Endogenous privacy and heterogeneous price sensitivity

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Abstract

This study analyzes a model in which two firms, one with profiling technology and one without, compete for old and new markets. In the old market, consumers leave their personal information online, whereas, in the new market, consumers do not. When a firm with profiling technology observes consumers' personal information, it sets personalized prices for them. Additionally, consumers can conceal their personal information by paying privacy costs. We introduce heterogeneity in price sensitivities among consumers into our model. We obtain the following result. For greater heterogeneity in price sensitivities, consumer and total surpluses are maximized with no privacy cost; for lower heterogeneity, a sufficiently high privacy cost is desirable for consumers and society; for intermediate heterogeneity, while consumers prefer no privacy cost, total surplus is maximized at a sufficiently high privacy cost. Therefore, when deciding on privacy policy, authorities should consider the heterogeneity in price sensitivities.

JEL codes: D43, L10, L13.

Keywords: personalized pricing, privacy, personal information, heterogeneous consumers, Hotelling model.

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1 Introduction

In recent years, with the spread of the Internet, firms have been able to personalize product offers based on consumers' personal information. For example, Netflix offers movie recommendations, and Amazon offers product recommendations. Orbitz, a hotel website, offers price discrimination (Mattioli 2012).[1] Accordingly, personalizing approaches based on consumers' personal information are diverse and wide-ranging. Particularly, price discrimination in personalizing has attracted the attention of many researchers because of its potential to disadvantage users (Choe et al. 2022, Esteves 2022, Taylor and Wagman 2014).

In terms of policy, the EU is providing consumers with an environment that makes it easier for consumers to protect their personal data through the General Data Protection Regulation (commonly known as GDPR). Compared to the EU, China has taken little action on data privacy (Valletti and Wu 2020). This is because the environment in China makes it difficult for consumers to conceal their personal information. Thus, various privacy policies exist in the real world. Conversely, in theoretical studies that have considered the horizontal differentiation models in the more realistic setting of two types of consumers who either leave or do not leave their personal information in online spaces, the main finding is that an environment in which it is difficult for consumers to conceal their personal information is most desirable (Montes et al. 2019, Valletti and Wu 2020).[2] Montes et al. (2019) have shown that, in a monopolistic market, an environment where it is easy for consumers to conceal their personal information may be desirable, but in a duopolistic market, an environment in which it is difficult to conceal personal

[1] Mattioli (2012) reported that Orbitz Worldwide, a travel agency, offers higher hotel prices to Mac users than other PC users.

[2] Montes et al. (2019) have shown that consumer surplus in a duopolistic market is an increasing function of the privacy cost, that is, the cost of concealing personal information. Valletti and Wu (2020) have shown that consumer surplus is a U-shaped function of the privacy cost; however, by substituting the lower and upper possible values of the privacy cost, we can easily confirm that consumer surplus is highest when the privacy cost is the largest.

information is always the most desirable. Thus, given the aforementioned context, few results justify privacy policies such as GDPR implemented in the EU.

This study proposes the price sensitivity of consumers as an important factor to consider when evaluating privacy policies across different countries worldwide. It is well-known that prices decrease when it becomes harder to conceal personal information, that is, when the cost of concealing personal information (hereafter, privacy cost) is higher (Montes et al. 2019, Valletti and Wu 2020). This is because higher privacy cost leads firms to lower their prices to compensate for the consumer disadvantage. Considering that this price reduction effect is dominant, the aforementioned studies have suggested that it is optimal for consumer surplus to make the privacy cost as high as possible. Therefore, if this price reduction effect is not sufficient, the high privacy cost should lead to a low consumer surplus. A factor that may contribute to this situation is the equilibrium prices being sufficiently low before the privacy cost increases. If consumers are price sensitive, the equilibrium prices are lower, and consequently, we can predict that the price reduction effect will be smaller. Thus, this study focuses on the heterogeneous price sensitivities of consumers and finds that we can justify the EU privacy policy even in a duopolistic market.

We consider competition between two firms, one with consumers' personal information and the other without.[3] We also consider a new market and an old market. The new market is the market of consumers who have not been active online in the past, while the old market is the market of consumers who have left information online in the past. We assume that the consumers in the old market can conceal their personal information from the firm by paying the privacy cost. Additionally, this study allows the degree of price sensitivity to be different between the two markets. We assume that the

[3] Microsoft and Salesforce, both with an active customer relationship management (CRM) market, were seeking to merge with LinkedIn, a provider of social networking services. After a fierce bidding war, Microsoft acquired LinkedIn. Salesforce argued that only Microsoft would have the ability to access LinkedIn's database. This is an example of a situation in which only a single firm can access a dataset instead of multiple firms (Montes et al. 2019).

new market consumers are more price sensitive than the old market consumers. This is based on Dedehayir et al. (2017) and Goldsmith and Newell (1997), who have analyzed the differences in price sensitivity among consumers based on Rogers' (1983) innovator theory, a concept that divides consumers by time of purchase.[4] The firm that receives access to personal information can offer personalized prices to these consumers. For old market consumers who conceal their information or new market consumers who have not left their personal information in online spaces, the firms offer uniform prices.

This study presents two findings. First, we find that the optimal privacy policy for consumers depends on the price sensitivity of the new market consumers. We consider that the new market consumers are sufficiently price sensitive. Consumer surplus is maximized when a government provides an environment in which consumers are more likely to conceal their personal information, such as in the EU. Conversely, if consumers' price sensitivity is sufficiently low, a policy restricting the concealment of personal information, such as that in China, leads to maximum consumer surplus.

Second, we provide conditions under which a privacy policy maximizing the consumer surplus differs from a policy maximizing the total surplus. This difference depends on the price sensitivity of the new market consumers. If they are sufficiently price sensitive, both consumer and total surpluses are maximized under a policy that makes it easier to conceal personal information. Conversely, if they are sufficiently price insensitive, like the old market consumers, both consumer and total surpluses are maximized under a policy that makes it harder to conceal personal information. However, under intermediate price sensitivity, the consumer surplus is maximized under a policy that makes it easier to conceal personal information, while the total surplus is maximized under a policy that makes it harder to conceal personal information.

This may be because privacy cost reduces the number of consumers who conceal their personal information. Therefore, the number of consumers who are offered personalized

[4] Dedehayir et al. (2017) and Goldsmith and Newell (1997) have empirically shown that consumers who purchase goods late are more price sensitive than consumers who purchase them earlier.

prices increases, resulting in decreasing consumer surplus. This occurs only in the case of consumer surplus, not total surplus. An increase in the privacy cost largely reduces consumer surplus to a greater degree than the total surplus. Thus, a policy that makes it easier to conceal personal information tends to maximize the consumer surplus instead of the total surplus.

In reality, we observe that the optimal policy in terms of consumer surplus is different from that of total surplus. In markets involving Big Data, the areas addressed by competition and consumer protection policies tend to overlap (Jin and Wagman 2021). Therefore, competition and data protection authorities may have different policies regarding the same issue. For example, the head of the EU's competition authority has stated that "as data becomes increasingly important for competition, it may not be long before the Commission [the EU-level competition authority] has to deal with cases where granting access to data is the best way to restore competition."[5] However, in terms of data privacy, this access may undermine consumer privacy (Douglas 2021). This study suggests that heterogeneity in consumer price sensitivity is a factor that causes this conflict.

The literature closely related to our research is the study of consumers' endogenous privacy choices. Particularly, our study is most closely related to Montes et al. (2019). They have considered a model in which firms with and without personal information compete in new and old markets. Our study incorporates heterogeneity in consumer price sensitivities and presents new results, while they have not considered these aspects. Several other studies have examined consumers' endogenous privacy choices (see Casadesus-Masanell and Hervas-Drane 2015, Conitzer et al. 2012, Koh et al. 2017, and Valletti and Wu 2020). These studies have also not considered the context discussed in this study, in which price sensitivity differs between two types of markets.

[5] Margrethe Vestager, Comm'r of Competition, Eur. Comm'n, Defending Competition in a Digitised World, Address at the European Consumer and Competition Day (Apr. 4, 2019), `https://wayback.archive-it.org/12090/20191129202059/https://ec.europa.eu/commission/commissioners/2014-2019/vestager/announcements/defending-competition-digitised-world_en`

Moreover, while this study considers endogenous privacy choices, the early privacy literature has studied exogenous consumer privacy choices (Acquisti and Varian 2005, Shy and Stenbacka 2016, Taylor 2004, Taylor and Wagman 2014). Most previous literature has analyzed the following two extreme cases: consumers cannot be anonymous, or they can conceal their personal information at no cost. Additionally, this study is also related to the literature on behavior-based price discrimination (Esteves 2010, Fudenberg and Tirole 2000, Fudenberg and Villas-Boas 2012, Villas-Boas 1999, Villas-Boas 2004).[6] In these studies, consumers make an implicit privacy choice regarding their purchasing decisions at the initial stage. Conversely, our study explicitly represents this choice and its associated costs in the model. For further literature on privacy, see Acquisti et al. (2016).

The remainder of this study is organized as follows. The next section describes the model. Section 3 provides the equilibrium calculations. Section 4 presents the comparative statics. Finally, Section 5 provides the conclusion.

2 Model

Consider a situation in which two firms, firm A and firm B, compete with each other. Following Montes et al. (2019), we assume that there are two markets, a new and an old market, and that consumers in both markets are uniformly distributed on $[0, 1]$.[7] Firm A is located at 0 and firm B is located at 1 in both the markets.

For the consumers in the new market, both firms observe only their distribution. Consumers who have not been active on the Internet in the past and whose personal information is not available in the online space, that is, new Internet users, belong to this category. They are offered uniform prices p_A, p_B because firms cannot observe any

[6]For other literature, comprehensive reviews can be found in Fudenberg and Villas-Boas (2006) and Esteves (2009).

[7]We can consider different market sizes. Even if we assume that the new market size is 1 and the old market size is λ, the main results of this study are robust in the range $0.5 \leq \lambda \leq 1.5$. Hence, for simplicity, we assume $\lambda = 1$.

personal information other than their distribution. Thus, the utilities of consumers at θ purchasing from firms A, B are $u_A^N = v - p_A - \alpha t(\theta - 0)$ and $u_B^N = v - p_B - \alpha t(1-\theta)$, respectively. Here, the superscript "N" means "new market." Furthermore, v is the utility of consuming the goods, and t is the parameter of transportation cost; $\alpha \in (0, 1]$ is the parameter for the new market consumers, which gives less weightage to the transportation cost.[8] Therefore, α represents the price insensitivity of the new market consumers, and when α is small (large), the consumers are price sensitive (insensitive). Note that several studies have interpreted the coefficient parameter of transportation costs as price sensitivity (Coughlan and Soberman 2005, Ishibashi and Matsushima 2009, Mehra et al. 2020, Shaffer and Zettelmeyer 2004). This allows for an alternative interpretation of α as the brand orientation of the new market consumers.

For the consumers in the old market, firm B only knows their distribution. Therefore, because firm B offers uniform price p_B, the utility of purchasing from firm B is $u_B^O = v - p_B - t(1-\theta)$. The superscript "$O$" means "old market." However, firm A may know both the distribution and personal information of the consumers, that is, their types. If the consumers do not protect their privacy, firm A observes their types and offers them personalized prices $p_A(\theta)$ accordingly. Thus, the utility of consumers who reveal personal information to firm A is $u_{RA}^O = v - p_A(\theta) - t(\theta - 0)$. Here, the subscript "$R$" means "revealing personal information." If the consumers pay the privacy cost c to conceal their types from firm A, firm A observes only their distribution and offers uniform price p_A. Thus, the utility of consumers concealing their types is $u_{CA}^O = v - p_A - t(\theta - 0) - c$. Here, the subscript "$C$" means "concealing personal information." We also assume $c < (3-\alpha)(2\alpha+1)t/(4\alpha+3) = c_H$ to guarantee that the number of consumers who conceal personal information is positive.

The behavioral categories of the consumers in both markets are as follows. In the new market, the consumers with $\theta \in [0, \theta_N]$ are close to firm A, which is why they purchase

[8] For simplicity, we assume $0 < \alpha \leq 1$. Although the calculation becomes highly complex, we can show similar results in $1 < \alpha < 3$.

from firm A. The consumers with $\theta \in (\theta_N, 1]$ purchase from firm B because they are closer to firm B. Here, θ_N is the type of consumer who is indifferent between purchasing from firms A and B in the new market. Similarly, in the old market, the consumers with $\theta \in [0, \theta_O]$ purchase from firm A, and consumers with $\theta \in (\theta_O, 1]$ purchase from firm B. Here, θ_O is the type of consumer who is indifferent between purchasing from firms A and B in the old market. Additionally, among the old market consumers purchasing from firm A, the consumers with $\theta \in [0, \theta_{CR}]$ have a higher willingness to pay for firm A because they are much closer to firm A. Therefore, they conceal their personal information. The consumers with $\theta \in (\theta_{CR}, \theta_O]$ are less willing to pay for firm A because they are relatively far away from firm A. Thus, they reveal their personal information. Here, θ_{CR} is the type of consumer who is indifferent between concealing and revealing their personal information. We assume that $\theta_{CR} < [(4\alpha+3)\sqrt{1+\alpha} - 6\alpha^2 - 3\alpha + 3]/[4\alpha(1+\alpha)] = \bar{\theta}$ as a condition for firm B to enter the old market.[9]

Firms A and B produce their goods without any cost. Firm A offers uniform price p_A to the consumers in the new market and the concealing consumers in the old market. It also offers personalized prices $p_A(\theta)$ to the revealing consumers in the old market. Accordingly, we define the profit of firm A as follows.

$$\pi_A = \int_0^{\theta_N} p_A d\theta + \int_0^{\theta_{CR}} p_A d\theta + \int_{\theta_{CR}}^{\theta_O} p_A(\theta) d\theta. \tag{1}$$

Firm B cannot observe the types of old market consumers. Therefore, because firm B only knows the distributions in both markets, it offers the uniform price p_B across both the new and old markets. Thus, the profit of firm B is expressed as follows.

$$\pi_B = \int_{\theta_N}^{1} p_B d\theta + \int_{\theta_O}^{1} p_B d\theta. \tag{2}$$

Finally, we define the consumer, producer, and total surpluses. Consumer surplus is

[9]For some parameter ranges in our analysis, we have more than two equilibria. We focus on their interior solution. In Online Appendix, we provide the condition that the interior solution exists.

defined as follows.

$$CS = \int_0^{\theta_N} u_A^N d\theta + \int_{\theta_N}^1 u_B^N d\theta + \int_0^{\theta_{CR}} u_{CA}^O d\theta + \int_{\theta_{CR}}^{\theta_O} u_{RA}^O d\theta + \int_{\theta_O}^1 u_B^O d\theta.$$

Furthermore, because the producer surplus is $PS = \pi_A + \pi_B$, the total surplus is $TS = CS + PS$.

The stages of this game are as follows. In the first stage, the consumers in the old market decide whether to conceal or reveal their personal information to firm A. In the second stage, each firm determines a uniform price for the consumers who do not leave or reveal their personal information. In the third stage, firm A determines the personalized prices for the consumers who have revealed their personal information. In the fourth stage, the consumers purchase and consume. Here, personalized pricing, after uniform pricing, reflects the firm's flexibility to adjust personalized prices. This pricing structure is standard in the literature on personalized pricing (Choe et al. 2018, Shaffer and Zhang 2002, Thisse and Vives 1988). Using backward induction, we solve this game.

3 Calculating Equilibrium

First, consider the fourth stage. We find the type θ_N of a consumer who is indifferent between purchasing from firms A and B in the new market. Considering that this consumer satisfies $u_A^N(\theta_N) = u_B^N(\theta_N)$, solving this equation yields θ_N as follows.

$$\theta_N = \frac{1}{2} + \frac{p_B - p_A}{2t\alpha}. \tag{3}$$

Therefore, in the new market, consumers at $\theta \leq \theta_N$ purchase from firm A, and consumers at $\theta > \theta_N$ purchase from firm B. Moreover, in the old market, the type θ_O of consumer who is indifferent between purchasing from firms A and B is established. The consumers at $\theta \leq \theta_O$ purchase from firm A, and consumers at $\theta > \theta_O$ purchase from firm B.

Next, in the third stage, we derive the personalized prices $p_A(\theta)$ that firm A offers to consumers who have revealed their personal information. Solving $u_{RA}^O = u_B^O$, we obtain the personalized prices $p_A(\theta)$ as follows.

$$p_A(\theta) = p_B + (1 - 2\theta)t. \tag{4}$$

In the old market, the type of last consumer purchasing from firm A is θ_O, which satisfies $p_A(\theta_O) = 0$. Therefore, by solving $p_A(\theta_O) = 0$, we can find θ_O as follows.

$$\theta_O = \frac{p_B + t}{2t}. \tag{5}$$

We consider the second stage. Substituting (3) and (5) into (1) and (2) for each firm's profit, we obtain the following maximization problems.

$$\max_{p_A} \int_0^{\frac{1}{2}+\frac{p_B-p_A}{2t\alpha}} p_A d\theta + \int_0^{\theta_{CR}} p_A d\theta + \int_{\theta_{CR}}^{\frac{p_B+t}{2t}} p_A(\theta) d\theta,$$
$$\max_{p_B} \int_{\frac{1}{2}+\frac{p_B-p_A}{2t\alpha}}^{1} p_B d\theta + \int_{\frac{p_B+t}{2t}}^{1} p_B d\theta.$$

Calculating the first-order condition for each firm, we obtain the uniform prices as follows.

$$p_A = \frac{2\alpha t[(2\alpha+2)\theta_{CR} + \alpha + 2]}{4\alpha + 3}, \quad p_B = \frac{\alpha t(\theta_{CR} + 5)}{4\alpha + 3}. \tag{6}$$

Finally, we consider the first stage. By substituting (6) into u^O_{CA} and u^O_{RA}, we obtain (7) and (8).

$$u^O_{CA} = v - \frac{2\alpha t[(2\alpha+2)\theta_{CR} + \alpha + 2]}{4\alpha + 3} - t\theta - c, \tag{7}$$

$$u^O_{RA} = v - \frac{\alpha t(\theta_{CR} + 5)}{4\alpha + 3} - t(1 - \theta). \tag{8}$$

From (7) and (8), we obtain the type θ^*_{CR} of the consumer who is indifferent between concealing and revealing personal information, as follows.

$$\theta^*_{CR} = \frac{(-2\alpha^2 + 5\alpha + 3)\, t - (4\alpha + 3)c}{2(\alpha + 1)(2\alpha + 3)t}. \tag{9}$$

Substituting (9) into (6), we obtain the equilibrium uniform prices as follows.

$$p^*_A = \frac{2\alpha(3t - c)}{2\alpha + 3}, \quad p^*_B = \frac{\alpha[2(\alpha + 3)t - c]}{(\alpha + 1)(2\alpha + 3)}. \tag{10}$$

Furthermore, by substituting (10) into (4), we obtain the equilibrium personalized prices as follows.

$$p_A^*(\theta) = \frac{\alpha[2(\alpha+3)t - c]}{(\alpha+1)(2\alpha+3)} + (1-2\theta)t.$$

From the aforementioned results, the equilibrium profit for each firm is as follows.

$$\pi_A^* = \frac{(8\alpha+9)c^2 - 12\alpha tc + 36\alpha(\alpha+2)t^2}{4(2\alpha+3)^2 t}, \quad \pi_B^* = \frac{\alpha[2(\alpha+3)t - c]^2}{2(\alpha+1)(2\alpha+3)^2 t}.$$

Similarly, we find the consumer and total surpluses in equilibrium as follows.

$$CS^* = 2v + \frac{\begin{bmatrix} (4\alpha^2+16\alpha+9)\,c^2 + 2\,(4\alpha^3+16\alpha^2-3\alpha-9)\,tc \\ -\,(4\alpha^4+68\alpha^3+229\alpha^2+174\alpha+9)\,t^2 \end{bmatrix}}{4(\alpha+1)(2\alpha+3)^2 t},$$

$$TS^* = 2v + \frac{\begin{bmatrix} (12\alpha^2+35\alpha+18)\,c^2 + 2\,(4\alpha^3+6\alpha^2-21\alpha-9)\,tc \\ -\,(4\alpha^4+24\alpha^3+73\alpha^2+30\alpha+9)\,t^2 \end{bmatrix}}{4(\alpha+1)(2\alpha+3)^2 t}.$$

4 Comparative Statics

We provide comparative statics for privacy cost c and price insensitivity α and investigate the optimal privacy policy. Differentiating the uniform prices p_A^*, p_B^*, and the personalized prices $p_A^*(\theta)$ with respect to c, we obtain the following.

Lemma 1 *As the privacy cost increases, firm A, which can observe personal information, reduces its uniform and personalized prices, and firm B, which cannot observe personal information, reduces its uniform price.*

Proof. We examine the signs of $\partial p_A^*/\partial c$, $\partial p_B^*/\partial c$, and $\partial p_A^*(\theta)/\partial c$. Differentiating each of the uniform and personalized prices calculated in Section 3 with respect to c, we obtain the following equations.

$$\frac{\partial p_A^*}{\partial c} = -\frac{2\alpha}{2\alpha+3}, \quad \frac{\partial p_B^*}{\partial c} = -\frac{\alpha}{(\alpha+1)(2\alpha+3)}, \quad \frac{\partial p_A^*(\theta)}{\partial c} = -\frac{\alpha}{(\alpha+1)(2\alpha+3)}.$$

Here, from $0 < \alpha < 1$, we find that the signs are all positive. Thus, we obtain Lemma 1.$\square$

The intuition behind Lemma 1 is as follows. As the privacy cost increases, the utility of consumers concealing their personal information in the old market decreases. Therefore, firm A lowers its uniform price to compensate accordingly. Similarly, firm B lowers its uniform price as a strategic complement. Moreover, the personalized prices of firm A decrease because they depend on the uniform price of firm B.

Next, by differentiating the uniform prices p_A^*, p_B^* and personalized price $p_A^*(\theta)$ with respect to α, the price insensitivity of the new market consumers, we obtain the following Lemma 2.

Lemma 2 *If the new market consumers are price sensitive (insensitive), the uniform and personalized prices of firm A, which can observe personal information, and the uniform price of firm B, which cannot observe personal information, are low (high).*

Proof. We check the signs of $\partial p_A^*/\partial\alpha$, $\partial p_B^*/\partial\alpha$, and $\partial p_A^*(\theta)/\partial\alpha$. Differentiating each price with respect to α, we obtain the following equations.

$$\frac{\partial p_A^*}{\partial\alpha} = \frac{6(3t-c)}{(2\alpha+3)^2}, \quad \frac{\partial p_B^*}{\partial\alpha} = \frac{-\left(3-2\alpha^2\right)c - 2\left(\alpha^2-6\alpha-9\right)t}{(\alpha+1)^2(2\alpha+3)^2},$$
$$\frac{\partial p_A^*(\theta)}{\partial\alpha} = \frac{-\left(3-2\alpha^2\right)c - 2\left(\alpha^2-6\alpha-9\right)t}{(\alpha+1)^2(2\alpha+3)^2}.$$

First, we examine the sign of $\partial p_A^*/\partial\alpha$. Solving $\partial p_A^*/\partial\alpha > 0$, we obtain $c < 3t$. From $3t - c_H = \left(2\alpha^2+7\alpha+6\right)t/(4\alpha+3)$, we immediately establish $c_H < 3t$ for any $\alpha \in (0,1]$. Therefore, we obtain $\partial p_A^*/\partial\alpha > 0$. Next, consider the signs of $\partial p_B^*/\partial\alpha$ and $\partial p_A^*(\theta)/\partial\alpha$. Since $\partial p_B^*/\partial\alpha = \partial p_A^*(\theta)/\partial\alpha$, solving $\partial p_B^*/\partial\alpha > 0$ or $\partial p_A^*(\theta)/\partial\alpha > 0$ yields $c < 2\left(-\alpha^2+6\alpha+9\right)t/(3-2\alpha^2)$. Since $2\left(-\alpha^2+6\alpha+9\right)t/(3-2\alpha^2) - c_H = (\alpha+1)(2\alpha+3)\left(-2\alpha^2+6\alpha+15\right)t/[(4\alpha+3)\left(3-2\alpha^2\right)]$, for any $\alpha \in (0,1]$ we obtain $\partial p_B^*/\partial\alpha > 0$ and $\partial p_A^*(\theta)/\partial\alpha > 0$. $\square$

The intuition behind Lemma 2 is as follows. When α is small, that is, when consumers in the new market are more price sensitive, the uniform prices of firms A and B are low because price competition in the new market is fierce. Therefore, the personalized prices of firm A are correspondingly low. The opposite is true when α is large, that is, when the consumers are price insensitive.

We investigate the effect of the price insensitivity α on the price reduction with the privacy cost. By further differentiating $\partial p_A^*/\partial c$, $\partial p_B^*/\partial c$, and $\partial p_A^*(\theta)/\partial c$ obtained in Lemma 1 with respect to α, we obtain the following Lemma 3.

Lemma 3 *The uniform and personalized prices of firm A, which can observe personal information, and uniform price of firm B, which cannot observe personal information, all slightly (significantly) decrease with the increasing privacy cost when the new market consumers are price sensitive (insensitive).*

Proof. We identify the signs of $\partial^2 p_A^*/(\partial\alpha\partial c)$, $\partial^2 p_B^*/(\partial\alpha\partial c)$ and $\partial^2 p_A^*(\theta)/(\partial\alpha\partial c)$. Differentiating $\partial p_A^*/\partial c$, $\partial p_B^*/\partial c$, and $\partial p_A^*(\theta)/\partial c$ with respect to α, we obtain the following equations.

$$\frac{\partial^2 p_A^*}{\partial\alpha\partial c} = -\frac{6}{(2\alpha+3)^2}, \quad \frac{\partial^2 p_B^*}{\partial\alpha\partial c} = -\frac{3-2\alpha^2}{[(\alpha+1)(2\alpha+3)]^2}, \quad \frac{\partial^2 p_A^*(\theta)}{\partial\alpha\partial c} = -\frac{3-2\alpha^2}{[(\alpha+1)(2\alpha+3)]^2}.$$

From $0 < \alpha < 1$, we obtain $\partial^2 p_A^*/(\partial\alpha\partial c) < 0$, $\partial^2 p_B^*/(\partial\alpha\partial c) < 0$ and $\partial^2 p_A^*(\theta)/(\partial\alpha\partial c) < 0$. □

The intuition for Lemma 3 is as follows. As shown in Lemma 2, when the new market consumers are sensitive (insensitive) to prices, the uniform and personalized prices of firm A and uniform price of firm B are low (high). Thus, the prices slightly (significantly) decline as the privacy cost rises.

Based on the aforementioned implications, we now provide comparative statics on the consumer surplus, profit of each firm, and total surplus based on privacy cost c. First, by differentiating the consumer surplus CS^* with respect to c and further examining c which yields the highest consumer surplus, we obtain the following Proposition 1.

Proposition 1 *(i) If the new market consumers are price sensitive, that is, if* $0 < \alpha < 0.7699$*, the consumer surplus is a U-shaped function of the privacy cost. Otherwise, that is, if* $0.7699 \leq \alpha \leq 1$*, the consumer surplus is an increasing function of the privacy cost. (ii) If the consumers are sufficiently price sensitive, that is, if* $0 < \alpha \leq \left(\sqrt{10} - 1\right)/6 \approx 0.3604$*, the consumer surplus is maximized at* $c = 0$*. Otherwise, that is, if* $\left(\sqrt{10} - 1\right)/6 < \alpha \leq 1$*, the consumer surplus is maximized at* $c = c_H$*.*

Proof. See the Appendix. □

We consider the intuition behind Proposition 1. First, we discuss Proposition 1 (i). An increase in c has two effects on the consumer surplus. The first effect is that it reduces the utility of consumers who conceal personal information. In the following section, we refer to this as the "direct effect". Note that the direct effect is small for the large c because few consumers conceal their personal information. The second effect is that the prices reduce, similar to Lemma 1. We refer to this effect as the "price reduction effect" below. The price reduction effect increases the consumer surplus. With small c, the direct effect dominates the price reduction effect, while for large c, the direct effect is dominated. This is because the direct effect decreases with c. Additionally, if the new market consumers are sufficiently price insensitive, from Lemma 3, the price reduction effect becomes larger. Thus, the consumer surplus takes an increasing function of c. Otherwise, the consumer surplus is a U-shaped function of c.

From Proposition 1 (i), the consumer surplus is a U-shaped or increasing function of c. Therefore, c maximizing consumer surplus yields either $c = 0$ or $c = c_H$. From Lemma 3, when α is large, the price reduction effect is large. Accordingly, the region in which the price reduction effect dominates the direct effect is large. Therefore, when drawing the consumer surplus with a U-shaped or increasing curve, an area of increasing consumer surplus expands. In this case, consumer surplus tends to be maximized at $c = c_H$. Conversely, when α is small, the direct effect dominates. Consumer surplus tends to be maximized at $c = 0$ because the area of increasing consumer surplus shrinks.

In the case of Montes et al. (2019), who have considered a duopolistic market, along with Valletti and Wu (2020), the price reduction effect always dominates. These previous studies show that a policy that makes consumers less likely to conceal their personal information, such as in China, maximizes consumer surplus. This study shows that the direct effect can dominate by considering the heterogeneity in price sensitivities among consumers. Hence, if the new market consumers are sufficiently price sensitive, a policy that allows consumers to easily conceal their personal information, such as the EU's GDPR, maximizes consumer surplus. Therefore, when discussing the optimal privacy policy for consumers, authorities should consider the heterogeneity of price sensitivities.

Second, we analyze the effect of increasing c on the firms' profits. By differentiating the firms' profits π_A^* and π_B^* with respect to c and further examining c, which maximizes each profit, we obtain the following proposition 2.

Proposition 2 *(i) The profit of firm A, which can observe consumers' personal information, is a U-shaped function of the privacy cost. The profit is always maximized at $c = c_H$, regardless of the price insensitivity. (ii) The profit of firm B, which cannot observe consumers' personal information, is a decreasing function of the privacy cost. The profit is always maximized at $c = 0$.*

Proof. See the Appendix. □

We discuss the intuition of Proposition 2. First, we consider Proposition 2 (ii). Considering that the price reduction effect decreases the profit π_B of firm B, π_B is a decreasing function of c. Therefore, π_B is maximized at $c = 0$. Next, consider Proposition

2 (i). An increase in c has two effects on the profit π_A of firm A: the price reduction effect and the "personalized price effect". The personalized price effect is that more consumers reveal their personal information and purchase at personalized prices. The price reduction effect reduces π_A; the personalized price effect increases π_A. For large c, the willingness to pay among consumers who have newly revealed their personal information is high. Thus, for large c, the personalized price effect dominates; for small c, the price reduction effect dominates. Based on the given information, π_A is a U-shaped function of c. Finally, we examine whether $c = 0$ or $c = c_H$ maximizes π_A. Regardless of the price insensitivity of the new market consumers, the personalized price effect always dominates the price reduction effect. Therefore, π_A is maximized with $c = c_H$.

Third, we consider the effect of increasing c on the total surplus. By differentiating the total surplus TS^* with respect to c and further investigating c, which maximizes the total surplus, we obtain Proposition 3.

Proposition 3 *The total surplus is a U-shaped function of the privacy cost. If the new market consumers are sufficiently price sensitive, that is, if* $0 < \alpha \leq 0.0638$*, the total surplus is maximized at* $c = 0$*. Conversely, if they are as price insensitive as the old market consumers, that is, if* $0.0638 < \alpha \leq 1$*, the total surplus is maximized at* $c = c_H$*.*

Proof. See the Appendix. □

The intuition behind Proposition 3 is as follows. We discuss that the total surplus is a U-shaped function of c. An increase in c has two effects on the total surplus. The first is the direct effect, which reduces the total surplus. If c is large, the direct effect is small because fewer consumers conceal their personal information. The second is that the price reduction effect improves the asymmetry of market share between firms. We refer to this effect as the "asymmetry improvement effect". The asymmetry improvement effect increases the total surplus because it reduces consumers' transportation costs. Based on the given information, for small c, the direct effect dominates, while for large c, the asymmetry improvement effect dominates.

Considering that the total surplus is a U-shaped function of c, we find that c maximizing total surplus yields either $c = 0$ or $c = c_H$. From Lemma 3, if α is large, the asymmetry improvement effect is large because the price reduction effect is large. Therefore, when drawing the total surplus, an area of increasing total surplus expands. In this case, the total surplus tends to be maximized at $c = c_H$. If α is small, the smallest privacy cost, $c = 0$, has the maximum total surplus because the direct effect dominates the asymmetry improvement effect.

Comparing Proposition 1 and Proposition 3, we obtain Corollary 1.

Corollary 1 *(i) If the price sensitivity of the new market consumers is intermediate, that is, if* $0.0638 < \alpha \leq \left(\sqrt{10} - 1\right)/6$, *the consumer surplus is maximized at* $c = 0$ *and total surplus is maximized at* $c = c_H$. *(ii) Otherwise, the arguments maximizing the consumer and total surpluses are the same.*

We consider the intuition behind Corollary 1. From Propositions 1 and 3, at $c = 0$, the consumer and total surpluses are maximized when α is small. For large α, the largest c yields the maximum consumer and total surpluses. Hence, we obtain Corollary 1 (ii). Next, we consider Corollary 1 (i). As noted in Proposition 3, an increase in c has two effects on total surplus: the direct and asymmetry improvement effects. When considering the consumer surplus, we also consider an additional effect. An increase in c decreases the consumer surplus because more consumers purchase at personalized prices. Therefore, an increase in c hampers the consumer surplus instead of the total surplus. Therefore, the area in which consumer surplus is maximized at $c = 0$ is wider than that in which total surplus is maximized at $c = 0$, leading to Corollary 1 (i).

As discussed before, a privacy policy that maximizes the consumer surplus may not coincide with a privacy policy that maximizes the total surplus. We find that the existence of this gap depends on the price insensitivity of the new market consumers.

Therefore, we suggest that authorities should carefully consider the heterogeneity of price sensitivities across the markets when determining privacy policies.

Finally, we conclude this section with a discussion of the effect of the price insensitivity of the new market consumers on the consumer surplus, profit of each firm, and total surplus. Differentiating each equilibrium value with respect to α, we obtain Proposition 4.

Proposition 4 *(i) The consumer surplus monotonically decreases as the new market consumers become price insensitive. (ii) As they become price insensitive, the profits of both firms monotonically increase. (iii) As they become price insensitive, the total surplus monotonically decreases.*

Proof. See the Appendix. □

The intuition behind Proposition 4 is as follows. First, consider Propositions 4 (i) and (ii). From Lemma 2, if the new market consumers are price insensitive, the firms set higher prices. Thus, if the consumers are price insensitive, the profits increase. Conversely, this leads to a decrease in consumer surplus.

Next, we discuss Proposition 4 (iii). In terms of the total surplus, a rise or fall in prices is simply a transfer of income between the consumers and firms. Therefore, the consumer's transportation costs determine the effect on the total surplus. In the new market, if the new market consumers are price insensitive, their transportation costs are relatively weighted and subsequently larger. Additionally, if the new market consumers are price insensitive, the uniform price of firm B is higher, and the share of firm B in the old market is smaller. This leads to a greater asymmetry of market shares in the old market, resulting in higher transportation costs in the old market. Accordingly, if the consumers become price insensitive, the total surplus decreases.

5 Conclusion

This study analyzes a model in which two firms compete across two markets, specifically, a new market without personal information and an old market with personal information. Additionally, one firm can observe personal information, while the other firm cannot. We also assume that consumers can choose their privacy. Furthermore, considering the heterogeneity of price sensitivities among consumers, we analyze how the optimal privacy policy varies with price sensitivity.

We get two main results. First, we find that the optimal privacy policy for consumers depends on the price sensitivity of the new market consumers. If the new market consumers are sufficiently price sensitive, the consumer surplus is maximized under a privacy policy in which consumers can easily conceal their personal information. Conversely, if the new market consumers are as price insensitive as the old market consumers, the consumer surplus is maximized under a privacy policy in which consumers are less likely to conceal their personal information. The total surplus shows similar characteristics. Second, we find that the privacy policy that maximizes the consumer surplus may or may not coincide with the privacy policy that maximizes the total surplus. When the price sensitivity of the new market consumer is intermediate, the consumer surplus is maximized at the smallest privacy cost, while the total surplus is maximized at the highest privacy cost. Based on the results, we argue that authorities should consider the heterogeneity of price sensitivities across consumers when deciding on policies.

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Appendix

Proof of Proposition 1.

We prove Proposition 1 (i). Differentiating the consumer surplus CS^* with respect to c yields the following equation.

$$\frac{\partial CS^*}{\partial c} = \frac{2\left(4\alpha^2+16\alpha+9\right)c+2\left(4\alpha^3+16\alpha^2-3\alpha-9\right)t}{4(\alpha+1)(2\alpha+3)^2 t}. \tag{A1}$$

Next, we consider the sign of (A1). Solving $\partial CS^*/\partial c < 0$ yields $c < (-4\alpha^3-16\alpha^2+3\alpha+9)t/(4\alpha^2+16\alpha+9) = c_{CS}$. Thus, if we ignore the range of c, CS^* is a U-shaped function with a minimum at $c = c_{CS}$.

Here, we consider the range of c. First, we check the sign of c_{CS}. The sign of c_{CS} corresponds to the sign of $-4\alpha^3-16\alpha^2+3\alpha+9$. Thus, solving $-4\alpha^3-16\alpha^2+3\alpha+9>0$ in the range $\alpha \in (0,1]$ gives the following result.

$$c_{CS} > 0 \quad \text{if} \quad 0 < \alpha < 0.7699. \tag{A2}$$

From (A2), if $0.7699 \leq \alpha \leq 1$, the consumer surplus is an increasing function of c.

Second, we compare c_H and c_{CS}. From $c_H - c_{CS}$, we obtain the following equation.

$$c_H - c_{CS} = \frac{2\alpha\left(4\alpha^3+32\alpha^2+55\alpha+24\right)t}{(4\alpha+3)\left(4\alpha^2+16\alpha+9\right)}. \tag{A3}$$

From (A3), we obtain $c_H > c_{CS}$ because $\alpha \in (0,1]$. Therefore, if $0 < \alpha < 0.7699$, the consumer surplus is a U-shaped function of c.

Finally, we prove Proposition 1 (ii). We examine whether $c = 0$ or $c = c_H$ maximizes CS^*. If $0.7699 \leq \alpha \leq 1$, the consumer surplus is an increasing function of c; therefore, the consumer surplus is maximized at $c = c_H$. Next, we consider when $0 < \alpha < 0.7699$. Let us denote the consumer surplus when $c = 0$ as CS_0^* and the consumer surplus when $c = c_H$ as CS_H^*. Calculating $CS_H^* - CS_0^*$ yields the following equation.

$$CS_H^* - CS_0^* = -\frac{\left(48\alpha^5+112\alpha^4-592\alpha^3-552\alpha^2+45\alpha+81\right)t}{4(2\alpha+3)^2(4\alpha+3)^2}. \tag{A4}$$

Examine the sign of (A4). The sign of (A4) is the same as the sign of $-48\alpha^5 - 112\alpha^4 + 592\alpha^3 + 552\alpha^2 - 45\alpha - 81$. Solving $-48\alpha^5 - 112\alpha^4 + 592\alpha^3 + 552\alpha^2 - 45\alpha - 81 > 0$ in the range $\alpha \in (0, 1]$ yields the following result.

$$CS_H^* > CS_0^* \quad \text{if} \quad \frac{1}{6}\left(\sqrt{10} - 1\right) < \alpha \leq 1. \tag{A5}$$

From (A2) and (A5), if the new market consumers are sufficiently price sensitive, that is, if $0 < \alpha \leq \left(\sqrt{10} - 1\right)/6$, the consumer surplus is maximized at $c = 0$. Additionally, if the consumers are as price insensitive as the old market consumers, that is, if $\left(\sqrt{10} - 1\right)/6 < \alpha \leq 1$, the consumer surplus is maximized at $c = c_H$. □

Proof of Proposition 2.

First, consider Proposition 2 (i). Differentiating the profit of firm A, π_A^*, with respect to c, we obtain the following equation.

$$\frac{\partial \pi_A^*}{\partial c} = \frac{(8\alpha + 9)c - 6\alpha t}{2(2\alpha + 3)^2 t}. \tag{A6}$$

Solving $\partial \pi_A^*/\partial c < 0$, we obtain $c < 6\alpha t/(8\alpha + 9) = c_A$ from (A6). Thus, if we ignore the range of c, we find that π_A^* is a U-shaped function with a minimum at $c = c_A$.

$$c_H - c_A = \frac{(2\alpha + 3)\left(-8\alpha^2 + 11\alpha + 9\right)t}{(4\alpha + 3)(8\alpha + 9)}. \tag{A7}$$

The sign of (A7) corresponds to the sign of $-8\alpha^2 + 11\alpha + 9$. Solving $-8\alpha^2 + 11\alpha + 9 > 0$ yields $\left(11 - \sqrt{409}\right)/16 < \alpha < \left(11 + \sqrt{409}\right)/16$. Therefore, $c_H > c_A$ holds in the range $\alpha \in (0, 1]$. Accordingly, for any $\alpha \in (0, 1]$, π_A^* is a U-shaped function of c.

Second, we examine whether $c = 0$ or $c = c_H$ maximizes π_A^*. Let us denote the profit of firm A at $c = 0$ as π_{A0}^* and profit of firm A at $c = c_H$ as π_{AH}^*. Calculating $\pi_{AH}^* - \pi_{A0}^*$, we obtain the following equation.

$$\pi_{AH}^* - \pi_{A0}^* = \frac{(3 - \alpha)(2\alpha + 1)\left(-16\alpha^3 - 26\alpha^2 + 33\alpha + 27\right)t}{4(2\alpha + 3)^2(4\alpha + 3)^2}. \tag{A8}$$

Examine the sign of (A8). This sign corresponds to the sign of $-16\alpha^3-26\alpha^2+33\alpha+27$. In the range $\alpha \in (0,1]$, $-16\alpha^3-26\alpha^2+33\alpha+27>0$. Thus, for any $\alpha \in (0,1]$, we obtain $\pi^*_{AH} > \pi^*_{A0}$. Therefore, the profit of firm A, π^*_A, is always maximized at $c = c_H$.

Finally, we prove Proposition 2 (ii). By differentiating the equilibrium profit of firm B, π^*_B, with respect to c, we obtain the following equation.

$$\frac{\partial \pi^*_B}{\partial c} = -\frac{\alpha[2(\alpha+3)t-c]}{(\alpha+1)(2\alpha+3)^2 t}. \tag{A9}$$

From (A9), if $c < 2(\alpha+3)t = c_B$, we obtain $\partial \pi^*_B/\partial c < 0$. Thus, if we ignore the range of c, we obtain that π^*_B is a U-shaped function with a minimum at $c = c_B$.

Here, we consider the range of c. Calculating $c_B - c_H$ yields the following equation.

$$c_B - c_H = \frac{5(\alpha+1)(2\alpha+3)t}{4\alpha+3}. \tag{A10}$$

From (A10), we find that $c_B > c_H$. Therefore, π^*_B is a decreasing function of c. Accordingly, we find that π^*_B is maximized at $c = 0$. □

Proof of Proposition 3.

Differentiating the equilibrium total surplus TS^* with respect to c yields the following equation.

$$\frac{\partial TS^*}{\partial c} = \frac{(12\alpha^2+35\alpha+18)\,c + (4\alpha^3+6\alpha^2-21\alpha-9)\,t}{2(\alpha+1)(2\alpha+3)^2 t}. \tag{A11}$$

Solving $\partial TS^*/\partial c < 0$, we obtain $c < (-4\alpha^3-6\alpha^2+21\alpha+9)\,t/(12\alpha^2+35\alpha+18) = c_{TS}$ from (A11). Therefore, if we ignore the range of c, we find that TS^* is a U-shaped function with a minimum at $c = c_{TS}$.

Next, we consider the range of c. We identify the sign of c_{TS}. This sign corresponds to the sign of $-4\alpha^3-6\alpha^2+21\alpha+9$. For $\alpha \in (0,1]$, we find that $-4\alpha^3-6\alpha^2+21\alpha+9>0$, which is why we obtain $c_{TS} > 0$.

Next, we compare c_H and c_{TS}. Calculating $c_H - c_{TS}$ yields the following equation.

$$c_H - c_{TS} = \frac{(2\alpha+3)\,(-4\alpha^3+19\alpha^2+26\alpha+9)\,t}{(3\alpha+2)(4\alpha+3)(4\alpha+9)}. \tag{A12}$$

The sign of (A12) corresponds to the sign of $-4\alpha^3 + 19\alpha^2 + 26\alpha + 9$. For $\alpha \in (0, 1]$, we find $-4\alpha^3 + 19\alpha^2 + 26\alpha + 9 > 0$, so we obtain $c_H > c_{TS}$. Therefore, we are certain that TS^* is a U-shaped function of c.

Finally, we examine whether TS^* is maximized at $c = 0$ or $c = c_H$. Let us denote the total surplus at $c = 0$ as TS_0^* and the total surplus at $c = c_H$ as TS_H^*. Calculating $TS_H^* - TS_0^*$, we obtain the following equation.

$$TS_H^* - TS_0^* = \frac{(3-\alpha)\alpha(2\alpha+1)\left(8\alpha^3 + 62\alpha^2 + 43\alpha - 3\right)t}{4(\alpha+1)(2\alpha+3)^2(4\alpha+3)^2}. \tag{A13}$$

The sign of (A13) corresponds to the sign of $8\alpha^3 + 62\alpha^2 + 43\alpha - 3$. Solving $8\alpha^3 + 62\alpha^2 + 43\alpha - 3 > 0$ for $\alpha \in (0, 1]$ yields the following result.

$$TS_H^* > TS_0^* \quad \text{if} \quad 0.0638 < \alpha \leq 1. \tag{A14}$$

From (A14), if the new market consumers are sufficiently price sensitive, that is, if $0 < \alpha \leq 0.0638$, the total surplus is maximized at $c = 0$. Conversely, if they are as price insensitive as the old market consumers, that is, if $0.0638 < \alpha \leq 1$, the total surplus is maximized at $c = c_H$. □

Proof of Proposition 4.

First, we prove Proposition 4 (i). By differentiating the consumer surplus CS^* with respect to α, we obtain the following equation.

$$\frac{\partial CS^*}{\partial \alpha} = \frac{\begin{bmatrix} -c^2(8\alpha^3 + 52\alpha^2 + 62\alpha + 15) + 12c(16\alpha^2 + 26\alpha + 9)t \\ -(\alpha+1)^2(8\alpha^3 + 36\alpha^2 + 54\alpha + 459)t^2 \end{bmatrix}}{4(\alpha+1)^2(2\alpha+3)^3 t}. \tag{A15}$$

The sign of (A15) corresponds to the sign of the numerator. The numerator is a convex upward quadratic function of c. Now, considering the discriminant D_{CS} of the numerator, we obtain the following result.

$$D_{CS} = -4(2\alpha+3)^2\left(16\alpha^6 + 160\alpha^5 + 552\alpha^4 + 1768\alpha^3 + 3205\alpha^2 + 2322\alpha + 441\right)t^2 < 0. \tag{A16}$$

From (A16), we obtain $\partial CS^*/\partial\alpha < 0$ for any $\alpha \in (0,1]$.

Second, consider Proposition 4 (ii). Differentiating the profit of firm A, π_A^*, with respect to α, we obtain the following equation.

$$\frac{\partial \pi_A^*}{\partial \alpha} = \frac{(3t-c)[(4\alpha+3)c + 6(\alpha+3)t]}{(2\alpha+3)^3 t}. \tag{A17}$$

(A17) is positive if $c < 3t$. Given that $c_H < 3t$, we obtain $\partial\pi_A^*/\partial\alpha > 0$. Next, differentiating the profit of firm B, π_B^*, with respect to α, we obtain the following equation.

$$\frac{\partial \pi_B^*}{\partial \alpha} = \frac{[2(\alpha+3)t - c]\left[\left(4\alpha^2+2\alpha-3\right)c + 2\left(-4\alpha^2+3\alpha+9\right)t\right]}{2(\alpha+1)^2(2\alpha+3)^3 t}. \tag{A18}$$

For (A18), the denominator is always positive. Additionally, because $c_H < 2(\alpha+3)t$, $2(\alpha+3)t - c > 0$. Therefore, the sign of (A18) corresponds to the sign of $\left[\left(4\alpha^2+2\alpha-3\right)c + 2\left(-4\alpha^2+3\alpha+9\right)t\right]$. It is clear that the second term in $\left[\left(4\alpha^2+2\alpha-3\right)c + 2\left(-4\alpha^2+3\alpha+9\right)t\right]$ is always positive. Next, if the first term of $\left[\left(4\alpha^2+2\alpha-3\right)c + 2\left(-4\alpha^2+3\alpha+9\right)t\right]$ is greater than or equal to 0, then $\left[\left(4\alpha^2+2\alpha-3\right)c + 2\left(-4\alpha^2+3\alpha+9\right)t\right] > 0$, we obtain $\partial\pi_B^*/\partial\alpha > 0$. Solving $4\alpha^2 + 2\alpha - 3 \geq 0$ for α yields $(\sqrt{13}-1)/4 \leq \alpha \leq 1$. Therefore, if $(\sqrt{13}-1)/4 \leq \alpha \leq 1$, then $\partial\pi_B^*/\partial\alpha > 0$. Next, consider when $0 < \alpha < (\sqrt{13}-1)/4$. Solving $\left[\left(4\alpha^2+2\alpha-3\right)c + 2\left(-4\alpha^2+3\alpha+9\right)t\right] > 0$, we obtain $c < 2\left(4\alpha^2-3\alpha-9\right)t/\left(4\alpha^2+2\alpha-3\right) = c_b$. Next, we compare c_H and c_b. Calculating $c_b - c_H$, we obtain the following equation.

$$c_b - c_H = \frac{\left(8\alpha^4+16\alpha^3-28\alpha^2-81\alpha-45\right)t}{(4\alpha+3)\left(4\alpha^2+2\alpha-3\right)}. \tag{A19}$$

In (A19), the denominator is negative because $4\alpha^2+2\alpha-3 < 0$. Therefore, the sign of (A19) is the same as the sign of $-8\alpha^4 - 16\alpha^3 + 28\alpha^2 + 81\alpha + 45$. Since $-8\alpha^4 - 16\alpha^3 + 28\alpha^2 + 81\alpha + 45 > 0$ is always positive in $\alpha \in (0,1]$, therefore $c_b > c_H$. Therefore, even for $4\alpha^2+2\alpha-3 < 0$, $\left[\left(4\alpha^2+2\alpha-3\right)c + 2\left(-4\alpha^2+3\alpha+9\right)t\right] > 0$. Therefore, for any $\alpha \in (0,1]$, we obtain $\partial\pi_B^*/\partial\alpha > 0$.

Finally, we prove Proposition 4 (iii). Differentiating the total surplus TS^* with respect to α, we obtain the following equation.

$$\frac{\partial TS^*}{\partial \alpha} = \frac{\left[\begin{array}{l} -c^2(24\alpha^3+104\alpha^2+106\alpha+21)+4c(10\alpha^2+69\alpha+66)\alpha t \\ -\left(8\alpha^5+52\alpha^4+94\alpha^3+315\alpha^2+324\alpha+27\right)t^2 \end{array}\right]}{4(\alpha+1)^2(2\alpha+3)^3 t}. \tag{A20}$$

The sign of (A20) corresponds to the sign of the numerator. Here, we find that the numerator is a convex upward quadratic function of c. Calculating the discriminant D_{TS} of the numerator, we obtain the following equation.

$$D_{TS} = -4(2\alpha+3)^2\left(48\alpha^6+376\alpha^5+792\alpha^4+1152\alpha^3+1579\alpha^2+990\alpha+63\right)t^2 < 0. \tag{A21}$$

Therefore, from (A21), we obtain $\partial TS^*/\partial\alpha < 0$. $\square$

Online Appendix (not for publication): condition for firm B to enter the old market.

In this part, we show the condition where firm B enters the old market.

Remark 1 *Firm B enters the old market if* $\theta_{CR} < [(4\alpha+3)\sqrt{1+\alpha} - 6\alpha^2 - 3\alpha + 3]/[4\alpha(1+\alpha)] = \bar{\theta}$.

Proof. In the case that firm B enters the old market, we define the profit of each firm in Section 2. Thus, we obtain the following candidate best response functions of the firms in the second stage.

$$p_A = \frac{1}{2}[p_B + \alpha t(2\theta_{CR} + 1)] = BR_A, \tag{B1}$$

$$p_B = \frac{p_A + 2\alpha t}{2(\alpha+1)} = BR_B. \tag{B2}$$

Next, we consider the case in which firm B does not enter the old market. In this case, the profit of each firm is expressed as follows.

$$\pi_A' = \int_0^{\theta_N} p_A d\theta + \int_0^{\theta_{CR}} p_A d\theta + \int_{\theta_{CR}}^1 p_A(\theta) d\theta, \tag{B3}$$

$$\pi_B' = \int_{\theta_N}^1 p_B d\theta. \tag{B4}$$

From (B3) and (B4), we obtain the following candidate best response functions of the firms when firm B does not enter the old market.

$$p_A = \frac{1}{2}[p_B + \alpha t(2\theta_{CR} + 1)] = BR_A \tag{B5}$$

$$p_B = \frac{1}{2}(p_A + \alpha t) = BR_B' \tag{B6}$$

From (B1) and (B5), the candidate best response functions of firm A are identical, regardless of whether firm B enters the old market or not. Therefore, whether or not firm B enters the old market in equilibrium depends on the shape of firm B's profit.

Here, the share of firm B in the old market is positive when $\theta_O = (p_B + t)/(2t) < 1$, indicating that it enters the old market when $p_B < t$. Therefore, the shape of the profit of firm B can be the case (a)∼(d) in Figure 1 below.

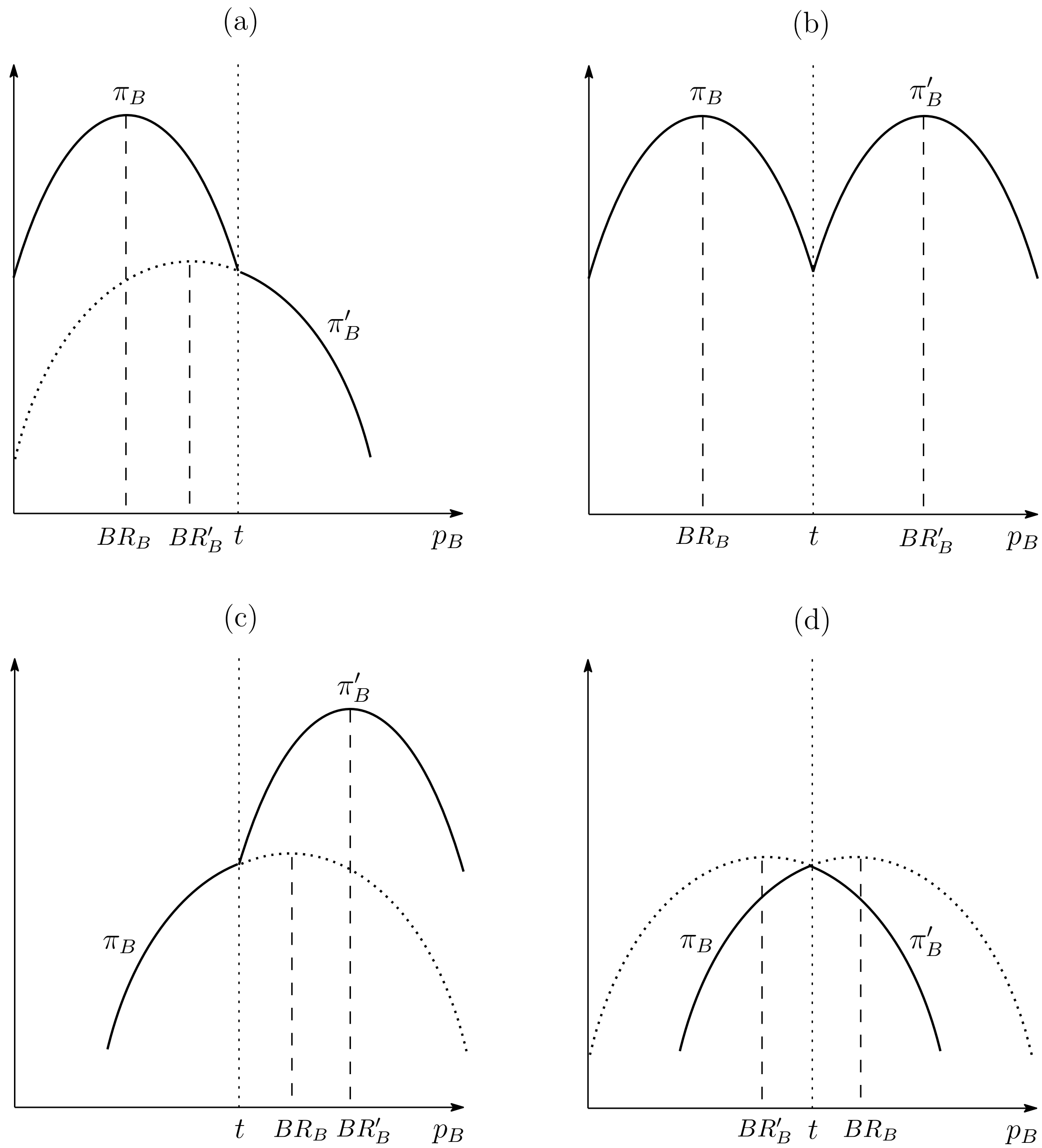

Figure 1. The profit of firm B

Find the condition for each case. First, let us consider Figure 1 (a). Case (a) is established when the following (B7) is satisfied.

$$BR_B < t \text{ and } BR'_B \leq t. \tag{B7}$$

Solving (B7), we obtain $p_A \leq t(2-\alpha)$. Therefore, if $p_A \leq t(2-\alpha)$, case (a) is established. In this case, the profit of firm B is maximized with $p_B = BR_B$.

Second, we consider Figure 1 (b). Case (b) holds if the following (B8) is satisfied.

$$BR_B < t \text{ and } t < BR'_B. \tag{B8}$$

Solving (B8), we obtain $t(2 - \alpha) < p_A < 2t$. Thus, if $t(2 - \alpha) < p_A < 2t$, case (b) is established. Next, we examine whether the profit of firm B is maximized when $p_B = BR_B$ or $p_B = BR'_B$. Substituting (B2) and (B6) into the profit of firm B, π_B, π'_B, respectively, we obtain the following two equations.

$$\pi_B\big|_{p_B=BR_B} = \frac{(p_A + 2\alpha t)^2}{8\alpha(\alpha+1)t}, \tag{B9}$$

$$\pi'_B\big|_{p_B=BR'_B} = \frac{(p_A + \alpha t)^2}{8\alpha t}. \tag{B10}$$

Solving (B9) $>$ (B10), we obtain $p_A < \left(1 - \alpha + \sqrt{\alpha+1}\right) t$. Thus, in case (b), if $p_A < \left(1 - \alpha + \sqrt{\alpha+1}\right) t$, the profit of firm B is maximized with $p_B = BR_B$.

Third, consider Figure 1 (c). Case (c) is established when the following (B11) is satisfied.

$$t \leq BR_B \text{ and } t < BR'_B. \tag{B11}$$

Solving (B11), we obtain $2t \leq p_A$. Therefore, if $2t \leq p_A$, case (c) is established, and the profit of firm B is maximized given $p_B = BR'_B$.

Finally, consider Figure 1 (d). Case (d) holds if the following (B12) is satisfied.

$$t \leq BR_B \text{ and } BR'_B \leq t. \tag{B12}$$

Here, we can immediately see that (B12) is not satisfied.

Accordingly, if $p_A < \left(1 - \alpha + \sqrt{\alpha + 1}\right) t = \bar{p}_A$, the profit of firm B is maximized with $p_B = BR_B$. Thus, the best response of firm B is $p_B = BR_B$ if $p_A < \bar{p}_A$.

Next, we find the intersection of the best response $p_A = BR_A$ for firm A and the best response $p_B = BR_B$ for firm B under $p_A < \bar{p}_A$. Solving this for p_A and p_B, we obtain the intersection as follows.

$$\tilde{p}_A = \frac{2\alpha t[(2\alpha + 2)\theta_{CR} + \alpha + 2]}{4\alpha + 3}, \quad \tilde{p}_B = \frac{\alpha t(2\theta_{CR} + 5)}{4\alpha + 3}.$$

Finding the condition that this intersection satisfies $p_A < \bar{p_A}$ and $p_B < t$, we obtain the following condition.

$$\theta_{CR} < \frac{(4\alpha + 3)\sqrt{\alpha + 1} - 6\alpha^2 - 3\alpha + 3}{4\alpha(\alpha + 1)} = \bar{\theta}.$$

Hence, firm B enters the old market if $\theta_{CR} < \bar{\theta}$. $\square$