Implementing the Process Tracing Technique using Combinatory Categorial Grammars: An Application to the Analysis of Economic Coordination within Firms

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Implementing the Process Tracing Technique using Combinatory Categorial Grammars: An Application to the Analysis of Economic Coordination within Firms[1]

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Abstract. This paper describes a method for analyzing the evolutionary path of a complex, dynamic, and contingent social phenomenon. Given empirical evidence of a surprising or anomalous fact that contradicts a widely acknowledged theory, the aim is to create a plausible explanation based on its context of occurrence, taking a holistic and historical point of view. The procedure begins by translating theoretical propositions into grammar rules that describe patterns of sequences of either individual actions or interactions carried out by a stable community of actors, such as types of decision-making events. Subsequently, applying a process tracing technique based on the logic of retroduction creates an extension of this initial process category, relying on configurations of contextual conditions that acknowledge the surprising fact as a new event outcome in a specific empirical setting. Finally, a structural comparison between pairs of representative instances may lead to the refinement of the theory.

Keywords: Critical Realism; Configurational Analysis; Generative Social Science; Pragmatism; Theory of the Firm.

1 Introduction

The difference between the objects of study in the natural sciences and the social sciences arises from the observation that people interpret the objective reality in many subjective ways and choose to act accordingly. Specifically, the human mind perceives and interprets external signals using a categorical framework of its own, which evidently often has both commonalities and differences with those of other peers. Additionally, due to the high uncertainty that results from subjectivity, explanation

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prevails over prediction in social research, and attempts to forecast future event outcomes are limited to contingent tendencies that may eventually become reality.

Despite their distinct natures, the transposition of methods of scientific research from the natural sciences into the social sciences without prior consideration of their ontological assumptions is common practice under the auspices of the epistemological guidance of the mainstream paradigm known as Social Positivism. This problem is particularly remarkable in economics, where the capability of prediction remains the main criterion for scientific assessment of works in this discipline. There is nothing inherently wrong with the mainstream research approach in economics and other social sciences, as they reflect both the goals and beliefs of their communities, which are often divergent. However, there are efforts of reconciliation among these fields of the social sciences towards such a philosophical and methodological framework that explains the reasons for their commonalities and differences and the context in which each approach is applicable.

The most promising trend in the social sciences seems to be towards formulating middle-range theories, which involve modeling a real-world phenomenon based on empirically testable propositions deduced from universal laws within a limited scope, rather than formulating such an abstract, grand theory (Merton, 1968). The diffusion of this approach to theorizing among the social sciences, due to the structural-functionalist tradition acknowledging the complex and contingent nature of social phenomena, promoted the use of empirical grounded methods. However, complexity is not fully intelligible using the methods of the so-called normal science, which rely on the assumptions of Social Positivism, particularly the so-called näive-realism, or the idea of the possibility of direct perception of the external world as it is.

Ironically but not surprisingly, the alternative to normal science still involves the transposition of recent results from fields of mathematics and physics, as shown in the book "A New Kind of Science" (Wolfram, 2002). The hard sciences are familiar with complexity as a justification for emergent phenomena, which are patterns of higher-level properties and behaviors that result from lower-level interactions among their constituent elements. It still seems ambitious to assume that the physical world operates using automata, but it is not so far-fetched to assume that the social world operates in this way also, especially social organizations, which often result from our deliberate rational actions under the specific context of their occurrence.

Social systems are epiphenomena of networks of individuals acting under the guidance of consensual rules, such that the result is not only collective, orderly behavior but also emergent macrostructures that seem to evolve and exhibit properties and behavior of their own. Even under the assumption of perfect knowledge of the individual decision rules, the evolutionary path of the emergent social forms and processes appear to remain largely independent from lower-level developments and is highly sensitive to any small differences in the initial conditions. Nonetheless, explanation still requires unveiling the links between the developments in the emergent and lower levels of social reality, particularly because emergent social forms do not exhibit the cohesion that is observable on emergent biological forms. There is no such thing like universal laws in social reality, but rather contingent tendencies that may or may not take place within specific empirical settings.

Nonetheless, mathematical models, such as complex adaptive systems and dynamical systems, still struggle to predict future event outcomes in the emergent level of social phenomena because of the structure and agency relation, which is absent in natural phenomena. The agents are not static reactive objects; they can actively learn from social situations, leading to potential changes in their own behavior and the surrounding social structures in unpredictable ways.

There is still a need for a proper definition of social science because of the particularities of its object of study, which implicates its own goals and methods, not necessarily the same as those of the natural sciences. For instance, while STEM disciplines rely on experimentation as the main general methodological approach to generalize universal laws, the social sciences should invest more on computational simulation to enable interventions on local phenomena. The computational power of modern digital computers already enables agent-based modeling and simulation of complex social systems (e.g., markets and organizations) relying on the assumption of the ontological equivalence of structures and agents to computational devices, but the approach is not widespread yet.

Generative Social Science is such an interdisciplinary effort towards the understanding of complex, dynamic, and contingent behaviors as the emergent result from interacting agents using computational models and simulation techniques (Epstein, 2006). The term “generative” still comes from Noam Chomsky’s Generative Grammar Theory (1957), which is a linguistic theory that relies upon rule systems to produce infinite valid constructions based upon a finite set of elements. Clearly, the elements and constructions that are the objects of studying in Generative Social Science are not alphabet symbols and sentences of a natural language, but the individual actions and social interactions that these elements are representing instead.

Generative Grammar Theory has influenced many fields of science, including computer science in the syntax specification of programming languages and computational linguistics, medicine in the study of the immune system (Jerne, 1985), and economics in the study of institutions for the governance of common goods (Crawford & Ostrom, 1995). This latter work proposes to use grammars like a metaphor rather than a model for the analysis of complex phenomena. The metaphorical utilization of grammars is also notable in Bourdieu (1977) and Salancik and Lebleblici (1988). Pentland (1995) uses grammar as a metaphor but also makes use of computational procedures for grammar induction from empirical data about organizational processes. Nonetheless, all works above that are empirical research rely strictly on positive evidence and are examples of the so-called positivist social science.

The assumptions of the philosophy of social sciences known as Social Positivism ignore some relevant differences between hard science and social science. Some of these include: (i) methodological monism, which argues for the existence of one scientific method; (ii) epistemological objectivism, which holds that true knowledge comes from perception, which is independent of subjective interpretations or biases; (iii) causal explanation as a provisional statement that is hypothetically deduced from universal principles of nature, including of human nature, which is true until being refuted by an empirical test; (iv) a demarcation criterion that discriminates between scientific and non-scientific statements relying on the logic of falsification rather than the logic of verification; and (v) methodological individualism, which holds that all scientific explanations must refer to empirical evidence about individuals and their interactions, rather than to emergent forms, properties, or behaviors.

The assumptions above contrast with a post-positivist epistemological perspective of making research, relying upon other assumptions about meaning, truth, and the nature of reality, including: (i) methodological dualism, which requires that the methods used to study human action be distinct from those used in natural sciences; (ii) epistemological relativism, which suggests that truth derives not only from the observation of an objective reality but also from conventions and frameworks of assessment that are specific to social contexts; (iii) causal explanation as an unobservable generative mechanism that is activated under a specific configuration of contextual conditions, which results in a socially and historically situated tendency rather than an universal law; (iv) a demarcation criterion that relies on the logic of retroduction rather than the logic of falsification; and (v) methodological holism, which assumes that an emergent reality exists independently of lower-level entities, relationships, and behaviors.

Among all the assumptions presented above, the choice for a logic of scientific research better typifies the difference between natural science and social science. Positivists revolutionized science by explaining the rationale behind opting for a logic of falsification rather than a logic of verification. While the latter naively argues for confirming a theory through exhaustive empirical observations or experiments, the former modifies this idea by asserting that a theory attains scientific status if it can be tested and potentially proven false by empirical evidence. However, refuting a social theory becomes a challenging task due to the structure and agency relation, as social phenomena evolve over time through individual learning of the social situation at hand. The logic of retroduction argues for inferring an explanatory hypothesis to provide a plausible explanation for any fact that contradicts the prediction of a widely acknowledged theory, rather than simply rejecting it outright.

1.1 The Logical Foundations for a Post-Positivist Methodology

Any prediction provided by a theory is essentially a claim about how reality works based on a set of assumptions. Nonetheless, any theory can fail after a single counterfactual example revealing its limitations. When a single counterfactual example contradicts a theoretical prediction, this situation means the *falsification* of the theory, and occurs for one of only two possible reasons (Popper, 1959).

If a theory relies on unrealistic underlying assumptions or contains a logical error, then it is an *incorrect theory*. This situation suggests that the theory's reasoning about the functioning of a system is either partially or completely wrong. The prospect of any incorrect theory either entails rejection in favor of another alternative theory or rectification if possible.

If a theory fails to elucidate all causal determinants of a phenomenon, then it is an *incomplete theory*. This situation suggests that the theory's reasoning has no internal problem, as many other predictions go through confirmation. However, there are causal determinants operating in some empirical settings that the currently acknowledged theory’s formulation did not consider. The prospect of any incomplete theory entails revision or refinement in favor of a more encompassing formulation.

In both cases, counterfactual examples provide empirical evidence that defies the validity of a theory. The theory's predictions do not align with concrete observations, indicating the need for the development of more accurate explanations of the phenomena.

Nonetheless, single cause-and-effect relationships do not characterize complex phenomena, which consist of a series of interconnected causal determinants taking place as either a chain or a network of interactions that together contribute to the observed result. In artificial, natural, or social systems, multiple variables, agents, or processes interact with each other, giving rise to emergent forms or behaviors as well as intricate causal relationships that affect the observed result in many unpredictable ways.

By analyzing causal chains in complex social phenomena, researchers gain insights into the underlying causal mechanisms of the hypothesized system. This understanding can help in predicting outcomes, designing interventions, and making informed decisions in real-life situations.

A technique that can help in the refinement of theory about complex phenomena must enable researchers to elucidate the underlying causal mechanisms and their contextual conditions of activation in particular empirical situations. The research procedure must involve tracing and systematic examining a sequence of events, whether they are individual actions or collective interactions that lead to an outcome of interest. As a result, the causal explanation for a complex phenomenon resembles a process instance representable in terms of rules of behavior, even though the phenomenon itself may not be representable in any rational way at all.

Process tracing (Bennett & Checkel, 2015; George & Bennett, 2004) is a qualitative research technique used to formulate or test hypotheses about the causal mechanisms involved in generating a sequence of event outcomes that are contingent on a specific empirical setting. As the given definition emphasizes, process tracing generally applies by means of one of two main approaches, for formulating and testing hypotheses, which relies on the mode of logical reasoning used in each of them.

On the one hand, the *theory-testing* approach relies on the deductive mode of reasoning, which starts with a general proposition ($A \rightarrow B$) and a sufficient condition (A) to determine a necessary observation (B) that is a logical consequence of the theory in use. In logic, deduction is the only syllogism that provides true logical consequences from a given pair of minor and major premises.

On the other hand, the *theory-formulation* approach relies on inductive reasoning, which starts with a set of repeating observations ($B_1, \dots, B_n$) and a hypothesized necessary condition (A) to infer a general proposition ($\forall i, A \rightarrow B_i$). However, it is important to note that this proposition may still not be consistent with the theory in use, as a single counter-example is sufficient to refute it. The operation of induction works as a tentative inverse operation to deduction, although deduction does not have any kind of inverse. Therefore, the theory-formulation approach is inherently speculative, as it involves formulating general propositions based on a set of observations and a hypothesized necessary condition, which may or may not align with the existing theory.

There is indeed a third mode of reasoning applicable, which is another tentative inverse operation to deduction. Abductive reasoning starts with a single observation (B) and a general proposition ($A \rightarrow B$) to infer a necessary condition (A). Applied to the process tracing technique as a third approach known as *theory-elaboration*, abductive inference makes use of a general proposition borrowed from a widely acknowledged theory, which, taken together with its necessary condition, provides a plausible explanation for the observation. Therefore, there may be other general propositions from other similarly widely acknowledged theories that would enable the inference of the same observation, such that the key issue for this inference procedure is to determine the best explanation among all equally plausible alternatives. In addition, there is no single or best way to accomplish this task.

Both inductive and abductive modes of reasoning apply to scientific research because they are the only ways to produce new knowledge from empirical evidence, as deduction only yields logical consequences that are implicit to the theory. Nonetheless, while inductive reasoning is always refutable by a single counter-example to its prediction, abductive reasoning is the weakest form of inference because it is less susceptible to refutation due to a lack of empirical reference. Abduction relates to diagnostic methods in the search for the causal determinants of any systematic unexpected result and to problem-solving techniques. While inductive reasoning enables the formulation of a completely new theory from scratch using empirical evidence, abductive reasoning seeks to *reformulate* or *elaborate* an existing theory by incorporating some new proposition regarded as true in another domain of inquiry to improve the explanation of an observation in the phenomenon under investigation.

The North-American pragmatist philosopher Charles Sanders Peirce proposed in the paper "*Illustrations of the Logic of Science: Deduction, Induction, and Hypothesis*" (1878) the abductive mode of reasoning to elucidate the logic of scientific inquiry since this mode of inference aims at theory elaboration as a response to the refutation of a hypothesis after empirical testing. In addition to rationalism, which relies on deductive reasoning to uncover logical consequences from universal principles, and empiricism, which relies only on inductive reasoning for hypothesis testing, a third mode of reasoning suggested an alternative approach to the scientific process. However, abduction alone cannot accomplish too much. Later in his life, Peirce perceived the complexity of scientific investigations and further developed this idea, introducing a joint use of abduction, induction, and deduction, which he referred to as the logic of retroduction.

Retroduction consists of the recursive analysis of empirical evidence to develop the best explanation for a surprising or anomalous fact that logically contradicts the predictions of a widely acknowledged theory. A retroductive research procedure uses abduction, deduction, and induction (in this order) to implement three phases of scientific inquiry: hypothesis formulation, demonstration, and evaluation.

The researcher begins formulating such a *theory-informed guess* or an *explanatory hunch* about what plausible causal condition may explain an unexpected observation that contradicts the prediction of the currently widely acknowledged theory, while considering other equally plausible explanations that may be either mutually-exclusive or concomitant in the situation under analysis. This is the first phase of *hypothesis formulation* using abduction.

The next phase of *hypothesis demonstration* using deduction involves predicting other necessary logical consequences that should be true along with the observed outcome of interest in the case of the hypothesized proposition and unobservable condition for its validity holds.

Finally, *hypothesis evaluation* using induction involves identifying configurations of the hypothesized necessary logical consequences in a collection of instances of the same situation under analysis, gathered in the same empirical setting, which suggests the existence of an operating causal mechanism that generated them all.

In addition to Charles Peirce, Roy Bhaskar (1975) also considered that retroduction is the logic *par excellence* for the social sciences rather than the hypothetical-deductive and deductive-inductive mixed approaches due to the complex, dynamic and contingent nature of social phenomena. While the hypothetico-deductive logic of research is *theory-laden* and the inductive logic of research is *data-laden*, the retroductive logic of making science is *subject-laden* because of the subjective nature of the task of hypothesis selection, which relies on the substantive knowledge of the researcher about the specific empirical setting of the phenomenon under inquiry.

Roy Bhaskar is the main proponent of Critical Realism (CR), a post-positivist philosophy of the social sciences that seeks to uncover the underlying powers and mechanisms that shape the observed reality using a logic of retroduction and acknowledging the subjective nature of knowledge, beliefs and experiences in shaping the scientific analysis and interpretation of social reality.

These influences of Pragmatism on CR, along with other ideas, such as the emphasis on contextual and historical analysis and reliance on a stratified social ontology in three levels of reality (i.e., real, actual, and empirical), contributes to make CR a relevant post-positivist philosophy of the social sciences. However, CR places little focus on formal logics and largely overlooks the phenomenon of language, which still holds an important position in the social sciences. CR also aligns more closely with a correspondence theory of truth rather than a pragmatic theory of the truth, as compared to pragmatist philosophy (Nellhaus, 1998). In this regard, a Pragmatist Critical Realism (Braga, 2023) emphasizing the practical consequences of beliefs and goals, the development of action-oriented theories, and helping in incorporating the other ignored pragmatist insights in relation to language and linguistic analysis can be helpful in guiding the development of a post-positivist methodology for social research.

1.2 The Theoretical Problem and the Practical Goal of this Work

A process tracing technique, based on the retroduction logic (Bennett & Checkel, 2015), allows for the tracking of a chain of event outcomes in a within-case study to uncover the underlying mechanisms that may generated them. Its goal is to reveal the contextual conditions that led to the activation of the causal mechanism, resulting in an unexpected event outcome in accordance with the rules of a hypothesized category of social process. The adjustment of the given process model considers the discrepancy between the predictions of the mainstream theory and the empirical data being analyzed, employing the logic of retroduction. It can even propose the analytical generalization of the contingent pattern of behavior to the theory itself through cross-case comparisons, thereby developing a new middle-range theory within a broader scientific paradigm. In this situation, there exists a process model representing the phenomenon, which is often not a formal model based on postulated social ontology and underlying mathematical foundations. Nonetheless, process tracing can acknowledge the complex, dynamic, and contingent nature of social phenomena, allowing for the adoption of a formal model and a systematic analytical procedure in alignment with this assumption.

Braga (2017a) proposed a retroductive methodological approach for analyzing complex, dynamic, and contingent phenomena as patterns of sequences of decision-making events in specific empirical settings using grammars. The goal is to refine a theory in response to a surprising or anomalous fact that is neither a prediction nor a logical consequence of its statements.

This qualitative research methodology, called Categorical-Generative Analysis (CGA), applies grammars as a tool to translate theories into a set of rules governing the behavior of a relatively stable group of actors within a hypothetical social system. Any process instance is analytically decomposable in terms of actions and interactions among elements of this system, but the emergent form or entity to which it is part of is not analytically decomposable in the same way. The social researcher can only enumerate and describe the processes necessary to generate an emergent form or entity, but no theory or model can make any kind of a definitive description of it.

Generative Grammar Theory, or simply Generativism, is the branch of linguistics concerned with a hypothesized innate grammatical structure, which is a biological capacity for language learning built into the human brain (Everaert et al., 2015). Consequently, the theory suggests there are innate constraints on the grammar that a human language could exhibit, which is the assumption of the existence of a *universal grammar*.

The generative approach to the study of language involves developing a grammar, which is a system of grammatical rules that generate valid sentences in the target language. It extends the paradigm of Linguistic Structuralism by arguing that language is an object of study within a branch of cognitive psychology, considering it as a form of reasoning and problem-solving (Chomsky, 2016).

In formal terms, *grammar* is a system of rules for producing sentences classified in relation to the computational limits imposed by these rules on the possible patterns of sequences of symbols in the alphabet of a language. Any sequential application of these production rules resulting in a sequence of symbols (i.e., a string) that represents a sentence that belongs to a language is a *derivation path*. The derivation procedure is a deductive proof technique to assert the set-membership relation between an elementary sentence and a language set. Consequently, the derivation path is not the recipe to construct a valid sentence, but the proof steps of its membership to the target language.

Chomsky hierarchy (1956) is a typology or containment arrangement of the classes of grammars (and languages) based on the set of sentences that each of them can produce (or comprise). This hierarchy establishes a containment relation ($\subseteq$) between each pair of classes of grammars. The levels of Chomsky's hierarchy divide the set of all languages into distinct classes based on the computational complexity of their system of production rules. Complexity refers to the available resources required to generate their set of grammatically valid sentences (i.e., in essence, recursion and context-sensitivity), but in terms of computation time and memory storage. Consequently, the Chomsky hierarchy asserts that the base computational level of regular languages is a subset of the next level of the so-called context-free languages, but with no pattern based on a recursion. Additionally, context-free languages are a subset of context-sensitive languages.

Most qualitative research methodologies lack a model and systematic analytical procedures that are capable of assessing empirical evidence under the assumptions of a post-positivist epistemology, although there is a trend for rigorous research quality standards, similar to those existing for quantitative methods. CGA relies on formal grammars for modeling and analyzing categories of social processes. Grammars represent the discrete-space, discrete-time version of dynamical systems. Just as the design and qualitative assessment of the behavior of dynamical systems are possible in mathematics, CGA seeks to enable a similar assessment for process-like phenomena.

Despite Formal Language Theory becoming a key branch of computer science by joining both Alan Turing (1936) and Noam Chomsky (1957) mathematically equivalent theories of computation, several linguists still reject Generativism for various reasons that are not relevant in this work. The theoretical problem at hand is that even generative linguists argue that the original Chomsky's formalism has certain inadequacies when it comes to analyzing natural languages (Genkin et al., 2010). One issue is that terminal and nonterminal symbols are structureless objects, meaning that syntactic relations rely only on the grammar rules. Another issue is that given a fixed set of alphabet symbols, language variation results only from grammar variation. Finally, concatenation over the alphabet set is the unique admissible syntactic operation.

In this way, there is a linguistic technique to eliminate these drawbacks called *lexicalization*, which involves creating a controlled vocabulary in a *lexicalized*, or *type logical grammar*. The *lexicon*, or vocabulary of the language, contains lexical items for words, set phrases, and word patterns that represent the units of meaning, or *lexemes*. Each syntactic structure regarded as a *type* or *category* has a lexical item in the alphabet set (Σ). This approach has two advantages: (i) most syntactic relations between words derive from the syntactically typed lexical items assigned to them; and (ii) the remaining syntactic relations derive from *type inference rules*, which are language-independent logical operations on types using deductive syllogism only.

In a lexicon, types (or categories) are syntactic structures, representing sets of strings. In turn, terminal symbols become informative syntactic structures mapping to either other terminals or non-terminals as complex types, waiting for evidence of the pattern of syntactic relations that they predict to occur in valid sentences of the language. Last of all, language variation now results from lexicon variation, which is language specific, while the fixed set of type inference rules, which extends the lexicon by assigning types to strings, becomes a kind of universal grammar since it is common to all languages.

Like Chomsky's formalism, lexicalized grammars are *constituency grammars*, which is a class of grammar formalisms that rely on a subject-predicate term logic. They rely on a binary division of non-terminals that result in a one-to-one-or-more correspondence between nodes in the derivation tree structure, known as *constituency relation*, such that the constituent structures become phrase-structure rules or rewrite rules. The alternative *dependency relation* is a one-to-one correspondence instead: for every word in a sentence, there is the same kind of node in the syntactic structure, which turns out to be a graph rather than a tree. This means that there is a structural difference in the derivation paths using constituency grammars and dependency grammars. However, even the derivation paths using distinct formalisms based upon the constituency relation are also completely different despite both of them presenting a tree-like structure. The selection between these types of formalisms relies only on technical issues such as efficiency of the syntactic parsing algorithm, ability to handle ambiguity in sentences, and the benefit of visualizing the syntactical structure of the sentence (in the particular case of constituency grammars).

There are several applied social research studies using combinatory categorial grammars. McMichael et al. (2004; 2005) developed multiple models of situation analysis using grammar induction methods based on various variations of the CCG formalism. In these studies, sequences of either actions or interactions, which manifest as command and control processes, become plan representations. Geib et al. (2009; 2018) also employed grammar induction methods based on the CCG formalism for plan recognition. However, Categorical-Generative Analysis takes a different approach by starting with a grammar model for the category of the process under analysis and utilizing a retroductive method to modify the grammar based on a surprising or anomalous fact, which becomes a new event outcome in the alphabet set.

In Categorical-Generative Analysis, there is another reason to choose constituency grammars, which relies on the assumption that the grammar serves as a hypothesis for the generative structure of the social processes through which actors interact within a social system. Although there is an ontological assumption of the existence of unobservable generative structures in the real domain that equivalent to linguistic structures and are only partially accessible to social researchers, the grammar itself does not directly correspond to this structure. Instead, it is merely a tentative approximation of reality. The adoption of a formalism that relies on the constituency relation also enables the assumption that the metaphysical properties of path dependence and contingency are equivalent to the computational properties of recursion and context-sensitivity.

In brief, modern computational linguistics techniques have eliminated the reliance on the Chomsky's formalism for grammar specification. For the same reasons, the present paper proposes the possibility and necessity of choosing between alternative formalisms for Categorical-Generative Analysis: substituting Chomsky's formalism with a lexicalized grammar whenever appropriate. In this sense, the goal of this paper is to introduce a new formalism to the CGA methodology and exemplify the research problem to which it is applicable. The following sections introduce the main classes of lexicalized grammars, pregroup grammars and categorial grammars, which are capable of providing the necessary lexicalization of process categories, and they explain why the latter class applies better than the former to the problem of social inquiry.

2 Pregroup Grammars

Pregroup grammar (PG) is a language formalism that is in the class of type logical grammars (Lambek, 1999). Precisely, it consists of a set of words L, a set of *basic* or *atomic* types T, the free pregroup P(B) that is generated by T, and a *dictionary* relation $\vdash : L \rightarrow T$ that relates each word to one type.

A type can be (i) *atomic* or *basic*, such as sentence (s) and noun (n); (ii) *simple*, which are iterated adjoints of basic types (e.g., n^l, n^r, n^{ll}, n^{rr}), and (iii) *composite*, which is a composition of the basic and simple types using the composite operation (.) on the set of simple types. Therefore, word patterns become compositions of grammatical functions, in which left-adjoint simple types represent symbols that must precede the denoted category, right-adjoints must succeed it, and there is always one basic type, which results from the function call.

If P is the set of simple types, then the set of all types T(P) satisfies (i) $P \subset T(P)$; (ii) if $\alpha \in T(P)$ and $\beta \in T(P)$ then $\alpha . \beta^l \in T(P)$; and (iii) $\alpha . \beta^r \in T(P)$. Due to this definition, there is a kind of *type hierarchy*: a type γ is a subtype of φ if and only if γ occurs within φ.

In the English language, the basic types are nouns and sentences, while the composite types are articles, prepositions and verbs. Complex types refer to positions in the structure of the sentence where other complex types and basic types must be located. Consider transitive verbs ($n^r.s.n^l$), which require a noun phrase to the right (n^r) as the subject and other noun phrase to the left (n^l) as the predicate, returning a sentence (s); it differs from intransitive verbs ($n^r.s$) that require no predicate. In its turn, articles ($n.n^l$) require a noun to the left (n^l) to return a noun phrase (n). This way the complex types refer one to each other. For example, in the sentence "cats eat mice", "cats", "mice" $\vdash$ n are the basic types, and "eat" $\vdash n^r.s.n^l$ is the composite type, which derives to "eat mice" $\vdash n^r.s$ (or to "cats eat" $\vdash s.n^l$), and then to "cats eat mice" $\vdash$ s. In the case of the sentence "the cats eat mice", "the" $\vdash n.n^l$ such that the reduction "the cats" $\vdash (n.n^l).(n)$

├ n must take place at the derivation path before the second reduction with “eat mice” ├ n^r.s completes the procedure, resulting in s.

2.1 Foundational Definitions of the Formalism on Group Theory

The mathematical foundation for the Pregroup Grammar formalism is Group Theory, which studies the algebraic structures known as groups. Many mathematical structures (e.g., cryptographic systems, grammars, vector spaces) and physical systems (e.g., molecular symmetry and the standard model of particle physics) are groups equipped with additional operations and axioms. This subsection introduces some necessary definitions to explain the concept of pregroup.

Definition 3.1.1. A *preorder* or *quasiorder* (S, *R*) is a binary relation *R* (eventually →, or ≤) on the set S, which is (i) *reflexive*, that is, it relates every element a in S to itself, such that a *R* a; and (ii) *transitive*, that is, for all elements a, b, c in S, whenever a *R* b and b *R* c hold, a *R* c also holds.

In Category Theory, the symbol → means an arbitrary binary relation, while ≤ use to mean an ordering relation. Precisely, a *partially ordered set* or *poset* (S, *) is a partial order relation ≤ on the set S that denotes the sequential arrangement of the elements of S, in which for some pairs of elements in S, one of the elements precedes (or succeeds) the other. It is reflexive, transitive, and antisymmetric. For example, the power set (S^*) of any given set is a poset.

In its turn, a *totally ordered set* (ℜ, ≤) is still a partial order relation that holds for every pair of elements in S, but the converse is not always true. For example, the relation “less than or equal to” (≤) on the set of real numbers ℜ is a total order relation.

Definition 3.1.2. A *group* is any tuple G = (S, →, •, 1), or simply (S, •), in which S is a set; • is a binary operation on S, that is, • : S × S → S, but satisfying three axioms: (i) *associativity*, that is, for all a, b, c in S, (a • b) • c = a • (b • c) holds; (ii) *identity element*, that is, there exists an element 1 in S such that for every element a in S, a • 1 = 1 • a = a holds; and (iii) every element a in S has an *inverse*.

A *monoid* or *semigroup* is a group M with no inverse. In other words, it is a set equipped with only an associative binary operation and identity element. In Category Theory, monoid is a category with a single object such that all morphisms depart from this single object to itself. For example, a *free monoid* is the monoid (Σ^*, •), in which Σ^* is the set of all finite sequences of symbols from the alphabet set Σ (strings), including the empty string ε; and • is the concatenation operation on S, that is • : $\Sigma^* \times \Sigma^* \to \Sigma^*$. Usually, concatenation may also use the symbols point (.), comma (,), or no symbol between the pair of terms in Σ^* at all.

In its turn, a *preordered monoid* is a preorder relation (S, ≤) along with a monoid (S, ≤, •), such that the binary operation • satisfies the axiom of *monotony* (i.e., given a, b, c, d in S, a ≤ c and b ≤ d imply a • b ≤ c • d), or equivalently the axiom of *substitution* (i.e., b ≤ d imply a • b • c ≤ a • d • c).

Finally, a *pomonoid* is a preordered monoid (S, ≤, •) with a partially ordered set (S, ≤), such that given a, b in S, a ≤ b ≤ a implies a = b, that is, the relation ≤ is *antisymmetric*.

Definition 3.1.3. A *pregroup* is a tuple P = (S, ≤, •, *l*, *r*, 1), in which the relation ≤ satisfies two axioms: (i) if a ≤ b then c • a ≤ c • b and a • c ≤ b • c; and (ii) *l* / *r* are unary operations called left and right *adjoints*.

Notice that (S, ≤) is a *preorder*, (S, ≤, •) is a *monoid*, and (S, ≤, •, 1) is a *partially-ordered monoid*. In addition, any pregroup is a *pomonoid*, such that every object a in S has left and right adjoints, satisfying four axioms known as the *Ajdukiewicz laws*: (i) and (ii) $a^l \bullet a \leq 1 \leq a \bullet a^l$ (i.e., contraction and expansion to the left); and (iii) and (iv) $a \bullet a^r \leq 1 \leq a^r \bullet a$ (i.e., contraction and expansion to the right).

Finally, a ≤ b if and only if $b^l \leq a^l$ if and only if $b^r \leq a^r$. The following equalities also hold in every pregroup: $1^l = 1^r = 1$; $a^{lr} = a^{rl} = a$; $(a \bullet b)^l = b^l \bullet a^l$; $(a \bullet b)^r = b^r \bullet a^r$.

Given a sentence type s in T and the types for a sequence of words t_1, …, t_n in T, the derivation procedure in pregroup grammars demonstrates $t_1, \ldots, t_n \to s$. The *switching lemma* (Lambek, 1999) makes the parsing problem for pregroups decidable: for any pair of types t, t' ∈ T, if t → t' then there is a type t"∈ T such that t → t" without expansions and t" → t' without contractions.

2.2 Limitations of the Formalism in Assigning Semantics

Between the context-free and context-sensitive classes of languages, there is infinite number of computational complexity levels of particular interest: the patterns found in natural languages that are not regular nor context-free result from the so-called *mildly context-sensitive grammars*, which are often efficiently parsable and inferable from positive evidence (Oates et al., 2006).

Definition 3.2.1 Any class of languages is mildly context-sensitive if and only if (Joshi, 1985): (i) it includes the class of all ε-free context-free languages, where ε is the empty string; (ii) all its languages are *semi-linear*, which means that they are of constant growth; (iii) it contains the languages in which there are sentences exhibiting the *multiple agreement* pattern, which is the set $\{a^n b^n c^n : a, b, c \in \Sigma^+ , n \geq 1\}$, the *cross-serial dependency* pattern, which is the set $\{a^m b^n c^m d^n : a, b, c, d \in \Sigma^+ , m, n \geq 1\}$, and the *duplication* pattern, which is the set $\{ww : w \in \Sigma^+\}$; and (iv) the membership problem for any sentence is decidable in polynomial time.

There are many alternative grammar formalisms available to describe languages at each level of mild context-sensitivity. Understanding the constraints on rules that generate these patterns and demonstrating their expressive power are central problems in Formal Language Theory. Two grammars have *weak equivalence* between themselves if and only if they generate the same set of valid sentences, meaning the resulting language sets are the same. Similarly, two grammars have *strong (or structural) equivalence* between themselves if and only if they generate the same set of derivation trees, which are abstract syntactic objects that represent the sequence of syllogistic proof steps required to verify if a given sentence belongs to a specific language.

Pregroup grammars are weakly equivalent to context-free grammars (Buszkowski, 2001), such that they do not have enough expressive power to parse natural languages. Although there are extensions to this class of grammars that are weakly equivalent to some mildly context-sensitive grammar (Genkin et al., 2010; Kobele & Kracht, 2005), they do not have a specific function type, and instead make use of inverse types combined with its monoidal operation. Therefore, pregroup grammars cannot use neither lambda-calculus nor function denotations to assign semantics, which makes this task quite complicated. Providentially, another weakly equivalent class of lexicalized grammar formalisms seems to be more suitable to characterize mild context-sensitivity: Combinatory Categorial Grammar.

3 Categorial Grammars

Categorial Grammar (CG) is a language formalism that belongs to the class of type logical grammars, establishing an interface between surface syntax and underlying semantic representation. Also called AB-grammar (Ajdukiewicz, 1935; Bar-Hillel, 1953; Lambek, 1958), it is a lexicalized grammar with two type inference rules, making it weakly equivalent to the class of Context-Free Grammar. Given a sequence of words with the syntactic types assigned to them, the type returned after the derivation procedure, consisting of a sequence of syllogistic proof steps, is the non-terminal symbol for the valid sentence (S).

In its turn, Combinatory Categorial Grammar (CCG) is an extension to CG (Steedman, 1987, 1996) with a slightly larger set of type inference rules borrowed from combinatory logic, which is weakly equivalent to the mildly context-sensitive class of grammars known as Linear Indexed Grammar (Vijay-Shanker & Weir, 1994).

A type can be either *primitive* or *complex*, which is a combination of the forms $\alpha\backslash\beta$ or α/β, where α is the resulting type and β is the type of the argument taking place either to the left or to the right, respectively. Thus, complex types are *functors* that take a type β as an argument, specified in the right side of the positioning symbol (either "\" or "/"), and return other type α, specified on the left side. Both the categories $\alpha\backslash\beta$ and α/β map β into α, but the former maps β to the left ($\beta^l \rightarrow \alpha$) and the latter maps β to the right ($\beta^r \rightarrow \alpha$).

When used for modeling languages, each type reflects a grammatical function, which results in a type (α) if it has as argument another type located to either the left side ($\alpha\backslash\beta$) or the right side (α/β) of the lexical item in the respective grammatical structure. Finally, there are operations for combining syntactic structures that derive new types, which work as type inference rules.

For the English language, primitive types are sentence (S), noun (N), and noun phrase (NP), whereas complex types represent types of verbs and other grammatical functions. For example, in some valid sentences (S), there is a transitive verb (V), which is a type for the syntactic structure with one noun phrase in its left side (NP^l) as the subject, and other noun phrase in its right side (NP^r) as the predicate – in the Pregroup Grammar notation, it is $v : np^l \,.\, np^r \rightarrow s$, while in CCG notation (i.e., $\alpha\backslash\beta/\beta$), it is $S\backslash NP^l/NP^r$. Consequently, the complex type for transitive verbs is (S\NP)/NP, which means that a verb of this kind forms a valid sentence (S) if and only if a noun phrase precedes it (NP^l), and another noun phrase (NP^r) follows it. In other words, a transitive verb is like a function that takes two instances of the same type as arguments (NP^l and NP^r) and returns the type of sentence S. For example, in "cats eat mice", there are "eat" $\vdash$ (S\NP)/NP and "cats", "mice" $\vdash$ NP, which derive to "eat mice" $\vdash$ (S\NP) and "cats eat mice" $\vdash$ S.

3.1 Combinatory Logic

In logic, a variable, which is a symbol that represents a value, can be classified as bounded or free depending on whether it is bound by a quantifier, such as "for all" (∀) or "there exists" (∃), or is not bound by any quantifier.

Combinatory logic (Curry, 1930; Schonfinkel, 1924) is a formal logic notation to write formulas without bounded variables by means of using such a limited set of primitive functions with no free variables. Known as *combinators* (Curry & Feys, 1958), these primitive functions with no free variables are higher-order functions that use function application and other previously defined combinators to define a result from its arguments. In this sense, combinators replace variable binding term operators and eliminates the need for quantifiers – denoted by the symbols ∀ ("for all") and ∃ ("there is at least one") –, which is an alternative solution to the so-called *problem of substitution*. For example, given a relation $\forall x_1, \ldots, x_n\ R(x_1, \ldots, x_n)$ with no free variable, the substitution of the bounded variables $x_1, \ldots, x_n$ for the terms $t_1, \ldots, t_n$ results in another term $R(t_1, \ldots, t_n)$.

Since Combinatory logic (CL) can specify recursive functions, which are computable functions, it is a model of computation like Lambda Calculus (Church, 1932) and Turing machines (Turing, 1936). In fact, it relies upon a pair of key notions from Lambda Calculus to provide a set of combinators: abstraction and application.

Define *abstraction* to be such a term of the form $\lambda v.E_i$, where v is a variable known as the *formal parameter* of the abstraction, and E_i is the *body* of the abstraction. This term symbolizes a function applied to an argument, binding the formal parameter v to this argument, and returning E_i with every occurrence of v replaced by the argument.

Define now *application* to be other term of the form (E_i E_j), which is the execution of the function E_i with E_j as its argument, where E_i is called *applicand*, and E_j is the argument to replace all occurrences of the formal parameter v in the body of the applicand. The result is a new term that is equivalent to the old one. In other words, given the abstraction $\lambda v.E_i$, the application $((\lambda v.E_i)\ a)$ is the same making v := a in all occurrences of v in E_i. For example, for any application, the identity combinator is (I x) = x.

Since substitution is a critical operation in any formal system that uses bounded variables, such as first-order logic and other high-order logics, CL imitates the λ abstraction although it does not offer a variable binding operator. Accurately, CL is a term rewriting system. A combinatory term is like a lambda term, but primitive functions are combinators, that is, functions with no free variables. Each combinator has a reduction rule like $(P\ _{x1}, \ldots, x_n) = E$.

3.2 Combinators

During the derivation procedure for a specific sentence, complex types result from the application of one *combinator* to an instance of the type provided as the argument. The differences between the classes of categorical grammars are the set of combinators and the computational complexity level of the class of languages that these combinators can generate.

Categorial grammars (Ajdukiewicz, 1935; Bar-Hillel, 1953; Lambek, 1958) make use of a pair of combinators: *forward application*, which is the logical operation (i) $> : \alpha/\beta\ ,\ \beta \Rightarrow \alpha$, that is, $\alpha/\beta : f\ ,\ \beta : x \Rightarrow \alpha : (f\,x)$; and *backward application*, which is (ii) $< : \beta\ ,\ \alpha\backslash\beta \Rightarrow \alpha$, that is, $\beta : x\ ,\ \alpha\backslash\beta : f\ , \Rightarrow \alpha : (f\,x)$. Both work as type inference rules.

Combinatory categorial grammars (Steedman, 1987, 1996) expand the CG's set of pairs of combinators by adding: two *function composition* combinators, (iii) $B> : \alpha/\beta\ ,\ \beta/\gamma \Rightarrow \alpha/\gamma$, that is, $\alpha/\beta : f\ ,\ \alpha/\gamma : g \Rightarrow \alpha/\gamma : \lambda x.f(g\ x)$, and (iv) $<B : \beta\backslash\gamma\ ,\ \alpha\backslash\beta \Rightarrow \alpha\backslash\gamma$, that is, $\alpha\backslash\beta : f\ ,\ \beta\backslash\gamma : g \Rightarrow \alpha/\gamma : \lambda x.g(f\ x)$; and two *type-raising* combinators (v) $T> : \alpha \Rightarrow T/(T\backslash\alpha)$, that is, $\alpha : x \Rightarrow T/(T\backslash\alpha) : \lambda f.(f\,x)$, and (vi) $<T : \alpha \Rightarrow T\backslash(T/\alpha)$, that is, $\alpha : x \Rightarrow T\backslash(T/\alpha) : \lambda f.(f\,x)$. Observe that the type-raising rules turn arguments into functions over functions over these arguments, which let arguments to compose.

Finally, many authors extended the classic version of CCG with new combinators, improving the expressive power of this of class of mildly context-sensitive formalisms while isolating all cross-linguistic variation in the lexicon and managing a universal set of inference rules (Kuhlmann et al., 2015; Wood, 1993). It is the case of an extended CCG for capturing long-range dependencies (Steedman, 2000), which enlarges the CCG's set of combinators with: two *substitution* combinators, (vii) $S> : (\alpha/\beta)/\gamma , \beta/\gamma \Rightarrow \alpha/\gamma$ and (viii) $<S : \beta\backslash\gamma , (\alpha\backslash\beta)\backslash\gamma \Rightarrow \alpha\backslash\gamma$; two *cross-substitution* combinators, (ix) $Sx> : (\alpha/\beta)\backslash\gamma , \beta\backslash\gamma \Rightarrow \alpha\backslash\gamma$ and (x) $<Sx : \beta/\gamma , (\alpha\backslash\beta)/\gamma \Rightarrow \alpha/\gamma$; and one *coordination* combinator, (xi) $<\&> : \alpha , CONJ , \alpha \Rightarrow \alpha$, in which CONJ is a simple type, or even the empty string ε. For the purpose of this work, this is the required set of combinators (Fig. 1).

```
(i)      > : α/β , β ⇒ α
(ii)     < : β , α\β ⇒ α
(iii)    B> : α/β , β/γ ⇒ α/γ
(iv)     <B : β\γ , α\β ⇒ α\γ
(v)      T> : α ⇒ T/(T\α)
(vi)     <T : α ⇒ T\(T/α)
(vii)    S> : (α/β)/γ , β/γ ⇒ α/γ
(viii)   <S : β\γ , (α\β)\γ ⇒ α\γ
(ix)     Sx> : (α/β)\γ , β\γ ⇒ α\γ
(x)      <Sx : β/γ , (α\β)/γ ⇒ α/γ
(xi)     <&> : α , CONJ , α ⇒ α
```

Fig. 1. A set of combinators for combinatory categorial grammars (Steedman, 2000).

4 Example: Analysis of an Economic Coordination Process

Neoclassical economics, which is the mainstream school of economic thought, asserts that the value of an economic good results from the maximization of utility by consumers with income constraints and profits by firms with budget and information constraints. Its neoclassical theory of the firm explains the existence, behavior, and structure of economic organizations based on their ability to make decisions to maximize the difference between revenue and costs. It is an alternative social system to the market-price mechanism whenever is more efficient to produce under a bureaucracy. The firm's behavior becomes manifest through decisions about what to produce and where to allocate capital, which depends on how profits increase thereafter.

Nonetheless, since real firms often do not behave in accordance with neoclassical predictions, economists and other social researchers created alternatives to the theory. Additionally, extensions to these theories offer better explanations to changing economic and market structures as well as specific empirical situations. As a result, there exists concurrent grand theories and middle-range theories to explain and predict the behavior of the firm.

Contemporary theories of the firm consider profit maximization as a short-run goal, while also acknowledging that firms exist to pursue long-run goals, such as growth and sustainability. The first alternative, known as the contract-based view of the firm, builds upon transaction costs, has its roots in the work of Ronald Coase (1937), but it still incorporates many neoclassical assumptions. Afterwards, several theories of the firm emerged, each focusing on specific issues. For example, the managerial view (Baumol, 1959) centers around the principal-agent relationship, the behavioral view (Cyert & March, 1963) challenges the assumptions of profit maximization and perfect information, and the resource-based view (Penrose, 1959) emphasizes idiosyncratic assets such as productive knowledge and inter-organizational relationships.

This paper is part of a research project that aims to develop a meta-theoretical framework for analyzing forms of economic coordination using a computational complexity approach (Braga, 2017b). Precisely, the proposed methodology involves using grammars for analyzing social processes that occur within a group of economic agents that maintain a stable membership configuration over time. The goal is to distinguish between the market and other forms of economic coordination, including the firm, based on the complexity of the causal relations between events outcomes generated by social structures operating in specific empirical settings.

4.1 The Assumption of Equivalence to Linguistic Structures

Consider the structural constraints from the market model of perfect competition: (a) a large number of buyers and sellers; (b) the product is homogeneous; (c) every participant is a price taker; (d) consumers and producers have perfect knowledge, such that they always known the price and the utilities they get from the product; (e) all participants are rational, which means that trades occur to maximize their economic utilities; (f) there are no barriers to entry or exit of market participants; (g) factors of production have perfect mobility; (h) there is no transaction costs. In this canonical model, interactions between buyers and sellers randomly generate a type of market event, which is *trade*, in such a way that each event outcome is independent from the others and the resulting perfect competition process is stochastic in nature.

Consider now that the market model has non-zero transaction costs. In this case, the existence of the firm is possible, but the limits of the firm may change over time in terms of the relationship between administrative costs and transaction costs. The decision-making process about which should be the configuration of internalized and externalized economic activities in a firm is stochastic in nature, such that there is no difference between firms and markets in this sense.

Given the assumption that both firms and markets are social systems to establish causal relations between their decision-making events over time, they typically do so in distinct ways. Markets, operating under their canonical structural constraints, randomly generate events. In contrast, firms and non-market forms of economic coordination do not exhibit the same structural constraints as markets. Instead, they can produce patterns of sequences of types of either individual action or social interaction at a computational complexity level that markets cannot achieve. This is due to the properties of path dependence and contingency inherent in many of their decision-making processes.

Therefore, under the assumption of equivalence between coordination and linguistic structures, patterns of sequences of decision-making event outcomes exhibit the same properties of sentences of a formal language. Consequently, this establishes computational complexity as a criterion for distinguishing between coordination processes that occur in markets and non-market structures. Market structures are weakly equivalent to linguistic structures at the regular level of computational complexity, while firms and all other non-market coordination structures are weakly equivalent to linguistic structures at the mildly context-sensitive level.

There is an ontological reason for opting for the "mildly" class instead of the context-sensitive class. This decision is in line with the pragmatist assumption that causal claims should never depend on emergent forms or processes, but rather admit that every system is a causal consequence of another distinct system only. Embracing the contrary notion would be akin to endorsing *downward causation*, which suggests that higher levels of emergent phenomena may sometimes causally influence their lower levels' developments. This belief reflects a form of radical holism.

The derivation of context-sensitive rules in Penttonen Normal Form, which is A, B → A, D, implies that a process A "causes" the substitution of a process B for D, even though A is not fully realized in terms of terminals for event outcomes yet. This mechanism is analogous to downward causation. The arrow of time always moves towards the future, which means that only unfolded event outcomes can causally influence processes or events that are yet to occur.

The goal is to achieve a holistic understanding of an emergent form or entity by describing each category of its constituent processes in terms of the underlying generative mechanism, rather than the opposite. This approach suggests the potential use of inductive algorithms to construct grammar rules for a chain of symbols that represents a process instance. Nonetheless, it is not surprising that the problem of grammar induction based solely on positive evidence remains unsolved for context-sensitive languages at computational complexity levels higher than that of mildly context-sensitivity.

4.2 The Hypothesis of Systemic Competence Development

Consider now the hypothesis of systemic competence development at the firm, which is the first research work (Braga, 2020) developed using Categorical-Generative Analysis. This multiple case study analyzed two credit unions that contributed to systemic competitiveness in the form of quasi-public economic goods. The paper demonstrated the need for a middle-range theory of the firm embedded in a socioeconomic system acknowledging contextual conditions of an apparently non-economically rational behavior. The difference between that paper and the current one is just the substitution of the Chomsky's formalism with a combinatory categorial grammar. This new initiative essentially involves the same theoretical model, the same body of empirical evidence derived from two pairs of cases, and the same analytical procedure using CGA, leading to identical conclusions. The difference still lies in the analytical procedure employed: the former adopts a top-down, iterative approach for parsing symbol sequences, while the latter employs a bottom-up, declarative approach.

The original CGA for the social process of competence development at the firm begins with a grammar (Fig. 2) in the Chomsky Normal Form for this category of process (S). It consists of the process of *relationships with partners* (RR) followed by *generation of capabilities and economic goods* (GG). The first social process involves a sequence of outcomes of either *combination of idiosyncratic resources* ({c}) or *information and knowledge exchange* ({i}), while the resulting event outcome is a single *economic good generation* ({g}) instance.

The hypothesis of the possibility of systemic competence development (H1) means mapping each type X into X[..], where the square brackets contain a configuration of contextual conditions allowing the surprising fact: the *systemic good generation* ({g'}) to satisfy systemic needs rather than market needs. First, this fact becomes a new event outcome for the type *generation of economic good* (G). Next, the retroductive procedure consists of hypothesizing knowledge exchange relationships that are not available to the rivals of the firm ({i'}) as a condition for the creation of a systemic economic good ({g'}), which in turn enables the creation of one or more economic goods ({g}) to the market in a future system state. In addition, the hypothesized exogenous contextual conditions are (i) the high impact of the systemic problem in the firm's performance and (ii) the appropriation by the firm of a part of the economic value created to the socioeconomic system in the form of a new source of rent.

```
S  → GG, CD
CD → GG, CD
   | GG
GG → RR, G
RR → R, RR
   | R
R  → {c} | {i}
G  → {g}
```

Fig. 2. A constituency grammar for the competence development process.

The goal is not predicting when (or if) a surprising fact is going to take place but tracing the decision-making process to identify the patterns of sequences of event outcomes and contextual conditions that enable the occurrence of the surprising fact, which then becomes a new event outcome. In the same social system, the hypothesized causal mechanism occurs repeatedly under the same contextual conditions (i.e., initial exogenous conditions and ordered configurations of unexpected past event outcomes), which characterizes an extension of the category of the social process under inquiry. An ordered configuration refers to a specific arrangement or sequence of event outcomes in a particular order. In ordered configurations, the relative positions or arrangement of the mutually exclusive alternative outcomes for a type of event matter and contribute to the overall structure or meaning.

The research seeks to identify contextual conditions using the Quine-McCluskey algorithm (McCluskey, 1959; Quine, 1952) implemented in a Configurational Analysis tool known as QCA (Ragin, 1987). The so-called *context* is any collection of ordered configurations of past event outcomes that represents a set-theoretic relation with one of the existing alternative rules generating the next event outcome in a system state. It involves the substitution of the alternative rules (e.g., producing g and g') for strictly context-sensitive rules (i.e., which are non-context-free). This task eliminates the ambiguity introduced by acknowledging the surprising or anomalous fact as a new event outcome, but it is important to note that every non-ambiguous, non-deterministic alternative rules may also turn out to be strictly context-sensitive.

After identifying the contextual conditions related to the instances of the surprising fact in within-case studies, the CGA procedure involves mapping the initial context-free grammar (Fig. 2) into a new mildly context-sensitive grammar using the Indexed Grammar formalism (Aho, 1968). The challenge with using this formalism is that each expected configuration of past event outcomes in a social process must be either copied or translated into another subsequent configuration. Specifically, $X[^\wedge\varphi..] \rightarrow Y_{(L)}[^\wedge\varphi..], Y_{(R)}[^\wedge\varphi..]$ or $X[^\wedge\varphi..] \rightarrow Y_{(L)}[^\wedge\lambda..], Y_{(R)}[^\wedge\lambda..]$, respectively, for each of its sub-processes until all the expected event outcomes occur. This requires changing both the rule set (i.e., the graph of the category of process) and the alphabet set (i.e., the set of all event outcomes). However, when replacing the Indexed Grammar formalism for the Combinatory Categorial Grammar formalism, there is a need to change the lexicon only. The lexicon becomes such an alphabet set in which each possible event outcome is equipped with additional structure.

Using the CCG formalism, the lexicon has event types for *economic good generation* ({g} ├ S\RR), *knowledge exchange relationship* ({i} ├ RR), and *combination of idiosyncratic resources* ({c} ├ RR\RR). The economic good (g) returns a valid sentence (S) if and only if preceded by an instance of the process of relationships with partners (RR), which is a sequence of zero or more event outcomes for combination of resources (RR\RR), preceded by one or more event outcomes for the knowledge exchange relationship (RR). These relations between event types follow the assumption A1 that instances of the type of event for economic good generation (g) are independent of each other. For example, the derivation path for the instance "i, c, g, i, i, g" is:

$$i , c , g \vdash RR , RR\backslash RR , S\backslash RR \Rightarrow_{(<)} RR , S\backslash RR \Rightarrow_{(<)} S$$

$$i, i, g \vdash RR, RR, S\backslash RR \Rightarrow_{(<\&>)} RR, S\backslash RR \Rightarrow_{(<)} S$$

$$i, c, g, i, i, g \vdash S, S \Rightarrow_{(<\&>)} S$$

Consider now the assumption A2 that the instances of the event type for economic good generation (g) are part of an evolutionary path with other instances of the same event type, such that there is interdependence between them (i.e., there is g_i that extends g_{i-1}). Using same example, $\{g\} \vdash S\backslash RR$ becomes $\{g\} \vdash (S\backslash S)\backslash RR$, such that the new derivation path is:

$$i, c, g_{i-1}, i, i, g_i \vdash S_{i-1}\backslash S_{i-2}, S_i\backslash S_{i-1} \Rightarrow_{(<B)} S_i\backslash S_{i-2}$$

The type “S\S” is absent in the lexicon, such that there is the need for the application of another combinator, which will apply to the event type for the initial exogenous contextual conditions (S). Consequently, the list of combinators in this derivation procedure – i.e., (<), (<), (<&>), (<B), and finally (<) for S, S\S – are the proof steps for the statement that this is an instance of the category of competence development (S).

Consider now the hypothesis H1 that implies the new event outcome, a systemic economic good (g’), which comes to solve a systemic problem hindering the generation of another economic good (g) in the future, but under the assumption A1. In the same example, the substitution of $\{g\} \vdash S\backslash RR$ for both $\{g'\} \vdash (S/S')\backslash RR'$ and $\{\bar{g}'\} \vdash S'\backslash RR$ still implies that economic goods are either systemic (g’), or non-systemic ($\bar{g}'$), such that the derivation path for the instance “i, c, g’, i, i, $\bar{g}'$” is now:

$$i', c, g', i, i, \bar{g}' \vdash S/S', S' \Rightarrow_{(>)} S$$

Finally, the same hypothesis H1, but under the assumption A2. In the example, the substitution of $\{g\} \vdash (S\backslash S)\backslash RR$ for $\{g'\} \vdash ((S/S')\backslash S)\backslash RR'$ and $\{\bar{g}'\} \vdash (S'\backslash S)\backslash RR$ is such that:

$$i', c, g', i, i, \bar{g}' \vdash (S/S')\backslash S, S'\backslash S \Rightarrow_{(Sx>)} S\backslash S$$

Or in the case that there are a previous economic good to be extended ($g_{i-1} \vdash S_{i-1}\backslash S_{i-2}$):

$$i', c, g_i', i, i, \bar{g}_i' \vdash (S_i/S'_i)\backslash S_{i-1}, S_i'\backslash S_{i-1} \Rightarrow_{(Sx>)} S_i\backslash S_{i-1}$$

The examples above represent a single instance of the social process of competence development. However, in real-life situations, there may be multiple concurrent instances of the same category of process or even distinct categories of processes occurring within a firm through the actions of a community of economic agents. As a result, the procedural memory of a process instance retains information about its execution because the community of actors remains relatively stable over time, and minor changes in its community do not influence the process execution.

In this situation of multiple concurrent instances, unlike the strictly sequential derivation procedure of categorial grammars, the left and right types of events may not be immediately adjacent, but rather causally antecedent or consequent types of events. The numbers in the types (e.g., S_i) are important for making causal links between pairs of instances, even though some instances may not have an index number, and thus may take part in more than one instance (e.g., the S for the initial exogenous contextual conditions).

5 Conclusions

This paper constitutes another chapter of the Categorical-Generative research project, which puts forth an analytic framework for formulating middle-range theories based on the ontological and epistemological assumptions of Pragmatist Critical Realism. The analytical procedure commences with the apprehension of a surprising or anomalous fact and elaboration of an informed guess or explanatory hunch. Utilizing the grammar model, this hunch is then deliberately and recursively taken backward in the given sequence of event outcomes, allowing for analysis and adjustment of the grammar, resulting in a hypothesis worthy of empirical testing. The proposed methodology has the capacity to address any concrete category of complex, dynamic, and contingent social process as a pattern of decision-making events of either individual action or interaction under the influence of the hypothesized configuration of past event outcomes. The result is a plausible theoretical explanation for a historical phenomenon that has taken place in a specific empirical setting.

The assumption that social systems are computational devices means that their constituent social processes are equivalent to Turing machines or partial recursive functions. Social systems emerge from concurrent complex, dynamic, and contingent social processes that occur in specific empirical settings. However, modeling social phenomena as an emergent result from the collective behavior of agents still entails interpreting how the properties of this mathematical model relate to the assumptions of the ontology in use. The acknowledgement of contradictions with the mainstream theory in the form of contingent developments also explains the heterogeneity of social phenomena observed by coexisting middle-range theories.

The first challenge of this methodology was the proposition of a meta-theoretical framework to explain the structures of economic coordination between markets and hierarchies from a computational complexity point of view. The ontological assumption of the equivalence of the structures of economic coordination with linguistic structures enables the analysis of sequences of decision-making event outcomes as chains of symbols belonging to a formal language. There is empirical evidence supporting the hypothesis of systemic competence development, which is a pattern of socioeconomic behavior that cannot occur in the market structure because of the constraint on the computational complexity level of its decision rules.

The main result is the proposition of a theorem: the firm and all non-market forms of economic coordination exist to generate categories of social processes exhibiting a pattern of causal relations between their event outcomes that the market can never support due to the structural constraints from the assumption of perfect competition. The systemic competence development process is an example of this kind of social phenomenon that cannot occur in market structures.

CGA provides a way to discover categories of social processes within real social systems according to the assumptions of Pragmatist Critical Realism, which allows scientific inferences that are not in the scope of the social positivist, normal science. The present work improves the CGA methodology with a kind of algebra based on an alternative grammar formalism, which achieves the same conclusions as the first empirical research but in a more precise and intelligible way.

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