently the inputs of the social welfare function vary over time according to the availability of commitment. Note that welfare increases when the overall level of consumption and leisure increases, and their inequality is reduced.

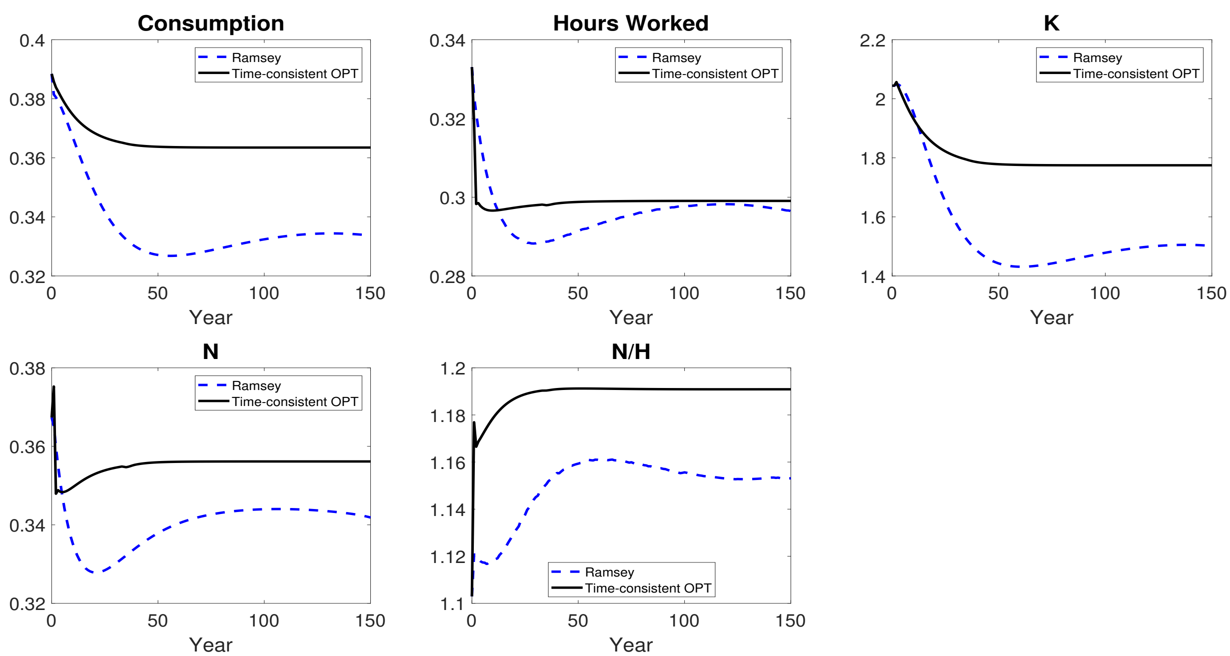


Figure 3: Time-Consistent and Time-Inconsistent Income Tax-UBI: Aggregate Outcomes

Figure 3 illustrates the changes in the levels of the aggregate variables. The figure suggests that the case without commitment generates more efficient outcomes. All the aggregate variables in the economy are larger with the time-consistent optimal policy than with the time-inconsistent optimal policy. This result may seem obvious because lower taxes in the time-consistent case cause fewer distortions. However, this finding might be unclear when examining the welfare consequences. Despite the fact that consumption is substantially larger in the case with the time-consistent policy, the gaps in hours worked are not significant and do not offer a complete understanding of welfare implications

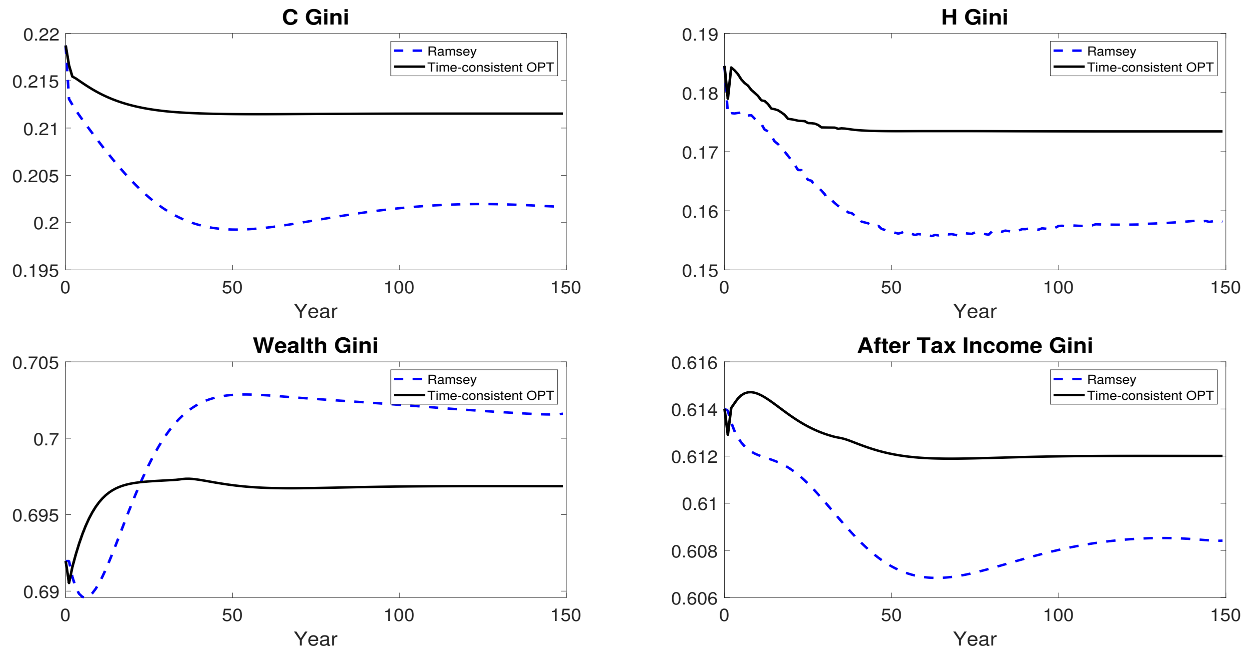


Figure 4: Time-Consistent and Time-Inconsistent Income Tax-UBI: Distributional Outcomes

Figure 4 illustrates the inequalities in consumption, hours worked, wealth, and after-tax income. The figure suggests that more significant welfare improvements in the case with the time-inconsistent optimal policy are driven mainly by larger reductions in inequalities in consumption and leisure. Although consumption inequality, measured by the Gini coefficient, decreases by less than 5 percent with the time-consistent optimal policy, it is reduced by approximately 10 percent with the time-inconsistent optimal tax. Similarly, inequality in hours worked is also reduced more in the case with the time-inconsistent optimal policy. These findings imply that the commitment instrument allows the Ramsey planner to better manage the evolution of inequality, leading to a better welfare outcome. However, this explanation does not provide a clear economic logic behind the quantitative outcomes. To better understand the economic logic behind these differences, I employ the theoretical findings in the previous chapter to interpret these quantitative outcomes.

Figure 5 shows the dynamics of the factor prices $w$ and $r$ depending on the availability of commitment. This figure suggests that the government with commitment (the Ramsey planner)

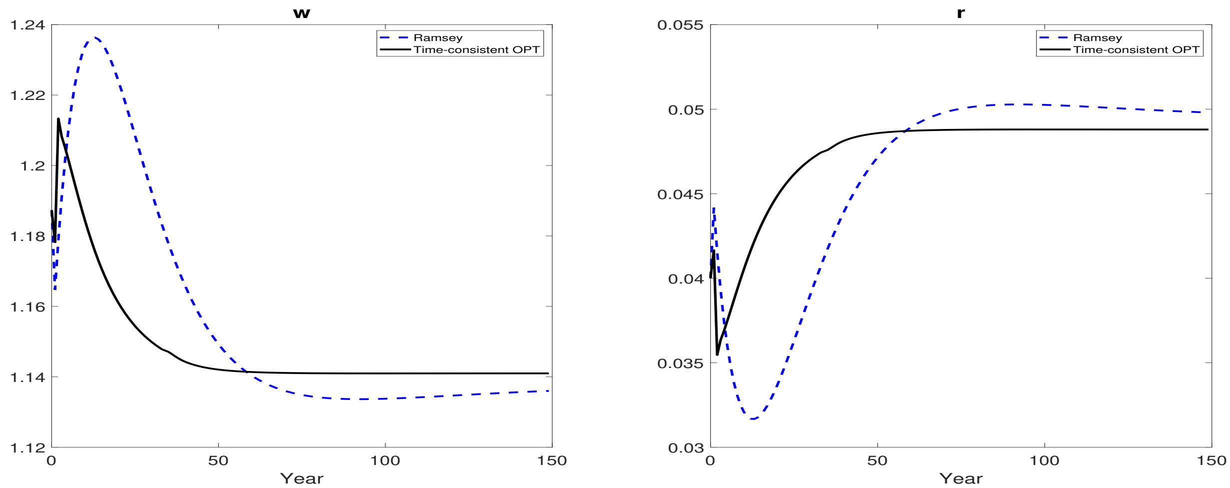


Figure 5: Time-Consistent and Time-Inconsistent Income Tax-UBI: Dynamics of $w$ and $r$

benefits from pecuniary externalities in the early phase of the transition by exploiting differences in the speed of adjustments between $K$ and $N$. Of course, in the long run, the increased tax rate in the case with commitment reduces the ratio of $K$ to $N$, leading to a decrease in $w$ and an increase in $r$. However, during their adjustment, the ratio of $K$ to $N$ increases in the early phase of the transition because the adjustment of $K$ is slower than that of $N$, leading to a rise in $w$ and a reduction in $r$. These price changes help improve the welfare of the consumption-poor, who are better represented by the government, through pecuniary externalities during the early transition.

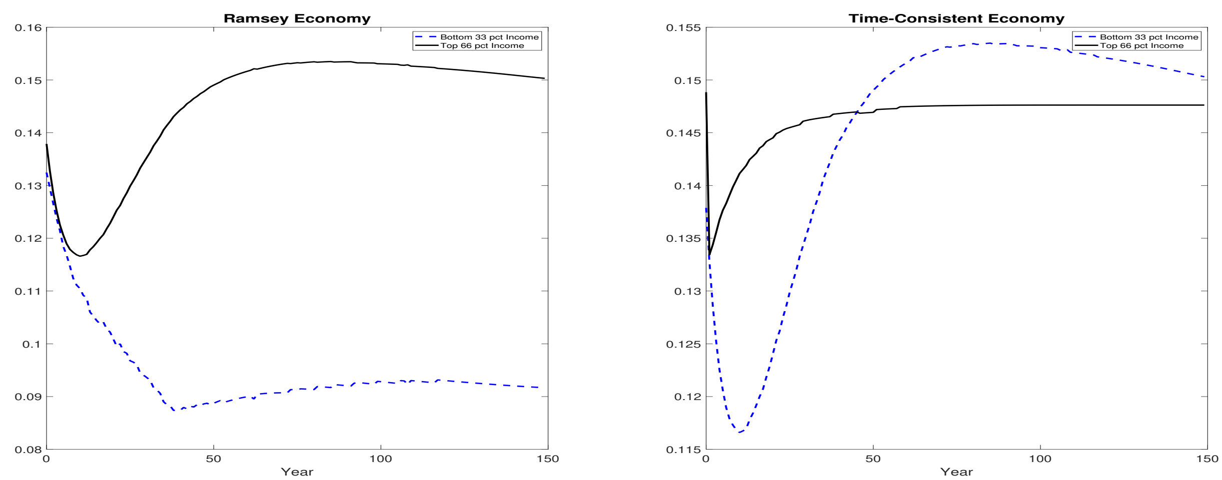


Figure 6: Time-Consistent and Time-Inconsistent Income Tax-UBI: Proportion of Capital Income

Figure 6 displays the proportion of capital income out of total disposable income across income groups, based on the government commitment technologies. The left panel of Figure 6 indicates that in the economy with commitment, low-income individuals reduce their precautionary savings, which is not observed for the middle- and high-income groups. Initially, all income groups experience a decrease in their capital income proportion as the market interest rate declines during the early transition. However, while middle- and high-income individuals increase their capital income proportion as the interest rate rises in the long run, low-income individuals maintain their reduced savings. This decrease in precautionary savings facilitates a reduction in after-tax income inequality among low-income individuals because inequality in capital income is more severe than inequality in other types of income, such as labor income and transfers. This adjustment leads to rapid reductions in inequality in after-tax income early in the transition, as shown in the bottom-right panel of Figure 4, generating front-loaded welfare gains from income redistribution.

The availability of government commitment technologies is the key to driving the difference in savings across income groups. The right panel of Figure 6 shows that without commitment, the economy does not have such permanent decline in savings for low-income individuals. Low-income individuals in the economy without commitment have larger precautionary savings because the government does not provide as substantial UBI as the government with commitment in the long run. These findings imply that the front-loaded welfare gains from both types of economic forces are driven by the government's ability to commit to future policies.

In exchange for these upfront welfare gains, the government with commitment endures welfare losses in the long run. In the long run, $w$ is lower and $r$ is higher than their initial values, and these changes become negative pecuniary externalities for the consumption-poor, as observed in Figure 5. Additionally, in the long run, as Figure (33) shows, there is no further reduction in after-tax income inequality, leading to minuscule welfare gains from income redistribution. Recall that, as (33) shows, the sum of all pecuniary externalities is equal to that of all income redistribution. These findings imply that the government with commitment takes front-loaded welfare gains via income redistribution and positive pecuniary externalities while enduring welfare losses from mitigated income redistribution and negative pecuniary externalities.

Note that the above strategy is not credible without commitment. Suppose that the government without commitment is in the long-run equilibrium of the economy with commitment. As shown in (29), a lack of commitment causes the government to measure the costs and benefits of changing a tax rate in a forward-looking manner. Since front-loaded welfare gains are disregarded, the marginal cost of raising the tax rate, which equals the aggregate pecuniary externalities, is greater than that of the initial economy, and the marginal benefit, which equals the aggregate income redistribution , is close to zero. Proposition 1 indicates that the government, in this case, will decide a one-time reduction its tax rate and UBI, which is all it can do due to the lack of commitment, in the next period. Individuals will recognize that the government cannot guarantee as substantial UBI as the government with commitment and will rationally anticipate the government's motive to reduce its tax rate and UBI in the next period. This change in expectations of the policy will cause individuals to increase precautionary savings. As a result, the long-run economy with commitment is no longer sustainable.

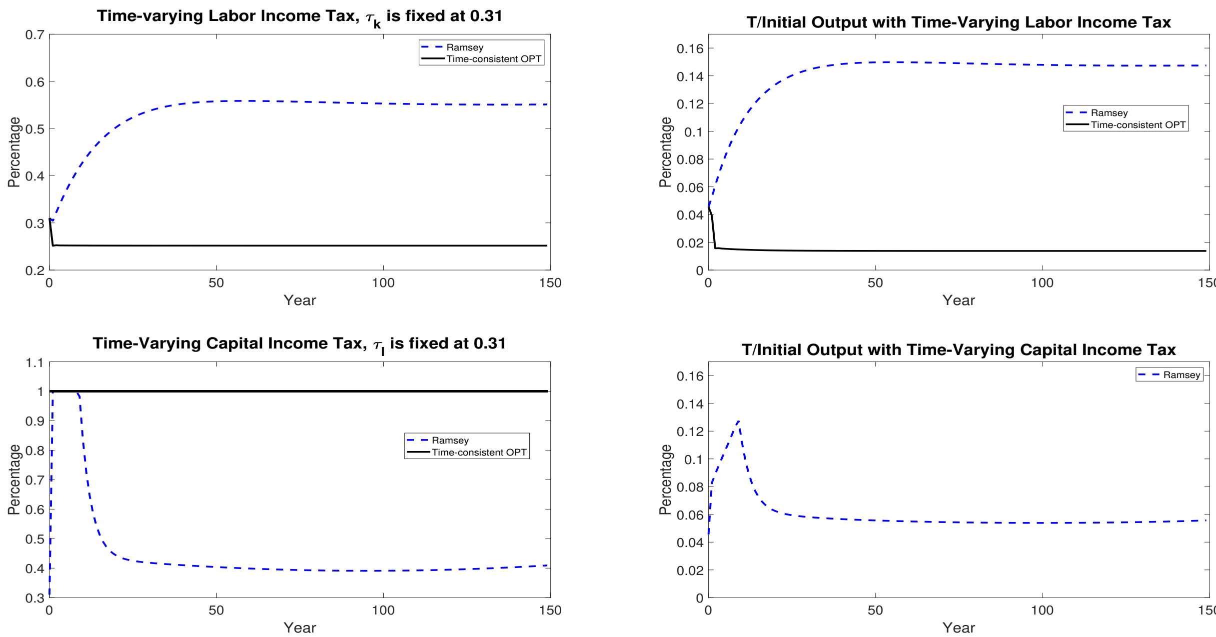


Figure 7: Time-Consistent and Time-Inconsistent Policies: Taxes/UBI by Tax Base

## 6.2 Time-Consistent versus Time-Inconsistent Policy by Tax Base

Figure 7 displays the time-consistent and time-inconsistent optimal taxes and transfers according to the tax base. The top (bottom) panels illustrate the outcomes when labor (capital) income taxes are permitted to vary over time while the capital (labor) income tax is held at the calibrated level of 0.31. In this section, I concentrate on the cases where labor income taxes change over time instead of the cases where capital income taxes change over time. The bottom panels demonstrate that when capital income tax is the only instrument available to the government, the time-consistent government raises it to the maximum rate of 100 percent. This extremely high capital income tax has been analyzed theoretically in previous studies, such as Chari and Kehoe (1990). These studies have shown that the time-consistent government makes the capital income tax confiscatory, resulting in no savings. In my quantitative exercise, time-consistent capital income taxes reduce the size of the economy to such an extent that the tax revenue cannot cover the exogenous government spending in its budget.[12] The results of the time-inconsistent capital income tax economy are

[12] To have sustainable capital income tax rates, the size of government spending might need to be endogenous by assuming households to value it, as in Klein et al. (2008). The current setting does not have this component.

presented in Appendix B, which are in line with those in Dyrda and Pedroni (2022).

Table 3: Welfare Outcomes According to Commitment (Labor Income Taxes)

| Welfare (CEV) | With Commitment | Wihtout Commitment |
|---|---|---|
| OPT Labor INC TAX ($\tau_k$ is given by 0.31) | +2.20% | -1.24% |

Based on the top panels of Figure 7, the economies with labor income taxes deliver similar messages as in the cases with proportional income taxes. The government with commitment raises its labor income taxes early in the transition, resulting in more substantial UBI. However, without commitment, the government reduces its labor income taxes, resulting in lower UBI. Table 3 shows that the welfare consequences are also similar to the cases with proportional income taxes: the time-inconsistent government brings more significant welfare improvements to the economy than the time-consistent government.

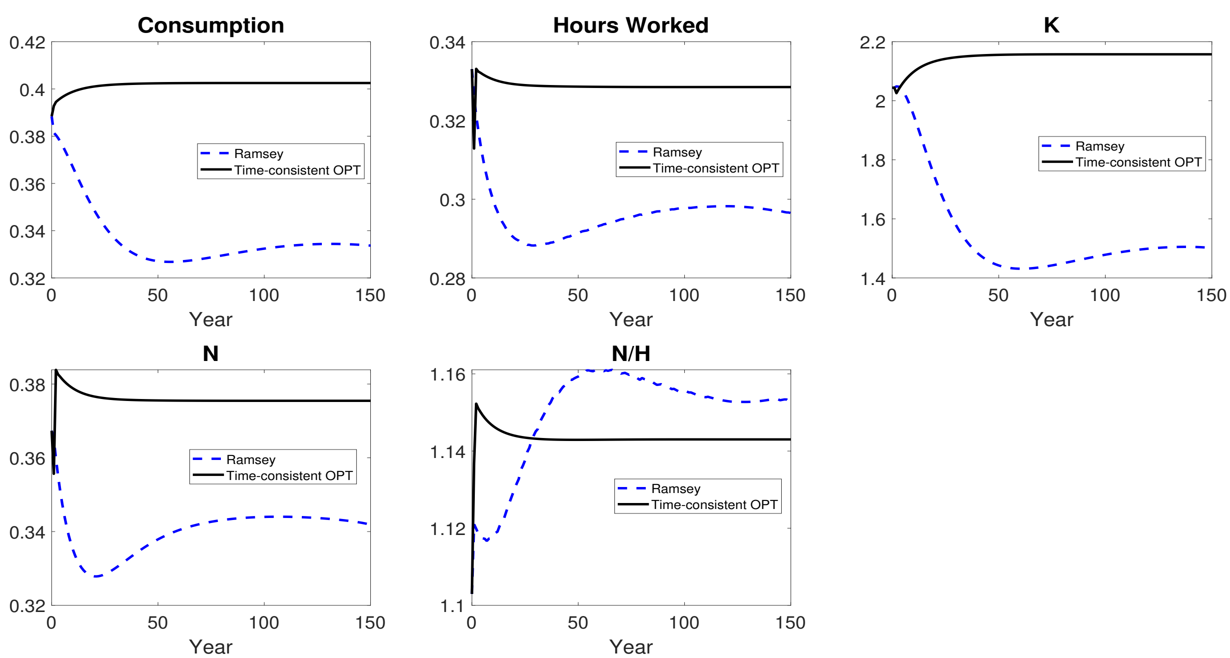


Figure 8: Time-consistent and Time-inconsistent Labor Income Tax-UBI: Aggregate Outcomes

Figure 8 shows that heavier labor income taxes in the case with commitment lead to a less efficient economy than without commitment, which is in line with the results in the cases with proportional income taxes. In the economy with commitment, aggregate consumption, hours worked, capital, and effective labor are lower than those in the case without commitment because heavier labor income taxes in the case with commitment lead to a greater loss in efficiency, causing the economy to shrink. Although the dynamics of the effective labor to hours worked ratio exhibit a different pattern, this gap is due to the larger differences in transfers driven by heavier labor income taxes inducing labor market selection for the case with commitment. The dynamics of overall aggregate variables are similar to those in the cases with proportional income taxes.

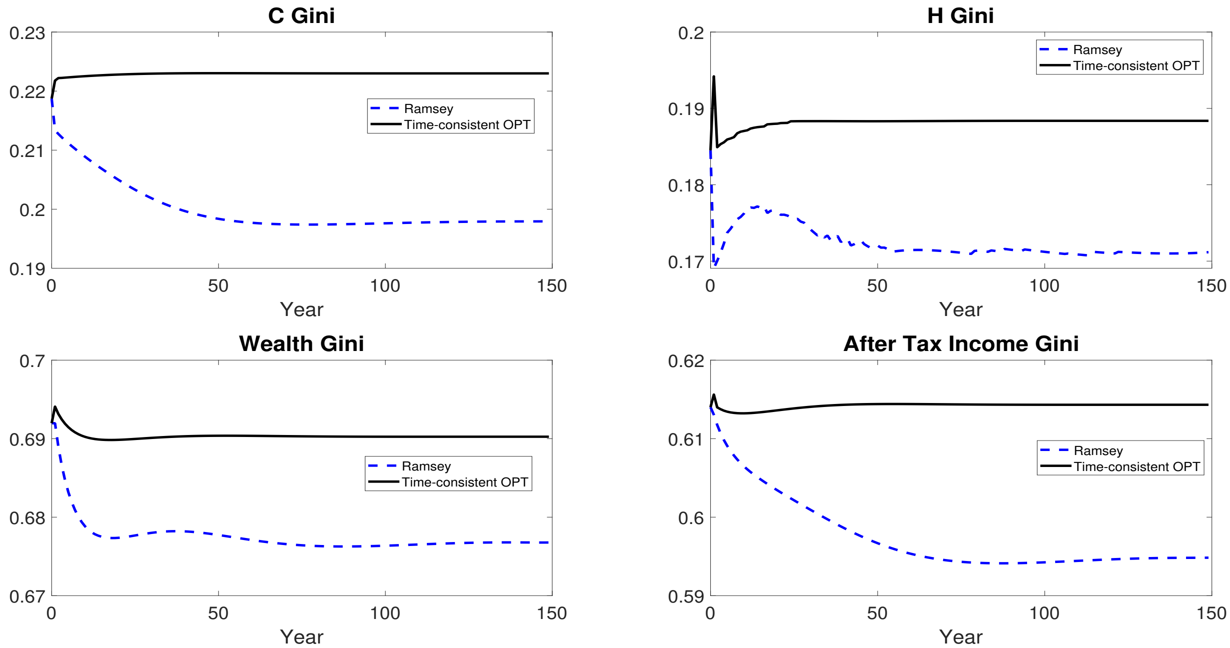


Figure 9: Time-consistent and Time-inconsistent Labor Income Taxes: Distributional Outcomes

Figure 9 shows that the dynamics of inequality play a crucial role in the welfare outcomes in both cases. The economy with commitment experiences more substantial UBI, which results in reduced inequality in after-tax income. This reduction in inequality leads to more equally distributed consumption and hours worked, ultimately improving welfare. In contrast, without commitment, the government reduces taxes and transfers, leading to more significant inequality in after-tax income. This increase in inequality worsens welfare in the economy. Overall, these findings are consistent with those in the cases with proportional income taxes. To obtain a better understanding, I apply the theoretical findings to the quantitative results.

Note that the government with commitment obtains positive externalities from reduced income inequality, as the bottom-right panel of Figure 9 shows. Furthermore, Figure 10 indicates that it also obtains positive pecuniary externalities from changes in the factor composition of income—increases in $w$ and decreases in $r$—in the early phase of the transition. These price changes seem inconsistent with the outcomes after increasing labor income taxes, which implies the opposite changes due to an increase in the capital-to-labor ratio in the long run. Nonetheless, as Figure 8 shows, increases in the labor income tax reduce the aggregate capital and labor supply in the early phase of the transition. Additionally, because the speed of adjustment in capital is slower than that

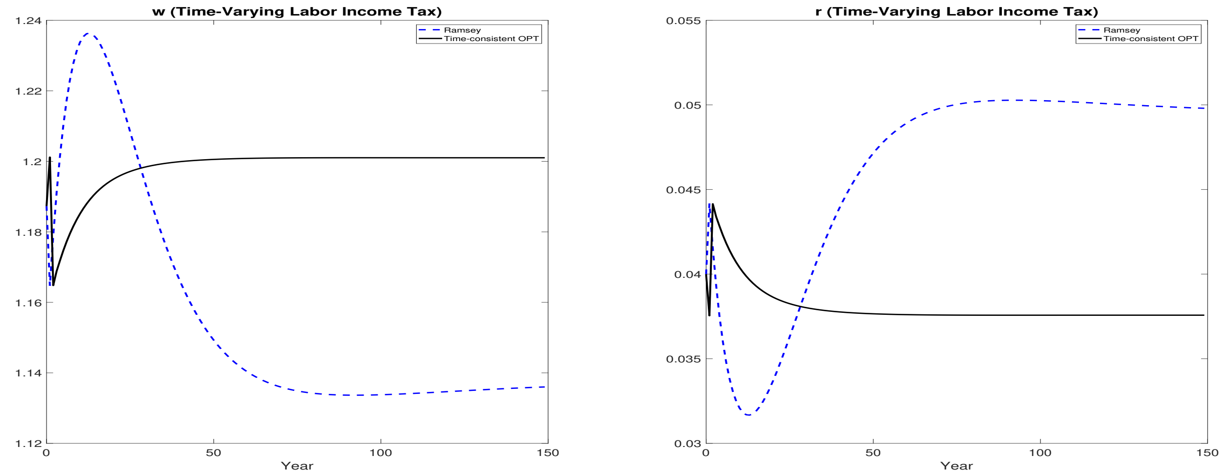


Figure 10: Time-Consistent and Time-Inconsistent Labor Income Tax-UBI : Dynamics of $w$ and $r$

in labor, the government obtains front-loaded positive pecuniary externalities from increases in $w$ and decreases in $r$, along with positive externalities from reduced income inequality.

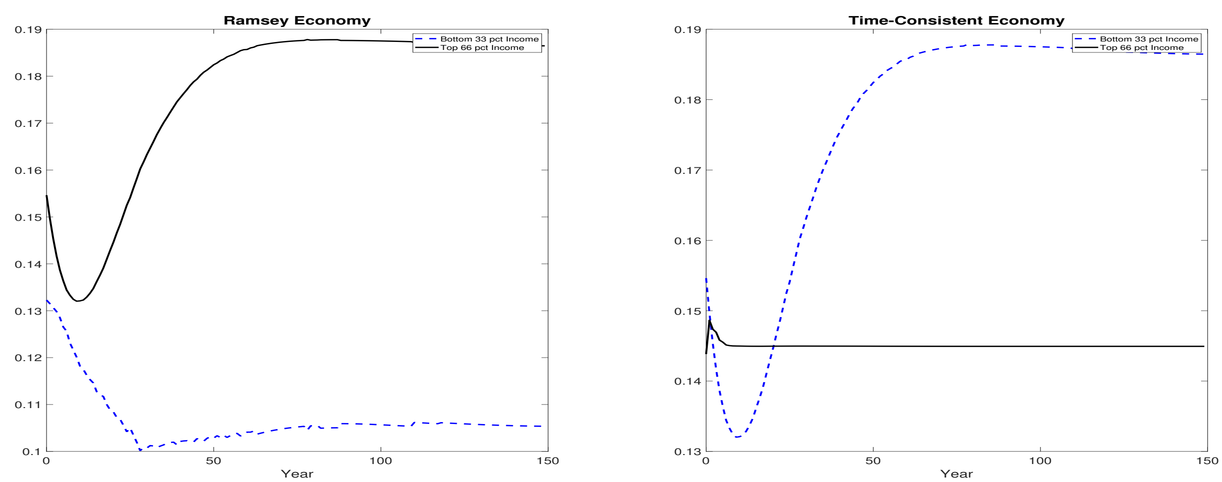


Figure 11: Time-Consistent and Time-Inconsistent Labor Income Tax-UBI : Proportion of Capital Income

Consistent with the results from the exercise with proportional income taxes, Figure 11 demonstrates that government commitment induces a reduction in precautionary savings for low-income individuals. With commitment, the government can provide substantial UBI, leading to a decrease in precautionary savings, as seen in the case of proportional income taxes. This reduction in precautionary savings induces overall inequality in after-tax income, as observed in Figure 9.

As mentioned previously, the reduction in precautionary savings decreases inequality in after-tax income because capital income has more severe inequality than labor income and lump-sum transfers. Therefore, as observed in Figure 9, this change contributes to redistributing income during the early transition—front-loaded income redistribution. However, for the economy without commitment, low-income individuals do not reduce their savings; instead, they increase their precautionary savings because UBI is smaller than transfers in the baseline economy. These differences lead to disparities in the distribution of the aggregate variables. These findings imply that commitment is the main driving force behind upfront welfare gains via both types of externalities, which is in line with that in the case with proportional income taxes.

For these upfront welfare gains, in the long run, the government with commitment must endure welfare losses from negative pecuniary externalities and mitigated income redistribution. As Figure 10 shows, the factor prices are unfavorable for low-income individuals—an increase in $r$ and a decrease in $w$. This change in the factor composition of income leads pecuniary externalities to be negative. At the same time, in the long run, because UBI increased during the initial transition and maintains its level thereafter, income redistribution becomes stagnant in the long run—mitigated income redistribution. These results imply that the government with commitment balances the two types of economic forces by allocating positive externalities in the early phase of the transition while putting negative externalities afterward. More substantial welfare improvements in the case with commitment suggest that this way of balancing is better for welfare consequences.

Without commitment, this strategy is not credible. Suppose that, as in the case with proportional income taxes, the government without commitment is in the long-run equilibrium of the economy with commitment. This government ignores the upfront welfare changes and compares the two types of economic forces in a forward-looking manner. From the government's perspective, negative pecuniary externalities from the factor composition of income are more significant than stagnant changes in income inequality. Therefore, the government is willing to do a one-time rebalancing between the two types of economic forces by reducing labor income taxes and UBI because doing so will alleviate negative externalities from lower wages and spare more income under the lack of commitment. individuals acknowledge that the government lacks the ability to commit to future policies and rationally anticipate this government's incentive, thereby changing their expectation on future policies. As a result, individuals have stronger precautionary motives, leading to increases in labor supply and savings. Consequently, the long-run equilibrium in the case of commitment is not maintained.

# 7 Conclusion

This paper examines how the availability of government commitment affects the optimal design of UBI along the transition. To this end, I arrange a dynamic game between heterogeneous individuals and a benevolent government in the standard incomplete-markets model and characterize and solve for its equilibria according to its commitment technologies. I find that when making policy decisions on income taxes and UBI, commitment affects how the government balance the two types of economic forces: income redistribution through taxes and UBI and pecuniary externalities from changing in the factor composition of income. Commitment allows the government to balance throughout the entire transition; however, without commitment, the government strikes a balance in a forward-looking manner in each period.

I assess the magnitude of this qualitative difference using a quantitative method in a calibrated economy. The quantitative analysis shows that commitment has significant impacts on the government's policy decisions along the equilibrium path. The key mechanism is that commitment enables the government to provide substantial UBI in the long run, resulting in upfront welfare gains while delaying welfare losses to the long run. Without commitment, the government lacks the ability to provide such long-run public insurance and cannot obtain as large welfare gains as the government with commitment.

Note that the solution method itself could provide many opportunities for studying unexplored research topics. Given the fundamental feature of Reiter (2010), this solution method can be compatible with aggregate uncertainty. This research direction makes it possible to extend the previous fiscal policy analyses with incomplete markets Bhandari et al. (2017a,b), investigating the implications of governments' political and commitment. Another exciting application of the method is addressing the interactions between policies and life-cycle dimensions. The Kim's (2021) method makes this direction reachable. She extends the Reiter's (2010) backward induction method to solve an overlapping generations model. Such analyses are deferred to future work.

# References

**Aiyagari, S Rao**, "Uninsured idiosyncratic risk and aggregate saving," *Quarterly Journal of Economics*, 1994, *109* (3), 659–684.

**Athey, Susan, Andrew Atkeson, and Patrick J Kehoe**, "The optimal degree of discretion in monetary policy," *Econometrica*, 2005, *73* (5), 1431–1475.

**Azzimonti, Marina**, "Barriers to investment in polarized societies," *American Economic Review*, 2011, *101* (5), 2182–2204.

**Barro, Robert J and David B Gordon**, "Rules, discretion and reputation in a model of monetary policy," *Journal of Monetary Economics*, 1983, *12* (1), 101–121.

**Bassetto, Marco, Zhen Huo, and José-Víctor Ríos-Rull**, "Institution Building without Commitment," *Working Paper*, 2020.

**Bhandari, Anmol, David Evans, Mikhail Golosov, and Thomas J Sargent**, "Fiscal policy and debt management with incomplete markets," *Quarterly Journal of Economics*, 2017, *132* (2), 617–663.

**__ , __ , __ , and __** , "Public debt in economies with heterogeneous agents," *Journal of Monetary Economics*, 2017, *91*, 39–51.

**Boar, Corina and Virgiliu Midrigan**, "Efficient redistribution," *Journal of Monetary Economics*, 2022, *131*, 78–91.

**Calvo, Guillermo A**, "On the Time Consistency of Optimal Policy in a Monetary Economy," *Econometrica*, 1978, *46* (6), 1411–1428.

**Carroll, Christopher D**, "The method of endogenous gridpoints for solving dynamic stochastic optimization problems," *Economics Letters*, 2006, *91* (3), 312–320.

**Chari, Varadarajan V and Patrick J Kehoe**, "Sustainable plans," *Journal of Political Economy*, 1990, *98* (4), 783–802.

**Cohen, Daniel and Philippe Michel**, "How should control theory be used to calculate a time-consistent government policy?," *Review of Economic Studies*, 1988, *55* (2), 263–274.

**Conesa, Juan Carlos Conesa, Bo Li, and Qian Li**, "A quantitative evaluation of universal basic income," *Journal of Public Economics, Forthcoming*, 2023.

**Corbae, Dean, Pablo D'Erasmo, and Burhanettin Kuruscu**, "Politico-economic consequences of rising wage inequality," *Journal of Monetary Economics*, 2009, *56* (1), 43–61.

**Daruich, Diego and Raquel Fernández**, "Universal basic income: A dynamic assessment," *American Economic Review, Forthcoming*, 2023.

**Davila, Julio, Jay H Hong, Per Krusell, and José-Víctor Ríos-Rull**, "Constrained efficiency in the neoclassical growth model with uninsurable idiosyncratic shocks," *Econometrica*, 2012, *80* (6), 2431–2467.

**Debortoli, Davide, Ricardo Nunes, and Pierre Yared**, "Optimal time-consistent government debt maturity," *Quarterly Journal of Economics*, 2017, *132* (1), 55–102.

**__ , __ , and __** , "Optimal fiscal policy without commitment: Revisiting Lucas-Stokey," *Journal of Political Economy*, 2021, *129* (5), 1640–1665.

**Domínguez, Begoña**, "On the time-consistency of optimal capital taxes," *Journal of Monetary Economics*, 2007, *54* (3), 686–705.

**__** , "Public debt and optimal taxes without commitment," *Journal of Economic Theory*, 2007, *135* (1), 159–170.

**Dyrda, Sebastian and Marcelo Pedroni**, "Optimal Fiscal Policy in a Model with Uninsurable Idiosyncratic Income Risk," *Review of Economic Studies*, 2022, *90* (2), 744–780.

**Ferriere, Axelle, Philipp Grübener, Gaston Navarro, and Oliko Vardishvili**, "On the optimal design of transfers and income-tax progressivity," *International Finance Discussion Paper*, 2022, (1350).

**Guner, Nezih, Remzi Kaygusuz, and Gustavo Ventura**, "Rethinking the welfare state," *CEPR*

*Discussion Paper No. DP16275*, 2021.

**Haan, Wouter J Den**, "Assessing the accuracy of the aggregate law of motion in models with heterogeneous agents," *Journal of Economic Dynamics and Control*, 2010, *34* (1), 79–99.

**Heathcote, Jonathan and Hitoshi Tsujiyama**, "Optimal income taxation: Mirrlees meets Ramsey," *Journal of Political Economy*, 2021, *129* (11), 3141–3184.

**Itskhoki, Oleg and Benjamin Moll**, "Optimal development policies with financial frictions," *Econometrica*, 2019, *87* (1), 139–173.

**Jang, Youngsoo, Takeki Sunakawa, and Minchul Yum**, "Heterogeneity, Transfer Progressivity and Business Cycles," 2021.

**Kim, Heejeong**, "Inequality, disaster risk, and the great recession," *Review of Economic Dynamics*, 2021.

**Klein, Paul and José-Víctor Ríos-Rull**, "Time-consistent optimal fiscal policy," *International Economic Review*, 2003, *44* (4), 1217–1245.

**_ , Per Krusell, and José-Víctor Ríos-Rull**, "Time-consistent public policy," *Review of Economic Studies*, 2008, *75* (3), 789–808.

**Krusell, Per and Anthony A Smith Jr**, "Income and wealth heterogeneity in the macroeconomy," *Journal of political Economy*, 1998, *106* (5), 867–896.

**_ and José-Víctor Ríos-Rull**, "On the size of US government: political economy in the neoclassical growth model," *American Economic Review*, 1999, *89* (5), 1156–1181.

**_ , Vincenzo Quadrini, and José-Víctor Ríos-Rull**, "Are consumption taxes really better than income taxes?," *Journal of Monetary Economics*, 1996, *37* (3), 475–503.

**Kydland, Finn E and Edward C Prescott**, "Rules rather than discretion: The inconsistency of optimal plans," *Journal of Political Economy*, 1977, *85* (3), 473–491.

**Laczó, Sarolta and Raffaele Rossi**, "Time-consistent consumption taxation," *Journal of Monetary Economics*, 2020, *114*, 194–220.

**Lucas, Robert and Nancy Stokey**, "Optimal fiscal and monetary policy in an economy without capital," *Journal of Monetary Economics*, 1983, *12* (1), 55–93.

**Luduvice, André Victor D**, "The macroeconomic effects of universal basic income programs," 2021.

**Park, Yena**, "Constrained efficiency in a human capital model," *American Economic Journal: Macroeconomics*, 2018, *10* (3), 179–214.

**Reiter, Michael**, "Recursive computation of heterogeneous agent models," *manuscript, Universitat Pompeu Fabra*, 2002, pp. 28–35.

_ , "Solving the incomplete markets model with aggregate uncertainty by backward induction," *Journal of Economic Dynamics and Control*, 2010, *34* (1), 28–35.

**Rogoff, Kenneth**, "The optimal degree of commitment to an intermediate monetary target," *Quar-*

*terly Journal of Economics*, 1985, *100* (4), 1169–1189.

**Rouwenhorst, K Geert**, "Asset Pricing Implications of Equilibrium Business Cycle Models. In: Cooley, T.F. (Ed.)," in "Frontiers of business cycle research," Princeton University Press, 1995, pp. 294–330.

**Santos, Marcelo Rodrigues and Christopher Rauh**, "How Do Transfers and Universal Basic Income Impact the Labor Market and Inequality?," *CEPR Discussion Paper No. DP16993*, 2022.

**Song, Zheng, Kjetil Storesletten, and Fabrizio Zilibotti**, "Rotten parents and disciplined children: A politico-economic theory of public expenditure and debt," *Econometrica*, 2012, *80* (6), 2785–2803.

**Straub, Ludwig and Iván Werning**, "Positive long-run capital taxation: Chamley-Judd revisited," *American Economic Review*, 2020, *110* (1), 86–119.

# Appendix A Numerical Solution Algorithm

Solving the Markov-Perfect Equilibria (MPE) of consecutive governments entails heavy computational burdens with heterogeneous agents. As in standard macroeconomic heterogeneous agent models, individual decisions should be consistent with the aggregate law of motion for the distribution of agents. On top of that, the aggregate tax policy function must be compatible with individual decisions and the aggregate law of motion for the distribution of agents. In other words, these three equilibrium objects—individual decisions, the law of motion for the distribution, and the tax policy function—have to be consistent with each other in the Markov-perfect equilibrium.

I address this computational issue by taking ideas from the Backward Induction method of Reiter (2010). This method discretizes the aggregate state into finite grid points. For each aggregate grid point, the Backward Induction algorithm allows updating the aggregate law of motion while solving the decision rules thanks to the existence of the proxy distribution. This means that for each aggregate grid point, the backward induction algorithm would make it possible to approximate not only the aggregate law of motion for the distribution; but also the tax policy function consistent with the choice of government lacking commitment. With the value function, this endogenous tax policy outcome can be directly obtained when the proxy distribution is explicitly available.

Unfortunately, the original Reiter's (2010) method cannot directly be applied to the MPE models because the existence of off-the-equilibrium paths makes it challenging to arrange the proxy distribution. In the model of Krusell and Smith (1998), for which the Reiter's (2010) method is originally designed, the distribution of TFP shocks $Z$ is stationary, thus all the aggregate states $Z$ are not measure zero. With a positive probability, all the states $Z$ are realized on the equilibrium path. However, the MPE economy does not have this property. Let us think about a case in a stationary distribution. In this equilibrium, this optimal policy is obtained by comparing among one-time deviation policies. Some tax paths would not be reached at all on the equilibrium path.

I have three variations from the original backward induction method. First, I have to approximate not only the aggregate law of motion for distributions but also the tax policy function that is endogenous. I find these mappings in a non-parametric way, as in Reiter (2010). Second, I arrange distributions for all types of off the equilibrium paths, taking the initial distribution of the simulations as the previous proxy distribution for each aggregate state. Finally, I modify the construction of the reference distributions in Reiter (2002, 2010), reflecting the features of economies in the MPE wherein how many times a policy off-equilibrium takes place is unknown before simulations.

Here, I show how to apply the algorithm to the case with proportional income taxes. Note that I solve all the value functions in the following steps with the Endogenous Grid Method of Carroll (2006).

## A.1 Notation and Sketch of the Solution Method

The aggregate law of motion $\Gamma$ and the tax policy function $\Psi$ are evolved with the distribution $\mu$ that is an infinite dimensional equilibrium object, and thus it not not feasible in computations. To handle this issue, the Backward Induction method replaces $\mu$ with $m$, a set of moments from the distribution and discretize it. Here, I take the mean of the distribution and discretize it, $M = \{m_1, \cdots, m_{N_m}\}$. Furthermore, I discretze the tax policy, $T = \{\tau_1, \cdots, \tau_{N_\tau}\}$. This setting allows me to define the aggregate law motion and the tax policy function on each grid $(m_{i_m}, \tau_{i_\tau})$ such that $m' = G(m_{i_m}, \tau_{i_\tau}, \tau')$ where $\tau' = P(m_{i_m}, \tau_{i_\tau})$. Note that $G$ and $P$ do not rely on a parametric law.

Across a grid of aggregate states $(m_{i_m}, \tau_{i_\tau})$, each point selecting a proxy distribution, the Backward Induction method simultaneously solves for households' decision rules and an intratemporally consistent end-of-period distribution. This implies a future approximate aggregate state consistent with households' expectation ($m' = G(m_{i_m}, \tau_{i_\tau}, \tau')$). Likewise, the backward induction can find the tax policy function that is consistent with the choice of the government, by using household's value functions and the proxy distribution ($\tau^m = \tau' = P(m_{i_m}, \tau_{i_\tau})$). Theses mappings imply that $G$ interacts with $P$. Given $P$, first, I find $G$ during the iteration of value functions, and then update $P$ with the value function and proxy distribution. I repeat this until $P$ is convergent.

Given a distribution over individual states at each aggregate grid point $(m_{i_m}, \tau_{i_\tau})$, my goal is to obtain the law of motion for households distribution $G$ and the tax policy function $P$ that are intratemporally consistent with the end-of-period distribution and the choice of the government. Explicitly,

$$m' = G(m_{i_m}, \tau_{i_\tau}, \tau') \tag{35}$$

$$\tau' = P(m_{i_m}, \tau_{i_\tau}) \tag{36}$$

$$\tau' = \tau^m(m_{i_m}, \tau_{i_\tau}) \tag{37}$$

$$w = W(m_{i_m}, \tau_{i_\tau}) \tag{38}$$

$$T = TR(m_{i_m}, \tau_{i_\tau}) \tag{39}$$

(35) is to approximate $\Gamma$, (36) is to do $\Psi$, (37) is for the choice of the government, (38) is the mapping for the market wage, and (39) is the mapping for transfers.

The backward induction method explicitly computes $G$, $P$, $\tau^m$, $W$, and $T$, given a set of proxy distributions before the simulation step. An issue is that computing $G(m_{i_m}, \tau_{i_\tau}, \tau')$ in solving the value is costly because it depends on $\tau'$ not only on the equilibrium path but also off the equilibrium path. To address this issue, I reduce $G(m_{i_m}, \tau_{i_\tau}, \tau')$ into $\tilde{G}(m_{i_m}, \tau_{i_\tau}) = G(m_{i_m}, \tau_{i_\tau}, P(m_{i_m}, \tau_{i_\tau}))$ while solving the value function; retrieve $G(m_{i_m}, \tau_{i_\tau}, \tau')$ with the converged value function and the proxy distribution. Note that $G(m_{i_m}, \tau_{i_\tau}, \tau')$ must also satisfy an intratemporal consistency.

## A.2 Computing the Aggregate Mappings given a Set of Proxy Distributions

(1) Given $v^n(a, \epsilon; m, \tau)$ and $\tau' = P^q(m, \tau)$, where $n = 1, 2, \cdots$ and $q = 1, 2, \cdots$ denote the rounds of iteration, at grid $(m_{i_m}, \tau_{i_\tau})$, where $i_m = 1, \cdots, N_m$ and $i_\tau = 1, \cdots, N_\tau$ are grid indexes, solve for intratemporally consistent $m'$.

a) Guess $m'$ . Using $v^n$ and $P^q$, solve for $a' = g_a^{n+1}(a, \epsilon; m_{i_m}, \tau_{i_\tau})$ and $n = g_n^{n+1}(a, \epsilon; m_{i_m}, \tau_{i_\tau})$ using

$$v^{n+1}(a, \epsilon; m_{i_m}, \tau_{i_\tau}) = \max_{c, a', n} u(c, 1-n) + \beta \sum_{\epsilon'} \pi_{\epsilon, \epsilon'} v^n(a', \epsilon', m', \tau') \tag{40}$$

such that

$$c + a' = (1 - \tau_{i_\tau}) w(m_{i_m}, \tau_{i_\tau}) \epsilon n + (1 + (1 - \tau_{i_\tau}) r(m_{i_m}, \tau_{i_\tau})) a + T(m_{i_m}, \tau_{i_\tau})$$

$$\tau' = P^q(m_{i_m}, \tau_{i_\tau})$$

b) Using the proxy distribution, $\mu(a, \epsilon; m_{i_m}, \tau_{i_\tau})$, compute the distribution consistent with capital stock in the end of period $\tilde{m}'$, wage $\tilde{w}$, and transfers $\tilde{T}$.

$$\tilde{m}' = \int g_a^{n+1}(a, \epsilon; m_{i_m}, \tau_{i_\tau}) \mu(\mathbf{d}(a \times \epsilon); m_{i_m}, \tau_{i_\tau}) \tag{41}$$

$$\tilde{w} = (1 - \theta) \left( \frac{m_i}{N} \right)^{\theta} \tag{42}$$

$$\tilde{T} = \tau_{i_\tau} (r(m_{i_m}, \tau_{i_\tau}) m_i + w(m_{i_m}, \tau_{i_\tau}) N) \tag{43}$$

where

$$N = \int g_n^{n+1}(a, \epsilon; m_{i_m}, \tau_{i_\tau}) \epsilon \, \mu(\mathbf{d}(a \times \epsilon); m_{i_m}, \tau_{i_\tau})$$

c) If $\max \left\{ |\tilde{m}' - m'|, |\tilde{w} - w|, |\tilde{T} - T| \right\}$ >precision, update $m', w$, and $T$; set $r = \theta \left( \frac{w}{1-\theta} \right)^{\frac{\theta - 1}{\theta}} - \delta$; and return to $a$).

(2) Having found $m' = \tilde{G}^q(m_{i_m}, \tau_{i_\tau})$, $w = W^q(m_{i_m}, \tau_{i_\tau})$, and $T = TR^q(m_{i_m}, \tau_{i_\tau})$, use (40) to define $v^{n+1}(a, \epsilon; m, \tau)$ consistent with $v^n(a', \epsilon'; G^q(m_{i_m}, \tau_{i_\tau}), P^q(m_{i_m}, \tau_{i_\tau}))$. If $||v^{n+1} - v^n|| >$ precision, $n = n + 1$ and return to (1).

(3) For each aggregate grid $(m_{i_m}, \tau_{i_\tau}, \tau'_{i_\tau})$, retrieve $G^q(m_{i_m}, \tau_{i_\tau}, \tau'_{i_\tau})$ by solving for intratempollay consistent $\hat{m}'$.

a) For each $(m_{i_m}, \tau_{i_\tau}, \tau'_{i_\tau})$, guess $\hat{m}'$. With $v^\infty$, solve for $a' = \hat{g}_a(a, \epsilon; m_{i_m}, \tau_{i_\tau}, \tau'_{i_\tau})$ and $n = \hat{g}_n(a, \epsilon; m_{i_m}, \tau_{i_\tau}, \tau'_{i_\tau})$ using

$$\hat{v}(a, \epsilon; m_{i_m}, \tau_{i_\tau}, \tau'_{i_\tau}) = \max_{c,a',n} u(c, 1-n) + \beta \sum_{\epsilon'} \pi_{\epsilon,\epsilon'} v^\infty(a', \epsilon', m', \tau'_{i_\tau})$$

such that

$$c + a' = (1 - \tau_{i_\tau})\hat{w}(m_{i_m}, \tau_{i_\tau}, \tau'_{i_\tau})\epsilon n + (1 + (1 - \tau_{i_\tau})\hat{r}(m_{i_m}, \tau_{i_\tau}, \tau'_{i_\tau}))a + \hat{T}$$

b) For each $(m_{i_m}, \tau_{i_\tau}, \tau'_{i_\tau})$, using the proxy distribution, $\mu(a, \epsilon; m_{i_m}, \tau_{i_\tau})$, compute the distribution consistent with the end of period aggregate capital stock.

$$\tilde{m}' = \int \hat{g}_a(a, \epsilon; m_{i_m}, \tau_{i_\tau}, \tau'_{i_\tau})\mu(\mathbf{d}(a \times \epsilon); m_{i_m}, \tau_{i_\tau})$$

$$\tilde{w} = (1-\theta)\left(\frac{m_i}{N}\right)^\theta$$

$$\tilde{T} = \tau_{i_\tau}(\hat{r} m_i + \hat{w} N)$$

where

$$N = \int \hat{g}_n(a, \epsilon; m_{i_m}, \tau_{i_\tau}, \tau'_{i_\tau})\epsilon\, \mu(\mathbf{d}(a \times \epsilon); m_{i_m}, \tau_{i_\tau})$$

c) If $\max\left\{|\tilde{m}' - \hat{m}'|, |\tilde{w} - \hat{w}|, |\tilde{T} - \hat{T}|\right\} >$ precision, update $\hat{m}', \hat{w}$, and $\hat{T}$; set $\hat{r} = \theta\left(\frac{\hat{w}}{1-\theta}\right)^{\frac{\theta-1}{\theta}} - \delta$; and return to $a)$.

(4) Having found $m' = G^q(m_{i_m}, \tau_{i_\tau}, \tau'_{i_\tau})$, keep $G^q(m_{i_m}, \tau_{i_\tau}, \tau'_{i_\tau})$. Note that here is no update of the value.

(5) For each aggregate grid $(m_{i_m}, \tau_{i_\tau})$, find $\tau^{m,q}(m_{i_m}, \tau_{i_\tau})$.

a) Given $(a, \epsilon; m_{i_m}, \tau_{i_\tau})$, using $\hat{v}(a, \epsilon; m_{i_m}, \tau_{i_\tau}, \tau'_{i_\tau})$ in (3) - a), solve $\Psi^q(m, \tau)$ as follows:

$$\Psi^q(m_{i_m}, \tau_{i_\tau}) = \operatorname*{argmax}_{\tilde{\tau}'} \hat{V}(m_{i_m}, \tau_{i_\tau}, \tilde{\tau}') = \int \hat{v}(a, \epsilon; m_{i_m}, \tau_{i_\tau}, \tilde{\tau}')\mu(\mathbf{d}(a \times \epsilon); m_{i_m}, \tau_{i_\tau}) \tag{44}$$

The golden section search is used to find $\Psi^q(m_{i_m}, \tau_{i_\tau})$ with a cubic spline for $\hat{V}$ over $\tau'$.

b) For each aggregate grid $(m_{i_m}, \tau_{i_\tau})$, if $P^q(m_{i_m}, \tau_{i_\tau}) = \Psi^q(m_{i_m}, \tau_{i_\tau})$, $G^q$ and $P^q$ are the solutions, given the proxy distribution. Then, go to the next step. Otherwise, they are not the solutions. Take $P^{q+1} = \omega \cdot P^q + (1-\omega) \cdot \Psi^q$, and go back to (1).

### A.3 Constructing the Reference Distributions

Until now, I have solved $G$ and $P$ for a given set of proxy distributions. In the following step, I will simulate the economy and update the distribution selection function, as in Reiter (2002, 2010); but, the simulation step in this paper is substantially different from that in his method. He addresses Krusell and Smith (1998) model where aggregate uncertainty exists. Thus, what matters in his papers is to obtain the Ergodic set that is not affected by the initial distribution.

However, in economies without government commitment, it is important to obtain not only distributions on the equilibrium path but also those off the equilibrium path. For example, let us think of an economy without commitment in the stationary equilibrium. Then, there will be a unique value of $\tau^* = P(m^*, \tau^*)$ and $m^* = G(m^*, \tau^*, \tau^*)$. In this case, I may not know the value of other alternatives because this economy has nothing but the unique equilibrium path. This difficulty might lead the previous studies to employ local solution methods in solving this type of the MPE. By constrast, my approach is a global solution method, which means I need proxy distributions over all types of off the equilibrium paths.

To reserve distributions off the equilibrium path, I use the proxy distributions in the previous step as the initial distribution for the simulation. For each $(m_{i_m}, \tau_{i_\tau})$, I run a simulations for $T$ periods from the proxy distribution $\mu_0 = \mu(a, \epsilon; m_{i_m}, \tau_{i_\tau})$, implying the number of simulations is $N_m \times N_\tau$ and that of simulation outcomes is $T \times N_m \times N_\tau$. Note that any type of $(m_{i_m}, \tau_{i_\tau})$ will be observed at least once in the simulations. For each $(m_{i_m}, \tau_{i_\tau})$, using $\mu_0 = \mu(a, \epsilon; m_{i_m}, \tau_{i_\tau})$ and $v^\infty$ from the previous step, I simulate the economy in a forward manner. I compute the market cleared $w_t$ and $r_t$ and transfers $T_t$ satisfying the government budget condition for each simulation period $t = 1, \cdots, T$. In addition, I solve the government's problem $\tau_t^m$ for each simulation period $t = 1, \cdots, T$ with the $m' = G(m_{i_m}, \tau_{i_\tau}, \tau'_{i_\tau})$ obtained in the previous step.

I gather all the simulated distributions and rearrange the index as $\tilde{t} = 1, \cdots, T \times N_m \times N_\tau$. In creating the reference distributions from the simulation, I need a measure of distance for the moments of a distribution. For $(m, \tau)$, define an inverse norm

$$d((m_0, \tau_0), (m_1, \tau_1)) = (m_0 - m_1)^{-4} + (\tau_0 - \tau_1)^{-4} \tag{45}$$

In contrast to an economy with uncertainty, the initial simulation results should be preserved, having to be used to construct the reference distributions off the equilibrium path (non-Ergodic set). For each $t$, when $(m_t, \tau_t)$ with $m_t \in [m_k, m_{k+1})$ and $\tau_t \in [\tau_s, \tau_{s+1})$,

$$
\begin{aligned}
d_1(m_k, \tau_s) &= d_1(m_k, \tau_s) + (m_t - m_k)^{-4} + (\tau_t - \tau_s)^{-4} \\
d_1(m_{k+1}, \tau_s) &= d_1(m_{k+1}, \tau_s) + (m_t - m_{k+1})^{-4} + (\tau_t - \tau_s)^{-4} \\
d_1(m_k, \tau_{s+1}) &= d_1(m_k, \tau_{s+1}) + (m_t - m_k)^{-4} + (\tau_t - \tau_{s+1})^{-4} \\
d_1(m_{k+1}, \tau_{s+1}) &= d_1(m_{k+1}, \tau_{s+1}) + (m_t - m_{k+1})^{-4} + (\tau_t - \tau_{s+1})^{-4}
\end{aligned}
$$

Above $m_k$ ($\tau_s$) is the $k$-th ($s$-th) grid point for $m$ ($\tau$). Note that distances between a given node and non-adjacent moments are not taken into account, which is different from the corresponding step in Reiter (2002, 2010).

I construct the reference distributions for each $(m_{i_m}, \tau_{i_\tau})$ using the above, when $(m_{\tilde{t}}, \tau_{\tilde{t}}) \in \Big([m_{i_m}, m_{i_m+1}), [\tau_{i_\tau}, \tau_{i_\tau+1})\Big)$,

$$
\mu^r(a, \epsilon; m_{i_m}, \tau_{i_\tau}) = \sum_{\tilde{t}=1}^{T \times N_m \times N_\tau} \frac{d((m_{i_m}, \tau_{i_\tau}), (m_{\tilde{t}}, \tau_{\tilde{t}}))}{d_1(m_{i_m}, \tau_{i_\tau})} \mu_{\tilde{t}}(a, \epsilon). \tag{46}
$$

Each reference distribution is a weighted sum of distributions over the simulation only when simulated moments are adjacent to a given pair of grid points $(m_{i_m}, \tau_{i_\tau})$. Since the simulation moments are not on an Ergodic set, this should be considered.

I arrange the finite grid, which is the distribution support, as explicit. The distribution over $(a, \epsilon)$ used below size $(N_a \times N_\epsilon)$ with $\epsilon \in E = \{\epsilon_1, \cdots, \epsilon_{N_\epsilon}\}$ and $a \in A = \{a_1, \cdots, a_{N_a}\}$. I represent $\mu^r(a, \epsilon; m_{i_m}, \tau_{i_\tau})$ using $\mu^r_{i_a, i_\epsilon}(i_m, i_\tau)$, indexing $(a_{i_a}, \epsilon_{i_\epsilon})$ over $A \times E$ for $(m_{i_m}, \tau_{i_\tau})$. The moment of a reference distribution, $\sum_{i_\epsilon}^{N_\epsilon} \mu^r_{i_a, i_\epsilon}(i_m, i_\tau) a_{i_a}$, will not be consistent with $m_{i_m}$. However, the proxy distribution at $(i_m, i_\tau)$ will have this property.

## A.4 Updating the Proxy Distributions

Following Reiter (2002, 2010), for each aggregate grid $(i_m, i_\tau)$, I solve for $\mu_{i_a, i_\epsilon}$, the proxy distribution, as the solution to a problem that minimizes the distance to the reference distribution while imposing that each type of sums to its reference value and moment consistency.

$$\min_{\{\mu_{i_a,i_\epsilon}\}_{i_a=1,i_\epsilon=1}^{N_a,N_\epsilon}} \sum_{i_a=1}^{N_a}\sum_{i_\epsilon=1}^{N_\epsilon}\left(\mu_{i_a,i_\epsilon}-\mu^r_{i_a,i_\epsilon}(i_m,i_\tau)\right)^2 \tag{47}$$

$$\sum_{i_a=1}^{N_a}\mu_{i_a,1_\epsilon}=\sum_{i_a=1}^{N_a}\mu^r_{i_a,i_\epsilon}(i_m,i_\tau) \text{ for } i=1,\cdots,N_\epsilon \tag{48}$$

$$\sum_{i_\epsilon=1}^{N_\epsilon}\sum_{i_a=1}^{N_a}\mu_{i_a,i_\epsilon}\cdot a_{i_a}=m_{i_m} \tag{49}$$

$$\mu_{i_a,i_\epsilon}\geq 0 \tag{50}$$

The first-order condition for $\mu_{i_a,i_\epsilon}$ with $\lambda_i$ as the LaGrange multiplier for (48) and $\omega$ the multiplier (49) is

$$2(\mu_{i_a,i_\epsilon}-\mu^r_{i_a,i_\epsilon}(i_m,i_\tau))-\lambda_i-\omega\cdot a_{i_a}=0 \tag{51}$$

If I ignore the non-negative constraints for probabilities in (50), I have $N_\epsilon$ constraint in (48). 1 constraint in (49) and $N_a\times N_\epsilon$ first-order conditions in (50). These are a system of $N_a\times N_\epsilon+N_\epsilon+1$ linear equations in $\left(\{\mu_{i_a,i_\epsilon}\}_{i_a=1,i_\epsilon=1}^{N_a,N_\epsilon},\{\lambda_{i_\epsilon}\}_{i_\epsilon}^{N_\epsilon},\omega\right)$.

I construct a column vector **x**. The first block of **x** are the stack of the elements from the proxy distribution, such that $\mathbf{x}(j)=\mu_{i_a,i_\epsilon}$ where $j=(i_\epsilon-1)\times N_a+i_a$. Next are the $N_\epsilon$ multipliers $\lambda_i$, followed by one multiplier $\omega$. I solve for **x** using a system of linear equations, $\mathbf{Ax}=\mathbf{b}$ in Figure 12. The non-zero element of **A** and **b** are described here. The coefficients for $\mu_{i_a,i_\epsilon}$ are entered into **A** as

$$\mathbf{A}((i_\epsilon-1)\times N_a+i_a,(i_\epsilon-1)\times N_a+i_a)=2 \tag{52}$$

$$\mathbf{A}(N_\epsilon\times N_a+i_\epsilon,(i_\epsilon-1)\times N_a+i_a)=1 \text{ for } i_\epsilon=1,\cdots,N_\epsilon \tag{53}$$

$$\mathbf{A}(N_\epsilon\times N_a+N_\epsilon+1,(i_\epsilon-1)\times N_a+i_a))=a_{i_a}. \tag{54}$$

The coefficient for $\lambda_i$ are entered in **A**, for $i_\epsilon=1,\cdots N_\epsilon$ and $i_a=1,\cdots,N_a$, as

$$\mathbf{A}((i_\epsilon-1)\times N_a+i_a,N_\epsilon\times N_a+i_\epsilon)=-1 \tag{55}$$

The coefficients for $\omega$ sets the following elements of **A**, for $i_\epsilon=1,\cdots N_\epsilon$ and $i_a=1,\cdots,N_a$,

$$\mathbf{A}((i_\epsilon-1)\times N_a+i_a,N_\epsilon\times N_a+N_\epsilon+1)=-a_{i_a}. \tag{56}$$

The elements of **b** are, for $i_\epsilon = 1, \cdots N_\epsilon$ and $i_a = 1, \cdots, N_a$,

$$\mathbf{b}((i_\epsilon - 1) \times N_a + i_a) = 2\mu^r_{i_a,i_\epsilon}(i_m, i_\tau) \tag{57}$$

$$\mathbf{b}(N_\epsilon \times N_a + i_\epsilon) = \sum_{i_a=1}^{N_a} \mu^r_{i_a,i_\epsilon}(i_m, i_\tau) \tag{58}$$

$$\mathbf{b}(N_\epsilon \times N_a + N_\epsilon + 1) = m_{i_m}. \tag{59}$$

I solve $\mathbf{x} = \mathbf{A}^{-1}\mathbf{b}$ iteratively using an active set method corresponding to probabilities that are not set to 0.

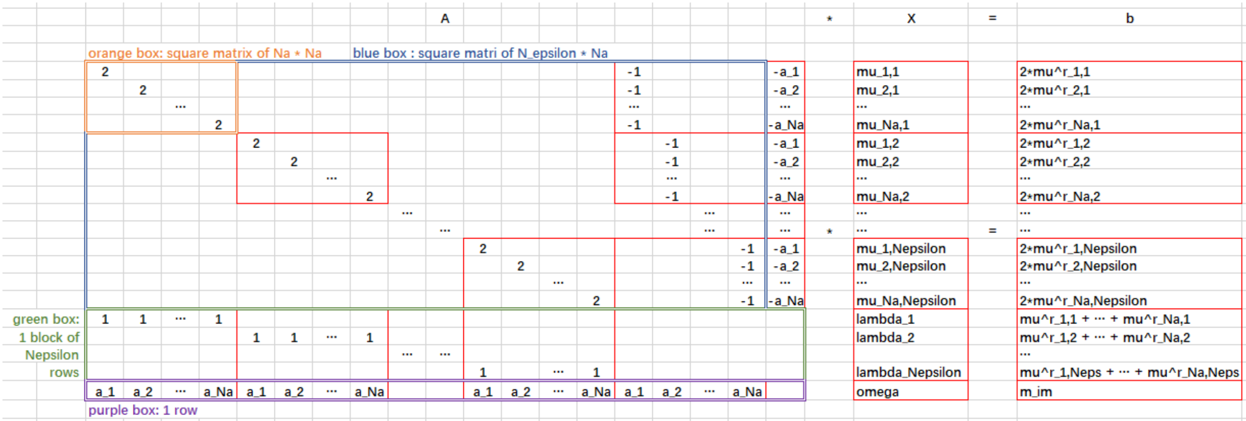


Figure 12: $\mathbf{A} \times \mathbf{x} = \mathbf{b}$

To solve the linear system, I use the active set approach to non-negative constraints in Reiter (2002, 2010). If any of the first $N_\epsilon \times N_a$ elements of **x** are negative, the constraint $\mu_{i_a,1_\epsilon} \geq 0$ has been violated for some $(i_\epsilon - 1)N_a + i_a = j \in J_0$ where

$$J_0 = \{j | 1 \leq j \leq N_a \times N_\epsilon \text{ and } \mathbf{x}(j) < 0\}. \tag{60}$$

For some $O > 0$, set the most negative $O$ elements indexed in $J_0$ to 0, $\mu_{i_a,i_\epsilon} = 0$. Remove the $j - th$ row and column of $A$ along with the $j - th$ element of **b**. Solve the reduced system with $O$ less rows. If any of the $N_\epsilon \times (N_a - O)$ elements are negative, again discard the most negative $O$. I repeat this procedure until the most negative elements of **x** is larger than a precision level. This iteratively implements the non-negativity of probabilities (60).

Table 4 shows the setting of the grids in this paper. With this setting, I continue to repeat the whole steps above until no improvement in accuracy statistic proposed by Den Haan (2010). I find that the mean errors on the equilibrium path are sufficiently small (considerably less than 0.02% for all cases) and the maximum of the errors are also reasonably small (not exceeding 0.03% for all cases). Furthermore, the method is substantially efficient in a usual personal computer.

Table 4: Setting for Computation

| | num. of nodes | Description |
|---|---|---|
| $N_a$ | 400(400) | asset (distribution) |
| $N_\epsilon$ | 10 | persistence wage process |
| $N_m$ | 10 | aggregate capital (aggregate) |
| $N_\tau$ | 9 | income tax (aggregate) |

Table 5: Accuracy and Efficiency of the Solution Method

| | Proportional Income Tax |
|---|---|
| Run time | 51.43 min |
| AVG(DH) of $m$ | 0.014% |
| AVG(DH) of $w$ | 0.004% |
| AVG(DH) of $\tau$ | 0.006% |
| MAX(DH) of $m$ | 0.018% |
| MAX(DH) of $w$ | 0.013% |
| MAX(DH) of $\tau$ | 0.022% |

$AVG(\cdot)$ and $MAX(\cdot)$ are computed on the equilibrium path.
Processor: AMD Ryzen Threadripper 3960X @ 3.8GHz, RAM: 256GB

# Appendix B Ramsey Problem with Capital Income Tax

The results are consistent with those in Dyrda and Pedroni (2022). This front-loaded capital income tax is to quickly reduce overall inequality. Afterward, the Ramsey planner balances between inequality and redistribution over the transition path.

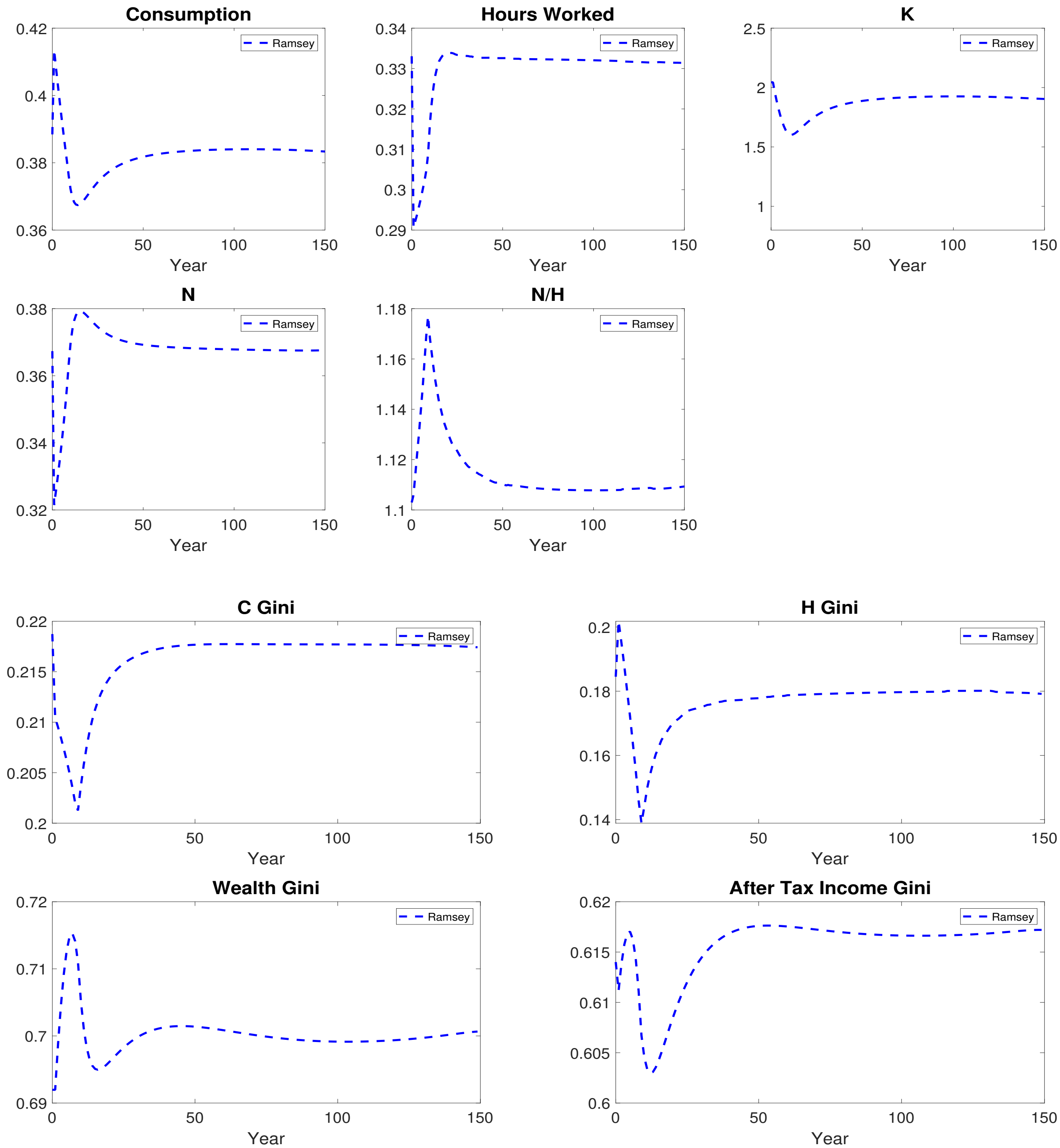


Figure 13: Results with Capital Income Tax in the Ramsey Problem

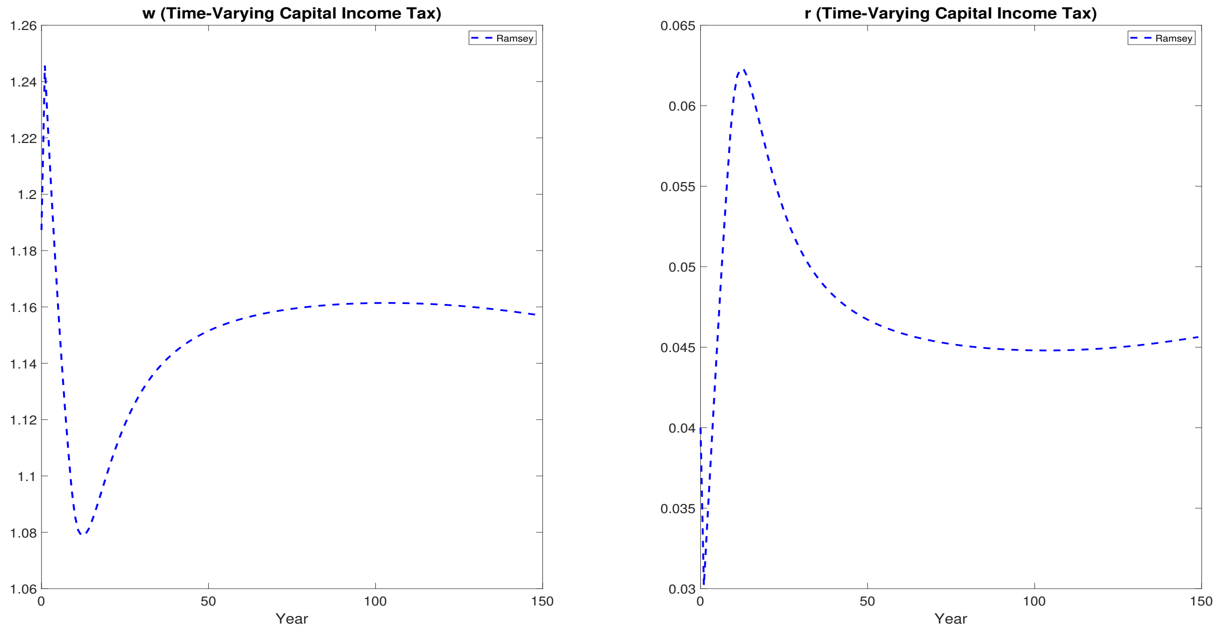


Figure 14: Time-Inconsistent Capital Income Taxes: Dynamics of $w$ and $r$

## Appendix C Definition of $\frac{\Delta K_{t+s}}{\Delta \tau_{t+1}}$ and $\frac{\Delta \tau_{t+s}}{\Delta \tau_{t+1}}$

$$\frac{\Delta K_{t+s}}{\Delta \tau_{t+1}} = \frac{\partial K_{t+1}}{\partial \tau_{t+1}} \cdot \frac{\Xi \Gamma_{t+s-1}}{\Xi K_{t+1}} + \frac{\Xi \Gamma_{t+s-1}}{\Xi \tau_{t+1}}$$
$$\frac{\Delta \tau_{t+s}}{\Delta \tau_{t+1}} = \frac{\partial K_{t+1}}{\partial \tau_{t+1}} \cdot \frac{\Xi \Psi_{t+s-1}}{\Xi K_{t+1}} + \frac{\Xi \Psi_{t+s-1}}{\Xi \tau_{t+1}}$$

where

$$\frac{\Xi \Gamma_{t+s-1}}{\Xi K_t} = F^K_{t+s}(F^K_{t+s-1}, G^K_{t+s-1}) = \begin{cases} \Gamma_{K_t} & \text{if s=1} \\ \Gamma_{K_{t+s-1}} F^K_{t+s-1} + \Gamma_{\tau_{t+s-1}} G^K_{t+s-1} & \text{if } s \geq 2 \end{cases}$$

$$\frac{\Xi \Psi_{t+s-1}}{\Xi K_t} = G^K_{t+s}(F^K_{t+s-1}, G^K_{t+s-1}) = \begin{cases} \Psi_{K_t} & \text{if s=1} \\ \Psi_{K_{t+s-1}} F^K_{t+s-1} + \Psi_{\tau_{t+s-1}} G^K_{t+s-1} & \text{if } s \geq 2 \end{cases}$$

$$\frac{\Xi \Gamma_{t+s-1}}{\Xi \tau_t} = F^\tau_{t+s}(F^\tau_{t+s-1}, G^\tau_{t+s-1}) = \begin{cases} \Gamma_{\tau_t} & \text{if s=1} \\ \Gamma_{K_{t+s-1}} F^\tau_{t+s-1} + \Gamma_{\tau_{t+s-1}} G^\tau_{t+s-1} & \text{if } s \geq 2 \end{cases}$$

$$\frac{\Xi \Psi_{t+s-1}}{\Xi \tau_t} = G^\tau_{t+s}(F^\tau_{t+s-1}, G^\tau_{t+s-1}) = \begin{cases} \Psi_{\tau_t} & \text{if s=1} \\ \Psi_{K_{t+s-1}} F^\tau_{t+s-1} + \Psi_{\tau_{t+s-1}} G^\tau_{t+s-1} & \text{if } s \geq 2 \end{cases}$$